Earth-Science Reviews 194 (2019) 283–305

Contents lists available at ScienceDirect

Earth-Science Reviews

journal homepage: www.elsevier.com/locate/earscirev

How to build a digital river

Rocko A. Brown[a,b], Gregory B. Pasternack[a,*]

[a] Department of Land, Air, and Water Resources, University of California, Davis, One Shields Avenue, Davis, CA, USA
[b] Cramer Fish Sciences, 3300 Industrial Blvd, Suite 100, West Sacramento, CA, USA

ARTICLE INFO

Keywords:
Topography
Rivers
Morphology
Landforms
Digital elevation modeling

ABSTRACT

There has been an increasing practice of creating Earth-like, realistic synthetic landscapes by Earth scientists and computer scientists for a variety of applications. While together these two fields have made significant scientific and social contributions to creating synthetic landscapes, it is presently infeasible to build artificial digital rivers that represent the diversity found on Earth. To understand and summarize the state of the science of rendering artificial river topography, we reviewed > 225 scientific articles and produced a road map for artificial synthesis of digital river topography. We broadly classify methods of digital river synthesis by whether they are driven by expert-based decisions or are strategic in the use of rules for objective rendering, with some rules being physics-based theories of river morphogenesis. Expert approaches include map, brush, geometric and interactive design. Strategic approaches include deterministic equilibrium models, morphodynamic models, and stochastic approaches. For each approach we discuss the conceptual basis for each method and how they can be applied. Readers can then identify what methods can create different types of digital riverscapes. We close by discussing how cross pollination can serve geomorphology and computer science, the role of digital rivers in furthering geoscience progress, and future directions in digital river synthesis.

1. Introduction

There has been a steady practice of creating synthetic (aka artificial) landscapes by Earth scientists, computer scientists, landscape architects, graphic artists, and civil engineers (Goodchild, 2008, 2012; most figures in this article illustrate such applications). Rivers are a key component of real and synthetic digital landscapes. Digital rivers are artificial rivers created and experienced using computers; they represent major elements of the digital Earth (Goodchild, 2012). Reviews exist for creating entire landscape terrains from geomorphology (Coulthard, 2001; Martin and Church, 2004; Willgoose, 2005; Tucker and Hancock, 2010) and computer science (Smelik et al., 2014), but none exist that combine these perspectives into a single scientific road map spanning theories and procedures for creation of synthetic river topography. The purpose of this article is to provide such a review.

The purpose of synthesizing artificial landscapes varies considerably, resulting in a diverse spectrum of theories and methods capable of creating different virtual realizations of artificial river corridors. Traditionally, Earth scientists, especially geomorphologists, have explored synthetic terrain generation through landscape evolution models (LEMs). In this context, the goal has been to understand the mathematical requirements for creating observed landscapes as well as how they will evolve in "what-if" scenarios, typically related to different tectonic and climatic regimes (Tucker and Slingerland, 1997). Independently, computer scientists and graphic artists approached terrain modeling with the goal of creating realistic virtual scenes with limited user input and computing resources (Doran and Parberry, 2010). In an applied sense, river scientists, engineers, and landscape architects also create digital river topography for a wide variety of uses, such as experimentation (Brown et al., 2014), irrigation (Lacey, 1929), navigation (Bhowmik and Adams, 1986), recreation, flow and sediment regime management (Chang and Osmolski, 1988), and river restoration (Pasternack, 2013). While these fields have made significant contributions to creating synthetic landscapes, there does not exist a review on this topic that spans disciplines. This remains a tremendous gap in a communal understanding of the state of ideas and practices in building digital rivers.

There are several reasons why artificial digital river topography is important for Earth scientists, engineers, computer scientists, landscape architects, graphic artists, and river restoration designers. First, fluvial geomorphologists already use synthetic channels to investigate form-process linkages (Wohl et al., 1999; Cao et al., 2003; Pasternack et al., 2008; Brown et al., 2014) and potentially to test the realism of landscape-scale morphodynamic models (Hillier et al., 2015). Second, in

* Corresponding author.
E-mail addresses: rokbrown@ucdavis.edu (R.A. Brown), gpast@ucdavis.edu (G.B. Pasternack).

https://doi.org/10.1016/j.earscirev.2019.04.028
Received 24 October 2018; Received in revised form 24 April 2019; Accepted 30 April 2019
Available online 07 May 2019

virtual scene generation, artificial river topography enables simulated water flow to dynamically interact with a non-trivial boundary so that water speed and water surface elevation can be spatially explicit variables as opposed to flat water terrains being used. With the advances in computational fluid mechanics and reduced complexity models, it is not unrealistic to expect for dynamic water flow to become an integral aspect of digital landscapes beyond their current use in video games and movies. Third, virtual scenes with synthetic rivers would enable scientists, engineers, and stakeholders (i.e., the role players) to interact with river topography and derivative environmental simulations in more realistic ways than having a smooth and uniform riverbed. Video game players already have such interactions with flow, fish, and other aquatic entities in digital rivers for fun, but this could be put to practical use.

Multiple disciplines would benefit from an overview of the various ways synthetic digital river topography can be generated. Moreover, it would benefit all communities by guiding future modeling efforts with an understanding of what the current palette of tools and methods can generate. There is a plethora of applications outside of fluvial geomorphology, such as virtual reality and scene generation, education, and river channel design, yet there is no comprehensive guidance that speaks to the multidisciplinary aspect of creating artificial river topography.

This review bridges the gap between scientists, who study linkages between process and form in the environment, and practitioners who create virtual landscapes for a variety of practical and entertainment applications. The objectives of this scientific review article are to: (i) present a road map for the synthesis of digital rivers from existing methods, (ii) discuss the conceptual basis for each method and how they can be applied, and (iii) discuss emerging methods and future directions for building digital rivers. This review focuses on nontidal rivers with water flow driven by gravity, although there is some mention of distributary channels that may occur on alluvial fans and deltas as well as in tidal coastal lowlands broadly. > 225 articles were reviewed. This list is not exhaustive, because so many different topics are reviewed. Rather, our approach has been to highlight key studies across the breadth of the scientific road map that help meet the article's goals.

2. Road map for building digital rivers

Fig. 1 is a flow chart to guide artificial synthesis of digital river topography based on current approaches. First, the overall rationale of the flow chart is discussed here along with nomenclature. Second, we briefly discuss the various routes for synthetic terrain generation. Third, we discuss river generation when a surrounding terrain is not present. Later in the article expert and strategic synthesis are discussed in more detail and then each method is reviewed.

The first step in building a digital river is determining whether a terrain outside of the river channel exists or is even needed. Primarily this serves to establish how the planimetric alignment of the river or river network is located on the Earth's surface. If there is an existing terrain with a channel on an alignment, then reach- (10^2–10^3 channel widths) or segment- scale (10^3–10^4 channel widths) characteristics used to scale the size of the river to the terrain need to be extracted for subsequent steps. For the rest of this review we will refer to only river reaches for brevity, but the concepts apply to segments, too, ideally by breaking them into reaches and proceeding to apply this framework on each one, with some transitional blending from reach to reach. If a terrain is needed, but does not exist, then one can be created from a variety of approaches discussed in section 3. When a terrain is not needed, the user should conceptualize the purpose of modeling along with the desired river typology, scale and resolution. At this stage, the user can create river channel topography using either expert or strategic approaches.

There are two broad approaches to digital river creation that we term as either expert or strategic synthesis. In expert synthesis a user explicitly describes each aspect of the river being created. It includes (i) geometric, (ii) object, (iii) map, and (iv) brush approaches (discussed in Section 5). In strategic synthesis a user specifies a set of initial attributes of the digital river, but subsequent modeling uses these attributes to yield a "heightmap" (i.e., a 2D grid whose cell value is a height, making it a 3D digital elevation model), analogous to procedural terrain generation methods in the computer science literature (discussed in Section 6). Strategic synthesis relies on the rules or specified probabilities to determine the final heightmap outcome and includes deterministic and stochastic models. The difference between these two approaches to strategic synthesis is that deterministic approaches are driven by underlying mathematical equations based on mechanistic physics (as revealed through theory and empiricism), while stochastic approaches represent terrains that have a chance to occur with set probabilities and begin with random seeds. Deterministic approaches include equilibrium models, traditional morphodynamic models, cellular automata variants, and discrete particle models. Each of these requires that initial conditions be specified such as the incoming flow and sediment load and initial channel geometry. There are similarities in that these all evaluate the time evolution of the initial conditions specified, but they differ in how the underlying mathematical rules are implemented, both conceptually and computationally. Stochastic models include inverse spectral, auto-regressive and object-based approaches.

3. Generating synthetic landscape terrains

We define three approaches to landscape terrain synthesis: (i) geomorphic landscape evolution models (LEMs), (ii) procedural models, and (iii) expert-based modeling (Table 1). Each of these approaches has different goals in creating terrains that have shaped their evolution through time. In the next paragraphs we first define each approach and provide a cursory overview. Many reviews exist for LEMs (e.g. Nicholas, 2005; Fonstad, 2006; Wilgoose, 2005; Tucker and Hancock, 2010) and procedural models (Hendrikx et al., 2013; Smelik et al., 2014), while expert techniques are rarely discussed in peer-reviewed literature. Hillier et al. (2015) discussed a few approaches to creating synthetic terrains as analogs for testing the realism in LEMs.

3.1. LEMs

Geomorphic modeling uses geomorphic transport equations for erosion, weathering, and deposition (Dietrich et al., 2013; Tucker and Hancock, 2010) to generate steady and unsteady terrain states. Commonly called landscape evolution models, these approaches aim to understand and replicate essential processes that shape landforms over geologic time. LEMs numerically model landscape-scale topographic change through geologic time, drawing on analytical and statistical geomorphology through mass conservation and heuristic transport equations (Wilgoose, 2005; Tucker and Hancock, 2010). Some LEMs include sub-models for soils and tectonics (Tucker and Slingerland, 1994), vegetation (Collins et al., 2004) and climate (Chase, 1992; Coulthard et al., 2002). While LEMs were founded on exploring Earth surface processes in broader space and time scales, they do offer a potential route for creating artificial terrains.

LEMs have employed varying approaches to deal with the fact that geomorphic processes can operate over variable spatial and temporal scales. Temporal scale variability can be controlled by simulation time steps, while spatial scale variability is often addressed through the computational domain of the landscape. The latter is a significant driver as to the resolution of river network typology and topography. Some LEMs use adaptive and irregular meshes, so more nodes are present in areas with more activity (e.g. Braun and Sambridge, 1997; Tucker et al., 2001b).

Commonly, channel widths are 3–4 orders of magnitude smaller than basin width so that channels are effectively sub-grid scale features (Tucker and Hancock, 2010). It is not that these models cannot create

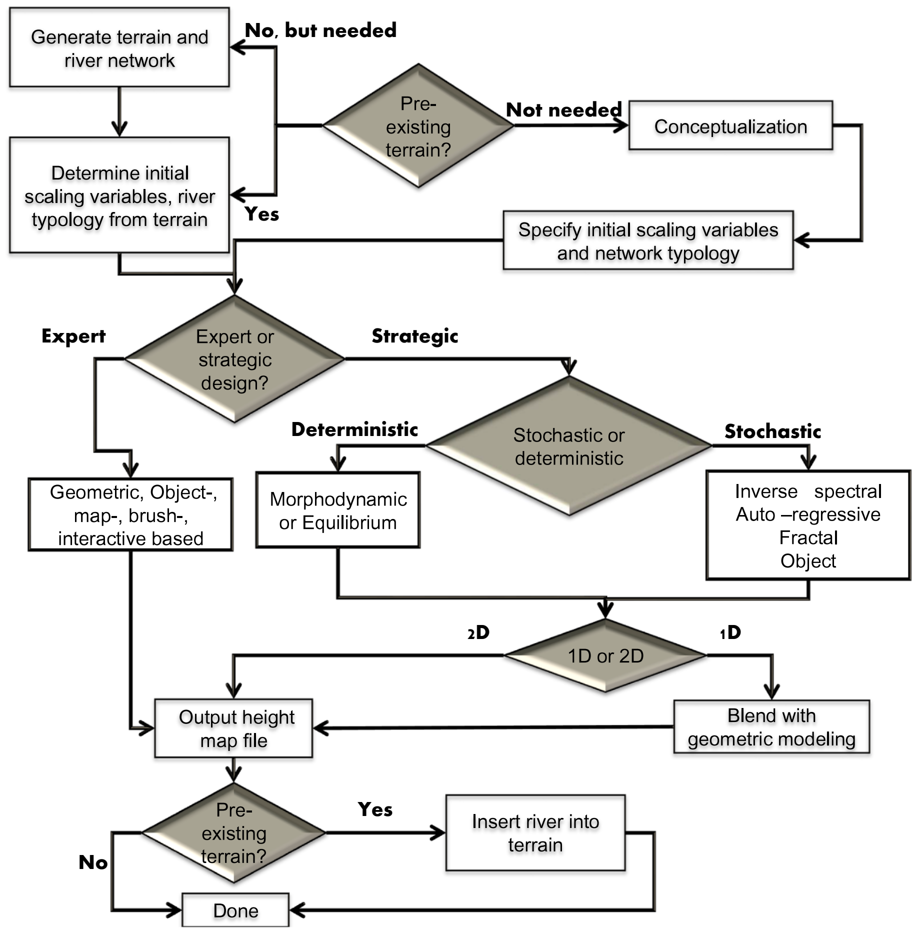

Fig. 1. A flow chart for building synthetic digital rivers.

Table 1
Conceptual basis and goals for the three primary approaches to terrain generation.

Type	Concept	Goals
Procedural	Uses algorithms that blend rules with randomness. Can incorporate geomorphic laws	Visual realism Rapid generation Some user control
Expert	Completely user defined	Visual realism or engineering design
Geomorphic	Driven by geomorphic theory on how landscapes form and evolve	Replicate essential physics Identify relevant processes and timescales

river topography per se, but they were never intended for that purpose, because many address broader space and time scale processes over entire landscapes (Willgoose et al., 1991; Chase, 1992; Tucker and Slingerland, 1994; Banavar et al., 1997; Rodriguez-Iturbe and Rinaldo, 1997). It is possible to indirectly resolve sub-channel width scale features in basin scale LEMs (e.g. Stark and Stark, 2001; Tucker and Slingerland, 1994; Willgoose et al., 1991), but this comes at the expense of more complicated parameterization of sub models. Despite the lack of emphasis on detailed channel dynamics related to river topography, most LEMs can at least create river network typology (Coulthard, 2001; Wiel et al., 2007; Tucker and Hancock, 2010). Lastly, some LEMs can nest small grid cells within larger meshes, so that model time can be concentrated on relevant areas of geomorphic change (Coulthard, 2001). Notably, the Cellular Automaton Evolutionary Slope and River model (CAESAR; Coulthard et al., 1996; Coulthard et al., 2007) is capable of modeling river topography at certain scales as discussed in more detail in Section 6.1.2.3.

Coulthard (2001) reviewed several free LEM software packages including CASCADE (Braun and Sambridge, 1997), SIBERIA (Willgoose, 2005), GOLEM (Tucker and Slingerland, 1994), CHILD (Tucker et al., 2001a), and CAESAR (Coulthard et al., 2002; also discussed in Section 6.1.2.3) and discusses tradeoffs and capabilities. Coulthard (2001) suggested that CASCADE and GOLEM are better suited for large-scale, long-term simulations, whereas SIBERIA, CAESAR and CHILD may be better for shorter periods requiring higher resolution. While many programs are free, they are in a variety of programming formats and offered for researchers without typical user interface elements necessary to be considered user-friendly (Coulthard, 2001). Interested readers are also recommended to visit the Community Surface Dynamic Modeling Systems website (http://csdms.colorado.edu/wiki/Main_Page), where they can license, upload, and share LEMs.

3.2. *Procedural*

Procedural modeling is a term used to describe the generation of 3D objects and environments automatically through rules, parameters and iterative algorithms (Ebert and Musgrave, 1998; Smelik et al., 2014). These methods have been pioneered by computer scientists to generate realistic synthetic landscapes at the individual mountain to regional scales for virtual reality, video games, and even flight simulations (Cerqueira et al., 2013). Commonly, procedural terrain methods strive for rapidly generating terrains with the goal of visual realism (Hendrikx et al., 2013; Smelik et al., 2014). Generally, one can further classify procedural methods as automated or semi-automated. Automated models create terrains from basic user inputs such as the type, scale, and extent of the desired landforms, while semi-automated models allow for user input during terrain generation. Procedural terrain generation has historically relied heavily on fractal geometry concepts (Mandelbrot and Van Ness, 1968; Mandelbrot, 1975; Fournier et al., 1982) for automated generation. Recent advances include interactive sketching (Smelik et al., 2010, 2011), software agents (Doran and Parberry, 2010), genetic algorithms (Saunders, 2006; Raffe et al., 2012), and procedural blocks (Genevaux et al., 2013).

Within procedural modeling there has been an emphasis on algorithms in which the development of river network typology is a significant driver for generating the surrounding terrain. Some algorithms create the river first and then surrounding terrains, while others work the opposite (Kelley et al., 1988), or create both in tandem (Musgrave et al., 1989; Prusinkiewicz and Hammel, 1993). Numerous algorithms of fractal river network synthesis within existing terrains have been explored under the term *fractal river basins* (Rodriguez-Iturbe et al., 1994; Banavar et al., 1997; Rodriguez-Iturbe and Rinaldo, 1997). Despite the benefits of fractal Brownian motion and iterative fractals in speed, Nagashima (1998) argued that modeled mountains and valleys were more realistic looking when they incorporated basic geomorphic processes such as fluvial erosion, rainfall, and weathering. For the case of an entire river network without the surrounding topography, Cieplak et al. (1998) review models for creating fractal river network typology of single thread rivers around which a landscape could be built. More recently, Zhang et al. (2016) used Tokunaga networks to generate large scale watersheds.

The ability to rapidly generate unique landscapes is often balanced with the level of user control (Raffe et al., 2012; Smelik et al., 2014). A detriment to most automated procedural terrain generators is that the user has no control over features until after the terrain is built. Expert techniques have been blended with procedural methods to allow for some expert-based feature design within procedural modeling. Examples include interactive procedural sketching (Teoh, 2009; Huijser et al., 2010; Jensen, 2011; Genevaux et al., 2013), procedural blocks (Genevaux et al., 2013), software agents (Doran and Parberry, 2010) and evolutionary algorithms (Saunders, 2006; Raffe et al., 2012). Smelik et al. (2010, 2011) advocated interactive procedural sketching, because it blends the automation of procedural design with the control of interactive sketching. For example, in the program RiverLand (Teoh, 2009; Jensen, 2011) the user defines the shape of an island with ridge lines by drawing on a 2D canvas. Within the island a meandering river is generated that does not cross the user-defined ridges. Combining geometric modeling with procedural terrain generation, Huijser et al. (2010) developed a procedural method that allows the user to define the path of the river and uses a predefined sub model for the cross section of the river to create simple meandering river topography. Another hybrid approach is to blend procedural sketching with evolutionary algorithms (Saunders, 2006; Raffe et al., 2012). For example, in the program Terrainosaurous (Saunders, 2006) the user can sketch regions in a layout that can be associated with different reference heightmaps. For each region a genetic algorithm melds together chunks of elevation data from the supplied examples creating a new terrain that has attributes of the example heightmaps. Genevaux et al. (2013) combine interactive sketching, procedural blocks, and basic concepts from hydrology and geomorphology, illustrating how procedural methods have evolved to allow for user control and rapid generation.

3.3. Expert

Expert-based techniques are the most open-ended avenue for creating terrains but are seldom discussed in the scientific literature. In fact, today over 100 million people around the world carry out landscape terrain manipulation by adding or subtracting individual 1 m^3 voxels in Minecraft and other similar video games. Expert-based techniques include (i) geometric, (ii) map, (iii) brush, and (iv) interactive methods. Geometric modeling is the mathematical representation of shapes. Map-based techniques include working in the XY plane and using contours, points, and/or break lines that have assigned elevation attributes, similar to how most civil engineering landscape grading occurs. Brush techniques also operate in the XY plane but use colored and textured "brushes" on terrain canvases, where colour scale of the brush has a prescribed elevation range (de Carpentier and Bidarra, 2009). Finally, interactive methods are embedded within software programs that allow the user to pull and stretch an initial terrain to create specific landforms manually. Because these approaches are so open-ended they are not discussed further for general terrain generation but will be elaborated in Section **Error! Reference source not found.** For creating river topography.

