ENSC 483 - Project 1
Magnetic Levitation System Analysis

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System Description

1.1 Configuration

The magnetic levitation system is considered in its SISO (single input, single output) system configuration as it levitates a magnet via repulsion. As such, only the bottom coil and magnet are considered as shown in the simplified diagram 1.1.

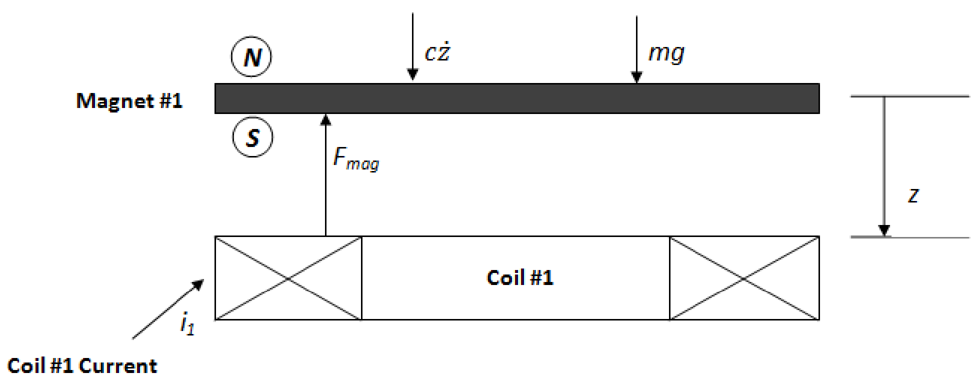

Figure 1.1: System Diagram

In this model, there are 3 forces acting on the magnet: the force of gravity (mg), the force of friction ($c\dot{z}$) and the magnetic force form the drive coil (F_{mag}).

1.2 State Space Model - Inputs, Outputs and State Variables

For the system modeling done in this report, y denotes the output variable, u denotes the input variable and the vector $\mathbf{x}$ denotes the state variables.

Inputs

The input to the system is the current in Coil #1, denoted in diagram 1.1 by i_1. This is given in equation 1.1.

$$\mathbf{u} = \begin{bmatrix} i_1 \end{bmatrix} \tag{1.1}$$

State Variables

The state variables for the system are chosen to be the the magnets velocity ($\dot{z}$) and its position (z). This is shown in equation 1.2

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} z \\ \dot{z} \end{bmatrix} \tag{1.2}$$

Output

The output of the system is the position of the disk magnet, denoted in diagram 1.1 by z. This is shown in equation 1.3:

$$\mathbf{y} = \begin{bmatrix} z \end{bmatrix} \tag{1.3}$$

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Nonlinear Model

2.1 System Dynamic

From figure 1.1, summing the forces yields equation 2.1

$$m\ddot{z} = F_{mag} - c\dot{z} - mg \tag{2.1}$$

The force F_{mag} is given by 2.2 as presented by the manual [1]. The constants a, b, and N are empirically determined properties of the system. For the purposes of this report their values are assumed based on a similar system [2] and given in table 2.1.

$$F_{mag} = \frac{i}{a\,(z+b)^N} \tag{2.2}$$

Table 2.1: System properties [2]

a	0.000105
b	6.2
N	4

It is assumed friction and air resistance are negligible, so $c = 0$ in equation 2.1. As such, equations 2.1 and 2.2 give equation 2.3

$$m\ddot{z} = \frac{i_1}{0.000105\,(z+6.2)^4} - 9.81\,m \tag{2.3}$$

2.2 State Space Model

Using the state variables defined in section 1.2, two system equations may be written. These are presented in equations 2.4 and 2.5.

$$\dot{x_1} = f_1 = x_2 \tag{2.4}$$

$$\dot{x_2} = f_2 = \frac{u}{0.000105\, m\ (x_1 + 6.2)^4} - 9.81 \tag{2.5}$$

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Linearized Model

3.1 Equilibrium Point

The system is linearized about an equilibrium point. This point is given by equation 3.1 where z_0 represents the initial position of the magnet. The input equilibrium may be found using equation 2.5 as shown in 3.3.

