The Economics of Green Consumption, Cultural Transmission and Sustainable Technological Change

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Abstract

A model which formalizes the interplay between green consumer culture and sustainable technology is used to revisit the trade-off between economic growth and environmental preservation. The theory includes (i) green preferences formed through cultural transmission which involves rational socialization actions, (ii) innovation endogenously directed to sustainable or unsustainable sectors depending on culture through market size effects. The model captures an important feature of sustainable innovation processes which is the existence of path dependency. The approach allows to examine implications for both market-based instruments (i.e., environmental taxes) and non-monetary interventions (i.e., environmental education). The two types of policies are either complements or substitutes depending on the substitutability between clean and dirty goods. Finally, an important disregarded issue is examined: the political sustainability of environmental taxes.

Keywords: Green consumption; Cultural transmission; Directed technological change, Environmental taxes
JEL Codes: Z13, D11, 033, H41

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1 Introduction

The attainment of serious environmental objectives along with sustained economic development has become a major challenge of current policymakers. Economists have emphasized that the long-term environmental impact of economic activity is profoundly affected by the rate and direction of technological change (see Nordhaus 2002, Popp 2004, Acemoglu et al. 2012). Another presumably significant channel in the trade-off between economic growth and environmental preservation is the change in preferences. For instance, some studies suggest that a shift in dietary preferences could allow to achieve significant greenhouses gases emissions reductions.[1] Survey research on national samples has provided evidence of an aggregate pro-environmental attitudes change since the early 70's (see Kanagy, Humphrey, and Firebaugh 1994, Inglehart 2008, Capstick et al. 2015). Moreover, recent experiments with environmental policies suggest that environmental preferences change in response to both economic incentives and non-monetary instruments (see D'Haultfœuille, Durrmeyer, and Février 2016, Thaler and Sustein 2008, Ferraro and Price 2013).

There is an increasing need to investigate how the response of preferences to economic changes and policies alter predictions about the cost of environmental preservation as well as to examine the potential for non-monetary instruments in reducing the environmental impact of economic growth. This paper contributes to this research agenda by developing a theory which formalizes dynamic interactions between innovation in sustainable technologies, changes in "green consumer culture" and pollution accumulation.

Empirical evidence indicates that sustainable innovation is sensitive to changing consumers' attitudes. For instance a European poll reveals that 88 % of firms surveyed mention increasing market demand for green products as an important driver of innovation (Flash Eurobarometer 315 2011). Furthermore, Popp, Hafner, and Johnstone (2011) use patents data to examine the determinants of the dramatic rise in chlorine free paper technologies which occurred during the 1990s. Their study indicates that innovation was mostly due to changes in consumers concern over chlorine in paper.

In addition, there is evidence that preferences respond to technological change (Alesina, Giuliano, and Nunn 2013; Talhelm et al. 2014; Galor and Özak 2016). For instance, Galor and Özak exploit a natural experiment associated with the expansion of suitable crops for cultivation during the Columbian Exchange. They find that pre-industrial agro-climatic characteristics associated with higher crop yield, have had a persistent effect on the distribution of time preference across societies. Other works more precisely suggest that changes in production technology and prices significantly alter preferences for green consumption. Teisl, Roe, and Hicks (2002) provide market-based evidence that the US dolphin-safe label has shifted consumers valuation of canned tuna. Moreover, D'Haultfœuille, Durrmeyer, and Février (2016) examine the impact of the French bonus-malus on consumers valuation of vehicle CO2 emissions.[2] Their results show that the policy triggered a substantial change in preferences towards low-emitting cars (it

[1] Tilman and Clark (2014) estimate that ruminant meats generate GHG emissions per gram of protein that are about 250 times those of legumes. Stehfest et al. (2009) find that a shift in dietary composition which would result in low-meat consumption standards (as recommended for health reasons) could reduce the mitigation costs to achieve a 450 ppm CO2 equivalent stabilization target by about 50 % in 2050.

[2] The French bonus malus consists of a feebate that provides a financial reward for low-CO2-emitting vehicles.

accounts for 40 % of the overall decrease in average CO_2 emissions of new cars in the period considered).

The interplay between green culture and sustainable technologies is also consistent with cross-country analysis revealing a positive correlation between green consumers' attitudes and clean technologies. Figure 1 displays a scatterplot illustrating the cross-country relationship between the share of people who embed an environmental dimension in their consumption decisions and the development of organic farming (on the left) and another scatterplot revealing the link between the fraction of people who agree with the idea of buying goods for environmental reasons and the share of EU firms who had introduced one eco-innovative product or service during the last two years (on the right). Both graphs highlight a positive correlation between green consumers attitudes and sustainable technologies. The correlation is significant and robust to the inclusion of several control variables (see Appendix A).

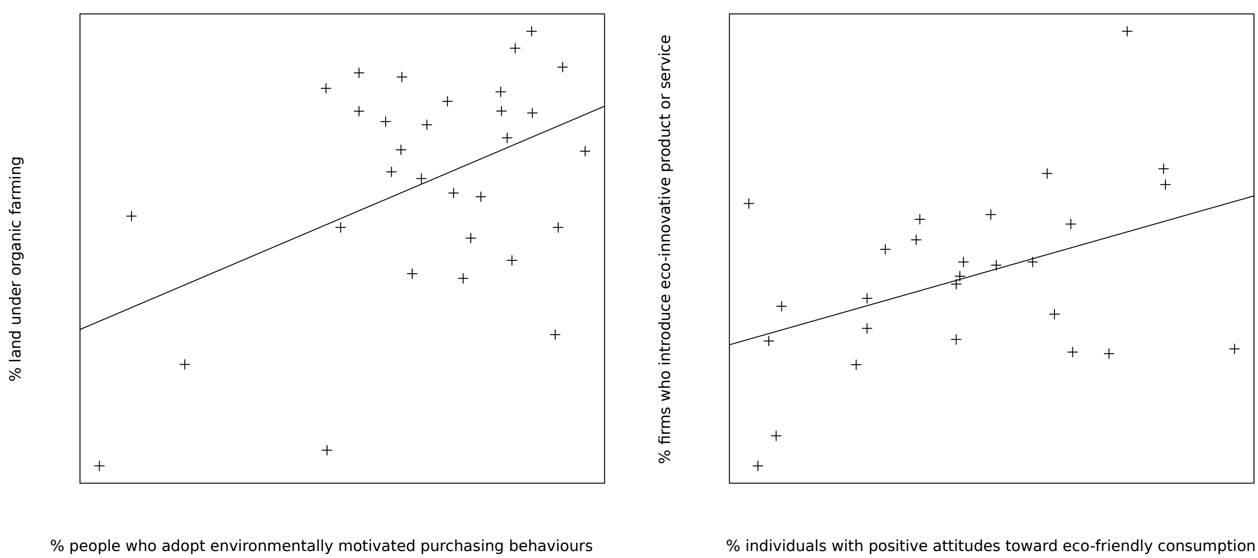

Figure 1: Cross-country correlation between the percentage of land under organic farming and the fraction of respondents to the ISSP survey who make consumption choices for environmental reasons (R-squared=0,19). Source: Research Institute of Organic Agriculture (FiBL) (2015) and ISSP Research Group (2010) (left). Correlation between the share of firms who have introduced an eco-innovative product or service during the last two years and the fraction of respondents to the Eurobarometer who agree with the statement "You are ready to buy environmentally friendly products even if they cost a little bit more." (R-squared=0,19). Source: Flash Eurobarometer 315 (2011) and European Commission (2011) (right).

This paper proposes a theory which formalizes the interplay between the formation of green consumer preferences, the direction of technological change and the accumulation of pollution. The theoretical set up builds on the three following blocks. First, some agents attach higher value to the consumption of non-polluting goods (i.e., goods which are less detrimental to environmental quality). This assumption is supported by various empirical evidence. For instance, a European survey reveals that 8 over 10 EU citizens felt that a product's impact on the environment is a critical element when deciding which product to buy (Flash Eurobarometer 256 2009). Studies on actual behaviors using market data confirm the existence of a willingness to pay for "cleaner" products (see Teisl *et al.* 2002, for the willingness to pay for dolphin-safe labeled canned tuna, Bjørner, Hansen, and Russell 2004 for the willingness to pay for the Swan-labeled paper). In particular, Bjørner et al. find that the willingness to pay for the Swan-labeled paper of Danish consumers ranges between 13% and 18% of the price.

Second, consumers' preferences are transmitted intergenerationally through role modeling and family socialization actions following the lines of Bisin and Verdier (2001). Works on environmentally friendly attitudes suggest that these attitudes are acquired during childhood (see Inglehart 1995; Inglehart and Baker 2000) and that family and peers matter in the socialization process (see Chawla 1998; Villacorta, Koestner, and Lekes 2003; Litina, Moriconi, and Zanaj 2016). In particular, Litina et al. base on the epidemiological approach proposed by Fernandez (2007) using variation associated with international migration flows. They find that the environmental preferences of migrants within Europe are significantly affected by environmental preferences in the origin country suggesting a transmission within family. Also, some sociological studies have revealed the existence of socialization actions (see Cairns, Johnston, and MacKendrick 2013, for socialization to organic food consumption practices, Maurer 2010; Boyle 2011 for socialization to vegetarianism).[3]

Finally, I assume that the direction of technological change, which depends on decisions by profit motivated agents, is influenced by green consumer culture through market size effects. This assumption is supported by evidence already detailed above such as the study by Popp et al. (2011).

The key insight is that innovation in clean sectors and green cultural change are co-determined. A first important result is that the dynamics generally exhibits path dependency. In this case, the economy can converge to two distinct long-term outcomes: a *green* equilibrium where both the fraction of green consumers and the relative productivity in the clean sector are high and a *brown* equilibrium where the fraction of green consumers is low and dirty technologies prevail. Path dependency in eco-innovation is in line with empirical evidence (see Newell, Jaffe, and Stavins 1999, Popp 2002, Aghion et al. 2016). Compared to the theoretical literature (e.g., Acemoglu et al. 2012), the present result holds true (i) whatever the value of the elasticity of substitution between clean and dirty goods and in particular when this elasticity is low, (ii) when there is decreasing returns at the firm level.[4] Hence, the theory proposed in this paper provides an alternative and complementary explanation for path dependency in eco-innovation.

The model involves new forces to complementarity in the dynamics of innovation. These forces arise from the cross effects which capture interactions between green culture and environmentally friendly technology. The first complementarity force is the *market size effect of cultural change*. The higher the fraction of green agents, the higher aggregate consumption of clean goods and the higher the incentives to innovate in the clean sector. The second force is the

[3] As an example, some respondent to the survey conducted by Cairns et al. clearly states : "I want my daughter to learn where things come from... I think it's the education, like setting examples for your kids". Parents also reported to consciously bring children to farmers markets so that they could develop a sense of connection to their food source.

[4] Both the high elasticity assumption and the existence of increasing returns at the firm level have been extensively discussed due to contradicting evidence (see Popp 2002, Pottier, Hourcade, and Espagne 2014).

impact of eco-innovation on green parenting. When productivity in the clean sector rises, clean goods become cheaper which makes green children better-off and increases incentives for altruistic parents to educate their child to green preferences. The strength of these cross effects reduces with the cost of green parenting and increases with the willingness to pay for clean goods, the elasticity of production to technological change and knowledge spillovers. Whenever both complementarity forces are sufficiently high, the dynamics exhibits multiple history-dependent equilibria.

In this set-up, some public intervention is welfare-improving since at the *laissez-faire*, the current generation does not internalize the negative impact of its own consumption choices on future generations through the reduction in environmental quality.[5] I examine different types of public intervention and study how classical economic instruments (here pollution taxes) interact with alternative non-monetary policies (i.e., environmental education which is comprehended as a decrease in the cost of green parenting). I show that pollution taxes and the non-monetary policy are either substitutes or complements depending on the elasticity of substitution between clean and dirty goods. Pollution taxes correct the market failure by giving a price to the negative production externality while environmental education promotes the green culture consumption. When the substitutability between clean and dirty goods is high, the cultural change due to environmental education entails a substantial reduction in dirty goods consumption. The resulting decrease in the size of dirty goods market implies a drop in profits in the dirty sector which redirects all innovation toward the clean sector. Hence, environmental education allows to stop the rise of dirty production. In such a case, both environmental education and pollution taxes allow to correct the market failure due to dirty goods production growth: they are *substitutes*. When the substitutability between clean and dirty goods is low, whatever the cultural change, the resulting decrease in consumption of dirty goods is limited. Profits in the dirty sector remain sufficiently high to encourage some innovation in the dirty sector. Dirty goods production grows and so does pollution. Only pollution taxes allow to regulate the growth of emissions. However, environmental education plays a key role. Because the substitutability between clean and dirty goods is low, the tax has a limited impact on the market of dirty goods. As a result, this tax must be initially high and continuously increasing over time (this result is in line with the literature, e.g., Acemoglu et al. 2012, Popp 2004). Environmental education causes a qualitative change in the dynamics and promotes the rise of the green culture consumption. This major cultural change counteracts the market rigidity to economic policies. As long as the share of green consumers rises, dirty goods consumption decreases so that pollution emissions are reduced and the tax can be decreasing. Hence, environmental education modifies the shape of the efficient tax rate: it implies a substantial reduction in the cost of the market-based instrument. Pollution taxes and environmental education are *complements*.

Finally, as a serious impediment to market-based instruments is their political acceptability, I study the political sustainability of environmental taxes. The question I ask is as follows: in which conditions will market-based instruments be politically feasible in the long-run? An important outcome is that non-monetary policies may be critical for the political sustainability of economic instruments. When the tax is implemented in the short-run, it positively impacts incentives to educate children to green preferences and incentives to innovate in the clean sector.

[5] Note that standard welfare assessments do not apply since preferences are endogenous. A criterion based on Pareto-type considerations is introduced in Section 4.

Whether the tax is sufficient to change the long-run equilibrium depends on the cost of green parenting which is affected by non-monetary policies such as environmental education. When the cost of green parenting is high, the impact of the tax on green parenting is weak and so is the market size effect of cultural change. In this case, the tax is not sufficient to change the long-run equilibrium of the economy which converges to the brown equilibrium where the fraction of green agents is too low to support the economic policy. Environmental education, which reduces the cost of green parenting, strengthens the positive impact of the tax on green culture which in turn increases the market size effect of cultural change. Whenever the tax is implemented in the short-run, then it is sufficient to shift the long-run outcome. The fraction of green agents converges to the green equilibrium in which the tax is supported by the majority.

Related literature This paper relates to four lines of work.

First, several theoretical papers investigate the interplay between specific cultural traits and some technology and show how the mechanism at play gives rise to path dependency and persistence (Bénabou, Ticchi, and Vindigni 2016; Galor and Özak 2016; Maystre et al. 2014). For instance, Bénabou et al. analyze the co-evolution of religious beliefs and innovation in a framework where religious beliefs affect innovation through the political economy channel (i.e., a government which controls technological change) and where beliefs are eroded by innovation but can also be influenced by a religious institution. They show that the tension between scientific knowledge and religion gives rise to three distinct long-term outcomes regarding religiosity and scientific progress.[6] A more closely related paper is that of Maystre et al. (2014) who build a framework interacting the cultural transmission of preferences for some differentiated consumption goods (in particular they focus on cultural goods such as movies and books) and the supply of these goods to analyze the impact of trade integration. Apart from the fact that they do not consider any negative environmental externality and implications for pollution accumulation, their framework differs from the one proposed in the present paper in several important ways. First, in their paper, any change on the production side is captured by increasing product variety. This modeling feature makes this framework unsuitable to discuss the role of the elasticity of substitution between two consumption goods (here clean and dirty ones) for economic outcomes. It also prevents the study of the impact of a tax on final good consumption such as the tax on polluting goods. Another important distinction is that, in their model, technology is not a state variable implying that there is no economic growth which is the focus of the present paper.

Second, the paper also contributes to the literature on growth and the environment with directed technological change (Smulders and De Nooij 2003; Acemoglu et al. 2012). A major difference with the models proposed so far is that preferences are endogenous. This is a crucial feature as (i) I show that it can be the source of path dependency in sustainable innovation processes, (ii) it provides novel implications for public policies.

A third theoretical literature has accounted for endogenous environmental preferences when studying long-run environmental issues (Bezin 2015, Schumacher 2015). These works do not

[6]More precisely, the economy can converge to a "Western-European" secularization regime (with declining religiosity and scientific progress), a "theocratic" regime (with knowledge stagnation and persistently high religiosity) and an "American" regime combining scientific progress and stable religiosity.

include endogenous technological progress. They neither study implications for sustainable innovation processes nor examine the trade-off between long-run economic growth and environmental preservation.

Fourth, the paper relates to a smaller literature analyzing the political sustainability of public good provision policy. Cremer, De Donder, and Gahvari (2004) emphasize the critical role of the refunding rule (i.e., the allocation of tax proceeds) for political support toward tax on polluting goods. Second, Bisin and Verdier (2000) examine the sustainability of a public good provision policy when preferences for that good are transmitted through socialization actions and role modeling. They highlight the importance of the initial distribution of preferences for the political sustainability of the policy. In their paper, this result is due to self-fulfilling expectations. In the present framework, the result lies on the existence of dynamic complementarities between culture and technology.

The rest of the paper unfolds as follows. Section 2 presents the model. Section 3 is devoted to the analysis of the dynamics and to the characterization of steady state equilibria. Section 4 examines efficient and sustainable environmental policies. In Section 5, I propose alternative modeling assumptions (one of which is developed in Online Appendix D). Section 6 focuses on the political sustainability of environmental taxes. Section 7 concludes. Appendix A provides more details about the cross-country correlation between attitudes and sustainable production methods. Appendix B contains the proofs of the key results, while Appendix C, which is available online, contains the remaining proofs.

2 The model

2.1 Preferences

2.1.1 The demand for clean and dirty goods

I consider an economy with an overlapping generations structure. Each cohort is a continuum of agents with measure normalized to one. Individuals live for two periods. During the first period, which is childhood, all members are identical and are subject to socialization. Once adult, agents differ according to their preferences which are defined over a *clean* good (associated to the index c) and a *dirty* good (associated to d) which negatively impacts the environment. The *brown* individuals, associated with the superscript B, are indifferent to the type of good they consume. The *green* individuals, associated with the superscript G, prefer consuming clean goods. At time t, utilities of agents G and B are respectively given by[7]

$$U^G(x_{ct}, x_{dt}) = \ln\left(\left(\theta^G x_{ct}^{\frac{\epsilon-1}{\epsilon}} + x_{dt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}\right), \tag{1}$$

$$U^B(x_{ct}, x_{dt}) = \ln\left(\left(x_{ct}^{\frac{\epsilon-1}{\epsilon}} + x_{dt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}\right), \tag{2}$$

[7] I choose log-utilities as it substantially simplify the analytics. These functional forms are not necessary for the results.

where x_{jt} is consumption of good $j \in \{c, d\}$ at time t, $\epsilon > 1$ is the elasticity of substitution between clean and dirty goods and $\theta^G > 1$ measures green individuals' taste for clean goods. Note that there exists a variety of approaches to describe green preferences (e.g., Brekke, Kverndokk, and Nyborg 2003, Rege 2004). The chosen specification amounts to assume some "warm glow of giving" as in Andreoni (1990). In Online Appendix D, I develop an alternative framework where green agents do not directly prefer clean goods (i.e., $\theta^G = 1$) but rather care about the environment. The results obtained under this alternative modeling assumption are summed up in Section 5.

