

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2017

MSc in Computing Science
for Internal Students of the Imperial College of Science, Technology and Medicine

PAPER M1

PROGRAM DESIGN AND LOGIC

Thursday 11 May 2017, 10:00

Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

Section A (Use a separate answer book for this Section)

Note: All natural deduction proofs must be presented clearly, with wff numbering, where appropriate, indentations, and explanations. Marks will be deducted for unclear and poorly presented proofs. When using natural deduction, unless otherwise stated, you may use any of the primitive and derived rules, but not equivalences, unless they are proved by natural deduction, themselves.

- 1 a Show by natural deduction that $p \vee q, p \rightarrow r, q \rightarrow r \vdash r$, using only the introduction and elimination rules of inference for the connectives \vee, \neg, \rightarrow .

- b Show by natural deduction that
 $m \vee n, m \rightarrow \neg c \vee d, n \rightarrow d \wedge e, (c \rightarrow d) \rightarrow k \vdash k$.
You may use any primitive or derived rules of natural deduction.

- c Which of the following two sentences *S1* and *S2* of propositional logic captures the following statements?

If you are on a train and there is an emergency, press the alarm signal.

But there is a fine for pressing the alarm signal if you are on a train and there is no emergency.

S1 $(t \wedge e \rightarrow p) \wedge (t \wedge p \wedge \neg e \rightarrow f)$

S2 $(t \wedge p \rightarrow e) \wedge (t \wedge p \wedge f \rightarrow \neg e)$

Here t, e, p, f , respectively, stand for *on train*, *there is an emergency*, *press the alarm signal*, and *there is a fine*.

- d Consider the following sentences (i)-(iv), where t, e, p, f are as given above in part (c).

- i) $t \wedge f \rightarrow p$
ii) $t \rightarrow (\neg e \vee p) \wedge (e \vee \neg p \vee f)$
iii) $t \wedge \neg f \rightarrow \neg e$
iv) $t \wedge \neg p \rightarrow \neg e$

Give the conjunctive normal form (CNF) of (ii).

Two of the sentences (i)-(iv) are entailed by the (English) statements in part (c). Which are they?

Show the entailment, using *S1* or *S2*, whichever is the correct logical characterisation of the statements in (c), and using any mix of natural deduction and equivalences you wish, but do not use truth tables.

Parts a, b, c, d carry 15%, 25%, 5%, 55% of the marks, respectively.

- 2 a Formalise in predicate logic the sentences (i)-(v), below, that concern student applications made to a university department. Use only the predicates listed below and the infix predicates = and \geq , if required. Ensure that you present your formulas clearly, using brackets to correctly identify the scope of quantifiers and disambiguate where necessary.

<i>applicant(PID, Name)</i>	<i>to mean an application has been made by a person named Name, who has been given a unique ID PID.</i>
<i>apply(PID, Degree)</i>	<i>to mean applicant PID has applied for Degree.</i>
<i>interview(PID)</i>	<i>to mean applicant PID is invited for interview.</i>
<i>dqual(PID, Degree, Subject, Class)</i>	<i>to mean applicant PID has a degree qualification Degree in Subject with the award class Class. An applicant may have zero or more degree qualifications.</i>
<i>lqual(PID, Avg)</i>	<i>to mean applicant PID has a language test qualification with average, Avg.</i>
<i>accept(PID, Degree)</i>	<i>to mean applicant PID is accepted for Degree.</i>
<i>accoffer(PID, D)</i>	<i>to mean applicant PID accepts the offer for degree D.</i>
<i>rejoffer(PID, D)</i>	<i>to mean applicant PID rejects the offer for degree D.</i>
<i>defoffer(PID, D)</i>	<i>to mean applicant PID defers the offer for degree D.</i>
<i>billed(PID, D)</i>	<i>to mean applicant PID is billed for a deposit towards degree D.</i>

- i) Each applicant applies for at least one degree, and that degree is MSc, MRes or PhD.
- ii) All applicants applying for the MRes or MSc degrees who have at least one degree qualification, are invited for an interview. Some, but not all, applicants to the PhD degree are invited for an interview.
- iii) No PhD applicant without an MSc degree qualification in computer science is invited for an interview or accepted for the PhD degree.
- iv) All accepted candidates (for any degree) have a 1st class degree qualification and an English language test average of at least 6.5.
- v) If and only if a candidate is accepted for a degree they may accept the offer, defer it or reject it. A candidate is billed for a deposit towards a degree if and only if they accept or defer the offer for that degree.

- b Answer either (i) or(ii) below. You do not have to do both.

- i) From the statements in part (a), together with the assumption that anyone who is accepted for a degree has applied for that degree, it can be proved that:
If a candidate is billed for a deposit towards a PhD degree then they have an MSc degree in computer science, and, moreover, they have a first class degree qualification.

Give a proof of this using any mix of natural deduction and equivalences you wish.

