A model of recycling and pollution control

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A model of recycling and pollution control. Consumption and production activities produce waste as a by-product. This waste is accumulated as a stock of polluted material which has a negative effect upon the utility of the consumer. To eliminate this negative effect we consider two ways of reducing the stock of pollution, recycling and disposal.

In many cases a decision must be made as to the resource allocation between the recycling of residues for reuse and the removal and disposal of polluted material. The latter, which reduces the negative effect of pollution is usually cheaper. In this paper we develop a recycling model that will both include the positive direct effect of the recycled good upon the consumer's utility function and determine the allocation of resources between the recycling activity and disposal. Thus, we will consider three activities: production of the original consumption good, production of the recycled good, and disposal.

The optimization problem is to allocate a given amount of labour between the three activities and maximize a stream of discounted utility, subject to dynamic constraints on the stock, production, and initial endowments.

Un modèle du recyclage et du contrôle de la pollution. Les activités de consommation et de production occasionnent des déchets. Ces sous-produits s'accumulent et forment un stock de matières polluantes qui produisent une utilité négative. L'auteur s'intéresse ici à deux façons de réduire ce stock et ses effets: le recyclage et l'élimination.

On doit souvent choisir entre consacrer des ressources au recyclage des résidus à des fins utiles ou en consacrer à l'enlèvement et à l'élimination des matières polluantes. Cette dernière façon de réduire les inconvénients de la pollution est ordinairement la moins coûteuse. L'article contient un modèle du recyclage dans lequel on tient compte à la fois de l'utilité positive des produits du recyclage et de l'utilité négative reliée aux activités de récupération. Le modèle détermine le partage des ressources entre le recyclage et l'élimination. On considère trois actions: la production initiale, la production au cours du recyclage et l'élimination.

L'optimisation consiste à répartir une certaine quantité de travail entre les

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trois activités tout en maximisant la valeur escomptée du flux d'utilité dans un cadre de contraintes relatives au stock de matières polluantes, à la production et aux ressources initiales.

La solution optimale comporte une diminution de la production et de la consommation du bien initial et une augmentation de celles du produit du recyclage. Comme l'article le montre, le gouvernement peut arriver à ces conditions optimales de façon décentralisée par une taxe à la consommation du bien initial, un subside pour le produit du recyclage et une garantie de couverture des coûts de l'élimination des matières polluantes. L'amélioration du marché des résidus réduit la nécessité de l'intervention gouvernementale.

Un marché parfait des résidus postule que le consommateur tienne compte dans ses achats des possibilités de revente des résidus de sa consommation. Les entreprises occupées au recyclage tiennent compte pour leur part des frais d'approvisionnement en résidus. De cette façon, il ne reste matière à intervention que pour les résidus polluants et non recyclables.

Two recent articles have addressed the question of waste accumulation and recycling. Plourde (1972) treats recycling as a productive process intended to decrease the stock of pollution. This stock results from the accumulation of waste, which accompanies production and consumption. Pollution, having undesirable effects on consumers, leads to a reallocation of resources to reduce its quantity. Thus, recycling is a waste disposal service. Plourde does not admit any positive direct effect on consumers or producers from recycled goods. Smith (1972) treats recycling as a reprocessing of the residue from consumption. The reprocessing activity represents a utility loss, i.e. a negative effect upon consumers' utility. Specifically, forcing consumers to retain pollutants such as aluminum cans or glass bottles can represent a loss of utility when disposal is considered a costless activity by the consumers. The pollution side of the problem is discussed by Keeler, Spence, and Zeckhauser (1972).

In this paper a recycling model is developed that will both include the positive direct effect of the recycled good upon the consumer's utility function and determine the allocation of resources between the recycling activity and disposal.

