Functional Dependencies

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What is wrong with this schema?

bank_data									
no	sortcode	bname	cash	type	cname	rate?	mid	amount	tdate
100	67	Strand	34005.00	current	McBrien, P.	null	1000	2300.00	1999-01-05
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1001	4000.00	1999-01-05
100	67	Strand	34005.00	current	McBrien, P.	null	1002	-223.45	1999-01-08
107	56	Wimbledon	84340.45	current	Poulovassilis, A.	null	1004	-100.00	1999-01-11
103	34	Goodge St	6900.67	current	Boyd, M.	null	1005	145.50	1999-01-12
100	67	Strand	34005.00	current	McBrien, P.	null	1006	10.23	1999-01-15
107	56	Wimbledon	84340.45	current	Poulovassilis, A.	null	1007	345.56	1999-01-15
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1008	1230.00	1999-01-15
119	56	Wimbledon	84340.45	deposit	Poulovassilis, A.	5.50	1009	5600.00	1999-01-18

```
SELECT cash
FROM   bank_data
WHERE  sortcode=67
```

⇨

cash
34005.00
34005.00
34005.00
34005.00
34005.00

What is wrong with this schema?

bank_data									
no	sortcode	bname	cash	type	cname	rate?	<u>mid</u>	amount	tdate
100	67	Strand	34005.00	current	McBrien, P.	null	1000	2300.00	1999-01-05
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1001	4000.00	1999-01-05
100	67	Strand	34005.00	current	McBrien, P.	null	1002	-223.45	1999-01-08
107	56	Wimbledon	84340.45	current	Poulovassilis, A.	null	1004	-100.00	1999-01-11
103	34	Goodge St	6900.67	current	Boyd, M.	null	1005	145.50	1999-01-12
100	67	Strand	34005.00	current	McBrien, P.	null	1006	10.23	1999-01-15
107	56	Wimbledon	84340.45	current	Poulovassilis, A.	null	1007	345.56	1999-01-15
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1008	1230.00	1999-01-15
119	56	Wimbledon	84340.45	deposit	Poulovassilis, A.	5.50	1009	5600.00	1999-01-18

```
SELECT DISTINCT cash
FROM   bank_data
WHERE  sortcode=67
```

⇒

cash
34005.00

What is wrong with this schema?

bank_data									
no	sortcode	bname	cash	type	cname	rate?	mid	amount	tdate
100	67	Strand	34005.00	current	McBrien, P.	null	1000	2300.00	1999-01-05
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1001	4000.00	1999-01-05
100	67	Strand	34005.00	current	McBrien, P.	null	1002	-223.45	1999-01-08
107	56	Wimbledon	84340.45	current	Poulovassilis, A.	null	1004	-100.00	1999-01-11
103	34	Goodge St	6900.67	current	Boyd, M.	null	1005	145.50	1999-01-12
100	67	Strand	34005.00	current	McBrien, P.	null	1006	10.23	1999-01-15
107	56	Wimbledon	84340.45	current	Poulovassilis, A.	null	1007	345.56	1999-01-15
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1008	1230.00	1999-01-15
119	56	Wimbledon	84340.45	deposit	Poulovassilis, A.	5.50	1009	5600.00	1999-01-18

```
SELECT DISTINCT rate
FROM   bank_data
WHERE  account=107
```

⇒

rate
null

Problems with Updates on Redundant Data

```
INSERT INTO bank_data
VALUES (100,67,'Strand',33005.00,'deposit','McBrien, P.',null,
        1017,-1000.00,'1999-01-21')

UPDATE bank_data
SET    rate=1.00
WHERE  mid=1007
```

bank_data

no	sortcode	bname	cash	type	cname	rate?	mid	amount	tdate
100	67	Strand	34005.00	current	McBrien, P.	null	1000	2300.00	1999-01-05
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1001	4000.00	1999-01-05
100	67	Strand	34005.00	current	McBrien, P.	null	1002	-223.45	1999-01-08
107	56	Wimbledon	84340.45	current	Poulovassilis, A.	null	1004	-100.00	1999-01-11
103	34	Goodge St	6900.67	current	Boyd, M.	null	1005	145.50	1999-01-12
100	67	Strand	34005.00	current	McBrien, P.	null	1006	10.23	1999-01-15
107	56	Wimbledon	84340.45	current	Poulovassilis, A.	1.00	1007	345.56	1999-01-15
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1008	1230.00	1999-01-15
119	56	Wimbledon	84340.45	deposit	Poulovassilis, A.	5.50	1009	5600.00	1999-01-18
100	67	Strand	33005.00	deposit	McBrien, P.	null	1017	-1000.00	1999-01-21

```
SELECT DISTINCT cash
FROM   bank_data
WHERE  sortcode=67
```