4. Creating digital rivers without a surrounding terrain

Digital rivers do not require surrounding terrains for many applications. When there is not a terrain to drive the type of river that is possible or desired, the user drives the synthesis process through conceptualization at the reach scale. Then, "scaling variables" are selected to be used in later steps (e.g., expert or strategic methods).

4.1. Conceptualization

In creating a synthetic river valley without an existing terrain, the purpose of modeling, type of river(s), scale and resolution should be conceptualized by the user. Conceptualization is important because it provides the broader template in which model components and their characteristics are envisioned by the user (Brown et al., 2014). Purposes of modeling could be to understand how specific channel and floodplain configurations affect ecological and geomorphic processes (Brown et al., 2016; Pasternack and Brown, 2016), to create prototypes of channel configurations for historical analysis (e.g., Jacobson and Galat, 2006), to develop river and stream rehabilitation scenarios (Elkins et al., 2007; Pasternack and Brown, 2013), to evaluate land management impacts and engineering scenarios, or for scene generation for virtual reality purposes, such as for video games (Nelson and Mateas, 2007; Hendrix et al., 2013), military training applications (Smelik, 2013) and flight simulators. The type of river planform has a strong bearing on subsequent steps because, as will be shown, not all methods are yet capable of creating all types of fluvial form.

Once the type of river planform is defined, then the scale and resolution of the synthetic river should be defined. Scale is important because geomorphologists are now learning more than ever that processes and landform variability are scale dependent (Drăguţ et al., 2011). For example, river profiles show varying statistical and mathematical characteristics depending on whether a single bedform, morphological unit, or entire river system is being considered (Brown et al., 2014). Resolution is important, too, because it can guide a user to the most effective approach. Resolution should be set to the coarsest level necessary to capture the features needed for the application. If bedforms, outcrops, and boulder clusters are needed, then a higher resolution will be required.

4.2. Defining scaling variables

To create a synthetic digital river without a terrain there are fundamental scaling variables that need to be determined, regardless of whether an expert or strategic route of river synthesis is desired. If a terrain exists, then these can be extracted, but if one does not exist, then they should be defined by the user. Fundamentally, fluvial geomorphology posits that there are relationships between landscape position, flow rate, sediment load, and the typology and geometry of a river (Leopold et al., 1964; Singh, 2003). Common scaling variables used for fluvial systems include bankfull discharge Q_{bf}, reach averaged slope $\overline{S}$, median sediment size $\overline{D_{50}}$, and bankfull channel width $\overline{W_{bf}}$ and depth

$\overline{H_{bf}}$ (Parker, 1976; Church, 2006; Parker et al., 2007). These can be specified outright by the user based on user conceptualization of the river under design or determined from empirical relationships to conform to evidence-based regional science. For example, if there is an existing terrain, the drainage area can be determined from relations between drainage area (and/or climate metrics) and Q_{bf} (e.g. Dury, 1976; Castro and Jackson, 2001). Then Q_{bf}, hydraulic geometry equations, and channel regime relationships can be used to determine $\overline{H_{bf}}$ and $\overline{W_{bf}}$ (Leopold and Maddock, 1953; see Williams et al., 2002 for tidal channel hydraulic geometry relations governed by tidal prism). Alternately, for single thread gravel and sand bedded rivers there exist several analytical and empirical equations from geoscience and engineering research that can determine $\overline{S}$, $\overline{W_{bf}}$, and $\overline{H_{bf}}$ from Q_{bf} and $\overline{D_{50}}$ (Parker et al., 2007; Wilkerson and Parker, 2011). Dodov and Foufoula-Georgiou (2004) provide a more theoretical foundation and procedure for rendering synthetic hydraulic geometry. Many other empirical functions suitable for scaling river designs exist among catchment scale and reach scale geomorphic variables customized to valley setting (Knighton, 1998; Shields Jr et al., 2003). Davidson et al. (2013) reviewed river patterns and processes for distributive fluvial systems that can be used to help select scaling variable values for synthetic river design.

4.3. Planform selection

In this section a brief overview is given on how to translate the scaling variables to channel planform typology and actual channel alignments for scenarios where there is no pre-existing terrain and one is not needed. In this situation, the user can determine what type of planform is possible or likely given the reach characteristics. In expert-based synthesis, the user would take the resulting planform type and then prescribe the spatial alignment of the channel(s). This can be achieved through subjective means where the user articulates the path of each channel within the synthetic domain. Time invariant deterministic equations or mathematical models for meandering rivers can also be utilized to prescribe an exact alignment. For strategic synthesis, deterministic and stochastic models are possible and for the former, need to be initially specified. Stochastic approaches rely on specifying the upstream and downstream limits and using a combination of random numbers and rules to determine the alignment between those two points. Time-varying deterministic models use input variables to generate an evolving planform.

River planforms are commonly classified as meandering, braided, anastomising, straight, and transitional (Leopold and Wolman, 1957; Schumm, 1985; Eaton et al., 2010). Clearly other fluvial planforms than these five exist in nature (Schumm, 1985), and there are a variety of distributive terminal channel planforms where rivers meet the sea in deltas, fjords, rias, and other estuaries (Perillo, 1995; Davidson et al., 2013). There do exist several empirical and analytical relationships to predict the type of channel planform a river would have depending on discharge, reach-averaged hydraulics, sediment size and type, and channel slope, width and depth (Parker, 1976; Eaton et al., 2010; Crosato and Mosselman, 2009). Parker (1976) derived a theoretical state space which discriminates between straight, meandering, and braided planforms based on the width, depth, slope, and bankfull discharge. Eaton et al. (2010) derived discriminate functions between the critical slope, relative bank strength, and dimensionless discharge that demarcate the transition from single thread to anabranching channels and another describes the transition from anabranching to braided channels. Crosato and Mosselman (2009) derived a physically based expression for the number of channels based on Q_{bf}, $\overline{S}$, $\overline{W_{bf}}$, a friction parameter, and a dimensionless sediment transport parameter. Overall, using any of these relationships one can objectively evaluate reach scale variables to determine whether or not they would likely be associated with single thread, anabranching, or braided planforms.

Once a planform typology is identified, it serves as the basis for developing the channel alignment(s) of the river. Different methods for creating static and dynamic planforms exist. Examples of meandering river models include sine generated curves (Langbein and Leopold, 1964), disturbed periodic models (Ferguson, 1976), fractal planforms (discussed in 6.2.1), and Kinoshita curves (Kinoshita, 1961). Mosselman (1995) completed a review of dynamic models of planform change and concluded that, while many approaches exist, they are not in software packages that facilitate broader use. This has changed somewhat since then with programs such as RVR Meander (Abad and Garcia, 2006), which is available as a standalone windows version and also for ArcGIS® 10.0.

5. Expert-based river design

Expert-based designs are driven by user creativity and knowledge in two avenues, understanding fluvial landforms as well as software preference and experience. Underlying all expert-based methodologies is a long history of scientific discovery and technological development whose modern "black box" software platforms may be taken for granted today, but which must be acknowledged in this review as foundational literature (Myers, 1998; Farin et al., 2002; de Carpentier and Bidarra, 2009; Li et al., 2015). The software with which a user is familiar heavily dominates what is achievable. User depth and breadth of skill is enhanced through time with creative play, attempting new challenges, and receiving software updates. As such, many theoretical and procedural advancements are not found in the peer-reviewed literature, but instead in software user forums- if publicized at all given constrains arising from proprietary commercial value. An exception is geometric modeling, which has been recently advocated by the authors to be a useful method for creating prescribed river topography for fluvial geomorphic inquiry as well as river rehabilitation design (Brown et al., 2014).

All of these methods- geometric, brush, and map- can be used to create a new river terrain or to modify an existing terrain. In addition, a surface can be transformed through scaling, filtering, and pushing and pulling one or several terrain nodes. Most terrain modeling software has diverse filters relevant for terrains, such as changing the surface roughness and adding directional gradients and curvatures. Next, we review each of the four types of expert-based synthesis and discuss their advantages and disadvantages.

5.1. Map river design

Map techniques require specifying the horizontal position and elevation of points and lines along a contiguous path of descent. Contours are isolines of constant elevation and are one of the oldest representations of landform topography. The generation of design contours for engineering purposes has been a staple of modern landform design (Schor and Grey, 1995). In this setting, contours of the existing Earth surface are generated from collected point or transect data either by eye or by computer. In landform design, new contours are generated manually over this existing template and the composite is then used as a basis for the new landform. An advantage of this approach is that valley and channel slopes are already accounted for in the pre-existing contours. Once topographic contours and points are developed, a surface is constructed, usually in the form of a triangulated irregular network that can then be turned into a heightmap.

In civil engineering, computer aided design (CAD) is the industry standard for map-based river design (Myers, 1998). Historically, CAD was a 2D plane-based method for drawing sections and profiles. Nowadays, skilled users apply CAD programs such as AutoCAD® Civil 3D® to yield sophisticated terrain models. More recently, Geographic Information System (GIS) software can also be used to do many of the same terrain generation steps as in CAD. Programming languages like Python and R can script these steps in GIS to automate them.

An example set of design surfaces for an actual river restoration

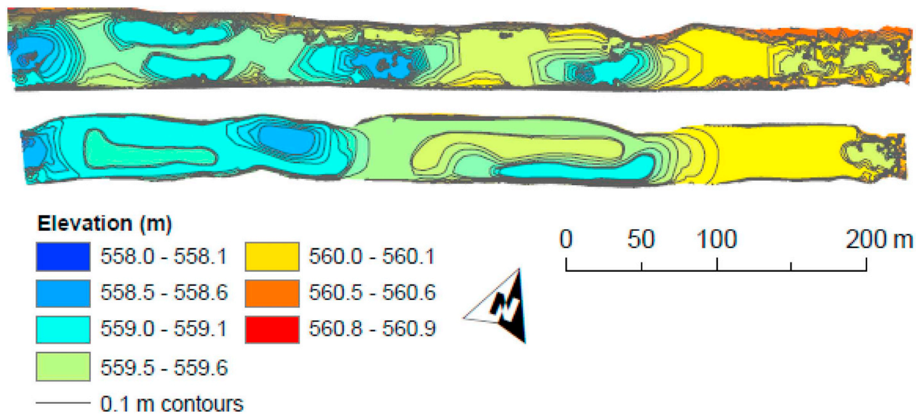

Fig. 2. Two contrasting map-based designs for a river restoration project on the Trinity River, CA (modified after Pasternack and Brown, 2013). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

design was built in CAD using contours and is shown in Fig. 2. To have control over the slopes the distance between contours needs to be considered. In most CAD programs this can be achieved by specifying horizontal offsets of existing contours in a specified direction. Breaklines are also sometimes used to delineate paths of constant elevation associated with specific features, such as walls or steep banks that can be used to guide interpolation.

Map techniques, such as contouring, are relatively quick to perform for experienced users. Further, this technique is embedded in many engineering disciplines as the de facto method for generating design surfaces. A drawback of using map-based approaches is that using contours to represent topography can be non-intuitive to some, just as brush-based approaches would be foreign to others. Further, creating contours, points, and breaklines are intermediate to creating a terrain because these features need to be interpolated. Interpolation can introduce an additional level of variability, depending on the resolution of created objects relative to the interpolation domain. Whether one or several contours are located within a single cell or multiple cells can have an effect on the final heightmap. Like brush-based methods, map techniques do not inherently and objectively specify key geomorphic values for the terrain but require iterative creation and analysis to see if it came out as desired.

5.2. Geometric river design

Geometric modeling of river channel topography is a method of synthesis where specific 2D geometric elements of river topography, such as the bed profile, cross section, and channel planform, are mathematically modeled in isolation and then combined to produce a 3D heightmap (Brown et al., 2014). Deutsch and Wang (1996) utilized aspects of this approach in developing a stochastic model for fluvial reservoirs that utilized a channel geometry model that incorporated the position along a centerline, the channel width, and an expression for variable cross section geometry. The use of kriging in modeling channel topography from field measurements (Legleiter and Kyriakidis, 2008) and synthetically (Legleiter, 2012) was founded on a similar approach whereby the channel alignment, bed profile, and cross section are modeled separately, and then coupled to produce channel topography.

Although CAD software was not originally intended for geometric design, it is increasingly adopting such capabilities. For example, the Corridors function in AutoCAD Civil 3D® can create channels by drawing an alignment and specifying a cross section that is projected through the alignment. This function was intended for roads, levees and other civil infrastructure components, but it can be used for rivers. Without additional information, Corridors yields highly simplistic canals, not natural channels. There is grey literature on creating river channels using Civil 3D®.

A recent method that was developed specifically for the geometric modeling of river corridors is called the synthetic river valleys (SRV) methodology (Brown et al., 2014). The basic steps in developing a geometric model of a synthetic river valley are (i) conceptualize, (ii) specify model domain, (iii) determine 2D fluvial geometric elements in the model, (iv) determine reach-average values of geometric elements, (v) develop geometric element equations, (vi) construct model, and (vii) parameterize. Two important aspects of geometric modeling are the selection and construction of appropriate geometric element equations, and their subsequent parameterization. Brown et al. (2014) review models used for basic geometric elements (Table 2). For single thread rivers there are a variety of models for planform alignments, longitudinal profiles, and channel cross sections that can be used to create digital rivers. The amount of control is driven by the types of mathematic models used within the geometric element equations. For example, planform alignments can be generated using deterministic sinusoid models or stochastic approaches such as auto-regressive models (as discussed in Section 6.2). Despite using relatively simple functions, such as sinusoids, the approach can yield remarkably diverse and complex river valleys. Parameterization is a key step whereby the parameters of the geometric element equations are adjusted to meet user-specified attributes defined through the conceptualization process. This includes specification of reach-average properties of the river corridor and also each control function parameter independently (e.g., the frequency of bed oscillations) and in some cases dependently (e.g., the relationship between thalweg elevation and bankfull width). The extent of independent and dependent parameterization will depend on the purpose of modeling, which fluvial elements are being included, the mathematical function used, and expert judgment.

A benefit of geometric modeling is that one can create channel and valley topography of prescribed conditions. For example, varying GCS parameterization between channel width and thalweg can yield rivers that have riffle and pool topography, while varying the channel and valley width GCS can yield confined or unconfined rivers (Fig. 3). Complex channel patterns, such as braided rivers, have not been explored to date. For the SRV approach, River Synth 1.1 is a Microsoft Excel® implementation available upon request from author Brown, while River Builder (currently version 0.1.1) is an open-source, free, public R package available from the Comprehensive R Archive Network.

5.3. Brush river design

Brush methods entail the digital "painting" of terrain canvases in the XY plane using artistic methods available in free and commercial software packages (de Carpentier and Bidarra, 2009). Recognizing that terrain is nothing more than a heightmap, any raster-based software

Table 2
Mathematical functions used to model the meander planform alignment, thalweg profile, and cross-section geometric elements drawn from the literature. M refers to morphologic unit scale (10^0–channel widths), R refers to reach scale (10^2–10^3 channel widths), and B refers to basin scale (> 10^1 channel widths).

Geometric element	Mathematical function/ model type	Scale	Sources
Channel profile	Exponential	B	Tanner, 1971; Yang, 1971; Snow and Slingerland, 1987
	Power	B	Yang, 1971; Snow and Slingerland 1987
	Logarithmic	B	Yang, 1971; Snow and Slingerland, 1987
	Hybrid	B	Schumm, 1960; Langbein and Leopold, 1964; Ohmori, 1991
	2nd order, autoregressive	R, M	Knighton, 1983; Richards, 1976a
	Variogram	R, M	Robert and Richards, 1988
	Regression	R, M	Anderson et al., 2005
	Linear trend	R, M	Leopold et al., 1964; Knighton, 1998
	Variogram	R	Legleiter and Kyriakidis, 2008; Legleiter, 2012
Cross section	Polynomial	NA	James, 1996
	Statistical distribution	NA	Merwade and Maidment, 2004; Jacobson and Galat, 2006
	Curvature based asymetry	NA	Deutch and Wang, 1996
	Analytical	NA	Bridge, 1977; Beck, 1988
	Rectangular	NA	Chow, 1959
	Semi-circle	NA	Chow, 1959
	Traiangular	NA	Chow, 1959
	Trapezoid	NA	Chow, 1959
Channel alignment	2nd order, autoregressive	M,R,B	Ferguson, 1976
	Analytical	M,R,B	Kinoshito, 1961
	Sinusoid	M,R,B	Langbein and Leopold, 1966

that can change the greyscale value of a blank digital canvas can be used to create digital terrain. That means programs such as Photoshop® and Gimp® are candidates for creating digital rivers. However, one can only get so far working entirely in a 2D view, so there exist software packages with more viewing perspectives and specific tools for manipulating what will ultimately be a terrain. Examples include Bryce3D®, SketchUp®, World Painter®, and Zbrush®. Video game engine software, such as the Unreal Development Kit®, CryEngine®, and Unity®, also offer brush methods for terrain generation and modification.

Brush-based river synthesis is commonly used for scene generation in artificial landscapes for video games and virtual reality. For example, the 2018 game Red Dead Redemption II© developed by Rockstar Games, Inc. has the most advanced and realistic synthetic rivers produced to date from an artistic approach (Fig. 4b,c), including a wide diversity spanning headwater to coastal settings. Though specific design tools and workflows are not publicized, investigation of the developer's global employee hiring advertisements for terrain development indicated that candidates should be versed in expert-based brush and geometric terrain methods, suggesting that these were the tools used to make those synthetic rivers. Brush-based methods have not been used for scientific inquiry to the authors' knowledge. With advances in geometric methods, brush techniques are no longer commonly the starting point for terrain generation but are used extensively to refine terrains and are an increasing part of hybridized toolsets (de Carpentier and Bidarra, 2009).

The use of brush-based software to create a river valley begins with designating terrain extent and resolution. Then an existing or blank terrain canvas is modified with digital brushes of varying size, shape, intensity, and texture/pattern to paint elevations and gradients. Brushes can be set to add or remove elevation. The upper elevations of the river valley are first painted with larger brushes, creating the broader valley template. Then, smaller brushes with lower elevation paint settings are used to place the river into the corridor (Fig. 4a). In this way, multiple inundation zones are hierarchically nested as would occur in nature. Finally, the resulting surface can be smoothed to remove brush irregularities.

