Computer Architecture

Tutorial 3 – Number Representation and Binary Arithmetic - Answers

1) Convert the following binary numbers to decimal:
   (a) 0110 = 6, (b) 1011 = 11, (c) 10101010 = 170

2) Convert the following binary numbers to hexadecimal:
   (a) 1110 = E, (b) 11011 = 1B, (c) 1010111101110010 = AF72

3) Convert the following decimal numbers to binary and hexadecimal:
   (a) 12 = 1100 & C, (b) 27 = 11011 & 1B, (c) 96 = 1100000 & 60

4) For an 8-bit group, work out the representation for -37_{10} in

   37_{10} = 100101

   a) Sign & Magnitude 10100101

   b) One's Complement 11011010

   c) Two's Complement 11011011

   d) Excess-255 -37 = -37 + 255 = 218 = 11011010

   e) Excess-128 -37 = -37 + 128 = 91 = 01011011

5) Express 9876510 in Binary Coded Decimal

9	8	7	6	5	1	0
1001	1000	0111	0110	0101	0001	0000

6) Form the negative equivalent of the following 8-bit Two's Complement numbers.

   (a) 00011001, (b) 00011110, (c) 01101000, (d) 01110100

(a) 00011001 = 16 + 8 + 1 = 25_{10}

“invert the bits and add 1” 11100110 + 1 = 11100111

check: 11100111 = -128 + (64 + 32 + 4 + 2 + 1) = -25_{10}

(b) 00011110 = 16 + 8 + 4 + 2 = 30_{10}

"invert the bits and add 1" 11100001 + 1 = 11100010

check: 11100010 = -128 + (64 + 32 + 2) = -30_{10}

(c) 01101000 = 64 + 32 + 8 = 104_{10}

"invert the bits and add 1" 10010111 + 1 = 10011000

check: 10011000 = -128 + (16 + 8) = -104_{10}

(d) 01110100 = 64 + 32 + 16 + 4 = 116_{10}

"invert the bits and add 1" 10001011 + 1 = 10001100

check: 10001100 = -128 + (8 + 4) = -116_{10}

by comparing the resulting bit patterns to the originals, can you spot a "short cut" method for the conversion?

Take another look at the bit patterns:

positive: 00011001 00011110 01101000 01110100
negative: 11100111 11100010 10011000 10001100

"starting from the rightmost bit (lsb), copy each bit unchanged up to and including the first 1 then invert all the remaining bits"

7) Perform the following 12-bit two's complement subtraction

1010 1010 1011 − 1011 0000 1101

Two's Complement subtraction: "negate the subtrahend and add"

Two's Complement negation: "invert the bits and add 1"

101100001101 = 010011110010 + 1 = 010011110011

```
  1010 1010 1011
+ 0100 1111 0011
  --------------------
  1111 1001 1110
```

Check your answer by determining the decimal representation of the numbers and the result

```
  1010 1010 1011 =   −2048 +  683 = −1365
 -1011 0000 1101 = −(−2048 +  781 = −1267)
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 1111 1001 1110 =   −2048 + 1950 =   -98
```

8) Perform the binary multiplication 10011 x 1101

```
      10011 x
        1101
     -----------
        1101 +
      1101
 ------------
     100111 +
  1101
------------------
  11110111
```

In decimal: 19 x 13 = 247

9) Divide the binary number 1011111 by 101

```
            10011
       ---------------
101 | 1011111
       - 101
      --------------
          001111-
             101
         ----------
               101
```

In Decimal: 95 / 5 = 19