⇒

cash
34005.00
33005.00

Problems with Updates on Redundant Data

```sql
INSERT INTO bank_data
VALUES (100,67,'Strand',33005.00,'deposit','McBrien, P.',null,
        1017,-1000.00,'1999-01-21')

UPDATE bank_data
SET    rate=1.00
WHERE  mid=1007
```

bank_data

no	sortcode	bname	cash	type	cname	rate?	mid	amount	tdate
100	67	Strand	34005.00	current	McBrien, P.	null	1000	2300.00	1999-01-05
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1001	4000.00	1999-01-05
100	67	Strand	34005.00	current	McBrien, P.	null	1002	-223.45	1999-01-08
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103	34	Goodge St	6900.67	current	Boyd, M.	null	1005	145.50	1999-01-12
100	67	Strand	34005.00	current	McBrien, P.	null	1006	10.23	1999-01-15
107	56	Wimbledon	84340.45	current	Poulovassilis, A.	1.00	1007	345.56	1999-01-15
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1008	1230.00	1999-01-15
119	56	Wimbledon	84340.45	deposit	Poulovassilis, A.	5.50	1009	5600.00	1999-01-18
100	67	Strand	33005.00	deposit	McBrien, P.	null	1017	-1000.00	1999-01-21

```sql
SELECT DISTINCT rate
FROM   bank_data
WHERE  account=107
```

⇒

rate
null
1.00

How do you know what is redundant?

Functional Dependency

A **functional dependency (fd)** $X \rightarrow Y$ states that if the values of attributes X agree in two tuples, then so must the values in Y.

Using an FD to find a value

If the FD no $\rightarrow$ rate holds then x in the table below must always take the value 5.25, but y and z may take any value.

bank_data		
no	mid	rate
101	1001	5.25
101	1008	x
119	1009	y
z	1010	5.25

Quiz 1: FDs that hold in bank_data

bank_data									
no	sortcode	bname	cash	type	cname	rate?	mid	amount	tdate
100	67	Strand	34005.00	current	McBrien, P.	null	1000	2300.00	1999-01-05
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1001	4000.00	1999-01-05
100	67	Strand	34005.00	current	McBrien, P.	null	1002	-223.45	1999-01-08
107	56	Wimbledon	84340.45	current	Poulovassilis, A.	null	1004	-100.00	1999-01-11
103	34	Goodge St	6900.67	current	Boyd, M.	null	1005	145.50	1999-01-12
100	67	Strand	34005.00	current	McBrien, P.	null	1006	10.23	1999-01-15
107	56	Wimbledon	84340.45	current	Poulovassilis, A.	null	1007	345.56	1999-01-15
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1008	1230.00	1999-01-15
119	56	Wimbledon	84340.45	deposit	Poulovassilis, A.	5.50	1009	5600.00	1999-01-18

Which set of FDs below do not hold for the data?

A

no → rate
no → bname

B

no → type
bname → no

C

no → type
mid → bname

D

amount → rate
bname → sortcode

Quiz 2: Deriving FDs from other FDs

sortcode $\rightarrow$ bname
no $\rightarrow$ sortcode
no $\rightarrow$ cname
no $\rightarrow$ rate
mid $\rightarrow$ no

Given the FDs above, which FD below might not hold?

A

no $\rightarrow$ bname

B

no,sortcode $\rightarrow$ cname,sortcode

C

amount,tdate $\rightarrow$ amount

D

amount,tdate $\rightarrow$ mid

Armstrong's Axioms

X,Y and Z are sets of attributes, and XY is a shorthand for $X \cup Y$

Reflexivity

$Y \subseteq X \models X \rightarrow Y$

- Such an FD is called a **trivial** FD

Applying reflexivity

If amount,tdate are attributes

By reflexivity

$\mathsf{amount} \subseteq \mathsf{amount}, \mathsf{tdate} \models \mathsf{amount}, \mathsf{tdate} \rightarrow \mathsf{amount}$

$\mathsf{tdate} \subseteq \mathsf{amount}, \mathsf{tdate} \models \mathsf{amount}, \mathsf{tdate} \rightarrow \mathsf{tdate}$

Armstrong's Axioms

X,Y and Z are sets of attributes, and XY is a shorthand for $X \cup Y$

Augmentation

$X \rightarrow Y \models XZ \rightarrow YZ$

Applying augmentation

If no,cname,sortcode are attributes and no $\rightarrow$ cname
By augmentation
$\mathsf{no} \rightarrow \mathsf{cname} \models \mathsf{no}, \mathsf{sortcode} \rightarrow \mathsf{cname}, \mathsf{sortcode}$

Armstrong's Axioms

X,Y and Z are sets of attributes, and XY is a shorthand for $X \cup Y$

Transitivity

$X \rightarrow Y, Y \rightarrow Z \models X \rightarrow Z$

Applying transitivity

If $\mathsf{no} \rightarrow \mathsf{sortcode}$ and $\mathsf{sortcode} \rightarrow \mathsf{bname}$
By transitivity
$\mathsf{no} \rightarrow \mathsf{sortcode}, \mathsf{sortcode} \rightarrow \mathsf{bname} \models \mathsf{no} \rightarrow \mathsf{bname}$

Union Rule

Armstrong's Axioms

Reflexivity: $Y \subseteq X \models X \rightarrow Y$
Augmentation: $X \rightarrow Y \models XZ \rightarrow YZ$
Transitivity: $X \rightarrow Y, Y \rightarrow Z \models X \rightarrow Z$

Union Rule

If $X \rightarrow Y, X \rightarrow Z$
By augmentation
$X \rightarrow Y \models XZ \rightarrow YZ$
$X \rightarrow Z \models X \rightarrow XZ$
By transitivity
$X \rightarrow XZ, XZ \rightarrow YZ \models X \rightarrow YZ$

If $X \rightarrow YZ$
By reflexivity
$YZ \models YZ \rightarrow Y, YZ \rightarrow Z$
By transitivity
$X \rightarrow YZ, YZ \rightarrow Y \models X \rightarrow Y$
$X \rightarrow YZ, YZ \rightarrow Z \models X \rightarrow Z$

$$\therefore X \rightarrow Y, X \rightarrow Z \equiv X \rightarrow YZ$$

- Note that the union rules means that we can restrict ourselves to FD sets containing just one attribute on the RHS of each FD without loosing expressiveness

Quiz 3: Deriving FDs from other FDs

Given a set $S = \{A \rightarrow BC, CD \rightarrow E, C \rightarrow F, E \rightarrow F\}$ of FDs

Which set of FDs below follows from S?

