ENSC 483 - Project 2: Magnetic Levitation Control

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1 Introduction

The objectives of the second project were to design an observer and state feedback controller for the linearized system from the first project. Parameters for the state feedback were chosen in order to obtain appropriate values for settling time and overshoot. State feedback was also used to stabilize the system as it would otherwise be unstable. The observer monitors the system and estimates state variables. The top magnet of the magnetic levitation apparatus was controlled; a system which is open-loop unstable.

The linearized model of the system is given in 1; it is unstable yet controllable. A steady state location of 2 cm was chosen; all inputs and position measurements are assumed to be relative to this point. Force exerted by the magnet is given by 2.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -6.32 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0.00537 \end{bmatrix} u \tag{1}$$

$$F_{mag} = \frac{u}{1.05\,(x_1 + 6.2)^4} \tag{2}$$

2 State Feedback

The system was first stabilized using state feedback. Input with state feedback is given by equation 3. The term u_g compensates for the effects of gravity and was calculated as 5588 based on equation 2 and a magnet mass of 120 grams.

$$u = -\mathbf{k}\,\mathbf{x} + u_g \tag{3}$$

System Model

The system model with state feedback is shown in equation 4. Parameters k_1 and k_2 are chosen to place the poles in the desired location using the the characteristic equation of the matrix. The characteristic equation is given in equation 5.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -6.32 + 0.00537\,k_1 & 0.00537\,k_2 \end{bmatrix} \mathbf{x} \tag{4}$$

$$s^2 + 0.00537\,k_2\,s + 6.33 + 0.00537\,k_1 \tag{5}$$

Pole Location

Overshoot and settling time for a second order system (with characteristic equation 6) are given by equations 7 and 8. An overshoot of 5% and settling time of 1 second, gives $\zeta = 0.15898$ and $\omega_n = 25.161$. As such, poles should be placed at at $P_1 = -4 + 24.84\,j$ and $P_2 = -4 - 24.84\,j$.

$$s^2 + 2\,\zeta\,\omega_n\,s + \omega_n^2 \tag{6}$$

$$T_s = \frac{y}{\zeta\,\omega_n} \tag{7}$$

$$\zeta = -\frac{\ln OS}{\sqrt{\pi^2 + \ln OS^2}} \tag{8}$$

Gain Determination

By equating the coefficients of the characteristic equation for a second order system (shown in equation 6) and that shown in 5, state feedback gain may be determined as $k_1 = 116690.3$ and $k_2 = 1489.5$.

3 Observer

An observer was designed to estimate state variable based on the output of the actual system.

System Model

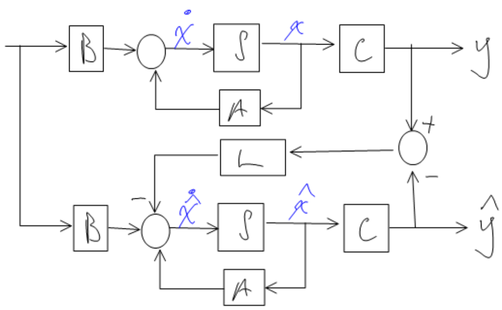

Figure 1: Observer Block Diagram

The observer block diagram is shown in figure 1. Estimated state variables are denoted by $\hat{\mathbf{x}}$. The dynamic of the estimated state variables is shown in 9. The error dynamic (with error defined in 10) is shown in equation 11.

$$\dot{\hat{\mathbf{x}}} = (\mathbf{A} - \mathbf{B}\,\mathbf{k} - \mathbf{L}\,\mathbf{C})\,\hat{\mathbf{x}} - \mathbf{L}\,y \tag{9}$$

$$e = x - \hat{x} \tag{10}$$

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{L}\,\mathbf{C})\,\mathbf{e} \tag{11}$$

Gain Determination

The elements of the $\mathbf{L}$ matrix were chosen to place the poles of the $(\mathbf{A} - \mathbf{L}\,\mathbf{C})$ matrix. In order for the observer to track the system state variables, the poles of the error dynamic should be placed in the left hand plane. In addition, the error dynamic should have a faster response than system with state feedback. As such, the poles were chosen to be four time larger than the poles of the state feedback system. This resulted in a stable response for the estimated state variables, $(\mathbf{A} - \mathbf{B}\,\mathbf{k} - \mathbf{L}\,\mathbf{C})$ as show in the figure 2, the magnitude response. This gives elements of the L matrix as $L_1 = 32$ and $L_2 = 249.67$

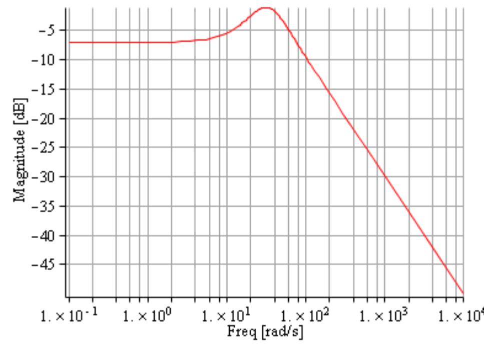

Figure 2: Estimated State Variable Magnitude Response

4 Results

The system was implemented as described in the previous sections. It was tested by applying a step function of 1.89 N, and recording the system response. The resulting magnet position can be seen in figure 3 and its velocity in 4. Observer results are also presented in this graph.

