IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2011

MSc in Computing Science
for Internal Students of the Imperial College of Science, Technology and Medicine

PAPER M1

PROGRAM DESIGN AND LOGIC

Thursday 19 May 2011, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

Section A *(Use a separate answer book for this Section)*

1a Formalise the sentences (i)-(v), below, about *power generating devices*, in predicate logic using '$\leq$' and $<$, if needed, and any of the predicates below:

pgd(X)	to mean X is a power generating device.
sp(X)	to mean X is a solar panel.
wt(X)	to mean X is a wind turbine.
weight(X,Y)	to mean the weight of X is Y.
b(X)	to mean X is a building.
has(X,Y)	to mean X has Y.
sup(X,Y)	to mean X supports maximum weight Y.

i) Power generating devices are either *solar panels* or *wind turbines.*

ii) Solar panels weigh at least 150 kg, and wind turbines weigh at least 100 kg.

iii) There is a solar panel that weighs more than any wind turbine.

iv) Some buildings have solar panels, and some have wind turbines, but none has both.

v) In any building that has a power generating device the weight of the device does not exceed the maximum weight the building can support.

b Using equivalences transform the wff below to an equivalent one where negation occurs only next to predicate symbols:

$$\neg(\forall X\ (p(X) \rightarrow \exists Y\ q(X, Y)) \vee \forall X\ \exists Y\ r(X, a, Y))$$

(To clarify, if the above was asked for the wff $\neg(m \wedge n)$, where m and n are predicate symbols, then the answer would be $\neg m \vee \neg n$.)

c Using inference rules of natural deduction only and the equivalence derived in (b), if required, show that:

$$\neg(\forall X\ (p(X) \rightarrow \exists Y\ q(X, Y)) \vee \forall X\ \exists Y\ r(X, a, Y)) \vdash \exists X \neg q(X, a)$$

At each step of the proof state clearly the wffs and the rule of inference used. Use indentation in your proof for ease of reading. Marks will be taken off for a proof which is presented poorly.

Parts a, b, c carry 70%, 15%, 15% of the marks, respectively.

2 a i) We would like to define a new connective in propositional logic, denoted as $\otimes$, such that $P \otimes Q$ is to be understood as "*P, but not Q*", where P and Q are any wff of propositional logic.
Give a definition for $P \otimes Q$ using any of the other connectives of propositional logic.

ii) Using the definition in (i) and equivalences, show that:
$(A \rightarrow B) \otimes (A \rightarrow C) \equiv (A \otimes C) \otimes (A \otimes B)$.

b i) Formalise the sentences S1-S4, below, in propositional logic, using the propositions: *Cloudy, Sunny, Rains, Snows, Warm, Cold, Windy*, with their obvious intended meanings.

S1 It never snows when it is warm or sunny.
S2 It does not rain if it is sunny, but if it is cloudy it either rains or it snows.
S3 If it is sunny it is warm, provided it is not windy.
S4 It snows only if it is cold.

ii) Give an alternative formalisation for S2 that has at least two occurrences of the new connective $\otimes$, introduced in a(i).

c i) Show $\neg(\neg A \wedge \neg B) \vdash A \vee B$ using only natural deduction rules of inference. Do not use equivalences. At each step of the proof state clearly the wffs and the rule of inference used. It is very important that you use indentation in your proof for ease of reading. Marks will be taken off for a proof which is presented poorly.

ii) Consider the following sentences:

L1 $\neg A \wedge \neg B \rightarrow P$
L2 $A \rightarrow (C \rightarrow P \vee (R \rightarrow N))$
L3 $A \rightarrow (\neg C \rightarrow P \vee (R \rightarrow N))$
L4 $B \rightarrow M \wedge N$
L5 $\neg P \vee S \rightarrow R$

Using only natural deduction rules of inference (and c(i), if required) show that
L1, L2, L3, L4, L5 $\vdash \neg P \rightarrow M \vee N$.