At time t, the budget constraint is given by

$$I_t = p_{ct}x_{ct} + p_{dt}x_{dt}, \tag{3}$$

where I_t is individual income and p_{ct} (resp. p_{dt}) the price of the clean (resp. dirty) good. Denote by q_t the fraction of agents of type G at time t, aggregate demands for clean and dirty goods are respectively given by

$$\begin{aligned} X_t^c &= \tfrac{I_t}{p_{ct}} \left(q_t \tfrac{\theta^{G\epsilon} \frac{p_{ct}}{p_{dt}}^{1-\epsilon}}{1+\theta^{G\epsilon} \frac{p_{ct}}{p_{dt}}^{1-\epsilon}} + (1-q_t) \tfrac{\frac{p_{ct}}{p_{dt}}^{1-\epsilon}}{1+\frac{p_{ct}}{p_{dt}}^{1-\epsilon}} \right), \\ X_t^d &= \tfrac{I_t}{p_{dt}} \left(q_t \tfrac{1}{1+\theta^{G\epsilon} \frac{p_{ct}}{p_{dt}}^{1-\epsilon}} + (1-q_t) \tfrac{1}{1+\frac{p_{ct}}{p_{dt}}^{1-\epsilon}} \right). \end{aligned} \tag{4}$$

2.1.2 Dynamics of Preferences

The distribution of preferences (or cultural traits) is endogenous: it is transmitted intergenerationally through a socialization and imitation process based on Bisin and Verdier (2001). Each parent has one child who is socialized to cultural traits both within family (i.e., vertical socialization) and by models met randomly within the society (i.e., oblique socialization). Compared to Bisin and Verdier (2001), there are several modifications. First, since trait B is the absence of preferences toward either type of goods, brown preferences are transmitted by default. By contrast, green preferences are costly transmitted through purposeful socialization actions. The transmission process is as follows. Suppose that the child is born in a green family. The green parent may exert a costly effort to transmit green preferences to its child. The child becomes green if both parents' socialization effort succeeds (with a probability denoted by d_t at time t) and the role model met randomly by the child is green (with probability q_t). Symmetrically, the child becomes brown if both the green parent fails to transmit its trait and the model met by the child is brown. If messages sent by parents and role models are contradictory the child is matched a second time with some member of the society and adopts her trait. Suppose that the child is born in a brown family. If the child meets a model with the brown trait then he becomes brown. If he meets a model with the green trait, he is matched a second time with some member of the society and adopts her trait. Note that the parent is not sufficient to transmit some trait which is different than Bisin and Verdier's framework.[8] This modeling

[8] In the standard model of Bisin and Verdier, an assumption is that parents matter more than peers which would imply, in the present framework that the fraction of green agents always converges to 1. This is not realistic.

approach amounts to strengthen the role of peers, which, in the case of environmental preferences, has strong empirical background. A growing empirical literature is stressing the impact of non-pecuniary strategies such as social norm-based approaches for changing environmental behaviors (see e.g., Thaler and Sustein 2008, Ferraro and Price 2013, Costa and Kahn 2013). For instance, Ferraro and Price (2013) rely on a natural field experiment allowing to examine the effect on social comparison messages on residential water demand. They find a significant impact of such messages on households' behavior : the social comparison effect is equivalent to that which would be expected if average prices were to increase approximately 12 to 15 percent.

Denote by $P_t^{ii'}$, the probability that a child born in a type-i family at time t adopts trait i', $i, i' \in \{G, B\}$, the transition probabilities write as

$$P_t^{GG} = 1 - P_t^{GB} = d_t q_t + (1 - d_t){q_t}^2 + d_t(1 - q_t)q_t, \tag{5a}$$
$$P_t^{BB} = 1 - P_t^{BG} = 1 - q_t + q_t(1 - q_t). \tag{5b}$$

The dynamics of the fraction of green agents is captured by the following equation,

$$q_{t+1} = q_t - q_t P_t^{GB} + (1 - q_t)P_t^{BG},$$
$$q_{t+1} = q_t + q_t(1 - q_t)(2d_t q_t - 1).$$

Socialization choices of parents Denote by e_t, the green parent's socialization effort at time t and assume $d_t = d(e_t)$, $d : \mathbb{R}^+ \to [0, 1]$, $d(0) = 0$, $\lim_{e_t \to +\infty} d(e_t) = 1$, $d' > 0$, $d'' < 0$.[9] The effort involves a disutility cost $C(e_t) = \frac{\kappa}{\nu} e_t^{\nu}$, with $\nu \geq 3$.[10]

Following Bisin and Verdier (2001), the incentive to engage in costly socialization actions is due to "imperfect empathy". Parents (i) care about their child's welfare, (ii) are able to anticipate the choices optimally made by the child depending on his future preferences (green or brown). However, (iii) they evaluate those choices through the filter of their own preferences, that is with their own utility function.[11] Also, suppose that parents are myopic: they assess the future welfare of their child with current economic variables rather than future ones[12]. Formally, denote by V_t^{GG} (resp. V_t^{GB}), the gain for a parent of type G to have a child of type G (resp. B) at time t, we have

$$V_t^{GB} = U^G(x_{ct}^B, x_{dt}^B),$$
$$\text{where } (x_{ct}^B, x_{dt}^B) = \text{argmax}_{(x_{ct}, x_{dt})} U^B(x_{ct}, x_{dt}) \quad \text{subject to} \quad I_t = p_{ct}x_{ct} + p_{dt}x_{dt}.$$

[9] One can relax the assumption $d(0) = 0$ and rather set $d(0) = \bar{d} > 0$, meaning that green parents who do not engage in costly socialization actions to transmit their preferences are more likely than brown parents to have green children. As long as $\bar{d} < \frac{1}{2}$, none of the results are affected. The intuition is that, supposing that $\bar{d}$ is low means that the costless transmission channel is not efficient enough to trigger the surge of the green culture. Hence, the response of parents to economic incentives is still a key driver of the cultural dynamics.

[10] This assumption is adopted to simplify the study of the dynamics, see Appendix B.2.

[11] I could alternatively assume that parents are perfectly altruistic, it would multiply cases when studying the dynamics but would not affect the results.

[12] This assumption allows to avoid multiple equilibria generating by self-fulfilling expectations. For an analysis of this mechanism in the cultural transmission framework, see Bisin and Verdier (2000).

At period t, the parent chooses e_t to maximize

$$P_t^{GG} V_t^{GG} + P_t^{GB} V_t^{GB} - \frac{\kappa}{\nu} e_t^{\nu},$$

where P_t^{GG} and P_t^{GB} are given by equations (5a) and (5b). The optimal socialization effort $e_t^\star$ is implicitly given by

$$2d^{'}(e_t^\star) q_t (1-q_t)(V_t^{GG} - V_t^{GB}) - \kappa e_t^{\star \nu - 1} = 0, \tag{6}$$

with

$$V_t^{GG} - V_t^{GB} = \log \left(\frac{\left(1 + \theta^{G\epsilon} \frac{p_{ct}}{p_{dt}}^{1-\epsilon}\right)^{\frac{1}{\epsilon-1}} \left(1 + \frac{p_{ct}}{p_{dt}}^{1-\epsilon}\right)}{\left(1 + \theta^{G} \frac{p_{ct}}{p_{dt}}^{1-\epsilon}\right)^{\frac{\epsilon}{\epsilon-1}}} \right).$$

First, one finds

$$\frac{\partial e_t^\star}{\partial q_t} \geq 0 \quad \Leftrightarrow \quad q_t \leq \frac{1}{2}.$$

The optimal socialization effort is a non-monotonic function of the fraction of green agents at time t. When this fraction is low, the effort increases with q_t, i.e., there is *cultural complementarity*. When the fraction of green agents is high, the optimal effort decreases with q_t, i.e., there is *cultural substitutability*. When the fraction of green agents is sufficiently high, transmission of the green trait by the society is so effective that green parents can rely on oblique transmission to transmit their preferences to their child and thus have lower incentives to exert a costly socialization effort. When the fraction of green agents is low enough, transmission of the brown trait is very strong. In this case, whatever their socialization effort, green parents hardly transmit their trait and so have lower incentives to exert a costly effort.

The socialization effort is also a function of relative prices. This feature is specific to the present framework. In particular,

$$\frac{\partial e_t^\star}{\partial \frac{p_{ct}}{p_{dt}}} < 0,$$

that is, a rise in the relative price of the clean good p_{ct}/p_{dt}, negatively impacts *green parenting*, i.e., the effort to transmit green preferences. When the relative price of the clean good reduces, consumption of clean goods is higher. This rise benefits more to green children since they have stronger preferences for clean goods. Hence, incentives for parents to transmit the green trait increases. Finally, the distribution of preferences is given by

$$q_{t+1} = q_t + q_t(1-q_t)(2d(e_t^\star)q_t - 1), \tag{7}$$

where $e_t^\star$ is implicitly given by equation (6).

2.2 Technology

2.2.1 The supply of clean and dirty goods

The production structure is in line with Acemoglu et al. (2012). Two sectors in perfect competition produce either a clean good or a dirty good using labor and a continuum of sector-specific machines. In the clean and dirty sector respectively, the production functions are given by

$$Y_{ct} = \Omega(E_t)L_{ct}^{1-\alpha}\int_0^1 A_{ckt}^{1-\alpha} z_{ckt}^{\alpha} dk, \qquad Y_{dt} = \Omega(E_t)L_{dt}^{1-\alpha}\int_0^1 A_{dkt}^{1-\alpha} z_{dkt}^{\alpha} dk, \tag{8}$$

with $\alpha \in [0, 1]$, L_{jt} is the quantity of labor used in sector $j \in \{c, d\}$ at time t, z_{jkt} is the quantity of machine k used in sector j at time t and A_{jkt} the productivity of this machine. Define aggregate productivity in sector $j \in \{c, d\}$ at time t as $A_{jt} = \int_0^1 A_{jkt} dk$. Productivity in both sectors is affected by the stock of environmental quality E_t. I assume $\Omega^{'} > 0$, $\Omega^{''} \leq 0$ and $\Omega(0) = 0$.

The variable E_t can be interpreted as the quality of soil, the cleanliness of rivers or oceans, biodiversity, the gas composition of the atmosphere. All these environmental variables matter for productivity. Soil quality is a clearly decisive factor for crop productivity. In India, for instance, the intensive rice wheat system of the Punjab has experienced a significant yield decline due to a loss in soil quality (Nambiar et al. 1994). In addition, plenty of evidence suggests that climate change and air quality negatively impact productivity (see Olesen and Bindi 2002 for impacts on agriculture productivity, Wang and Feng 2015 for the productive sector at large). There also exists evidence which indicates that biodiversity loss is detrimental for productivity of fisheries (e.g., Worm et al. 2006). I suppose that if environmental quality goes to zero, then production becomes impossible. This assumption captures the notion of environmental disaster as introduced in Acemoglu et al. (2012).

I assume that production generates pollution. The dynamics of environmental quality is described by the following equation

$$E_{t+1} = \begin{cases} (1+b)E_t - \sigma Y_{dt}, & \text{if} \quad (1+b)E_t - \sigma Y_{dt} \in]0, \bar{E}], \\ \bar{E}, \quad \text{if} \quad (1+b)E_t - \sigma Y_{dt} > \bar{E}, \\ 0 \quad \text{if} \quad (1+b)E_t - \sigma Y_{dt} < 0. \end{cases} \tag{9}$$

where $(1+b)E_t$ measures the regeneration rate of environmental quality (i.e., the capacity of the environment to absorb pollution).[13] The term σY_{dt}, is pollution from dirty goods production, with σ the marginal environmental damage. One can think of Y_d as intensive agriculture (characterized by intensive land management, culture specialization, use of chemicals inputs such as fertilizers and persticides), while Y_c can be broadly interpreted as ecologically-based agricultural systems such as agroforestry or organic agriculture. In this case, one interpretation of E_t

[13] One can show that when $\Omega^{''} \leq 0$, then $dE_{t+1}/dE_t \geq 0 \; \forall E_t \in \mathbb{R}^+$. Indeed, we have $dE_{t+1}/dE_t = (1 + b) - \Omega^{'}(E_t)L_{dt}^{1-\alpha}\int_0^1 A_{dkt}^{1-\alpha} z_{dkt}^{\alpha} dk \equiv M_t$ if $E_{t+1} > 0$ and $dE_{t+1}/dE_t = 0$ otherwise. Note that $M_t < 0$ is equivalent to $(1 + b)E_t - E_t\Omega^{'}(E_t)L_{dt}^{1-\alpha}\int_0^1 A_{dkt}^{1-\alpha} z_{dkt}^{\alpha} dk < 0$ (for $E_t \neq 0$). The concavity of the function Ω implies $\Omega(E_t) > \Omega^{'}(E_t)E_t$ which, with $M_t < 0$, imply $(1 + b)E_t - \Omega(E_t)L_{dt}^{1-\alpha}\int_0^1 A_{dkt}^{1-\alpha} z_{dkt}^{\alpha} dk < 0$ that is $E_{t+1} < 0$. Hence $dE_{t+1}/dE_t \geq 0 \; \forall E_t \in \mathbb{R}^+$.

is the quality of soil. There is considerable evidence suggesting that agriculture intensification has negatively impacted soil quality by increasing erosion or lowering natural soil fertility (Matson et al. 1997). Reversely, organic agriculture for instance utilizes carbon-based amendments, diverse crop rotations, and cover crops which allow to preserve soil fertility (see Mäder et al. 2002). Another interpretation of E_t is the inverse stock of greenhouse gases in the atmosphere. The agricultural sector is responsible for about one quarter of global greenhouse gas emissions (IPCC 2014) and agriculture intensification has been identified as a major contributor notably through deforestation, the use of fertilizers and intensive livestock farming. If one thinks of E_t as the gas composition of the atmosphere, Y_d can also be more specifically interpreted as livestock production as the sector is responsible alone for 14% to 18% of all greenhouse gases emissions depending on whether land use changes are included (Steinfeld et al. 2006).

Finally, machines are supplied by firms in monopolistic competition. These firms use labor to produce machines: producing one unit of machine requires Ψ units of labor.

The supply of labor is inelastic. Labor market clearing requires

$$L_{ct} + L_{dt} + \Psi \int_0^1 z_{ckt} dk + \Psi \int_0^1 z_{dkt} dk \leq 1. \tag{10}$$

2.2.2 The innovation possibility frontier

I consider a continuum of scientists with mass normalize to 1, who have to decide whether to direct their research toward the clean or the dirty sector. At the beginning of period t, a scientist decides to undertake research in sector j. He is then randomly allocated to one machine, indexed by k, and is successful in innovation with some probability $\omega A_{jt-1}{}^{\frac{1+\delta}{2}} A_{j't-1}^{\frac{1-\delta}{2}} / A_{jkt-1}$, with $\omega \in [0,1]$ and $\delta \in [0,1]$. In such a case, he increases the productivity of machine k by a factor η and becomes the monopolist producer of this machine. The probability of success captures two important features. First, there exist positive aggregate knowledge spillovers (including cross-sector spillovers), i.e., aggregate productivity in both sectors, c and d, has a positive impact on the future productivity of the firm. The degree of cross-sector spillovers is inversely related to the parameter δ. Second, I assume decreasing returns to innovation at the firm level, i.e., a negative effect of the firm's own past research on the return to future innovation (i.e., the higher the past productivity of the firm, A_{jkt-1}, the lower the probability of being successful in innovation). If the scientist is not successful, monopoly rights are randomly allocated to entrepreneurs drawn from the pool of all potential entrepreneurs who then use the old technology.[14]

I denote by r_t, the share of scientists who work in sector c at time t, the dynamics of relative

[14]The specification of the innovation possibility frontier follows Acemoglu et al. (2012). It allows to simplify the exposition while reproducing the structure of equilibrium in continuous time models. The authors have shown that this specification leads to similar results than an alternative framework using patents and free entry.

(aggregate) productivity in sector c is given by

$$\frac{A_{ct+1}}{A_{dt+1}} = \frac{1+\eta\omega\; r_{t+1}\frac{A_{ct}}{A_{dt}}^{-\frac{1-\delta}{2}}}{1+\eta\omega\;(1-r_{t+1})\frac{A_{ct}}{A_{dt}}^{\frac{1-\delta}{2}}}\frac{A_{ct}}{A_{dt}} \tag{11}$$

2.3 Equilibrium

2.3.1 Temporary equilibrium

Definition 1 *For $j \in \{c,d\}$, $i \in \{G,B\}$, a temporary equilibrium consists in prices for final goods (p_{jt}), individual demands for final goods (x^i_{jt}), socialization effort ($e^\star_t$), prices for machines (p_{jkt}), demands for machines (z_{jkt}), labor demands (L_{jt}), research allocation $\{r_t\}$, distribution of preferences $\{q_t\}$ and level of environmental quality, E_t, such that at each t, (i) (x^G_{ct}, x^G_{dt}, $e^\star_t$) maximize utility of individuals of type G, (x^B_{ct}, x^B_{dt}) maximize utility of individuals of type B, (ii) (p_{jkt}, z_{jkt}) maximize the profit for the producer of machine k in sector j, (iii) L_{jt} maximizes the profit of the producer of the final good j, (iv) (p_{jt}, w_t) respectively clears the market for final good j and the labor market, (v) r_t maximizes the expected profit of scientists.*

Let the wage be the numeraire. Profit maximization by producer in final sector $j \in \{c,d\}$ delivers

$$1 = \Omega(E_t)\,(1-\alpha)\,p_{jt}L_{jt}^{-\alpha}\int_0^1 A_{jkt}^{1-\alpha}z_{jkt}^{\alpha}dk, \tag{12}$$

$$z_{jkt} = \left(\Omega(E_t)\frac{\alpha p_{jt}}{p_{jkt}}\right)^{\frac{1}{1-\alpha}} A_{jkt}L_{jt}. \tag{13}$$

Equalizing equation (12) for all j leads to

$$\frac{p_{ct}}{p_{dt}} = \frac{A_{ct}}{A_{dt}}^{-(1-\alpha)}. \tag{14}$$

The producer of machine k in sector j maximizes its profit given by $\pi_{jkt} = (p_{jkt}-\Psi)z_{jkt}$. Given the iso-elastic demand curve for machines, the price of machine k in sector j is set to $p_{jkt} = \Psi/\alpha$. Without loss of generality, I assume $\Psi = \alpha^2$, the equilibrium demand for machine k in sector j is given by[15]

$$z_{jkt} = \Omega(E_t)^{\frac{1}{(1-\alpha)}}p_{jt}{}^{\frac{1}{(1-\alpha)}}A_{jkt}L_{jt}.$$

Hence, the equilibrium profit of the producer of machine k in sector j is

$$\pi_{jkt} = \alpha(1-\alpha)\Omega(E_t)^{\frac{1}{(1-\alpha)}}p_{jt}{}^{\frac{1}{(1-\alpha)}}A_{jkt}L_{jt},$$

[15] Note that this assumption has no impact on the dynamics. As long as Ψ is the same in sectors c and d, the constant Ψ/α^2 does not play any role as it simplifies in relative terms and all that matters are relative profits rather than absolute ones. If one relaxes the symmetry assumption (i.e., having $\Psi_c \neq \Psi_d$), the results are qualitatively unchanged. In this case, there are two additional parameters, Ψ_c and Ψ_d, which matter for the dynamics.

The expected profit of a scientist who engages in sector j at time t is given by

$$\Pi_{jt} = \omega \frac{A_{jt-1}}{A_{j't-1}}^{-\frac{(1-\delta)}{2}} \alpha(1-\alpha)(1+\eta)\Omega(E_t)^{\frac{1}{(1-\alpha)}} p_{jt}^{\frac{1}{1-\alpha}} L_{jt} A_{jt-1}.$$

Note that whenever $\delta < 1$, through the term $(A_{jt-1}/A_{j't-1})^{-\frac{(1-\delta)}{2}}$, an increase in the relative productivity in sector j negatively impacts the expected profit in sector j. This feature is due to decreasing returns, introduced through the probability of success for scientists, and measured by the parameter δ. When $\delta = 0$, decreasing returns in sector j are maximal while when $\delta = 1$, there is no decreasing returns.[16] Using final good j production function and the equilibrium demand for machines one obtains

$$L_{jt} = \frac{Y_{jt}}{A_{jt} p_{jt}^{\frac{\alpha}{(1-\alpha)}} \Omega(E_t)^{\frac{1}{(1-\alpha)}}},$$

which, after equalizing the demand and supply for final goods in both sectors leads to

$$L_{ct} = \frac{X_{ct}}{A_{ct} p_{ct}^{\frac{\alpha}{(1-\alpha)}} \Omega(E_t)^{\frac{1}{(1-\alpha)}}},$$

$$L_{dt} = \frac{X_{dt}}{A_{dt} p_{dt}^{\frac{\alpha}{(1-\alpha)}} \Omega(E_t)^{\frac{1}{(1-\alpha)}}},$$

where X_{ct} and X_{dt} are given as in equation (4). Replace L_{ct} and L_{dt} by their expression in the expected profit of scientists. At equilibrium, the ratio of expected profits for scientists writes as

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-(1-\delta)+\gamma} f(q_t, \frac{A_{ct-1}}{A_{dt-1}}, r_t) \left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}})\right)^{\gamma-1}, \tag{15}$$

with $\gamma = (\epsilon - 1)(1-\alpha)$,

$$g(r_t, \frac{A_{ct-1}}{A_{dt-1}}) = \frac{1 + \eta\omega\ r_t \frac{A_{ct-1}}{A_{dt-1}}^{-\frac{1-\delta}{2}}}{1 + \eta\omega\ (1-r_t) \frac{A_{ct-1}}{A_{dt-1}}^{\frac{1-\delta}{2}}}$$

$$f(q_t, \frac{A_{ct-1}}{A_{dt-1}}, r_t) = \frac{q_t(\theta_G^\epsilon - 1) + 1 + \theta_G^\epsilon \left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}}) \frac{A_{ct-1}}{A_{dt-1}}\right)^\gamma}{-q_t \left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}}) \frac{A_{ct-1}}{A_{dt-1}}\right)^\gamma (\theta_G^\epsilon - 1) + 1 + \theta_G^\epsilon \left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}}) \frac{A_{ct-1}}{A_{dt-1}}\right)^\gamma}.$$

The higher the ratio Π_{ct}/Π_{dt}, the more profitable it is to undertake research in the clean sector. At equilibrium, incentives to conduct R&D in the clean sector at date t depends both on the past ratio of productivities A_{ct-1}/A_{dt-1} and the fraction of green agents q_t.

