A Distribution Fitting Approach for Localization of Multiple Scattered Sources with Very Large Arrays

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***Abstract*—In this paper, a distribution fitting approach is proposed to estimate the angular parameters of multiple incoherently distributed sources with large antenna arrays. The estimator utilizes the approximate orthogonality of the array response vectors of large arrays to obtain a vector which is the superposition of the probability distributions of the direction-of-arrivals (DOAs) of the sources. Then, the angular parameters can be estimated by fitting distribution functions to this vector successively. Hence, the complexity of the proposed approach is much lower than that of other well-known estimators which require the fitting of a matrix. Analysis and numerical results show that the proposed estimator can achieve better performance than other estimators.**

I. INTRODUCTION

Multiuser multiple-input multiple-output (MU-MIMO) systems have been shown to provide multiplexing gain and diversity gain [1]. In large-scale MIMO (LS-MIMO) systems, very large antenna arrays are equipped at the base station (BS), and the spatial gains can be significantly improved [2], [3]. However, the antenna spacing is restricted in LS-MIMO systems, thus the performance relies on large angular spreads, which is not realistic in rural or suburban environments with high BSs [4]. As a result, beamforming with large antenna arrays has been proposed to achieve directional antenna gain rather than diversity gain in [4].

Because the estimation accuracy of the nominal direction-of-arrival (DOA) and the angular spread is crucial to the performance of beamforming systems, the estimation of DOA has been extensively investigated [5]-[13]. The maximum likelihood (ML) estimator [5] and the approximate ML estimator [6] are optimal, but are too complicated because the search dimension is very high. In contrast to the ML approaches, the covariance fitting approaches [7]-[9] are suboptimal but of lower complexity. However, these estimators are either limited to the single source assumption or complicated for multiple sources. The simplest estimation approaches are the subspace fitting approaches [10]-[12] and the beamforming approach [13], which only require a two-dimensional search. The subspace fitting approaches exploit the approximate orthogonality of the pseudonoise subspace and the noise-free covariance matrix of the received signals, and is similar to the beamforming approach. However, these estimators need matrix multiplication operation, the complexity of which is proportional to the square of the number of receiving antennas. Moreover, all these estimators are derived from the approximation of the covariance matrix of the received signals for small angular spreads, the performance of which deteriorates as the angular spreads increase.

In this paper, a distribution fitting approach is proposed to reduce the computational load and improve performance. The proposed approach approximates the integration with the summation in estimating the covariance matrix of the received signals by dividing the integral interval into smaller intervals. Then the approximate orthogonality of the array response vectors of large antenna arrays is utilized to obtain an observation vector from the covariance matrix, which is the superposition of the probability distributions of the DOAs. By fitting the observation vector to the distributions, the nominal DOA and the angular spread of one user terminal (UT) can be estimated by a two-dimensional search. After that, the impact of the estimated angular parameters on the observation vector is eliminated. This process can be executed successively until the angular parameters of all the UTs are estimated. To be specific, the main contributions of this paper are summarized as follows. First, an estimator of angular parameters is proposed for large antenna array systems. The estimator is derived from the fitting of a vector rather than that of a matrix, and only requires a two-dimensional search for estimating the angular parameters of each UT. Hence, the computational complexity of the proposed estimator is much lower than that of other well-known estimators. Second, the proposed estimator can achieve good performance in ideal scenarios. It is proved that the estimates converge to the true values for large antenna array when the DOA separations are large. In addition, the distribution fitting in the proposed estimator is derived from the approximation of the covariance matrix for large arrays rather than for small angular spreads, thus the performance of the proposed estimator is better than that of other estimators when the angular spreads are large.

Notations: Lower-case (upper-case) boldface symbols denote vectors (matrices); $\mathbf{I}_K$ is the dimensional-K identity matrix; $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and $\mathbb{E}\{\cdot\}$ denote the conjugate, the transpose, the conjugate transpose, and the expectation, respec-

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tively; $\det(\cdot)$, $\mathrm{tr}\{\cdot\}$, and $||\cdot||_{\mathrm{F}}$ are the determinant, the trace, and the Frobenius norm of a matrix; $\delta(\cdot)$ is the Kronecker delta function.

