DOA-based Beamforming for Multi-Cell Massive MIMO Systems

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Abstract: **This paper proposes a direction-of-arrival (DOA)-based beamforming approach for multi-cell massive multiple-input multiple-output systems with uniform rectangular arrays (URAs). The proposed approach utilizes the steering vectors of the URA to form a basis of the spatial space and selects the partial space for beamforming according to the DOA information. As a result, the proposed approach is of lower computational complexity than the existing methods which utilize the channel covariance matrices. Moreover, the analysis demonstrates that the proposed approach can eliminate the interference in the limit of infinite number of the URA antennas. Since the proposed approach utilizes the multipaths to enhance the signal rather than discarding them, the proposed approach is of better performance than the existing low-complexity method, which is verified by the simulation results.**

Index Terms: **Beamforming, direction-of-arrival (DOA), interference, massive multiple-input multiple-output (MIMO), uniform rectangular array (URA).**

I. INTRODUCTION

MULTIPLE -input multiple-output (MIMO) technology is efficient in improving spectral efficiency in wireless communication systems [1]. By employing a hundred or a few hundred antennas at the base station (BS), the spectrum efficiency and the energy efficiency can be greatly improved in comparison to traditional MIMO systems. This system is termed as the large-scale or massive MIMO system and has been extensively investigated recently [2]–[4].

Despite these benefits, the inter-cell interference (ICI) is one of the constraints in multi-cell massive MIMO systems that limit the performance [4]–[6]. Although the ICI has been tackled by various approaches in massive MIMO systems [6]–[11], these approaches cannot be employed in scenarios with correlated channel responses and two-dimensional (2D) antenna arrays. Meanwhile, the channels are correlated when the angular spreads are not large enough. Additionally, 2D arrays are more feasible for massive MIMO systems because of the space constraints at the BSs [12]–[14].

For correlated channels in massive MIMO systems, beamforming is a feasible technology [15]. In traditional beamforming approaches, the generalized eigenvector of the channel correlation matrices [16], [17] and the projection of the signal subspace [18]–[20] have been utilized to cancel interference. However, the computational complexity of these approaches is high for massive MIMO systems because of the high dimension of the channel vectors. In order to reduce the computational complexity of the beamforming approaches, the direction-of-arrival (DOA) information is utilized in recent researches. The approach in [21] directly employs the array steering vector, which is a function of the DOA, as the beamforming weight vector, but the multipaths are taken as interference. Thus, this approach only applies to line-of-sight (LOS) transmission scenarios [21]. The approaches in [22] and [23] utilize the array steering vectors and the DOA information to reduce the dimension of the channel vectors. However, the instantaneous channel state information (CSI), which is harder to be acquired than the DOA information in static or low-speed scenarios, is still necessary for beamforming. Therefore, DOA-based beamforming for correlated channels still needs research.

On the other hand, for 2D antenna arrays, the DOA information is also utilized in the known beamforming approaches to mitigate the interference. The approaches in [24]–[26] adjust antenna downtilt in the vertical plane with the DOA information and perform beamforming in the horizontal plane with the instantaneous CSI. Furthermore, the beamforming approach forms multiple beams in the vertical dimension [27]. However, all these approaches necessitate the instantaneous CSI to form multiple beams. Therefore, research on DOA-based beamforming for 2D arrays is necessary.

In this work, a DOA-based beamforming approach for multi-cell massive MIMO systems with uniform rectangular arrays (URAs) is proposed. The proposed approach utilizes the steering vectors of the URA to form a basis of the Euclidean space. Then, the signal-to-interference-plus-noise-ratio (SINR) in each dimension of the space is calculated with the aid of the DOA information. Finally, part of the basis vectors corresponding to larger SINRs are selected to form the beamforming weight vector. More specifically, the main contributions of this work are three-fold.

1) The proposed approach only utilizes the DOA information in forming multiple beams in both the horizontal and the elevation directions. In contrast, the instantaneous CSI is essential for the approaches in [22]–[27]. Moreover, the proposed approach collects the multipaths for enhancing signal power rather than treating the multipaths as interference. Therefore, the proposed approach performs better than the approach in [21] in scattering scenarios.
2) The proposed approach is of much lower computational complexity than the approaches in [16]–[20]. Because the basis of the space is constructed by the array steering vectors, the DOA information is directly utilized in selecting the basis

Manuscript received June 28, 2015; approved for publication by Huaping Liu, Division II Editor, February 9, 2016.

This research was supported by Zhejiang Provincial Natural Science Foundation of China under Grant No. LQ16F010007 and Scientific Research Starting Foundation of Hangzhou Dianzi University under Grant No. KYS085614054.

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Digital object identifier 10.1109/JCN.2016.000103

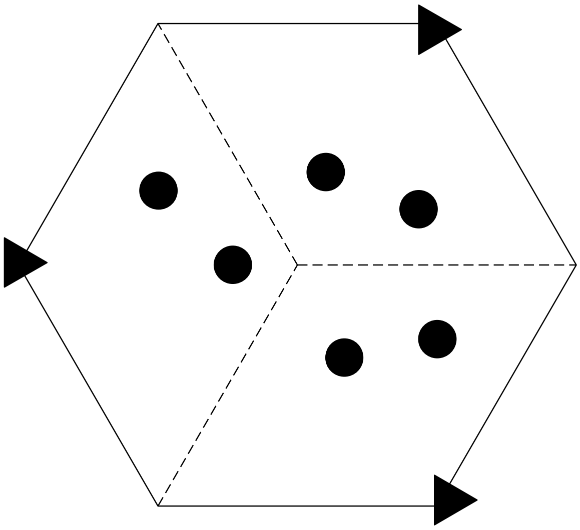

Fig. 1. Cellular layout and cell geometry. The triangles represent the BSs, the circles represent the UTs.

vectors. Therefore, there is not any calculation of the channel covariance matrices, which is essential in [16]–[20].

3) The asymptotic lower bound of the sum rate with the proposed approach is analyzed. It is proved that the lower bound of the sum rate increases as the number of the URA antennas increases.

This paper is organized as follows. Section II presents the system model and the assumptions. Section III contains the proposed DOA-based beamforming approach. In Section IV, the asymptotic performance of the proposed approach is analyzed and the computational complexity of the proposed approach is compared with that of other approaches. The simulation results are presented in Section V and the concluding remarks are provided in Section VI.

Notations: Lower-case (upper-case) boldface symbols denote vectors (matrices); $\mathbf{I}_K$ represents the $K \times K$ identity matrix; $(\cdot)^H$ and $\mathbb{E}\{\cdot\}$ denote the conjugate transpose and the expectation, respectively; $||\cdot||$ is the Euclidean norm of a vector; $[\cdot]_j$ is the jth element of a vector or the jth column of a matrix; $\delta(\cdot)$ is the Kronecker delta function; and i is the imaginary unit.

II. SYSTEM MODEL

The system considered consists of three rhombic cells[1], as shown in Fig. 1. The triangles represent the BSs, and are placed at the vertex of the cell. The circles represent the user terminals (UTs), and are distributed uniformly in the cell. All the cells share the same frequency band, which leads to severe ICI. The cells are indexed by $l = 1, 2, \cdots, L$, and the BS in the jth cell is denoted as the jth BS in the rest of the paper. Obviously, there is $L = 3$ in the system considered.

