

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2016

MSc in Computing Science
for Internal Students of the Imperial College of Science, Technology and Medicine

PAPER C580

ALGORITHMS

Thursday 5 May 2016, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1 a For a worst case input of size N , Algorithm X has $\Theta(N)$ time complexity.

- i) Give a full explanation of the statement above.
- ii) What can you deduce about the time complexity of Algorithm X for any input?
- iii) Using the formal definition of Big Theta, show whether

$$7N + 5 = \Theta(2N)$$

is true or not.

b Describe the space complexity of Quicksort, Merge Sort and Counting Sort.

c Given an array A of integers, the *maximum sub-array sum* problem, is to find the value of the greatest sum

$$A_i + \dots + A_j$$

where $[A_i, \dots, A_j]$ can be any (contiguous) sub-array of A . The problem can be decomposed into subproblems as follows. If S_k is defined to be the maximum sum of those sub-arrays that *end at index k* , then S_k is the greater of:

$$\begin{aligned} &A_k \\ &S_{k-1} + A_k \end{aligned}$$

Write a $\Theta(N)$ algorithm which, given an array A of N integers, calculates the maximum sub-array sum of A . You can use either pseudocode or Java.

The three parts carry, respectively, 40%, 20%, and 40% of the marks.

- 2a An application needs to be able to search for a given object x in a set S .
- The average time to search the set for an object should be constant (i.e. independent of the size of S). What type of data structure would you use to store S ? Explain your answer.
 - Describe the insert procedure, including any steps required to ensure the average time for search remains constant. What would be the (time) performance of insert?
 - Suppose the objects in S are strings, and a search should return all strings in S that are *anagrams* of a string x . Describe how you would store the data in this case, and what changes you would need to make to the search and insert procedures.
- b The following set of linear programming equations, where x_1 and x_4 are basic variables and x_2 and x_3 are non-basic variables, represent a stage of the Simplex algorithm. The objective of the problem is to find a solution that maximizes z .

$$\begin{aligned} z &= 12000 - 25x_2 + 350x_3 \\ x_1 &= 40 - \frac{1}{12}x_2 - \frac{1}{2}x_3 \\ x_4 &= 250 - \frac{5}{12}x_2 - \frac{25}{2}x_3 \end{aligned}$$

- What is the basic solution given by these equations?
- Which of the non-basic variables x_2 and x_3 should be selected for the next pivot of the algorithm? Justify your answer.
- Which of the basic variables x_1 and x_4 should be selected for the next pivot of the algorithm? Justify your answer.

The two parts carry, respectively, 70% and 30% of the marks.

- 3 a The following keys are added to an empty binary search tree in order: 4, 6, 23, 7, 2, -8, 46, 36, 5. Draw the resulting tree.
- b You are implementing a binary search tree that allows duplicate keys. In such a tree both the left and the right subtree of a node may contain other nodes with the same key. A tree is composed of node objects that have public fields *left*, *right* (subtrees) and *key* (an integer). The tree has a public field *root* (the root node).
- i) Write a procedure ADD that adds a new node x to such a binary search tree T . You can use either pseudocode or Java.
 - ii) Describe the (runtime) performance of your ADD procedure given inputs with and without duplicate keys.
- c An application searches English language “texts” (strings) for occurrences of certain “patterns” (also strings) using the Knuth–Morris–Pratt algorithm.
- i) Write out a table containing the Knuth–Morris–Pratt π function for the pattern “amanadamanages”.
 - ii) How will the input text and input pattern affect the running time?
 - iii) If the same application was used with texts and patterns both composed from the alphabet $\mathcal{A} = \{0, 1\}$, how would this affect your answer to (ii)?
 - iv) How would the input pattern and the input text affect the running time of the application if it used the Boyer–Moore algorithm instead? Compare the English language and the $\mathcal{A} = \{0, 1\}$ cases.

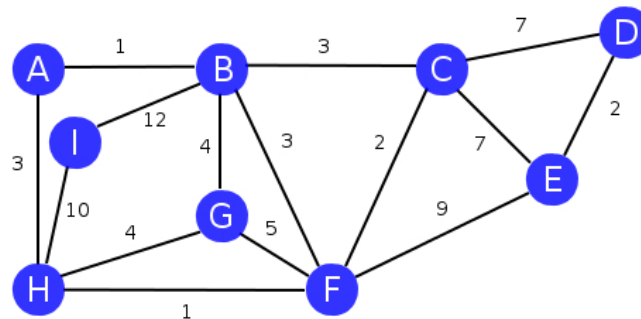
The three parts carry, respectively, 15%, 40%, and 45% of the marks.

- 4a Write a $O(V + E)$ algorithm to find the number of paths from a vertex s to a vertex t in a directed acyclic graph $G = (V, E)$. HINT — the number of paths from vertex u to t is the sum of:

- 1 for each edge (u, t) , plus
- the number of paths from v to t for each other edge (u, v) .

Your algorithm can be in pseudocode or Java. You can assume the existence of a function $G.adj(u)$ that returns the list of vertices adjacent to vertex u .

- b i) Use Kruskal's algorithm to find a minimum spanning tree (MST) for the graph below. List the edges that are added, in the order they are added, and give the total weight of the MST. If you have a choice of which edge to add, choose the edge (u, v) with the (alphabetically) lowest value for u .



- ii) Consider this statement: *if a graph G contains a cycle C , and C contains an edge e whose weight is less than the weight of all other edges in C , then e is in an MST for G .* Is this true or false? HINT — consider how Kruskal's algorithm would construct the MST for relevant examples.
- c To find the shortest paths *containing at most Q edges* from a vertex s to all other vertices of a directed graph G with positively weighted edges, you are told to modify either the Dijkstra or the Bellman–Ford algorithm. You limit the main loop to run exactly Q times. In iteration i , your modified RELAX procedure:

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1: procedure RELAX( $(u, v), i$ )
2:   if  $dist[u][i - 1] + w(u, v) < dist[v][i]$  then
3:      $dist[v][i] = dist[u][i - 1] + w(u, v)$ 

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where (u, v) is an edge in G , is used to update $dist[v][i]$, the distance to vertex v using at most i edges, after initialising this to $dist[v][i - 1]$. Would you choose the Dijkstra or the Bellman–Ford algorithm to adapt, and why?

The three parts carry, respectively, 40%, 35%, and 25% of the marks.