Answers to Some Predicate Logic Formalisation Exercises from the Predicate Logic Notes

Predicates to be used:

lecTheatre/1

office/1

contains/2

lecturer/1

has/2

same/2

phd/1

supervises/2

happy/1

completePhd/1

1.

311 is a lecture theatre and 447 is an office.

$\underbrace{\text{311 is a lecture theatre}}$ and $\underbrace{\text{447 is an office}}$

$\wedge$

lecTheatre(311) $\wedge$ office(447)

2. Every lecture theatre contains a projector.

Every lecture theatre contains a projector.

$\forall \mathbf{x}$ ($\rightarrow$)

Every

Any

All

Everyone

….

Map to $\forall$

Every lecture theatre contains a projector.

$\forall X\ (lecTheatre(X) \rightarrow \qquad\qquad)$

$\forall X\ (lecTheatre(X) \rightarrow contains(X, projector))$

Alternatively:

$\forall X\ (lecTheatre(X) \rightarrow$

$\exists Y\ (projector(Y) \wedge contains(X, Y)))$

A universally quantified wff is usually like this:

$\forall \mathbf{x}\ (\qquad \rightarrow \qquad)$

Or like this:

$\forall \mathbf{x}\ (\qquad \leftrightarrow \qquad)$

That is their principle connective is

$\rightarrow$ or $\leftrightarrow$.

3. Every office contains a telephone and either a desktop or a laptop computer.

Every office contains a telephone and either a desktop or a laptop computer.

$\forall X\ (office(X) \rightarrow ....)$

... contains a telephone and either a desktop or a laptop computer.

$\forall X\ (office(X) \rightarrow (.... \wedge\ ...))$

$\forall X\ (office(X) \rightarrow (contains(X, telephone) \wedge (...)))$

... either a desktop or a laptop computer.

$\forall X\ (office(X) \rightarrow (contains(X, telephone) \wedge (... \vee ...)))$

$\forall X\ (office(X) \rightarrow (contains(X, telephone) \wedge$
$(contains(X, desktop) \vee contains(X, laptop))))$

4. Every lecturer has at least one office.

Every lecturer has at least one office.

$\forall X$ (lecturer(X) $\rightarrow$ …)

… has at least one office.

$\forall X$ (lecturer(X) $\rightarrow$ $\exists Y$ (office(Y) $\wedge$ has(X, Y)))

At least one

Some

One

....

map to ∃

An existentially quantified wff is usually like this:

$\exists \mathbf{X}\ (\qquad \wedge \qquad)$

Or like this:

$\exists \mathbf{X}\ (\qquad \vee \qquad)$

That is their principle connective is

$\wedge$ or $\vee$.

5. No lecturer has more than one office.

No lecturer has more than one office.

Not any lecturer has more than one office.

(There does not exist a lecturer who has more than one office.)

$\neg\exists$L (lecturer(L) $\wedge$ *L has more than one office*)

$\neg\exists$L (lecturer(L) $\wedge$ *L has at least 2 offices that are not the same*)

$\neg\exists$L (lecturer(L) $\wedge$ there are *at least 2 offices that L has and are not the same*)

$\neg\exists$L (lecturer(L) $\wedge$

$\exists$O1 $\exists$O2 (office(O1) $\wedge$ office(O2) $\wedge$

has(L, O1) $\wedge$ has(L, O2) $\wedge$

$\neg$same(O1,O2)))

There are other ways of doing 5. For example:

$$\forall L\ (lecturer(L) \rightarrow \neg\ (\exists O1\ \exists O2\ (office(O1) \wedge office(O2) \wedge has(L, O1) \wedge has(L, O2) \wedge \neg same(O1,O2)))$$

$$\forall L\ \forall O1\ \forall O2\ (lecturer(L) \wedge office(O1) \wedge office(O2) \wedge has(L, O1) \wedge has(L, O2) \rightarrow same(O1,O2))$$

6. No lecturers share offices with anyone.

Try it .....

7. Some lecturers supervise PhD students and some do not.

$(\exists L \;\; .....) \wedge (\exists L \; .....)$ or

$(\exists L1 \;\; .....) \wedge (\exists L2 \; .....)$

Some lecturers supervise PhD students and some do not.

(∃L (lecturer(L) ∧ *there is at least one PhD student that L supervises*)) ∧

(∃L (lecturer(L) ∧ *there is not at least one PhD student that L supervises*))

$(\exists L\ (lecturer(L) \land \exists S\ (phd(S) \land supervises(L,S)))) \land$

$(\exists L\ (lecturer(L) \land \neg\exists S\ (phd(S) \land supervises(L,S))))$

$(\exists L\ (lecturer(L) \land \exists S\ (phd(S) \land supervises(L,S)))) \land$

$(\exists L\ (lecturer(L) \land \neg\exists S\ (phd(S) \land supervises(L,S))))$

Drop some unneccesary brackets:

$\exists L\ (lecturer(L) \land \exists S\ (phd(S) \land supervises(L,S))) \land$

$\exists L\ (lecturer(L) \land \neg\exists S\ (phd(S) \land supervises(L,S)))$

8. Each PhD student has an office, but all PhD students share their office with at least one other PhD student.

Try it ……

9. A lecturer is happy if the PhD students he/she supervises successfully complete their PhD.

$\forall$L (lecturer(L) $\wedge$ *his/her PhD students successfully complete* → happy(L))

$\forall$L (lecturer(L) $\wedge$ *all his/her PhD students successfully complete* → happy(L))

$\forall$L (lecturer(L) $\wedge$ $\forall$S *if S is L's PhD student then S successfully completes*→ happy(L))

$\forall$L (lecturer(L) $\wedge$

$\forall$S *if S is L's PhD student then S successfully completes* → happy(L))

$\forall$L (lecturer(L) $\wedge$

$\forall$S (phd(S) $\wedge$ supervises(L,S) → completePhD(S))

→ happy(L))

10. Not all PhD students complete their PhD.

Try it ....