In this system, each BS is equipped with one URA, and each UT is equipped with a single antenna. The system considered is termed as full-dimension MIMO in [13], and is regarded as a promising realization of massive MIMO systems. Only uplink transmission is considered in this paper, i.e., the UTs transmit signals to the BSs. At the tth time slot, $t = 1, 2, \cdots, T$, the signal received by the jth BS is

$$\mathbf{y}_j(t) = \sum_{l=1}^{L}\sum_{k=1}^{K} \mathbf{h}_{j,l_k}(t) s_{l,k}(t) + \mathbf{n}_j(t) \in \mathbb{C}^{M\times 1}, \quad (1)$$

where K is the number of the UTs in each cell, M is the number of the antenna elements in each URA; $s_{l,k}(t)$ is the symbol transmitted by the kth UT in the lth cell at the tth time slot; $\mathbf{n}_j(t) \in \mathbb{C}^{M\times 1}$ is the noise received by the jth BS at the tth time slot; $\mathbf{h}_{j,l_k}(t) \in \mathbb{C}^{M\times 1}$ is the channel vector from the kth UT in the lth cell to the BS of the jth cell at the tth time slot, and is expressed as

$$\mathbf{h}_{j,l_k}(t) = \sum_{n=1}^{N} \frac{\gamma_{j,l_k,n}(t)}{\sqrt{N}} \mathbf{a}(\theta_{j,l_k,n}(t), \phi_{j,l_k,n}(t)), \quad (2)$$

where N is the number of the multipaths received by each BS from each UT; $\gamma_{j,l_k,n}(t)$, $\theta_{j,l_k,n}(t)$, and $\phi_{j,l_k,n}(t)$ are the channel response, the azimuth DOA, and the elevation DOA of the nth path from the kth UT in the lth cell to the jth BS at the tth time slot, respectively; $\mathbf{a}(\theta_{j,l_k,n}(t), \phi_{j,l_k,n}(t)) \in \mathbb{C}^{M\times 1}$ is the array response vector of the URA to the DOAs $\theta_{j,l_k,n}(t), \phi_{j,l_k,n}(t)$. Since the URA can separate signals from one side of the array, the overall ranges of the DOAs are $0 \le \theta_{j,l_k,n}(t) < \pi$ and $-\pi/2 \le \phi_{j,l_k,n}(t) < \pi/2$. In fact, the UTs are inside the cell, and the DOAs of the UTs satisfy $\pi/6 \le \theta_{j,l_k,n}(t) \le 5\pi/6$ and $0 < \phi_{j,l_k,n}(t) < \pi/2$. In addition, $\mathbf{a}(\theta_{j,l_k,n}(t), \phi_{j,l_k,n}(t))$ is the response vector of the URA, and is defined as [28]

$$\begin{aligned}
[\mathbf{a}(\theta_{j,l_k,n}(t), \phi_{j,l_k,n}(t))]_m &= \exp\Big(iu \sin(\phi_{j,l_k,n}(t)) \big[(m_\mathrm{x} - 1) \\
&\quad \times \cos(\theta_{j,l_k,n}(t)) + (m_\mathrm{y} - 1)\sin(\theta_{j,l_k,n}(t))\big]\Big), \\
m &= (m_\mathrm{y} - 1)M_\mathrm{x} + m_\mathrm{x},\ m_\mathrm{x} = 1, 2, \cdots, M_\mathrm{x}, \\
m_\mathrm{y} &= 1, 2, \cdots, M_\mathrm{y},
\end{aligned} \quad (3)$$

where $u = 2\pi d/\lambda$, d is the distance between two adjacent antenna elements in the URA, and λ is the wavelength of the signals transmitted by the UTs. Here, the distance between any two adjacent antennas is set as $d = \lambda/2$, which means $u = \pi$. In addition, M_x and M_y are the numbers of the antenna elements of the URA in the azimuth direction and the elevation direction, respectively. Obviously, M_x and M_y satisfy $M = M_\mathrm{x} M_\mathrm{y}$.

Before presenting the proposed beamforming approach, the initial assumptions are given as follows.

1) The symbols, $s_{l,k}(t), l = 1, 2, 3, k = 1, 2, \cdots, K, t = 1, 2, \cdots, T$, are independent and identically distributed (i.i.d.) random variables with covariance one. In addition, the noise vectors, $\mathbf{n}_j(t), t = 1, 2, \cdots, T$, are composed of i.i.d. Gaussian random variables with zero mean, and the covariance matrix of these noise vectors is $\mathbf{I}_M$. Moreover, the elements of the noise vectors, $\mathbf{n}_j(t), t = 1, 2, \cdots, T$, and the symbols, $s_{l,k}(t), l = 1, 2, 3, k = 1, 2, \cdots, K, t = 1, 2, \cdots, T$, are uncorrelated.

[1]The rhombic cell is a 120° sector of a hexagonal cell, and is relevant to systems of cell sectoring.

2) The channel responses, $\gamma_{j,l_k,n}(t), l = 1,2,3, k = 1,2,\cdots,K, n = 1,2,\cdots,N, t = 1,2,\cdots,T$, are composed of large scale fading and small scale fading, and are i.i.d. random variables with covariance, $\sigma^2_{\gamma_{j,l_k}}$.
3) The DOAs, $\theta_{j,l_k,n}(t), \phi_{j,l_k,n}(t), n = 1,2,\cdots,N, t = 1,2,\cdots,T$, are i.i.d. random variables, the distributions of which can be described by the first- and the second-order statistics. Examples of the distributions include the uniform, the Gaussian, the Laplace, and the Von Mises distributions. The means of $\theta_{j,l_k,n}(t), \phi_{j,l_k,n}(t)$ are denoted as $\bar{\theta}_{j,l_k}$, $\bar{\phi}_{j,l_k}$, which are also known as the nominal azimuth DOA and the nominal elevation DOA, respectively. The variances of $\theta_{j,l_k,n}(t), \phi_{j,l_k,n}(t)$ are denoted as $\sigma^2_{\theta_{j,l_k}}$, $\sigma^2_{\phi_{j,l_k}}$. Additionally, $\sigma_{\theta_{j,l_k}}$ and $\sigma_{\phi_{j,l_k}}$ are also known as the azimuth angular spread and the elevation angular spread, respectively.
4) The elements of the channel vector, $\mathbf{h}_{j,l_k}$, are correlated. Following the correlated channel model in [11] and [29], the channel vector in (2) can be re-expressed as

$$\mathbf{h}_{j,l_k}(t) = \mathbf{R}^{1/2}_{j,l_k}\tilde{\mathbf{h}}_{j,l_k}(t), \tag{4}$$

where

$$\mathbf{R}_{j,l_k} = \mathbb{E}\{\mathbf{h}_{j,l_k}(t)\mathbf{h}^H_{j,l_k}(t)\} \in \mathbb{C}^{M\times M} \tag{5}$$

is the channel correlation matrix, and $\tilde{\mathbf{h}}_{j,l_k}(t) \in \mathbb{C}^{M\times 1}$ has i.i.d. complex entries of zero mean and unit variance. Additionally, the channel undergoes block fading. Thus, the channel correlation matrix, $\mathbf{R}_{j,l_k}$, the nominal DOAs, and the angular spreads are constant in the channel coherence period, and $\tilde{\mathbf{h}}_{j,l_k}(t)$ is fixed in each time slot but changes over time slots. Additionally, it is assumed that the UTs are static or low-speed so that each channel coherence period is composed of multiple time slots, which means the channel correlation matrix is constant in multiple time slots and is easier to be acquired than the instantaneous CSI.