Because this technique is artistic-expert-based, the created river valley can have a wide range of topographic characteristics that is bounded only by the user, operating software, and time. For example, the mountain meadow in Fig. 4b shows cutbanks, point bars, riffle-pool undulations, islands, floodplains, and a large secondary channel. An important aspect of using brushes to create synthetic rivers is relating brush dimensions to actual river dimensions, both horizontally and vertically, which can be done afterwards by applying scaling factors to convert to real-world coordinates. Further, it is difficult to design

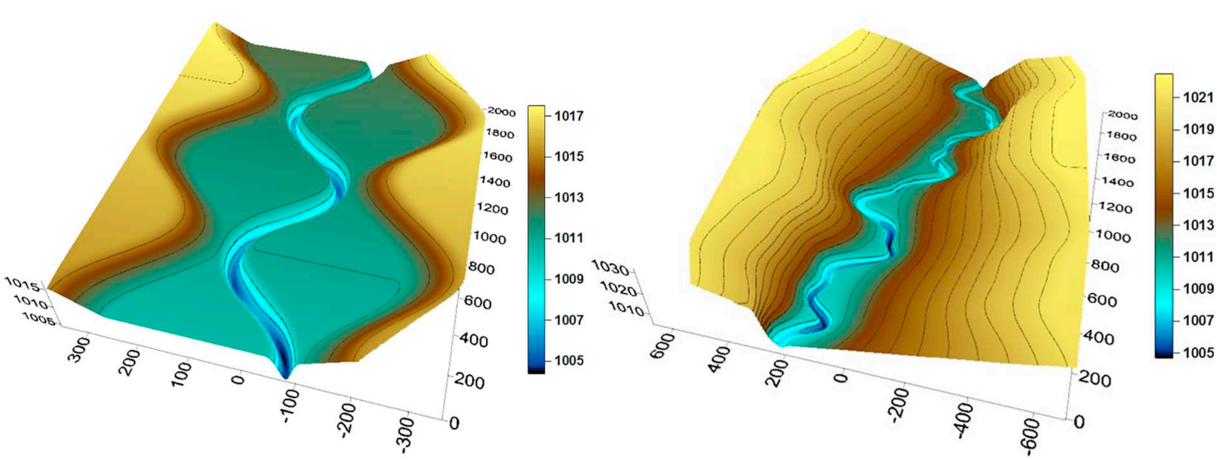

Fig. 3. Surface topography for unconfined (A) and confined (B) rivers valleys created using the synthetic river valley framework of Brown et al. (2014). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

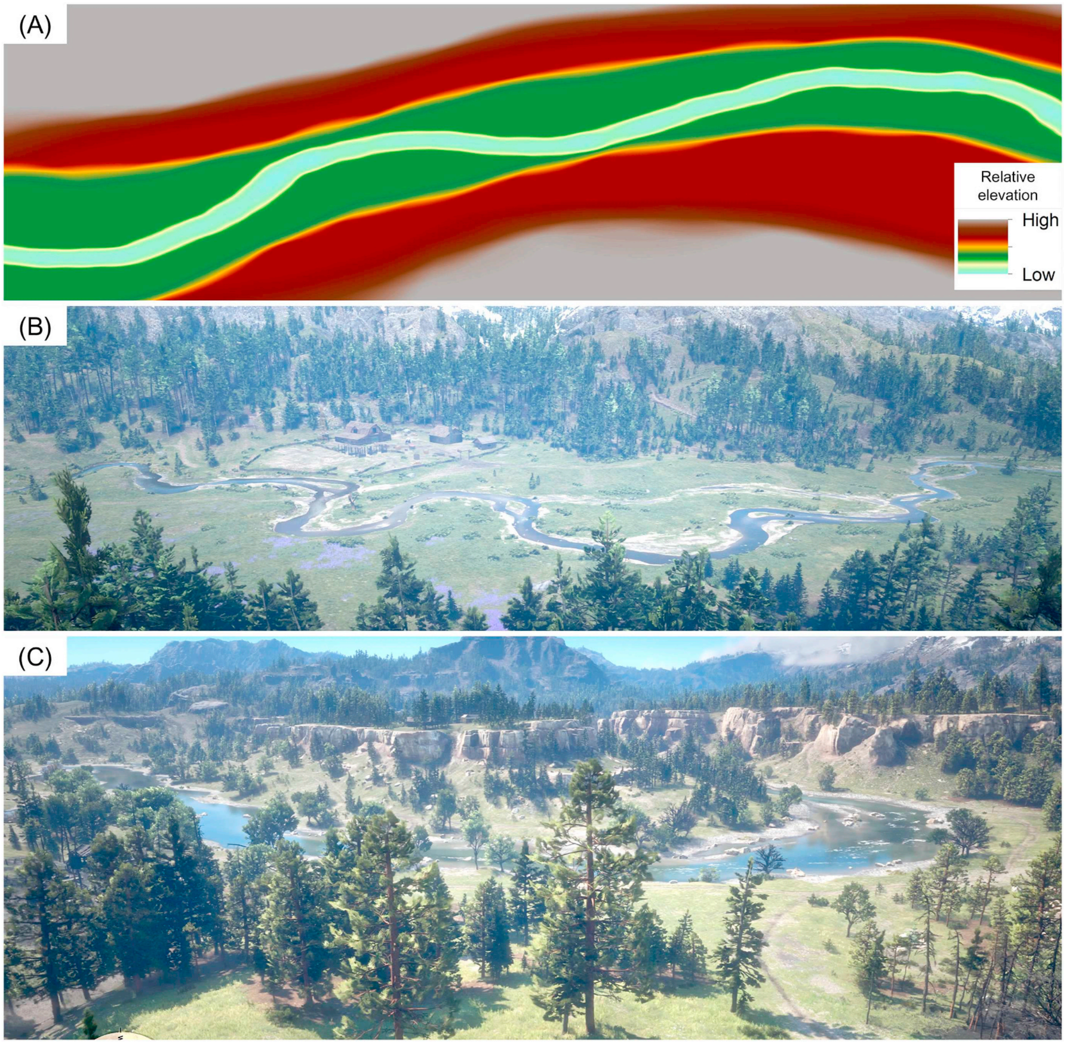

Fig. 4. Example digital river created using brush techniques (A). The heightmap shown was created in Bryce 3D® as explained in the text. Datum and scale are arbitrary.Also shown are two riverscapes (B,C) created using brush techniques from the game Red Dead Redemption. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

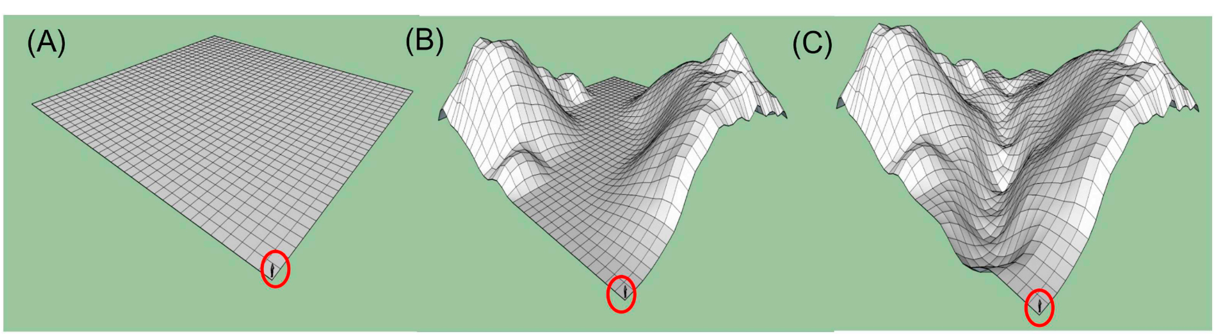

Fig. 5. Example digital river created using interactive technique in Sketchup®. First a blank grid is produced with 1 m spacing (A). Next, mountains are created by extruding cells upwards along the edge of the grid (B). Finally, cells in the middle are pulled downward to create the river channel (C). The circled object is a 1.68-m tall person for scale.

specific river planform types and morphologies, because the brush is driven by hand operation, for which the precision is limited by drawing device (e.g., mouse, trackball, trackpad, or stylus). Artists commonly use digital drawing tablets with a precision stylus. A benefit to brush synthesis is that built-in filters can be used to smooth and sharpen brush strokes.

A key challenge to brush-based methods is the difficulty in matching specifications for a variety of river metrics. This necessitates iterative brushing and terrain evaluation. Note that even industry-standard CAD is unable to prescriptively control several channel metrics and thus also requires iteration between artistry and terrain analysis.

An improvement to the brush method could include fluvial-specific brush types and surface material textures that are specifically tailored to creating riverine landforms. For example, a brush could be designed with a lateral fall-off profile to create the desired cross-sectional shape as one moves along the centerline. Also, brush texture with grain-scale roughness and organization to include sedimentary facies could be created.

5.4. Interactive

Interactive approaches are used in programs that allow a user to create an initial terrain and then use a variety of other tools to do further manipulation (de Carpentier and Bidarra, 2009). To provide an example, the "sandbox" tool in SketchUp® was used to generate a blank grid of 100 one-meter cells (Fig. 5A). To create a river channel in a valley the surrounding cells were then extruded upwards to create mountains and hillsides (Fig. 5B). The channel is created by pulling the grid downwards between the valley (Fig. 5C). This inevitably brings to light the issue of constant grid spacing in interactive and brush-based terrain methods. A user may want finer scale topographic detail in the channel than the surrounding hillsides, and to do this the channel grid cells would have to be subdivided further in the channel. While creating a domain as in Fig. 2 is relatively simple and straightforward, it would be very time consuming and difficult to create scientifically meaningful and realistic terrain with sediment grain scale variability using this method.

6. Strategic river design

6.1. Deterministic

Deterministic methods include equilibrium, morphodynamic, cellular automata, and discrete particle models, and each of these are possible in multiple dimensions, although most common are one (1D) and two (2D) dimensional models. Models in which elevation is the variable of interest as a function of distance along a river are termed 1D, while those in which it is a function of both longitudinal and lateral distances are termed 2D. A 3D model would be a terrain that has multiple elevations for an {X,Y} position, which would happen with overhangs and undercuts. This article does not address such 3D problems. For 1D models (typically long profiles or channel alignments), outputs would have to be used with geometric modeling to create 3D topography. For example, there exists a plethora of mathematical models for longitudinal profiles, but a profile model would have to be linked to a model for the cross section and alignment to create a heightmap. The benefit of these types of models is their foundation in fluvial geomorphology. Similarly, a detriment is that these approaches are limited by the existing palette of what fluvial geomorphologists can model. For example, there are several methods for modeling single thread meandering river alignments but far fewer exist for braided, anabranching, or anastomosing rivers. Next, each of these deterministic approaches is discussed along with advantages and disadvantages.

6.1.1. Equilibrium models

Fluvial geomorphology has produced a considerable amount of research related to the idea of equilibrium in river systems. Equilibrium refers to the idea that a river maintains a modal state with respect to one or all of its geometric variables, while adjusted to stable landscape parameters, such as water and sediment supply and base level (Leopold et al., 1964). Many of these approaches have their basis in the concept of a "graded" river (Mackin, 1948), which is defined as a river that has become adjusted to water and sediment discharge over a modest period of time. Some use the term "dynamic equilibrium" (Hack, 1975) whereby the river system is adjusted to exogenic controls but change still occurs in metastable states. Most equilibrium models are for single thread rivers in 1D, with an emphasis on straight and meandering planforms. There are numerous deterministic relationships for physical characteristics of equilibrium single-thread river topography that are founded on analytical and empirical fluvial geomorphology. Namely, the longitudinal profile, channel alignment, and cross section can all be modeled using deterministic equations. Some of these are purely empirical, where the parameters of mathematical functions are fit from field data, while others are simplified solutions to theoretical treatments of flow and sediment transport relationships. In this section a few of these types of models are discussed for generating watershed to reach scale longitudinal profiles, followed by analytical models for equilibrium topography for single thread meandering rivers.

6.1.1.1. 1D longitudinal profiles. One-dimensional longitudinal profiles of rivers are one of the most studied attributes of river topography, and approaches exist for their generation at watershed to morphologic unit scales. Methodologically, modeling has encompassed approaches that (i) model basic geometric shape using mathematical equations with empirical coefficients, (ii) provide deterministic equilibrium solutions based on 1D flow and sediment transport relationships, and (iii) predict dynamic solutions modeling profile shape as governed by a diffusive process or morphodynamic interactions. Mathematical and diffusion models are used most commonly for generating entire watershed profiles of a mainstem whereas the coupling of 1D flow and sediment transport relationships are used to generate reach and sub-reach scale variability. The benefit of these types of models is that they are computationally efficient and can generate long sections relatively fast. However, an obvious detriment is that as a 1D series there is no lateral variability for the river profile.

The use of geometric mathematical equations to model watershed scale longitudinal profiles has been widespread and is considered a staple in fluvial geomorphology. Linear, exponential, logarithmic and power functions have all been used to model and describe river profiles (Leopold and Langbein, 1962; Langbein and Leopold, 1964; Tanner, 1971; Shepherd, 1985). These types of geometric models are useful when one knows a priori the type of profile desired to be created and they can also be easily adjusted by simple parameter manipulation. Further, they can be contextualized with mathematical functions for different physiographic conditions such as lithology (Brush, 1961), grain size (Yatsu, 1955) as well as fluvial regimes related to aggradation and degradation (Ohmori, 1991).

In such cases where a fluvial foundation is desired, simple analytical models of open channel flow and sediment transport can be used strategically to determine time-invariant equilibrium solutions for watershed scale longitudinal profiles. An example of this approach is from Snow and Slingerland (1987), who developed a model for graded stream profiles using open channel flow and sediment transport equations coupled with empirical relations for the downstream variation in flow discharge, sediment discharge and size and channel width. The initial sets of equations in their model were time-dependent and would thus be considered morphodynamic (e.g geometric or morphologic properties change with time, as explained later in Section 6.1.2). However, by explicitly analyzing for equilibrium geometry over graded time the equations were simplified, allowing the determination of relatively simple analytical expressions. Their comparison of model outputs to known mathematical models of profiles, such as exponential, logarithmic, and power functions, showed that these functions do provide representative descriptions of river profile shapes depending on substrate and external controls.

Analytical models of equilibrium bed profiles have been used to attempt to model channel-width-scale longitudinal undulations for gravel and sand bedded rivers. For gravel bed rivers, Cao et al. (2003) developed a simple dynamic equilibrium model in an effort to reconstruct the bed topography of a riffle-pool unit. Specifically, equations for 1D steady, uniform fluid mass and energy conservation, a flow resistance equation (e.g. the Manning equation), and a sediment transport relationship, (e.g. the Meyer-Peter Muller equation) were coupled to determine the equilibrium bed elevation for a river reach with fixed channel width. The utility of this approach is that it illustrates that variable bed topography at the sub-reach scale, such as riffles and pools, can be created using 1D analytical equations, so long as the channel width series is specified a priori. For sand-bed rivers, Julien and Klaassen (1995) developed analytical and empirical approximations of dune height and steepness based on a dimensionless particle diameter and transport stage. While these models only give the bedform geometry (e.g. height and steepness), which approximates topography, they can still guide the synthesis of these types of bedforms using other techniques. For example, based on calculated bedform geometry parameters, mathematical models can be used to generate profiles with those dimensions.

Mathematical profile models for riverbeds are advantageous in that they are widespread and span a wide domain of approaches. These types of models are appropriate for mostly the river reach scale or greater, so finer scale topographic variability would have to be incorporated separately. The main detriment is that only the profile is generated, so other models for the alignment and cross section of the river still need to be specified. However, this can be accommodated by combining these models within a geometric modeling framework (e.g. Section 5.2).

6.1.1.2. 2D and 3D equilibrium meander bed topography. While 1D models for simulating longitudinal profiles are numerous, 2D equilibrium models that directly generate heightmaps are less prevalent and are primarily restricted to meandering rivers. For example, Bridge (1976, 1982, 1992) developed an equilibrium model of flow, bed topography, and grain size based on analytical and empirical relationships for individual meander bends. Bridge and Gable (1992) also showed that this general model could be applied to either side of anabranches. Beck (1988) developed a simplified analytical model for meandering rivers in equilibrium that can generate topography from simplified expressions for the transverse bed slope and maximum depth that require only the channel half width, curvature of the channel, and average depth. Fluvial geomorphologists have already been using this model to develop synthetic topography to evaluate computational fluid dynamics models within meandering rivers (Abad and Garcia, 2008). These 2D equilibrium models are advantageous in that for meandering rivers, bed topography can be predicted from a modest amount of reach-averaged input variables (as in Section 4.2) with relatively low computational expense. A detriment is that they produce very simple topographies that are much smoother than real rivers, but this could be dealt with by superimposing random variability in the bed topography from stochastic models (described in Section 6.2). The 2D models of equilibrium bed topography by Bridge (1976, 1982) have also been extended to model the 3D sedimentary structure of point bar deposits (Bridge, 1977; Willis, 1989; Willis and Tang, 2010), incorporating lateral and vertical variations in sediment size through meander evolution.

6.1.2. Morphodynamic modeling

Morphodynamic models of river topography explicitly consider the relationship between water flow, sediment transport, and changes in boundary geometry over computational grids to determine time varying solutions of riverbed topography (Mosselman, 2012). These types of models can be formulated in 1D as for a longitudinal profile or in 2D for planform pattern. The former must be combined with alignment and channel cross section models through geometric modeling to create a heightmap. 2D morphodynamic models can generate river topography from steady-state solutions or taking the output of unsteady solutions. As they are non-equilibrium, evolutionary models, any resulting topography is a product of (i) initial conditions, (ii) grid type and resolution (Doeschl-Wilson and Ashmore, 2005; Nicholas et al., 2006; Nicholas et al., 2013), (iii) boundary conditions (Murray and Paola, 1997; Nicholas et al., 2013), and (iv) the processes considered in the model's structure (Nicholas, 2013). Note that boundary conditions include dynamic hydrologic and sediment flux regimes, which are often challenging to specify to characterize future conditions for real-world design. A common approach is to use historical discharge time series as representative of future flows. There is rarely any sediment flux data, so inputs have to be designed from scratch. While it is not possible to prescribe the exact creation of river topography desired, these models are powerful in their ability to simulate interactions between channel flows, sediment transport, and vegetation to produce emergent forms. Below 1D and 2D morphodynamic models are discussed. We exclude explicit 3D models for brevity, recognizing that the concepts associated with 2D models are sufficiently similar to provide the context.

6.1.2.1. 1D Morphodynamic models. One-dimensional morphodynamic models predict the evolution of the channel bed profile of rivers from coupling open channel flow and sediment transport capacity equations, and in some cases the grain size distribution is also predicted. Since the 1970's 1D morphodynamic models have been applied to both sand and gravel bed rivers at reach and watershed scales (USACE, 1993; Havis et al., 1996). Most commonly, channel hydraulics are computed from the energy equation using the standard step-method, so that backwater effects are incorporated. For each time step a water surface profile is calculated, thereby providing energy slope, velocity, and depth at each cross section node. Next, the sediment transport capacity is computed and when combined with the duration of the flow, permits a volumetric accounting of sediment. Changes in sediment transport capacity between nodes are translated into changes in bed elevation via the Exner equation for the continuity of sediment flux. With updated cross section bed elevations, the computations then proceed to the next flow in the sequence and the cycle is repeated beginning with the updated geometry. The sediment calculations are performed by grain size fraction thereby allowing the simulation of hydraulic sorting and armoring.

At smaller spatial scales, analytical 1D morphodynamic models can be developed using the basic relationships of flow and sediment transport, analogous to the Cao et al. (2003) model but for dynamic simulations. For example, Wallerstein (2003) developed a dynamic model that determines the equilibrium or time dependent pool scour from channel constrictions. Again, equations for fluid and sediment mass conservation, conservation of energy, and sediment transport were coupled to determine the bed elevation between two rectangular cross sections, where the second section is constricted. The model varies from Cao et al. (2003) in that specific energy is calculated between the two sections to determine the change in water depth and thus energy slope, sediment transport, and ultimately bed elevation. Rather than explicitly accounting for water flow and how that drives sediment transport, another approach is to treat topography as a flowing media unto itself in consideration of the time-averaged behavior of landforms when viewed over decades to millennia. If one could watch a time lapse movie of a landscape at those scales, water flows would not be seen and just the resultant landscape movements would be seen. The type of analytical model that achieves this dynamism uses the diffusion equation to model river and watershed scale longitudinal profiles. Begin (1988) for example, used the diffusion equation to simulate river longitudinal profiles in response to base level lowering at the basin scale. Diffusion models are governed by only two parameters- an initial height profile and the "diffusivity" of topography, making them very simple to implement. Good approaches exist for constraining and quantifying diffusivity (Paola et al., 1992; Pasternack et al., 2001).

One-dimensional morphodynamic models can be used to create longitudinal profiles in two different ways. First, they can generate a river profile from an initially flat surface or highly simplified channel network. Second, they can start with an existing profile and evolve that over a time period to obtain a subsequent profile given the model inputs a user wants to specify. Examples of 1D morphodynamic models include HEC-RAS (Hydrologic Engineering Center River Analysis System; http://www.hec.usace.army.mil/software/hec-ras/), FLUVIAL-12 (http://chang.sdsu.edu/fluvial.html), and various Excel workbooks by Gary Parker (http://hydrolab.illinois.edu/people/parkerg/

morphodynamics_e-book.htm). HEC-RAS, in particular, is a widely used 1D morphodynamic model in civil engineering and fluvial geomorphology. Initially known as HEC-6 (USACE, 1993) the model has been successfully used to model changes in bed elevation in large river systems (Havis et al., 1996) and even replicate riffle-pool bedforms (deAlmeida and Rodríguez, 2012). Using this type of model for synthesizing 1D river topography requires more information and computational effort than the 1D equilibrium models described in Section 6.1.1. Namely, a hydrograph needs to be generated and a bed sediment distribution needs to be specified for the incoming sediment load and at each node. To generate a hydrograph, the selected discharge from the conceptualization step (e.g. Section 4) can be used with hydrologic methods that convert peak discharge to storm events (Clark, 1945; Aron and White, 1982). Similarly, a grain size distribution can be generated from the previously defined median sediment size and a sediment distribution relation using the equation presented by Fuller and Thompson (1906).

