Dynamic Data Structures

Dr Timothy Kimber

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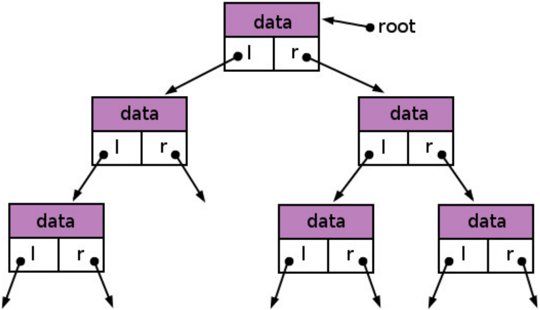


Dynamic Data Structures

Having efficient data structures is crucial for successful algorithms.

- The problems seen so far involved fixed length lists
- In most languages we have a simple way to implement this efficiently — arrays
- Our algorithms assumed some sort of array type was available

Other problems require dynamic data structures such as

- Lists, Stacks and Queues
- Sets and Dictionaries

These are designed to hold variable, essentially unlimited amounts of data.

Ordered Data Structures

A *list* is an ordered collection of {nodes, items, elements}.

- The key property of a list is the ordering of the nodes
- A list might support operations such as

push	adds an element to the end of the list
pop	removes the last element of the list
shift	removes the first element of the list
unshift	adds an element to the front of the list
insert	adds an element at a given position
remove	removes the element at a given position
iterate	returns the items in order

- Plus sorting, searching, copying, joining, splitting ...
- The most appropriate implementation depends on which operations are needed.

Stacks

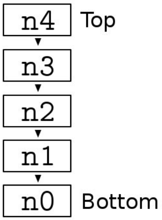
A *stack* is a last-in first-out (LIFO) list.

- Stacks support only

  push for adding elements

  pop for removing elements

- Stacks are usually pictured as a vertical (stacked!) structure



- Stacks support recursive algorithms including fundamental operations such as calling subprocedures and evaluating arithmetic expressions

Stacks

Question

How would you implement a stack?

- Must be able to add “unlimited” objects
- Push and Pop must implement LIFO behaviour

Performance of Push

Question

Given a stack containing N objects, what is the worst case time complexity of push?

- Assume: time to insert (copy, add) one object to array is c
- Assume: initial capacity is 4

Performance of Push

Revised Question

Given an empty stack, what is the worst case time to push N objects?

- Assume: initial capacity is 4
- Assume: time to insert (copy, add) one object to array is c

Amortisation

The time for N pushes is $O(N)$, so:

- A single push is *effectively* a constant time operation
- More correctly: `push` is amortised $\Theta(1)$
- NOT the same as $\Theta(1)$

Amortisation

- Related to accountancy method used to defer large costs
- Amortised analysis considers a sequence of operations
- Cost of individual ops is "amortised" across the sequence
- Unlike accountancy, must never be in debt

Amortised Analysis

Rather than calculating cost of full sequence of N steps we can

- Pick a representative subsequence
- Subsequence is some "cycle" that repeats
- Pick an amortised cost for operations
- Show that paying amortised cost covers all costs (never in debt)

Exercise

Find a representative cycle (subsequence) of pushes into the stack and show that the amortised cost of $3c$ covers all costs.

Amortised Analysis

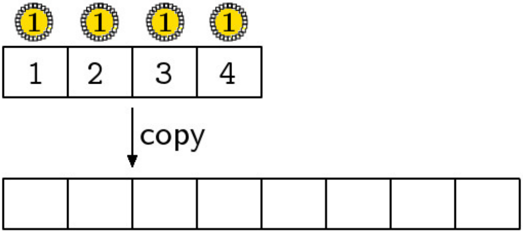
Begin cycle when ...

End cycle when ...