[16]While Acemoglu et al. (2012) assume increasing returns in R&D activities, some empirical studies find evidence for decreasing returns (e.g., Popp 2002). I purposefully assume decreasing returns in order to show that in the present framework, path dependency does not anymore requires increasing returns in R&D activities.

3 Dynamics at equilibrium

Lemma 1 *There exists a threshold value of ϵ, denoted $\tilde{\epsilon}$, such that:*

(i) when $\epsilon < \tilde{\epsilon}$, there exists an increasing map $X : [0,1] \to \mathbb{R}^+$, such that

$$\begin{aligned} &\frac{A_{ct+1}}{A_{dt+1}} \geq \frac{A_{ct}}{A_{dt}}, \\ \Leftrightarrow &\frac{A_{ct}}{A_{dt}} \leq X(q_t), \end{aligned}$$

(ii) when $\epsilon > \tilde{\epsilon}$, there exists a decreasing map $X : [0,1] \to \mathbb{R}^+$, such that[17]

$$\begin{aligned} &\frac{A_{ct+1}}{A_{dt+1}} \geq \frac{A_{ct}}{A_{dt}}, \\ \Leftrightarrow &\frac{A_{ct}}{A_{dt}} \geq X(q_t). \end{aligned}$$

Proof. See Appendix B.1 to B.3. ■

First, Lemma 1 reveals that the dynamics of relative productivity critically depends on the elasticity of substitution between clean and dirty goods. When the elasticity of substitution between clean and dirty goods is low (resp. high), relative productivity in the clean sector increases whenever past productivity in this sector is low (resp. high). Actually, a change in relative productivity in the clean sector has two opposite effects. Due to decreasing returns in R&D, a rise in relative productivity in the clean sector decreases incentives to undertake research in this sector which negatively affects relative productivity in this sector. However, an increase in past productivity in the clean sector also positively impacts future productivity in this sector through a market size effect. More precisely, the rise in A_{ct}/A_{dt} implies a reduction in p_{ct}/p_{dt} which in turn increases consumption of clean goods. Depending on the value of the elasticity of substitution between clean and dirty goods, either the decreasing returns effect (ϵ low) or the market size effect (ϵ high) prevails.

The dynamics of aggregate productivities is also shaped by changes in green consumer culture. Lemma 1 reveals that whatever the value of the elasticity of substitution, the relative productivity in the clean sector increases when the fraction of green agents is high. There is a positive *market size effect of cultural change*. The more prevalent is green consumer culture, the larger aggregate consumption of clean goods, the higher the profits in the clean sector and so is the incentive to undertake research in this sector.

[17] Actually, case (ii) is divided in two sub-cases depending on the value of ϵ. Since these two cases are very similar, in order to simplify the exposition, I will consider only one of these two cases (called case A. in the Appendix). Results for the other case, Case B., are available in Appendix B.

Lemma 2 *There exists* $Y :]\frac{1}{2}, 1] \to \mathbb{R}^+$ *and* q^* *such that*

$$q_{t+1} \geq q_t$$
$$\Leftrightarrow \frac{A_{ct}}{A_{dt}} \geq Y(q_t),$$

with

$$\lim_{q_t \to 1/2} Y(q_t) = +\infty,$$
$$\lim_{q_t \to 1} Y(q_t) = +\infty,$$
$$q^* = \arg\min_{q_t \in]\frac{1}{2},1]} Y(q_t)$$

Proof. See Appendix B.4. ■

The dynamics of preferences is shaped by several forces. First, the fraction of green agents is affected by changes in the past fraction of green agents. Lemma 2 reveals that the function Y is decreasing (resp. increasing) for low (resp. high) values of q_t. For low values of q_t, any rise in q_t positively affects the future fraction of green agents (i.e., the threshold function Y is decreasing). When q_t is higher, rises in q_t have a negative impact on the future fraction of green agents (i.e., the threshold function Y is increasing). The result relies on the existence of two opposite forces. First, there is a population composition effect: the higher the fraction of green agents, the more efficient is transmission of the green trait within the society. Second, parents optimally respond to a shift in q_t by changing their socialization effort. When q_t is high there is cultural substitutability: the higher the fraction of green agents, the lower the transmission of the green trait because green parents reduce their socialization effort. When q_t is low enough, the composition effect outweighs cultural substitutability. In such a case, rises in q_t positively impact the future fraction of green agents. When q_t is high, cultural substitutability prevails so that rises in q_t have a negative impact on the future fraction of green agents.

Second, the dynamics of the fraction of green agents is shaped by changes in the technology. The higher the past productivity in the clean sector, the higher the future fraction of green agents. This is the *impact of eco-innovation on green parenting.* The higher the relative productivity in the clean sector, the lower the price of the clean good and the higher consumption of clean goods by each type of agent. As green agents have higher preferences for clean goods, the rise in clean consumption benefits more to green children. Hence, this rise positively impacts the relative gain to have green children and so the incentives for green parents to transmit their green trait.

From both previous Lemmas, I deduce the dynamics of the economy which is summed up in the following Proposition.

Proposition 1

I- Suppose that the elasticity of substitution between clean and dirty goods is low (i.e., $\epsilon < \tilde{\epsilon}$).

(i) When the willingness to pay for clean goods (i.e., θ^G) is low, the elasticity of production to technical change (i.e., $1-\alpha$) is low, knowledge spillovers (i.e., δ) are low, the socialization cost (i.e., κ) is high, the system admits a unique globally attracting steady state $(q, A_c/A_d) = (0, 1)$.

(ii) When the willingness to pay for clean goods is high, the elasticity of production to technical change is high, knowledge spillovers are high, the socialization cost is low, the system admits three steady states $(q, A_c/A_d) = (0, 1)$, $(\underline{q}, \underline{A_c}/\underline{A_d})$, $(\overline{q}, \overline{A_c}/\overline{A_d})$, $\underline{q} < \overline{q}$ and $1 < \underline{A_c}/\underline{A_d} < \overline{A_c}/\overline{A_d}$. For all $A_{c0}/A_{d0} < Y(q_0)$ and $q_0 < \underline{q}$, the economy converges to $(0, 1)$. For all $A_{c0}/A_{d0} > Y(q^\star)$ and $q_0 > q^\star$, or $A_{c0}/A_{d0} > Y(\underline{q})$ and $q_0 > \underline{q}$ the economy converges to $(\overline{q}, \overline{A_c}/\overline{A_d})$.

II- Suppose that the elasticity of substitution between clean and dirty goods is high (i.e., $\epsilon > \tilde{\epsilon}$),

(i) For all $A_{c0}/A_{d0} > \max\{X(q_0), Y(q^*)\}$, the economy converges to $(1, +\infty)$, (ii) for all $A_{c0}/A_{d0} < \min\{X(q_0), Y(q^\star)\}$, the economy converges to $(0, 0)$.

Proof. See Appendix B.5. ■

Proposition 1 characterizes the long-run fraction of green agents and the long-run ratio of aggregate productivity.

A first, important result is that whatever the value of the elasticity of substitution, for some parameters combination, the long-run equilibrium depends on initial conditions. In other words, whatever the value of the elasticity, there exists path dependency in the direction of innovation. This theoretical result is in line with empirical evidence (see Newell, Jaffe, and Stavins 1999, Popp 2002, Aghion et al. 2016). For instance, Newell, Jaffe, and Stavins (1999) find that the two energy shocks of the 1970s have had a strong impact on energy efficiency of air conditioners and gas water heaters. These shocks account for one-quarter to one-half of the observed improvements in the mean energy efficiency over a twenty years period. Popp (2002) uses U.S. patent data from 1970 to 1994 to study the determinants of energy-efficient innovation. He emphasizes a positive impact of scientific advancements, though he finds decreasing returns to knowledge. A formal argument has been developed in Acemoglu et al. (2012) where path dependency arises due to technological-side mechanisms (in particular they assume increasing returns to knowledge) and demand-side factors (market size effects due to a high elasticity of substitution between clean and dirty goods). This result is in line with case II of Proposition 1.[18]

By contrast in case I, there are decreasing returns to knowledge. Furthermore, market size effects due to a strong elasticity of substitution disappear. Interestingly, for some combinations of the model parameters, the dynamics still exhibits path dependency. This result is illustrated in Figure 2 which depicts the phase diagram associated to the dynamics of the economy for distinct parameters combinations (i.e., case I-(i) at the top, case I-(ii) at the bottom). In

[18]Note that empirical studies find that the current elasticity of substitution between some clean and dirty goods is low (i.e. close to one), see Pottier, Hourcade, and Espagne 2014.

the top graph, one sees that the economy globally converges to a unique steady state while in the bottom one, the economy converges to two distinct steady states depending on initial conditions.

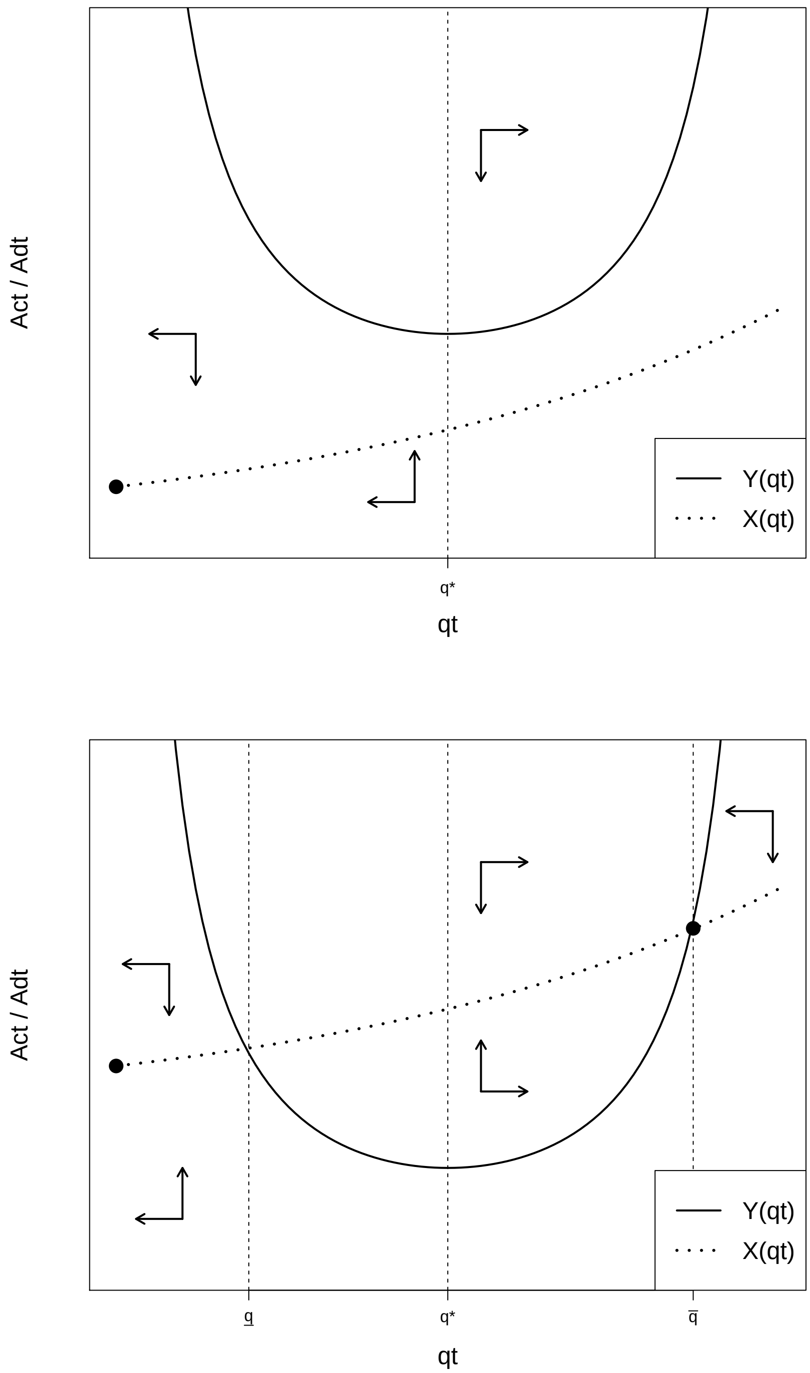

Figure 2: Phase diagram in the $(q_t, A_{ct}/A_{dt})$ space when $\epsilon < \tilde{\epsilon}$ and (i) θ^G, $1-\alpha$, δ low, κ high (top) and (ii) θ^G, $1-\alpha$, δ high, κ low (bottom). Steady states are represented by a solid circle.

The intuition for case I goes as follows. Several forces shape the global dynamics of the economy. These forces can be divided into two groups: forces to substitutability and forces to complementarity. The first set includes the cultural substitution effect and decreasing returns to knowledge. Due to the cultural substitution effect, a rise in the share of green consumers, q_t, negatively affects the future share of these consumers. As well, given decreasing returns in past research (measured by $1-\delta$), an increase in the relative productivity in the clean sector, A_{ct}/A_{dt}, has a negative impact on future relative productivity in this sector. Forces to complementarity are the novelty of this model. These forces arise from the cross effects which capture interactions between green culture and environmentally friendly technology. The first complementarity force is the market size effect of cultural change. The second force is the impact of eco-innovation on green parenting. For a given value of ϵ, both forces are enhanced by a rise in (i) the willingness to pay for clean goods, θ^G, (ii) the elasticity of production with respect to technological change, $1-\alpha$. The second force is negatively affected by an increase in the socialization cost, κ.

When forces to global complementarity outweigh forces to substitutability, the model generates path dependency in innovation processes. In other words, when the elasticity of substitution is low, path dependency occurs when interactions between green culture and sustainable technologies - the market size effect of cultural change and the impact of eco-innovation on green parenting - are high. Hence, compared to Acemoglu et al. (2012), the present paper provides a complementary explanation for path dependency in eco-innovation which is based on the interplay between the green consumer culture and environmentally friendly technologies.

When there is path dependency (cases I-(ii) and II), the interplay between cultural transmission and clean innovation generally gives rise to a positive relationship between the share of green consumers and environmentally friendly technologies. Depending on initial conditions, the economy either converges to a *green* equilibrium where green consumers prevail and productivity in the clean sector is relatively high, or to a *brown* equilibrium where brown consumers prevail and productivity in the clean sector is low. This theoretical result is consistent with the cross-country correlation between environmental attitudes and environmentally friendly technologies displayed in Figure 1.

A numerical example The dynamics of green culture and production technologies is illustrated in the figures below for parameters combinations corresponding to case I-(ii). Figures 4 and 3 depict the dynamics of relative productivity (left) and the dynamics of green culture (right) for identical parameters but different initial conditions. In particular, I consider a change in q_0, the initial fraction of green agents. Figure 4 displays the dynamics of the economy for $(A_{c0}/A_{d0}, q_0) = (0.5, 0.3)$ while Figure 3 corresponds to the dynamics for $(A_{c0}/A_{d0}, q_0) = (0.5, 0.7)$. Clearly, a cultural shock (here a rise in q_0 from 0.3 to 0.7) has a persistent impact on both green culture and the technology. This result is consistent with some empirical findings. For instance, a study by Popp *et al.* (2011) reveals that a Greenpeace report published in 1987 (devoted to inform consumers about water pollution generated by the use of chlorine in paper production) was the main determinant of the sustained rise in chlorine

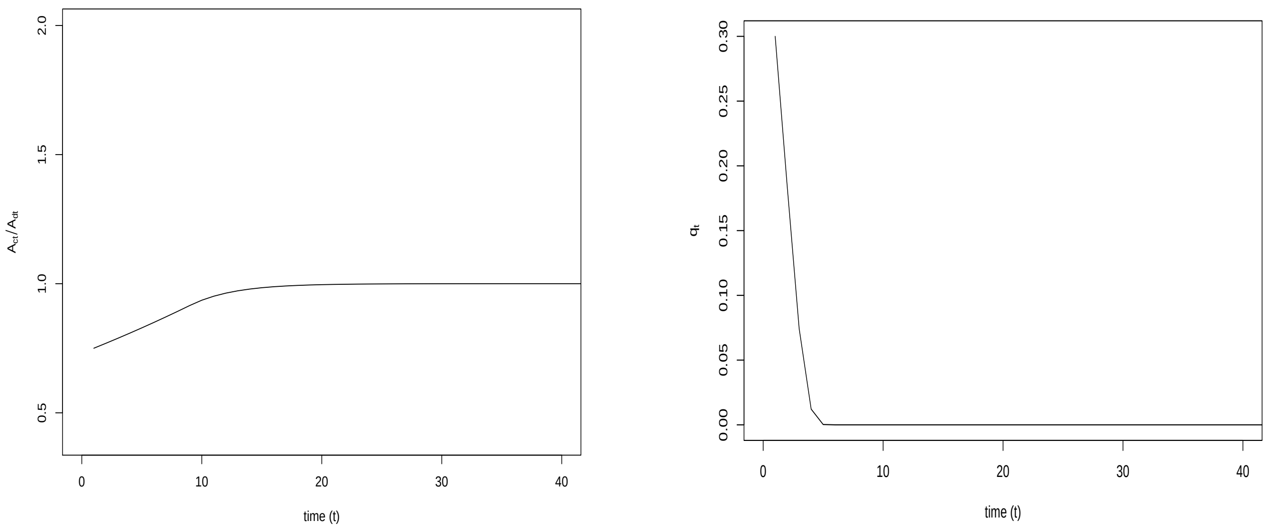

Figure 3: Evolution of relative productivity A_{ct}/A_{dt} (left) and the fraction of green agents q_t (right), for $\epsilon = 1.1$, $\theta^G = 4$, $\nu = 3$, $\kappa = 0.1$, $\alpha = 0.1$, $\delta = 0.5$, $\eta = 0.5$, $\omega = 0.05$, $\bar{E} = 4$, $b = 0.8$, $\sigma = 3$, $\Omega(E_t) = E_t/(1 + E_t)$, $d(e) = 3e/(1 + 3e)$, $q_0 = 0.3$, $A_{c0} = 1$, $A_{d0} = 2$, $E_0 = 0.15$.

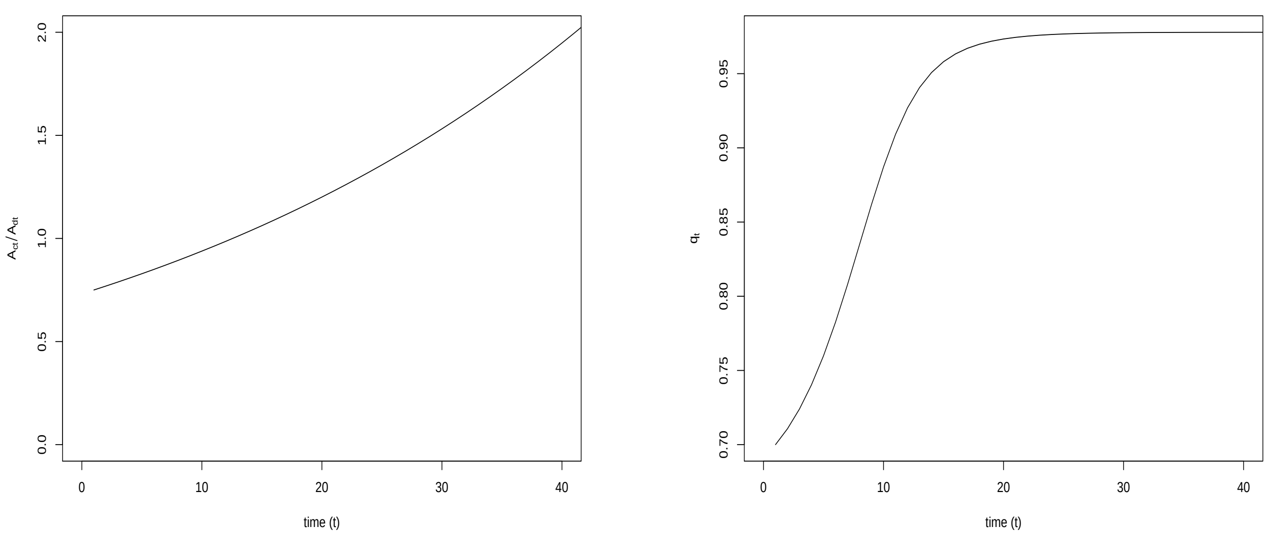

Figure 4: Evolution of relative productivity A_{ct}/A_{dt} (left) and the fraction of green agents q_t (right), for $\epsilon = 1.1$, $\theta^G = 4$, $\nu = 3$, $\kappa = 0.1$, $\alpha = 0.1$, $\delta = 0.5$, $\eta = 0.5$, $\omega = 0.05$, $\bar{E} = 4$, $b = 0.8$, $\sigma = 3$, $\Omega(E_t) = E_t/(1 + E_t)$, $d(e) = 3e/(1 + 3e)$, $q_0 = 0.7$, $A_{c0} = 1$, $A_{d0} = 2$, $E_0 = 0.15$.

free patents experienced by several countries until mid 2000's.[19]

[19]Some works also suggest that the BSE (Bovine spongiform encephalopathy) crisis which occurred in Europe during the 1990's has had a long-term impact on food preferences (see Corsi and Novelli 2011).