At each time slot, linear beamforming is employed at the BS to suppress the interference and enhance the signal. For the kth UT in the jth cell, the received signal vector, $\mathbf{y}_j(t)$ in (1), is processed by the jth BS with the linear beamforming, and the resulting signal is denoted as

$$\tilde{s}_{j,k}(t) = \mathbf{w}^H_{j,k}\mathbf{y}_j(t), \tag{6}$$

where $\mathbf{w}_{j,k} \in \mathbb{C}^{M\times 1}$ is the linear beamforming weight vector for the kth UT in the jth cell, and is of unit norm, i.e., $||\mathbf{w}_{j,k}|| = 1$. Then, the resulting signal $\tilde{s}_{j,k}(t)$ is employed for signal detection.

Substituting (1) into (6), the SINR for the kth UT in the jth cell can be derived, which is shown in (7). Correspondingly, with an ergodic channel, the achievable ergodic sum rate is given by

$$R = \sum_{j=1}^{L}\sum_{k=1}^{K}\mathbb{E}\left\{\log_2\left(1+\xi_{j,k}\right)\right\}. \tag{8}$$

To be specific, the focus of this work is the design of the beamforming weight vector, $\mathbf{w}_{j,k}$ in (6), with only the DOA information to maximize the ergodic uplink sum rate of the system. With the system model and the aim of this work presented, the proposed beamforming approach will be presented in the next section.

III. THE PROPOSED BEAMFORMING APPROACH

With the knowledge of the angular parameters, DOA-based beamforming can be employed to mitigate the interference. The beamforming approaches in [16]–[20] are of high computational complexity for massive MIMO systems. Additionally, the known DOA-based beamforming approaches cannot collect signal power efficiently in scattered scenarios [21] or cannot be employed with only the DOA information available [22]–[27]. In order to cancel the interference with lower computational complexity and enhance the signal, a beamforming approach that only utilizes the DOA information is proposed. In this section, first the lower bound and the limit of the ergodic sum rate are derived. Then, the DOA information is utilized to maximize the lower bound of the sum rate, and this leads to the proposed beamforming approach.

A. Sum Rate Analysis

With the Jensen's inequality, the lower bound of the ergodic sum rate in (8) can be derived, i.e.,

$$\begin{aligned} R &= \sum_{j=1}^{L}\sum_{k=1}^{K}\mathbb{E}\left\{\mathbb{E}\left\{\log_2\left(1+\xi_{j,k}\right)|\mathbf{h}_{j,j_k}(t)\right\}\right\} \\ &\geq \sum_{j=1}^{L}\sum_{k=1}^{K}\mathbb{E}\left\{\log_2\left(1+\frac{1}{\mathbb{E}\left\{1/\xi_{j,k}|\mathbf{h}_{j,j_k}(t)\right\}}\right)\right\} \\ &= \sum_{j=1}^{L}\sum_{k=1}^{K}\mathbb{E}\left\{\log_2\left(1+\bar{\xi}_{j,k}\right)\right\} \triangleq \bar{R}, \end{aligned} \tag{9}$$

where $\bar{\xi}_{j,k}$ is shown in (10), and $\bar{R}$ is the lower bound of the ergodic sum rate. Substituting (5) into (10) yields (11).

Since the lower bound of the ergodic sum rate is more concise than the ergodic sum rate, it is feasible to derive a corresponding beamforming approach to maximize the lower bound of the sum rate, which will be presented in the next subsection.

B. DOA-based Beamforming

According to the lower bound of the ergodic sum rate in (9), it can be seen that the beamforming weight vector, $\mathbf{w}_{j,k}$, that maximizes the sum rate is exactly the generalized eigenvector of the matrices, $\mathbf{R}^{1/2}_{j,j_k}\tilde{\mathbf{h}}_{j,j_k}(t)\tilde{\mathbf{h}}^H_{j,j_k}(t)\mathbf{R}^{1/2}_{j,j_k}$ and $\sum_{\substack{k'=1\\k'\neq k}}^{K}\mathbf{R}_{j,j_{k'}} + \sum_{\substack{l=1\\l\neq j}}^{L}\sum_{k'=1}^{K}\mathbf{R}_{j,l_{k'}}$. However, this calculation necessitates the knowledge of the instantaneous CSI, $\tilde{\mathbf{h}}_{j,l_k}(t)$, the acquisition of which consumes more resource than that of the statistical CSI, such as the channel correlation matrix, $\mathbf{R}_{j,l_k}$, and the DOA information.

In order to further simplify the lower bound of the ergodic sum rate, the channel correlation matrix, $\mathbf{R}_{j,l_k}$, is represented in the form of singular value decomposition (SVD) as

$$\mathbf{R}_{j,l_k} = \mathbf{U}\mathbf{D}_{j,l_k}\mathbf{U}^H, \tag{12}$$

$$\xi_{j,k} = \frac{\left|\left|\mathbf{w}_{j,k}^H \mathbf{h}_{j,j_k}(t)\right|\right|^2}{\sum_{\substack{k'=1\\k'\neq k}}^K \left|\left|\mathbf{w}_{j,k}^H \mathbf{h}_{j,j_{k'}}(t)\right|\right|^2 + \sum_{\substack{l=1\\l\neq j}}^L \sum_{k'=1}^K \left|\left|\mathbf{w}_{j,k}^H \mathbf{h}_{j,l_{k'}}(t)\right|\right|^2 + 1} \tag{7}$$

$$\bar{\xi}_{j,k} = \frac{\left|\left|\mathbf{w}_{j,k}^H \mathbf{h}_{j,j_k}(t)\right|\right|^2}{\mathbb{E}\left\{\sum_{\substack{k'=1\\k'\neq k}}^K \left|\left|\mathbf{w}_{j,k}^H \mathbf{h}_{j,j_{k'}}(t)\right|\right|^2 + \sum_{\substack{l=1\\l\neq j}}^L \sum_{k'=1}^K \left|\left|\mathbf{w}_{j,k}^H \mathbf{h}_{j,l_{k'}}(t)\right|\right|^2\right\} + 1} \tag{10}$$

$$= \frac{\mathbf{w}_{j,k}^H \mathbf{R}_{j,j_k}^{1/2} \tilde{\mathbf{h}}_{j,j_k}(t) \tilde{\mathbf{h}}_{j,j_k}^H(t) \mathbf{R}_{j,j_k}^{1/2} \mathbf{w}_{j,k}}{\mathbf{w}_{j,k}^H \left(\sum_{\substack{k'=1\\k'\neq k}}^K \mathbf{R}_{j,j_{k'}} + \sum_{\substack{l=1\\l\neq j}}^L \sum_{k'=1}^K \mathbf{R}_{j,l_{k'}}\right) \mathbf{w}_{j,k} + 1} \tag{11}$$

where $\mathbf{U} \in \mathbb{C}^{M\times M}$ is composed of the singular vectors, and $\mathbf{D}_{j,l_k} \in \mathbb{R}^{M\times M}$ is a diagonal matrix of singular values. Because the columns of $\mathbf{U}$ form a basis of the M-dimensional Euclidean space, the instantaneous channel vector, $\tilde{\mathbf{h}}_{j,l_k}(t)$, and the weight vector, $\mathbf{w}_{j,k}$, can be represented as

$$\tilde{\mathbf{h}}_{j,l_k}(t) = \mathbf{U}\mathbf{g}_{j,l_k}^{\mathrm{h}}, \tag{13}$$

$$\mathbf{w}_{j,k} = \mathbf{U}\mathbf{g}_{j,k}^{\mathrm{w}}, \tag{14}$$

where $\mathbf{g}_{j,l_k}^{\mathrm{h}} \in \mathbb{C}^{M\times 1}$ and $\mathbf{g}_{j,k}^{\mathrm{w}} \in \mathbb{C}^{M\times 1}$ are composed of weight values.