In many cases a decision must be made as to the resource allocation between the recycling of residues and the removal and disposal of polluted material. The latter, which reduces the negative effect of pollution, is usually cheaper. Consider the disposal of water waste, which is a residue from the commercial or household use of clean water. Without recycling or disposal it would be accumulated into a stock of pollution that had negative health and aesthetic effects upon the community. This negative effect creates an incentive for recycling and for disposal. One method to reduce the pollutant stock is to build and maintain a drainage system. Another method, using a more sophisticated and expensive technology, refines and purifies the waste water into a reusable product for cooling electric turbines, watering fields, or even drinking. In some cases the cost of recycling consists mainly of collection and accumulation, as in the use of old newspapers as a wrapping and packing material and the use of wood residue as a fuel. In order to analyse several of these alternatives we shall construct a model that will describe the alternatives in recycling from a stock of pollution by either removal and disposal or by recycling to final consumption.

Following Smith and Plourde, the problem will be to maximize a discounted stream of benefits subject to dynamic constraints on the stock, production, and initial endowments. First, the model will be presented and the nature of the optimal solution described and interpreted. The second section will contain the efficiency gains from the internalization of pollution externalities by comparing the optimal centralized solution to a laissez-faire solution. The dynamic solution and its characteristics will be described in the third section. The fourth section will contain a discussion of various pollution control policies designed to diminish the quantity of refuse discarded by consumers.

THE MODEL

Consumers enjoy a consumption good c, a certain proportion of which γ becomes a waste. The waste is accumulated in the environment. Let G represent the amount of this stock of polluted material. The stock G enters into the consumers' utility function and represents the negative effect of pollution. The effect of pollution on consumers has the nature of a common property resource since individual consumers usually will not act unilaterally to reduce the stock G. There are two alternatives for reducing pollution. The first is disposal of the pollutant. The second, more expensive, method employs a technique which recycles the stock of pollution into a reusable consumption good y. In general we shall be able to distinguish clearly between the original consumption good c and the recycled good y. For example, under certain conditions, drinking water refuse can be disposed via drainage or recycled via a purification technique for use in watering lawns.

Let the utility function be represented by $U(c,y,G)$, where c is the original commodity, y is the recycled commodity, and G is the stock of pollution. Furthermore, let $U_1 = \partial U/\partial c > 0$, $U_2 = \partial U/\partial y > 0$, and $U_3 = \partial U/\partial G < 0$. Assume that there is a constant amount of labour L over time which serves as an input into various production activities. Let $c = f(L_1)$, $y = h(L_2)$, where $f' > 0$, $h' > 0$, $f'' < 0$, $h'' < 0$ represent the production functions of commodities c and y respectively.[1] Let $g(L_3)$,

1 Note that y is also a function of the refuse of c as an input. For expository argument we assume here that the stock of pollutant has the nature of a common property resource, that is, every recycling firm has free access to it.

where $g' > 0$, $g'' < 0$, represent the amount disposed of from the stock of pollution. At the optimal levels of L_2 and L_3 it will be assumed that $g'(L_3) > h'(L_2)$, reflecting the assumption that disposal is less expensive than recycling.

If, in general $g'(L_3) < h'(L_2)$ for all L_2 and L_3, that is, the marginal productivity of labour in recycling the refuse is greater than the marginal productivity of labour in the disposal activity, then the result will be a corner solution where $g = 0$. In this case no disposal activity will take place because recycling not only eliminates pollution but provides a consumption product as well.

The optimization problem is to allocate the given amount of labour L between the three activities to maximize a stream of discounted utility:

$$\text{Max } J = \int_0^\infty U(c,y,G)\, e^{-rt}\, dt, \tag{1}$$

subject to

$$c - f(L_1) = 0, \tag{2}$$
$$y - h(L_2) = 0, \tag{3}$$
$$L - L_1 - L_2 - L_3 = 0, \tag{4}$$
$$dG/dt = \gamma c - g(L_3) - y, \quad 0 < \gamma < 1.^2 \tag{5}$$