4 Efficiency, sustainability and environmental policy

In models of economic growth with environmental externalities, efficient policies are deduced from the maximization of the discounted sum of utilities supposing that individual well-being is measured by an exogenously given utility function and that all individuals in a generation are identical. Here, since preferences evolve due to changes in economic variables, classical welfare assessments do not apply. Preferences endogeneity gives rise to intertemporal inconsistency, i.e., we cannot compare social welfare between generations since utilities are not the same across generations (see Goldman 1979 for a discussion). In order to be able to discuss efficiency, one needs to introduce some refinements of the classical approach. I define an alternative welfare criteria which is based on Pareto-type considerations and avoids interpersonal welfare comparison.

Definition 2 *Situation A is a welfare improvement over Situation B if and only if in Situation A discounted utilities of each agent type - green and brown - is at least as great as it is in situation B, and, moreover, at least one of the agent types has strictly higher discounted utility in A than in B.*

This criteria avoids intertemporal inconsistency as it does not compare the sum of utilities over the whole population which is changing over time. Note, that it does not allow to rank all situations. However, using Definition 2, I am able to characterize efficient policies. First, let me impose the following reasonable assumption.

Assumption 1 *The clean sector is initially backward, i.e.,* $A_{c0}/A_{d0} < \min\{X(q_0), Y(q^*)\}$.

Lemma 3 *Suppose that Assumption 1 holds and the discount rate* β *is in* $]0, 1[$. *Any policy which implies* $\sigma Y_{dt} \in]0, bE_0[$, $\forall t$ *is efficient.*

Proof. See Appendix C.1. ■

Lemma 3 reveals that restricting emissions to a sufficiently low level (but not to zero) is efficient.[20] In line with the literature, public intervention is needed because there exists an intergenerational externality. Due to innovation in the dirty sector, the price of dirty goods decreases over time which encourages the current generation to increase dirty goods production. Doing so, this cohort does not internalize the negative impact she has on future generations by rising pollution emissions.

Any policy which satisfied the condition of Lemma 3 is efficient. Now, I will examine different types of intervention. First, I consider a tax levied on dirty goods production (redistributed to each household through a lump sum transfer).

[20] Note that this type of policy objective is the one which has been commonly proposed in climate change international negotiations. It has also been examined in Popp (2004).

Proposition 2 (Tax on dirty goods) *Suppose that Assumption 1 holds.*

(i) When ϵ is low (i.e., $\epsilon < \tilde{\epsilon}$), there exists an efficient sequence of tax $\{T_t\}_{t \in \mathbb{N}}$, for all $\tilde{t}$, there exists $t > \tilde{t}$ such that $T_t > 0$.

(ii) When ϵ is high (i.e., $\epsilon > \tilde{\epsilon}$), there exists an efficient sequence of tax $\{T_t\}_{t \in \mathbb{N}}$ and some threshold $\tilde{t}$, such that $\forall t > \tilde{t}$, $T_t = 0$.

Proof. See Appendix C.2. ■

Pollution taxes correct the market failure by giving a price to the negative production externality. Proposition 2 states that efficient taxes are either permanent or temporary depending on the elasticity of substitution between clean and dirty goods. Intuitively, when clean and dirty goods are weak substitutes (i.e., $\epsilon < \tilde{\epsilon}$), even if research carried out in the clean sector is substantial, consumption of dirty goods remains sufficient to maintain some research in the dirty sector so that dirty goods production grows and so does pollution, negatively affecting future generations. When clean and dirty goods are strong substitutes (i.e., $\epsilon > \tilde{\epsilon}$), higher research in the clean sector implies a substantial drop of dirty consumption at equilibrium which in turn prevents research in this sector so that the growth of dirty goods production stops and environmental quality stabilizes. This result, which links the type of efficient policy to the elasticity of substitution is in line with the literature on environmental policies with endogenous directed technological change (see Acemoglu et al. 2012, Pottier, Hourcade, and Espagne 2014). Interestingly, the present framework allows to examine a different type of intervention which targets green culture rather than purely economic incentives. More precisely, I now investigate how classical market-based instruments interact with a particular non-monetary type of policy, i.e., environmental education. I comprehend the policy as a decrease in κ, the socialization cost.[21]

Proposition 3 *Suppose that Assumption 1 holds.*

(i) When ϵ is low (i.e., $\epsilon < \tilde{\epsilon}$), environmental education is not efficient.

(ii) When ϵ is high (i.e., $\epsilon > \tilde{\epsilon}$), there exists a threshold $\tilde{\eta}$ such that $\forall \eta < \tilde{\eta}$, if $A_{c0}/A_{c0} > (1 + \eta\omega\,(A_{c0}/A_{c0})^{-\frac{1-\delta}{2}})X(q_0 + q_0(1-q_0)(2q_0-1))$ and $q_0 > \frac{1}{2}$, environmental education is efficient.

Proof. See Appendix C.3. ■

The efficiency of environmental education policies, which favor the green culture consumption, is critically affected by the value of ϵ. When the substitutability between clean and dirty goods is high, the cultural change due to environmental education involves a substantial reduction in dirty goods consumption (as long as q_0 is not too low). The resulting decrease in the size of dirty goods market implies a drop in profits in the dirty sector. Note however, that while the cultural change encourages innovation toward the clean sector, since the clean sector is initially

[21] A way to interpret this modeling choice is to think about environmental education at school as a complement to green parenting reducing the cost of teaching green preferences at home.

backward (due to Assumption 1), there is a counteracting force which favors innovation in the dirty sector. When η is low and $A_{c0}/A_{c0} > (1 + \eta\omega\,(A_{c0}/A_{c0})^{-\frac{1-\delta}{2}})X(q_0 + q_0(1-q_0)(2q_0-1))$, the market size effect of cultural change outweighs the negative impact due to the technological gap so that environmental education redirects all innovation toward the clean sector. Hence environmental education allows to stop the rise of dirty production which is responsible for the negative intergenerational externality. In such a case, both environmental education and pollution taxes allow to correct the market failure due to dirty goods production growth: they are *substitutes*.

When the substitutability between clean and dirty goods is low, the reduction of dirty goods consumption due to the greening of preferences is limited. Even though, the fraction of green agents is high, profits in the dirty sector remain sufficiently high to encourage some innovation in the dirty sector. Dirty goods production grows and so does pollution. Only pollution taxes allow to regulate the growth of emissions. Nevertheless, environmental education plays a key role. In Figure 5 below, the left-hand graph displays the evolution of the efficient tax rate T_t for three different values of the socialization cost κ. When the socialization cost is high (i.e., $\kappa = 0.2$), the tax is monotonically increasing over time. When κ decreases to 0.15 or 0.1, there is a qualitative change in the behavior of the efficient tax rate over time. *Environmental education modifies the shape of the efficient pollution tax.* In particular, one can see that for lower values of κ, the tax rate is deceasing over time. Hence, environmental education fosters a substantial reduction in the cost of efficient taxes.

The intuition goes as follows. When the elasticity of substitution, ϵ is low, the tax has a limited (positive) impact on the market for clean goods. Moreover, when the socialization cost is low, the tax also has a weak impact on green parenting. The effort of green parents is not sufficient to trigger a surge in the green culture (see Figure 5). Both the rigidity of the market (i.e., the weak elasticity to a variation in the pollution tax) and the weakening of the green culture overtime imply a growth in dirty goods production. The tax must be increasing so as to counteract the resulting rise in pollution. Due to environmental education, the socialization cost is lower. In this case, the tax has a strong impact on green parenting which implies a qualitative change in the dynamics. The green culture spreads over time which creates a decrease in the demand for dirty goods and favors the development of sustainable technologies. In other words, the cultural change counteracts the rigidity of the market. Pollution emissions are reduced for some time so that the tax can be decreasing. Hence, environmental education modifies the shape of the efficient tax rate and implies a substantial reduction in the cost of the market-based policy. In such a case, pollution taxes and environmental education are *complements*.

So far, I have focused on the efficiency of environmental policies. To so, I have defined a welfare criteria based on discounted utility. However, the criterion of discounted utility, rooted in the so-called utilitarian principle, has been heavily criticized on the ground that it treats successive generations differently (see Asheim 2010 for a survey). Consequently, other criteria have been introduced to address intergenerational concern. A very popular one is the sustainability criterion which is inspired from the definition proposed in the Bruntland report (*Our Common future*, WCED, 1987) and which consists of ensuring that future generations are not worth off than today's (see Pezzey et al. 2002, for a rich discussion about this approach). Just as discounted utility, the sustainability criterion has generally been applied to models with a

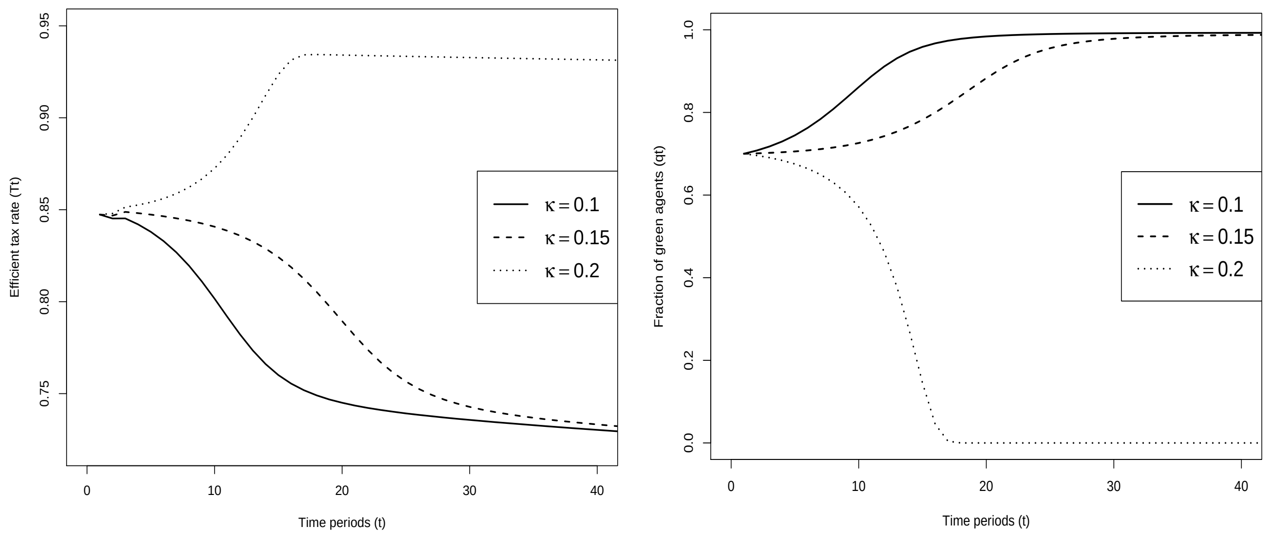

Figure 5: Evolution of the tax rate (left) and the fraction of green agents (right) under different values of κ, for $\epsilon = 1.1$, $\theta^G = 4$, $\nu = 3$, $\alpha = 0.1$, $\delta = 0.5$, $\eta = 0.5$, $\omega = 0.05$, $\bar{E} = 4$, $b = 0.8$, $\sigma = 3$, $q_0 = 0.7$, $A_{c0} = 1$, $A_{d0} = 2$, $E_0 = 0.15$, $\Omega(E_t) = E_t/1 + E_t$, $d(e) = 3e/(1 + 3e)$.

representative agent and fixed preferences.[22] In the present framework where preferences are heterogeneous, one needs to introduce a modified sustainability criteria which avoids interpersonal welfare comparisons.

Definition 3 (Sustainability) *A path is sustainable if and only if*

$$U^i(x^i_{ct+1}, x^i_{dt+1}) \geq U^i(x^i_{ct}, x^i_{dt}) > 0 \quad \forall i \in \{G, B\}, \quad \forall t.$$

The present sustainability criteria requires non-decreasing utility streams for each agents' type.

Lemma 4 *Under Assumption 1, the laissez-faire is not sustainable.*

Proof. See Appendix C.4. ■

As for efficiency, the criteria of Definition 3 calls for intervention. I now interest in the compatibility between efficiency and sustainability. To do so, I draw in Figure 6 the evolution of utility of type-B agents under efficient policies (a similar picture holds for type-G agents).[23]

[22] A number of other criteria have been proposed in the literature, some of which solves the conflict between efficiency and intergenerational equity (see Asheim and Mitra 2010, Zuber and Asheim 2012, and Asheim 2017 for an application of these criteria to various technological environments.). Likewise, they generally assume away preferences heterogeneity. Few exceptions include Fleurbaey (2007), Isaac and Piacquadio (2015).

[23] Also, I present the low elasticity case which is similar to the high elasticity case.

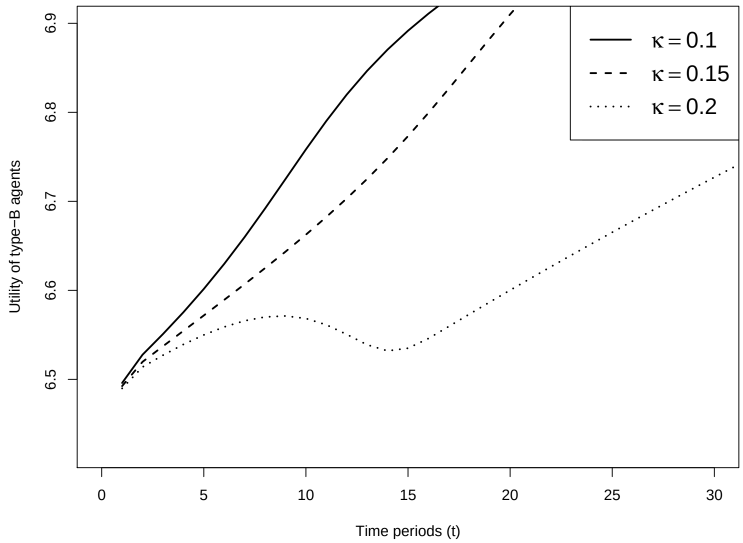

Figure 6: Evolution of the utility of type-B agents under different values of κ, for $\epsilon = 1.1$, $\theta^G = 4$, $\nu = 3$, $\alpha = 0.1$, $\delta = 0.5$, $\eta = 0.5$, $\omega = 0.05$, $\bar{E} = 4$, $b = 0.8$, $\sigma = 3$, $q_0 = 0.7$, $A_{c0} = 1$, $A_{d0} = 2$, $E_0 = 0.15$, $\Omega(E_t) = E_t/1 + E_t$, $d(e) = 3e/(1 + 3e)$.

Interestingly, in this numerical example, one sees that efficient policies may or may not be sustainable: it depends on the value of the socialization cost κ. Efficient taxes are not sustainable for a high value of the socialization cost κ (i.e., $\kappa = 0.2$), while they are sustainable for a lower value of κ (i.e., $\kappa = 0.15$ or 0.1). Hence, environmental education policies, which reduce the socialization cost, critically affect the relationship between efficiency and sustainability : they allow to *reconcile efficiency and sustainability.*

Efficient policies limit pollution emissions by increasing the after-tax cost of dirty goods so as to stop the growth of dirty production. With directed technological change, by doing so, those policies also favor innovation in the clean sector as they reduce profits in the dirty sector. Hence, they yield a reduction of the price of clean goods over time which has a positive effect on the growth of utilities.

There are two cases depending on the value of the socialization cost.

When this cost is high, the fraction of green agents converges to a low equilibrium level and is decreasing over time. Due to the population change, there is an extensive margin effect which increases the demand for dirty goods and thus dirty goods production. To control the increase in dirty goods, the policy must decrease pollution per capita. Hence there are two competing effects which affect the growth of utilities over time. One the one hand, the policy implies a rise of clean goods consumption, which positively affects utility growth. One the other hand, the policy generates a decrease in dirty goods consumption which negatively affects future utilities. Whenever the impact of the growth of clean goods consumption on utility is not high enough, the decrease in dirty goods consumption reduces utility at the subsequent period which contradicts Definition 3.

When the socialization cost decreases below a certain threshold, there is a qualitative change in the dynamics. The fraction of green agents then converges to a high equilibrium value and is increasing over time. The negative extensive margin effect disappears. Only the positive effect due to the growth of clean consumption remains so that the effect on utility growth is unambiguously positive.

The present framework brings light to important conclusions regarding the relationship between classical market-based instruments and non-monetary instruments such as environmental education. The relationship between the two types of policy critically depends on the substitutability between clean and dirty goods. Also, the use of both types of policy may allow to reconcile efficiency and intergenerational equity. Note that whenever market-based instruments are efficient does not mean that they will be implemented as they are subject to strong political acceptability constraints. In Section 6, I investigate the feasibility of environmental policies by adopting a positive point of view about pollution taxes.

5 Alternative modeling assumptions

5.1 Green preferences

Instead of assuming that green households prefer clean goods, i.e., $\theta^G > 1$, one could assume that they care about the stock of environmental quality E_t. This alternative framework is developed in Online Appendix D. In that case, green agents have lower incentives to consume dirty goods than brown ones as they internalize the negative impact that dirty goods have on environmental quality. In contrast to the baseline model, their consumption choices do not only depend on prices but also on past environmental quality and the fraction of green agents. This second approach yields similar results regarding the long-run behavior of the economy (see Proposition D1). I show that when knowledge spillovers (i.e., δ) are high enough, there exists path dependency in the direction of innovation. The economy either converges to a *brown* equilibrium (defined by a low fraction of green individuals, weak productivity of the clean sector and low environmental quality) or to a *green* equilibrium (defined by a high fraction of green consumers, weak productivity of the dirty sector and high level of environmental quality). In that second framework, again two sets of forces shape the dynamics of the economy: forces to substitutability (namely decreasing returns to research) and forces to complementarity. Interestingly, compared to the baseline model, there are two additional forces to complementarity: *the impact of environmental quality changes on green parenting* and *the market size effect of environmental degradation.*

Regarding efficiency issues, what is interesting is that one could easily show that the efficiency of education policies would still depend on initial conditions but not anymore on the elasticity of substitution between clean and dirty goods. This is because, in the framework proposed in Online Appendix D, green agents are assumed to have strong preferences for the environment (i.e., they internalize the possibility of an environmental disaster and its consequences on welfare). Whenever the green culture prevails (which may be possible thanks to an environmental education policy), private actions (i.e., restriction of dirty goods consumption) impedes a major environmental degradation so that economic policies are not necessary.

I examine the implications of this alternative preferences assumption for the political sustainability of environmental taxes in Section 6.

5.2 Pollution technology

In the baseline model, environmental degradation is caused by final goods production. Innovation in dirty sectors increases production of these goods which in turn negatively impacts the environment. This type of modeling captures important aspects of environmental degradation. Going back to the examples mentioned before, intensive agriculture has been enhanced by continuous technological change such as innovation in machinery, the introduction of high-yielding crop variety or the development of chemical fertilizers. As well, the tremendous growth of the livestock sector, responsible for a significant part of greenhouse gases emissions, has undergone a growing production intensity, characterized by increasing use of feed cereals, use of advanced genetics and feeding systems, animal health protection and enclosure of animals (Steinfeld et al. 2006).

In other common cases, pollution is caused by the use of dirty capital (e.g., fossil fuels). In this case, pollution is not directly increased by innovation into dirty sectors. Rather it is improvements in the technology of final goods, increasing the relative demand for dirty capital (i.e., machines), which in turn negatively impacts the environment. This type of pollution-generating mechanism is related to the so-called "reboud effect" whereby a rise in polluting inputs is induced by efficiency gains in the dirty sector. Suppose that the law of motion of the environmental stock takes the following form

$$E_{t+1} = \begin{cases} (1+b)E_t - \sigma \int_0^1 z_{dkt} dk, & \text{if} \quad (1+b)E_t - \sigma \int_0^1 z_{dkt} dk \in]0, \bar{E}], \\ \bar{E}, \quad \text{if} \quad (1+b)E_t - \sigma \int_0^1 z_{dkt} dk > \bar{E}, \\ 0 \quad \text{if} \quad (1+b)E_t - \sigma \int_0^1 z_{dkt} dk < 0. \end{cases} \tag{16}$$

In this case, rises in the aggregate productivity of the dirty good at time t positively impacts the demand for good d which in turn increases the demand for machines of type d at equilibrium.

