IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2013

MSc in Computing Science
for Internal Students of the Imperial College of Science, Technology and Medicine

PAPER M1

PROGRAM DESIGN AND LOGIC

Monday 13 May 2013, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

Section A *(Use a separate answer book for this Section)*

Note: All natural deduction proofs must be presented clearly, with wff numbering, where appropriate, indentations, and explanations. Marks will be deducted for unclear and poorly presented proofs. When using natural deduction you may use any of the primitive and derived rules.

1 a Define a new connective $\otimes$ for *exclusive-or*, using any (combination) of the usual connectives, $\wedge$, $\vee$, $\neg$, $\rightarrow$, $\leftrightarrow$. Thus $p \otimes q$ is to mean either p or q but not both.

b Use the new connective $\otimes$ together with any of the other usual connectives to express the following sentences in propositional logic, where *either ... or* is to be understood as *exclusive-or*. The propositions to be used are given in the text in Italics inside brackets.

> Either John will leave the company *(jL)* or Mary will *(mL)*. If John leaves then either the tax department will close *(closeTax)*, or Peter will be shared between two departments *(pShare)* and an administrator will be recruited *(recruitA)*. If Mary leaves then either an administrator will be recruited or a secretary will be recruited *(recruitS)*, provided John is shared between two departments *(jShare)*.

c Show $p \otimes q, \neg(p \otimes r) \vdash q \rightarrow \neg r$, using only the rules of inference of natural deduction and the definition of $\otimes$ (i.e. wherever you have a sub-formula of the form $m \otimes n$ you can replace it by the definition you have provided in (a)).

d Show $p \otimes q, \neg(p \otimes r) \vdash p \rightarrow (\neg q \wedge r)$, using any combination of equivalences and the rules of inference of natural deduction and the definition of $\otimes$.

e Consider the equivalences alleged below:

Eq1 $\quad p \leftrightarrow q \equiv \neg(p \otimes q)$

Eq2 $\quad (p \vee q) \otimes (p \vee r) \equiv \neg p \wedge (q \otimes r)$

Eq3 $\quad p \wedge q \leftrightarrow r \equiv (p \otimes q) \leftrightarrow r$

i) Of these three alleged equivalences two hold and one does not. Which one does not hold? By an assignment of truth values to p, q, r show that your chosen equivalence does not hold.

ii) Of the two equivalences that do hold prove the one that has fewer occurrences of the $\leftrightarrow$ connective, using the definition of $\otimes$ and any techniques you wish except truth tables. (If you use rules of inference they must be natural deduction rules.)

Parts a, b, c, d, e carry 10%, 15%, 20%, 20%, 35% of the marks, respectively.

2 a Companies Registration House (CRH) keeps a record of registered companies and a register of disqualified directors. Formalise in predicate logic the sentences (i)-(iv), below, that govern the activities of CRH. Use any of the predicates listed below and $<, \leq, =$ that you can use both for comparing dates and for comparing share values, and in the case of $=$ for comparing names as well.

Ensure that you present your formulas clearly, using brackets to correctly identify the scope of quantifiers and disambiguate where necessary.

reg(C)	*to mean C is a registered company.*
dir(D, C)	*to mean D is the director of company C.*
reqReg(ID, C, D, S)	*to mean request ID is for registering a company by the name C, with director D and share capital S.*
reject(ID)	*to mean the request identified by ID for a company registration is rejected.*
accept(ID)	*to mean the request identified by ID for a company registration is accepted.*
disq(D)	*to mean D is disqualified.*
insolvent(C)	*to mean C is insolvent.*
share(C,Value)	*to mean the share capital of company C is set to Value.*
acc_due(C, P, D)	*to mean the accounts of company C for period P are due on date D.*
acc_filed(C, P, D)	*to mean the accounts of company C for period P are filed on date D.*
warn(D, C)	*to mean director D is warned about company C.*

i) Every registered company has exactly one director.

ii) Any request for registering a company is rejected if a company of that name is already registered, or if the director is disqualified, or if the share capital of the company is less than £5000, otherwise the request is accepted.

iii) A director is disqualified and the share capital of all of his registered companies is set to zero if at least one of his registered companies becomes insolvent.

iv) The director of a registered company who is not disqualified is warned if on at least two occasions the company's accounts have not been filed by the due deadlines.

b i) Show $\forall X\,(p(X) \rightarrow q(X)),\ \neg\,\exists Y\,q(Y) \vdash \neg\,\exists X\,p(X)$ using only the rules of inference of natural deduction.

ii) Given sentences *I-V*, below, show *I, II, III, IV, V* $\vdash \forall X\,(s(X) \rightarrow \neg r(X))$ using only the rules of inference of natural deduction and 2b(i), above, if needed.