Using the Pontryagin maximum principle,[3] the present value Hamiltonian is

$$H = U(c,y,G) + p(\gamma c - g(L_3) - y) + \lambda_1(f(L_1) - c) \\ + \lambda_2(h(L_2) - y) + w(L - L_1 - L_2 - L_3), \tag{6}$$

where r is the discount rate, λ_1, λ_2, and w are the Lagrange multipliers. The costate variable p represents the value, at the optimum, of adding an extra unit of pollution to the stock of pollution G, i.e. if $J^* = \text{Max } J$ (the maximand) then $p = \partial J^*/\partial G$. Since the stock of pollution is a nuisance to consumers, this price is negative. For optimality, the following first-order conditions must hold at each moment of time:

$$\partial H/\partial c = U_1 + \gamma p - \lambda_1 = 0, \tag{7}$$

$$\partial H/\partial y = U_2 - p - \lambda_2 = 0, \tag{8}$$

2 To simplify the presentation we assume here that the recycled commodity y leaves no residue after consumption. In a more general case, if the portion of the residue from consuming y is ϵ, the dynamic equation (5) will become

$$dG/dt = \gamma c + \epsilon y - g(L_3) - y.$$

As a result, the only change in the optimal condition will be in equation (8), which will read

$$\partial U/\partial y - (1 - \epsilon)p - \lambda_2 = 0.$$

We also neglect here the possibility of natural biodecomposition.

3 For an exposition of this technique see Arrow and Kurz (1970), Quirk and Smith (1970), or Pontryagin (1962).

$$\partial H/\partial L_1 = \lambda_1 f' - w = 0, \tag{9}$$

$$\partial H/\partial L_2 = \lambda_2 h' - w = 0, \tag{10}$$

$$\partial H/\partial L_3 = - pg' - w = 0, \tag{11}$$

$$dp/dt = rp - \partial H/\partial G = rp - U_3. \tag{12}$$

The transversality conditions are

$$\lim_{t\to\infty} e^{-rt}(-p(t)) \geqslant 0,$$

$$\lim_{t\to\infty} e^{-rt}(-p(t))G(t) = 0.$$

A sufficient condition for optimality according to Arrow and Kurz (1970, 49, Proposition 8) is the concavity of

$$H^0 = \operatorname*{Max}_{c,y,L_1,L_2,L_3} H \quad ,$$

with respect to G for given p. In our case the solutions for the control variables c, y, L_1, L_2, and L_3 do not involve G. Thus a sufficient condition in our model is the concavity of the utility function with respect to G.

From equations (10) and (11) we can solve for $\lambda_2 = - pg'/h'$. Substituting this into equation (8) yields

$$U_2 = - p(g'/h' - 1).$$

Given that $p < 0$ (from equation 11), an interior solution can only be formed with $U_2 > 0$, when $g' > h'$.

In the above context the following corner solutions are possible: when $\lambda_2 h' - w < 0$, then $L_2 = 0$ and there is no recycling activity. When $- pg' - w < 0$, then $L_3 = 0$ and there is no disposal activity. Since c can be regarded as essential, $L_1 > 0$ always. The following analysis will concentrate only on the interior solution.

Taking w as the implicit wage rate, equations (9) and (10) show that λ_1 is the marginal private cost of producing the consumption good c, while λ_2 is the marginal private cost of producing the recycled commodity y. Equation (11) also explains the economic meaning of $- p$, the marginal cost of the disposal activity. From equation (1) we see that $\partial U/\partial c = \lambda_1 - \gamma p > \lambda_1$. At the optimum, the marginal utility of the consumption good is not equal to the marginal private cost of producing the consumption good λ_1, but rather to λ_1 plus the value of the stock of pollution, which has been increased by γ, after consuming one extra unit of the consumption good c. Because of the concavity of the utility function, the consumption of c is lower, at the optimum, relative to the amount which would have been consumed had marginal utility been equated to marginal cost λ_1. Equation (8) states the condition that, at the optimum, the marginal utility of consumption will be less than the marginal private cost of production of the recycled good y. The reason for the differences in the marginal utilities of consumption between both original and recycled goods and their respective marginal private costs is that the controlled solution internalizes the external effects of the stock of pollution. This is a result of the fact that the recycled commodity y increases consumers' utility indirectly by reducing the negative effect of pollution.

The intertemporal dynamic equation (12) states that along the optimal path the net rate of price change over time, $(dp/dt)/p - r$, will be equal to the value of the marginal utility, in terms of consumption, of removing an extra unit of pollution from the stock U_3.

