IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2012

BSc Honours Degree in Mathematics and Computer Science Part II
MSci Honours Degree in Mathematics and Computer Science Part II
MSc in Computing Science
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER MC202

SOFTWARE ENGINEERING - ALGORITHMS

Thursday 10 May 2012, 14:30
Duration: 75 minutes
(Reading time 5 minutes)

Answer ALL TWO questions

Paper contains 2 questions
Calculators required

Section A (Use a separate answer book for this Section)

1 a i) Complete the following sentence: "For a given algorithm, Big Omega (Ω) characterizes the ..."

ii) Is the following claim true or false? Explain your answer.

Let u be a string of length m and v a string of length n. Then the Boyer-Moore method to determine whether u is a substring of v is asymptotically faster than the brute force method.

iii) Suppose that the Knuth-Morris-Pratt algorithm is searching for the pattern "senses" in some larger text, and that at some point in the search it has matched the first five characters "sense" but finds a mismatch in the next position. How many units would the algorithm shift? Show fully how you came to this number.

b For each of the following, argue why or why not an adversary can elicit a worst-case behavior (assuming the input values are unique):

i) Ordinary Quicksort

ii) Randomized Quicksort

iii) Hash table

iv) Skiplist

v) Digital search tree

vi) Existence trie

c Let A be an array containing n elements, all distinct. The successor of an element x in A is the element y such that x is smaller than y but no element z is strictly between x and y. Every element except the maximum has a successor.

i) Explain how to find the successor of x in $O(n)$ time whether A is a sorted array or not.

ii) Suppose you have a table allowing you to find the successor of any element in $O(1)$ time. Explain how to sort A in $O(n)$ time.

d Draw a suffix tree for the text "senses". (Keys start on character boundaries.)

The four parts carry, respectively, 25%, 30%, 30%, and 15% of the marks.

Section B (Use a separate answer book for this Section)

2 Throughout this question, $G = (V; E)$ is a flow network with capacity function c, source vertex s and sink vertex t.

Consider the specific flow network G with vertex set $V = \{s, \alpha, \beta, \gamma, \delta, t\}$ and with capacities c and flows f as given in the following tables:

c	s	α	β	γ	δ	t
s	0	9	3	0	0	0
α	0	0	2	0	7	0
β	0	0	0	6	0	0
γ	0	0	0	0	3	6
δ	0	0	0	0	0	5
t	0	0	0	0	0	0

f	s	α	β	γ	δ	t
s	0	5	3	0	0	0
α	−5	0	2	0	3	0
β	−3	−2	0	5	0	0
γ	0	0	−5	0	2	3
δ	0	−3	0	−2	0	5
t	0	0	0	−3	−5	0

a Draw the residual network associated with this network and flow, explaining how you arrived at it.

b Using the residual network from part (a), write down the (unique) augmenting path in G for the given flow. What is the maximum extra flow that can be routed along this augmenting path?

c Draw the flow in G that results from routing the maximum possible additional flow through the augmenting path found in part (b). Identify a cut in G that proves (via the min-cut/max-flow theorem) that the flow is maximal.

d In general a network has many maximum flows. In this case, however, there is just one. Demonstrate, using an ad-hoc (though clear and concise) argument, that the flow constructed in part (c) is unique. (Certain edges must carry exactly the flows they do. Now fix the flows in these edges and argue that some further edges must carry the flows they do, and so on.)

e It can be shown that a maximum flow f is unique if and only if the associated residual network has no non-trivial simple cycles. (A cycle is simple if it has no repeated vertices; it is "non-trivial" if its length is greater than two.) Construct the residual network for the maximum flow found in part (c) and verify that it has no non-trivial simple cycles.

The five parts carry, respectively, 25%, 20%, 20%, 15%, and 20% of the marks.