Amortised Analysis

Argument only works because array is initially empty and size is doubled

- Say we have N objects on stack after a copy
- Before next copy we always push N more
- This is how cost is covered



- Multiplying by any factor will do - will affect amortisation constant

Queues

A *queue* is also a list, but the next object removed is either:

- The earliest one added (FIFO Queue)
- The one with highest priority (Priority Queue)

Questions

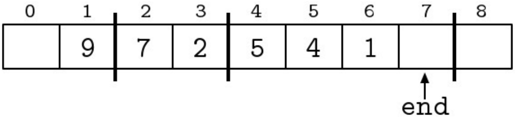
- How could you implement a priority queue (PQ)?
- Given a PQ following your design that contains N objects, what would be the worst case time to add a new object? (Each object has a key attribute that determines its priority.)

Priority Queue Design

Heap: a Tree in an Array

We want to know where the "end" of the tree is:

- Build a tree within an array

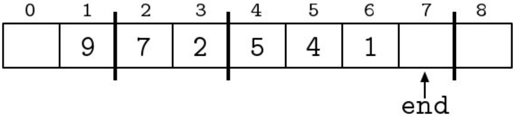


- Track end using "stack pointer"
- Navigate by indices
- Leaving `a[0]` blank means:
    - parent of `a[n]` is `a[n/2]`
    - children of `a[n]` are `a[2*n]` and `a[2*n+1]`

Exercise

How should a new object be added to a max binary heap? (i.e. the greatest key should be at the root).

Heap: a Tree in an Array

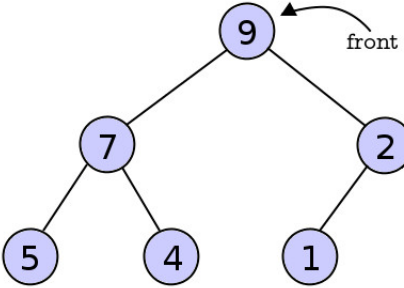


- Track end using "stack pointer"
- Leaving a[0] blank means:
  - parent of a[n] is a[n/2]
  - children of a[n] are a[2*n] and a[2*n+1]

Exercise

How should the object with the greatest key be removed from a max binary heap?

Binary Heap Performance



Question

Given a heap containing N objects, what is the time complexity for adding or removing one object?

Heapsort

Heaps also provide us with the Heapsort algorithm (JWJ Williams, 1964)

Heapsort (given a list L)

- Create an empty heap H
- Remove each element of L and add it to H
- Remove each element of H and add it to L
- HALT

- What could be simpler?!
- Performance is again $\Theta(N log_2 N)$
- Can also be implemented in place by setting up list and heap partitions within a single array

Sets

A *set* is an unordered collection of objects each having a unique key.

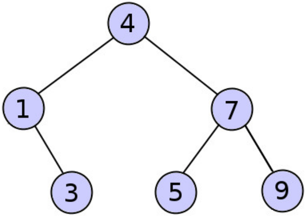
- Should have "unlimited" capacity
- Want to put and get by key
- A key could be any type that defines $<$ and $=$

Questions

- How could you implement a set?
- Given a set following your design that contains N objects, what would be the worst case time to get the object with key k?

A Search Tree?

A tree will divide the data but need a different ordering



- Start at the root (it's a tree)
- Go right: find/add larger keys
- Go left: find/add smaller keys

Binary Search Tree

In a Binary Search Tree

- Go right: find/add larger keys
- Go left: find/add smaller keys

Exercise

- Draw the (integer) binary search tree implied by the following code:

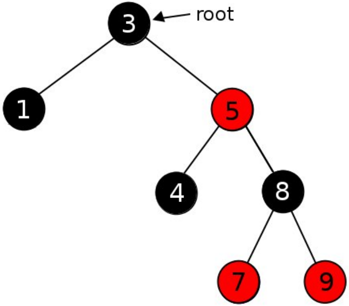
```
bst = new BST
keys = [5,3,10,1,6,9,8,0,4]
for i = 0 to 8
  bst.put(keys[i])
```

- What is the worst case time complexity of the put procedure?