A

$A \rightarrow BF, A \rightarrow CF, A \rightarrow ABCF$

B

$A \rightarrow BD, A \rightarrow CF, A \rightarrow ABCF$

C

$A \rightarrow BD, A \rightarrow BF, A \rightarrow ABCF$

D

$A \rightarrow BD, A \rightarrow BF, A \rightarrow CF$

Pseudotransitivity Rule

Armstrong's Axioms

Reflexivity: $Y \subseteq X \models X \rightarrow Y$
Augmentation: $X \rightarrow Y \models XZ \rightarrow YZ$
Transitivity: $X \rightarrow Y, Y \rightarrow Z \models X \rightarrow Z$

Pseudotransitivity Rule

If $X \rightarrow Y, WY \rightarrow Z$
By augmentation
$X \rightarrow Y \models WX \rightarrow WY$
By transitivity
$WX \rightarrow WY, WY \rightarrow Z \models WX \rightarrow Z$

$$\therefore X \rightarrow Y, WY \rightarrow Z \models WX \rightarrow Z$$

Decomposition Rule

Armstrong's Axioms

Reflexivity: $Y \subseteq X \models X \rightarrow Y$
Augmentation: $X \rightarrow Y \models XZ \rightarrow YZ$
Transitivity: $X \rightarrow Y, Y \rightarrow Z \models X \rightarrow Z$

Decomposition Rule

If $X \rightarrow Y, Z \subseteq Y$
By reflexivity
$Z \subseteq Y \models Y \rightarrow Z$
By transitivity
$X \rightarrow Y, Y \rightarrow Z \models X \rightarrow Z$

$$\therefore X \rightarrow Y, Z \subseteq Y \models X \rightarrow Z$$

FDs and Keys

Super-keys and minimal keys

- If a set of attributes X in relation R functionally determines all the other attributes of R, then X must be a **super-key** of R
- If it is not possible to remove any attribute from X to form X', and X' functionally determine all attributes, then X is a **minimal key** of R

Determining keys of a relation

Suppose $\mathsf{branch}(\mathsf{sortcode}, \mathsf{bname}, \mathsf{cash})$ has the FD set
$\{\mathsf{sortcode} \rightarrow \mathsf{bname}, \mathsf{bname} \rightarrow \mathsf{sortcode}, \mathsf{bname} \rightarrow \mathsf{cash}\}$

1. $\{\mathsf{sortcode}, \mathsf{bname}\}$ is a super-key since $\{\mathsf{sortcode}, \mathsf{bname}\} \rightarrow \mathsf{cash}$
2. However, $\{\mathsf{sortcode}, \mathsf{bname}\}$ is not a minimal key, since $\mathsf{sortcode} \rightarrow \{\mathsf{bname}, \mathsf{cash}\}$ and $\mathsf{bname} \rightarrow \{\mathsf{sortcode}, \mathsf{cash}\}$
3. $\mathsf{sortcode}$ and bname are both minimal keys of branch

Quiz 4: Deriving minimal keys from FDs

Suppose the relation $R(A, B, C, D, E)$ has functional dependencies
$S = \{A \to E, B \to AC, C \to D, E \to D\}$

Which of the following is a minimal key?

A	B	C	D
A	AB	BC	B

Quiz 5: Keys and FDs

Suppose the relation $R(A, B, C, D, E)$ has minimal keys AC and BC

Which FD does not necessarily hold?

A	B	C	D
$ABC \rightarrow DE$	$AC \rightarrow BDE$	$AB \rightarrow DE$	$BC \rightarrow DE$

Closure of a set of attributes with a set of FDs

Closure X^+ of a set of attributes X with FDs S

1. Set $X^+ := X$
2. Starting with X^+ apply each FD $X_s \rightarrow Y$ in S where $X_s \subseteq X^+$ but Y is not already in X^+, to find determined attributes Y
3. $X^+ := X^+ \cup Y$
4. If Y not empty goto (2)
5. Return X^+

Closure of attributes

Relation $R(A, B, C, D, E, F)$ has FD set $S = \{A \rightarrow BC, CD \rightarrow E, C \rightarrow F, E \rightarrow F\}$
To compute A^+

- Start with $A^+ = A$, just $A \rightarrow BC$ matches, so $Y = BC$
- $A^+ = ABC$, just $C \rightarrow F$ matches, so $Y = F$
- $A^+ = ABCF$, no FDs apply, so we have the result