By replacing the matrices in (11) with (12)–(14), $\bar{\xi}_{j,k}$ can be re-written as (15),

where $d_{j,l_k,m} = [\mathbf{D}_{j,l_k}]_{m,m}$ is the singular value of $\mathbf{R}_{j,l_k}$, $g_{j,k,m}^{\mathrm{w}} = [\mathbf{g}_{j,k}^{\mathrm{w}}]_m$ is related to the weight vector, cf. (14), and $g_{j,l_k,m}^{\mathrm{h}} = [\mathbf{g}_{j,l_k}^{\mathrm{h}}]_m$ is related to the instantaneous channel vector, $\tilde{\mathbf{h}}_{j,l_k}(t)$, cf. (13).

Because the lower bound of the ergodic sum rate is directly related to $\bar{\xi}_{j,k}$ in (15), the object of the design changes into the maximization of (15). Meanwhile, the weight vector $\mathbf{w}_{j,k}$ is of unit norm. Thus, $||\mathbf{g}_{j,k}^{\mathrm{w}}|| = 1$ can be derived according to (14). For clarity, the object of the design of the beamforming weight vector can be expressed as

$$\max_{\mathbf{g}_{j,k}^{\mathrm{w}}} \quad \bar{\xi}_{j,k}$$
$$\text{s.t.} \quad ||\mathbf{g}_{\mathrm{j,k}}^{\mathrm{w}}|| = 1. \tag{18}$$

According to the assumption that the elements of $\tilde{\mathbf{h}}_{j,l_k}(t)$ are i.i.d. Gaussian distributed and unknown, it is obvious that $g_{j,l_k,m}^{\mathrm{h}}, m = 1, 2, \cdots, M$ are also i.i.d. Gaussian variables and unknown. Thus, the maximization of $\bar{\xi}_{j,k}$ is not feasible. Instead, with only the knowledge of $d_{j,l_k,m}$, the vector, $\mathbf{g}_{j,k}^{\mathrm{w}}$, can be designed to achieve a relatively large $\bar{\xi}_{j,k}$ rather than the maximal $\bar{\xi}_{j,k}$. With the unit norm constraint of $\mathbf{g}_{j,k}^{\mathrm{w}}$, the expression of $\bar{\xi}_{j,k}$ in (15) can be re-expressed as (16), and further simplified as

$$\bar{\xi}_{j,k} = \sum_{m=1}^M f_{j,k,m}^{\mathrm{w}} \zeta_{j,k,m} \left|g_{j,j_k,m}^{\mathrm{h}}\right|^2, \tag{19}$$

where $f_{j,k,m}^{\mathrm{w}}$ is defined as (17) and

$$\zeta_{j,k,m} = \frac{d_{j,j_k,m}}{\sum_{\substack{k'=1\\k'\neq k}}^K d_{j,j_{k'},m} + \sum_{\substack{l=1\\l\neq j}}^L \sum_{k'=1}^K d_{j,l_{k'},m} + 1}. \tag{20}$$

It can be seen that $f_{j,k,m}^{\mathrm{w}}$ satisfies the constraint $\sum_{m=1}^M f_{j,k,m}^{\mathrm{w}} = 1$ and $f_{j,k,m}^{\mathrm{w}}$ is a monotonically increasing function of $\left|g_{j,k,m}^{\mathrm{w}}\right|^2$. In order to present the setting of the weight vector $\mathbf{g}_{j,k}^{\mathrm{w}}$, the following lemma is necessary.

Lemma 1: When the range of the azimuth or the elevation DOAs from the kth UT in the lth cell to the jth BS is shorter than the overall range of the DOAs in the corresponding direction, part of the singular values in $d_{j,l_k,m}, m = 1, 2, \cdots, M$, tend to be zero as $M_{\mathrm{x}} \to \infty, M_{\mathrm{y}} \to \infty$.

Proof: Refer to Appendix A for details.

Remark 1: For correlated channels, the angular spreads of the UTs are much smaller than the ranges of the DOAs, which means the DOAs of each UT satisfy the condition in the lemma. Hence, only part of the singular values are nonzero in massive MIMO systems.

According to Lemma 1, it is favorable to set $f_{j,k,m}^{\mathrm{w}}$ to be nonzero when $d_{j,j_k,m}$ is nonzero and $d_{j,l_{k'},m}, l \neq j, \mathrm{or}\ \mathrm{k}' \neq \mathrm{k}$ is zero, so as to maximize $\bar{\xi}_{j,k}$ in (19). For the case that both the value $d_{j,j_k,m}$ and $d_{j,l_{k'},m}, l \neq j, \mathrm{or}\ \mathrm{k}' \neq \mathrm{k}$ are nonzero, $f_{j,k,m}^{\mathrm{w}}$ should be nonzero when the ratio, $\zeta_{j,k,m}$ in (20), is relatively large. According to the relation between $f_{j,k,m}^{\mathrm{w}}$ and $g_{j,k,m}^{\mathrm{w}}$, $g_{j,k,m}^{\mathrm{w}}$ should also be nonzero in these cases. Because the region of the DOAs of each UT determines the number of nonzero singular values in $d_{j,l_k,m}, m = 1, 2, \cdots, M$, the angular spreads, $\sigma_{\theta_{j,j_k}}$ and $\sigma_{\phi_{j,j_k}}$, can be utilized to determine the number of the nonzero values in $d_{j,l_k,m}, m = 1, 2, \cdots, M$. According to the ranges of azimuth and elevation DOAs, $[0, \pi)$ and $[-\pi/2, \pi/2)$, it is reasonable to pick out the largest $\lceil M\sigma_{\theta_{j,j_k}} \sigma_{\phi_{j,j_k}}/\pi^2\rceil$ values in $\zeta_{j,k,m}, m = 1, 2, \cdots, M$, cf. (20), and set the corresponding $g_{j,k,m}^{\mathrm{w}}$ to be nonzero values. Then, the beamforming weight vector can be obtained with (14).

In contrast to the method in [21], the scattered component in the received signal is also utilized to enhance signal. Therefore, the proposed approach can achieve higher sum rate in scattering scenarios. Moreover, the proposed approach is feasible for

$$\begin{aligned}\bar{\xi}_{j,k} &= \frac{\left|\left|(\mathbf{g}^{\mathrm{w}}_{j,k})^H \mathbf{D}^{1/2}_{j,j_k} \mathbf{g}^{\mathrm{h}}_{j,j_k}\right|\right|^2}{\sum_{\substack{k'=1\\k'\neq k}}^{K} \left|\left|(\mathbf{g}^{\mathrm{w}}_{j,k})^H \mathbf{D}^{1/2}_{j,j_{k'}}\right|\right|^2 + \sum_{\substack{l=1\\l\neq j}}^{L} \sum_{k'=1}^{K} \left|\left|(\mathbf{g}^{\mathrm{w}}_{j,k})^H \mathbf{D}^{1/2}_{j,l_{k'}}\right|\right|^2 + 1} \\ &= \frac{\sum_{m=1}^{M} d_{j,j_k,m} \left|g^{\mathrm{h}}_{j,j_k,m}\right|^2 \left|g^{\mathrm{w}}_{j,k,m}\right|^2}{\sum_{m=1}^{M} \left(\sum_{\substack{k'=1\\k'\neq k}}^{K} d_{j,j_{k'},m} + \sum_{\substack{l=1\\l\neq j}}^{L} \sum_{k'=1}^{K} d_{j,l_{k'},m}\right) \left|g^{\mathrm{w}}_{j,k,m}\right|^2 + 1}\end{aligned} \tag{15}$$