Red-Black Trees

Red-Black Trees are binary search trees that maintain balance

- A BST can become (very) unbalanced, resulting in long branches
- Searching a BST takes $O(N)$ time in the worst case
- The branches of a balanced tree remain as short as possible



Red-Black Tree Properties

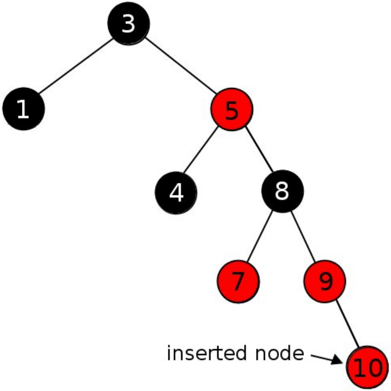
Definition (Red-Black Tree)

A binary search tree T is a red-black tree iff T satisfies the following five properties:

1. All nodes (including nulls) are either red or black
2. The root node is black
3. Every leaf (all null) is black
4. Both children of a red node are black
5. All paths from a node to a descendant leaf contain the same number of black nodes

Insertion

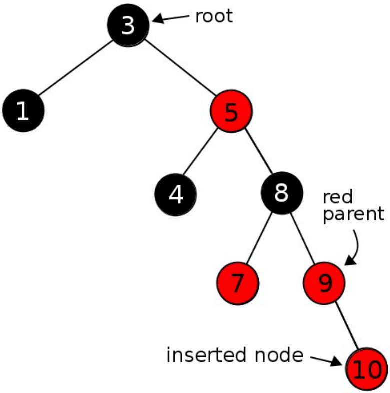
A node is inserted using the ordinary BST procedure



- A new node is always colored red

Insertion

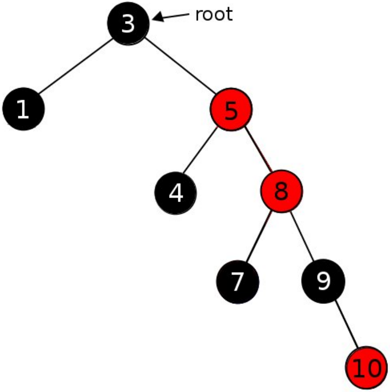
The insertion may result in a violation of the red-black tree properties



- The root might be coloured red
- A red node might have a red child

Insertion

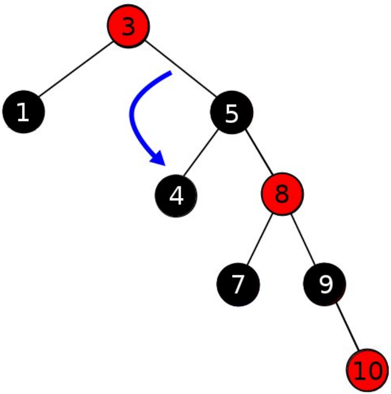
Either recolour $\Theta(1)$ nodes



- There is still a red node with a red parent
- The problem has moved closer to the root (continue)

Insertion

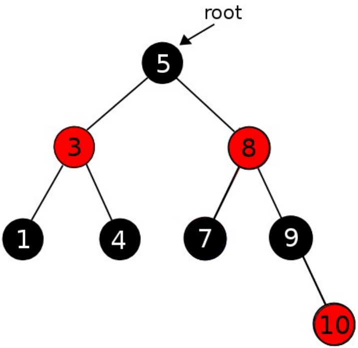
Or perform a rotation of $\Theta(1)$ nodes and Stop



- Reduces height of the tree
- Preserves key ordering

Insertion

The properties are restored



Performance

By maintaining the red-black tree properties, we have $h \leq 2log_2(N + 1)$

- Get procedure is the same as for BST
- Height constraint means it is now $O(\log_2 N)$

For Put, only the last part is different

- The extra work is still localised to one branch
- So, Put also runs in $O(log_2 N)$ time