LAISSEZ FAIRE VS CONTROLLED SOLUTIONS

Given that the utility effects of the stock of pollution G have the nature of a public good (bad), no private action is expected which will reduce the quantity of consumption of c, the pollution-causing commodity. Moreover, as there is no marketable good in the disposal activity there is no private incentive to engage in disposal services. Thus a laissez-faire condition which does not internalize the stock of pollution externalities will lead to a suboptimal solution.

We can compare the laissez-faire solution with a controlled solution. Take the market prices of c and y under laissez faire as q_1 and q_2 (in utility units) respectively. In the production of c, producers maximize the discounted stream of profits over time,

$$\text{Max} \int_0^\infty (q_1 f(L_1) - wL_1)e^{-rt}dt,$$

which yields the usual optimal condition of equality between the value of marginal product and input price:

$$q_1 f' = w. \tag{14}$$

The recycling firm maximizes an analogous function,

$$\text{Max} \int_0^\infty (q_2 h(L_2) - wL_2)e^{-rt}dt,$$

with the optimal condition

$$q_2 h' = w. \tag{15}$$

A consumer with initial endowments of labour who maximizes his discounted sum of utility will equate his marginal utility to the market prices. The laissez faire optimal conditions will be

$$U_1 = w/f' = q_1 = \lambda_1, \tag{16}$$

$$U_2 = w/h' = q_2 = \lambda_2. \tag{17}$$

The marginal utilities of consumption goods are equal to the marginal costs of producing them. Comparing these with the control solutions (7) and (8), we observe that the laissez faire solution is suboptimal and that consumers should refrain from their excessively high consumption of commodity c, which produces pollution along the way. At the same time they should increase consumption of the recycled good y because consuming it improves the environment. Since the disposal activity is not marketable, no private firms will be engaged in this activity unless some central authority compensates the firm by subsidizing its cost of production wL_3 or by paying this firm the social price $-p$ for every unit of pollution it disposes. Alternatively, the government can decentralize the economy with a tax-cum-subsidy scheme. A comparison of equations (7) and (16) calls for a unit tax of $t_c = -\gamma p$ on the consumption good c. A comparison between equations (8) and (17) calls for a subsidy of $t_y = - p$ per unit of recycled commodity y as well as a subsidy of $t_d = - p$ per unit of removed pollution for disposal.[4] Again, as demonstrated by Plourde (1972), if the economy starts where the stock of pollution is smaller than the steady-state solution i.e. $dG/dt > 0$, then $- p\gamma c + pg(L_3) + py > 0$, and the government will produce a budget surplus in the transition toward the steady state. When the movement towards steady state involves a reduction in the initial stock of pollution, i.e. $dG/dt < 0$, then the government suffers a transitory budget deficit. In the steady state, i.e. when $dp/dt = 0$ and $dG/dt = 0$, the governmental budget of these taxes and subsidies will be in balance. Also, in the laissez case since $p = 0 < p^*$, dG/dt is always positive. This result stems from our simplifying assumption of omitting natural biodecomposition from our model.

DYNAMIC SOLUTIONS

The solution for the optimal trajectory and the optimal path of the control and state variables involve solving L_1, L_2, L_3, c, and y as functions of p and G from equations (7) to (11), and substituting these solutions into equations (5) and (12). Using the initial conditions for the stock G and the transversality conditions enables one to solve for $G(t)$ and $p(t)$ and thus the allocation of inputs and outputs over time.

For simplicity and for illustrative purposes in the dynamic exposition, let us follow Plourde and Smith by assuming that the utility function is additively separable, i.e. the utility function takes the form

$$U(c,y,G) = U^1(c) + U^2(y) + U^3(G). \tag{18}$$

4 A government intervention in changing the pattern of consumption and production of the recycled commodity y is not needed if there exists a perfect market for consumption residue.