- ii) Show that

$$T1, T2, T3, T4 \vdash p(a) \rightarrow \exists Y \neg m(a, Y))$$

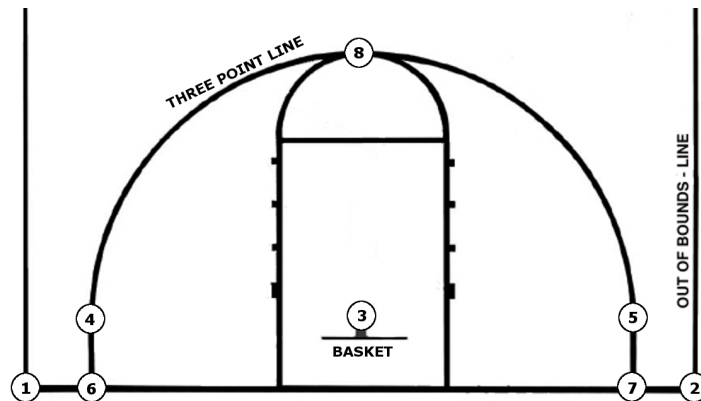
using any mix of natural deduction and equivalences you wish, where $T1$ - $T4$ are predicate logic sentences as follows:

- $T1 \quad \forall X (p(X) \leftrightarrow q(X))$
- $T2 \quad \forall X (q(X) \vee r(X) \rightarrow \exists Y s(X, Y))$
- $T3 \quad \forall X \forall Y (s(X, Y) \rightarrow t(X, Y))$
- $T4 \quad \forall X \forall Y \neg(t(X, Y) \wedge m(X, Y))$

Parts a, b carry 80%, 20% of the marks, respectively.

Section B (Use a separate answer book for this Section)

- 3 Consider one end of a basketball court as shown below. Note also the table of corresponding (x,y) point coordinates where the units used are metres.



Point	2D court coordinates
1	(0, 0)
2	(15, 0)
3	(7.5, 1.575)
4	(0.9, 1.575)
5	(14.1, 1.575)
6	(0.9, 0)
7	(14.1, 0)
8	(7.5, 8.175)

Suppose a simplified game of basketball is played on this area as follows:

- Basketball players are defined by their current position (in 2D court coordinates), their skill factor (a higher-is-better real number from 0 to 1) and whether or not they are currently have the ball.
- When they have the ball, a player may *pass* the ball to another player – in which case that player obtains possession of the ball – or may *shoot* at the basket – in which case the probability that the shot succeeds is:

$$1 - \frac{1}{1 + \exp(-a(x - bs))}$$

where $a = 0.62$, $b = 6.84$, x is the distance (in metres) between the position of player and the centre of the basket (Point 3), s is the player's skill factor and \exp is the exponential function.

- Successful shots taken inside the three point line earn 2 points; successful shots taken outside the three point line earn 3 points. Failed shots earn 0 points.
- Write C++ class declarations (i.e. no function bodies) to support the above.
 - Write a test function where:
 - Larry is a basketball player with skill level 0.9 at (4.5, 3.25). Michael is a basketball player with skill level 0.95 at (10.5, 5.1). Michael has the ball.
 - Larry moves to court coordinates (0.25, 2.2).
 - Michael passes the ball to Larry, who shoots at the basket. The number of points scored by Larry is printed.
 - Write function bodies for your classes.

Hint: The function `drand48()` returns a random number between 0 and 1.

The three parts carry, respectively, 35%, 25%, and 40% of the marks.

- 4 Consider the description of the following scenario:
- Your task is to package nuts, which includes putting nuts in small bags and sticking labels with basic nutrition information (BNI) on them. The BNI provides information about the energy and the fat content of all nuts in a bag. There are two types of nuts: walnuts and almonds. Every nut has a weight in grams. Bags come in different sizes and can only hold a limited number of nuts. Bags also have space for one label, which can be replaced with a new one if necessary. Labels are created with text on them; this text cannot be altered in any way later.
 - To fill a bag, you can add one nut at a time. At any time you should be able to obtain the BNI of all nuts in a bag in the format:
Total of XX kcal (YY g fat) [high in fat]
where XX is the total energy (in kcal) and YY is the total fat content (in grams) of all nuts in a bag. The string between [and] should only be printed if the total fat content exceeds 20g. This BNI goes on a label, which can be stuck on a bag.
 - Walnuts contain 65.21% fat and Almonds contain 50.64% fat. One gram of fat gives 9 kcal of energy whereas all non-fat components give only 4 kcal per gram. You may assume the availability of the following template class:

```
template <typename T> class list {
public:
    list(); //constructor that creates an empty list
    void push_back(const T& item); //adds item to the end
    int size(); //returns the number of items
    //members that return a static iterator to the
    //beginning or the end of the list:
    list<T>::const_iterator begin();
    list<T>::const_iterator end();
};
```
- Draw a UML class diagram to describe the above.
 - Write C++ class declarations to support the above.
 - Write a test function as follows:
 - You are given a bag that can hold 5 nuts, 2 walnuts that weigh 8g and 13g, and one Almond that weighs 1.5g.
 - After adding one walnut and one almond to the bag, you create a label with the BNI and stick it on the bag.
 - You add the remaining walnut to the same bag and stick a new sticker with the updated BNI on it.
 - Write the bodies of the functions from part (b). You might want to use the function `string to_string(float)` to convert numbers to strings.

The four parts carry, respectively, 25%, 40%, 10%, and 25% of the marks.