# Designing HAL

ENSC 384

Dan Hendry (30113878) Brian Hook (200046894) John Jamel (301080425)

Simon Fraser University School of Engineering Science

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Course Instructor: Professor Golnaraghi

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## 1 Introduction

Controller design and simulation is a critical stage in the development of the system as a whole. This report details four major steps associated with the design process. First, determination of accurate model and system parameters; specifically truss inertia. Second, choice of PID controller gains using the mathematical system model based on specifications for maximum overshoot and settling time. Third, system simulation to verify and tune controller parameters. Finally, amplifier control convert the PID output signal to motor voltage accurately.

## 2 Problem Definition

#### 2.1 Determination of Truss Inertia

The project's truss inertia,  $J_{truss}$  (mass moment of inertia about the rotational axis), is a fundamental quantity that is dependent upon the design specifications of the truss and is initially unknown. Since  $J_{truss}$  appears in mathematical equations associated with the controller, it must be determined in order to allow for proper position/speed control of the system. One method for calculating the truss inertia is to model and analyze a given truss design using 3-D CAD software. A second way, which can also be used to check modeled values, it to create an experimental test, using parameters provided by the motor manufacturer, to estimate the  $J_{truss}$  value. A possible inaccuracy with this method is that not all system parameters are necessarily known with reasonable certainty. In this report, an experimental method is described and the results are compared with theoretical values determined from the modeling performed at an earlier stage of the project.

## 2.2 Controller Design

In order to obtain a suitable overall system response, a feedback controller of some type is needed. This report describes the design of a proportional, integral, derivative controller. Such controller is accurate and relatively easier to implement and simulate than other types. Furthermore, in most cases the more complex controller is, the more it costs, the less reliable it is, and it is more difficult to design. A PID controller requires careful choice of its gains,  $K_p$ ,  $K_d$ , and  $K_i$ , to achieve the desired response and to meet design specifications.

#### 2.3 Simulation

To verify the system response given controller gains determined using system theory, a simulation is invaluable. Simulation allows additional experimentation based on a more realistic model which incorporates saturation in an effort tune gain parameters. A simulation also allows experimentation with no possibility of damaging physical components.

## 2.4 Amplifier Control

The PWM amplifier being used was found to be generally nonlinear, and difficult to control. A new approach to amplifier control is presented which eliminates a large portion of the circuitry.

## 3 Proposed Solution

#### 3.1 Determination of Truss Inertia

In order to accurately determine the truss inertia, the truss needs to be analyzed under actual operational conditions. According to speed-control equations for an armature-controlled DC motor, velocity is a function of a variety of either known or determinable system parameters  $(A, K_m, K_b, K_t, R_a, B, J_{motor}, J_{magnet})$  and the unknown  $J_{truss}$ . In general, by solving for  $J_{truss}$  and measuring velocity, the truss inertia can be calculated. To measure velocity, the truss was attached to the motor apparatus and operated at different set voltages while simultaneously recording encoder values for angular velocity. As will be discussed in the following section, the details of the truss inertia calculations involve time t (before the system reaches final velocity), velocity at time t, final velocity  $w_{fv}$ , and input motor voltage A.

#### 3.2 Controller Design

The following steps were taken to find values for controller gains.

- 1. Find system transfer function with PD controller (shown in Equation 1).
- 2. Substitute actual values for system parameters. These parameters are listed in Table 1.
- 3. Equate the coefficients of the characteristic equation to the prototype second order system such that an  $\omega_n$  and  $\zeta$  may be found.
- 4. Use the equation for percent overshoot to find  $\zeta$ . This is shown in Equation 2.
- 5. Use the equation for settling time to find  $\omega_n$ . This is shown in Equation 3.
- 6. Solve for  $K_p$  and  $K_d$ .

Since  $\zeta < 0.69$ , Equation 3 may be used. When solving a settling time of 0.6 seconds and maximum overshoot of 15% was chosen. This resulted in desired controller gains of  $K_p = 1.380778087$  and  $K_d = 0.1141262430$ . A pole-zero plot for the system with these values may be seen in Figure 1.