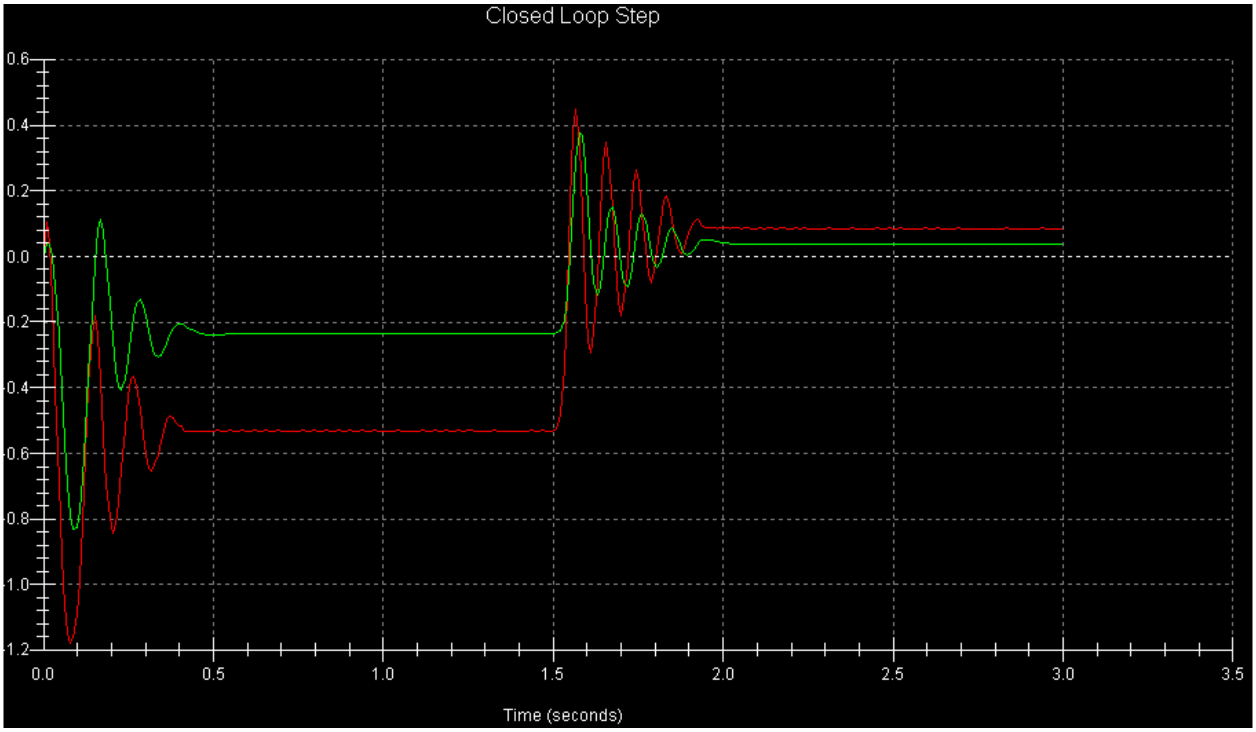

Figure 3: System Response: Position - Actual (Red) and Observed (Green)