Note that to examine the implications of this alternative modeling approach for long-run growth related issues, one has to introduce a refinement of the theoretical framework. In the baseline model, the total quantity of machines does not grow in the long-run since a scarce factor, i.e., labor, is used to produce machines (whereas final good production grows since the productivity of machines grows in spite of a constant mass of scientists due to knowledge spillovers). This is because I assume that there is no population growth. For this alternative modeling case, let me relax this assumption. At time t, the population is of mass L_t with $q_t L_t$ individuals of type G. At any date t, each parent, whatever its preferences type, has n children.

Aggregate demands for good c and d are now given by

$$X_t^c = \frac{I_t}{p_{ct}} L_t \left(q_t \frac{\theta^{G^\epsilon} \frac{p_{ct}}{p_{dt}}^{1-\epsilon}}{1+\theta^{G^\epsilon} \frac{p_{ct}}{p_{dt}}^{1-\epsilon}} + (1-q_t) \frac{\frac{p_{ct}}{p_{dt}}^{1-\epsilon}}{1+\frac{p_{ct}}{p_{dt}}^{1-\epsilon}} \right),$$

$$X_t^d = \frac{I_t}{p_{dt}} L_t \left(q_t \frac{1}{1+\theta^{G^\epsilon} \frac{p_{ct}}{p_{dt}}^{1-\epsilon}} + (1-q_t) \frac{1}{1+\frac{p_{ct}}{p_{dt}}^{1-\epsilon}} \right).$$

Suppose that the socialization cost increases proportionally with the number of children, the socialization effort is the same as in the baseline model (since the gain also increases proportionally with n).[24] The dynamics of the fraction of green agents is still given by equation 7. Given the aggregate demand, the ratio of expected profits for scientists is unchanged as the new state variables L_t simplifies in this ratio. Therefore, Lemma 1, 2 and Proposition 1 hold.

It is interesting to examine implications for efficient environmental policies. The dynamics of environmental quality is negatively affected by the quantity of dirty machines used by sector d which, at equilibrium, is now given by

$$\int_0^1 z_{dkt} dk = \frac{1}{(1-\alpha)} I L_t \Omega(E_t) \left(q_t \frac{1}{1+\theta^{G^\epsilon} \frac{A_{ct}}{A_{dt}}^\gamma} + (1-q_t) \frac{1}{1+\frac{A_{ct}}{A_{dt}}^\gamma}. \right).$$

For the analysis of efficiency, one could use similar arguments than those developed for the baseline case to show under Assumption 1 that Lemma 3, Proposition 2 and Proposition 3 hold. Again, the elasticity of substitution between clean and dirty goods plays a key role for efficient environmental policies, both taxes and environmental education. Let me simply draw a sketch of the proof.

Under Assumption 1, the pollution flows $\sigma \int_0^1 z_{dkt} dk$ grows to infinity since (i) $\Omega(E_t)$ is bounded, (ii) the term in brackets converges to a positive constant and (iii) L_t grows exponentially. This result implies that from some date t, E_t reaches zero which has severe welfare consequences. Therefore, any sequence of tax T_t which is set such that the term in brackets decreases to zero at a higher rate than n (the growth rate of L_t) implies that $\int_0^1 z_{dkt} dk \to 0$ so that the long-run environmental quality remains positive and the sequence T_t is efficient.

Taxes When $\epsilon < \tilde{\epsilon}$. Suppose that taxes are temporary. When $T_t = 0$, Proposition 1 holds, which means that the sequence A_{ct}/A_{dt} converges to some constant so that the term in brackets converges to a strictly positive constant. Since L_t grows exponentially, then $\int_0^1 z_{dkt} dk$ grows exponentially meaning that E_t reaches zero which contradicts efficiency. Hence, taxes must be permanent. When $\epsilon > \tilde{\epsilon}$, for any sequence of taxes such that $A_{ct} + 1/A_{dt} + 1 > A_{ct}/A_{dt}$, from some t we have $A_{ct}/A_{dt} > \min\{X(q_0), Y(q^*)\}$). Suppose that the tax rate is then set to zero, given Proposition 1 it implies that A_{ct}/A_{dt} grows to infinity which, in turn, implies that the term in brackets tends to zero. Then, as long as n is sufficiently low, $\int_0^1 z_{dkt} dk$ converges to zero so that environmental quality can stabilize to a strictly positive level.

[24]Otherwise, it depends on n but since n is a parameter it does not impact the dynamics and thereby the results.

Environmental education Environmental education increases incentives to transmit the green trait which affects the long-run equilibrium if those policies are set such that $A_{c0}/A_{d0} > \min\{X(q_0), Y(q^*)\})$. Given Proposition 1, when $\epsilon < \tilde{\epsilon}$, the term in brackets cannot go to zero so that the flow of pollution grows exponentially meaning that E_t reaches zero which contradicts efficiency. Hence, education policies are not efficient. However when $\epsilon > \tilde{\epsilon}$, then A_{ct}/A_{dt} grows to infinity so that the term in brackets goes to zero. Again, as long as n is sufficiently low, $\int_0^1 z_{dkt}dk$ converges to zero so that environmental quality can stabilize to a strictly positive level. Environmental education is efficient.

6 The political sustainability of environmental taxes

In this part, I adopt a positive point of view about pollution taxes. I assume that in each period, a tax on dirty goods production is decided by majority voting of the mature generation. Individuals first make their consumption choices with perfect foresight regarding the outcome of the voting process. Second, given the chosen consumption bundles, agents vote for their optimal level of tax. The tax is redistributed to any household through a lump sum transfer denoted by $R(T_t)$. Given profit maximization by final producers, prices of good c and d are given by

$$p_d(T_t) \equiv \Omega(E_t)^{-1}(1-T_t)^{-1}{A_{dt}}^{-(1-\alpha)}(1-\alpha)^{-(1-\alpha)},$$
$$p_{ct} = \Omega(E_t)^{-1}{A_{ct}}^{-(1-\alpha)}(1-\alpha)^{-(1-\alpha)},$$

which leads to

$$\frac{p_{ct}}{p_d(T_t)} = (1-T_t)^{-1}\frac{A_{ct}}{A_{dt}}^{-(1-\alpha)}.$$

At period t, green and brown consumers vote for a tax T_t so as to respectively maximize[25]

$$V^G(T_t) = \ln\left(\frac{I+R(T_t)}{p_d(T_t)}\left(1+\theta^{G\epsilon}\left(\frac{p_{ct}}{p_d(T_t)}\right)^{1-\epsilon}\right)^{\frac{1}{\epsilon-1}}\right) \tag{17}$$

$$V^B(T_t) = \ln\left(\frac{I+R(T_t)}{p_d(T_t)}\left(1+\left(\frac{p_{ct}}{p_d(T_t)}\right)^{1-\epsilon}\right)^{\frac{1}{\epsilon-1}}\right). \tag{18}$$

Solving this program for type B one obtains

$$\frac{dR_t/dT_t}{I+R_t} - \frac{1}{(1-T_t)}\frac{1}{\left(1+\left(\frac{P_{ct}}{P_{dt}(T_t)}\right)^{1-\epsilon}\right)} < 0,$$

[25]To simplify the analytics, I skip the altruistic motive in the utility function of green parents. Since parents want to increase the indirect utility of their child which is similar to their own indirect utility this simplification does not affect the results.

so that type-B vote for a tax $T_t = 0$. Define

$$T_t^G = \text{argmax}_{T_t \in [0,1]} V^G(T_t).$$

The optimal tax rate for type-G, T_t^G is implicitly given by

$$\frac{dR_t/dT_t}{I + R_t} - \frac{1}{(1 - T_t^G)} \frac{1}{\left(1 + \theta^{G\epsilon} \left(\frac{P_{ct}}{P_{dt}(T_t^G)}\right)^{1-\epsilon}\right)} = 0,$$

or is equal to zero. The first order conditions for type B and G reveal that the tax (i) positively impacts utilities of both types of agent through the rise in income due to the transfer, (ii) negatively impacts utilities by reducing consumption of good d.

Lemma 5 *The equilibrium tax rate is given by*

$$T_t^\star = \begin{cases} 0 & \text{if} \quad q_t < \frac{1}{2}, \\ T_t^G > 0 & \text{if} \quad q_t > \frac{1}{2}. \end{cases} \qquad \qquad \text{with} \qquad \frac{dT_t^G}{dq_t} < 0.$$

Proof. See Appendix C.5. ■

Green agents vote for a positive tax rate while brown agents vote for a tax rate equal to zero. This result lies on two specific features of the framework: (i) green agents consume a lower amount of polluting goods, (ii) the tax is refunded lump sum. Both features imply that the tax causes a redistribution from brown agents to green ones. This result is in line with Cremer et al. (2004) who study the political support for a pollution tax under different refunding rules. They find that environmental preferences are critical for the preferred tax level because these preferences actually determine whether an individual receives more or less than the amount he pays in tax. The result also echoes some empirical evidence. For instance, Cragg et al. (2013) use a US county-level dataset based on 2002 carbon emissions and show that the geography of carbon emissions across the United States is a strong determinant of congressional voting pattern on carbon legislation. Lemma 5 also reveals that the equilibrium tax rate is decreasing in q_t. The higher the fraction of green agents, the lower the amount green agents receive in tax because the tax base decreases.

Contrary to what has been done so far, the present framework allows to adopt a long-run perspective on pollution taxes. Let me introduce the following definition.

Definition 4 *A tax is* ***politically sustainable*** *if it is supported by a majority in the long-run.*

When the tax is not voted initially, then one trivially shows that it is not politically sustainable. Therefore, I will focus on cases in which the tax is initially voted. Also remind that I have shown in Section 4 that when clean and dirty goods are strong substitutes (i.e., $\epsilon > \tilde{\epsilon}$) efficient environmental taxes are to be temporary. The political sustainability of environmental taxes is relevant only when efficient taxes are permanent. These restrictions are summed up in Assumption 2.

Assumption 2 *Suppose that*

(i) the tax is voted initially, i.e., $q_0 > \frac{1}{2}$,

(ii) clean and dirty goods are weak substitutes, i.e., $\epsilon < \tilde{\epsilon}$.

Proposition 4 *Suppose that Assumption 2 holds. For any* $(A_{c0}/A_{d0}, q_0) : A_{c0}/A_{d0} > X(q_0)$, *there exists two threshold levels of the socialization cost,* κ, *such that*

(i) when the socialization cost is high (i.e., κ *is higher than the largest threshold), the tax is not politically sustainable,*

(ii) when the socialization cost is low (i.e. κ, *is lower than the smallest threshold) the tax is politically sustainable.*

Proof. See Appendix C.5. ∎

Proposition 4 stresses another key role for non-monetary interventions which is to foster the political sustainability of economic instruments. This result relies on the existence of substitutability/complementarity between green culture and clean technologies which determine uniqueness versus multiplicity of long-run outcomes. Complementarity which arises when both the market size effect of culture and the impact of eco-innovation on green parenting are high, gives rise to multiple history-dependent equilibria (one with a high fraction of green agents, i.e. $q > \frac{1}{2}$ and the other one with a low fraction of green agents, i.e., $q < \frac{1}{2}$). The tax favors complementarity as it positively impacts both green parenting and eco-innovation (because it increases the welfare of green children and the profits of scientists who engage in sector c). However, when κ is low (i.e., $\kappa < \tilde{\kappa}$), socialization to green preferences is so costly that the positive tax effect on green parenting is weak which in turn implies that the market size effect of culture is low. The tax is not sufficient to change the long-run equilibria of the economy which converges to a state in which the fraction of green agents is too low to support the economic instrument. In this case, non-monetary instruments are critical for the political sustainability of environmental taxes. Environmental education (by increasing κ) strengthens the impact of the tax on green parenting and eco-innovation. The tax becomes sufficient to change the long-run outcome of the economy by triggering complementarities between green culture and sustainable technologies. If the initial fraction of green agents and the productivity in the clean sector are both sufficiently high, the economy thereby converges to the green equilibrium in which the population supports the tax in the long-run.

Alternative modeling Let me come back to the alternative preferences assumption where green agents do not directly prefer clean goods but rather care about environmental quality. As shown in Appendix D, this second approach yields the same results regarding the equilibrium tax, although the intuition is different (see Lemma D2). In the baseline model, green agents vote for a positive tax since the tax causes a redistribution from brown agents to green ones. In the second case, green agents vote for a positive tax in order to reduce the negative environmental externality caused by dirty goods. Interestingly, in this political economy framework, I show that their consumption of clean and dirty goods is the same as brown agents'. Hence, to tackle with the negative environmental externality green agents entirely substitute on public contributions to private ones. In other words, public intervention completely crowds out private incentives to contribute to the public good.

To study implications for the political sustainability of environmental taxes, I rely on numerical simulations. In this alternative framework, education policy still plays a critical role.

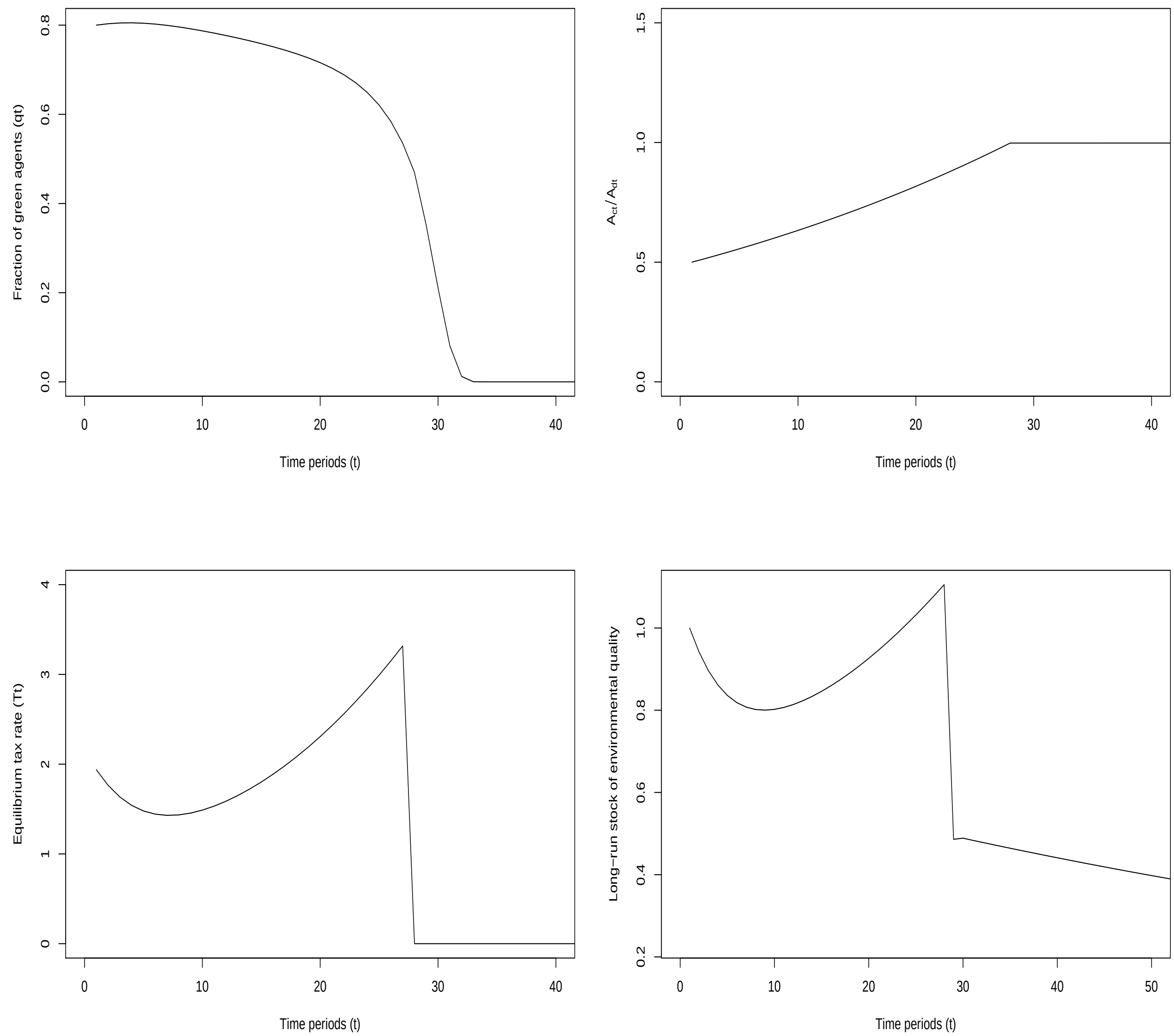

Figure 7: Evolution of q_t, $\frac{A_{ct}}{A_{dt}}$, E_t and $T_t^\star$ in the low κ case (i.e., $\kappa = 0.05$), with $(A_{c0}, A_{d0}, q_0, E_0) = (4, 8, 0.8, 1)$, $\nu = 3$, $\alpha = 0.1$, $\delta = 0.9$, $\eta = 0.5$, $\omega = 0.05$, $\bar{E} = 2$, $b = 0.1$, $\sigma = 0.5$, $d(e) = 3e/(1 + 3e)$, $\Omega(E_t) = E_t$.

Figure 7 shows the evolution of the fraction of green agents, the relative productivity in the clean

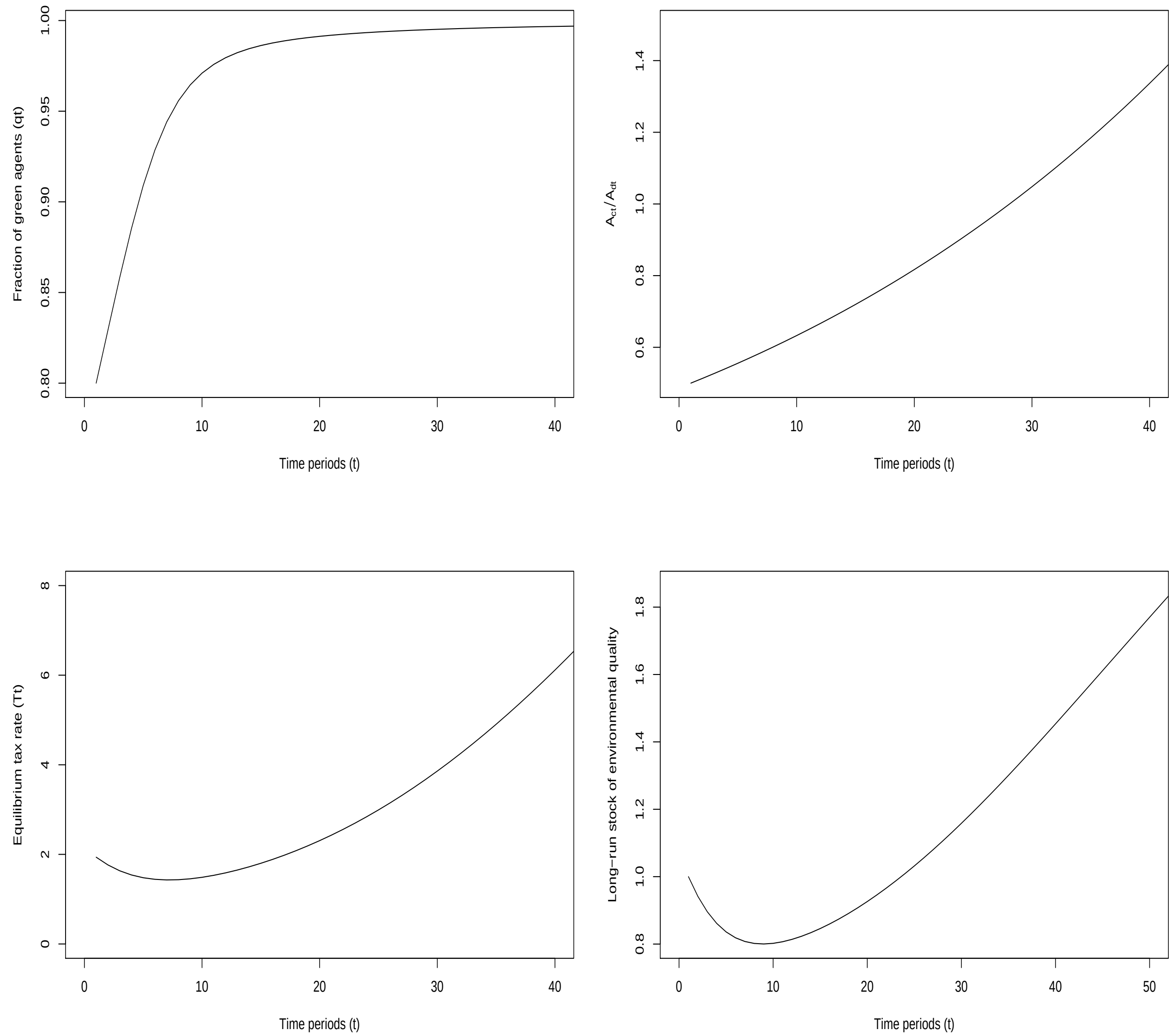

Figure 8: Evolution of q_t, $\frac{A_{ct}}{A_{dt}}$, E_t and $T_t^\star$ in the low κ case (i.e., $\kappa = 0.01$), with $(A_{c0}, A_{d0}, q_0, E_0) = (4, 8, 0.8, 1)$, $\nu = 3$, $\alpha = 0.1$, $\delta = 0.9$, $\eta = 0.5$, $\omega = 0.05$, $\bar{E} = 2$, $b = 0.1$, $\sigma = 0.5$, $d(e) = 3e/(1 + 3e)$, $\Omega(E_t) = E_t$.

sector, the equilibrium tax rate and the level of environmental quality when the socialization cost is high (i.e., $\kappa = 0.05$). Under Assumption 2, the tax is initially voted which has a positive impact on green parenting. However, the socialization cost is so high that the positive effect of the tax is not sufficient to promote the development of the green culture. The fraction of green agents decreases to drop below the threshold of one half so that the tax is not voted anymore. From that time, environmental quality also starts to decline over time. Figure 7 depicts the evolution of the same variables when the socialization cost is lower (i.e., $\kappa = 0.01$). In this case, the positive impact of the tax on green parenting is sufficient to trigger the rise of the green culture which in turn allows to sustain pollution taxes in the long-run. As a consequence, pollution stop to grow and environmental quality stabilizes to some positive level.