$$\mathbf{x}^\star = \begin{bmatrix} z_0 \\ 0 \end{bmatrix} \tag{3.1}$$

$$\frac{u^\star}{0.000105\ (z_0 + 6.2)^4} = 9.81\, m \tag{3.2}$$

$$u^\star = 0.00103005\, m\, (z_0 + 6.2)^4 \tag{3.3}$$

3.2 Linearizing About the Equilibrium Point

Modal Matrix

Equations f_1 and f_2 from 2.4 and 2.5 may be used to obtain a linearized modal matrix. The modal matrix general form is given in equation 3.4. Using equations 3.5, this gives 3.6.

$$\mathbf{A} = \begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} \\ \frac{df_2}{dx_1} & \frac{df_2}{dx_2} \end{bmatrix} \Bigg|_{x^\star, u^\star} \tag{3.4}$$

$$\frac{df_1}{dx_1} = 0 \qquad \frac{df_1}{dx_2} = 1 \qquad \frac{df_2}{dx_1} = -\frac{38095.23810\,u}{m\,(x_1 + 6.2)^5} \qquad \frac{df_2}{dx_2} = 0 \tag{3.5}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\frac{39.24}{z_0+6.2} & 0 \end{bmatrix} \tag{3.6}$$

Input Matrix

Similarly, f_1 and f_2 from 2.4 and 2.5 may be used to obtain a linearized input matrix. The input matrix general form is given in equation 3.7. Using equations 3.8, this gives 3.9.

$$\mathbf{B} = \begin{bmatrix} \frac{df_1}{du} \\ \frac{df_2}{du} \end{bmatrix} \Bigg|_{x^\star, u^\star} \tag{3.7}$$

$$\frac{df_1}{du} = 0 \qquad \frac{df_2}{du} = \frac{9523.809524}{m\ (x_1 + 6.2)^4} \tag{3.8}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ \frac{9523.809524}{m\,(z_0+6.2)^4} \end{bmatrix} \tag{3.9}$$

Output Matrix

The output matrix is simply given by equation 3.10

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{3.10}$$

3.3 Linearized System Model

The final linearized model of the is given by equations 3.11.

$$\dot{\mathbf{x}} - \dot{\mathbf{x}}^{\star} = \begin{bmatrix} 0 & 1 \\ -\frac{39.24}{z_0+6.2} & 0 \end{bmatrix} (\mathbf{x} - \mathbf{x}^{\star}) + \begin{bmatrix} 0 \\ \frac{9523.809524}{m\,(z_0+6.2)^4} \end{bmatrix} (u - u^{\star}) \tag{3.11}$$

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Open Loop Analysis

4.1 Stability

System stability depends on the eigenvalues of matrix **A** from equation 3.6. The eigenvalues are shown in equation 4.1. For simplicity, initial position is assumed to be $z_0 = 1$. This yields equation 4.2.

$$\lambda_1 = \frac{65.4}{\sqrt{-109\, z_0 - 675.8}} \qquad \lambda_2 = -\frac{65.4}{\sqrt{-109\, z_0 - 675.8}} \tag{4.1}$$

$$\lambda_1 = 0 + 2.33\, j \qquad \lambda_2 = 0 - 2.33\, j \tag{4.2}$$

Since the real part of these values is zero, the system is stable in the sense of Laypunov. This result is expected as friction was assumed to be zero.

4.2 Controllability

The controllability matrix for this system is shown in equation 4.3. It can easily be seen that the matrix is full rank. As such, the system is controllable.

$$\mathbf{\Phi_c} = \left[\begin{array}{c|c} \mathbf{B} & \mathbf{AB} \end{array} \right] = \begin{bmatrix} 0 & \frac{0.138}{m} \\ \frac{0.138}{m} & 0 \end{bmatrix} \tag{4.3}$$

4.3 Observability

The observability matrix for this system is shown in equation 4.4. It can easily be seen that the matrix is full rank. As such, the system is observable.

$$\mathbf{\Phi_o} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4.4}$$

References

[1] E. C. Products, *Manual For Model 730 Magnetic Levitation System*, 1 Buckskin Court, Bell Canyon, California 91307, 1999.

[2] P. S. V. Nataraj and M. D. Patil, “Robust Control Design for Nonlinear Magnetic Levitation System using Quantitative Feedback Theory (QFT),” *IEEE*, 2008.