II. Signal Model

Let us consider K UTs transmitting narrowband signals to a BS on the same plane. These signals reflect in the vicinity of the UTs, and impinge on the uniform linear array (ULA) of M antennas at the BS, where $M \gg K$, as shown in [14]. With phase reference at the first element of the array, the array response vector $\mathbf{a}(\theta) \in \mathbb{C}^{M\times 1}$ for a point source at direction θ is denoted as $\mathbf{a}(\theta) = [1, \exp(i\Delta\sin(\theta)), \exp(i2\Delta\sin(\theta)), \cdots, \exp(i(M-1)\Delta\sin(\theta))]^T$, where M is the number of antenna sensors of the ULA, and θ satisfies $-\pi/2 < \theta \leq \pi/2$; $\Delta = 2\pi d/\lambda$, d is the distance between adjacent antennas, and λ is the wavelength. Then, the baseband signals received at the antenna array at time instant t can be represented as

$$\mathbf{y}(t) = \sum_{k=1}^{K} s_k(t) \sum_{j=1}^{N_k} \alpha_{k,j}(t)\mathbf{a}(\phi_{k,j}(t)) + \mathbf{n}(t) \in \mathbb{C}^{M\times 1}, \quad (1)$$

where $s_k(t)$ is the signal transmitted from the kth UT, N_k is the number of paths, $\alpha_{k,j}(t)$ and $\phi_{k,j}(t)$ are the path gain and the DOA of the jth path of the kth UT, and $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$ is the received noise.

Then, the following assumptions are made on the signal model. 1) The signals are incoherently distributed as in [10], and the path gains for different UTs and time instants are assumed to be independent of each other. Hence, the covariance of the path gain $\alpha_{k,j}(t)$ satisfies $\mathbb{E}\{\alpha_{k,j}(t)\alpha^*_{\tilde{k},\tilde{j}}(\tilde{t})\} = \sigma^2_{\alpha_k}\delta(k-\tilde{k})\delta(j-\tilde{j})\delta(t-\tilde{t})$, where $\sigma^2_{\alpha_k}$ is the path gain variance for any path from the kth UT. 2) The DOA $\phi_{k,j}(t)$ satisfies the Gaussian angular distribution with the angular power density $\rho(\theta, \boldsymbol{\psi}_k)$, where $\boldsymbol{\psi}_k = [\phi_k, \sigma_{\phi_k}]^T \in \mathbb{R}^{2\times 1}$ is composed of the nominal DOA ϕ_k and the angular spread σ_{ϕ_k}. 3) The scattering parameters $\alpha_{k,j}(t)$ and $\phi_{k,j}(t)$ vary more rapidly than $\boldsymbol{\psi}_k$. 4) The transmitted signal powers $|s_k(t)|^2$ are constant for $t = 1, 2, \cdots, T$, and are approximately equal for the UTs. 5) The received noise $\mathbf{n}(t)$ is composed of complex Gaussian random variables with zero mean and covariance matrix $\mathbb{E}\{\mathbf{n}(t)\mathbf{n}^H(\tilde{t})\} = \sigma^2\delta(t-\tilde{t})\mathbf{I}_M$.

It should be noticed that the localization problem considered in this paper is to estimate $\boldsymbol{\psi}_k, k = 1, 2, \cdots, K$, with the received signals $\mathbf{y}(t), t = 1, 2, \cdots, T$, and the known angular power density $\rho(\theta, \boldsymbol{\psi})$, where $\boldsymbol{\psi} \in \mathbb{R}^{2\times 1}$ is composed of a nominal DOA and an angular spread as $\boldsymbol{\psi}_k$. Like other estimators, the covariance matrix of the received signals is used to estimate these parameters. Based on the assumptions, the covariance matrix $\mathbf{R}_y = \mathbb{E}\{\mathbf{y}(t)\mathbf{y}^H(t)\} \in \mathbb{C}^{M\times M}$ is given by

$$\mathbf{R}_y = \sum_{k=1}^{K} P_k \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{a}(\theta)\mathbf{a}^H(\theta)\rho(\theta, \boldsymbol{\psi}_k)\,\mathrm{d}\theta + \sigma^2\mathbf{I}_M, \quad (2)$$

where $P_k = \sigma^2_{\alpha_k}|s_k(t)|^2 N_k$ is the received signal power at each antenna from the kth UT, and t is an arbitrary number in the set $\{1, 2, \cdots, T\}$. In addition, the covariance matrix can only be estimated with the average matrix

$$\hat{\mathbf{R}}_y = \frac{1}{T}\sum_{t=1}^{T} \mathbf{y}(t)\mathbf{y}^H(t). \quad (3)$$