Finally, let me focus on the role of another critical parameter, σ, which measures the marginal environmental damage from dirty goods production. The literature has generally stressed a monotonic positive relationship between the marginal environmental damage and the long-run stock of pollution (e.g., John et al. 1995). Here, the impact of σ on the long-run environmental quality is ambiguous. There is a direct negative effect: for a given level of dirty goods production, a rise in σ decreases environmental quality since it increases the amount of pollution per unit of production. On the other hand, a rise in the marginal damage σ increases the tax rate which favors an increase in the long-run environmental quality. Figure 9 displays the long-run level of environmental quality as a function of σ. When σ is high, any decrease in σ has a positive impact on environmental quality through the first direct effect. However, for lower values of σ (in this numerical example, σ close to 0.27), a rise in σ has a strong negative impact on the long-run level of environmental quality (which equals zero). This numerical example shows that, in the present framework, the second effect can be substantial compared to the first one. The change in σ has a qualitative impact on the long-run dynamics. The decrease in σ lowers the equilibrium tax rate which negatively affects both incentives to transmit the green preferences and incentives to innovate in the clean sector. In spite of favorable initial conditions (i.e. relatively high fraction of green agents), transmission of the green trait and clean innovation are so low that the green culture disappears and environmental quality converges to zero. This result cautions against policies aiming at reducing σ as they may backfire through lowering the positive interactions between green culture, eco-innovation and environmental quality.

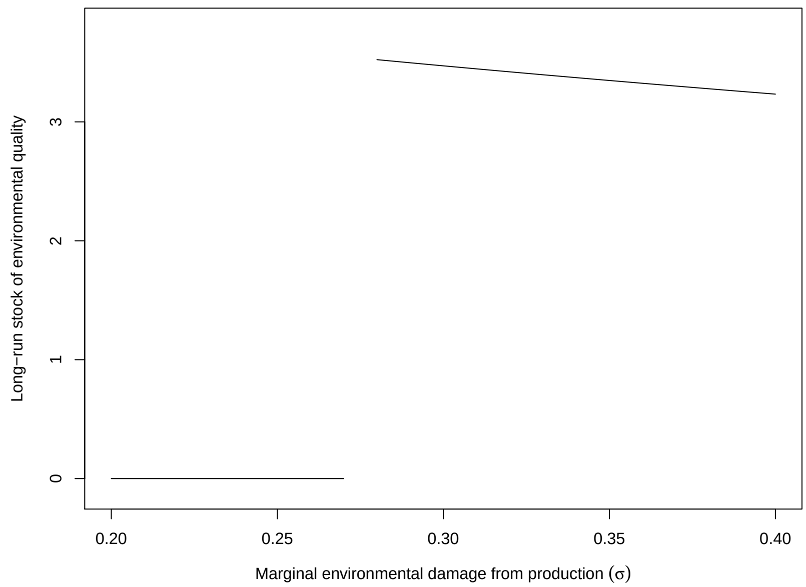

Figure 9: Long-run stock of environmental quality as a function of the marginal environmental damage σ for $\kappa = 0.01$, $\nu = 3$, $\alpha = 0.1$, $\delta = 0.9$, $\eta = 0.5$, $\omega = 0.05$, $\bar{E} = 2$, $b = 0.1$, $q_0 = 0.8$, $A_{c0} = 4$, $A_{d0} = 8$, $E_0 = 1$, $d(e) = 3e/(1 + 3e)$.

7 Concluding remarks

This paper develops a theory which formalizes the interplay between green consumer culture and clean technologies. The idea that interactions with environmental culture should be more systematically considered in the analysis of sustainable technological development has received attention beyond the economic field. Some scholars argue that "carbon lock-in" (i.e. the persistent use of carbon-intensive technologies) arise through a process which involves the co-evolution between values and norms and innovation (e.g. Unruh 2000). This paper proposes an economic approach where the transmission of preferences is rooted in the rational choice paradigm and innovation responds to profit incentives and market size effects.

First, the model captures an important feature of sustainable innovation processes which is the existence of path dependency. Compared to the theoretical literature, path dependency arises under more plausible values for the elasticity of substitution between clean and dirty goods and knowledge spillovers. This result suggests that focusing on the technological side may not be sufficient to explain the mechanics of sustainable innovation processes.

The approach allows to examine the impact of both market-based instruments and an alternative type of tool : non-monetary policies. In general the results call for a wider use of non-monetary instruments although they do not necessarily substitute to traditional economic tools. An important finding is that market-based and non-monetary instruments can be complements or substitutes depending on the substitutability between clean and dirty goods. Also, one shows that non-monetary policies are a mean to reach the twofold objective of efficiency and intergenerational equity. Finally, as market-based instruments, such as taxes, have been politically unpopular, the model is extended to a political economy framework to study the long-run feasibility of such policies. One shows that non-monetary policies are critical for the political sustainability of market-based instruments.

This framework could be extended in several ways. Here, socialization is realized by parents and role models. I could enrich the model by introducing other socialization agents (e.g., mass media, NGO leaders, school). A particularly promising avenue for further research is to include a cultural leader, as formalized in Verdier and Zenou (2018), motivated by some long-run environmental objective. This approach would allow to endogenize environmental education policies. Finally, the mechanism analyzed extend to the interplay of any preferences for private consumption of a public good and technologies more or less harmful to the public good (it includes any type of what is usually referred to as ethical consumer preferences).[26] The theory then potentially allows to discuss a variety of different problems. It could be extended to international trade to contribute to the fair trade debate (generally focused on human rights and working conditions) where it is argued that trade could have detrimental effects by lowering the values supporting nationally more sustainable production methods (Rodrik 1997).

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[26]The expression ethical consumer has come to describe consumption practices which integrate a concern for collective or social issues, be it environmental protection, animal welfare or human rights.

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A Appendix: the correlation between green consumer atttitudes and clean technologies

As Table A1 shows the correlation between the percentage land under organic farming and the share of the population who adopt environmentally motivated purchasing behaviors is significant and robust to controlling for GDP per capita and a measure of support for environmental policies. This result indicates that the relationship between the two variables does not arise through a "technological effect" as described in Grossman and Krueger (1995) nor through the political economy channel. Note that according to the technological effect, higher income nations, who can afford greater spending on R&D, are better able to develop cleaner technologies. If so, and if green consumer attitudes are positively correlated with income, the observed correlation could go through this technological effect.

Table A1: Percentage land under organic farming and fraction of the population who adopt environmentally motivated purchasing behaviors: cross-country estimates.

Dependant variable: *% land under organic farming*	1	2	3
% who adopt environmentally motivated purchasing behaviours	3,99*** (1,42)	3,22** (1,53)	3,42** (1,43)
GDP per capita		0,47 (0,34)	0,66** (0,33)
% willing to pay much higher taxes for the environment			-1,5** (0,66)
R-squared	0,19	0.23	0.40

Source: GDP per capita from World Development Indicators and percentage of individuals who are willing to pay much higher taxes to protect the environment from ISSP Research Group (2010).
OLS Estimates. Standard errors in parentheses. **Significant at 10%,* ***Significant at 5%,* ****Significant at 1%.*

There also exists a positive correlation between the fraction of people who agree with the idea of buying goods for environmental reasons and the share of EU firms who had introduced one eco-innovative product or service during the last two years. As Table A2 shows, the correlation is significant and robust to controlling for GDP per capita and a measure of environmental policies, i.e., green taxes as a percentage of GDP.[27]

[27] Green taxes include energy taxes, transport taxes, pollution taxes and resource taxes.

Table A2: Percentage firms who introduced an eco-innovative product or service and fraction of the population with positive attitudes toward eco-friendly consumption: cross-country estimates.

Dependant variable:	*% firms who introduced an eco-innovative product or service*		
	1	2	3
% with positive attitudes toward eco-friendly consumption	0,33** (0,14)	0,44** (0,18)	0,56*** (0,19)
GDP per capita		$6{\cdot}10^{-5}$ ($6{\cdot}10^{-5}$)	3,2* (0,18)
Green taxes			$-8{\cdot}10^{-5}$ ($6{\cdot}10^{-5}$)
R-squared	0,19	0.22	0.31

Source: GDP per capita from World Development Indicators and Green taxes from EUROSTAT. OLS Estimates. Standard errors in parentheses. **Significant at 10%,* ***Significant at 5%,* ****Significant at 1%.*

B Appendix: proofs

In subsections B.1 and B.2, I introduce two different Lemmas which will be useful for the following proofs.

B.1 Equilibrium for scientists

Let me define the map $F : [0,1] \times [0,1] \times \mathbb{R}^+ \to \mathbb{R}^+$ such that

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-(1-\delta)+\gamma} f(q_t, \frac{A_{ct-1}}{A_{dt-1}}, r_t) \ \left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}})\right)^{\gamma-1} \equiv F(r_t, q_{t-1}, \frac{A_{ct-1}}{A_{dt-1}}).$$

with $\gamma = (\epsilon - 1)(1 - \alpha)$,

$$g(r_t, \frac{A_{ct-1}}{A_{dt-1}}) = \frac{1 + \eta\omega r_t \ \frac{A_{ct-1}}{A_{dt-1}}^{-\frac{1-\delta}{2}}}{1 + \eta\omega(1 - r_t)\frac{A_{ct-1}}{A_{dt-1}}^{\frac{1-\delta}{2}}}$$

$$f(q_t, \frac{A_{ct-1}}{A_{dt-1}}, r_t) = \frac{q_t(\theta_G^\epsilon - 1) + 1 + \theta_G^\epsilon \left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}})\frac{A_{ct-1}}{A_{dt-1}}\right)^\gamma}{-q_t\left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}})\frac{A_{ct-1}}{A_{dt-1}}\right)^\gamma (\theta_G^\epsilon - 1) + 1 + \theta_G^\epsilon \left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}})\frac{A_{ct-1}}{A_{dt-1}}\right)^\gamma}.$$

1. To determine equilibria for scientists, let me examine the derivative of F with respect to r_t.

$$\frac{\partial F}{\partial r_t} = \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-(1-\delta)+\gamma} \frac{\partial f}{\partial r_t} \left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}})\right)^{\gamma-1}$$
$$+ \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-(1-\delta)+\gamma} f(q_t, \frac{A_{ct-1}}{A_{dt-1}}, r_t)(\gamma-1)\left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}})\right)^{\gamma-2} \frac{\partial g}{\partial r_t}$$

One easily finds

$$\frac{\partial g}{\partial r_t} > 0.$$

Furthermore,

$$\frac{\partial f}{\partial r_t} = \frac{\partial f}{\partial g}\frac{\partial g}{\partial r_t}.$$

One can perform

$$\frac{\partial f}{\partial g} = \gamma\left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}})\right)^{\gamma-1} \frac{-(1-q_t)q_t(\theta_G^\epsilon - 1)^2}{\left(-q_t\left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}})\frac{A_{ct-1}}{A_{dt-1}}\right)^{\gamma}(\theta_G^\epsilon - 1) + 1 + \theta_G^\epsilon\left(g(r_t, \frac{A_{ct-1}}{A_{dt-1}})\frac{A_{ct-1}}{A_{dt-1}}\right)^{\gamma}\right)^2} < 0.$$

I deduce that when ϵ is low, i.e., $\epsilon < \frac{1}{1-\alpha} + 1$, one has $\partial F/\partial r_t < 0$. When ϵ is higher, i.e., $\gamma > 1$, we can have both $\partial F/\partial r_t < 0$ and $\partial F/\partial r_t > 0$. I will deal with the two cases: case A. $\partial F/\partial r_t < 0$ and case B. $\partial F/\partial r_t > 0$.

2. Equilibria for scientists $r_t^\star$.

In case A. I deduce that there exists a unique equilibrium given by

-$r_t^\star = 1$, iif $F(1, q_{t-1}, \frac{A_{ct-1}}{A_{dt-1}}) \geq 1$,

-$r_t^\star = 0$, iif $F(0, q_{t-1}, \frac{A_{ct-1}}{A_{dt-1}}) \leq 1$,

-$r_t^\star \in]0,1[$, iif $F(1, q_{t-1}, \frac{A_{ct-1}}{A_{dt-1}}) < 1$ and $F(0, q_{t-1}, \frac{A_{ct-1}}{A_{dt-1}}) > 1$.

In case B. I deduce that there exists a unique equilibrium given by

-$r_t^\star = 1$, iif $F(0, q_{t-1}, \frac{A_{ct-1}}{A_{dt-1}}) > 1$,

-$r_t^\star = 0$, iif $F(1, q_{t-1}, \frac{A_{ct-1}}{A_{dt-1}}) < 1$.

-There exists several equilibria $r_t^\star = 1$, $r_t^\star = 0$ and $r_t^\star \in]0,1[$ iif $F(1, q_{t-1}, \frac{A_{ct-1}}{A_{dt-1}}) \geq 1$ and $F(0, q_{t-1}, \frac{A_{ct-1}}{A_{dt-1}}) \leq 1$. The interior equilibrium $r_t^\star \in]0,1[$ is unstable.

B.2 Some properties of the dynamical system

Define $Q : [0,1] \times \mathbb{R}^+ \to [0,1]$, $A : [0,1] \times \mathbb{R}^+ \to \mathbb{R}^+$,

$$q_{t+1} = q_t + q_t(1-q_t)(2d(e^\star)q_t - 1) \equiv Q(q_t, \frac{A_{ct}}{A_{dt}}),$$

$$\frac{A_{ct+1}}{A_{dt+1}} = \frac{1 + \eta\omega r^*_{t+1} \, \frac{A_{ct}}{A_{dt}}^{-\frac{1-\delta}{2}}}{1 + \eta\omega(1 - r^*_{t+1}) \frac{A_{ct}}{A_{dt}}^{\frac{1-\delta}{2}}} \frac{A_{ct}}{A_{dt}} \equiv A(q_t, \frac{A_{ct}}{A_{dt}}),$$

where $e^\star_t$ is implicitly given by

$$2d^{'}(e^\star_t)q_t(1-q_t)(V^{GG}_t - V^{GB}_t) - \kappa(e^\star_t)^{\nu-1} = 0,$$

with

$$V^{GG}_t - V^{GB}_t = \log\left(\frac{\left(1 + \theta^{G\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^\gamma\right)^{\frac{1}{\epsilon-1}}\left(1 + \left(\frac{A_{ct}}{A_{dt}}\right)^\gamma\right)}{\left(1 + \theta^{G}\left(\frac{A_{ct}}{A_{dt}}\right)^\gamma\right)^{\frac{\epsilon}{\epsilon-1}}}\right),$$

and $r^\star_{t+1}$ is given as in subsection B.1.

Lemma B1 *Suppose that variables are such that $2d^{'}(e^\star) - 1 < 0$, we have*

$$\frac{\partial Q}{\partial q_t} > 0, \qquad \textit{and} \qquad \frac{\partial A}{\partial \frac{A_{ct}}{A_{dt}}} > 0.$$

Moreover, one has

$$\frac{\partial Q}{\partial \frac{A_{ct}}{A_{dt}}} > 0, \qquad \textit{and} \qquad \frac{\partial A}{\partial q_t} > 0.$$

Proof of Lemma B1.

$$\frac{\partial Q}{\partial q_t} = 1 + q_t(1-q_t)(2d^{'}(e^\star)\frac{\partial e^\star}{\partial q_t}q_t + 2d(e^\star)) + (1-2q_t)(2d(e^\star)q_t - 1),$$

with

$$\frac{\partial e^\star}{\partial q_t} = -\frac{2d^{'}(e^\star)(1-2q_t)(V^{GG}_t - V^{GB}_t)}{2d''(e^\star)q_t(1-q_t)(V^{GG}_t - V^{GB}_t) - \kappa(\nu-1)(e^\star_t)^{\nu-2}} > 0 \Leftrightarrow q < \frac{1}{2}.$$

Re-write

$$\begin{aligned}\frac{\partial Q}{\partial q_t} =&1 + (1-2q_t)(2d(e^\star)q_t - 1) + 2d(e^\star)q_t(1-q_t)\\ &- 2d^{'}(e^\star)q_t\frac{2d^{'}(e^\star)(1-2q_t)q_t(1-q_t)(V_t^{GG}-V_t^{GB})}{2d^{''}(e^\star)q_t(1-q_t)(V_t^{GG}-V_t^{GB}) - \kappa(\nu-1)(e_t^\star)^{\nu-2}}\\ =&1 + (1-2q_t)(2d(e^\star)q_t - 1) + 2d(e^\star)q_t(1-q_t)\\ &- 2d^{'}(e^\star)q_t\frac{(1-2q_t)}{\frac{d^{''}(e^\star)}{d^{'}(e^\star)} - \frac{\kappa(\nu-1)(e_t^\star)^{\nu-2}}{2d^{'}(e^\star)q_t(1-q_t)(V_t^{GG}-V_t^{GB})}}\end{aligned}$$

Note that we have

$$\frac{\kappa(\nu-1)(e_t^\star)^{\nu-2}}{2d^{'}(e^\star)q_t(1-q_t)(V_t^{GG}-V_t^{GB})} = \frac{(\nu-1)}{2}(e_t^\star)^{-1},$$

hence,

$$\begin{aligned}\frac{\partial Q}{\partial q_t} =&1 + (1-2q_t)(2d(e^\star)q_t - 1) + 2d(e^\star))q_t(1-q_t)\\ &+ 2d^{'}(e^\star)q_t\frac{(1-2q_t)}{\frac{(\nu-1)}{2}(e_t^\star)^{-1} - \frac{d^{''}(e^\star)}{d^{'}(e^\star)}}.\end{aligned}$$

We have

$$\begin{aligned}&\frac{(\nu-1)}{2}(e_t^\star)^{-1} - \frac{d^{''}(e^\star)}{d^{'}(e^\star)} > \frac{(\nu-1)}{2}(e_t^\star)^{-1}\\ \Leftrightarrow&\frac{1}{\frac{(\nu-1)}{2}(e_t^\star)^{-1} - \frac{d^{''}(e^\star)}{d^{'}(e^\star)}} < \frac{2}{(\nu-1)}e_t^\star.\end{aligned}$$

When $q_t > \frac{1}{2}$, this leads to

$$\frac{2q_t d^{'}(e_t^\star)(1-2q_t)}{\frac{(\nu-1)}{2}(e_t^\star)^{-1} - \frac{d^{''}(e^\star)}{d^{'}(e^\star)}} > 2q_t d^{'}(e_t^\star)(1-2q_t)\frac{2}{(\nu-1)}e_t^\star.$$

Furthermore, since d is concave we have

$$\begin{aligned}&d^{'}(e_t^\star) < \frac{d(e_t^\star)}{e_t^\star}\\ \Leftrightarrow&d^{'}(e_t^\star)e_t^\star < d(e_t^\star)\\ \Leftrightarrow&(1-2q_t)d^{'}(e_t^\star)e_t^\star > (1-2q_t)d(e_t^\star).\end{aligned}$$

Therefore, I deduce

$$\frac{2q_t d^{'}(e_t^\star)(1-2q_t)}{\frac{(\nu-1)}{2}(e_t^\star)^{-1} - \frac{d^{''}(e^\star)}{d^{'}(e^\star)}} > 2q_t d^{'}(e_t^\star)(1-2q_t)\frac{2}{(\nu-1)}e_t^\star > 2q_t d(e_t^\star)(1-2q_t)\frac{2}{(\nu-1)}.$$

Then suppose that $\nu \geq 3$, we have

$$\begin{aligned}\frac{\partial Q}{\partial q_t} >& 1+(1-2q_t)(2d(e^\star)q_t-1)+2d(e^\star)q_t(1-q_t)\\&+2q_t d(e_t^\star)(1-2q_t)=1+(1-2q_t)(2d(e^\star)q_t-1)+2d(e^\star)q_t(2-3q_t)\\&>1+(1-2q_t)(2d(e^\star)q_t-1)-2d(e^\star)q_t=-2q_t(2d(e^\star)q_t-1).\end{aligned}$$

Suppose that $2d(e^\star)q_t < 1$, then $-2q_t(2d(e^\star)q_t-1) > 0$ so that $\frac{\partial Q}{\partial q_t} > 0$.

$$\begin{aligned}\frac{\partial A}{\partial \frac{A_{ct}}{A_{dt}}} =& \frac{1+\eta\omega r^*_{t+1}\frac{A_{ct}}{A_{dt}}^{-\frac{1-\delta}{2}}}{1+\eta\omega(1-r^*_{t+1})\frac{A_{ct}}{A_{dt}}^{\frac{1-\delta}{2}}}\\&+\frac{\frac{\partial r^*_{t+1}}{\partial \frac{A_{ct}}{A_{dt}}}\left[1+\eta\omega\right]+\frac{\partial p}{\partial \frac{A_{ct}}{A_{dt}}}\left[1+\eta\omega r^*_{t+1}(1-r^*_{t+1})\right]}{\left(1+\eta\omega(1-r^*_{t+1})\frac{A_{ct}}{A_{dt}}^{\frac{1-\delta}{2}}\right)^2}\frac{A_{ct}}{A_{dt}} > 0.\end{aligned}$$

Finally, using the fact that $d(e^\star)$ is increasing in $\frac{A_{ct}}{A_{dt}}$ and $\frac{\Pi_c}{\Pi_d}$ is increasing in q_t, one easily shows that

$$\frac{\partial Q}{\partial \frac{A_{ct}}{A_{dt}}} > 0, \quad \frac{\partial A}{\partial q_t} > 0.$$

■

B.3 Proof of Lemma 1

Given $r_t^\star$, I examine the dynamics of relative productivity.