6.1.2.2. 2D morphodynamic models. Morphodynamic models in two dimensions are an avenue for autogenically deriving a heightmap. They differ from 1D models in that more sophisticated relationships are used to model water flow that account for local spatial accelerations and decelerations as well as 2D flow fields. Compared to LEMs, 2D morphodynamic models are different in that only channel processes are considered, typically in computational grids that explicitly are channel orientated as opposed to Cartesian coordinates and with grid cells that are much smaller than a channel width (Struiksma, 1985; Ikeda and Nishimura, 1986; Nelson and Smith, 1989; Sun et al., 1996; Vasquez et al., 2007). Moreover, they differ from cellular automata models in that the 2D St. Venant equations are solved numerically, rather than simplified through abstracted rules (Nicholas, 2010). While morphodynamic models all have the basic attributes of combining partial differential equations for flow and sediment transport to predict bed change and or equilibrium conditions, they have some common types of differences that influence the type of topography produced. For example, models can differ in the coordinate systems used, the type of grid, specific hydrodynamic components such as secondary flows and convective accelerations, the type of sediment transport mechanisms and empirical functions used to estimate sediment transport. In many of these early models, bank erosion is absent and only bed topography is predicted for fixed width (Nelson and Smith, 1989) or small width variations (Struiksma, 1985). Since then, models are now capable of having variable channel widths and also now can incorporate processes such as bank erosion (Mosselman, 1998; Duan and Julien, 2010) as well as geotechnical bank failure processes. In addition, many models are striving to incorporate the effects of vegetation (Li and Millar, 2011; van Oorschot et al., 2016). However, many potentially important processes are also commonly neglected, including riverbank freeze-thaw (Wolman, 1959; Yumoto et al., 2006) and stochastic events.

Recently, 2D morphodynamic models have become more successful in simulating braided rivers (Jang and Shimizu, 2005; Williams et al., 2016) as well as the ability to model both meandering and braided river planforms (Nicholas et al., 2013). The Hydrodynamics and Sediment Transport in Alluvial Rivers model (HSTAR) is a depth-averaged morphodynamic model based on the shallow water equations, with a two-fraction sediment transport scheme and relatively simple treatments of bank erosion and vegetation growth (Nicholas et al., 2013) shown to simulate a wide array of channel planforms with realistic process dynamics (Fig. 6). In comparing morphodynamic models, Nicholas (2013) highlighted five important model components necessary to model dynamic planforms: (i) simple grid structure capable of representing channel-floodplain dynamics without the need for mesh refinement, (ii) limiting diffusion of the bank line migration in the bank erosion sub-model, (iii) including momentum conservation in the hydrodynamic sub-model while including secondary circulation, (iv) at least two grain size fractions, and (v) a simple vegetation sub-model that incorporates stabilization of new floodplains by vegetation.

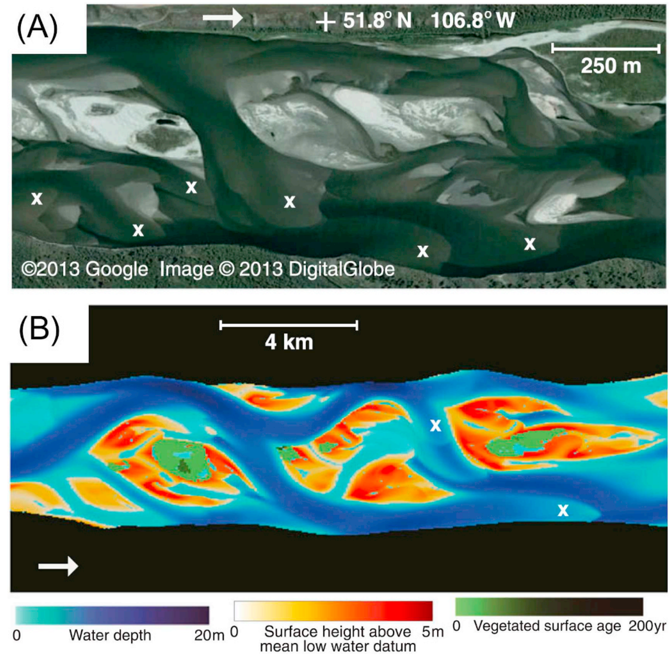

Fig. 6. Examples of morphodynamic model outputs and real rivers from Nicholas (2013). Natural and modeled braided sand-bed rivers: (A) South Saskatchewan River, Canada; (B) simulated morphology after 150 years of channel evolution. Colour scale bars indicate water depth (blue), surface height above a fixed datum (mean low flow water level) at dry channel locations (yellow to red), and age of vegetated surfaces (green). Labels ‘x’ indicate unit bars. Key model parameter values for this simulation are listed in Table I in Nicholas, 2013. All model results are shown at low flow (discharge ~10,000–11,000 m3 s–1). Flow is left to right (indicated by arrow). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The explicit treatment of 2D morphodynamically derived models of river topography have shown considerable promise (Engelund, 1974; Struiksma, 1985; Ikeda and Nishimura, 1986; Nelson and Smith, 1989; Seminara, 2006; Vasquez et al., 2007; Wang et al., 2010a). However, these models are still in their infancy when it comes to simulating large river reaches with multiple scales of material heterogeneity with modest computing capabilities. Early morphodynamic models were built to determine interactions between multiple dependent variables and not necessarily to completely represent all aspects of river topography (e.g., Sun et al., 1996). Therefore, similar to LEMs they are strong methods for directed artificial synthesis obeying transparent process characterizations, even if they have yet to prove valid for predicting changes at real sites with real events in light of the inherent stochasticity of real dynamic phenomena. Rather, they do have the capability of autogenically simulating river topography over time and are considered a potential avenue of artificial river topographic synthesis. The utility of morphodynamic models is that these tools can be used as an autogenic method to determine the bed topography given some specified set of boundary conditions. For example, Wang et al. (2010b) showed that by altering initial and boundary conditions, varying channel patterns including meandering, braided, and anabranching could be produced. Similarly, Nicholas et al. (2013) illustrate how model parameters can also affect the final planform generated. The construction of morphodynamic models requires skill sets not familiar to most fluvial geomorphologist and this may be a barrier that prohibits the widespread development and use of these types of models in favor of more simplified approaches. Moreover, to generate diverse channel types models may have to be run for long periods that may pose computational constraints on their use. A potentially difficult aspect of using morphodynamic models to create synthetic rivers is determining when to stop the model. That is, a user needed to determine a priori when to stop a model, which is difficult to objectively constrain in the virtual sense. Some morphodynamic models are freely offered such as River2D-Morph (http://river2dm.wordpress.com/about/) Delft3D, SRH2D V2 (http://www.usbr.gov/pmts/sediment/model/srh2d/), and Nays2DH within the IRIC platform (http://i-ric.org/). Other models can also be found through the Community Surface Dynamics and Modeling System, a community sharing website at https://csdms.colorado.edu/wiki/Main_Page.

6.1.2.3. Cellular automata. Cellular automata (CA) modeling is an emerging tool within geomorphology (Nicholas, 2005; Fonstad, 2006; Tucker and Hancock, 2010). Cellular automata models differ from LEMs and traditional morphodynamic models in that they use expert-based rules that are simplified abstractions of geomorphic transport laws and/or hydraulic and hydrodynamic equations of motion. However, because they incorporate time dependent interactions of flow and sediment transport, they are grouped under morphodynamic models in this article. The rule-based representation of fluvial and geomorphic processes has a large bearing on the types of outputs generated (Murray and Paola, 1997; Nicholas, 2010). Cellular automata models operate almost exclusively on discrete grids and are favorable because of their ability to implement deterministic, probabilistic, and rule-based expressions that while simplistic, can be constructed in ways that mimic the complexity of many natural phenomena (Wolfram, 2002). A CA model consists of an array of cells or nodes either in 1D or 2D, whereby the state of each cell evolves based on transition rules that mediate the dynamics of the model on a moving neighborhood within the model domain (Wolfram, 2002).

Since its inception, CA models have blossomed into modeling river morphodynamics. The first CA model applied to river topography was the braided river model of Murray and Paola (1994, 1997) using simple water flow and sediment routing schemes. Over the computational neighborhood, water flow is routed to 3 downstream cells according to the topographic gradients, in that flow is proportional to the cell-to-cell gradient. Then, sediment flux is determined based on water flow rate and a discretized and simplified version of the Exner equation. Since then, other studies have provided further refinements in cellular automata models including modeling vegetation (Murray and Paola, 2003), unsteady effects (Parsons and Fonstad, 2006), accounting for bank erosion (Coulthard and Van de Wiel, 2006), multiple grain sizes (Hodge et al., 2013), and also refinements to compete with physics-based 2D and 3D hydrodynamic models (Nicholas, 2010; Nicholas et al., 2013). A well-documented and freely available cellular automata model that is capable of basin and reach-scale topographic simulations is the Cellular Automaton Evolutionary Slope and River model (CAESAR; Coulthard et al., 2013; Wiel et al., 2007). CAESER now uses the LisFlood routine to model 2D water flow (Seybold et al., 2007; Bates et al., 2010; Coulthard et al., 2013). CAESAR can handle bedload and suspended load and uses two different routing schemes for each of these types of sediment transport. At the catchment scale, CAESER can simulate meandering and braiding planforms (Fig. 7). Bank erosion is possible, but it is calculated independent of flow and sediment routing (Coulthard and Van de Wiel, 2006). While not currently publicly available, the model of Nicholas (2010) has excellent hydrodynamic capabilities compared with earlier schemes and has been shown to (i) compete with 2D and 3D CFD models (Nicholas, 2010; Nicholas et al., 2013) and (ii) simulate the initiation and growth of free bars within straight channel geometries (Nicholas, 2010).

Overall, CA models have shown how simple rules can be utilized to construct models capable of synthesizing relatively complex river topography, ranging from meandering rivers to river deltas (Seybold et al., 2007; Nicholas, 2010; Liang et al., 2015; Schurmann et al. 2013; Nicholas et al., 2013). Fonstad (2006) argued that cellular automata models are good for multidisciplinary studies, such as between fluvial geomorphology and ecology, because of the differences in the type and complexity of conceptual schemas employed by various fields are readily incorporated into these types of models as transition rules. To date it has been demonstrated that CA models can create the topography of specific river planforms but are limited at the reach scale and catchment scales (Coulthard and Van De Wiel, 2006; Van De Wiel et al., 2007; Nicholas, 2009, 2010). Some freely available CA models are CAESAR (http://www.coulthard.org.uk/CAESAR.html) and an Excel version of the Murray-Paola braided river model (http://www.coulthard.org.uk/downloads/murray_and_paola.htm).

6.1.2.4. Discrete particle modeling. Discrete particle models operate at the grain scale. They differ from LEM and CA models in that model cells represent individual particles, rather than sediment mass (Naden, 1987, Jiang and Haff, 1993, Tribe and Church, 1999, Malmaeus and Hassan, 2002, Schmeeckle and Nelson, 2003, MacVicar et al., 2006, Hodge et al., 2007). These models have been constructed in both 2D vertical and horizontal grids. The generation of bed topography using these models has been primarily focused on modeling sub channel width scale features such as pebble clusters, transverse bedforms, and steps. Commonly, probabilistic rules are used that determine particle trajectories and interactions and these can further be related to flow hydraulics that dictate the probability of erosion and deposition. Most models have a similar computational algorithm, with deviations related to whether or how flow calculations are performed and the exact rules for particle entrainment and flow and sediment feedback. To provide further detail a brief summary of several discrete particle models is presented next.

One of the earliest particle models developed was by Naden (1987) who modeled sub-channel width scale gravel bed river topography from sediment transport as particle queuing. Arranged within a 2D vertical grid of sediments, the model was able to simulate profiles with characteristics of step-pools and antidunes. Tribe and Church (1999) developed a 2D kinematic model of gravel stream beds focusing on particle interactions rather than flow-based transport and deposition (e.g. Naden, 1987). Within the 2D planform model domain, gravel particles are modeled as discrete circular disks and particle entrainment and deposition are not based on modeled flow hydrodynamics but the local configuration and interactions of particles. Advancement to this model was made by Malmaeus and Hassan (2002) by allowing particle interactions without direct contact and also allowing for particle skimming. The model was found to be able to simulate realistic particle interactions and bed sediment structures (Hassan et al., 1992) reported in the literature and represent an avenue for further exploration in these types of channels. MacVicar et al. (2006) developed a 2D discrete particle model for gravel-bed rivers that considers turbulence, flow accelerations, and feedbacks between both the flow and sediment bed. Structurally, the model domain is similar to Naden (1987) in that a 2D vertical matrix is used along the channel centerline, but the model differs in that flow and sediment interactions are not strictly empirical. Instead, the model allows for feedback. Because of these modifications to prior particle-based models, such as the inclusion of feedback rules between flow and sediment, larger scale emergent bedforms can be created such as pools and riffles. With the goal of nesting discrete particle models within reach-scale cellular automata modeling, Hodge et al. (2007) developed a 3D grain DEM based model of bedload transport. The input to this model is an artificial 3D grain DEM. Grain movement is determined probabilistically with weights based on shear stress. The exact flow model was not specified in their study, so flexibility does exist in coupling the bedload grain model with more sophisticated flow models. A key benefit of this modeling approach is the treatment of fractional bedload transport and its ability to model changes to grain size distributions at the grain scale.

Discrete particle models have been useful to geomorphologists in understanding how bedforms are generated. Particularly, these models have been successfully applied to steeper channels (e.g. > 1%) whereas

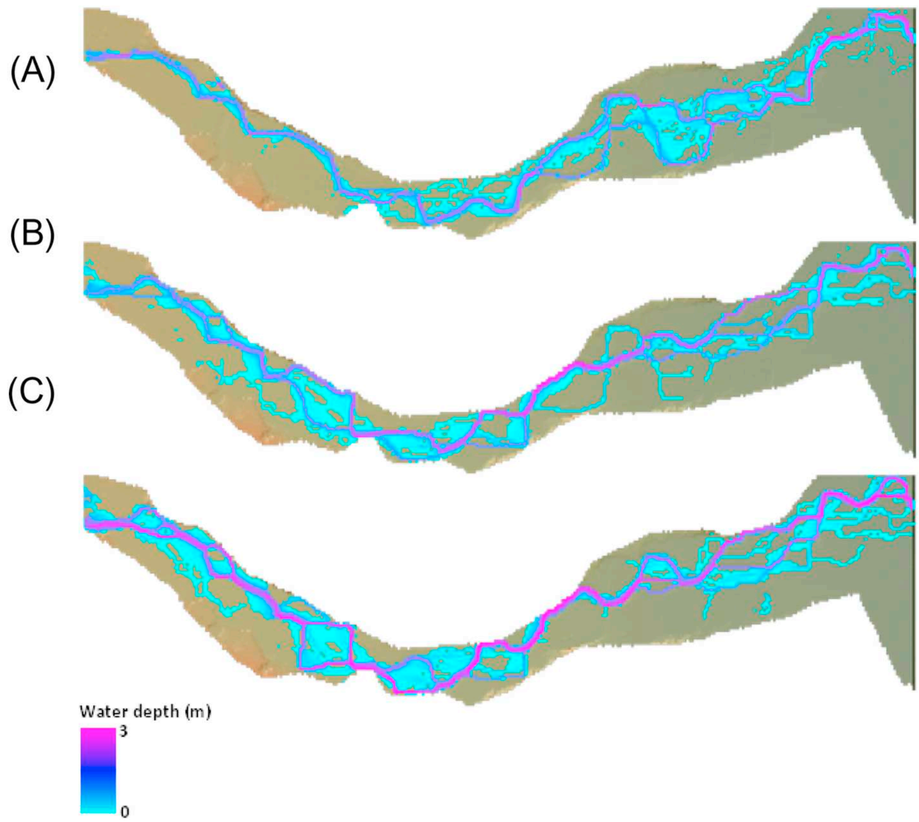

Fig. 7. CAESER model outputs of a 12-km stretch of the Upper Severn, UK at 1000 (A), 3000 (B), and 6000 (C) days of simulation illustrating planform changes. The model grid resolution is 20 m. Image courtesy of Tom Coulthard, University of Hull.

traditional morphodynamic models have not. Translating 1D profiles generated from discrete particle models to topography would require hybridizing with geometric modeling, as described in Section 5.2. It seems that 2D discrete particle model outputs could be easily translated to a heightmap, but the authors have not tried these themselves. To the authors knowledge, there are no publicly available discrete particle models, but models may be available from authors. Overall, outside of geomorphic inquiry, these types of models may not have much utility because similar outputs could be generated with far less user complexity.

6.2. Stochastic

Using statistical models, it is possible to create spatial series associated with geometric elements of river topography and less commonly discrete polygon objects. These approaches are primarily based on (i) fractal, (ii) auto-regressive, (iii) inverse spectral, and (iv) object-based methods. Each of these approaches makes inherent assumptions of the overall statistical structure of the data that limits the potential variability of the output. For inverse spectral methods, additional criteria, such as the frequency composition, are further specified, either on the basis of observational data or as artificial constructs.

6.2.1. Fractal modeling

Fractals have played a large role in general terrain synthesis and procedural modeling. In fluvial geomorphology, fractals have been primarily utilized as an analytical tool for investigating longitudinal profiles (Robert, 1988), planform geometry (Nikora, 1991; Sapozhnikov and Foufoula-Georgiou, 1996; Stolum, 1998), and river networks (Rodriguez-Iturbe and Rinaldo, 1997). While fractal terrain and river network algorithms exist for heightmaps (as described above in Section 2.1) no such approach exists for the creation of river topography. Overall fractal methods are currently limited to the simulation of 1D meandering planforms. Nikora and Sapozhnikov (1993) developed a random walk method of simulating fractal river meanders by using rule-based probabilities. The novelty in this method is that it explicitly accounted for valley width constraints on meander wavelengths and was also capable of simulating planforms with similar fractal dimensions of real rivers. This could be highly useful as an input for a combination-geometric approach.

6.2.2. Inverse spectral modeling

Commonly the analysis of 1D spatial series (e.g., bed-elevation, width, and/or width as a function of elevation series) and 2D fields is achieved through spectral analysis, whereby measurements in the space domain are transformed to the frequency domain via a convolution (Newland, 2012). Typically, such a convolution is performed using the Fourier transform, although wavelets offer another avenue for nonstationary series. Since the 1960's geographers have applied spectral methods to Earth surface landforms (Rayner, 1971). As Pike and Rozema (1975) state, spectral analysis can quantify the characteristics of general landforms, such as the presence of nonrandom periodic features, the roughness or power of specific frequencies, and the relationship between high and low frequency content, which implies how important large and small landforms are.

Although periodic signals are most commonly constructed by adding sine and cosine functions with different amplitude, angular frequency, and phase, it is possible to begin with a complex spectral pattern and then invert the spectral analysis procedure. The value comes from expert-based knowledge of how different spatial series interact across a range of flows to yield different hydrogeomorphic processes (e.g., Brown and Pasternack, 2016; Brown and Pasternack, 2017). To do this, a power spectral density function is first synthesized for the variables of interest in the frequency domain. This is where geomorphic interpretation of stage-dependent processes is needed- one a set of generic, end-member power spectral density functions is well-known for different hydrogeomorphic regimes, then individual random realizations (i.e., synthetic surrogates) are created by randomly re-assigning phases between 0 and 2π to the Fourier Transform, and then returning the data to the space domain using the inverse transform algorithm (Newland, 2012). The inverse Fourier transform allows one to exactly recover the series x_r and is given by:

$$x_r = \sum_{k=0}^{N-1} X_k e^{2\pi i k r/N} \tag{1}$$

It is rather straightforward to generate a random signal using the inverse DFT approach. First, X_k is calculated from a defined set of spectral data. Then, the phases, θ_k, are randomly selected and re-assigned. Finally, the inverse transform is used to reconstruct a realization based on the original data series.

If one wants to avoid the comprehensive frequency domain it is also possible to determine the inverse autocorrelation sequence using auto-regressive modeling while remaining in the space domain (Cleveland, 1972; Chatfield, 1979), though this yields far fewer periodic functions than inverse spectral modeling, as only the biggest 1–3 fluctuations are used. Regardless of whether inversion occurs in the space or frequency domain, it represents a compact method of statistically synthesizing 1D spatial series. If 1D series are generated, then these can be utilized within geometric modeling, along with other fluvial geometric element spatial series to create 2D topography. To date, this approach has not been used to model or create 1D spatial series or 2D height fields of river topography. This procedure can be extended to correlations between other random variables and for 2D processes as well (Newland, 2012). However, direct application to 2D processes would need to address the fact that this approach assumes that data is spatially isotropic, while river topography is inherently anisotropic (Merwade, 2009). One way to address this is to simply switch from sine and cosine functions to longitudinally anisotropic periodic functions, such as the cnoidal wave function. Cnoidal waves can be parameterized to have any shape, ranging from nearly sinusoidal to nearly flat-bottomed. This could be highly useful for step-pool and even riffle-pool longitudinal profiles, as well as for river width profiles dominated by periodic bedrock or manmade constrictions.