In the following analysis, it is assumed that T is large enough, which is realistic, so that $\hat{\mathbf{R}}_y$ approximates $\mathbf{R}_y$.

III. Distribution Fitting Estimation

The known estimators are complicated with large antenna arrays. Hence, in this section, the properties of large antenna arrays are utilized to derive an estimator of angular parameters, which is of good performance and low complexity. Then the corresponding estimation algorithm is presented, and the time complexity of the proposed estimator is compared with that of other well-known estimators.

A. Theoretical Derivation

It is known that the integration can be approximated by the summation when the integral interval is evenly divided into intervals that are small enough. With large antenna arrays, i.e., the number of BS antennas $M \gg K$, the covariance matrix in (2) can be approximated as

$$\mathbf{R}_y \approx \sum_{k=1}^{K} P_k \frac{\pi}{M}\sum_{j=1}^{M} \mathbf{a}(\theta_j)\mathbf{a}^H(\theta_j)\rho(\theta_j, \boldsymbol{\psi}_k) + \sigma^2\mathbf{I}_M, \quad (4)$$

where $\theta_j = -\pi/2 + j\pi/M, j = 1, 2, \cdots, M$. For large ULAs, the array response vector has the property [14]

$$\begin{aligned} \frac{1}{M}\mathbf{a}^H(\theta_j)\mathbf{a}(\theta_k) &= \frac{1}{M}\frac{1-\exp(i\Delta\sin(\theta_j-\theta_k)M)}{1-\exp(i\Delta\sin(\theta_j-\theta_k))} \\ &\to \delta(j-k), \text{ as } M \to \infty. \end{aligned} \quad (5)$$

Hence, the covariance matrix $\mathbf{R}_y$ can be transformed into the vector $\mathbf{z} = [z_1, z_2, \cdots, z_M]^T \in \mathbb{R}^{M\times 1}$, where

$$\begin{aligned} z_j &= \frac{1}{M}\mathbf{a}^H(\theta_j)\mathbf{R}_y\mathbf{a}(\theta_j) \\ &\approx \sum_{k=1}^{K} P_k\pi\rho(\theta_j, \boldsymbol{\psi}_k) + \sigma^2, \end{aligned} \quad (6)$$

and θ_j is defined below (4). From (5), it can be seen that the vectors $(1/\sqrt{M})\mathbf{a}(\theta_j), j = 1, 2, \cdots, M$, approximately constitute a basis of the M-dimensional complex space. Hence, the elements of $\mathbf{z}$ can be regarded as the eigenvalues of $\mathbf{R}_y$, and the vectors $(1/\sqrt{M})\mathbf{a}(\theta_j), j = 1, 2, \cdots, M$, are the corresponding eigenvectors. As a result, when $z_j, j = 1, 2, \cdots, M$, are different, there is approximately a one-to-one correspondence between the matrix $\mathbf{R}_y$ and the vector $\mathbf{z}$ with M large enough.

From the above analysis, it is known that the parameter vectors $\boldsymbol{\psi}_k, k = 1, 2, \cdots, K$, can be estimated from $\mathbf{z}$ with little loss of precision when M is large enough. There are $3K + 1$ unknown parameters in $\mathbf{z}$, including the received signal powers $P_k, k = 1, 2, \cdots, K$, the noise power σ^2, and the nominal DOAs and the angular spreads in $\boldsymbol{\psi}_k, k =$

