Solutions to Logic Tutorial 4

Q1a.
1. A∧B   Given
2. A   1, ∧E
3. B   1, ∧E
4. B∧A   2, 3, ∧I

Q1b.
1. A∧B   Given
2. A   1, ∧E
3. A∨B   2, ∨I

Q1c.
1.   P∧Q   Given
2.   P→R   Given
3.   Q→S   Given
4.   P   1, ∧E
5.   Q   1, ∧E
6.   R   2, 4, →E
7.   S   3, 5, →E
8.   R∧S   6, 7, ∧I

Q1d.

1.   P → (Q→R) Given
    2.   P∧Q   Assume
    3.   P   2, ∧E
    4.   Q   2, ∧E
    5.   Q→R   1, 3, →E
    6.   R   4, 5, →E
7.   P∧Q→R   2,6, →I

Q1e.
You have to show
(P→Q) → (¬Q→¬P) and
(¬Q→¬P) → (P→Q).
I will just do the first here.The second is similar.

1. (P→Q)   assume
        2. ¬Q   assume
                3. P   assume
                4. Q   1,3, →E
        5. ¬P   RAA, 3,4,2
6. ¬Q→¬P   →I, 2,5
7. (P→Q) →(¬Q→¬P)   →I,1,6

Q2. Using L for "PM loses next vote", C for "PM's leadership is challenged", E for "PM will call a general election":
    i)   L→(¬C→E)
    ii)   (L∧¬C)→E

Showing i |- ii:
1. L→(¬C→E)   Given
      2. L∧¬C   assume
      3. L   2, ∧E
      4. ¬C→E   1, 3, →E
      5. ¬C   2, ∧E
      6. E   4, 5, →E
7. (L∧¬C)→E   2, 6, →I

Showing ii |- i:
1. (L∧¬C)→E   Given
      2. L   assume
            3. ¬C   assume
             4. L∧¬C   2,3, ∧I
             5. E   1, 4, →E
      6. ¬C→E   3, 5, →I
7. L→(¬C→E)   2, 6, →I

Q3.
a. Murderer
b. Formalising the information:
1. Murderer∨Blackmailer
2. Murderer→Violent
3. Blackmailer→Rich
4. Rich→Spends∨Account
5. ¬Account
6. ¬Spends

Abbreviate to:
1. M∨B   Given
2. M→V   Given
3. B→R   Given
4. R→S∨A   Given
5. ¬A   Given
6. ¬S   Given

Deriving M:
      7. B   assume
      8. R   3, 7, →E
      9. S∨A   4, 8, →E
      10. S   9, 5, ∨E
11. ¬B   7, 10, 6, RAA
12. M   1, 11, ∨E

Q4

a) Showing $\neg(p \rightarrow q) \equiv p \wedge \neg q$

$\neg(p \rightarrow q) \equiv \neg(\neg p \vee q) \equiv \neg\neg p \wedge \neg q \equiv p \wedge \neg q$

b)
I will use (a) and also
lemma 1 (I leave the proof of lemma 1 to you.):
$A, A \wedge \neg B \rightarrow C \vdash \neg B \rightarrow C$

1. $A \wedge \neg B \rightarrow C$	Given	
2. $B \rightarrow C$	Given	
3. $C \rightarrow \neg(B \rightarrow A)$	Given	
4. $A \vee B \vee C$	Given	
5. A	Assume	
6. $\neg B \rightarrow C$	1, 5, lemma1	
7. C	2, 6, dilemma	
8. $\neg(B \rightarrow A)$	3, 7, $\rightarrow$E	
9. $B \wedge \neg A$	8, (a)	
10. $\neg A$	9, $\wedge$E	
11. $\neg A$	5, 10, RAA	
12. $B \vee C$	4, 11, $\vee$E	
13. B	assume	
14. C	2, 13, $\rightarrow$E	
15. $B \wedge C$	13, 14, $\wedge$I	
16. $B \rightarrow B \wedge C$	13, 15, $\rightarrow$I	
17. C	assume	
18. $\neg(B \rightarrow A)$	17, 3, $\rightarrow$E	
19. B	18, (a), $\wedge$E	
20. $B \wedge C$	17, 19, $\wedge$I	
21. $C \rightarrow B \wedge C$	17, 20, $\rightarrow$I	
22. $B \wedge C$	12, 16, 21, proof by cases	