6.2.3. Auto-regressive modeling

Auto-regressive modeling is another method of statistical simulation capable of creating 1D spatial series of topographic attributes, such as the planform alignment and longitudinal profile. An auto-regressive model qualitatively states that current values are related to both past values and some level of randomness in the form of white noise (Newland, 2012). Auto-regressive models are considered random process models that use linear prediction formulas to predict an output of a system based on the previous outputs. These types of models are used when a trend is not assumed a priori and have been used extensively to analyze and model riverbed profiles (Bennett, 1976; Richards, 1976a, 1976b; Knighton, 1983) and river planforms (Ferguson, 1976; Phillips and Robert, 2007).

There are no readily available models to download that the authors are aware of, but programming a 1D auto-regressive model is elementary. Auto-regressive modeling can be used in two primary ways to create synthetic longitudinal profiles and planforms. First, coefficients from existing studies can be utilized insomuch as they represent landscape characteristics that are of interest. Second, for 2nd order auto-regressive models, Ferguson (1976) has cast the coefficients in terms of wavelength and a damping factor so that there is some control over the spatial series being created. For this first case, one would need to draw on the existing body of literature that is limited by the types of streams analyzed, the sampling distances in each study, and the scale of the rivers analyzed. From several existing studies, there is some context to how one could expect the AR coefficients would change with discharge, sediment size, and land use that could guide their use in synthesis (Bennett, 1976; Richards, 1976a, 1976b; Knighton, 1983). Given that model coefficients can provide a simple stochastic model for spatial series oscillations, these studies show that these coefficients can also be used to model changes associated with differing bed material, sediment sizes, and water discharge. Thus, AR modeling is a simple and compact method of modeling 1D profiles and alignments.

6.2.4. Object-based synthesis

Object-based synthesis rests on the idea that attributes of river topography, primarily morphologic units (e.g. quasi-discrete fluvial geomorphic units such as riffles and pools), can be treated as discrete objects. To date this has been performed as stochastic object synthesis, where probabilities of occurrence and even adjacency probabilities are assigned to differing morphologic units from specified distributions based on empirical studies of morphological unit organization (e.g. Grant et al., 1990; Myers and Swanson, 1997; Thompson, 2001; Wyrick and Pasternack, 2014). These types of models assume that specific morphologic units are preceded by other units, analogous to auto-regressive modeling, and that for certain combinations, exclusions may occur. For example, Myers and Swanson, 1997 developed a stochastic model of pool-to-pool spacing and widths in small, rangeland streams in Nevada, USA using a compound Poisson process. Later, Thomas and Nicholas (2001) modeled pool-to-pool spacing in coarse bedded streams whose pools are dominated by channel constrictions. A fundamental assumption of the Thomas and Nicholas (2002) model is the minimum length assumption, whereby there is a minimum length, and thus spacing, of pools related to hydraulic factors that lead to their formation, such as a backwater effect. The result of such an assumption is that there exists an exclusion length driven by local hydraulics where a new pool cannot exist (Thomas and Nicholas, 2002). In the model, the location of pool forming elements (PFE) are generated from uniformly and randomly distributed numbers and the sorted distances used to represent PFE locations within a simulation reach. A pool is assumed present at the first PFE and its length determined from a probability distribution based on empirical values. After the pool, a riffle is assumed to form with a set spacing. At the next PFE a determination is made whether or not the PFE is located within an existing pool-riffle couplet. A new pool-riffle couplet is then added only when it does not occupy an existing one. Overall, the modeling procedure creates a series of pool-riffle couplets as a function of distance. The impact of this study was that the synthetic modeling of pool spacing allowed insights into how regular pool spacing values commonly reported could exist, despite random controls on pool locations.

To date, only discrete units in 1D have been generated, as opposed to 2D object maps and heightmaps, so this approach has not been fully demonstrated. However, Wyrick and Pasternack (2014) analyzed the adjacency of laterally explicit morphological units and generated both abundance and size statistics as well as the probability of each unit type being adjacent to each other one. They also showed that each unit type has characteristic hydraulics, and to the extent that depth is a type of slope-detrended elevation, it would be possible to assign characteristic heights to each morphological unit type. That points toward the feasibility of translating this approach to yield heightmaps but would require additional development. Namely, the statistical and rule-based models for object location could be coupled with statistical models for bed topography for specific morphologic units.

7. Discussion

7.1. Digital realism

Digital rivers are constructed for a wide range of purposes by individuals with backgrounds drawing from graphic arts, computer sciences, earth sciences, engineering and architecture. Currently, this diversity in user background and purpose yields an inherent conflict in digital realism in that there exists no universal standard as to what

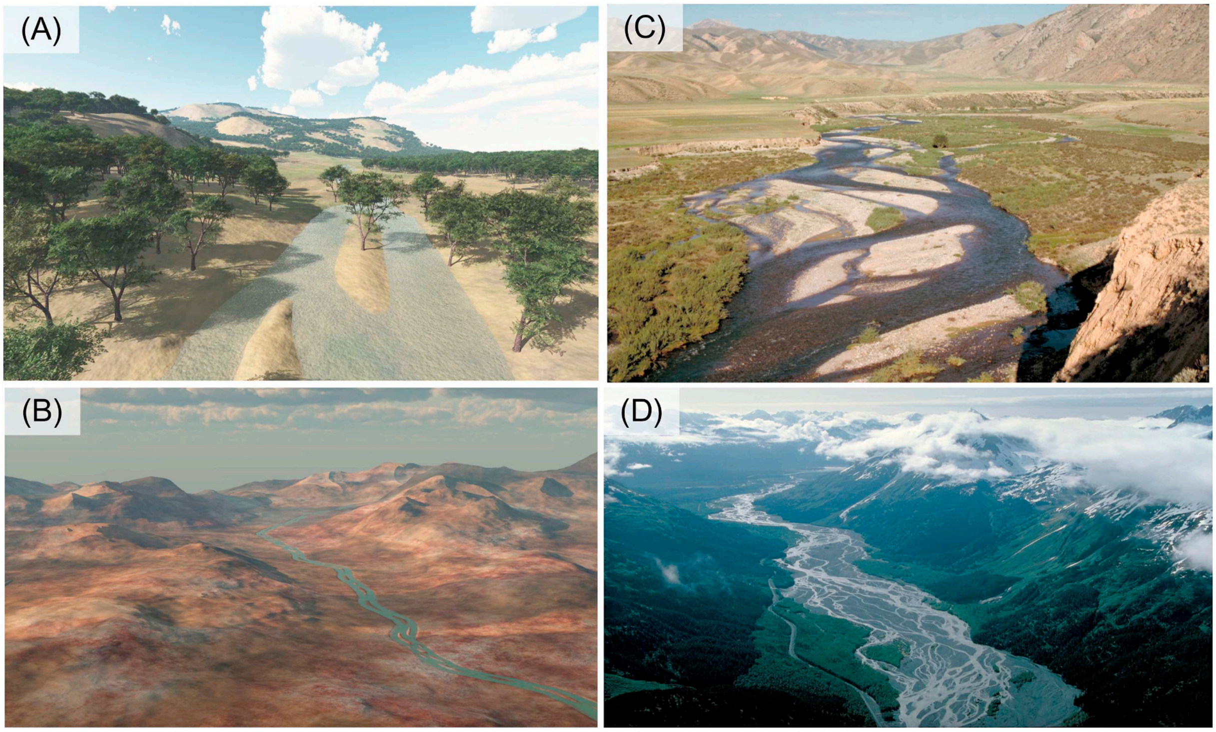

Fig. 8. Examples of procedural modeling at channel (A,B) and valley scales based on Geneveaux et al., 2013 compared with photographs of real rivers (C,D). The braid bars in (B) lack anisotropy in bar geometry found in real rivers (C). Further, there is no transition in the braid plain width (B) with increasing valley width (D) as commonly occurs in nature. The photograph in (C) is the Son-Kul River in Kyrgyzstan and in (D) is the Resurrection River, Kenai Peninsula, Alaska. Images A and B are courtesy of Eric Galin, University of Lyon and photographs C and D are courtesy of Marli Miller, University of Oregon.

makes a digital river adequately realistic. Part of this is intrinsic to visual assessments that are classically in the eye of the beholder. While geomorphologist may develop quantitative topographic, stratigraphic, statistical and morphologic metrics of realism in the scientific context, most people experience rivers without this background. An observer could inspect a river corridor surface constructed using 2D morphodynamic modeling that embodies the state of the art in fluvial geomorphology (e.g. Fig. 6), but the untrained eye may not even recognize it as a river due to the lack of surficial sedimentary texture, or the presence of vegetation and animals. Conversely, an artistically derived river corridor surface with those three types of elements (e.g. Fig. 4b) could have no underlying physical basis, yet appear more realistic than something created using a morphodynamic model that even includes stratigraphic layering. Trained geomorphologists may view these artistic representations as lacking the basic physical attributes of real river corridors (Fig. 8A).

Given the value of digital rivers to multiple applications, such as science, engineering, entertainment and art, there need not be a singular standard as to what constitutes a real river for all purposes, but this idea does deserve some attention. In a scientific context, one may want to test one or more unrealistic and realistic digital rivers to test the presence/absence of specific processes in different contexts. The juxtaposition of results from different designs can provide powerful insights about why rivers with specific features function as they do (Jackson et al., 2015). However, in an engineering context, one might only test realistic designs, but each with slight variations and embellishments to layer on unique features serving different management goals. In a video game context, realism is often outweighed by playability, leading to fluvial landscapes with much higher relief and vastly more discharge than naturally occur. A goal of this paper has been to provide a review of these different approaches so that ultimately digital rivers can be created that are realistic to scientists and casual observers who interact with digital rivers in through entertainment.

7.2. From headwater to Sea –what is possible?

This section aims to discuss what types of river systems are currently possible to simulate using different approaches, and then, what approaches could simulate the longitudinal diversity of channel form within a watershed. From headwater streams down to river deltas there is a continuum of fluvial form, ranging from hillslope hollows to step-pool streams, meandering rivers and ultimately to distributary channel networks through depositional terrain. Hypothetically, expert methods can be used to create any type of river morphology and planform, provided the user is knowledgeable in both fluvial geomorphology and the software platform(s) being used; Red Dead Redemption II© boldly illustrates the achievable scope given enough resources. For the strategic approaches discussed in this article, Table 3 shows what is currently possible in terms of stream morphology and planform typology. In terms of river planforms, straight, meandering, braided, anabranching, and distributary can be created from a variety of methods with varying assumptions and complexities (Table 3). Straight channels are geometrically and topographically simple and can be created in a simple and straightforward manner in most approaches. Other than straight rivers, meandering rivers have the most methods available and can be simulated rather quickly, while other planforms, such as braided and anabranching, are more limited. With regards to channel morphology, dune-ripple, riffle-pool, and step-pool profiles can be simulated in 1D from statistical and analytical methods. Discrete particle models have been shown to simulate a plethora of channel profiles, such as those just mentioned, as well as channel forms associated with steeper gradients (e.g. > 1%), such as transverse ribs, sediment clasts, and cascades. However, both of these would need to be combined with geometric modeling to create 3D river channel topography.

Presently, there is no single tool or approach to simulate the continuum of channel form from headwaters to the sea. Most planforms associated with lowland river valleys, including river distributary networks, have been simulated using 2D morphodynamics within the

Table 3
Examples of current strategic approaches to topographic synthesis by channel morphology and planform typology. P denotes that it is possible but no peer-reviewed studies to date and N denotes not able to be created.

Channel Morphology	1D Spatial Series	2D Height Map
Dune-ripple	van Rijn,1984; Karim, 1999; Macvicar et al., 2006	Paarlberg et al., 2009; Nabi et al., 2013
Riffle-pool	Cao et al., 2003; Macvicar et al., 2006; deAlmeida and Rodriquez, 2012	Beck, 1988; Nicholas, 2010, 2013
Plane bed	P	P
Step-pool	P	Tribe and Church, 1999; Malmaeus and Hassan 2002
Cascade	N	N
Planform		
Meandering	Langbein and Leopold, 1966; Nikora and Sapozhnikov, 1993	Coulthard and Van de Wiel, 2006; Wang et al., 2010a,b; Sylvester et al. 2011; Nicholas et al., 2013
Braided	N	Murray and Paola, 1994; Nicholas et al., 2013; Schuurman et al. 2013
Anastomising	N	Murray and Paola, 2003; Wang et al., 2010; Nicholas et al., 2013
Distributary	N	Pyrcz, 2009; Edmonds and Slingerland, 2007; Seybold et al. 2007; Liang et al. 2015

Delft3D platform (e.g. see Table 3), but this approach has not been used to model or create the continuum of these forms within a catchment. To create the topography of rivers and stream networks within complete landscapes, a mosaic of techniques appears to be needed (de Carpentier and Bidarra, 2009). By analogy with global climate models, river synthesis models may require multiple modules connected within a larger framework to achieve the range of outcomes needed, as any one single algorithm does not capture the diversity of processes and forms across multiple scales at this time.

When considering the headwater to sea problem, expert approaches can be used to create multiscalar surfaces with as much detail as one wants to invest time to create, and with the outcomes as good as the user, software platform, and time investment. In many regards, this is the current state-of-the art for professional practice in engineering and landscape architecture with CAD, though engineers rarely design at the catchment scale unless they are addressing a problem like reclamation design for mountain mining (DePriest et al., 2015) or large-scale housing development. Engineers tend to limit their efforts to just essential topographic design at a scale that is practical for construction. In contrast, landscape architects aim to convey more detail since their work interacts with the public to gain support for implementation. Most of their efforts tend to be in planform view or cross-sectional view, both with feature-based elements, but they can include intensive 3D design as well. However, from a practical level the extreme cost of implementing laborious brush and map expert approaches is never going to be affordable for environmental problem solving using traditional consultant-based funding approaches. Thus far, this has only been affordable for open-world video game design where billion-dollar revenues justify such effort. The construction of entire terrains, including rivers, in video games such as Skyrim, Dragon Age Inquisition, Assassins Creed III, and countless user-generated maps in Minecraft are all testimony to what people can achieve with these tools at the catchment scale if they want to invest the time into it and when working as a large collaborative team. Nevertheless, for traditional business use with low labor investment, it is essential to move beyond these traditional methods and get at automated approaches.

7.3. Cross-pollinating among disciplines

A key outcome of this review is that real and artificial rivers are generated for digital environments from a variety of applied and scientific disciplines. An interesting aspect of cross pollination is that, broadly speaking, Earth science and computer science applications have different measures of success. While computer scientists strive to create landforms that are visually realistic and are driven by aesthetics (e.g. Smelik et al., 2014), fluvial geomorphologists seek to understand how and why specific forms originate as well as how they change spatially and temporally. River engineers want to build real analogues to geomorphologist ideals. Put another way, fluvial geomorphologists seek hydrogeomorphic process realism over landform realism, with terrains generated by systems of equations that can be simplified (CA models) or highly complex (3D morphodynamic process models). Inevitably models simplify real world processes and forms. Some models do aim to achieve as much realism as possible, while others aim for parsimonious methods (Willgoose, 2005). Modeling of fluvial form and process has focused on developing, calibrating and validating models to real world conditions. To this, many mathematical models of river flow and sediment transport suffer from scientific criticisms related to underlying model assumptions, lack of validation, and unrealistic outputs (Cao and Carling, 2002). However, their utility in computer science may be unbounded because those applications do not have to adhere to the constraints of real-world calibration and validation. Therefore, many models that are considered inadequate for understanding fluvial geomorphologic processes may be useful computer science applications in creating digital rivers. Fluvial geomorphologists can identify processes and mathematical relationships for specific types of topography, but computer scientists can help those ideas be implemented in user friendly and dynamic platforms for uses in other fields.

An opportunity for both disciplines to collaborate is to develop ways of relating visual realism and landscape aesthetics to quantitative measures of river corridor variability. Leopold (1969) developed an objective approach to evaluate landscape aesthetics that could be used to forward this idea. Physical, biological and human interests are used as organizing elements in developing metrics for characterization. For example, the amount of trash in a river corridor is a human centric attribute that can be quantified by direct measurement. If geomorphologists and computer scientists could agree on which attributes have the most utility in providing a link between the physical form of river corridor and their aesthetics, then there could possibly be a greater exchange between disciplines. River channel classification may provide an adequate bridge in this context, as the more advanced procedural models of rivers have already shown their utility (Genevaux et al., 2013). Habitat typing classifications that consider biotic forms, such as vegetation, over those that are strictly geomorphic may be more useful to graphic designers, because of the role vegetation plays in most real and artificial scenes (Fig. 4).

We posit that much can be gained from cross-disciplinary collaborations, especially considering the fiscal motivation behind each discipline. Comparatively, the total economic motivation to have tools capable of synthetic terrain generation and modification for use in river restoration, engineering, and science is on par with that for use in computer games and movie animation, but the latter are far more visible to and used by the public. Of course, the societal value of these different uses is debatable, but ultimately both are driving advances that can benefit each other if there were cross-pollinating efforts. Costs for restoration projects are highly concentrated and centralized, whereas those for video games and animations are distributed among a wide user base, making it more feasible to expend more and adapt to the latest technologies. The annual expenditure for river restoration activity worldwide is poorly documented, but for the United States Bernhardt et al. (2005) estimated it to be ~ $1 billion. A primary cost associated with large, marquee river restoration projects is land purchase, such as the expenditure of ~ $300 million to buyback land for the Kissimmee River Project in Florida, which is currently estimated to have an eventual total cost of $980 million, though this is spread over many years (Bousquin, 2010). The Kissimmee River Project is an excellent example where the river's terrain was heavily altered. As another example the cost of the Elwha Dam Restoration Project, including but not limited to the removal of two large dams, has been estimated at $324.7 million (Callis, 2011).

Meanwhile, individual video games have sales on par with the cost of the largest restoration projects. The most heavily used terrain generating and modifying video game ever is Minecraft®, which uses procedural generation to create infinitely sized worlds. Users can modify generated worlds either by adding or subtracting individual 1^3 m blocks or using external third-party software, such as the free World Painter®. As of June 2016, there were over 100 million registered users of Minecraft on sales of over 106 million units. Sales revenue in 2013 alone was $330 million (Grundberg and Hansegard, 2014). Minecraft® uses simple volumetric pixels, so in contrast to that consider a premiere exemplar for the application of synthetic terrain and river in advanced graphics video games from the same vintage– The Elder Scrolls V: Skyrim®, an open-world fantasy adventure game with stark mountain terrain, waterfalls, and many rivers covering an estimated ~ 17 km^2 of horizontal terrain and 3.2 km of height (Sutton, 2012). This game has sold > 30 million copies with over $1.3 billion in revenues. Another open-world historical science fiction game with realistic terrain and rivers, Assassin's Creed 3®, sold 12 million units in its first four months on the market. With a fixed retail price of ~ $60 at that time, gross income was ~ $720 million. It is being remastered and resold with improved graphics in 2019. Red Dead Redemption II sold 23 million units in its first fiscal quarter available, generating $1.38 billion in revenues. Other examples in recent years include the Far Cry® series, Supersonic Sled®, and Rigs of Rods® (a vehicle simulator). Usually a handful of high-revenue games (> $100 million) are produced each year along with many low-budget games using terrain generators. Comparatively, the profit motive for advancing methods for synthetic river terrain generation and modification clearly resides with the video game and animation industries over scientific and engineering ones.

Outside of collaborations between geomorphologists and computer scientists, further exploration and development of digital river design could benefit the growing research and applied science of how to restore or recreate heavily impacted rivers and streams. This is an area where both computer and earth scientists have a lot to offer. In developing designs for restoration projects, rivers are often recast in light of significant anthropogenic impacts, such as flow regulation, floodplain development, stream burial, dredger and dredge mining, to name a few. For example, in river restoration design, ideas and concepts are typically conveyed to public stakeholders using 2D rendered conceptual sketches, idealizations using landscape architecture methods, or abstract CAD drawings meant for construction. While these are helpful, they do not convey the full topography of what is envisioned, let alone the associated processes. Artistic renderings can be time consuming and are only as good as the artist. Design details and drawings are common staples of engineering, but primarily due to technological deficiencies as the field developed over time - not because they are the best way to convey ideas. One way that this process could be improved is through Virtual Geographic Environments (VGEs, Goodchild, 2009, Konecny, 2011, Lin et al., 2013) whereby stakeholders can experience and interact with designs before they are constructed in the real world. VGE's can nest not only terrain but other sub models, too, that users can virtually interact with, such as vegetation and human infrastructure. While VGEs are currently in their infancy in terms of widespread usage, they are a prime example of cross-pollination that should be pursued by river restoration scientists and practitioners.