Case A.

$$\begin{aligned}&\frac{A_{ct+1}}{A_{dt+1}} \geq \frac{A_{ct}}{A_{dt}}\\ \Leftrightarrow & r^\star_{t+1} \geq r(\frac{A_{ct}}{A_{dt}}),\\ \Leftrightarrow & F(r^\star_{t+1}, q_t, \frac{A_{ct}}{A_{dt}}) \leq F(r(\frac{A_{ct}}{A_{dt}}), q_t, \frac{A_{ct}}{A_{dt}}),\\ \Leftrightarrow & f\left(Q(q_t, \frac{A_{ct}}{A_{dt}}), \frac{A_{ct}}{A_{dt}}, r(\frac{A_{ct}}{A_{dt}})\right)\cdot\left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\delta)+\gamma} \geq 1,\end{aligned}$$

Define

$$G(\frac{A_{ct}}{A_{dt}}, q_t) \equiv f\left(Q(q_t, \frac{A_{ct}}{A_{dt}}), \frac{A_{ct}}{A_{dt}}, r(\frac{A_{ct}}{A_{dt}})\right)\cdot\left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\delta)+\gamma}.$$

Note that we have

$$f\left(Q(q_t,\frac{A_{ct}}{A_{dt}}),\frac{A_{ct}}{A_{dt}},r(\frac{A_{ct}}{A_{dt}})\right)\in[0,1].$$

Suppose that $\gamma < 1-\delta$. When $\frac{A_{ct}}{A_{dt}}$ tends to zero, then G tends to $+\infty$. When $\frac{A_{ct}}{A_{dt}}$ tends to infinity, then G tends to 0. I deduce that there exists some value of $\frac{A_{ct}}{A_{dt}}$ such that $G(\frac{A_{ct}}{A_{dt}},q_t)=1$.

Moreover, we have

$$\begin{aligned}\frac{\partial G}{\partial\frac{A_{ct}}{A_{dt}}}=&\left(\frac{\partial f}{\partial q_{t+1}}\frac{\partial Q}{\partial\frac{A_{ct}}{A_{dt}}}+\frac{\partial f}{\partial\frac{A_{ct}}{A_{dt}}}\right)\left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\delta)+\gamma}\\&+f\left(Q(q_t,\frac{A_{ct}}{A_{dt}}),\frac{A_{ct}}{A_{dt}},r(\frac{A_{ct}}{A_{dt}})\right)\cdot(-(1-\delta)+\gamma)\left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\delta)+\gamma-1}\end{aligned}$$

When ϵ tends to one

$$\begin{aligned}&\frac{\partial Q}{\partial\frac{A_{ct}}{A_{dt}}}\to 0,\\&\frac{\partial f}{\partial\frac{A_{ct}}{A_{dt}}}\to 0,\\&\gamma\to 0,\end{aligned}$$

so that $\frac{\partial G}{\partial\frac{A_{ct}}{A_{dt}}}<0$. By continuity of $\frac{\partial G}{\partial\frac{A_{ct}}{A_{dt}}}$ in ϵ, I deduce that there exists a threshold $\tilde{\epsilon}\in]1,\frac{(1-\delta)}{(1-\alpha)}+1[$ such that $\forall\epsilon<\tilde{\epsilon}$, $\frac{\partial G}{\partial\frac{A_{ct}}{A_{dt}}}<0$. I deduce that $\forall\epsilon<\tilde{\epsilon}$, there exists a function $X:[0,1]\to\mathbb{R}^+$ such that

$$\begin{aligned}&\frac{A_{ct+1}}{A_{dt+1}}\geq\frac{A_{ct}}{A_{dt}}\\\Leftrightarrow&\frac{A_{ct}}{A_{dt}}\leq X(q_t),\quad\text{where } X(q) \text{ is implicitly given by } \; G\left(X,q\right)=1.\end{aligned}$$

Moreover, one easily finds

$$\frac{\partial G}{\partial q_t}>0.$$

The implicit function theorem gives

$$\frac{dX}{dq}=-\frac{\frac{\partial G}{\partial\frac{A_c}{A_d}}}{\frac{\partial G}{\partial q}}>0.$$

When $\epsilon\in[\frac{(1-\delta)}{(1-\alpha)}+1,\frac{1}{(1-\alpha)}+1]$, $\frac{\partial G}{\partial\frac{A_{ct}}{A_{dt}}}>0$.

Using similar arguments, I deduce that $\forall\epsilon \in [\frac{(1-\delta)}{(1-\alpha)}+1, \frac{1}{(1-\alpha)}+1]$, there exists a function X such that

$$\frac{A_{ct+1}}{A_{dt+1}} \geq \frac{A_{ct}}{A_{dt}}$$
$$\Leftrightarrow \frac{A_{ct}}{A_{dt}} \geq X(q_t).$$

The implicit function theorem gives

$$\frac{dX}{dq} = -\frac{\frac{\partial G}{\partial \frac{A_c}{A_d}}}{\frac{\partial G}{\partial q}} < 0.$$

Case B. (i.e., $\partial F/\partial r_t > 0 \Rightarrow \epsilon > \frac{1}{(1-\alpha)}+1$).

$$\frac{A_{ct+1}}{A_{dt+1}} \geq \frac{A_{ct}}{A_{dt}}$$
$$\text{if} \quad F(0, q_t, \frac{A_{ct}}{A_{dt}}) > 1.$$

Define

$$H(q_t, \frac{A_{ct}}{A_{dt}}) \equiv F(0, q_t, \frac{A_{ct}}{A_{dt}}) = \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\delta)+\gamma} f\left(Q(q_t, \frac{A_{ct}}{A_{dt}}), \frac{A_{ct}}{A_{dt}}, 0)\right) \left(g(0, \frac{A_{ct}}{A_{dt}})\right)^{\gamma-1}.$$

When $\epsilon > \frac{1}{(1-\alpha)}+1$, we have

$$H(q_t, 0) = 0,$$
$$\lim_{\frac{A_{ct}}{A_{dt}} \to +\infty} H = +\infty,$$

so that there exists some values of $\frac{A_{ct}}{A_{dt}}$ such that $H(q_t, \frac{A_{ct}}{A_{dt}}) = 1$.

Moreover,

$$\begin{aligned}\frac{\partial H}{\partial \frac{A_{ct}}{A_{dt}}} =& (-(1-\delta)+\gamma)\left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\delta)+\gamma-1} f(q_{t+1}, \frac{A_{ct}}{A_{dt}}, 0) \left(g(0, \frac{A_{ct}}{A_{dt}})\right)^{\gamma-1} \\ &+ \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\delta)+\gamma} \left(\frac{\partial f}{\partial q_{t+1}} \frac{\partial Q}{\partial \frac{A_{ct}}{A_{dt}}} + \frac{\partial f}{\partial \frac{A_{ct}}{A_{dt}}}\right) \left(g(0, \frac{A_{ct}}{A_{dt}})\right)^{\gamma-1} \\ &+ \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\delta)+\gamma} f(q_{t+1}, \frac{A_{ct}}{A_{dt}}, 0)\ (\gamma-1)\frac{\partial g}{\partial \frac{A_{ct}}{A_{dt}}} \left(g(0, \frac{A_{ct}}{A_{dt}})\right)^{\gamma-2}\end{aligned}$$

When $\gamma > 1$, the first two terms of this sum are positive while the third one is negative. In what follows, I just consider the sum of the first and third term and show that it is positive. This sum equals

$$(-(1-\delta)+\gamma)\left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\delta)+\gamma-1} f(q_{t+1},\frac{A_{ct}}{A_{dt}},0)\ \left(g(0,\frac{A_{ct}}{A_{dt}})\right)^{\gamma-1}$$
$$+\left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\delta)+\gamma} f(q_{t+1},\frac{A_{ct}}{A_{dt}},0)\ (\gamma-1)\frac{\partial g}{\partial\frac{A_{ct}}{A_{dt}}}\left(g(0,\frac{A_{ct}}{A_{dt}})\right)^{\gamma-2} =$$
$$\left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\delta)+\gamma-1} f(q_{t+1},\frac{A_{ct}}{A_{dt}},0)\left(g(0,\frac{A_{ct}}{A_{dt}})\right)^{\gamma-1}\left((-(1-\delta)+\gamma)-(\gamma-1)\frac{1-\delta}{2}\frac{\eta\omega(\frac{A_{ct-1}}{A_{dt-1}})^{-\frac{1-\delta}{2}}}{1+\eta\omega(\frac{A_{ct}}{A_{dt}})^{\frac{1-\delta}{2}}}\right)$$

Note that

$$\frac{1-\delta}{2}\frac{\eta\omega(\frac{A_{ct}}{A_{dt}})^{-\frac{1-\delta}{2}}}{1+\eta\omega(\frac{A_{ct}}{A_{dt}})^{\frac{1-\delta}{2}}}<1,$$

so that the sum of the two terms is positive if $(-(1-\delta)+\gamma)-(\gamma-1)=\delta>0$ which is true. One deduces

$$\frac{\partial H}{\partial\frac{A_{ct}}{A_{dt}}}>0.$$

Hence, there exists a function $X_1:[0,1]\to\mathbb{R}^+$ such that

$$\frac{A_{ct+1}}{A_{dt+1}}\geq\frac{A_{ct}}{A_{dt}}$$
$$\Leftrightarrow\frac{A_{ct}}{A_{dt}}\geq X_1(q_t),\quad \text{where } X_1(q) \text{ is implicitly given by } \ H\left(X_1,q\right)=1.$$

The implicit function theorem gives

$$\frac{dX_1}{dq}=-\frac{\frac{\partial H}{\partial q}}{\frac{\partial H}{\partial\frac{A_c}{A_d}}}<0.$$

Using the same reasoning, one can show that there exists a function $X_2:[0,1]\to\mathbb{R}^+$ such that

$$\frac{A_{ct+1}}{A_{dt+1}}\geq\frac{A_{ct}}{A_{dt}}$$
$$\text{if}\quad F(1,q_t,\frac{A_{ct}}{A_{dt}})<1,$$
$$\Leftrightarrow\frac{A_{ct}}{A_{dt}}\geq X_2(q_t),$$

with

$$\frac{dX_2}{dq}<0.$$

B.4 Proof of Lemma 2

One has

$$
\begin{aligned}
& q_{t+1} \geq q_t \\
\Leftrightarrow & 2d(e_t^\star)q_t \geq 1,
\end{aligned}
$$

where $e_t^\star$ is implicitly given by

$$2d^{'}(e_t^\star)q_t(1-q_t)(V_t^{GG} - V_t^{GB}) - \kappa(e_t^\star)^{\nu-1} = 0,$$

with

$$V_t^{GG} - V_t^{GB} = \log\left(\frac{\left(1+\theta^{G\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\gamma}\right)^{\frac{1}{\epsilon-1}}\left(1+\left(\frac{A_{ct}}{A_{dt}}\right)^{\gamma}\right)}{\left(1+\theta^{G}\left(\frac{A_{ct}}{A_{dt}}\right)^{\gamma}\right)^{\frac{\epsilon}{\epsilon-1}}}\right)$$

Define the functions $e : [0,1]\times\mathbb{R}^+ \to \mathbb{R}^+$ and $\Phi : [0,1]\times\mathbb{R}^+ \to [0,2]$ such that

$$
\begin{aligned}
& 2d^{'}(e(q_t, \frac{A_{ct}}{A_{dt}}))q_t(1-q_t)(V_t^{GG} - V_t^{GB}) - \kappa(e(q_t, \frac{A_{ct}}{A_{dt}}))^{\nu-1} = 0, \\
& \Phi(q_t, \frac{A_{ct}}{A_{dt}}) \equiv 2d(e(q_t, \frac{A_{ct}}{A_{dt}}))q_t.
\end{aligned}
$$

We have

$$\frac{\partial\Phi}{\partial\frac{A_{ct}}{A_{dt}}} = 2q_t d^{'}(e(q_t, \frac{A_{ct}}{A_{dt}}))\frac{\partial e}{\partial\frac{A_{ct}}{A_{dt}}} > 0.$$

For any given $q_t \in [\frac{1}{2}, 1[$, when $\frac{A_{ct}}{A_{dt}}$ tends to infinity, $\Phi(q_t, \frac{A_{ct}}{A_{dt}})$ tends to $2q_t$ which is higher than 1, when $\frac{A_{ct}}{A_{dt}}$ tends to zero, then $\Phi(q_t, \frac{A_{ct}}{A_{dt}})$ tends to zero. Finally Φ is continuous in $\frac{A_{ct}}{A_{dt}}$. One deduces that for each $q_t \in [\frac{1}{2}, 1[$, there exists a function $Y : [\frac{1}{2}, 1[\to \mathbb{R}^+$, implicitly given by $\Phi(q_t, Y(q_t)) = 1$ such that

$$
\begin{aligned}
& \forall\frac{A_{ct}}{A_{dt}} \leq Y(q_t), \quad \Phi(q_t, \frac{A_{ct}}{A_{dt}}) \leq 1, \\
& \forall\frac{A_{ct}}{A_{dt}} \geq Y(q_t), \quad \Phi(q_t, \frac{A_{ct}}{A_{dt}}) \geq 1.
\end{aligned}
$$

Let me examine the limits of the function Y.

Note that $\lim_{q_t\to\frac{1}{2}} \Phi(q_t, \frac{A_{ct}}{A_{dt}}) = d(e(\frac{1}{2}, \frac{A_{ct}}{A_{dt}})$. We have

$$\lim_{\frac{A_{ct}}{A_{dt}}\to+\infty} d(e(\frac{1}{2}, \frac{A_{ct}}{A_{dt}}) = 1$$

so that

$$\Leftrightarrow \lim_{q_t \to \frac{1}{2}} Y(q_t) = +\infty.$$

Second consider the limit when q_t tends to one.

Suppose that there exists a real number A such that the variable $\frac{A_{ct}}{A_{dt}} < A$, then $\lim_{q_t \to 1} e(q_t, \frac{A_{ct}}{A_{dt}}) = 0$. I deduce that

$$\lim_{q_t \to 1} \sup_{\frac{A_{ct}}{A_{dt}} \leq A} d(e(q_t, \frac{A_{ct}}{A_{dt}})) = 0.$$

Then, one shows that $\lim_{q_t \to 1} Y(q_t) = +\infty$. To do so, use a reduction ad absurdum.

Suppose that $\lim_{q_t \to 1} Y(q_t) \neq +\infty$. Hence, there exists a sequence $(q_{tn})_n$ and a real number A such that $(q_{tn})_n \to 1$ and $Y(q_{tn}) \leq A$. We have

$$d(e(q_{tn}, Y(q_{tn})) < \sup_{X \leq A} d(e(q_t, \frac{A_{ct}}{A_{dt}})) = 0.$$

Due to the definition of $Y(q)$, $\Phi(q_t, Y(q_t)) = 1$, this is impossible. I deduce that $\lim_{q_t \to 1} Y(q_t) = +\infty$.

Now, consider q_t equal to some real number $1/2 < B < 1$. Suppose that for all $C \in \mathbb{R}^+$, $Y(q_t) > C$. It implies that $\Phi(q_t, Y(q_t)) = 2q_t > 1$. This is impossible given the definition of $Y(q_t)$. Hence, for each q_t equal to some real number $1/2 < B < 1$, there exists a real number C such that $Y(q_t) < C$. Since furthermore $\lim_{q_t \to 1/2} Y(q_t) = +\infty$ and $\lim_{q_t \to 1} Y(q_t) = +\infty$, it implies that there exists at least one q such that $\frac{\partial Y}{\partial q_t} = 0$ and $\frac{\partial^2 Y}{\partial {q_t}^2} > 0$. Denote by $q^* \in]\frac{1}{2}, 1[$, the value of q which globally minimizes Y, that is, $q^* = \text{argmin}_q Y(q)$.

B.5 Proof of Proposition 1

The proof is divided in two main parts. In part I, I consider the case where ϵ is low. In step 1, I show that for some parameters combinations there exists a unique globally attracting steady state (in (i) I show there exists only two steady states, in (ii) I show that only one of these steady states is globally stable). In step 2, I show that for some other parameters combinations, there exists two stable steady states (in (i) I show that there exists at least four steady states, in (ii) I show that two of these steady states are locally stable). In part II, I consider the case where ϵ is high. I show that in this case, there exists multiple attracting steady states.

Part I of Proposition 1 (i.e., $\epsilon < \tilde{\epsilon}$).

Steady states are such that

$$\begin{cases} q_{t+1} = q_t \\ \frac{A_{ct+1}}{A_{dt+1}} = \frac{A_{ct}}{A_{dt}}. \end{cases}$$

$$\Leftrightarrow \begin{cases} q_{t+1} = q_t \\ \frac{A_{ct}}{A_{dt}} = X(q_t), \end{cases}$$

$$\Leftrightarrow \begin{cases} q = 0, \text{ or } q = 1 \text{ or } \frac{A_{ct}}{A_{dt}} = Y(q_t) \\ \frac{A_{ct}}{A_{dt}} = X(q_t). \end{cases}$$

Two obvious steady states of the dynamical system are $(0, X(0))$, and $(1, X(1))$. Other steady states are such that

$$Y(q_t) = X(q_t).$$

Let me study the existence of solutions to this equation.

Step 1: (i) uniqueness. One shows that when κ is high, θ^G is low, $1-\alpha$ is low, δ is low, the equation

$$Y(q_t) = X(q_t)$$

admits a unique solution.

The function X being increasing with q_t, one has $X(q_t) \leq X(1)\ \forall q_t \in [0,1]$. Let me determine $X(1)$:

$$(X(1))^{-\frac{(1-\delta)}{2}+\gamma} f(1, X(1), r_t^*) = 1,$$

with

$$f(1, X(1), r_t^*) = \theta_G^{\epsilon}.$$

Hence

$$X(1) = \theta_G^{\overline{(1-\delta)-\gamma}^{\epsilon}}.$$

The function Y is decreasing with q_t for all $q_t \in]\frac{1}{2}, q^*]$ and increasing for all $q_t \in [q^*, 1]$.