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{\frac{(K_p + K_d \, s) \, K_m \, K_s}{R \, J}}{s^2 + \frac{R \, B + K_m \, K_b + K_m \, K_s \, K_d}{R \, J} \, s + \frac{K_m \, K_s \, K_p}{R \, J}} \tag{1}$$

$$PO = e^{-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}} \tag{2}$$

$$T_s = \frac{3.2}{\zeta \,\omega_n} \tag{3}$$

Table 1: System Parameters

Parameter	Name	Value	Source
$K_b$	Back EMF Constant	$0.02424 \frac{V}{rad/s}$	Motor Data Sheet
$K_m$	Motor Constant	$0.02424 \frac{N m}{A}$	Motor Data Sheet
R	Armature Resistance s	$2.23~\Omega$	Motor Data Sheet
B	Viscous Damping Friction	$0.00035 \frac{kg m^2}{s}$	Experimentally Determined
J	Total Inertia	$0.00141  kg  m^2$	Experimentally Determined

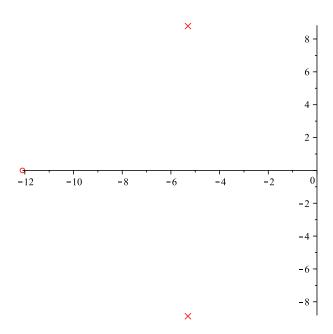


Figure 1: Pole-Zero Plot

## 3.3 Simulation

The ACSYS software system was used to simulate system behavior. Correct system parameters were entered including gains to represent the gearbox. A simulink block diagram for controller design may be seen in Figure 2.

## 3.4 Amplifier Compensation

Due to nonlinearities and difficulties controlling the amplifier with an analog input and on-board oscillator with peak detectors, the majority of the circuitry was bypassed. Two digital to analog output of the computer controller were used and connected at the previous peak detector outputs. Software PWM was

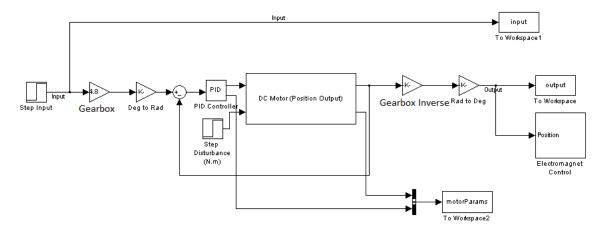


Figure 2: ACSYS Control

then implemented; a simulink block model may be seen in Figure 3. Two levels of feedback were also implemented correct for motor back EMF and simple error.

## 4 Theory of Operation

#### 4.1 Determination of Truss Inertia

#### **Essential Equations**

The truss inertia was calculated from the time domain equations for system speed-control. Equation 4 is the general equation for DC motor speed control closed-loop response, with zero disturbance torque.

$$w(t) = AKK_m K_t \left( 1 - e^{-\frac{t(K_m K_b + R_a B + K_t K_m K)}{Ra (J_{truss} + J_{mag} + J_{mot})}} \right) (K_m K_b + R_a B + K_t K_m K)^{-1}$$
(4)

Equation 5 is the equation for final velocity,  $w_{fv}$ , that has been solved for the unknown viscous damping constant, B.

$$B = \left(\frac{AKKm\ Kt}{w_f\ v} - Km\ Kb - Kt\ Km\ K\right)Ra^{-1} \tag{5}$$

By substituting 5 into 4, the simplified equation for w(t) is produced.

$$w(t) = w_f v \left( 1 - e^{-\frac{tAKK_m Kt}{w_f v Ra (Jtruss + Jmag + Jmot)}} \right)$$
 (6)

In 6, the variables that remain after substituting for constants are t, w(t),  $w_{fv}$ , and A.

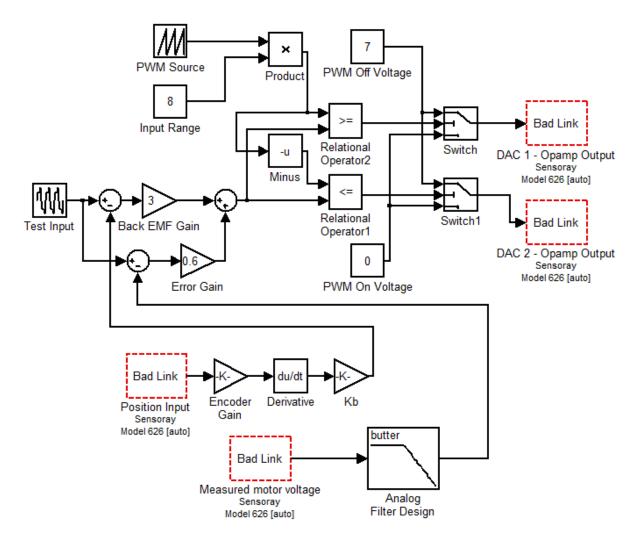


Figure 3: Simulink Software PWM With Velocity and Error Feedback Control

#### Determining the Variables

Using a sampling time of 0.001s the velocity was recorded, and to be thorough, testing was done at A values of 5V, 7.5V, and 9V. By taking an average of w(t) values (over a roughly one second period) after final velocity was achieved, a value for  $w_{fv}$  was obtained. Finally, t was chosen at 2/3 final velocity, which provided a corresponding w(t) value.