7.4. Future directions

Future research into building digital rivers is relatively open ended depending on purpose. Here we discuss several future directions that may hold promise including: morphodynamic, hierarchical, procedural, object based, hybrid modeling, and machine learning.

Recent developments in morphodynamic modeling using reduced complexity and traditional physics suggest that process-based models capable of producing emergent digital river topography will improve dramatically with time and become more accessible to users across engineering, geomorphology, and landscape animation. The recent shift of the DELTARES morphodynamic model to be open source marks a potential turning point along this line and will surely pressure other developers to open their sources as well, though such models can have steep learning curves. Using morphodynamic models has the advantage over most strategic methods in that they are based on physical processes as we perceive and model them today. They can develop emergent forms that are related to exogenous terrain features. Morphodynamic models have commonly been criticized for excessive computational demands, a limited inclusion of relevant processes, lack of stochastic processes, simplified bed and bank material heterogeneity, lack of vegetation feedbacks, and divergent outcomes when different models of the same type are used with the same starting inputs. Also, it is challenging to know what hydrologic and sediment flux regimes to use and how long to run unsteady models in light of dynamic boundary conditions. Advances in parallel computing and/or graphical processing units and associated software should address computational aspects making larger and more detailed models more feasible. Further, research has over time identified key physical processes, such as convective accelerations and secondary flow needed to produce emergent fluvial forms such as meanders and anabranches. These processes can be reduced to more simplified forms allowing use over greater spatial domains in reduced complexity models. Simplifications of bed material heterogeneity are often used to limit computational demands, but this too should continue to evolve. Considering how far morphodynamic modeling has come over the last 30 years we expect this growth to not only continue but accelerate.

Fluvial systems exhibit hierarchical organization in that smaller scale processes and features are nested within larger scale features (Hallet, 1990; de Boer, 1992). The implications of hierarchical organization in river systems have significant bearing on how they can be modeled. Larger features typically operate over larger time scales, while smaller scale features operate over shorter time scales (de Boer, 1992; Werner, 1999). Werner (1999) advocated for the use of hierarchical modeling for simulating complex landforms because the smaller scales processes are "slaved" to larger scale dynamics. Drawing on this, Doeschl-Wilson and Ashmore (2005) demonstrated conceptually how hierarchical modeling could be used to overcome some of the pitfalls of the Murray and Paola (1994). One way to perform hierarchical modeling is to represent features, and not necessarily the dynamics, with models that describe variability at specific scales. As an example, Clarke (1988) proposed a scale dependent method for creating terrain that married fractal geometry and inverse spectral synthesis. While Clarke's model (1988) has not been used to create river topography, it is worth considering here because of the potential for this method to yield spatially dynamic river topography. The elevation field of the model considers both scale-dependent and scale-independent features defined by the equation:

$$Z_{x,y} = T_{x,y} + F_{x,y} + H_{x,y} + E_{x,y} \tag{2}$$

where $T_{x,\ y}$ is a trend model, $F_{x,\ y}$ are the scale dependent surface components related to the Fourier coefficients, $H_{x,\ y}$ is the scale independent surface components, and $E_{x,\ y}$ is an erosion component. Each of these components are argued by Clarke (1988) to be vital aspects of natural terrain and together, create terrains with both visual and realistic properties by merging stochastic and deterministic models. Thus, this model shows how a reductionist analysis of topography can be inverted by considering the total elevation field to be a linear combination of scale independent and scale dependent attributes. While it hasn't been explored in great detail for river topography to date, hierarchical, scale dependent modeling represents a relatively untapped area of research for digital rivers. In particular, this approach fits within the synthetic river valley approach, whereby each geometric element can be designed by creating linear combinations of scale independent and scale dependent attributes (Brown et al., 2014).

Procedural modeling has been successfully applied to the creation of terrains and river networks. However, it has also recently been applied to more complex problems such as structurally sound masonry buildings (Whiting et al., 2009) and entire cities (Parish and Müller, 2001). While it has begun to incorporate geomorphic theory, such as in Genevaux et al. (2013), this is still very simplistic compared to natural river topography (Fig. 8). Fluvial geomorphology has produced a wealth of knowledge related to how rivers and stream look and function, yet most of this is untapped in the realm of procedural modeling, likely because the coders and users are experts in math and art, not geoscience and engineering.

Technological advances in data collection have begun to yield high-resolution surveys of real rivers in a variety of settings (Bangen et al., 2014), the sharing of topographic data sources is now possible through websites such as www.opentopography.org, which allows researchers to have access to a wide array of meter-scale topographic data sources. A key area of research should be distilling data for specific channel typologies (e.g. both planform and morphology) into adjustable and scalable ‘topographic templates’ that can be used in evolutionary algorithms, procedural blocks and in object-based modeling for river restoration and computer science applications. Using topographic templates as a basis for procedural modeling primitives could be a way to capture the benefits of procedural modeling while also incorporating data on actual river topography. For example, terrain simulation techniques using evolutionary algorithms, such as Saunders (2006), rely on the user providing a reference heightmap. If topographic templates for specific types of rivers could be generated, then they could be used in video games if a library of topographic templates for specific types of rivers was also available. Further, object based procedural modeling may be a way to model boulders and stream wood for more diverse channel typologies of smaller order streams in mountainous regions (Fig. 9). Many civil engineers use 3D “blocks' for these types of features in river restoration plans, so the technology already exists. What is needed is developing object blocks and merging them with models of channel form, so they can hierarchically nested as they are in nature (Hallet, 1990).

Landscape terrain modeling software that hybridizes procedural, morphodynamic, and geometric modeling methods already exists and can be further enhanced to provide the best balance between realism (both process and form) and computational speed. For example, it is presently possible to begin with the “blue line” hydrography of a river network and a sloped plane representing the frontal slope of the mountain that the hydrography will sit in, combine them mathematically, and then hit them with LEM-style hillslope and channel erosion processes already built into the software to yield a dendritic watershed on a mountain front. The recent development of an expert-based geometric method/model for synthetic river valleys shows that it is highly feasible to take the principles of geomorphology and apply them to create realistic terrains with geometric modeling that in turn have real physical processes and associated ecological functions (Brown et al., 2014). With modest effort, this river-centric approach can be built into landscape-scale and reach-scale terrain generators, such as World Machine® and Bryce®. An example of this approach is shown in Fig. 10. Processing time to generate such a model would be seconds to a few hours depending on terrain size and complexity, which would be far faster than pure CA and morphodynamic modeling. Meanwhile, geometric modeling has the value of being able to create unconstrained terrains as well as constrained ones without having to recode and re-run the way a morphodynamic model would require you to really delve into its innards to deviate from the pre-programming. Recognizing that there is still more we don't know about rivers than we do know, it is highly important to have this capability. Another important advantage of this approach over pure morphodynamic modeling is that it is explicitly multi-scalar, with a wide limit on the range and resolution of scales to address; only constrained by computation time. Overall, the value of pursuing procedural hybrid modeling appears to be high, even as morphodynamic models continue to advance.

On 29 October 2018, the video game publisher Electronic Arts announced Project Atlas, which may prove to be a fundamentally

Fig. 10. Oblique view of an artificial river valley created using the Synthetic River Valley geometric modeling approach now built into World Machine®.

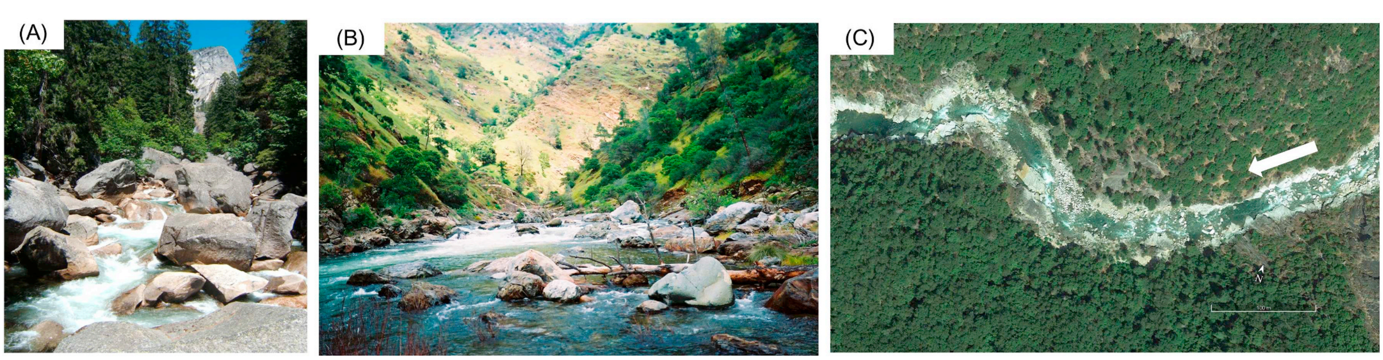

Fig. 9. Examples of fluvial topography with boulders and streamwood. These local 3D objects yield complex surfaces that cannot currently be created procedurally from any as-of-yet articulated methods other than expert-based techniques that rely on artistry. Future efforts could solve this problem if attended to.

disruptive scientific and technological development for digital river synthesis, if not for geoscience as a whole. Project Atlas recognizes that expert-based synthesis involving manual labor is reaching its limit of scalability due to labor cost, necessitating adoption of widespread automation throughout video game development. At the core of this automation lies Artificial Intelligence (AI). Quoting Ken Moss, Electronic Arts' Chief Technology Officer, “With Project Atlas, we are starting to put the power of AI in the creative's hands… we are using high-quality LIDAR data about real mountain ranges, passing that data through a deep neural network trained to create terrain-building algorithms, and then creating an algorithm which will be available within the platform's development toolbox. With this AI-assisted terrain generation, designers will within seconds generate not just a single mountain, but a series of mountains and all the surrounding environment with the realism of the real-world.” Already, AI has transformed many industries, and now it is moving into geosciences. Shen (2018) reviewed the potential applications for hydrologic sciences, while specific applications of AI for geomorphic analysis are emerging (Brungard et al., 2015; Perry and Dickson, 2018). Yet, Project Atlas is offering something far beyond mere subdisciplinary analysis of any one geoscientific phenomenon; it is a comprehensive and integrated AI system that will phenotypically understand and produce holistic combinations of synthetic natural worlds, not only with hyper-realistic terrain, but also biota, ecological interactions, human infrastructure, and human culture. Further, it will not just produce outputs, but more importantly provide tools to those who want to use this power to make their own customized outputs. In theory, AI could hybridize all pre-existing toolsets, bring in new deep learning toolsets, as well as invent entirely unforeseeable new approaches. Can and will such creative capability be available to and taken up by scientists and engineers? Not only that, but under Project Atlas, which also includes a suite of video game engines, there is the opportunity to fully integrate synthetic river development with immersive experiential visualization systems to put stakeholders into those river corridors before construction to fully experience a wide range of geophysical processes and ecological functions. This could allow for troubleshooting problems interactively before spending large sums on construction. Thus far that has not been possible due to the manual labor of video game engines, but Project Atlas would change that, too. Overall, the future is exciting and bright for synthetic river development.

8. Concluding remarks

This paper shows that there are many avenues to building artificial digital rivers, and we have made the first attempt to synthesize a wide array of approaches drawing on Earth and computer sciences and video games. From the review, it was found that while a diverse array of channel types can be simulated for river reaches or segments, complex and spatially dynamic models do not exist to date that can represent the full topography of rivers within watersheds. Expert-based strategies that rely on artistry are unbounded, but laborious and often not grounded in how rivers are shaped and formed. Strategic approaches are grounded in deterministic or stochastic physical rules but are limited by our current knowledge of riverine geomorphology, which of course is still evolving. To bridge this gap, we proposed a now obvious need for multidisciplinary collaborations that will improve all approaches for creating synthetic river topography. Importantly, we do not advocate for one approach over another, as the spectrum of methods presented affords a wide range of approaches to an inherent multidisciplinary problem. Ultimately this begs the question as to what is considered realistic or accurate in depicting the Earth, which is application specific. Considering how much of our world is being recast in digital formats, having realistic rivers for scientific work – and play - is an area of research that should be prioritized in both fluvial geomorphology and computer science.

Acknowledgments

Financial support was provided by a Henry A. Jastro Graduate Research Award to R. A. Brown. Funding for G.B. Pasternack was provided by the USDA National Institute of Food and Agriculture [Hatch project number # CA-DLAW- 7034-H]. We thank the editor and anonymous expert reviewers for their constructive guidance that improved the depth and clarity of the manuscript.

References

Abad, J., Garcia, M., 2006. RVR Meander: a toolbox for re-meandering of channelized streams. Comput. Geosci. 32, 92–101. https://doi.org/10.1016/j.cageo.2005.05.006.

Abad, J.D., Garcia, M.H., 2008. Bed morphology in Kinoshita meandering channels: Experiments and numerical simulations. in River, Coastal and Estuarine Morphodynamics: RCEM 2007. In: Proceedings of the 5th IAHR Symposium on River, Coastal and Estuarine Morphodynamics, vol. 2, pp. 869-875, 5th IAHR-Symposium on River, Coastal and Estuarine Morphodynamics, RCEM 2007, Enschede, Netherlands, 9/17/07.

Anderson, J.K., Wondzell, S.M., Gooseff, M.N., Haggerty, R., 2005. Patterns in stream longitudinal profiles and implications for hyporheic exchange flow at the H.J. Andrews Experimental Forest, Oregon, USA. Hydrological Processes 19 (15), 2931–2949. https://doi.org/10.1002/hyp.5791.

Aron, G., White, E.L., 1982. Fitting a Gamma distribution over a Synthetic Unit Hydrograph. J. Am. Water Resour. Assoc. 18, 95–98. https://doi.org/10.1111/j.1752-1688.1982.tb04533.x.

Banavar, J., Colaiori, F., Flammini, A., Giacometti, A., Maritan, A., Rinaldo, A., 1997. Sculpting of a Fractal River Basin. Phys. Rev. Lett. 78, 4522–4525. https://doi.org/10.1103/physrevlett.78.4522.

Bangen, S.G., Wheaton, J.M., Bouwes, N., Bouwes, B., Jordan, C., 2014. A methodological intercomparison of topographic survey techniques for characterizing wadeable streams and rivers. Geomorphology 206, 343–361. https://doi.org/10.1016/j.geomorph.2013.10.010.

Bates, P.D., Horritt, M.S., Fewtrell, T.J., 2010. Simple Inertial Formulation Of The Shallow Water Equations For Efficient Two-Dimensional Flood Inundation Modelling. Journal of Hydrology 387, 33–45.

Beck, S.M., 1988. Computer-Simulated Deformation of Meandering River Patterns. PhD Thesis. University of Minnesota, Minneapolis, MN.

Begin, Z.B., 1988. Application of a diffusion-erosion model to alluvial channels which degrade due to base-level lowering. Earth Surf. Process. Landf. 13, 487–500. https://doi.org/10.1002/esp.3290130603.

Bennett, R.J., 1976. Adaptive adjustment of channel geometry. Earth Surf. Process. Landf. 1, 131–150. https://doi.org/10.1002/esp.3290010204.

Bernhardt, E.S., Palmer, M.A., Allan, J.D., Alexander, G., Barnas, K., Brooks, S., Carr, J., Clayton, S., Dahm, C.N., Follstad-Shah, J., Galat, D.L., Gloss, S., Goodwin, P., Hart, D.D., Hassett, B., Jenkinson, R., Katz, S., Kondolf, G.M., Lake, P.S., Lave, R., Meyer, J.L., O'Donnell, T.K., Pagano, L., Powell, B., Sudduth, E., 2005. Synthesizing U.S. River Restoration efforts. Science 308, 636–637. https://doi.org/10.1126/science.1109769.

Bhowmik, N.G., Adams, J.R., Sparks, 1986. Fate of Navigation Pool on Mississippi River. J. Hydraul. Eng. 112, 967–970. https://doi.org/10.1061/(ASCE)0733-9429(1986)112:10(967).

Bousquin, S.G., 2010. Kissimmee River Restoration Project Fact and Tour Sheet. http://my.sfwmd.gov/portal/page/portal/xrepository/sfwmd_repository_pdf/krr_krrep_factstour_sheet.pdf.

Braun, J., Sambridge, M., 1997. Modelling landscape evolution on geological time scales: a new method based on irregular spatial discretization. Basin Res. 9, 27–52. https://doi.org/10.1046/j.1365-2117.1997.00030.x.

Bridge, J.S., 1976. Mathematical model and FORTRAN IV program to predict flow, bed topography and grain size in open-channel bends. Comput. Geosci. 2, 407–416. https://doi.org/10.1016/0098-3004(76)90036-4.

Bridge, J.S., 1977. Flow, bed topography, grain size and sedimentary structures in open channel bends : a three dimensional model. Earth Surface Processes 2, 401–416.

Bridge, J.S., 1982. A revised mathematical model and FORTRAN IV program to predict flow, bed topography, and grain size in open-channel bends. Comput. Geosci. 8 (1), 91–95.

Bridge, J.S., 1992. A revised model for water flow, sediment transport, bed topography and grain size sorting in natural river bends. Water Resour. Res. 28, 999–1013. https://doi.org/10.1029/91WR03088.

Bridge, J.S., Gabel, S.L., 1992. Flow and sediment dynamics in a low sinuosity, braided river: Calamus River, Nebraska Sandhills. Sedimentology 39, 125–142. https://doi.org/10.1111/j.1365-3091.1992.tb01026.x.

Brown, R.A., Pasternack, G.B., 2017. Bed and width oscillations form coherent patterns in a partially confined, regulated gravel–cobble-bedded river adjusting to anthropogenic disturbances. Earth Surface Dynamics 5, 1–20. https://doi.org/10.5194/esurf-5-1-2017.

Brown, R.A., Pasternack, G.B., Wallender, W.W., 2014. Synthetic River Valleys: Creating Prescribed Topography for Form-Process Inquiry and River Rehabilitation Design. Geomorphology 214, 40–55. https://doi.org/10.1016/j.geomorph.2014.02.025.

Brown, R.A., Pasternack, G.B., Lin, T., 2016. The topographic design of river channels for form-process linkages. Environ. Manag. https://doi.org/10.1007/s00267-015-0648-0.