$$\bar{\xi}_{j,k} = \sum_{m=1}^{M} \frac{\left|g^{\mathrm{w}}_{j,k,m}\right|^2}{\sum_{m'=1}^{M} \left(\sum_{\substack{k'=1\\k'\neq k}}^{K} d_{j,j_{k'},m'} + \sum_{\substack{l=1\\l\neq j}}^{L} \sum_{k'=1}^{K} d_{j,l_{k'},m'} + 1\right) \left|g^{\mathrm{w}}_{j,k,m'}\right|^2} d_{j,j_k,m} \left|g^{\mathrm{h}}_{j,j_k,m}\right|^2 \tag{16}$$

$$f^{\mathrm{w}}_{j,k,m} = \frac{\left|g^{\mathrm{w}}_{j,k,m}\right|^2 \left(\sum_{\substack{k'=1\\k'\neq k}}^{K} d_{j,j_{k'},m} + \sum_{\substack{l=1\\l\neq j}}^{L} \sum_{k'=1}^{K} d_{j,l_{k'},m} + 1\right)}{\sum_{m'=1}^{M} \left(\sum_{\substack{k'=1\\k'\neq k}}^{K} d_{j,j_{k'},m'} + \sum_{\substack{l=1\\l\neq j}}^{L} \sum_{k'=1}^{K} d_{j,l_{k'},m'} + 1\right) \left|g^{\mathrm{w}}_{j,k,m'}\right|^2} \tag{17}$$

systems with only the DOA information while the methods in [22]–[27] cannot be applied in such systems.

Till now, the calculation of the beamforming weight vector has been presented. However, the employment of the matrix of singular vectors, i.e., $\mathbf{U}$ in (12), requires the SVD of $\mathbf{R}_{j,l_k}$ and the calculation of $\mathbf{R}_{j,l_k}$, cf. (5), which are of high computational complexity in massive MIMO systems. In fact, the proposed approach can be further simplified, and the simplification will be shown in the next subsection.

C. Simplification

Inspired by the beamspace MIMO system in [22] and [23], the property of URAs is employed for simplifying the calculation of the matrix $\mathbf{U}$ and the singular values. According to (3) and the assumptions in Section II, the array steering vector can be re-expressed as

$$[\mathbf{b}(\eta_{\mathrm{x}}, \eta_{\mathrm{y}}))]_m \triangleq \exp\Big(i\pi\left[(m_{\mathrm{x}}-1)\eta_{\mathrm{x}} + (m_{\mathrm{y}}-1)\eta_{\mathrm{y}}\right]\Big), \tag{21}$$

where

$$\begin{aligned}\eta_{\mathrm{x}} &= \sin(\phi_{j,l_k,n}(t))\cos(\theta_{j,l_k,n}(t)),\\ \eta_{\mathrm{y}} &= \sin(\phi_{j,l_k,n}(t))\sin(\theta_{j,l_k,n}(t)),\end{aligned}$$

and $\mathbf{b}(\eta_{\mathrm{x}}, \eta_{\mathrm{y}})) \in \mathbb{C}^{M\times 1}$ is another notation of the array steering vector. Obviously, for the steering vector $\mathbf{a}(\theta_{j,l_k,n}(t), \phi_{j,l_k,n}(t))$, $\eta_{\mathrm{x}} \in [-1,1]$ and $\eta_{\mathrm{y}} \in [-1,1]$ corresponds to a point source from the azimuth direction, $\theta_{j,l_k,n}(t) \in [0,\pi]$ and the elevation direction, $\phi_{j,l_k,n}(t) \in [-\pi/2, \pi/2]$. Then, there is

$$\begin{aligned}&\frac{1}{M}\mathbf{b}^H(f_{\mathrm{x}}(n_{\mathrm{x}}), f_{\mathrm{y}}(n_{\mathrm{y}}))\mathbf{b}(f_{\mathrm{x}}(\tilde{n}_{\mathrm{x}}), f_{\mathrm{y}}(\tilde{n}_{\mathrm{y}})\\ &\qquad = \delta\left(n_{\mathrm{x}} - \tilde{n}_{\mathrm{x}}\right)\delta\left(n_{\mathrm{y}} - \tilde{n}_{\mathrm{y}}\right),\end{aligned} \tag{22}$$

where $f_{\mathrm{x}}(n_{\mathrm{x}}) = -1 + 2n_{\mathrm{x}}/M_{\mathrm{x}}$, $f_{\mathrm{y}}(n_{\mathrm{y}}) = -1 + 2n_{\mathrm{y}}/M_{\mathrm{y}}$, and $n_{\mathrm{x}} \in \{0,1,\cdots,M_{\mathrm{x}}-1\}$, $\tilde{n}_{\mathrm{x}} \in \{0,1,\cdots,M_{\mathrm{x}}-1\}$, $n_{\mathrm{y}} \in \{0,1,\cdots,M_{\mathrm{y}}-1\}$, $\tilde{n}_{\mathrm{y}} \in \{0,1,\cdots,M_{\mathrm{y}}-1\}$.

From the property of the URA presented in (22), it is known that the vectors, $\mathbf{b}(f_{\mathrm{x}}(n_{\mathrm{x}}), f_{\mathrm{y}}(n_{\mathrm{y}}))/\sqrt{M}$, $n_{\mathrm{x}} = 0,1,\cdots,M_{\mathrm{x}}-1$, $n_{\mathrm{y}} = 0,1,\cdots,M_{\mathrm{y}}-1$, form a basis for the M-dimensional Euclidean space. Because the columns of $\mathbf{U}$ in (12) can be any M vectors that form a basis for the M-dimensional Euclidean space, they can be formed with $\mathbf{b}(f_{\mathrm{x}}(n_{\mathrm{x}}), f_{\mathrm{y}}(n_{\mathrm{y}}))/\sqrt{M}$, $n_{\mathrm{x}} = 0,1,\cdots,M_{\mathrm{x}}-1$, $n_{\mathrm{y}} = 0,1,\cdots,M_{\mathrm{y}}-1$. Note that this property is employed to reduce the dimension of the instantaneous channel vector in [22], [23], but is utilized to simplify the calculations of the singular values and the basis for the M-dimensional Euclidean space here.

Then, the n_{a}th diagonal element of $\mathbf{D}_{j,l_k}$ in (12) can be obtained with

$$\begin{aligned}[\mathbf{D}_{j,l_k}]_{n_{\mathrm{a}},n_{\mathrm{a}}} = \frac{1}{M}&\mathbf{b}^H(f_{\mathrm{x}}(n_{\mathrm{x}}), f_{\mathrm{y}}(n_{\mathrm{y}}))\mathbf{R}_{j,l_k}\\ &\times\mathbf{b}(f_{\mathrm{x}}(n_{\mathrm{x}}), f_{\mathrm{y}}(n_{\mathrm{y}})),\end{aligned} \tag{23}$$

where $n_{\mathrm{a}} = n_{\mathrm{y}}M_{\mathrm{x}} + n_{\mathrm{x}} + 1$. According to the expression of $\mathbf{R}_{j,l_{k'}}$ in (24), the calculation in (23) can be simplified into an integral. The detailed derivations are shown in Appendix B for brevity.

For clarity, the algorithm corresponding to the proposed DOA-based beamforming approach is presented as follows.

Algorithm 1: The proposed DOA-based beamforming approach

Step 1) Calculate $\mathbf{D}_{j,l_k}$ as (26).