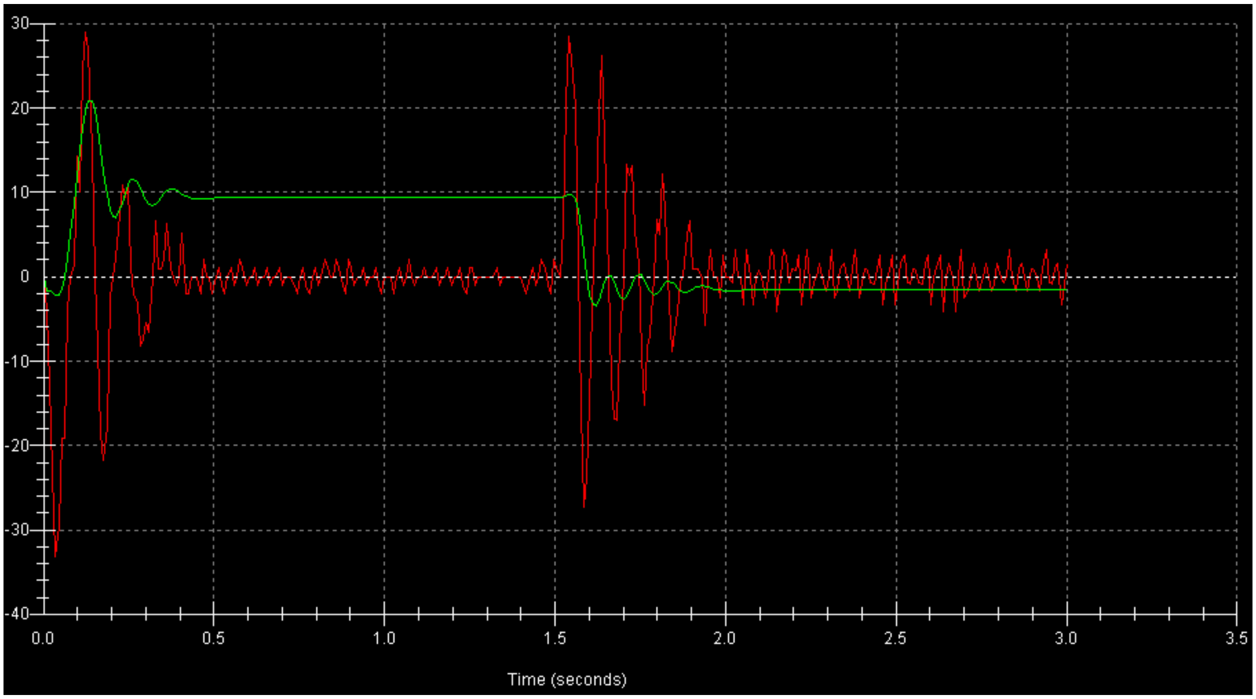

Figure 4: System Response: Velocity - Actual (Red) and Observed (Green)

A Controller Code

```
;############### Variables - State Feedback ###############
#define y_eq q1
#define ts  q2

#define control_1 q3
#define control_2 q4

#define y_1 q20
#define y_last_1 q21
#define x1_sys_1 q22
#define x2_sys_1 q23

#define y_2  q30
#define y_last_2  q31
#define x1_sys_2  q32
#define x2_sys_2  q33

#define k_1  q5
#define k_2  q6
#define magic_number_1  q7
#define magic_number_2  q8

#define gravity_offset q9

;############### Variables - Observer ###############
#define x1_hat  q40
#define x2_hat  q41
#define x1_hat_dot  q42
#define x2_hat_dot  q43

#define a_11  q50
#define a_12  q51
#define a_21  q52
#define a_22  q53

#define l_1  q54
#define l_2  q55

;############### Initialize ###############

;Which one are we controlling?
control_1=0
control_2=1

ts=0.001768
control_effort1 = 0
control_effort2 = 0

;############### Initialize - State feedback ###############
y_eq=2

x1_sys_1=0
x2_sys_1=0
x1_sys_2=0
x2_sys_2=0
y_last_1=0
y_last_2=0

gravity_offset=5588 ;for eq point

k_1=116690.2776
k_2=1489.45
magic_number_1=5
magic_number_2=-7

;############### Initialize - Observer feedback ###############
x1_hat=0
x2_hat=0
x1_hat_dot=0
x2_hat_dot=0

a_11=-32
a_12=1.000000
a_21=-882.754369
a_22=-8

l_1=32
l_2=249.670968

;############### Control Loop ###############
```

```
begin
;State feedback - bottom (1)
if (control_1 != 0)
y_1=enc1_pos/10000-y_eq
x1_sys_1= y_1
x2_sys_1=((y_1 - y_last_1)/ts)
y_last_1=y_1
control_effort1 = (k_1*(x1_sys_1)+k_2*(x2_sys_1))/magic_number_1-gravity_offset
endif

;State feedback - top (2)
if (control_2 != 0)
y_2=enc2_pos/10000+y_eq
x1_sys_2=y_2
x2_sys_2=(y_2 - y_last_2)/ts
y_last_2=y_2
control_effort2 = (k_1*(x1_sys_2)+k_2*(x2_sys_2))/magic_number_2+gravity_offset-cmd2_pos
endif

;Observer - top
x1_hat_dot = a_11*(x1_hat) + a_12*(x2_hat) + l_1*(y_2)
x2_hat_dot = a_21*(x1_hat) + a_22*(x2_hat) + l_2*(y_2)

x1_hat = x1_hat + ts*x1_hat_dot
x2_hat = x2_hat + ts*x2_hat_dot

;Observations
q10=x1_sys_2
q11=x1_hat
q12=x2_sys_2
q13=x2_hat
end
```