$1, 2, \cdots, K$. With far DOA separations, it can be seen that $\rho(\theta, \boldsymbol{\psi}_k)\rho(\theta, \boldsymbol{\psi}_m) \approx 0, \forall k \neq m$ holds, hence the minimum element of $\mathbf{z}$ approximately equals σ^2, and is denoted as $\hat{\sigma}^2_{\min}$ here. Then, z_j in (6) can be further processed into

$$\begin{aligned} z_j^{(1)} &= z_j - \hat{\sigma}^2 \\ &\approx \sum_{k=1}^{K} P_k \pi \rho(\theta_j, \boldsymbol{\psi}_k), \end{aligned} \tag{7}$$

where $z_j^{(k)}, k = 1, 2, \cdots, K$, is the processed data in the kth iteration of the estimation process. With far DOA separations, it can be seen that the received signal power of one UT can be easily eliminated by normalization, thus $z_j^{(1)}$ can be processed into

$$\begin{aligned} \tilde{z}_j^{(1)} &= \frac{z_j^{(1)}}{z_{m_1}} \\ &\approx \tilde{\rho}(\theta_j, \boldsymbol{\psi}_{n_1}) + \frac{1}{z_{m_1}} \sum_{k \neq n_1} P_k \pi \rho(\theta_j, \boldsymbol{\psi}_k), \end{aligned} \tag{8}$$

where $z_{m_1} \approx P_{n_1} \pi \rho(\theta_{m_1}, \boldsymbol{\psi}_{n_1}), m_k \in \{1, 2, \cdots, M\}, n_k \in \{1, 2, \cdots, K\}, k \in \{1, 2, \cdots, K\}$, is the maximum element of $z_j^{(1)}, j = 1, 2, \cdots, M$, and $\tilde{\rho}(\theta, \boldsymbol{\psi})$ is the normalized angular power density with the maximum value being one. It should be noticed that the values of m_1 and n_1 are set according to the system parameters so that z_{m_1} is the maximum element of $z_j^{(1)}, j = 1, 2, \cdots, M$, and corresponds to the angular power density of the n_1th UT. When the DOA separations are far, the nominal DOA and the angular spread in $\boldsymbol{\psi}_{n_1}$ can be estimated with the least squares criterion, and the proposed estimator is given by

$$\hat{\boldsymbol{\psi}}_{n_1} = \arg\min_{\boldsymbol{\psi}} \sum_{j=1}^{M} |\tilde{z}_j^{(1)} - \tilde{\rho}(\theta_j, \boldsymbol{\psi})|^2. \tag{9}$$

It can be seen that this estimator derives from the fitting of the angular power density, which depends on the angular distribution of the sources, thus is termed as the distribution fitting estimator in this paper. In contrast to other two-dimensional estimators [10]-[13], this estimator avoids the matrix multiplication in each search, thus can significantly reduce the computational complexity. Moreover, the estimator does not need the approximation of the covariance matrix $\mathbf{R}_y$ for small angular spreads in the existing estimators. Hence, the proposed estimator is more accurate for sources with large angular spreads.

In order to show that the estimate $\hat{\boldsymbol{\psi}}_{n_1}$ converges to the true vector $\boldsymbol{\psi}_{n_1}$, a proposition is presented below.

Proposition 1: When the number of BS antennas $M \to \infty$, and the correlation of the angular power density $\rho(\theta, \boldsymbol{\psi}_k)\rho(\theta, \boldsymbol{\psi}_m) \to 0, \forall k \neq m$, then we have $\hat{\boldsymbol{\psi}}_{n_1} \to \boldsymbol{\psi}_{n_1}$.

Proof: From the previous analysis, it is known that $\tilde{z}_j^{(1)} \to \tilde{\rho}(\theta_j, \boldsymbol{\psi}_{n_1}) + (1/z_{m_1}) \sum_{k \neq n_1} P_k \pi \rho(\theta_j, \boldsymbol{\psi}_k)$ as $M \to \infty$ and $\rho(\theta, \boldsymbol{\psi}_k)\rho(\theta, \boldsymbol{\psi}_m) \to 0, \forall k \neq m$. Then, the function

$$f(\boldsymbol{\psi}) = \sum_{j=1}^{M} |\tilde{z}_j^{(1)} - \tilde{\rho}(\theta_j, \boldsymbol{\psi})|^2 \tag{10}$$

is defined, which is part of the proposed estimator in (9). When $\rho(\theta, \boldsymbol{\psi}_k)\rho(\theta, \boldsymbol{\psi}_m) \to 0, \forall k \neq m$, it can be found that

