Propositional Logic Tutorial 4
Derivations

1. Show the Following:

a) $A \wedge B \vdash B \wedge A$

b) $A \wedge B \vdash A \vee B$

c) $P \wedge Q, P \rightarrow R, Q \rightarrow S \vdash R \wedge S$

d) $P \rightarrow (Q \rightarrow R) \vdash P \wedge Q \rightarrow R$

e) $\vdash (P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$

2. Consider the following statement:

> If the Prime Minister loses the next vote, then if his leadership is not challenged by his own party he will call a general election.

a. Formalise the statement in propositional logic in two different ways.
b. Show that each of the two wffs in (a) can be derived from the other.

3. Either John is a murderer or he is a blackmailer. If he is a murderer he is violent. If he is a blackmailer he is rich. If he is rich he either spends a lot or he has a lot of money in his bank account. But his bank account is nearly empty. He is also quite mean with his money.

a. Is John a murderer or a blackmailer?
b. Formalise the information given in propositional logic and formally derive your answer to (a).

4. Remember Tutorial 1?
Sergeant James knew that at least one of Adam, Bill and Charles was a thief. He investigated all three and he learned the following additional information:
If Adam was a thief then Charles would also be a thief, provided Bill was not one. If Bill was a thief then so would Charles be a thief. If Charles was a thief then it would not be the case that if Bill was a thief then so would Adam be a thief.

Representing the information above in propositional logic using the propositional symbols
A: for Adam is a thief
B: for Bill is a thief
C: for Charles is a thief.

a) Show $\neg(p \rightarrow q) \equiv p \wedge \neg q$

b) Show by natural deduction, and (a), if you wish, who the thieves are.