Brungard, C.W., Boettinger, J.L., Duniway, M.C., Wills, S.A., Edwards Jr., T.C., 2015. Machine learning for predicting soil classes in three semi-arid landscapes. Geoderma 239, 68–83. https://doi.org/10.1016/j.geoderma.2014.09.019.
Brush, L.M., 1961. Drainage basins, channels and flow characteristics of selected streams in Central Pennsylvania. In: Professional Paper 282F. United States Geological Survey, Washington, DC, pp. 145–181.
Callis, T., 2011. Elwha River Dam Removal Project Cost Estimate Shrinks. Peninsula Daily News, pp. 2011. March 22. http://www.peninsuladailynews.com/article/20110323/NEWS/303239989/elwha-river-dam-removal-project-cost-estimate-shrinks.
Cao, Z., Carling, P., 2002. Mathematical modelling of alluvial rivers: Reality and myth. Part 1: General review. In: Proceedings of the Institution of Civil Engineers - Water and Maritime Engineering. vol. 154. pp. 207–219. https://doi.org/10.1680/wame.2002.154.3.207.
Cao, Z., Carling, P., Oakey, R., 2003. Flow reversal over a natural pool-riffle sequence: a computational study. Earth Surf. Process. Landf. 28, 689–705. https://doi.org/10.1002/esp.466.
Castro, J., Jackson, P., 2001. Bankfull discharge recurrence intervals and regional hydraulic geometry relationships: patterns in the Pacific Northwest, USA. J. Am. Water Resour. Assoc. 37, 1249–1262. https://doi.org/10.1111/j.1752-1688.2001.tb03636.x.
Cerqueira, C.S., Santos, W.A., Ambrosio, A.M., 2013. Serious Game Interaction Techniques Applied to an Operational Satellite Simulator. In: Art & Design Track at the XII Simpósio Brasileiro de Games e Entretenimento Digital. SBGames, São Paulo.
Chang, H.H., Osmolski, Z., 1988. Computer-aided Design for Channelization. J. Hydraul. Eng. 114, 1377–1389. https://doi.org/10.1061/(ASCE)0733-9429(1988)114:11(1377).
Chase, C., 1992. Fluvial landsculpting and the fractal dimension of topography. Geomorphology 5, 39–57. https://doi.org/10.1016/0169-555X(92)90057-U.
Chatfield, C., 1979. Inverse Autocorrelations. Journal of the Royal Statistical Society. Series A (General) 142, 363–377.
Chow, V.T., 1959. Open-Channel Hydraulics. McGraw-Hill Publishers, New York.
Church, M., 2006. Bed Material Transport and the Morphology of Alluvial River Channels. Annu. Rev. Earth Planet. Sci. 34, 325–354. https://doi.org/10.1146/annurev.earth.33.092203.122721.
Cieplak, M., Giacometti, A., Maritan, A., Rinaldo, A., Rodriguez-Iturbe, I., Banavar, J.R., 1998. Models of fractal river basins. J. Stat. Phys. 91, 1–15. https://doi.org/10.1023/A:1023069201470.
Clark, C., 1945. Storage and the Unit Hydrograph. Trans. Am. Soc. Civ. Eng. 110, 1419–1446.
Clarke, K.C., 1988. Scale-Based Simulation of Topographic Relief. The American Cartographer 15, 173–181.
Cleveland, W.S., 1972. The Inverse Autocorrelations of a Time Series and their applications. Technometrics 14, 277–293. https://doi.org/10.1080/00401706.1972.10488914.
Collins, D.B.G., Bras, R.L., Tucker, G.E., 2004. Modeling the effects of vegetation-erosion coupling on landscape evolution. J. Geophys. Res. 109, F03004. https://doi.org/10.1029/2003JF000028.
Coulthard, T.J., 2001. Landscape evolution models: a software review. Hydrol. Process. 165, 165–173. https://doi.org/10.1002/hyp.426.
Coulthard, T., Van de Wiel, M., 2006. A cellular model of river meandering. Earth Surf. Process. Landf. 2731, 123–132. https://doi.org/10.1002/esp.1315.
Coulthard, T.J., Kirkby, M.J., Macklin, M.G., 1996. A cellular automaton landscape evolution model. In: Abrahart, R.J. (Ed.), Proceedings of the First International Conference on GeoComputation. vol. Volume 1. School of Geography, University of Leeds, pp. 248–281.
Coulthard, T.J., Macklin, N.G., Kirkby, M.J., 2002. A cellular model of Holocene upland river basin and alluvial fan evolution. Earth Surf. Process. Landf. 27, 269–288. https://doi.org/10.1002/esp.318.
Coulthard, T., Hicks, D., Van De Wiel, M., 2007. Cellular modelling of river catchments and reaches: Advantages, limitations and prospects. Geomorphology 90, 192–207. https://doi.org/10.1016/j.geomorph.2006.10.030.
Coulthard, T.J., Neal, J.C., Bates, P.D., Ramirez, J., de Almeida, G.A.M., Hancock, G.R., 2013. Integrating the LISFLOOD-FP 2D hydrodynamic model with the CAESAR model: implications for modelling landscape evolution. Earth Surf. Process. Landf. 38, 1897–1906. https://doi.org/10.1002/esp.3478.
Crosato, A., Mosselman, E., 2009. Simple physics-based predictor for the number of river bars and the transition between meandering and braiding. Water Resour. Res. 45, W03424. https://doi.org/10.1029/2008WR007242.
Davidson, S.K., Hartley, A.J., Weissmann, G.S., Nichols, G.J., Scuderi, L.A., 2013. Geomorphic elements on modern distributive fluvial systems. Geomorphology 180-181, 82–95. https://doi.org/10.1016/j.geomorph.2012.09.008.
de Boer, D.H., 1992. Hierarchies and spatial scale in process geomorphology: a review. Geomorphology 4, 303–318. https://doi.org/10.1016/0169-555X(92)90026-K.
de Carpentier, G.J., Bidarra, R., 2009. Interactive GPU-based procedural heightfield brushes. In: Proceedings of the ACM 4th International Conference on Foundations of Digital Games, Orlando, Florida, April 26, pp. 55–62. https://doi.org/10.1145/1536513.1536532.
deAlmeida, G.A.M., Rodríguez, J.F., 2012. Spontaneous formation and degradation of pool-riffle morphology and sediment sorting using a simple fractional transport model. Geophys. Res. Lett. 39, L06407. https://doi.org/10.1029/2012GL051059.
DePriest, N.C., Hopkinson, L.C., Quaranta, J.D., Michael, P.R., Ziemkiewicz, P.F., 2015. Geomorphic landform design alternatives for an existing valley fill in central Appalachia, USA: Quantifying the key issues. Ecol. Eng. 81, 19–29. https://doi.org/10.1016/j.ecoleng.2015.04.007.
Deutsch, C., Wang, L., 1996. Hierarchical object-based stochastic modeling of fluvial reservoirs. Math. Geol. 28, 857–880. https://doi.org/10.1007/BF02066005.
Dietrich, W.E., Bellugi, D.G., Sklar, L.S., Stock, J.D., Heimsath, A.M., Roering, J.J., 2013. Geomorphic Transport Laws for predicting Landscape form and Dynamics. In: Wilcock, P.R., Iverson, R.M. (Eds.), Prediction in Geomorphology. American Geophysical Union, Washington, DC. https://doi.org/10.1029/135GM09.
Dodov, B., Foufoula-Georgiou, E., 2004. Generalized hydraulic geometry: Insights based on fluvial instability analysis and a physical model. Water Resour. Res. 40, W12201. https://doi.org/10.1029/2004WR003196.
Doeschl-Wilson, A., Ashmore, P., 2005. Assessing a numerical cellular braided-stream model with a physical model. Earth Surf. Process. Landf. 30, 519–540. https://doi.org/10.1002/esp.1146.
Doran, J., Parberry, I., 2010. Controlled Procedural Terrain Generation using Software Agents. IEEE Transactions on Computational Intelligence and AI in Games 2, 111–119. https://doi.org/10.1109/TCIAIG.2010.2049020.
Drăguţ, L., Eisank, C., Strasser, T., 2011. Local variance for multi-scale analysis in geomorphometry. Geomorphology 130, 162–172. https://doi.org/10.1016/j.geomorph.2011.03.011.
Duan, J.G., Julien, P.Y., 2010. Numerical simulation of meandering evolution. J. Hydrol. 391, 34–46. https://doi.org/10.1016/j.jhydrol.2010.07.005.
Dury, G., 1976. Discharge prediction, present and former, from channel dimensions. J. Hydrol. 30, 219–245. https://doi.org/10.1016/0022-1694(76)90102-5.
Eaton, B.C., Millar, R.G., Davidson, S., 2010. Channel patterns: Braided, anabranching, and single-thread. Geomorphology 120, 353–364. https://doi.org/10.1016/j.geomorph.2010.04.010.
Ebert, D.S., Musgrave, F.K., 1998. Texturing & Modeling a Procedural Approach, Second edition. AP Professional (ISBN 0-12-228730-4).
Edmonds, D.A., Slingerland, R.L., 2007. Mechanics of river mouth bar formation: Implications for the morphodynamics of delta distributary networks. J. Geophys. Res. Earth Surf. 112. https://doi.org/10.1029/2006JF000574.
Elkins, E.M., Pasternack, G.B., Merz, J.E., 2007. The use of slope creation for rehabilitating incised, regulated, gravel bed rivers. Water Resour. Res. 43, W05432. https://doi.org/10.1029/2006WR005159.
Engelund, F., 1974. Flow and bed topography in channel bends. J. Hydraul. Div. 100, 1631–1648.
Farin, G., Hoschek, J., Kim, M.S., 2002. Handbook of Computer Aided Geometric Design. Elsevier, Amsterdam (ISBN: 9780444511041).
Ferguson, R.I., 1976. Disturbed periodic model for river meanders. Earth Surface Processes 1, 337–347. https://doi.org/10.1002/esp.3290010403.
Fonstad, M.A., 2006. Cellular automata as analysis and synthesis engines at the geomorphology–ecology interface. Geomorphology 77, 217–234. https://doi.org/10.1016/j.geomorph.2006.01.006.
Fournier, A., Fussell, D., Carpenter, L., 1982. Computer rendering of stochastic models. Commun. ACM 25, 371–384. https://doi.org/10.1145/358523.358553.
Fuller, W.B., Thompson, S.E., 1906. The laws of proportioning concrete. Trans. Am. Soc. Civ. Eng. 57, 67–143.
Genevaux, J.D., Galin, É., Guérin, E., Peytavie, A., Beneš, B., 2013. Terrain generation using procedural models based on hydrology. ACM Trans. Graph. 32, 143. https://doi.org/10.1145/2461912.2461996.
Goodchild, M.F., 2008. The use cases of Digital Earth. Int. J. Digital Earth 1, 31–42. https://doi.org/10.1080/17538940701782528.
Goodchild, M.F., 2009. Virtual geographic environments as collective constructions. In: Lin, H., Batty, M. (Eds.), Virtual Geographic Environments. Science Press, Beijing, pp. 15–24.
Goodchild, M.F., 2012. The future of Digital Earth. Ann. GIS 18, 93–98. https://doi.org/10.1080/19475683.2012.668561.
Grant, G.E., Swanson, F.J., Wolman, M.G., 1990. Pattern and origin of stepped-bed morphology in high-gradient streams, western Cascades. Oregon. Geol. Soc. Am. Bull. 102 (3), 340–352. https://doi.org/10.1130/0016-7606(1990)102<0340:PAOOSB>2.3.CO;2.
Grundberg, S., Hansegard, J., 2014. Minecraft' Maker's Profit Soars. Wall Street J March 18, 2014. https://www.wsj.com/articles/minecraft-makers-profit-soars-1395140019.
Hack, J.T., 1975. Dynamic equilibrium and landscape evolution. In: Melhorn, W.N., Flemal, R.C. (Eds.), Theories of Landform Development. Allen and Unwin, Winchester, Mass, pp. 87–102.
Hallet, B., 1990. Spatial Self-organization in Geomorphology: from Periodic Bedforms and Patterned Ground to Scale-invariant Topography. Earth Sci. Rev. 29, 57–75. https://doi.org/10.1016/0012-8252(0)90028-T.
Hassan, M.A., Church, M., Ashworth, P.J., 1992. Virtual rate and mean distance of travel of individual clasts in gravel-bed channels. Earth Surf. Process. Landf. 17, 617–627.
Havis, R., Alonso, C., King, J., 1996. Modeling Sediment in Gravel-Bedded Streams using HEC-6. J. Hydraul. Eng. 122, 559–564. https://doi.org/10.1061/(ASCE)0733-9429(1996)122:10(559).
Hendrikx, M., Meijer, S., Van Der Velden, J., Iosup, A., 2013. Procedural content generation for games: a survey. ACM Trans. Multimed. Comput. Commun. Appl. 9 (1). https://doi.org/10.1145/2422956.2422957.
Hillier, J.K., Sofia, G., Conway, S.J., 2015. Perspective–synthetic DEMs: a vital underpinning for the quantitative future of landform analysis? Earth Surf. Dyn. 3, 587–598. https://doi.org/10.5194/esurf-3-587-2015.
Hodge, R., Richards, K., Brasington, J., 2007. A physically-based bedload transport model developed for 3-D reach-scale cellular modelling. Geomorphology 90, 244–262.
Hodge, R.A., Sear, D.A., Leyland, J., 2013. Spatial variations in surface sediment structure in riffle–pool sequences: a preliminary test of the Differential Sediment Entrainment Hypothesis (DSEH). Earth Surf. Process. Landf. 38, 449–465. https://doi.org/10.1002/esp.3290.
Huijser, R., Dobbe, J., Bronsvoort, W.F., Bidarra, R., 2010. Procedural Natural Systems for

Game Level Design. In: 2010 Brazilian Symposium on Games and Digital Entertainment, Florianapolis, Santa Catarina Brazil, November 8–10, pp. 189–198. https://doi.org/10.1109/SBGAMES.2010.31.
Ikeda, S., Nishimura, T., 1986. Flow and bed profile in meandering sand-silt rivers. J. Hydraul. Eng. 112, 562–579. https://doi.org/10.1061/(ASCE)0733-9429(1986)112:7(562).
Jackson, J.R., Pasternack, G.B., Wheaton, J.M., 2015. Virtual manipulation of topography to test potential pool-riffle maintenance mechanisms. Geomorphology 228, 617–627. https://doi.org/10.1016/j.geomorph.2014.10.016.
Jacobson, R.B., Galat, D.L., 2006. Flow and form in rehabilitation of large-river ecosystems: an example from the lower Missouri River. Geomorphology. 77, 249–269. https://doi.org/10.1016/j.geomorph.2006.01.014.
James, A.L., 1996. Polynomial and power functions for glacial valley cross-section morphology. Earth Surf. Process. Landf. 21, 413–432.
Jang, C., Shimizu, Y., 2005. Numerical Simulation of Relatively Wide, Shallow Channels with Erodible Banks. Journal of Hydraulic Engineering 131, 565–575.
Jensen, J., 2011. Riverland 2.0: Blending of Multiple User-Defined Slopes in a Procedurally Modeled Terrain. M.S. Thesis. San Jose State University, CA.
Jiang, Z., Haff, P.K., 1993. Multiparticle simulation methods applied to the micromechanics of bed load transport. Water Resour. Res. 29, 399–412. https://doi.org/10.1029/92WR02063.
Julien, P., Klaassen, G., 1995. Sand-Dune Geometry of large Rivers during Floods. J. Hydraul. Eng. 121, 657–663. https://doi.org/10.1061/(ASCE)0733-9429(1995)121:9(657).
Karim, F., 1999. Bed-Form Geometry in Sand-Bed Flows. Journal of. Hydraulic. Engineering 125, 1253–1261.
Kelley, A., Malin, M., Nielson, G., 1988. Terrain Simulation Using a Model of Stream Erosion. vol. 22. Association for Computing Machinery Siggraph Computer Graphics, pp. 263–268. https://doi.org/10.1145/54852.378519.
Kinoshita, R., 1961. Investigation of Channel Deformation in Ishikari River. Publication 36. Naural Resources Division, Ministry of Science and Technology of Japan, Tokyo, pp. 139.
Knighton, A.D., 1983. Models of stream bed topography at the reach scale. J. Hydrol. 60, 105–121. https://doi.org/10.1016/0022-1694(83)90016-1.
Knighton, D., 1998. Fluvial Forms and Processes: A New Perspective. Edward Arnold Publishers Ltd, London, UK.
Konecny, M., 2011. Review: Cartography: challenges and potential in the virtual geographic environments era. Ann. GIS 17, 135–146. https://doi.org/10.1080/19475683.2011.602027.
Lacey, G., 1929. Stable channels in alluvium. Proceedings of the Institution of Civil Engineers 229, 259–384.
Langbein, W.B., Leopold, L.B., 1964. Quasi-equilibrium states in channel morphology. Am. J. Sci. 262, 782–794. https://doi.org/10.2475/ajs.262.6.782.
Legleiter, C.J., 2012. A geostatistical framework for quantifying the reach-scale spatial structure of river morphology: 1. Variogram models, related metrics, and relation to channel form. Geomorphology 205, 65–84. https://doi.org/10.1016/j.geomorph.2012.01.016.
Legleiter, C., Kyriakidis, P., 2008. Spatial prediction of river channel topography by kriging. Earth Surf. Process. Landf. 33, 841–867. https://doi.org/10.1002/esp.1579.
Leopold, L.B., 1969. Quantitative Comparison of some Aesthetic Factors among Rivers, Circular 620. United States Geological Survey, Washington, DC, pp. 1–14.
Leopold, L., Langbein, W., 1962. The Concept of Entropy in Landscape Evolution. In: United States Geological Survey Professional Paper: 500-a, (Washington, DC).
Leopold, L., Maddock, T., 1953. The hydraulic geometry of stream channels and some physiographic implications. In: Professional Paper 252. United States Geological Survey, Washington, DC.
Leopold, L.B., Wolman, M.G., 1957. River Channel Patterns: Braided, Meandering, and Straight. Professional Paper 282-B. United States Geological Survey, Washington, DC.
Leopold, A., Wolman, M., Miller, J., 1964. Fluvial Processes in Geomorphology. WH Freeman and Company, San Francisco, CA.
Li, S.S., Millar, R.G., 2011. A two-dimensional morphodynamic model of gravel-bed river with floodplain vegetation. Earth Surf. Process. Landf. 36, 190–202. https://doi.org/10.1002/esp.2033.
Li, Y., Hedlind, M., Kjellberg, T., 2015. Usability Evaluation of CADCAM: State of the Art. Procedia CIRP 36, 205–210. https://doi.org/10.1016/j.procir.2015.01.053.
Liang, M., Voller, V.R., Paola, C., 2015. A reduced-complexity model for river delta formation–part 1: Modeling deltas with channel dynamics. Earth Surface Dynamics 3, 67–86.
Lin, H., Chen, M., Lu, G., Zhu, Q., Gong, J., You, X., Wen, Y., Xu, B., Hu, M., 2013. Virtual Geographic Environments (VGEs): a New Generation of Geographic Analysis Tool. Earth Sci. Rev. 126, 74–84. https://doi.org/10.1016/j.earscirev.2013.08.001.
Mackin, J., 1948. Concept of the Graded River. Geol. Soc. Am. Bull. 59, 463–512. https://doi.org/10.1130/0016-7606(1948)59[463:COTGR]2.0.CO;2.
Macvicar, B., Parrott, L., Roy, A., 2006. A two-dimensional discrete particle model of gravel bed river systems. J. Geophys. Res. 111, F03009. https://doi.org/10.1029/2005JF000316.
Malmaeus, J., Hassan, M., 2002. Simulation of individual particle movement in a gravel streambed. Earth Surf. Process. Landf. 27, 81–97. https://doi.org/10.1002/esp.305.
Mandelbrot, B., 1975. Stochastic models for the Earth's relief, the shape and the fractal dimension of the coastlines, and the number-area rule for islands. Proc. Natl. Acad. Sci. U. S. A. 72, 3825–3828. https://doi.org/10.1073/pnas.72.10.3825.
Mandelbrot, B., Van Ness, J., 1968. Fractional Brownian motions, fractional noises and applications. SIAM Rev. 10, 422–437.
Merwade, V., 2009. Effect of spatial trends on interpolation of river bathymetry. J. Hydrol. 371, 169–181. https://doi.org/10.1016/j.jhydrol.2009.03.026.
Merwade, V., Maidment, D., 2004. A GIS framework for describing river channel bathymetry. Submitted to the Texas Water Development Board.
Mosselman, E., 1995. A review of mathematical models of river planform changes. Earth Surf. Process. Landf. 20, 661–670. https://doi.org/10.1002/esp.3290200708.
Mosselman, E., 1998. Morphological modelling of rivers with erodible banks. In: Hydrologic Processes. vol. 12. pp. 1357–1370. https://doi.org/10.1002/(SICI)1099-1085(19980630)12:8 < 1357::AID-HYP619 > 3.0.CO;2-7.
Mosselman, E., 2012. Modelling Sediment Transport and Morphodynamics of Gravel-Bed Rivers. In: Church, M., Biron, P.M., Roy, A.G. (Eds.), Gravel-Bed Rivers: Processes, Tools, Environments. John Wiley & Sons, Ltd, Chichester, UK. https://doi.org/10.1002/9781119952497.ch9.
Murray, A.B., Paola, C., 1994. A cellular model of braided rivers. Nature 371, 54–57. https://doi.org/10.1038/371054a0.
Murray, A.B., Paola, C., 1997. Properties of a cellular braided-stream model. Earth Surf. Process. Landf. 22, 1001–1025. https://doi.org/10.1002/(SICI)1096-9837(199711)22:11 < 1001::AID-ESP798 > 3.0.CO;2-O.
Murray, A.B., Paola, C., 2003. Modeling the effect of vegetation on channel pattern in bedload rivers. Earth Surf. Process. Landf. 28, 131–143. https://doi.org/10.1002/esp.428.
Musgrave, F., Kolb, C., Mace, R., 1989. The synthesis and rendering of eroded fractal terrains. In: Association for Computing Machinery SIGGRAPH Computer Graphics. vol. 23. pp. 41–50. https://doi.org/10.1145/74334.74337.
Myers, B.A., 1998. A brief history of human-computer interaction technology. Interactions 5, 44–54. https://doi.org/10.1145/274430.274436.
Myers, T.J., Swanson, S., 1997. Stochastic modeling of pool-to-pool structure in small Nevada rangeland streams. Water Resour. Res. 33, 877–889. https://doi.org/10.1029/96WR03975.
Nabi, M., de Vriend, H.J., Mosselman, E., Sloff, C.J., Shimizu, Y., 2013. Detailed simulation of morphodynamics: 3. Ripples and dunes. Water Resour. Res. 49, 5930–5943. https://doi.org/10.1002/wrcr.20457.
Naden, P., 1987. Modelling gravel-bed topography from sediment transport. Earth Surf. Process. Landf. 12, 353–367. https://doi.org/10.1002/esp.3290120403.
Nagashima, K., 1998. Computer generation of eroded valley and mountain terrains. Vis. Comput. 13, 456–464. https://doi.org/10.1007/s003710050117.
Nelson, M., Mateas, M., 2007. Towards Automated Game Design. Artificial Intelligence and Human-Oriented Computing Lecture Notes in Computer Science. vol. 4733. pp. 626–637. https://doi.org/10.1007/978-3-540-74782-6_54.
Nelson, J.M., Smith, J.D., 1989. Flow in meandering channels with natural topography. In: Ikeda, S., Parker, G. (Eds.), River Meandering. American Geophysical Union, Washington, D. C. https://doi.org/10.1029/WM012p0069.
Newland, D.E., 2012. An Introduction to Random Vibrations, Spectral & Wavelet Analysis. Dover Publications, New York, NY.
Nicholas, A.P., 2005. Cellular modelling in fluvial geomorphology. Earth Surf. Process. Landf. 30, 645–649. https://doi.org/10.1002/esp.1231.
Nicholas, A.P., 2009. Reduced-complexity flow routing models for sinuous single-thread channels: intercomparison with a physically-based shallow-water equation model. Earth Surf. Process. Landf. 34, 641–653. https://doi.org/10.1002/esp.1761.
Nicholas, A.P., 2010. Reduced-complexity modeling of free bar morphodynamics in alluvial channels. J. Geophys. Res. 115, F04021. https://doi.org/10.1029/2010JF001774.
Nicholas, A.P., 2013. Modeling the continuum of river channel patterns. Earth Surf. Process. Landf. 38, 1187–1196. https://doi.org/10.1002/esp.3431.
Nicholas, A.P., Thomas, R., Quine, T., 2006. Cellular modelling of braided river form and process. In: Sambrook Smith, G.H., Best, J.L., Bristow, C.S., Petts, G.S. (Eds.), Braided Rivers: Process, Deposits, Ecology and Management. Blackwell Publishing Ltd., Oxford, UK. https://doi.org/10.1002/9781444304374.ch6.
Nicholas, A.P., Ashworth, P.J., Smith, G.H.S., Sandbach, S.D., 2013. Numerical simulation of bar and island morphodynamics in anabranching megarivers. J. Geophys. Res. Earth Surf. 118, 2019–2044. https://doi.org/10.1002/jgrf.20132.
Nikora, V.I., 1991. Fractal structures of river plan forms. Water Resour. Res. 27, 1327–1333. https://doi.org/10.1029/91WR00095.
Nikora, V.I., Sapozhnikov, V.B., 1993. River network fractal geometry and its computer simulation. Water Resour. Res. 29, 3569–3575. https://doi.org/10.1029/93WR00966.
Ohmori, H., 1991. Change in the Mathematical Function Type describing the Longitudinal Profile of a River through an Evolutionary Process. J. Geol. 99, 97–110. https://doi.org/10.1086/629476.
Paarlberg, A.J., Dohmen-Janssen, C.M., Hulscher, S.J., Termes, P., 2009. Modeling river dune evolution using a parameterization of flow separation. J. Geophys. Res. Earth Surf. 114 (F1).
Paola, C., Heller, P.L., Angevine, C.L., 1992. The large-scale dynamics of grain-size variation in alluvial basins, 1: theory. Basin Res. 4, 73–90.
Parish, Y., Müller, P., 2001. Procedural modeling of cities. In: Proceedings of the 28th Annual Conference on Computer Graphics and Interactive Techniques, pp. 301–308. https://doi.org/10.1145/383259.
Parker, G., 1976. On the cause and characteristic scales of meandering and braiding in rivers. J. Fluid Mech. 76, 457–480. https://doi.org/10.1017/S0022112076000748.
Parker, G., Wilcock, P.R., Paola, C., Dietrich, W.E., Pitlick, J., 2007. Physical basis for quasi-universal relations describing bankfull hydraulic geometry of single-thread gravel bed rivers. J. Geophys. Res. 112, F04005. https://doi.org/10.1029/2006JF000549.
Pasternack, G.B., 2013. Geomorphologist's guide to participating in river rehabilitation. In: Wohl, E. (Ed.), Treatise on Geomorphology, Volume 9, Fluvial Geomorphology. Academic Press, San Diego, pp. 843–860. https://doi.org/10.1016/B978-0-12-374739-6.00268-2.
Pasternack, G.B., Brown, R., 2013. Ecohydraulic Design of Riffle-Pool Relief and