Steady-state solutions are obtained when $dG/dt = 0$ and $dp/dt = 0$:

$$dp/dt = rp - \partial H/\partial G = rp - \partial U/\partial G = 0, \tag{19}$$

$$dG/dt = \gamma c - g(L_3) - y = 0. \tag{20}$$

For $dp/dt = 0$, $dp/dG = (\partial^2 U/\partial G^2)/r < 0$. For $dG/dt = 0$, $dp/dG = 0$.[5] If $U^{3\prime}(0) > rp^*$ in the steady state, the optimal amount of the stock of polluted material will be positive. The optimal trajectory is described in Figure 1, from which it is clear that if we start with a stock of pollution $G < G^*$, then $dG/dt > 0$ and $dp/dt < 0$. On the other hand if we start with a quantity of pollution $G > G^*$ then the movement towards a steady state is associated with $dG/dt < 0$ and $dp/dt > 0$. The pattern of allocating labour into the three activities along the optimal trajectory towards a steady-state position can be determined. In particular, labour inputs change as the value of removing pollution $(-p)$ changes. It can be shown[6] that $dL_1/d(-p) < 0$, $dL_2/d(-p) \gtreqless 0$, $dL_3/d(-p) > 0$. When consumers are willing to pay more for pollution removal, i.e. when $-p$ is higher, allocation of L_3 for pollution removal activity is increased and at the same time the production of the consumption good which contributes to pollution decreases. The sign of $dL_2/d(-p)$ is ambiguous. However, around the steady-state point the sign of $dL_2/d(-p)$ is negative. This result is obtained from equation (20). At the steady state $dG/dt = 0$. Thus,

$$\gamma f' dL_1/d(-p) - g' dL_3/d(-p) - h' dL_2/d(-p) = 0;$$

since $dL_1/d(-p) < 0$, $dL_3/d(-p) > 0$, we have $dL_2/d(-p) < 0$.

POLLUTION CONTROL

In the foregoing analysis the portion of consumption that accumulates as pollution γ is a constant parameter, i.e. a technological coefficient. In this section, some pollution-control policies that can affect and determine the optimal level of γ will be discussed.

Following Smith, assume that it is in the consumer's interest to litter, so that a constraint which forces him to retain used beer cans or to return used bottles produces a disutility. In this context let γ, the fraction of consumption that is added to pollution, be the factor that affects the con-

5 From our additively separable assumption, equation (20) is not a function of G. Thus, implicit derivation yields this result.

6 Use equations (7) to (11) to form the following system:

$$\partial U/\partial c + \gamma p + pg'/f' = 0.$$
$$\partial U/\partial y - p + pg'/h' = 0,$$
$$L_1 + L_2 + L_3 - L = 0,$$

and apply implicit derivation.

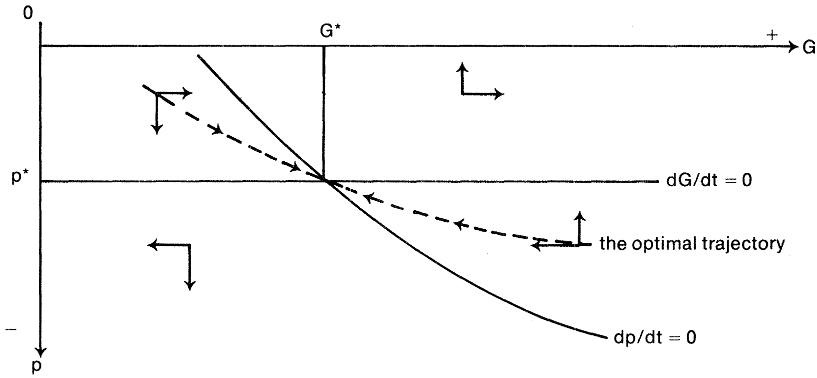

FIGURE 1

sumer's utility. Assume that the consumer is interested in maximizing the discounted stream of the following utility function:

$$U(c,y,G) + \phi(\gamma),$$

where $\phi'(\gamma) > 0$ and $\phi(\gamma)$ represents the 'utility of littering.' The sign of $\phi'(\gamma)$ is positive because the larger the proportion of residue to total consumption c, the greater the disutility of having to retain the residue from consumption.