By solving 6 in terms of  $J_{truss}$  and substituting the previous constant/variable values, three truss inertia values were calculated for two different arrangements, a) truss alone and b) full truss-magnet. By averaging the results, an experimental determination of  $J_{truss}$  is achieved for each arrangement.

#### 4.2 Controller Design

The transient response is characterized by behavior such as maximum overshoot, rise time, delay time, and settling time and parameters such as damping ratio ( $\zeta$ ) and natural undamped frequency ( $\omega_n$ ). The main

objective is to reach the steady state stage as soon as possible. In order to achieve this goal there certain steps must be taken.

The response of the system becomes more oscillatory with larger percent overshoot as  $\zeta$  decreases. Also setting time is affect by  $\zeta$  and  $\omega_n$  and a practical way to reduce to its to increase  $\omega_n$  while keeping  $\zeta$  constant. However, this increased the overshoot which depends only on  $\zeta$ . To simplify our calculations, the assumption was made to ignore new zero in the transfer function numerator and to use conservative specifications. By increasing or decreasing settling time,  $\omega_n$  will decrease or increase accordingly (see Equation 3).

In addition, the overshoot changes as  $\zeta$  increases or decreases. The gains  $K_p$  and  $K_d$  were calculated using  $\zeta$  and  $\omega_n$ . These gains made a significant impact on the response of the system. For instance,  $K_p$  increases overshoot while it has a small change of the setting time. Furthermore,  $K_d$  has minor affect on both the settling time and overshoot. Therefore, to obtain better overall response it is important to add  $K_d$  to improve overshoot. Finally, the overall performance is generally measured by the step response and steady- state error.

The reason is that  $K_i$  is not considered because there is no factors to cause steady-state error in the response such as disturbance torque which assumed to be zero and also it causes an increase in the settling time. Therefore, it was desired that unless integral gain was needed, it be left out.

## 4.3 Amplifier Compensation

A simulink model to implement software PWM including motor back EMF compensation and simple error correction feedback is shown in figure 3. The core component of this model is the PWM generator; a ramp generator which outputs between zero and one and repeats ever 20 discreet steps. For a system timestep of 0.001 seconds, this gives the software PWM output a nominal frequency of 50 Hz. Two feed back mechanisms have been implemented to improve output accuracy. The first is motor back EMF compensation. This compensator estimates voltage generated by the spinning motor based on its encoder measured speed and back EMF constant (from Table 1. The second mechanism is error feedback using a proportional controller. The input to this compensator is the actual voltage measured at the motor terminals.

## 5 Software Model

#### 5.1 Determination of Truss Inertia

A SolidWorks model exists from initial design of the truss. Using this model and a material specified as brass, a new coordinate system was created on the top surface of the back mounting washer. The corresponds to the axis of rotation for the truss when mounted on the motor. Then the selected axis was used to determine the value in the Moment of Inertia using MASSPROP function.

## 5.2 Controller Design

The PID controller available in the ACSYS software was used to simulate the system. An overall block diagram may be seen in Figure 2. The built in simulink PID controller is used in the model. A block diagram for the motor subsystem model is shown in Figure 4 It should be noted that the correct values for voltage saturation and current saturation were used based on the motor datasheet.

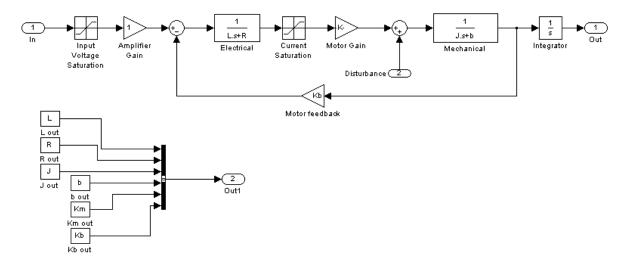


Figure 4: Motor Model

#### 5.3 Simulation

MATLAB and simulink were used to simulate the motor model and PID controller. Models available in the ACSYS software package were used.

