

Predicate Logic

Some Exercises in Semantics

With my thanks to everyone who has
taught logic in the Department

Assume the Universe of integers ≥ 1

For each of the following decide if it is true or false.

1. $\exists X$ (X is the product of two odd integers)

$$\exists X, Y, Z (X=Y*Z \wedge \text{odd}(Y) \wedge \text{odd}(Z))$$

2. $\forall X$ (X is the product of two odd integers)

$$\forall X \exists Y, Z (X=Y*Z \wedge \text{odd}(Y) \wedge \text{odd}(Z))$$

3. $\forall Y \exists X$ (X is an even factor of Y)

$$\forall Y \exists X, Z (Y=X*Z \wedge \text{even}(X))$$

4. $\forall X \exists Y (Y > X)$

5. $\forall X \forall Y (X * Y \geq X)$

6. $\exists Y \forall X (Y \leq X)$

7. $\exists Y \forall X (X \leq Y)$

8. $\forall X (\exists Y (X = Y^2) \rightarrow \text{even}(X))$

9. $\forall X (\forall Y \neg (X = Y^2) \rightarrow \text{odd}(X))$

10. $\forall X (\neg \forall Y (X \text{ divides } Y) \rightarrow \neg (X = 1))$

Answers

1. True. Take $15=3*5$.
2. False. Take 20.
3. False. Take 15.
4. True. For every X take $X+1$.
5. True.
6. True. $Y=1$.
7. False.

8. False. Take $X = 25$.

$25 = 5^2$, but 25 is not even.

9. False. Take 24.

10.

$\forall X (\neg \forall Y (X \text{ divides } Y) \rightarrow \neg (X=1)) \equiv$

$\forall X (X=1 \rightarrow \forall Y (X \text{ divides } Y))$

True.