IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2012

MSc in Computing Science
for Internal Students of the Imperial College of Science, Technology and Medicine

PAPER M1

PROGRAM DESIGN AND LOGIC

Thursday 17 May 2012, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

Section A *(Use a separate answer book for this Section)*

Please note: All natural deduction proofs must be presented clearly, with wff numbering, where appropriate, indentations, and explanations. Marks will be deduced for unclear and poorly presented proofs.

1 a Consider the following formulas:

Transitivity: $\forall X \forall Y \forall Z\, (r(X,Y) \wedge r(Y,Z) \rightarrow r(X,Z))$
Asymmetry: $\forall X \forall Y\, (r(X,Y) \rightarrow \neg r(Y,X))$
Irreflexivity: $\forall X\, \neg r(X,X)$

In the case of each formula, consider whether it is a consequence of one or both of the other two. If it is then provide a proof using inference rules of natural deduction only.

b A formula *W* is valid if $\vdash W$ holds. Which of the following formulas is valid? For any that is valid provide a proof.

i) $(\forall X \exists Y\, r(X,Y)) \rightarrow (\exists Y \forall X\, r(X,Y))$

ii) $(\exists Y \forall X\, r(X,Y)) \rightarrow (\forall X \exists Y\, r(X,Y))$

c Let *W1* and *W2* be wffs. Let $W1 \vdash W2$. One of the following statements is not correct. Which one? For the one that is not correct illustrate its incorrectness by an example.

i)	$W1 \rightarrow W2$	is valid.
ii)	$\neg(W1 \wedge \neg W2)$	is valid.
iii)	$W1 \vee W2$	is valid.

d i) Explain what is meant by soundness of predicate logic.

ii) Using only inference rules of natural deduction and soundness of predicate logic show the following:

$\exists Y\, r(c,Y),\ \forall X \forall Y\, (r(X,Y) \rightarrow \forall Z\, r(X,Z)),\ \forall X(\forall Z\, (r(X,Z)) \rightarrow \forall Y\, (r(Y,X))) \models \forall X \forall Y\, r(X,Y).$

Parts a, b, c, d carry 35%, 20%, 10%, 35% of the marks, respectively.

2 a i) Show $P \rightarrow (Q \rightarrow R) \equiv P \wedge Q \rightarrow R$

using any techniques you wish, except truth tables.

ii) Show that

$$(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow (C \rightarrow D)) \rightarrow (A \rightarrow (B \rightarrow D)))$$

is a tautology. Do this in two different ways: Once using inference rules of natural deduction only, and once using equivalences only, including the one in part a(i), if required. Do not use truth tables.

b Formalise the sentences (i)-(iii), below, in predicate logic using "=", if needed, and any of the predicates below.
Ensure that you present your formulas clearly, using brackets to correctly identify the scope of quantifiers and to disambiguate where necessary.

m(X)	to mean X is a manager.
a(X)	to mean X is an administrator.
d(X)	to mean X is a director.
em(X, Y)	to mean X earns more than Y.
c(X)	to mean X is a contract.
sc(X)	to mean X is a special contract.
sign(X,Y)	to mean X signs Y.
app(X,Y, Z)	to mean X appoints Y to oversee contract Z.
worksFor(X, Y)	to mean X works for Y.
delegate(X,Y,Z)	to mean X delegates contract Z to Y.

i) Every manager earns more than every administrator. But there is a director who earns more than every manager.

ii) Every contract is signed by two (different) managers (and no others), unless it is special contract, in which case it is signed by a director only.

iii) Directors who sign contracts have two options. Either for each contract they sign they appoint at least one manager to oversee it, or they have an administrator working for them to whom they delegate all the contacts they sign. (You do not need to represent that these two options are exclusive from one another.)

Parts a, b carry 40%, 60% of the marks, respectively.

Section B (Use a separate answer book for this Section)

3 Consider the following simplified description of a tracking system designed to protect animals (including domestic pets):

- Each animal is fitted with a collar containing an *active tracking tag*. Each such tag is identified by a (unique) tag number and regularly transmits a series of *tag readings*. A tag reading consists of a tag number, a position (given as an (x, y) coordinate) and a battery level (a real number from 0 to 100).
- For each animal, tracking system users can specify a rectangular area known as a *protected area* within which the animal should remain at all times.
- When a tag reading is received by the tracking system, it checks two conditions: firstly, whether the animal corresponding to the tag number is outside its protected area, and secondly whether the battery level is below 20. In either case, an appropriate alert message is displayed.

You may assume the availability of the following template class:

```
template <class T1, class T2>
class Table { // lookup table associating T1s to T2s
  ...
public:
  Table(); // an empty table
  void set(const T1 &x, T2 &y);
    // inserts an association from x to y into the table
  T2 *get(const T1 &x);
    // returns a pointer to the T2 associated with x
    // returns NULL if there is no associated element
};
```

a Write C++ class declarations (i.e. no function bodies) to support the above.

b Write a test function where:

- Animal "Lassie" wears active tag number 111. Lassie's protected area extends from (0,0) to (5,5).
- Animal "Garfield" wears active tag number 222. Garfield's protected area extends from (3,4) to (7,8).
- A tag reading for tag number 111 at position (3,3) with battery level 15 is transmitted and received by the tracking system.
- A tag reading for tag number 222 at position (8,8) with battery level 50 is transmitted and received by the tracking system.

c Write function bodies for your animal, tag reading and tracking system classes.

The three parts carry, respectively, 40%, 25%, and 35% of the marks.

4 Consider the following simplified description of the power consumption behaviour of electronic devices:

- Electronic devices can be digital cameras or smartphones.
- Every electronic device has a battery to provide its power needs. Battery capacity is measured in mW h (milliwatt hours). Batteries have a maximum capacity, an available capacity (initially set to the maximum capacity) and a recharge power measured in mW (milliwatts). When used for x hours at power y, the available capacity of a battery decreases by the product of x and y. When recharging for x hours the available capacity of a battery increases by the product of the recharge power and x.
- The *anticipated operating life* of a device (in hours) is given by the ratio of the available capacity of its battery and the device's power consumption. The power consumption of a digital camera is 1500 mW. Smartphones have multiple CPU cores used to run applications. The power consumption of a smartphone is 500 mW (for screen display and network connectivity) plus 500 mW per active CPU core. The number of active CPU cores is the minimum of the number of running applications and the number of CPU cores.

a Draw a UML class diagram to describe the above.

b Write C++ class declarations (i.e. no function bodies) to support the above.

c Write a test function as follows:

- A 6000 mW h battery with a recharge power of 500 mW is inserted into a Canon digital camera.
- The Canon digital camera operates for 1.5 hours, and is then recharged for 0.5 hours.
- The anticipated operating life of the Canon digital camera is displayed.
- A 4000 mW h battery with a recharge power of 800 mW is inserted into a Samsung smartphone that has two CPU cores.
- 3 applications are started on the Samsung smartphone.
- The Samsung smartphone operates for 2 hours.
- The anticipated operating life of the Samsung smartphone is displayed.

d Write the bodies of the functions from part (b) that relate to your classes that represent batteries, digital cameras and smartphones.

The four parts carry equal marks.