Closure of a set of attributes with a set of FDs

Closure X^+ of a set of attributes X with FDs S

1. Set $X^+ := X$
2. Starting with X^+ apply each FD $X_s \rightarrow Y$ in S where $X_s \subseteq X^+$ but Y is not already in X^+, to find determined attributes Y
3. $X^+ := X^+ \cup Y$
4. If Y not empty goto (2)
5. Return X^+

Closure of a set of attributes

Relation $R(A, B, C, D, E, F)$ has FD set $S = \{A \rightarrow BC, CD \rightarrow E, C \rightarrow F, E \rightarrow F\}$
To compute AD^+

- Start with $AD^+ = AD$, just $A \rightarrow BC$ matches, so $Y = BC$
- $AD^+ = ABCD$, $CD \rightarrow E, C \rightarrow F$ matches, so $Y = EF$
- $AD^+ = ABCDEF$, no FDs apply, so we have the result

Quiz 6: Closure of Attribute Sets

Given a relation $R(A, B, C, D, E, F)$ and FD set
$S = \{A \rightarrow BC, C \rightarrow D, BA \rightarrow E, BD \rightarrow F, EF \rightarrow B, BE \rightarrow ABC\}$

Which closure of attributes of S does not cover R?

A	B	C	D
A^+	BC^+	BE^+	EF^+

Closure of a set of Functional Dependencies

Closure of the FD Set

- The closure S^+ of a set of FDs S is the set of all FDs that can be infered from S
- Two sets of FDs S, T are equivalent if $S^+ = T^+$
- For speed, we can ignore
  - trivial FDs (*e.g.* ignore $A \rightarrow A$)
  - LHS that are not minimal (*e.g.* ignore $AB \rightarrow C$ if $A \rightarrow C$)
  - flatten all FDs to have just one attribute in RHS (*e.g.* consider $A \rightarrow CD$ as $A \rightarrow C$ and $A \rightarrow D$)
- Apart from calculating equivalence, do not normally need to compute closure

Equivalent FDs

$S = \{A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow D\}$
$T = \{A \rightarrow B, A \rightarrow C, A \rightarrow D, B \rightarrow A\}$
$S^+ = T^+ = \{A \rightarrow B, A \rightarrow C, A \rightarrow D, B \rightarrow A, B \rightarrow C, B \rightarrow D\}$
$\therefore S \equiv T$

Minimal cover of a set of FDs

Minimal cover S_c of S

A minimal cover S_c of FD set S has the properties that:

- All the FDs in S can be derived from S_c ($i.e.$ $S^+ = S_c^+$)
- It is not possible to form a new set S'_c by deleting an FD from S_c or deleting an attribute from an FD in S_c, and S'_c can still derive all the FDs in S

In general, a set of FDs may have more than one minimal cover

Deriving a minimal cover

Suppose $S = \{A \to B, BC \to A, A \to C, B \to C\}$

1 Since $B \to C$
$BC \to A \Rightarrow B \to A$
Leaves $S' = \{A \to B, B \to A, A \to C, B \to C\}$

2_a Since $A \to B, B \to C \models A \to C$
$A \to C \Rightarrow \emptyset$
Leaves $S_c = \{A \to B, B \to A, B \to C\}$

2_b Since $B \to A, A \to C \models B \to C$
$B \to C \Rightarrow \emptyset$
Leaves $S_c = \{A \to B, B \to A, A \to C\}$

Quiz 7: Minimal Cover of a Set of FDs

Given an FD set $S = \{A \rightarrow BC, C \rightarrow D, BA \rightarrow E, BD \rightarrow F, EF \rightarrow B, BE \rightarrow ABC\}$

Which is a minimal cover of S?

A

$A \rightarrow BC, C \rightarrow D, BA \rightarrow E, BD \rightarrow F, EF \rightarrow B, BE \rightarrow ABC$

B

$A \rightarrow BC, C \rightarrow D, BA \rightarrow E, BD \rightarrow F, EF \rightarrow B, BE \rightarrow A$

C

$A \rightarrow BCE, C \rightarrow D, BD \rightarrow F, EF \rightarrow B, BE \rightarrow A$

D

$A \rightarrow BC, C \rightarrow D, B \rightarrow E, B \rightarrow F, EF \rightarrow B, BE \rightarrow A$

Worksheet: Minimal Cover

$R(A, B, C, D, E, F, G, H)$

$S = \{AB \rightarrow DEH, BEF \rightarrow A, FGH \rightarrow C, D \rightarrow EG, EG \rightarrow BF, F \rightarrow BH\}$

1. Rewrite S to an equivalent set of FDs which only have a single attribute on the RHS of each FD.
2. Consider each FD $X \rightarrow A$, and for each $B \in X$, consider if $X \rightarrow B$ from the other FDs. If so, replace $X \rightarrow A$ by $(X - B) \rightarrow A$ in S.
3. Consider each FD $X \rightarrow A$, and compute X^+ without using $X \rightarrow A$. If $A \subseteq X^+$, delete $X \rightarrow A$ since it is rundundant. This will give a minimal cover S_c of S.
4. Justify what are the minimal candidate keys of R constrained by S_c

Worksheet: Minimal Cover (Step 3)

1. $AB^+ = ABDEHGFC$

   Try removing $AB \to D$: find $AB^+ = ABEH$, so can't remove.
   Try removing $AB \to E$: find $AB^+ = ABDHEGFC$, so remove it from S'' to get S'''
   Try removing $AB \to H$: find $AB^+ = ABDEGFHC$, so remove it from S''' to get $S'''' = \{AB \to D, EF \to A, FG \to C, D \to E, D \to G, EG \to B, EG \to F, F \to B, F \to H\}$

2. $EF^+ = EFABHDGC$

   Try removing $EF \to A$: find $EF^+ = EFBH$, so can't remove.

