IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2013

BSc Honours Degree in Mathematics and Computer Science Part II
MSci Honours Degree in Mathematics and Computer Science Part II
MSc in Computing Science
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER MC202

SOFTWARE ENGINEERING - ALGORITHMS

Wednesday 8 May 2013, 14:30
Duration: 75 minutes
(Reading time 5 minutes)

Answer ALL TWO questions

Paper contains 2 questions
Calculators not required

Section A (Use a separate answer book for this Section)

1 a Consider the problem of computing *set difference* (also called *relative complement*) as a primitive operation (or method) for a data structure used to store sets. The set difference \ of two sets a and b is defined as follows:

$$a \backslash b = \{x \in a | x \notin b\}$$

i) Assume the elements of the sets are stored using the Skip list data structure. Write the pseudo-code (***neatly and readably!***) for the algorithm `Diff(a,b,c)`, where the set difference of Skip lists `a` and `b` is returned in Skip list `c`. *(Note: You cannot assume the presence of any sort of iterator, print, or dump method.)*

ii) Give a short description of your algorithm, including a characterization of its asymptotic run-time complexity.

b This question concerns search tries.

i) Draw a standard trie that contains these strings: APE, APPEND, APPLE, CALF, CAT, CATBIRD, CATFISH, CATFOOD, COW, COWBELL, COWBIRD.

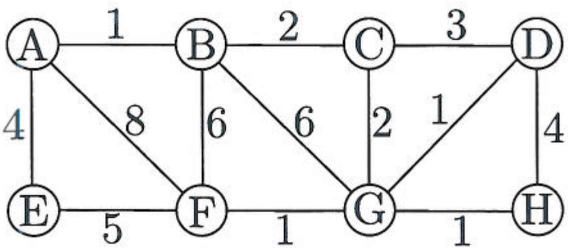
ii) Draw an optimized trie that contains these strings: APE, APPEND, APPLE, CALF, CAT, CATBIRD, CATFISH, CATFOOD, COW, COWBELL, COWBIRD.

iii) Given n non-fixed-length keys (i.e., strings) drawn from an alphabet Σ, what is the worst case complexity of search in a basic TST? Explain your answer. Your explanation should include a sketch of an example illustrating the worst case.

The two parts carry equal marks.

Section B (Use a separate answer book for this Section)

2a Consider the following graph,



Compute the Minimum Spanning Tree (MST) of the graph above using Prim's algorithm. Whenever there is a choice of nodes, always use alphabetic ordering (i.e. start from node A). Draw a table showing the edge included and the cost at each iteration of the algorithm.

b The greatest common divisor of two positive integers a, and b with $a \geq b > 0$ is defined as the largest integer which divides both numbers, and is denoted by $\text{gcd}(a,b)$. It can be shown that the following rule is true,

$$\text{gcd}(a,b) = \begin{cases} 2\text{gcd}(a/2, b/2) & \text{if } a \text{ and } b \text{ are even} \\ \text{gcd}(a, b/2) & \text{if } a \text{ is odd}, b \text{ is even} \\ \text{gcd}((a-b)/2, b) & \text{if } a \text{ and } b \text{ are odd.} \end{cases}$$

Use the result above to design a divide-and-conquer algorithm that computes the greatest common divisor between two positive integers.

c Use dynamic programming to design an algorithm that computes the probability to obtain exactly k heads when n biased coins are tossed independently. The input to the algorithm are the two integers n, k, and $p_1, \ldots, p_n \in [0,1]$ where p_i is the probability that the ith coin comes up heads. Show that your algorithm runs in $O(nk)$. You may assume that multiplication and addition of two numbers in $[0,1]$ take $O(1)$ time.

d For the same task as in part (c) design a divide-and-conquer algorithm that takes $O(n\log^2(n))$ time.

The four parts carry, respectively, 20%, 20%, 30%, and 30% of the marks.