Logic Tutorial 2

1. An Exercise taken from Year 1 Logic – with my thanks to Prof. Ian Hodkinson

Consider a set of objects labelled A, B, C, … placed on a 3*3 grid, and the following atomic formulas talking about the objects:

[x next-to y] means (that is, it is true if) x and y are adjacent (horizontally or vertically, but not diagonally);

[x sees y] means x and y are in the same row or the same column;

[x left-of y] means x is in a column to the left of the column of y;

[x above y] means x is in a row above the row of y.

(a) For the placements shown in figure below, which of the following evaluate to true, and why?

i. [A sees B] $\leftrightarrow$ [B sees C]
ii. [B next-to D] $\vee$ [B next-to E]
iii. $\neg$ ([A left-of F] $\wedge$ [F above A])
iv. $\neg$ ([E left-of D] $\rightarrow$ $\neg$[D next-to C]) $\rightarrow$ $\neg$[A sees E]
v. ([E sees D] $\vee$ [F sees E]) $\rightarrow$ $\neg$([B above E] $\leftrightarrow$ [B next-to C])

A		D
C	F	B
	E	

(b) Place the 6 objects A, … , F on the grid so that all the formulas above are true.

2. For each of the following determine if it is a tautology, inconsistency or contingency by drawing the truth table.

a. $P \wedge (P \vee Q)$
b. $(P \vee Q) \wedge (P \rightarrow Q)$
c. $Q \wedge \neg P \wedge (P \vee (Q \rightarrow P))$
d. $(P \wedge (Q \vee P)) \leftrightarrow P$
e. $(P \rightarrow Q) \rightarrow (\neg P \vee Q)$
f. $((P \rightarrow Q) \wedge (R \rightarrow S) \wedge (P \vee R)) \rightarrow (Q \vee S)$