3. $FG^+ = FGCBH$

   Try removing $FG \to C$: find $FG^+ = FGBH$, so can't remove.

4. $D^+ = DEGBFHAC$

   Try removing $D \to E$: find $D^+ = DG$, so can't remove.
   Try removing $D \to G$: find $D^+ = DE$, so can't remove.

5. $EG^+ = EGBFHADC$

   Try removing $EG \to B$: find $EG^+ = EGFBHADC$, so remove it from S'''' to get S'''''
   Try removing $EG \to F$: find $EG^+ = EG$, so can't remove.

6. $F^+ = FBH$

   Try removing $F \to B$: find $F^+ = FH$, so can't remove.
   Try removing $F \to H$: find $F^+ = FB$, so can't remove.
   Thus S''''' is a minimal cover

$$S_c = \{AB \to D, EF \to A, FG \to C, D \to EG, EG \to F, F \to BH\}$$

Normalisation

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Using FDs to Formalise Problems in Schemas

bank_data									
no	sortcode	bname	cash	type	cname	rate?	mid	amount	tdate
100	67	Strand	34005.00	current	McBrien, P.	null	1000	2300.00	1999-01-05
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1001	4000.00	1999-01-05
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107	56	Wimbledon	84340.45	current	Poulovassilis, A.	null	1007	345.56	1999-01-15
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1008	1230.00	1999-01-15
119	56	Wimbledon	84340.45	deposit	Poulovassilis, A.	5.50	1009	5600.00	1999-01-18

Formalise the intuition of redundancy by the statements of FDs

$\mathsf{mid} \rightarrow \{\mathsf{tdate}, \mathsf{amount}, \mathsf{no}\}$,
$\mathsf{no} \rightarrow \{\mathsf{type}, \mathsf{cname}, \mathsf{rate}, \mathsf{sortcode}\}$,
$\{\mathsf{cname}, \mathsf{type}\} \rightarrow \mathsf{no}$,
$\mathsf{sortcode} \rightarrow \{\mathsf{bname}, \mathsf{cash}\}$
$\mathsf{bname} \rightarrow \mathsf{sortcode}$

1st Normal Form (1NF)

Every attribute depends on the key

Quiz 8: 1st Normal Form

bank_data									
no	sortcode	bname	cash	type	cname	rate?	mid	amount	tdate
100	67	Strand	34005.00	current	McBrien, P.	null	1000	2300.00	1999-01-05
101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1001	4000.00	1999-01-05
100	67	Strand	34005.00	current	McBrien, P.	null	1002	-223.45	1999-01-08
107	56	Wimbledon	84340.45	current	Poulovassilis, A.	null	1004	-100.00	1999-01-11
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101	67	Strand	34005.00	deposit	McBrien, P.	5.25	1008	1230.00	1999-01-15
119	56	Wimbledon	84340.45	deposit	Poulovassilis, A.	5.50	1009	5600.00	1999-01-18

$\mathrm{mid} \rightarrow \{\mathrm{tdate}, \mathrm{amount}, \mathrm{no}\}$,
$\mathrm{no} \rightarrow \{\mathrm{type}, \mathrm{cname}, \mathrm{rate}, \mathrm{sortcode}\}$,
$\{\mathrm{cname}, \mathrm{type}\} \rightarrow \mathrm{no}$,
$\mathrm{sortcode} \rightarrow \{\mathrm{bname}, \mathrm{cash}\}$
$\mathrm{bname} \rightarrow \mathrm{sortcode}$

Is bank_data in 1st Normal form?

True

False

Prime and Non-Prime Attributes

Prime Attribute

An attribute A of relation R is **prime** if there is some minimal candidate key X of R such that $A \subseteq X$

Any other attribute $B \in Attrs(R)$ is **non-prime**

Prime and non-prime attributes of bank_data

bank_data(no,sortcode,bname,cash,type,cname,rate,mid,amount,tdate)

Has FDs $\mathsf{mid} \rightarrow \{\mathsf{tdate}, \mathsf{amount}, \mathsf{no}\}$, $\mathsf{no} \rightarrow \{\mathsf{type}, \mathsf{cname}, \mathsf{rate}, \mathsf{sortcode}\}$, $\{\mathsf{cname}, \mathsf{type}\} \rightarrow \mathsf{no}$, $\mathsf{sortcode} \rightarrow \{\mathsf{bname}, \mathsf{cash}\}$, $\mathsf{bname} \rightarrow \mathsf{sortcode}$

Then

1. the only minimal candidate key is mid
2. the only prime attribute is mid
3. non-prime attributes are no,sortcode,bname,cash,type,cname,rate,amount,tdate

3rd Normal Form (3NF)

3rd Normal Form (3NF)

For every non-trivial FD $X \rightarrow A$ on R, either

1. X is a super-key
2. A is prime

Every non-key attribute depends on the key, the whole key and nothing but the key