$$\begin{aligned} f(\boldsymbol{\psi}_{n_1}) &\to \sum_{j=1}^{M} \sum_{k \neq n_1} \Big| \frac{1}{z_{m_1}} P_k \pi \rho(\theta_j, \boldsymbol{\psi}_k) \Big|^2, \\ f(\boldsymbol{\psi}_{n_2}) &\to \sum_{j=1}^{M} \Big| \tilde{\rho}(\theta_j, \boldsymbol{\psi}_{n_1}) + \sum_{k \neq n_1} \frac{P_k \pi}{z_{m_1}} \rho(\theta_j, \boldsymbol{\psi}_k) \\ &\qquad - \tilde{\rho}(\theta_j, \boldsymbol{\psi}_{n_2}) \Big|^2. \end{aligned}$$

Then, we have

$$\begin{aligned} f(\boldsymbol{\psi}_{n_2}) - f(\boldsymbol{\psi}_{n_1}) \to \sum_{j=1}^{M} \Bigg(& \Big| \frac{P_{n_2} \pi}{z_{m_1}} \rho(\theta_j, \boldsymbol{\psi}_{n_2}) + \tilde{\rho}(\theta_j, \boldsymbol{\psi}_{n_1}) \\ & - \tilde{\rho}(\theta_j, \boldsymbol{\psi}_{n_2}) \Big|^2 - \Big| \frac{P_{n_2} \pi}{z_{m_1}} \rho(\theta_j, \boldsymbol{\psi}_{n_2}) \Big|^2 \Bigg). \end{aligned}$$

When $n_1 \neq n_2$, it can be seen that

$$\begin{aligned} f(\boldsymbol{\psi}_{n_2}) - f(\boldsymbol{\psi}_{n_1}) &> \frac{\pi^2}{z_{m_1}^2} \sum_{j=1}^{M} \Big(P_{n_1}^2 |\rho(\theta_j, \boldsymbol{\psi}_{n_1})|^2 \\ &\qquad - P_{n_2}^2 |\rho(\theta_j, \boldsymbol{\psi}_{n_2})|^2 \Big). \end{aligned}$$

Because the received signal powers $P_k, k = 1, 2, \cdots, K$, are approximately equal and $z_{m_1} \approx P_k \pi \rho(\theta_{m_1}, \boldsymbol{\psi}_{n_1})$ is the maximum element of $\mathbf{z}$, the angular spread of the n_1th UT is smaller than that of the n_2th UT. Because $\rho(\theta, \boldsymbol{\psi})$ is the angular power density for the Gaussian angular distribution, $\sum_{j=1}^{M} |\rho(\theta_j, \boldsymbol{\psi})|^2$ is inversely proportional to the angular spread with large M. Hence, $f(\boldsymbol{\psi}_{n_2}) > f(\boldsymbol{\psi}_{n_1})$ is true. For $\boldsymbol{\psi} \notin \{\boldsymbol{\psi}_1, \boldsymbol{\psi}_2 \cdots, \boldsymbol{\psi}_K\}$, $f(\boldsymbol{\psi}) > f(\boldsymbol{\psi}_{n_1})$ can be easily verified because the DOA separations of the sources are far. Therefore, the estimate $\hat{\boldsymbol{\psi}}_{n_1}$ in (9) converges to $\boldsymbol{\psi}_{n_1}$ as $M \to \infty$, $\rho(\theta, \boldsymbol{\psi}_k)\rho(\theta, \boldsymbol{\psi}_m) \to 0, \forall k \neq m$. ■

Remark 1: The proposition shows the requirements for the convergence of the estimate $\hat{\boldsymbol{\psi}}_{n_1}$ to the true vector $\boldsymbol{\psi}_{n_1}$. With a large antenna array and far DOA separations, these requirements can be approximately satisfied. Hence, the proposed estimator can achieve good performance in such scenarios.

B. Implementation of the Algorithm

Although the proposed estimator only considers the estimation of the nominal DOA and the angular spread of the n_1th UT, it can be easily extended to the estimation of the angular parameters of other UTs. The corresponding processes are presented as follows.