Step 2) Pick out the largest $\lceil M\sigma_{\theta_{j,j_k}}\sigma_{\phi_{j,j_k}}/\pi^2\rceil$ values in $\zeta_{j,k,m}$, $m = 1,2,\cdots,M$, and set the corresponding $g^{\mathrm{w}}_{j,k,m}$ to be $1/\sqrt{\lceil M\sigma_{\theta_{j,j_k}}\sigma_{\phi_{j,j_k}}/\pi^2\rceil}$.

Step 3) Form the columns of $\mathbf{U}$ with $\mathbf{b}(f_{\mathrm{x}}(n_{\mathrm{x}}), f_{\mathrm{y}}(n_{\mathrm{y}}))/\sqrt{M}$, $n_{\mathrm{x}} = 0, 1, \cdots, M_{\mathrm{x}}-1$, $n_{\mathrm{y}} = 0, 1, \cdots, M_{\mathrm{y}}-1$, using (21).

Step 4) Form the DOA-based beamforming weight vector, $\mathbf{w}_{j,k}$, using (14).

Remark 2: The selected elements in $g^{\mathrm{w}}_{j,k,m}$, $m = 1, 2, \cdots, M$, are set to be $1/\sqrt{\lceil M\sigma_{\theta_{j,j_k}}\sigma_{\phi_{j,j_k}}/\pi^2\rceil}$ to satisfy the unit norm constraint on $\mathbf{g}^{\mathrm{w}}_{j,k}$. Meanwhile, other values that satisfy the constraint are also feasible. According to equation (19), $\bar{\xi}_{j,k}$ is a random variable because of the randomness of $g^{\mathrm{h}}_{j,j_k,m}$. Therefore, $g^{\mathrm{w}}_{j,k,m}$ as well as $f^{\mathrm{w}}_{j,k,m}$ are designed to obtain a relatively large value of $\bar{\xi}_{j,k}$ according to $\zeta_{j,k,m}$. In order to maximize $\zeta_{j,k,m}$, the value of random variables, $g^{\mathrm{h}}_{j,j_k,m}, m = 1, 2, \cdots, M$, should be known a priori, i.e., the instantaneous CSI is needed. This paper only focuses on the utilization of the DOA information in designing the weight vector, and the instantaneous CSI will be employed in designing the weight components more finely in the future research.

Since only part of the elements in $\mathbf{g}^{\mathrm{w}}_{j,k}$ are nonzero and the nonzero elements are of the same value, the calculation in (14) is actually a summation of part of the columns of $\mathbf{U}$.

As can be seen, the proposed DOA-based beamforming approach utilizes the DOA information to form the weight vector without any calculation of matrices, while the approaches in [16]–[20] necessitate calculations of matrices. Therefore, the proposed approach is of much lower computational complexity than the approaches in [16]–[20] in massive MIMO systems. In the next section, a detailed analysis of the computational complexity and the performance of the proposed approach in the large system limit will be presented.

IV. ANALYSIS IN THE LARGE SYSTEM LIMIT

In this section, the lower bound of the sum rate will be analyzed to demonstrate the performance of the proposed approach. Additionally, the computational complexity of the proposed approach will be compared with that of the approaches in [16]–[21]. Note that all these analyses are performed in the large system limit, i.e., with infinite number of the BS antennas.

A. Asymptotic Lower Bound of the Sum Rate

The lower bound of the sum rate can be utilized to evaluate the performance of the proposed approach. Moreover, the asymptotic lower bound in the large system limit indicates the performance of the proposed approach in the massive MIMO system. The following theorem shows the analysis result of the asymptotic lower bound.

Theorem 1: When the ranges of the azimuth and the elevation DOAs from each UT to the serving BS are not completely overlapped by the the ranges of the azimuth and the elevation DOAs from other UTs to this BS, the lower bound of the sum rate tends to infinity as the number of the BS antennas tends to infinity.

Proof: Refer to Appendix C for details.

Remark 3: The UTs are usually separated from each other, which means there exists a unique range of the DOAs of each UT that is not in the range of the DOAs of other UTs. Hence, the sum rate is not constrained by the interference with the proposed approach in massive MIMO systems.

B. Computational Complexity

In this analysis, the big O notation, $O(n)$, is used to denote the computational complexity that is linear in $n \in \mathbb{R}^+$ [30]. Meanwhile, the computational complexity analyzed here corresponds to the calculation of the beamforming weight vector for one UT.

For the approach in [16], [17], firstly, the covariance matrices of the channel vectors are calculated, cf. (24), which is of $O(M^2)$. Then, the generalized eigenvector of the signal covariance matrix and the interference covariance matrix is calculated, which corresponds to $O(M^3)$. Hence, the asymptotic computational complexity of the approach in [16], [17] is $O(M^3)$ in the limit of the number of the BS antennas.

Similarly, the approach in [18]–[20] calculates the covariance matrices of the interference channels first and necessitates $O(M^2)$. Then, the eigenvectors that correspond to the largest τ eigenvalues are selected to calculate the null space of the interference, this results into $O(M^3)$. Finally, the array steering vector is projected onto the null space, which is of $O(M)$. Therefore, the approach in [18]–[20] is of $O(M^3)$ in the large system limit.

In contrast to these approaches in [16]–[20], the approach in [21] form the weight vector with the array steering vector, which is of $O(M)$. For the proposed approach, the steps in Algorithm 1 are of $O(M)$. Note that the forth step of Algorithm 1 is of $O(M)$ rather than $O(M^2)$ because of the simplification presented in Remark 2.

Obviously, the proposed approach and the approach in [21] are of much lower computational complexity than the approaches in [16]–[20] in the limit of infinite number of BS antennas. Therefore, the proposed approach and the approach in [21] are more feasible for massive MIMO systems than the approaches in [16]–[20] considering the burden of computations.

V. SIMULATION RESULTS

In this section, numerical results are provided to compare the performance of the proposed approach and that of the existing approaches in [18]–[21], and a zero-forcing (ZF) approach which is a modification of the approach in [21]. The system parameters are listed in Table 1. These parameters are fixed except the case they are variables in the simulations. Additionally, other system configurations are given as follows.

1) The angular spreads out of the cell $\sigma_{\theta_{j,l_k}}, \sigma_{\phi_{j,l_k}}, l \neq j$, are zeros.
2) The distance from the UT to the BS ranges from 30 m to 240 m.
3) The distance between two adjacent URA antennas is 0.5λ m.
4) The numbers of the URA antennas in the azimuth direction and the elevation direction are equal.
5) The UTs are distributed uniformly in the cell.
6) The azimuth DOAs and the elevation DOAs are Gaussian distributed.
7) Each BS employs power control to compensate the large scale fading of the UTs in the coverage area of that BS.

Table 1. System parameters.

Cell radius (from center to edge)	300 m
Height of the BS	30 m
Decay exponent	3.8
Shadow fading standard deviation	8 dB
Number of the UTs per-cell K	20
Number of the URA antennas M	100
Number of multipaths N	50
Received SNR $10\log_{10}(\sigma^2_{\gamma_{j,l_k}} N)$	10 dB
Number of time slots T	10
Angular spreads in the cell $\sigma_{\theta_{j,j_k}}, \sigma_{\phi_{j,j_k}}$	π/9 radians

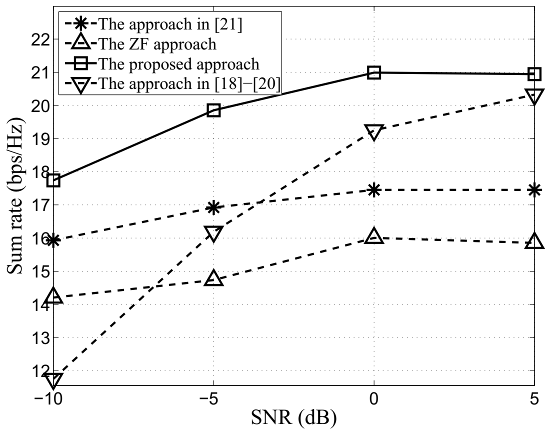

Fig. 2. Illustration of the sum rates versus the received SNR with the proposed beamforming approach, the beamforming approach in [21], the ZF approach, and the approach in [18]–[20].