Failure of bank_data to meet 3NF

bank_data(no,sortcode,bname,cash,type,cname,rate,mid,amount,tdate)

- Has the following FDs where the LHS is not a super-key:
  $\mathsf{no} \rightarrow \{\mathsf{type}, \mathsf{cname}, \mathsf{rate}, \mathsf{sortcode}\}$, $\{\mathsf{cname}, \mathsf{type}\} \rightarrow \mathsf{no}$,
  $\mathsf{sortcode} \rightarrow \{\mathsf{bname}, \mathsf{cash}\}$, $\mathsf{bname} \rightarrow \mathsf{sortcode}$
- Each of the above FD causes the relation not to meet 3NF since the RHS contains non-prime attributes

Quiz 9: Prime and nonprime attributes

Given a relation $R(A, B, C, D, E, F)$ and an FD set
$A \to BCE, C \to D, BD \to F, EF \to B, BE \to A$

What are the nonprime attributes?

A

DEF

B

BC

C

CDF

D

CD

Quiz 10: 3rd Normal Form

Given a relation $R(A, B, C, D, E, F)$ and an FD set
$A \to BCE, C \to D, BD \to F, EF \to B, BE \to A$

Which decomposition is not in 3NF?

A

$R_1(B, D, F), R_2(A, B, C, D, E)$

B

$R_1(A, B, C, E, F), R_2(C, D)$

C

$R_1(A, B, C, E, F), R_2(C, D), R_3(B, D, F)$

D

$R_1(B, E, F), R_2(A, C, E), R_3(C, D)$

Lossless-join decomposition of relations

Lossless-join decomposition of a Relation

A **lossless-join** decomposition of a relation R with respect to FDs S into relations $R_1, \ldots, R_n$ has the properties that:

- $Attrs(R_1) \cup \ldots \cup Attrs(R_n) = Attrs(R)$
- For all possible extents of R satisfying S, $\pi_{Attrs(R_1)} R \bowtie \ldots \bowtie \pi_{Attrs(R_n)} R = R$

Lossless-join decomposition of bank_data

bank_data(no,sortcode,bname,cash,type,cname,rate,mid,amount,tdate)

- Has FDs $\mathsf{mid} \rightarrow \{\mathsf{tdate}, \mathsf{amount}, \mathsf{no}\}$, $\mathsf{no} \rightarrow \{\mathsf{type}, \mathsf{cname}, \mathsf{rate}, \mathsf{sortcode}\}$, $\{\mathsf{cname}, \mathsf{type}\} \rightarrow \mathsf{no}$, $\mathsf{sortcode} \rightarrow \{\mathsf{bname}, \mathsf{cash}\}$, $\mathsf{bname} \rightarrow \mathsf{sortcode}$
- Decomposing bank_data into
  $\mathsf{branch} = \pi_{\mathsf{sortcode},\mathsf{bname},\mathsf{cash}}\ \mathsf{bank\_data}$
  $\mathsf{account} = \pi_{\mathsf{no},\mathsf{type},\mathsf{cname},\mathsf{rate},\mathsf{sortcode}}\ \mathsf{bank\_data}$
  $\mathsf{movement} = \pi_{\mathsf{mid},\mathsf{amount},\mathsf{no},\mathsf{tdate}}\ \mathsf{bank\_data}$
  satisfies the lossless-join decomposition property

Problems if not a lossless-join decomposition

If a decomposition of R into $R_1, \ldots, R_n$ is not lossless, then some tuples spread over $R_1, \ldots, R_n$ can result in phantom tuples appearing

$R(A, B, C, D)$, $S = \{A \rightarrow B, B \rightarrow CD\}$

R

A	B	C	D
1	1	2	6
2	2	3	4
3	3	3	5

⇒

R_1

A	B	C
1	1	2
2	2	3
3	3	3

R_2

C	D
2	6
3	4
3	5

⇒

$R_1 \bowtie R_2$

A	B	C	D
1	1	2	6
2	2	3	4
3	3	3	5
2	2	3	5
3	3	3	4

Quiz 11: Lossless join decomposition

Given a relation $R(A, B, C, D, E, F)$ and an FD set
$A \to BCE, C \to D, BD \to F, EF \to B, BE \to A$

Which is not a lossless-join decomposition of R?

A

$R_1(B, D, F), R_2(A, B, C, D, E)$

B

$R_1(A, B, C, E, F), R_2(C, D)$

C

$R_1(A, B, C, E, F), R_2(C, D), R_3(B, D, F)$

D

$R_1(B, E, F), R_2(A, C, E), R_3(C, D)$

Worksheet: Lossless Join Decomposition

1. $R(A, B, C, D, E)$ has the FDs $S = \{AB \to C, C \to DE, E \to A\}$.
   Which of the following are lossless join decompositions?
   1. $R_1(A, B, C), R_2(C, D, E)$
   2. $R_1(A, B, C), R_2(C, D), R_3(D, E)$
2. Derive a lossless join decomposition into three relations of $R(A, B, C, D, E, F)$ with FDs $S = \{AB \to CD, C \to E, A \to F\}$.
3. Derive a lossless join decomposition into three relations of $R(A, B, C, D, E, F)$ with FDs $S = \{AB \to CD, C \to E, F \to A\}$.