With the estimate $\hat{\boldsymbol{\psi}}_{n_1}$, the observation $\tilde{z}_j^{(1)}$ in (8) can be processed into $z_j^{(2)} = \tilde{z}_j^{(1)} - \tilde{\rho}(\theta_j, \hat{\boldsymbol{\psi}}_{n_1})$. Then $z_j^{(2)}, j = 1, 2, \cdots, M$, are further normalized by the maximum value, resulting into $\tilde{z}_j^{(2)}, j = 1, 2, \cdots, M$. After that, the proposed estimator can be used to estimate the parameters of another UT by replacing $\tilde{z}_j^{(1)}$ in (9) with $\tilde{z}_j^{(2)}$. This process can be successively implemented until the nominal DOAs and the angular spreads of all the K UTs are estimated. For example, in the kth iteration, where $k = 1, 2, \cdots, K$, the normalized value of $z_j^{(k)}$ is given by

$$\tilde{z}_j^{(k)} = \frac{z_j^{(k)}}{z_{m_k}}, \tag{11}$$

where z_{m_k} is the maximum value of $z_j^{(k)}, j = 1, 2, \cdots, M$. Then, the proposed estimator in (9) can be generalized into

$$\hat{\boldsymbol{\psi}}_{n_k} = \arg\min_{\boldsymbol{\psi}} \sum_{j=1}^{M} |\tilde{z}_j^{(k)} - \tilde{\rho}(\theta_j, \boldsymbol{\psi})|^2. \tag{12}$$

After that, the observation $\tilde{z}_j^{(k)}$ is transformed into

$$z_j^{(k+1)} = \tilde{z}_j^{(k)} - \tilde{\rho}(\theta_j, \hat{\boldsymbol{\psi}}_{n_k}). \tag{13}$$

Now, we summarize our algorithm for clarity.

Algorithm 1: Distribution Fitting Estimation of the Nominal DOAs and the Angular Spreads of Multiple Sources

Step 1) Compute the average matrix $\hat{\mathbf{R}}_y$ using (3).

Step 2) Transform $\hat{\mathbf{R}}_y$ into $z_j^{(1)}, j = 1, 2, \cdots, M$, using (6) and (7). Set $k = 0$.

Step 3) $k = k + 1$.

- *Normalize $z_j^{(k)}, j = 1, 2, \cdots, M$, using (11).*
- *Search the nominal DOA and the angular spread of the n_kth UT using (12).*
- *Calculate $z_j^{(k+1)}, j = 1, 2, \cdots, M$, using (13).*

Step 4) Repeat step 3 until $k = K$.

Remark 2: The proposed estimator can also be implemented in a simpler way without successive estimation, as the subspace fitting estimators of [11], [12] and the beamforming estimator of [13]. However, the algorithm will need to pick K local minimum values rather than K minimum values with the criterion in (9), and the performance relies on the accuracy of the known angular parameters. Therefore, the successive estimation is proposed for better performance.

C. Complexity Analysis

In this subsection, the time complexity of the proposed estimator is compared with that of other benchmarking estimators, and the analysis is derived from the assumption that the individual complex multiplication has complexity $O(1)$.