Morphological Unit Geometry in support of Regulated Gravel-Bed River Rehabilitation. In: Maddock, I., Harby, A., Kemp, P., Wood, P. (Eds.), Ecohydraulics: An Integrated Approach. John Wiley & Sons, Ltd, Chichester, UK. https://doi.org/10.1002/9781118526576.ch20.
Pasternack, G.B., Brown, R.A., 2016. Designing rivers with multiple scales of channel and floodplain variation to yield diverse processes and ecosystem services. In: 11th International Conference on Ecohydraulics, February 7–12, Melbourne, Australia.
Pasternack, G.B., Brush, G.S., Hilgartner, W.B., 2001. Impact of Historic Land-Use Change on Sediment delivery to an Estuarine Delta. Earth Surf. Process. Landf. 26, 409–427. https://doi.org/10.1002/esp.189.
Pasternack, G.B., Bounrisavong, M.K., Parikh, K.K., 2008. Backwater control on riffle–-pool hydraulics, fish habitat quality, and sediment transport regime in gravel-bed rivers. J. Hydrol. 357, 125–139. https://doi.org/10.1016/j.jhydrol.2008.05.014.
Perillo, G.M.E., 1995. Geomorphology and Sedimentology of Estuaries. Elsevier Science B.V, New York.
Perry, G.L.W., Dickson, M.E., 2018. Using machine learning to predict geomorphic disturbance: the effects of sample size, sample prevalence, and sampling strategy. J. Geophys. Res. Earth Surf. 123, 2954–2970. https://doi.org/10.1029/2018JF004640.
Phillips, R.T.J., Robert, A., 2007. Hydrologic control of waveforms on small meandering rivers. Earth Surf. Process. Landf. 32, 1533–1546. https://doi.org/10.1002/esp.1483.
Pike, R.J., Rozema, W.J., 1975. Spectral analysis of landforms. Ann. Assoc. Am. Geogr. 65, 499–516. https://doi.org/10.1111/j.1467-8306.1975.tb01058.x.
Prusinkiewicz, P., Hammel, M., 1993. A fractal model of mountains and rivers. In: Proceeding of Graphics Interface. vol. 93. pp. 174–180.
Pyrcz, M.J., Boisvert, J.B., Deutsch, C.V., 2009. ALLUVSIM: a program for event-based stochastic modeling of fluvial depositional systems. Comput. Geosci. 35, 1671–1685. https://doi.org/10.1016/j.cageo.2008.09.012.
Raffe, W.L., Zambetta, F., Li, X., 2012. A survey of procedural terrain generation techniques using evolutionary algorithms. In: Proceedings of Congress of Evolutionary Computation (CEC 2012), United States, pp. 2090–2097.
Rayner, J., 1971. Introduction to Spectral Analysis. Pioneer Limited, London, UK.
Richards, K., 1976a. Channel width and the riffle-pool sequence. Geol. Soc. Am. Bull. 87, 883–890. https://doi.org/10.1130/0016-7606(1976)87<883:CWATRS>2.0.CO;2.
Richards, K.S., 1976b. The morphology of riffle-pool sequences. Earth Surf. Process. Landf. 1, 71–88. https://doi.org/10.1002/esp.3290010108.
Robert, A., 1988. Statistical properties of sediment bed profiles in alluvial channels. Math. Geol. 20, 205–225. https://doi.org/10.1007/BF00890254.
Rodriguez-Iturbe, I., Rinaldo, A., 1997. Fractal River Basins. Cambridge University Press, Cambridge, UK.
Rodriguez-Iturbe, I., Marani, M., Rigon, R., Rinaldo, A., 1994. Self-organized river basin landscapes: Fractal and multifractal characteristics. Water Resour. Res. 30, 3531–3539. https://doi.org/10.1029/94WR01493.
Sapozhnikov, V., Foufoula-Georgiou, E., 1996. Self-Affinity in Braided Rivers. Water Resour. Res. 32, 1429–1439. https://doi.org/10.1029/96WR00490 =.
Saunders, R., 2006. Terrainosaurus Realistic Terrain Synthesis Using Genetic Algorithms. MS Thesis. Texas A&M University, TX.
Schmeeckle, M.W., Nelson, J.M., 2003. Direct numerical simulation of bedload transport using a local, dynamic boundary condition. Sedimentol. 50, 279–301. https://doi.org/10.1046/j.1365-3091.2003.00555.x.
Schor, H.J., Grey, D.H., 1995. Landform Grading and Slope Evolution. J. Geotech. Eng. 121, 729–734. https://doi.org/10.1061/(ASCE)0733-9410(1995)121:10(729).
Schumm, S.A., 1960. The shape of alluvial channels in relation to sediment type: U.S. Geological Survey Professional Paper 352-B: 17–30.
Schumm, S., 1985. Patterns of alluvial rivers. Annu. Rev. Earth Planet. Sci. 13, 5–27. https://doi.org/10.1146/annurev.ea.13.050185.000253.
Schuurman, F., Marra, W.A., Kleinhans, M.G., 2013. Physics-based modeling of large braided sand-bed rivers: Bar pattern formation, dynamics, and sensitivity. J. Geophys. Res. Earth Surf. 118, 2509–2527. https://doi.org/10.1002/2013JF002896.
Seminara, G., 2006. Meanders. J. Fluid Mech. 554, 271–297. https://doi.org/10.1017/s0022112006008925.
Seybold, H., Andrade, J.S., Herrmann, H.J., 2007. Modeling river delta formation. Proc. Natl. Acad. Sci. 104, 16804–16809. https://doi.org/10.1073/pnas.0705265104.
Shen, C., 2018. A transdisciplinary review of deep learning research and its relevance for water resources scientists. Water Resour. Res. 54, 8558–8593. https://doi.org/10.1029/2018WR022643.
Shepherd, R., 1985. Regression Analysis of River Profiles. J. Geol. 93, 377–384. https://doi.org/10.1086/628959.
Shields Jr., F.D., Copeland, R.R., Klingeman, P.C., Doyle, M.W., Simon, A., 2003. Design for stream restoration. J. Hydraulic Engineering-ASCE 129, 575–584. https://doi.org/10.1061/(ASCE)0733-9429(2003)129:8(575). (Simpson G, Castelltort S. 2006. Coupled model of surface water flow, sediment transport and morphological evolution. Computers & Geosciences 32: 1600–1614. DOI: 10.1016/j.cageo.2006.02.020).
Singh, V.P., 2003. On the Theories of Hydraulic Geometry. Int. J. Sediment Res. 18, 196–218.
Smelik, R.M., 2013. Declarative terrain modeling for military training games. Int. J. Computer Games Technol. 2010, 360458. https://doi.org/10.1155/2010/360458.
Smelik, R.M., Tutenel, T., de Kraker, K.J., Bidarra, R., 2010. Interactive creation of virtual worlds using procedural sketching. In: Proceedings of Eurographics. vol. 2010 The Eurographics Association.
Smelik, R.M., Tutenel, T., de, Kraker K.J., Bidarra, R., 2011. A declarative approach to procedural modeling of virtual worlds. Comput. Graph. 35, 352–363. https://doi.org/10.1016/j.cag.2010.11.011.
Smelik, R.M., Tutenel, T., Bidarra, R., Benes, B., 2014. A survey on Procedural Modelling for Virtual Worlds. Comput. Grap. Forum 33, 31–50. https://doi.org/10.1111/cgf.12276.
Snow, R., Slingerland, R., 1987. Mathematical Modeling of Graded River Profiles. J. Geol. 95, 15–33. https://doi.org/10.1086/629104.
Stark, C., Stark, G., 2001. A channelization model of landscape evolution. Am. J. Sci. 301, 486–512. https://doi.org/10.2475/ajs.301.4-5.486.
Stolum, H., 1998. Planform geometry and dynamics of meandering rivers. Geol. Soc. Am. Bull. 110, 1485–1498. https://doi.org/10.1130/0016-7606(1998)110<1485:PGADOM>2.3.CO;2.
Struiksma, N., 1985. Prediction of 2-D bed topography in rivers. J. Hydraul. Eng. 111, 1169–1182. https://doi.org/10.1061/(ASCE)0733-9429(1985)111:8(1169).
Sun, T., Meakin, P., Jøssang, T., Schwarz, K., 1996. A simulation Model for Meandering Rivers. Water Resour. Res. 32, 2937–2954. https://doi.org/10.1029/96WR00998.
Sutton, M., 2012. http://www.quora.com/The-Elder-Scrolls-V-Skyrim/How-large-is-Skyrims-overworld?q=how+large+skyrim+size.
Sylvester, Z., Pirmez, C., Cantelli, A., 2011. A model of submarine channel-levee evolution based on channel trajectories: Implications for stratigraphic architecture. Mar. Pet. Geol. 28, 716–727. https://doi.org/10.1016/j.marpetgeo.2010.05.012.
Tanner, W.F., 1971. The river profile. J. Geol. 79, 482–492. https://doi.org/10.1086/627653.
Teoh, S.T., 2009. RiverLand: An Efficient Procedural Modeling System for Creating Realistic-looking Terrains. In: Bebis, G., Boyle, R., Parvin, B., Koracin, D., Kuno, Y., Wang, J., Pajarola, R., Lindstrom, P., Hinkenjann, A., Encarnação, M.L., Silva, C.T., Coming, D. (Eds.), Advances in Visual Computing, ISVC 2009, Lecture Notes in Computer Science. Springer, Berlin, Heidelberg, pp. 468–479. https://doi.org/10.1007/978-3-642-10331-5_44.
Thomas, R., Nicholas, A.P., 2002. Simulation of braided river flow using a new cellular routing scheme. Geomorphology 43, 179–195. https://doi.org/10.1016/S0169-555X(01)00128-3.
Thompson, D.M., 2001. Random controls on semi-rhythmic spacing of pools and riffles in constriction-dominated rivers. Earth Surf. Process. Landf. 26, 1195–1212.
Tribe, S., Church, M., 1999. Simulations of cobble structure on a gravel streambed. Water Resour. Res. 35, 311–318. https://doi.org/10.1029/98WR01141.
Tucker, G.E., Hancock, G.R., 2010. Modelling landscape evolution. Earth Surf. Process. Landf. 35, 28–50. https://doi.org/10.1002/esp.1952.
Tucker, G., Slingerland, R., 1994. Erosional dynamics, flexural isostasy, and long-lived escarpments: a numerical modeling study. J. Geophys. Res. 99, 12229–12243. https://doi.org/10.1029/94JB00320.
Tucker, G., Slingerland, R., 1997. Drainage basin responses to climate change. Water Resour. Res. 33, 2031–2047.
Tucker, G., Lancaster, S., Gasparini, N., Bras, R., Doe III, W.W., 2001a. The channel-hillslope integrated landscape development model (CHILD). In: Harmon, R.S. (Ed.), Landscape Erosion and Evolution Modeling. Springer, New York, NY, pp. 349–388.
Tucker, G., Lancaster, S., Gasparini, N., Bras, R., Rybarczyk, S., 2001b. An object-oriented framework for distributed hydrologic and geomorphic modeling using triangulated irregular networks. Comput. Geosci. 27, 959–973. https://doi.org/10.1016/S0098-3004(00)00134-5.
United States Army Corps of Engineers, Hydrologic Engineering Center, 1993. HEC-6. Scour and Deposition in Rivers and Reservoirs, User's Manual. Davis, CA.
van Oorschot, M., Kleinhans, M.G., Geerling, G., Middelkoop, H., 2016. Distinct patterns of interaction between vegetation and morphodynamics. Earth Surf. Process. Landf. 41, 791–808. https://doi.org/10.1002/esp.3864.
van Rijn, L.C., 1984. Sediment Transport, Part III Bed Forms and Alluvial Roughness. J. Hydraul. Eng., ASCE 110 (12), 1733–1754 December 1984.
Vasquez, J.A., Millar, R.G., Steffler, P.M., 2007. Two-dimensional finite element river morphology model. Can. J. Civ. Eng. 34, 691–702. https://doi.org/10.1139/L06-170.
Wallerstein, N., 2003. Dynamic model for constriction scour caused by large woody debris. Earth Surf. Process. Landf. 28, 49–68. https://doi.org/10.1002/esp.426.
Wang, H., Zhou, G., Shao, X., 2010a. Numerical simulation of channel pattern changes part I: Mathematical model. Int.l J. Sediment Res. 25, 366–379. https://doi.org/10.1016/S1001-6279(11)60004-8.
Wang, H., Zhou, G., Shao, X., 2010b. Numerical simulation of channel pattern changes Part II: Application in a conceptual channel. Int.l J. Sediment Res. 25, 380–390. https://doi.org/10.1016/S1001-6279(11)60005-X.
Werner, B.T., 1999. Complexity in Natural Landform Patterns. Science (5411), 102–104.
Whiting, E., Ochsendorf, J., Durand, F., 2009. Procedural modeling of structurally-sound masonry buildings. In: Association for Computing Machinery Transactions on Graphics. vol. 28. pp. 112. https://doi.org/10.1145/1618452.1618458.
Wiel, M.J.V.D., Coulthard, T.J., Macklin, M.G., Lewin, J., 2007. Embedding reach-scale fluvial dynamics within the CAESAR cellular automaton landscape evolution model. Geomorphology 90, 283–301. https://doi.org/10.1016/j.geomorph.2006.10.024.
Wilkerson, G.V., Parker, G., 2011. Physical Basis for Quasi-Universal Relationships describing Bankfull Hydraulic Geometry of Sand-Bed Rivers. J. Hydraul. Eng. 137, 739–753. https://doi.org/10.1061/(ASCE)HY.1943-7900.0000352.
Willgoose, G., 2005. Mathematical modeling of whole landscape evolution. Annu. Rev. Earth Planet. Sci. 33, 443–459. https://doi.org/10.1146/annurev.earth.33.092203.122610.
Willgoose, G., Bras, R.L., Rodriguez-Iturbe, I., 1991. A coupled channel network growth and hillslope evolution model, 1, Theory. Water Resour. Res. 27. https://doi.org/10.1029/91WR00935.
Williams, P.B., Orr, M.K., Garrity, N.J., 2002. Hydraulic geometry: a geomorphic design tool for tidal marsh channel evolution in wetland restoration projects. Restor. Ecol. 10, 577–590. https://doi.org/10.1046/j.1526-100X.2002.t01-1-02035.x.
Williams, R.D., Brasington, J., Hicks, D., 2016. Numerical Modelling of Braided River Morphodynamics: Review and Future challenges. Geogr. Compass 10 (3), 102–127. https://doi.org/10.1111/gec3.12260.
Willis, B.J., 1989. Palaeochannel reconstructions from point bar deposits: a three-dimensional perspective. Sedimentology 36, 757–766.

Willis, B.J., Tang, H., 2010. Three-dimensional connectivity of point-bar deposits. J. Sediment. Res. 80, 440–454.
Wohl, E.E., Thompson, D.M., Miller, A.J., 1999. Canyons with undulating walls. Geol. Soc. Am. Bull. 111, 949–959. https://doi.org/10.1130/0016-7606(1999)111<0949:CWUW>2.3.CO;2.
Wolfram, S., 2002. A New Kind of Science. Wolfram Media, Chicago, IL.
Wolman, M.G., 1959. Factors influencing erosion of a cohesive river bank. Am. J. Sci. 257, 204–216.
Wyrick, J.R., Pasternack, G.B., 2014. Geospatial organization of fluvial landforms in a gravel-cobble river: beyond the riffle-pool couplet. Geomorphology 213, 48–65. https://doi.org/10.1016/j.geomorph.2013.12.040.
Yatsu, E., 1955. On the longitudinal profile of the graded river. Trans. Am. Geophys. Union 36, 655–663. https://doi.org/10.1029/TR036i004p00655.
Yumoto, M., Ogata, T., Matsuoka, N., Matsumoto, E., 2006. Riverbank freeze-thaw erosion along a small mountain stream, Nikko volcanic area, Central Japan. Permafr. Periglac. Process. 17, 325–339. https://doi.org/10.1002/ppp.569.
Zhang, H., Qu, D., Hou, Y., Gao, F., Huang, F., 2016. Synthetic Modeling Method for Large Scale Terrain Based on Hydrology. IEEE Access 4, 6238–6249.