## 5.4 Amplifier Compensation

The amplifier compensation block diagram may be found in Figure 3. It assumes a sampling time of 0.001 seconds.

## 6 Results and Observations

#### 6.1 Determination of Truss Inertia

The truss inertia as estimated by SolidWorks is  $0.00192 \ kg \ m^2$ . The truss inertia determined through various experiments is listed in Table 2.

The difference between theoretical (0.00192) and experimental (0.00347) values is roughly a factor of 2. There are at least two explanations for the difference. Firstly, the truss rods were soldered at the joints, which adds extra weight that the computer model does not account for. Secondly, the actual truss inertia

Table 2: Truss Inertia Experimental Results

Applied Voltage	Magnet As Load	Inertia	Average
5 V	No	0.00398	0.00347
7.5 V	No	0.00361	
9 V	No	0.00349	
5 V	Yes	0.00392	0.00369
7.5 V	Yes	0.00329	
9 V	Yes	0.00319	

is an experimental value using all the data sheet parameters provided by the motor manufacturer. These parameters cannot be assumed to be accurate for every individual motor.

### 6.2 Controller Design

System response to position control may be seen in Figure 5 on page 10. Although settling time and overshoot are set to be  $T_s = 0.6$  and PO = 0.15, the response is observed to have larger the settling time and maximum overshoot values. There are many possible causes for this discrepancy. First, the new zero in the transfer function, created by adding derivative gain, was ignored. The reason that we made this assumption to ignore the zero is to equate the ideal transfer function with our PD transfer function to determine  $\zeta$  and  $\omega_n$ . Secondly, it is known that  $\zeta$  has significant effect percent overshoot and also  $\zeta$  and  $\omega_n$  affect the settling time. For these reasons overshoot and settling time of the system response are different for their original values.

## 6.3 Amplifier Compensation

Figure 6 shows amplifier output voltage for a given input voltage over time. Data was captured with the motor connected to the amplifier. Throughout the experiment variable load was applied to the motor; it can be seen that load had little effect on output voltage and therefore speed.

## 7 Conclusion

This report explains the method which was used to accurately determine the model and system parameters such as arm inertia. Secondly, it discusses the reasons for choosing PID and specifically PD based on specifications for maximum overshoot and settling time. Moreover, the report discusses the effects of  $\zeta$  and  $\omega_n$  on the overshoot and settling time values. Third, it illustrates the method which was used to verify system simulation and fine tune the controller parameters. Furthermore, the report describes the conversion of the PID output signal to motor voltage using the amplifier control. Finally, it concludes that all of the objectives of this project were successfully accomplished and future work ideas for improvements were proposed.

## 8 Future Work

Conducting experiment with actual system and observe its response. If the system appear to be large steady-state error then may be it requires adding integral gain. PI controller will improve the response and performance by reducing the steady-state error to zero. The reasons that cause the steady-state to increase are disturbance when the system is not level. In addition, steady-state errors are contributed to non-linear friction or dead zone. For instance, when an amplifier is used in a control system has the input signal which occurs within the dead zone, there will be zero the output of the amplifier, and the control would not be able to able to correct this error. Moreover, the output signals of digital components used in control systems take on only discrete or quantized levels. Therefore, the system generates an error in the output. It must be realized that there is no error-free control systems in the real world this because many physical systems have nonlinear characteristics; therefore, steady-state errors can be reduced but never completely eliminated.

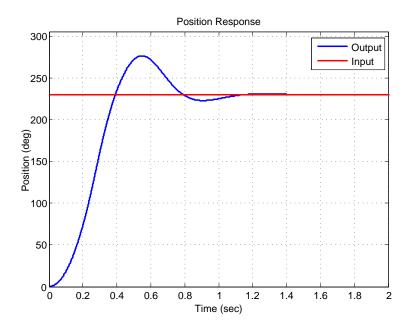


Figure 5: Position Response

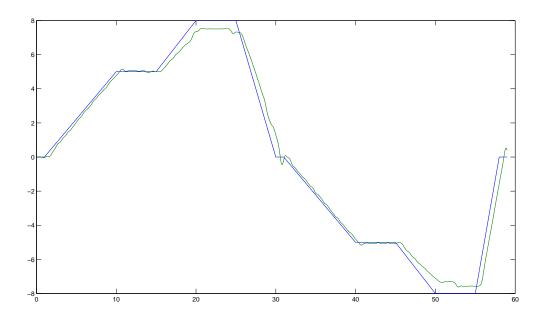


Figure 6: Amplifier Input and Measured Voltage vs Time