Generating 3NF

Generating 3NF

1. Given R and a set of FDs S, find an FD $X \to A$ that causes R to violate 3NF (*i.e.* for which A is not a prime attribute and X is not a superkey).
2. Decompose R into $R_a(Attr(R) - A)$ and $R_b(XA)$ (Note because the two relations share X and $X \to A$ this is lossless)
3. Project the S onto the new relations, and repeat the process from (1)

Note that step (2) ensures that the decomposition is lossless since joining R_a with R_b will share X, and $X \to A$

Canonical Example of 3NF Decomposition

Suppose $R(A, B, C)$ has FD set $S = \{A \to B, B \to C\}$

- The only key is A, and so $B \to C$ violates 3NF (since B is not a superkey and C is nonprime).
- Decomposing R into $R_1(A, B)$ and $R_2(B, C)$ results in two 3NF relations.

Example: Decomposing bank_data into 3NF

Bank Database as a Single Relation

bank_data(no,sortcode,bname,cash,type,cname,rate,mid,amount,tdate)

$S = \{\mathsf{mid} \rightarrow \{\mathsf{tdate}, \mathsf{amount}, \mathsf{no}\}, \mathsf{no} \rightarrow \{\mathsf{type}, \mathsf{cname}, \mathsf{rate}, \mathsf{sortcode}\},$
$\{\mathsf{cname}, \mathsf{type}\} \rightarrow \mathsf{no}, \mathsf{sortcode} \rightarrow \{\mathsf{bname}, \mathsf{cash}\}, \mathsf{bname} \rightarrow \mathsf{sortcode}\}$

Since $\mathsf{sortcode} \rightarrow \{\mathsf{bname}, \mathsf{cash}\}$ and sortcode is not superkey and $\mathsf{bname}, \mathsf{cash}$ nonprime, we should decompose bank_data into

1. $\mathsf{branch}(\mathsf{sortcode}, \mathsf{bname}, \mathsf{cash})$ with FDs $\mathsf{sortcode} \rightarrow \{\mathsf{bname}, \mathsf{cash}\}$, $\mathsf{bname} \rightarrow \mathsf{sortcode}$
2. $\mathsf{bank\_data}'(\mathsf{no}, \mathsf{sortcode}, \mathsf{type}, \mathsf{cname}, \mathsf{rate}, \mathsf{mid}, \mathsf{amount}, \mathsf{tdate})$ with FDs $\mathsf{mid} \rightarrow \{\mathsf{tdate}, \mathsf{amount}, \mathsf{no}\}$, $\mathsf{no} \rightarrow \{\mathsf{type}, \mathsf{cname}, \mathsf{rate}, \mathsf{sortcode}\}$, $\{\mathsf{cname}, \mathsf{type}\} \rightarrow \mathsf{no}$

branch is in 3NF, but $\mathsf{no} \rightarrow \{\mathsf{type}, \mathsf{cname}, \mathsf{rate}, \mathsf{sortcode}\}$ makes $\mathsf{bank\_data}'$ violate 3NF, so we should decompose $\mathsf{bank\_data}'$ into:

3. $\mathsf{account}(\mathsf{no}, \mathsf{type}, \mathsf{cname}, \mathsf{rate}, \mathsf{sortcode})$ with FDs $\mathsf{no} \rightarrow \{\mathsf{type}, \mathsf{cname}, \mathsf{rate}, \mathsf{sortcode}\}$, $\{\mathsf{cname}, \mathsf{type}\} \rightarrow \mathsf{no}$
4. $\mathsf{movement}(\mathsf{mid.amount}, \mathsf{no}, \mathsf{tdate})$ with FD $\mathsf{mid} \rightarrow \{\mathsf{tdate}, \mathsf{amount}, \mathsf{no}\}$

The relations branch, account, and movement are all in 3NF

Preserving FDs during decomposition

FD preserving decomposition

A lossless decomposition of R with FDs S into R_a and R_b preserves functional dependencies S if the projection of S^+ onto R_a and R_b is equivalent to S

FD preserving decomposition

Suppose $R(ABC)$ with $S = \{A \to B, B \to C, C \to A\}$ is decomposed into $R_a(AB)$ and $R_b(BC)$.

- $S^+ = \{A \to B, A \to C, B \to A, B \to C, C \to A, C \to B\}$
- The projection of S^+ onto R_a gives $S_a^+ = \{A \to B, B \to A\}$
- The projection of S^+ onto R_b gives $S_b^+ = \{B \to C, C \to B\}$
- Note that the union S_u of the two subsets of S^+ (*i.e.* $S_u = S_a^+ \cup S_b^+$) has the property that $S_u^+ = S^+$, and hence the decomposition preserves functional dependencies.

3NF

There is always possible to decompose a relation into 3NF in a manner that preserves functional dependencies. Thus any *good* 3NF decomposition of a relation must also preserve functional dependencies.

Quiz 12: Preserving FDs during Decomposition

Given a relation $R(A, B, C, D, E, F)$ and an FD set
$A \rightarrow BCE, C \rightarrow D, BD \rightarrow F, EF \rightarrow B, BE \rightarrow A$

Which decomposition preserves FDs?