Explain your proof fully. At each step of the proof state clearly the wffs and the rule of inference used. Do not use equivalences. You can use proof by cases, modus tollens, and other derived rules of inference. It is very important that you use indentation in your proof for ease of reading. Marks will be taken off for a proof which is presented poorly.

Parts a, b, c carry 20%, 35%, 45% of the marks, respectively.

Section B (Use a separate answer book for this Section)

3 Suppose you are designing a part of an online gaming platform. The platform hosts a number of games in which players compete with each other, and keeps track of the players' scores.

- Games are identified by their titles, and players by unique logins. Scores are recorded per player per game. All scores are initially zero.
- Initially, all players are marked as *non-cheaters*. A player will be marked as a *cheater* if the system deems appropriate.
- A player can initiate a match by specifying a game and the opponent player. A predefined function 'compete' determines who wins the match. The winning player's score in that game is incremented by 1 if and only if he or she is marked as a non-cheater, otherwise no score is changed.
- For each game, the best player can be retrieved. A player becomes the best player of the game when his or her score *exceeds* that of the previous best player.

a Write C++ class declarations (i.e. no function bodies) to support the above. You may use the following predefined template class and global function:

```
template < class K, class V >
class Map { // a collection of key (of type K) to value (of type V) mappings
public:
    Map();                  // creates an empty map
    void set(K key, V val); // creates/updates map from key to val.
    V get(K key) const;     // returns value mapped from key, or default value
};                          // of type V if nothing is mapped from key.
bool compete(Game* g, Player* a, Player* b); // runs game g for a & b;
                            // returns true if a wins and false if b wins.
```

b Write a test function simulating the following scenario:

- NFS and WOW are two of the games available in the platform. Cycl0n3, Typh00n and Hurr1c4ne are the logins of three of the registered players.
- Cycl0n3 plays NFS with Typh00n, and then Typh00n plays WOW with Hurr1c4ne. Then, Cycl0n3 is marked as a cheater.
- Typh00n's score in WOW is retrieved, and then the best player in NFS is retrieved.

c Write the bodies of the functions of your classes, which are used in part (b) and relate to game playing, score keeping, and best player tracking.

The three parts carry, respectively, 40%, 20%, and 40% of the marks.

4 Consider the following simplified description of the operation of a law firm:

- A law firm has *employees* who carry out work for *clients*.

- Each client has a name. When an employee does a certain number of hours of work for a client, the cost – calculated as the number of hours multiplied by the hourly rate of the employee – is billed to the client. Further, the employee is paid 10% of this amount as a bonus.

- Each employee has a name and is either a *solicitor* or a *barrister*. The hourly rate for solicitors is the same for all solicitors and is set by the Law Society. Currently this rate is £150 per hour. The hourly rate for a barrister depends on the grade of the barrister. A Grade 1 barrister earns £150 per hour, a Grade 2 barrister £200 per hour, and a Grade 3 barrister £400 per hour. Grade 1 and 2 barristers advance to the next grade by passing bar grade advancement examinations.

a Draw a UML class diagram to describe the above.

b Write C++ class declarations (i.e. no function bodies) to support the above.

c Write a test function as follows:

- Solicitor Fiona Shackleton and Grade 2 barrister Nicholas Mostyn are employees of the law firm.
- Paul McCartney and Prince Andrew are clients of the law firm.
- Fiona Shackleton works for Paul McCartney for 10 hours.
- The Law Society decides to adjust the hourly rate for solicitors to be £175 per hour.
- Fiona Shackleton works for Prince Andrew for 5 hours.
- Nicholas Mostyn works for Paul McCartney for 15 hours.
- Nicholas Mostyn passes his bar grade advancement examinations.
- The amount owing to the law firm by Paul McCartney is calculated.
- The bonus due to Fiona Shackleton is calculated.

d Write the bodies of the functions from part (b) that relate to your classes that represent clients, solicitors and barristers.

The four parts carry equal marks.