Moreover, the regions of the DOAs are lessened in the simulations for reducing computational complexity, i.e., the limits of the integrals in (24) and (26) are $\bar{\theta}_{j,l_k} - \sigma_{\theta_{j,l_k}}$, $\bar{\theta}_{j,l_k} + \sigma_{\theta_{j,l_k}}$, $\bar{\phi}_{j,l_k} - \sigma_{\phi_{j,l_k}}$, and $\bar{\phi}_{j,l_k} + \sigma_{\phi_{j,l_k}}$.

Fig. 2 shows the relation between the sum rates of the beamforming approaches and the received SNR. It can be seen that the sum rates increase with the increasing of the SNR and saturate at high SNR regions. This coincides with the performance of these beamforming approaches. Take the proposed approach as an example. When the SNR increases, the singular values of the channel covariance matrices, $d_{j,j_k,m}$, $d_{j,j_{k'},m}$, and $d_{j,l_{k'},m}$, increase. Thus, the SINR, $\bar{\xi}_{j,k}$, in (15) as well as the sum rate increases with the increasing of the SNR and reaches the maximum when the SNR is so high that the interference constrains the performance. Additionally, the sum rate of the proposed approach is higher than that of the approach in [21] and the ZF approach. The reason is the proposed approach utilizes the multipaths to enhance the signal when cancelling the interference, while the approach in [21] and the ZF approach only take the specular component as the desired signal and discards the multipaths. Meanwhile, the proposed approach performs better than the approach in [18]–[20] and the approach in [18]–[20] tends to perform similar to the proposed approach in the high SNR region. This is because the proposed approach can collect more signal power than the approach in [18]–[20] while the approach in [18]–[20] can mitigate more interference than the proposed approach.

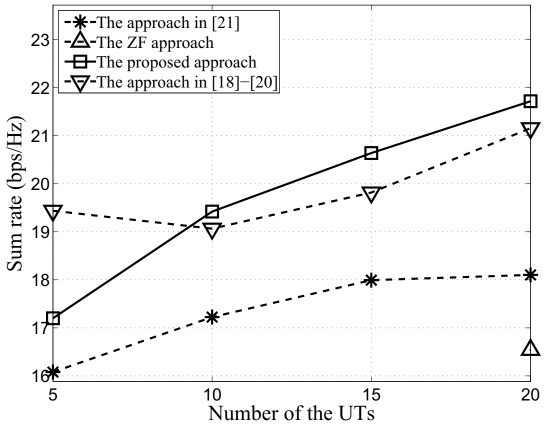

Fig. 3. Illustration of the sum rates versus the number of the UTs in each cell with the proposed beamforming approach, the beamforming approach in [21], the ZF approach, and the approach in [18]–[20].

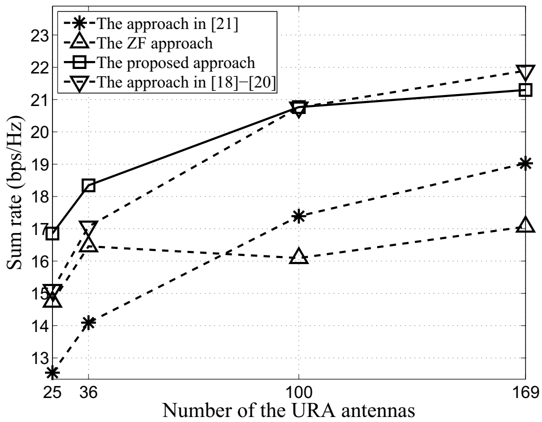

Fig. 4. Illustration of the sum rates versus the number of the URA antennas with the proposed beamforming approach, the beamforming approach in [21], the ZF approach, and the approach in [18]–[20].

In Fig. 3, the change of the sum rate with varying number of the UTs in each cell is illustrated. When the proposed approach is employed, it can be seen that the sum rate increases with the increase of the number of the UTs. However, the sum rate saturates with a relatively large number of the UTs when the approach in [21] is employed. When the number of the UTs increases, the power of the interference also increases. Since the multipaths are not collected by the approach in [21], the sum rate of each UT decreases fast with the increase of the number of the UTs. Because the multipaths are utilized to enhance the signal power in the proposed approach, the sum rate of each UT decreases slowly with the increase of the number of the UTs. Hence, this result also verifies that the proposed approach can achieve better performance than the approach in [21]. Meanwhile, the approach in [18]–[20] performs better than the proposed approach when there is a small number of the UTs but worse than the proposed approach when there is a large number of the UTs. This is because the approach in [18]–[20] can estimate the interference subspace more accurately when the number of the UTs is small. It can also be seen that the ZF approach is not applicable when the number of the UTs is small. For the ZF approach, the beamforming vector for the

kth UT in the jth cell is formed as $\mathbf{w}_{j,k} = [\mathbf{A}_j(\mathbf{A}_j^H\mathbf{A}_j)^{-1}]_k$, where $\mathbf{A}_j \in \mathbb{C}^{M\times K}$ is the matrix of array response vectors, i.e., $[\mathbf{A}_j]_k = \mathbf{a}(\bar{\theta}_{j,j_k}, \bar{\phi}_{j,j_k})$, cf. equation (3) and assumption (3). As the difference between the nominal DOAs, i.e., $\bar{\theta}_{j,j_k}, \bar{\phi}_{j,j_k}$, of two UTs decreases, the correlation between the array response vectors, i.e., $\mathbf{a}(\bar{\theta}_{j,j_k}, \bar{\phi}_{j,j_k})$, of the two UTs increases. Thus, it is of high probability that some of the singular values of $\mathbf{A}_j$ is close to zero when the number of UTs is small, which means that $\mathbf{A}_j^H\mathbf{A}_j$ is more likely to be ill-conditioned. Hence, the ZF approach is not applicable when there is not enough number of the UTs.

In Fig. 4, the number of the URA antennas is taken as the variable parameter to evaluate the sum rate performance. As can be seen, the sum rate of the proposed approach increases without bound when the number of the URA antennas increases. This result coincides with the asymptotic performance analysis of the proposed approach presented in Theorem 1, which shows that the sum rate tends to infinity as the number of the URA antennas tends to infinity in favorable conditions. Meanwhile, the proposed approach performs better than the approach in [21] and the ZF approach. However, the approach in [18]–[20] has better performance than the proposed approach when the number of the URA antennas is large. This is because the approach in [18]–[20] can estimate the interference subspace more accurately with more number of the URA antennas. This result verifies that the proposed approach can achieve better performance-complexity tradeoff than the approach in [18]–[20].