The ML estimator of [5] is optimal in performance, the criterion of which is $\arg\min_{\boldsymbol{\eta}} \ln\det(\mathbf{R}_y(\boldsymbol{\eta})) + \mathrm{tr}\{\mathbf{R}_y^{-1}(\boldsymbol{\eta})\hat{\mathbf{R}}_y\}$, where $\boldsymbol{\eta} = [P_1, P_2, \cdots, P_K, \sigma, \boldsymbol{\psi}_1^T, \boldsymbol{\psi}_2^T, \cdots, \boldsymbol{\psi}_K^T]^T \in \mathbb{C}^{(3K+1)\times 1}$, and $\mathbf{R}_y(\boldsymbol{\eta}) \in \mathbb{C}^{M\times M}$ is defined the same as $\mathbf{R}_y$ in (2). Hence, the time complexity is $O(b_1 D_1 M^3)$, where $b_1 > 2$ and the specific value of b_1 depends on the matrix inversion algorithm and the matrix determinant algorithm implemented, and D_1 is the search dimension of the received signal powers, the noise power, and the nominal DOAs and the angular spreads of all the UTs. The covariance matching estimator (COMET) of [9] is a covariance fitting approach, and is asymptotically efficient [5] with the criterion $\arg\min_{\boldsymbol{\eta}} \mathrm{tr}\{((\mathbf{R}_y(\boldsymbol{\eta}) - \hat{\mathbf{R}}_y)\hat{\mathbf{R}}_y^{-1})^2\}$, and the time complexity is $O(D_1 M^3)$. The subspace based estimator of [12] is derived from the subspace fitting, and avoids determining the dimension of the pseudosignal subspace with the criterion $\arg\min_{\boldsymbol{\psi}} ||\hat{\mathbf{R}}_y^{-1}\boldsymbol{\Psi}(\boldsymbol{\psi})||_F^2$, where $\boldsymbol{\Psi}(\boldsymbol{\psi}) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{a}(\theta)\mathbf{a}^H(\theta)\rho(\theta, \boldsymbol{\psi})\,\mathrm{d}\theta \in \mathbb{C}^{M\times M}$. Therefore, the time complexity is $O(D_2 M^3)$, where D_2 is the search dimension of the angular parameters of one UT. The generalized Capon (GC) estimator of [13] is a beamforming approach, which minimizes the variance of the distortion power with the criterion $\arg\min_{\boldsymbol{\psi}} \lambda_{\max}\{\hat{\mathbf{R}}_y^{-1}\boldsymbol{\Psi}(\boldsymbol{\psi})\}$, where $\lambda_{\max}\{\cdot\}$ is the maximum eigenvalue of a matrix argument. Hence, the time complexity is $O(b_2 D_2 M^3)$, where $b_2 > 1$ and the specific value of b_2 depends on the eigenvalue decomposition algorithm implemented. The dispersed signal parametric estimation (DISPARE) of [11] is also derived from the subspace fitting, and the criterion is $\arg\min_{\boldsymbol{\psi}} ||\hat{\mathbf{E}}_n^H \boldsymbol{\Psi}(\boldsymbol{\psi})||_F^2$, where $\hat{\mathbf{E}}_n \in \mathbb{C}^{M\times b_3}$ and $b_3 \geq 1$. As a result, the time complexity is $O(b_3 D_2 M^2)$. For the proposed estimator, the time complexity is $O(2D_3 MK)$, where D_3 is the partial search dimension of the angular parameters of one UT. Because the coarse estimates of the nominal DOAs can be obtained by searching the K local maximum values in the vector $\mathbf{z}$, c.f. (6), $D_2 \gg D_3$ holds. Additionally, $D_1 \gg D_2$ holds because of the additional search of the received signal powers and the noise power. It should be noticed that only the calculations in the searching procedure are taken into account, which is because other calculations are of much lower complexity.

TABLE I
TIME COMPLEXITY COMPARISON

ML estimator	$O(b_1 D_1 M^3)$
COMET	$O(D_1 M^3)$
Subspace estimator	$O(D_2 M^3)$
GC estimator	$O(b_2 D_2 M^3)$
DISPARE	$O(b_3 D_2 M^2)$
Proposed estimator	$O(2D_3 MK)$

*$b_1 > 2, b_2 > 1, b_3 \geq 1$, $D_1 \gg D_2 \gg D_3$, $M \gg K$.

The time complexities of these estimators are summarized in Table I. Because the number of BS antennas $M \gg K$, and $D_1 \gg D_2 \gg D_3$, the proposed approach is of much lower complexity than other well-known approaches.