A sufficient condition for $X(q_t) < Y(q_t)\ \forall q_t$ is $\theta_G^{\overline{(1-\delta)-\gamma}^{\epsilon}} < Y(q^*)$.

Denote by Γ the vector of the model parameters with Γ_i being one coordinate of this vector. Now, let me study the variations of $Y(q^*)$ with respect to Γ_i. First,

$$\frac{dY(q^*)}{d\Gamma_i} = \frac{\partial Y}{\partial q}\frac{\partial q^*}{\partial \Gamma_i} + \frac{\partial Y}{\partial \Gamma_i} = \frac{\partial Y}{\partial \Gamma_i}.$$

Remind that $Y(q)$ is implicitly given by

$$2qd(e(q,Y)) - 1 = 0,$$

where $e(q, \frac{A_c}{A_d}) = e^\star$ is given by

$$2d^{'}(e^\star)q(1-q)(V^{GG} - V^{GB}) - \kappa = 0.$$

Then, one finds

$$\frac{dY(q^*)}{d\Gamma_i} = -\frac{\frac{\partial e}{\partial \Gamma_i}}{\frac{\partial e}{\partial Y}} = -\frac{\frac{\partial e}{\partial \Gamma_i}}{\frac{\partial e}{\partial \frac{A_{ct}}{A_{dt}}}}.$$

From proof of Lemma 2 we know that $\frac{\partial e}{\partial \frac{A_{ct}}{A_{dt}}} > 0$ so that $\frac{\partial Y}{\partial \Gamma_i}$ is the opposite sign of $\frac{\partial e}{\partial \Gamma_i}$.

Consider $\Gamma_i = \kappa$.

One has $\frac{\partial e}{\partial \kappa} < 0$ which implies $\frac{dY(q^*)}{d\kappa} > 0$.

When κ tends to infinity, for any finite value of $\frac{A_c}{A_d}$, $e^\star$ tends to zero so that $d(e^\star)$ tends to zero. I deduce that Y tends to infinity. When κ tends to zero, Y tends to zero. Given the continuity of Y in κ for $\kappa \in \mathbb{R}^+$, there exists a unique $\tilde{\kappa}$ such that

$$\forall \kappa \leq \tilde{\kappa}, \theta_G^{\overline{\frac{\epsilon}{(1-\delta)-\gamma}}} \geq Y(q^*),$$
$$\forall \kappa \geq \tilde{\kappa}, \theta_G^{\overline{\frac{\epsilon}{(1-\delta)-\gamma}}} \leq Y(q^*).$$

Consider $\Gamma_i = \theta^G$. One has $\frac{\partial e}{\partial \theta^G} > 0$ which implies $\frac{dY(q^*)}{d\theta^G} < 0$. When θ^G tends to infinity, for any positive value of $\frac{A_c}{A_d}$, $d(e^\star)$ tends to one. I deduce that Y tends to zero. When θ^G tends to 1, for any finite value of $\frac{A_c}{A_d}$, $d(e^\star)$ tends to zero so that Y tends to infinity. Furthermore, one has $\frac{d\theta_G^{\overline{\frac{\epsilon}{(1-\delta)-\gamma}}}}{d\theta_G} > 0$. When θ_G tends to one, $\theta_G^{\overline{\frac{\epsilon}{(1-\delta)-\gamma}}}$ tends to one, when θ_G tends to infinity $\theta_G^{\overline{\frac{\epsilon}{(1-\delta)-\gamma}}}$, tends to infinity. Hence, given the continuity of $Y(q^*)$ in θ_G, there exists a unique $\tilde{\theta_G} > 1$ such that

$$\forall \theta_G \leq \tilde{\theta_G}, \theta_G^{\overline{\frac{\epsilon}{(1-\delta)-\gamma}}} \leq Y(q^*),$$
$$\forall \theta_G \geq \tilde{\theta_G}, \theta_G^{\overline{\frac{\epsilon}{(1-\delta)-\gamma}}} \geq Y(q^*).$$

Consider $\Gamma_i = \delta$. We have $\frac{d\theta_G^{\overline{\frac{\epsilon}{(1-\delta)-\gamma}}}}{d\delta} > 0$. When δ tends to $\bar{\delta}$ such that $(1-\bar{\delta}) - \gamma = 0$, then $\theta_G^{\overline{\frac{\epsilon}{(1-\delta)-\gamma}}}$ tends to infinity. If κ and θ_G are such that $\tilde{\theta_G}, \theta_G^{\overline{\frac{\epsilon}{(1)-\gamma}}} < Y(q^*)$, then there exists $\tilde{\delta}$ such that

$$\forall \delta \leq \tilde{\delta}, \theta_G^{\overline{\frac{\epsilon}{(1-\delta)-\gamma}}} \leq Y(q^*),$$
$$\forall \delta \geq \tilde{\delta}, \theta_G^{\overline{\frac{\epsilon}{(1-\delta)-\gamma}}} \geq Y(q^*).$$

Finally one can also show that $\frac{dY(q^*)}{d\alpha} > 0$, $\frac{d\theta_G^{\overline{\frac{\epsilon}{(1-\delta)/2-\gamma}}}}{d\alpha} < 0$. This implies that $\frac{\partial \tilde{\theta_G}}{\partial \alpha} < 0$, $\frac{\partial \tilde{\kappa}}{\partial \alpha} > 0$, $\frac{\partial \tilde{\delta}}{\partial \alpha} < 0$.

Step 1: (ii) basin of attraction. The sequence q_t belongs to the interval $[0, 1]$ so that it is bounded. The sequence $\frac{A_{ct}}{A_{dt}}$ belongs to $\mathbb{R}^+$ so that it is bounded below by zero. For any $X(q_0) < \frac{A_{c0}}{A_{d0}}$, the sequence $\frac{A_{ct}}{A_{dt}}$ is decreasing so that it is bounded above by $\frac{A_{c0}}{A_{d0}}$. The sequences q_t and $\frac{A_{ct}}{A_{dt}}$ are bounded. Furthermore, whenever $\theta^G < \tilde{\theta^G}$ and/ or $\kappa < \tilde{\kappa}$, the dynamic system $(q_t, \frac{A_{ct}}{A_{dt}})$ admits a unique steady states. Since both maps Q and A are continuous in q_t and $\frac{A_{ct}}{A_{dt}}$, the dynamic system $(q_t, \frac{A_{ct}}{A_{dt}})$ converges to its unique steady state $(0, 1)$.

Step 2: (i) multiplicity. One shows that when κ is low, θ^G is high, α is high, δ is high, the system admits multiple globally attracting steady states.

The function X being increasing with q_t, one has $X(q_t) \geq X(0)$ $\forall q_t \in [0, 1]$. One easily shows that $X(0) = 1$.

A sufficient condition for $X(q_t) = Y(q_t)$ admits some solution over the interval $]\frac{1}{2}, 1]$ is $Y(q^*) < 1$. Then, one uses the same arguments as in step 1 to show that the condition $Y(q^*) < 1$ holds whenever κ is low, θ^G is high, α is high, δ is high.

When κ is low, θ^G is high, α is high, δ is high, the equation $X(q_t) = Y(q_t)$ admits at least two solutions $\underline{q} < q^*$ and $\bar{q} > q^*$.[28] In what follows, to alleviate the presentation, I focus on the case for which there exist two and only two solutions. All the results extend to cases for which there are more than two solutions. The dynamic system $(q_t, \frac{A_{ct}}{A_{dt}})$ admits two additional steady states $(\underline{q}, X(\underline{q}) \equiv \underline{\frac{A_{ct}}{A_{dt}}})$ and $(\overline{q}, X(\overline{q}) \equiv \overline{\frac{A_{ct}}{A_{dt}}})$.

Step 2: (ii) basin of attraction.

• Suppose that $\frac{A_{c0}}{A_{d0}} > Y(q^\star)$ and $q_0 > q^\star$.

Define the following sets:

$$\mathbf{A} = \{(q_t, \frac{A_{ct}}{A_{dt}}) : q_t \in]\underline{q}, \bar{q}] \ \&\ \frac{A_{ct}}{A_{dt}} \in]Y(q_t), X(q_t)]\}.$$
$$\mathbf{B} = \{(q_t, \frac{A_{ct}}{A_{dt}}) : q_t > \underline{q} \ \&\ Y(q^\star) < \frac{A_{ct}}{A_{dt}} < \min\{X(q_t), Y(q_t)\}\}.$$
$$\mathbf{C} = \{(q_t, \frac{A_{ct}}{A_{dt}}) : q_t \in]\underline{q}, \bar{q}] \ \&\ \frac{A_{ct}}{A_{dt}} > X(q_t) > Y(q_t)\}.$$
$$\mathbf{D} = \{(q_t, \frac{A_{ct}}{A_{dt}}) : q_t > \underline{q} \ \&\ \frac{A_{ct}}{A_{dt}} \in [X(q_t), Y(q_t)]\}.$$

1. Consider $(q_0, \frac{A_{c0}}{A_{d0}}) \in \mathbf{A}$.

Both sequences q_t and $\frac{A_{ct}}{A_{dt}}$ are increasing so that they are respectively bounded below by q_0 and $\frac{A_{c0}}{A_{d0}}$.

[28] Note that $\underline{q}$ is unique while there may exist several $\bar{q}$ depending on the shape of X.

There are two cases. Either $(q_t, \frac{A_{ct}}{A_{dt}}) \in \mathbf{A}\ \forall t$ which implies that both q_t and $\frac{A_{ct}}{A_{dt}}$ are bounded above. Or, there exists some t such that $(q_t, \frac{A_{ct}}{A_{dt}}) \in \mathbf{B} \cup \mathbf{C} \cup \mathbf{D}$.

2. Consider $(q_0, \frac{A_{c0}}{A_{d0}}) \in \mathbf{C}$.

The sequence $\frac{A_{ct}}{A_{dt}}$ is decreasing so that it is bounded above by $\frac{A_{c0}}{A_{d0}}$. The sequence q_t is increasing so that it is bounded below by $q_0 > \underline{q}$ (and above by one).

Furthermore, one can show that $\frac{A_{ct}}{A_{dt}}$ is bounded below by $X(q_0) > Y(q^*)$. Whenever $(q_0, \frac{A_{c0}}{A_{d0}}) \in \mathbf{A}$, we have

$$\frac{A_{c0}}{A_{d0}} > X(q_0).$$

Since the map A is increasing in $\frac{A_{ct}}{A_{dt}}$ the previous inequality is equivalent to

$$\begin{aligned}
&A(\frac{A_{c0}}{A_{d0}}, q_0) > A(X(q_0), q_9),\\
\Leftrightarrow &\frac{A_{c1}}{A_{d1}} > X(q_0),\\
\Leftrightarrow &A(\frac{A_{c1}}{A_{d1}}, q_0) > X(q_0).
\end{aligned}$$

Furthermore $q_1 > q_0$ and the map A is increasing in q_t so that

$$\frac{A_{c2}}{A_{d2}} = A(\frac{A_{c1}}{A_{d1}}, q_1) > A(\frac{A_{c1}}{A_{d1}}, q_0) > X(q_0).$$

By recurrence I deduce that $\forall t,\ \frac{A_{ct}}{A_{dt}} > X(q_0)$. The sequence $\frac{A_{ct}}{A_{dt}}$ is bounded below by $X(q_0) > Y(q^*)$.

3. Consider $(q_0, \frac{A_{c0}}{A_{d0}}) \in \mathbf{B}$.

The sequence $\frac{A_{ct}}{A_{dt}}$ is increasing so that it is bounded below by $\frac{A_{c0}}{A_{d0}}$. The sequence q_t is decreasing so that it is bounded above by q_0.

One can show that the sequence q_t is bounded below by $Y^{-1}(\frac{A_{c0}}{A_{d0}}) > q^*$. Whenever $(q_0, \frac{A_{c0}}{A_{d0}}) \in \mathbf{B}$, we have

$$\begin{aligned}
&\frac{A_{c0}}{A_{d0}} < Y(q_0),\\
\Leftrightarrow &Y^{-1}(\frac{A_{c0}}{A_{d0}}) < q_0.
\end{aligned}$$

When $(q_0, \frac{A_{c0}}{A_{d0}}) \in \mathbf{B}$ (implying to $2d(e^\star)q_t - 1 < 0$), the map Q is increasing in q_t. Hence we have

$$\begin{aligned}
&Q(Y^{-1}(\frac{A_{c0}}{A_{d0}}), \frac{A_{c0}}{A_{d0}}) = q_1 < Q(q_0, \frac{A_{c0}}{A_{d0}}) = Y^{-1}(\frac{A_{c0}}{A_{d0}}),\\
\Leftrightarrow &Q(Y^{-1}(\frac{A_{c0}}{A_{d0}}), \frac{A_{c1}}{A_{d1}}) < Q(q_1, \frac{A_{c1}}{A_{d1}}) = q_2
\end{aligned}$$

Furthermore in **B** we have $\frac{A_{c1}}{A_{d1}} > \frac{A_{c0}}{A_{d0}}$ and the map Q is increasing in $\frac{A_{ct}}{A_{dt}}$ so that

$$Q(Y^{-1}(\frac{A_{c0}}{A_{d0}}), \frac{A_{c0}}{A_{d0}}) = Y^{-1}(\frac{A_{c0}}{A_{d0}}), \frac{A_{c0}}{A_{d0}} < Q(Y^{-1}(\frac{A_{c0}}{A_{d0}}), \frac{A_{c1}}{A_{d1}}).$$

This in turn implies

$$Y^{-1}(\frac{A_{c0}}{A_{d0}}) < q_2.$$

By recurrence, I deduce that $\forall t$, $q_t > Y^{-1}(\frac{A_{c0}}{A_{d0}})$. The sequence q_t is bounded below by $Y^{-1}(\frac{A_{c0}}{A_{d0}}) > q^*$.

Finally, I deduce that $\forall (q_0, \frac{A_{c0}}{A_{d0}}) \in \mathbf{B}$, we either have $(q_t, \frac{A_{ct}}{A_{dt}}) \in \mathbf{B}$ $\forall t$, or there exists t such that $(q_t, \frac{A_{ct}}{A_{dt}}) \in \mathbf{D}$.

4. Consider $(q_0, \frac{A_{c0}}{A_{d0}}) \in \mathbf{D}$.

Both q_t and $\frac{A_{ct}}{A_{dt}}$ are decreasing so that they are bounded above by q_0 and $\frac{A_{c0}}{A_{d0}}$.

We have $q_0 > \overline{q}$ and $\frac{A_{c0}}{A_{d0}} > \overline{\frac{A_c}{A_d}}$. The map Q is increasing in q_t (since $(q_0, \frac{A_{c0}}{A_{d0}}) \in \mathbf{D}$ implies to $2d(e^{\star})q_t - 1 < 0$) and in $\frac{A_{ct}}{A_{dt}}$. The map A is increasing in $\frac{A_{ct}}{A_{dt}}$ and q_t. Hence, we have

$$Q(q_0, \frac{A_{c0}}{A_{d0}}) = q_1 > Q(q_0, \overline{\frac{A_c}{A_d}}) > Q(\overline{q}, \overline{\frac{A_c}{A_d}}) = \overline{q},$$
$$A(q_0, \frac{A_{c0}}{A_{d0}}) = \frac{A_{c1}}{A_{d1}} > A(q_0, \overline{\frac{A_c}{A_d}}) > A(\overline{q}, \overline{\frac{A_c}{A_d}}) = \overline{\frac{A_c}{A_d}}.$$

By recurrence, I deduce that $\forall t$, $q_t > \overline{q}$ and $\frac{A_{ct}}{A_{dt}} > \overline{\frac{A_c}{A_d}}$.

5. One can conclude that for any $\frac{A_{c0}}{A_{d0}} > Y(q^{\star})$ and $q_0 > q^{\star}$, $(q_t, \frac{A_{ct}}{A_{dt}}) \in]q^*, q_0] \times]Y(q^*), \frac{A_{c0}}{A_{d0}}]$, that is both sequences q_t and $\frac{A_{ct}}{A_{dt}}$ are bounded. Furthermore, in $]q^*, q_0] \times]Y(q^*), \frac{A_{c0}}{A_{d0}}]$, there exists a unique steady state $(\overline{q}, \overline{\frac{A_c}{A_d}})$. Since both maps Q and A are continuous, one deduces that for any $\frac{A_{c0}}{A_{d0}} > Y(q^{\star})$ and $q_0 > q^{\star}$ $(q_t, \frac{A_c}{A_d})$ converges to $(\overline{q}, \overline{\frac{A_c}{A_d}})$.

• For $\frac{A_{c0}}{A_{d0}} > Y(\underline{q})$ and $q_0 > \underline{q}$, a similar reasoning leads to conclude that $(q_t, \frac{A_c}{A_d})$ converges to $(\overline{q}, \overline{\frac{A_c}{A_d}})$.

• Suppose that $\frac{A_{c0}}{A_{d0}} < Y(q_0)$ and $q_0 < \underline{q}$.

The sequence q_t is decreasing, we have $q_1 < q_0$ which implies $Y(q_0) < Y(q_1)$ so that $\frac{A_{c0}}{A_{d0}} < Y(q_1)$. Then there are two cases. Either $\frac{A_{ct}}{A_{dt}}$ is decreasing so that $\frac{A_{c1}}{A_{d1}} < Y(q_1)$. Or $\frac{A_{ct}}{A_{dt}}$ is increasing which is true only if $\frac{A_{c0}}{A_{d0}} < X(q_0) < X(\underline{q})$. Since the map A is increasing in $\frac{A_{ct}}{A_{dt}}$ we have

$$A(\frac{A_{c0}}{A_{d0}}) < A(X(\underline{q})),$$
$$\Leftrightarrow \frac{A_{c1}}{A_{d1}} < X(\underline{q}) < Y(q_1).$$

In both cases one concludes $\frac{A_{c1}}{A_{d1}} < Y(q_1)$. By recurrence we hence have $\frac{A_{ct}}{A_{dt}} < Y(q_t)\ \forall t$. The sequence q_t decreases over time. The sequence $\frac{A_{ct}}{A_{dt}}$ is bounded above by $Y(q_0)$ so that it converges. Since both A and Q are continuous, one deduces that $(q_t, \frac{A_{ct}}{A_{dt}})$ converges to the fixed point $(0, 1)$.

Part II of Proposition 1 (i.e., $\epsilon > \tilde{\epsilon}$).

• Suppose that $\frac{A_{c0}}{A_{d0}} > \max\{X(q_0), Y(q^*)\}$ (case A.), or $\frac{A_{c0}}{A_{d0}} > \max\{X_2(q_0), Y(q^*)\}$ (case B.).

Then, the sequence $\frac{A_{ct}}{A_{dt}}$ is increasing which implies $\frac{A_{ct}}{A_{dt}} > \max\{X(q_0), Y(q^*)\}\ \forall t$ (case A.) or $\frac{A_{ct}}{A_{dt}} > \max\{X_2(q_0), Y(q^*)\}\ \forall t$ (case B.). The sequence $\frac{A_{ct}}{A_{dt}}$ grows exponentially and tends to infinity.

Furthermore, since $\frac{A_{c0}}{A_{d0}} > Y(q^*)$. Then q_t is bounded below by q_0 and above by one. Or, q_t is decreasing which is equivalent to

$$\frac{A_{c0}}{A_{d0}} < Y(q_0),$$
$$\Leftrightarrow q^* < Y^{-1}(\frac{A_{c0}}{A_{d0}}) < q_0.$$

Using similar arguments than for part I (step 2. (ii)), the later inequality implies that $\forall t$ $q_t > Y^{-1}(\frac{A_{c0}}{A_{d0}}) > q^*$. One concludes that the sequence $q_t \in]q_0, 1]\ \forall t$.

The sequence q_t is bounded so that it converges to its steady state 1 or $\overline{q}$ (note that while the former steady state always exists, the later one exists under some parameter restrictions). The sequence $\frac{A_{ct}}{A_{dt}}$ is unbounded and converges to $+\infty$ which implies that $d(e(q_t, \frac{A_{ct}}{A_{dt}}))$ tends to one so that q_t tends to one.

• For $\frac{A_{c0}}{A_{d0}} < \min\{X(q_0), Y(q^*)\}$ (case A.) or for $\frac{A_{c0}}{A_{d0}} < \min\{X_1(q_0), Y(q^*\}$ (case B.), one can use similar arguments to show that $(q_t, \frac{A_{ct}}{A_{dt}})$ converges to $(0, 1)$.