A

$R_1(B, D, F), R_2(A, B, C, D, E)$

B

$R_1(A, B, C, E, F), R_2(C, D)$

C

$R_1(A, B, C, E, F), R_2(C, D), R_3(B, D, F)$

D

$R_1(B, E, F), R_2(A, C, E), R_3(C, D)$

Preserving FDs, lossless join, and 3NF

Given a relation $R(A, B, C, D, E, F)$ and an FD set
$A \to BCE, C \to D, BD \to F, EF \to B, BE \to A$

Decomposition	lossless join	3NF	Preserves FDs
$R_1(B, D, F), R_2(A, B, C, D, E)$	✓	✗	✗
$R_1(A, B, C, E, F), R_2(C, D)$	✓	✓	✗
$R_1(A, B, C, E, F), R_2(C, D), R_3(B, D, F)$	✓	✓	✓
$R_1(B, E, F), R_2(A, C, E), R_3(C, D)$	✗	✓	✗

Decomposing to 3NF

Since it is always possible to decompose a relation into a 3NF form that is both a lossless join decomposition, and preserves FDs, you should always do so.

Quiz 13: Preserving FDs during Decomposition to 3NF

Suppose the relation $R(A, B, C, D, E)$ has functional dependencies $S = \{AC \rightarrow DBE, BC \rightarrow DE, B \rightarrow A, E \rightarrow D\}$ (and hence has minimal keys AC and BC)

Which is a lossless join decomposition to 3NF that preserves FDs?

A

$R_a(B, C, E), R_b(A, B, C), R_c(D, E)$

B

$R_a(A, B, C), R_b(A, C, D, E)$

C

$R_a(A, C, D), R_b(A, C, E), R_c(A, B)$

D

$R_a(A, C, E), R_b(B, D, E)$

Boyce-Codd Normal Form (BCNF)

Boyce-Codd Normal Form (BCNF)

For every non-trivial FD $X \rightarrow A$ on R, X is a super-key.
Every attribute depends on the key, the whole key and nothing but the key

BCNF schema

$\mathsf{branch}(\mathsf{sortcode}, \mathsf{bname}, \mathsf{cash})$ with FDs $\mathsf{sortcode} \rightarrow \{\mathsf{bname}, \mathsf{cash}\}$, $\mathsf{bname} \rightarrow \mathsf{sortcode}$ is in BCNF since sortcode and bname are both candidate keys

$\mathsf{account}(\mathsf{no}, \mathsf{type}, \mathsf{cname}, \mathsf{rate}, \mathsf{sortcode})$ with FDs $\mathsf{no} \rightarrow \{\mathsf{type}, \mathsf{cname}, \mathsf{rate}, \mathsf{sortcode}\}$, $\{\mathsf{cname}, \mathsf{type}\} \rightarrow \mathsf{no}$ is in BCNF since no and $\mathsf{cname}, \mathsf{type}$ are both candidate keys

$\mathsf{movement}(\mathsf{mid.amount}, \mathsf{no}, \mathsf{tdate})$ with FD $\mathsf{mid} \rightarrow \{\mathsf{tdate}, \mathsf{amount}, \mathsf{no}\}$ is in BCNF since mid is key

Decomposition of Relations into BCNF

Generating BCNF

1. Given R and a set of FDs S, find an FD $X \rightarrow A$ that causes R to violate BCNF ($i.e.$ for which X is not a superkey).
2. Decompose R into $R_a(Attr(R) - A)$ and $R_b(XA)$ (Note because the two relations share X and $X \rightarrow A$ this is lossless)
3. Project the S onto the new relations, and repeat the process from (1)

Difference between 3NF and BCNF

Suppose the relation $\mathsf{address}(\mathsf{no}, \mathsf{street}, \mathsf{town}, \mathsf{county}, \mathsf{postcode})$ has FDs $\{\mathsf{no}, \mathsf{street}, \mathsf{town}, \mathsf{county}\} \rightarrow \mathsf{postcode},\ \mathsf{postcode} \rightarrow \{\mathsf{street}, \mathsf{town}, \mathsf{county}\}$,

- The relation is in 3NF (alternative keys $\mathsf{no}, \mathsf{street}, \mathsf{town}, \mathsf{county}$ and $\mathsf{no}, \mathsf{postcode}$).
- The relation is not in BCNF since $\mathsf{postcode} \rightarrow \{\mathsf{street}, \mathsf{town}, \mathsf{county}\}$ has a non-superkey as the determinant
  - Decompose the relation $\mathsf{address}$ on $\mathsf{postcode} \rightarrow \{\mathsf{street}, \mathsf{town}, \mathsf{county}\}$ to:
    $\mathsf{postcode}(\mathsf{postcode}, \mathsf{street}, \mathsf{town}, \mathsf{county})$
    $\mathsf{streetnumber}(\mathsf{no}, \mathsf{postcode})$
  - Note FD $\{\mathsf{no}, \mathsf{street}, \mathsf{town}, \mathsf{county}\} \rightarrow \mathsf{postcode}$ cannot be projected over the relations.

Worksheet: Normal Forms

$$S_c = \{AB \rightarrow D, EF \rightarrow A, FG \rightarrow C, D \rightarrow EG, EG \rightarrow F, F \rightarrow BH\}$$

1. Decompose the relation into 3NF
2. Decompose the relation into BCNF
3. Determine if your decompositions in (1) and (2) preserve FDs, and if they do not, suggest how to amend you schema to preserve FDs.