I	$\forall X\,(p(X) \rightarrow q(X))$	*II*	$\forall X\,(s(X) \rightarrow m(X) \vee n(X))$
III	$\neg\,\exists X\,q(X)$	*IV*	$\forall X\,(n(X) \rightarrow \forall Y\,(r(Y) \rightarrow p(Y))$
V	$\exists X\,r(X) \rightarrow \neg\exists Y\,m(Y)$		

Parts a, b carry 60%, 40% of the marks, respectively.

Section B (Use a separate answer book for this Section)

3 Consider the following description of a shop which sells and services bicycles:

- Each bicycle has a (unique) frame identifier, a make and a price. Each customer has a name and an account balance. Customers start with an account balance of zero but can deposit money to increase their account balance.
- The shop keeps a record of each bicycle sold to customers, and of the number of times each bicycle has been serviced. The first service is free and the second incurs a charge of £20. All subsequent services are charged at £40 each.
- When a customer attempts to buy a bicycle, the customer's account balance is compared to the price of the bicycle. If the account balance is less than the price of the bicycle, the sale does not take place. Otherwise the sale takes place and the account balance is reduced by the price of the bicycle.
- When a customer brings a bicycle to the shop for a service, the service is only performed if the bicycle was sold by the shop, and the customer has a large enough account balance to pay the charge for the service (if there is one).

You may assume the availability of the following template class:

```
template <class T1, class T2>
class Table { // lookup table associating T1s to T2s
  ...
public:
  Table(); // an empty table
  void set(const T1 &x, T2 y);
    // inserts an association from x to y into the table
  T2 *get(const T1 &x);
    // returns a pointer to the T2 associated with x
    // returns NULL if there is no associated element
};
```

a Write C++ class declarations (i.e. no function bodies) to support the above.

b Write a test function where:

- Customer Lance is created. Lance deposits £2000 into his account.
- Bicycle b1, with frame identifier "TK15895", make "Trek" and price £475 is created. Bicycle b2, with frame identifier "SS82300", make "Specialized" and price £1600 is created.
- Lance attempts to buy b1. Lance attempts to buy b2.
- Lance takes b1 for a service. Lance takes b2 for a service.

c Write function bodies for your classes (excluding the template class).

The three parts carry, respectively, 35%, 30%, and 35% of the marks.

4 Consider the following description of a Women's Heptathlon competition:

- Each heptathlon competitor has a name and an age. The competitors take part in seven events in the following (strict) order: 100 metre hurdles, high jump, shot put, 200 metres, long jump, javelin throw and 800 metres.
- Competitors earn points in each event according to their performance. The total points earned by a competitor can be computed at any time.
- The events can be classified into three main types:
  - Running events (100 metre hurdles, 200 metres, 800 metres). Completing a running event in time of T seconds earns $a(b - T)^c$ points.
  - Jumping events (high jump and long jump). Achieving a height/length of M centimetres earns $a(M - b)^c$ points.
  - Throwing events (shot put and javelin). Throwing a distance of D metres earns $a(D - b)^c$ points.
- a, b and c have different values for each event as follows:

Event	a	b	c
100 metre hurdles	9.23	26.7	1.835
High jump	1.85	75	1.348
Shot put	56.02	1.5	1.05
200 metres	4.99	42.5	1.81
Long jump	0.1889	210	1.41
Javelin throw	15.98	3.8	1.04
800 metres	0.112	254	1.88

a Draw a UML class diagram to describe the above.

b Write C++ class declarations (i.e. no function bodies) to support the above.

c Write a test function as follows:
- Jessica Ennis is a 28 year old heptathlon competitor.
- Jessica runs the 100 metre hurdles in 12.54 seconds, achieves 186 centimetres in the high jump, throws 14.28 metres in the shot put and runs the 200 metres in 22.88 seconds.
- Jessica's points total is computed.
- Jessica leaps 648 centimetres in the long jump, throws 47.49 metres in the javelin throw, and runs the 800 metres in 2 minutes 8.65 seconds.
- Jessica's points total is computed.

d Write the bodies of the functions from part (b).

The four parts carry, respectively, 25%, 30%, 20%, and 25% of the marks.