VI. CONCLUSION

In this paper, a beamforming approach utilizing only the DOA information is proposed to maximize the sum rate of multi-cell massive MIMO systems. The sum rate expression is derived first and its relation with the singular values of the channel covariance matrices is further investigated. Then, the asymptotic property of the singular values of the channel covariance matrices in the large system limit is utilized to design the beamforming weight vector. Furthermore, the orthogonality of the steering vectors of the URA is employed to simplify the calculation of the beamforming vector. The asymptotic analysis in the large system limit demonstrates that the proposed approach is able to obtain performance gains from the increased number of the BS antennas with lower computational complexity than the existing methods. Moreover, the proposed approach can achieve better performance than the known approach of comparable computational complexity and achieve similar performance as the other more complicated approach.

ACKNOWLEDGMENT

The author gratefully acknowledge the many helpful suggestions of the two anonymous reviewers and Editor Prof. Huaping Liu.

APPENDIX A
PROOF OF LEMMA 1

Substituting (2) into (5) results into

$$\mathbf{R}_{j,l_k} = \sigma_{\gamma_{j,l_k}} \int_{\theta_{j,l_k}^{\min}}^{\theta_{j,l_k}^{\max}} \int_{\phi_{j,l_k}^{\min}}^{\phi_{j,l_k}^{\max}} \mathbf{a}(\theta,\phi)\mathbf{a}^H(\theta,\phi) \times \rho_{j,j_k}(\theta,\phi)\mathrm{d}\theta\mathrm{d}\phi, \quad (24)$$

where $\rho_{j,l_k}(\theta,\phi)$ is the distribution function of the DOAs from the k'th UT in the lth cell to the jth BS; $\theta_{j,l_k,\min} \leq \theta_{j,l_k,n}(t) \leq \theta_{j,l_k,\max}$ and $\phi_{j,l_k,\min} \leq \phi_{j,l_k,n}(t) \leq \phi_{j,l_k,\max}$ are the ranges of the DOAs.

When the range of the azimuth or the elevation DOAs from the kth UT in the lth cell to the jth BS is shorter than the overall range of the DOAs in the corresponding direction, i.e., $[0,\pi)$ for the azimuth direction and $[-\pi/2,\pi/2)$ for the elevation direction, there are $\tilde{\theta} \in [0,\pi)$, $\tilde{\phi} \in [-\pi/2,\pi/2)$ that satisfy at least one of the following four inequalities, which are $\tilde{\theta} < \theta_{j,l_k,\min}$, $\tilde{\theta} > \theta_{j,l_k,\max}$, $\tilde{\phi} < \phi_{j,l_k,\min}$, and $\tilde{\phi} > \phi_{j,l_k,\max}$. Then, there is

$$\mathbf{a}^H(\tilde{\theta},\tilde{\phi})\mathbf{R}_{j,l_k}\mathbf{a}(\tilde{\theta},\tilde{\phi})/M \to 0, \text{as } M_{\mathrm{x}} \to \infty, M_{\mathrm{y}} \to \infty, \quad (25)$$

which is based on $\left|\mathbf{a}^H(\tilde{\theta},\tilde{\phi})\mathbf{a}(\theta,\phi)\right|/M \to 0$. As a result, the channel covariance matrix, $\mathbf{R}_{j,l_k}$, tends to be rank-deficient as $M_{\mathrm{x}} \to \infty, M_{\mathrm{y}} \to \infty$. In other words, part of the singular values in $d_{j,l_k,m}, m = 1,2,\cdots,M$, tend to be zero as $M_{\mathrm{x}} \to \infty, M_{\mathrm{y}} \to \infty$.

APPENDIX B
SIMPLIFIED CALCULATION OF THE DIAGONAL ELEMENTS

Substituting (24) into (23) yields

$$[\mathbf{D}_{j,l_k}]_{n_{\mathrm{a}},n_{\mathrm{a}}} = \frac{\sigma_{\gamma_{j,l_k}}}{M} \times \int_{\theta_{j,l_k}^{\min}}^{\theta_{j,l_k}^{\max}} \int_{\phi_{j,l_k}^{\min}}^{\phi_{j,l_k}^{\max}} \left|\mathbf{b}^H(f_{\mathrm{x}}(n_{\mathrm{x}}), f_{\mathrm{y}}(n_{\mathrm{y}}))\mathbf{a}(\theta,\phi)\right|^2 \mathrm{d}\theta\mathrm{d}\phi.$$

According to the expressions of $\mathbf{a}(\theta,\phi)$ and $\mathbf{b}(f_{\mathrm{x}}(n_{\mathrm{x}}), f_{\mathrm{y}}(n_{\mathrm{y}}))$ in (3) and (21), respectively, the above equation can be re-expressed as

$$[\mathbf{D}_{j,l_k}]_{n_{\mathrm{a}},n_{\mathrm{a}}} = \sigma_{\gamma_{j,l_k}} M \int_{\theta_{j,l_k}^{\min}}^{\theta_{j,l_k}^{\max}} \int_{\phi_{j,l_k}^{\min}}^{\phi_{j,l_k}^{\max}} \left|\frac{\mathrm{sinc}(\delta_{\mathrm{x}} M_{\mathrm{x}})}{\mathrm{sinc}(\delta_{\mathrm{x}})}\right|^2 \times \left|\frac{\mathrm{sinc}(\delta_{\mathrm{y}} M_{\mathrm{y}})}{\mathrm{sinc}(\delta_{\mathrm{y}})}\right|^2 \mathrm{d}\theta\mathrm{d}\phi, \quad (26)$$

where $\mathrm{sinc}(x) = \sin(x)/x$, $\delta_{\mathrm{x}} = \pi(\sin(\phi)\cos(\theta) - f_{\mathrm{x}}(n_{\mathrm{x}}))/2$, and $\delta_{\mathrm{y}} = \pi(\sin(\phi)\sin(\theta) - f_{\mathrm{y}}(n_{\mathrm{y}}))/2$. As a result, the calculation of matrices in (23) is simplified into an integral.

APPENDIX C
PROOF OF THEOREM 1

Because the result of Theorem 1 is the asymptotic performance in the large system limit, the derivations below are based on the condition that $M_{\rm x} \to \infty, M_{\rm y} \to \infty$. When the ranges of the azimuth and the elevation DOAs from the kth UT in the jth cell to the jth BS are not completely overlapped by the the ranges of the azimuth and the elevation DOAs from other UTs to this BS, these exist an azimuth angle, $\theta_{j,k}$, and an elevation angle, $\phi_{j,k}$, that are only in the range of the DOAs of the kth UT in the jth cell. Correspondingly, there exist $n_{\rm x} \in \{1, 2, \cdots, M_{\rm x}\}$ and $n_{\rm y} \in \{1, 2, \cdots, M_{\rm y}\}$ that satisfy $f_{\rm x}(n_{\rm x}) = \sin(\phi_{j,k})\cos(\theta_{j,k})$ and $f_{\rm y}(n_{\rm y}) = \sin(\phi_{j,k})\sin(\theta_{j,k})$. Moreover, these two equations hold only for these two angles. According to (26), there is

$$[\mathbf{D}_{j,l_{k'}}]_{n_{\rm a},n_{\rm a}} \to \sigma_{\gamma_{j,l_{k'}}}\delta(j-l)\delta(k-k'),$$

which means $\zeta_{j,k,n_{\rm a}} \to \infty$, where $\zeta_{j,k,n_{\rm a}}$ is defined in (20) and $d_{j,l_{k'},m}$ in (20) equals $[\mathbf{D}_{j,l_{k'}}]_{m,m}$. Consequently, $\bar{\xi}_{j,k}$ in (19) satisfies $\bar{\xi}_{j,k} \to \infty$. Then, it is easy to find that the lower bound of the sum rate, $\bar{R}$, tends to infinity according to the definition of $\bar{R}$ in (9).

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