IV. NUMERICAL RESULTS

In this section, we provide numerical results to illustrate the performance of the proposed estimator as well as comparison with the DISPARE, the subspace based estimator, the GC estimator, and the Cramér-Rao bound (CRB) [15]. The ML approaches and the covariance matching approaches are not simulated because of their high complexity. For the DISPARE, the dimension of the pseudosignal space is the number of eigenvalues that contains 95% of the sum of the eigenvalues. In the simulations, the distance between adjacent antennas is $d = 0.5\lambda$. There are $K = 2$ UTs emitting BPSK modulated signals to the BS, and the transmitted signal powers $|s_k(t)|^2, k = 1, 2$, are the same. The path gain variances and the numbers of paths for the UTs are $\sigma_{\alpha_k}^2 = 1, N_k = 50, k = 1, 2$, and the power of the received noise is $\sigma^2 = 1$. Hence, the received

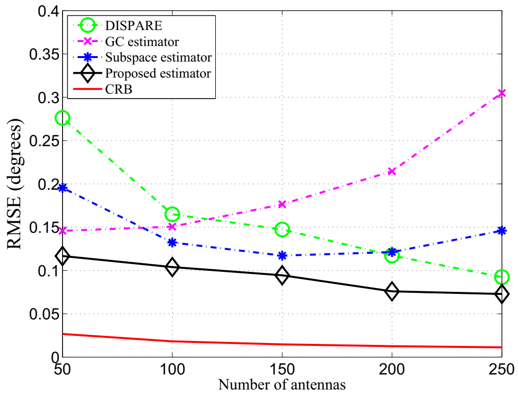

Fig. 1. RMSE versus M for the nominal DOA estimates of two sources, $\boldsymbol{\psi}_1 = [0, 2]^T$, $\boldsymbol{\psi}_2 = [20, 3]^T$.

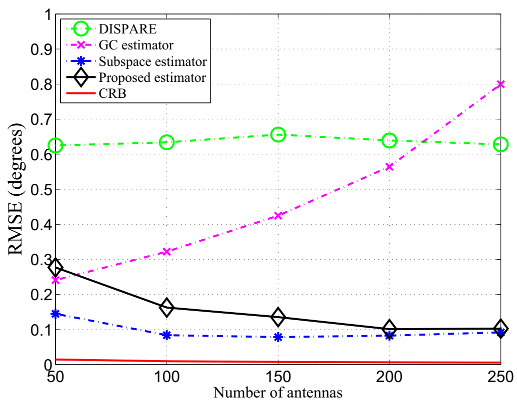

Fig. 2. RMSE versus M for the angular spread estimates of two sources, $\boldsymbol{\psi}_1 = [0, 2]^T$, $\boldsymbol{\psi}_2 = [20, 3]^T$.

signal-to-noise ratio (SNR) from each UT is $50|s_k(t)|^2$, and is set to 10 dB in the simulations. The nominal angles and the angular spreads of the two sources are $\boldsymbol{\psi}_1 = [0, 2]^T$ and $\boldsymbol{\psi}_2 = [20, 3]^T$, which are measured in degrees. The number of snapshots is $T = 300$. For saving the computing time, the search range of the nominal DOA is $\phi_1 - 4^\circ$ to $\phi_1 + 4^\circ$ and $\phi_2 - 4^\circ$ to $\phi_2 + 4^\circ$, the search range of the angular spread is 0.1° to 4°, and the searching resolution is 0.1°. Each simulated point is averaged over the root mean square errors (RMSEs) of the estimates of the sources. In real applications, when the received signal powers and the noise power are known *a priori* and the maximum angular spread is 4°, $D_1 = D_2 = 4/0.1 \times 180/0.1 = 72000$, $D_3 = 4/0.1 \times 8/0.1 = 3200$. With $M = 100$, the complexity of the proposed estimator is lower than 1% of the complexity of other estimators.

In Fig. 1 and Fig. 2, the RMSEs versus the number of BS antennas, M, for the nominal DOA estimates and the angular spread estimates are shown, respectively. It can be seen that the proposed estimator has a substantially better estimation performance than other estimators considering both the nominal DOAs and the angular spreads. This is because these estimators are derived from the approximation of the covariance matrix $\mathbf{R}_y$ for small angular spreads, which deteriorates the performance when the angular spreads are large. In contrast, the proposed estimator is derived from the approximation of the covariance matrix for a large number of BS antennas, thus the performance improves and comes close to the CRB as M increases.

V. CONCLUSION

In this paper, we present a distribution fitting approach to estimate the angular parameters of incoherently distributed sources with large antenna arrays. The estimator utilizes the approximate orthogonality of the array response vectors to estimate the angular parameters with low complexity. The numerical results show that the performance of the proposed approach is better than that of other well-known approaches.

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