The optimization problem now becomes

$$\text{Max } J = \int_0^\infty [U(c, y, G) + \phi(\gamma)]e^{-rt}dt, \tag{21}$$

subject to

$$\begin{aligned} &c - f(L_1) = 0,\\ &y - h(L_2) = 0,\\ &L - L_1 - L_2 - L_3 = 0,\\ &dG/dt = \gamma c - g(L_3) - y \end{aligned} \tag{22}$$

with $0 \leq \gamma \leq \gamma_0$ (we assume that γ has a certain technical maximum, $\gamma_0 < 1$). In this formulation the system of first-order conditions (7) to (12) holds, plus the addition of the following equation:

$$\partial H/\partial\gamma = \phi'(\gamma) + pc = 0. \tag{23}$$

Thus equilibrium occurs when the marginal utility of γ is equal to the price society is willing to pay for the removal of an extra unit of pollution caused by this changing γ.

To analyse the laissez faire solution, reformulate the consumer's maximization problem as

$$\text{Max } J = \int_0^\infty [U(c, y, G) + \phi(\gamma)]e^{-rt}dt,$$

subject to

$$0 \leq \gamma \leq \gamma_0.$$

The first order conditions are again (taking into account the budget constraint) $U_1 = q_1$, $U_2 = q_2$.

Assuming that the consumer ignores his own action of pollution upon himself, he will choose $\gamma^* = \gamma_0$, where γ^* is the optimal laissez faire portion of consumption waste. However, the government's tax-cum-subsidy scheme, discussed in the second section, will bring about social optimality in this case as well. Since the budget constraint now reflects the tax-cum-subsidy,

$$(q_1 - p\gamma)c + (q_2 + p)y - wL = 0,$$

the optimal conditions (7) and (8) are met. Moreover, with respect to γ, $\partial U/\partial\gamma + cp = 0$ or $\phi'(\gamma) + pc = 0$, which is the optimal condition (23). Alternatively, it can be argued that the proportion of consumption discarded γ affects production (costs) rather than a consumer's utility. For this case, assume that γ affects the production of the recycled commodity y. Specifically, let $y = h(L_2, \gamma)$ with $h_1 = \partial h/\partial L_2 > 0$ and $h_2 = \partial h/\partial\gamma > 0$, i.e. the marginal productivities of L and γ are positive. This means that a larger rate of accumulation of waste augments the production of the recycled good. Therefore, in addition to the optimal conditions (7) to (11),

$$pc + \lambda_2 h_2 = 0, \tag{24}$$

which is similar to condition (23). The value of the marginal productivity of γ is equal to the price society is willing to pay in order to remove an additional unit of pollution caused by a change in the value of γ.

CONCLUSION

In the foregoing discussion the optimal allocation of productive inputs has been solved for over time among the activities of producing original goods, recycling, and disposal. The recycling activity reclaims refuse which would have been disposed of and thereby offers an alternative means of goods production. The usual allocation conditions that equate the value of the marginal product of the input to its rental price at each moment of time are a result of the optimization procedure. The shadow prices employed to evaluate marginal products in various activities differ from private marginal costs. This difference is a consequence of the fact that the optimal centralized solution internalizes the consumption externality and the existence of inefficiencies, if any, in the pollutant markets. The optimal solution calls for a decrease of the consumption of the original good, together with an increase in the production and consumption of the recycled good. The government can achieve these optimal conditions by decentralizing the economy using a tax on the consumption of the original good, a subsidy for the recycled good, and a guarantee that the cost of disposing pollution in the disposal activity will be covered. Improving the market for refuse will eliminate the necessity of governmental intervention in this market. In a perfect market for refuse, the consumer, in his buying decision, will take into account the possibility of selling residues resulting from his consumption activities. Recycling firms will include this cost of purchasing inputs in their marginal cost considerations. This procedure leaves the need for intervention only for the non-recyclable pollutant that has negative effects upon consumer's utilities. Aspects of this process are evident in the markets for scrap metals. To a lesser extent, markets exist in the recycling of used paper and pulp products, aluminum cans, and glass bottles. Currently, however, these reclamation programs may be ecologically motivated community actions rather than active markets for residues.

Water and some general refuse, both of which can be recycled commercially, have not developed viable markets. When refuse markets are not forthcoming, government intervention is necessary in order to reduce pollution and its effects upon consumers' utility functions.

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