PS 4

Thursday, April 23, 2020 3:10 PM

Problem 1

$[R] = R_T - [R^*]$
$[X] = X_T - [X^*]$
$[Y] = Y_T - [Y^*]$

$$\frac{d[Y^*]}{dt} = -\frac{d[Y]}{dt} = \frac{\gamma_3[X^*](Y_T - [Y^*])}{K_3 + (Y_T - [Y^*])} - \frac{V_4[Y^*]}{K_4 + [Y^*]}$$

$$\frac{\gamma_3[X^*](Y_T - [Y^*])}{K_3 + (Y_T - [Y^*])} = \frac{V_4[Y^*]}{K_4 + [Y^*]} \qquad y^* = \frac{[Y^*]}{Y_T}$$

Solving R^*:

$$\frac{d[R^*]}{dt} = k_{on}(R_T - [R^*])[L] - k_{off}[R^*]$$

$$\frac{d[R^*]}{dt} = k_{on}R_T[L] - k_{on}[R^*][L] - k_{off}[R^*]$$

$\div R_T$ @ S.S. $\quad \theta_B = [R^*]/R_T \quad K_D = k_{off}/k_{on}[L]$

$$0 = k_{on}[L] - k_{on}\theta_B[L] - k_{off}\theta_B$$

$$0 = k_{on}[L] - \theta_B(k_{on}[L] + k_{off})$$

$$k_{on}[L] = \theta_B(k_{on}[L] + k_{off})$$

$$\theta_B = \frac{k_{on}[L]}{k_{on}[L] + k_{off}}$$

$$\boxed{\theta_B = \frac{1}{1 + K_D}} \Rightarrow \theta_B(1 + K_D) = 1$$

$$\frac{\theta_B(1+K_D)}{K_D} = \frac{1}{K_D}$$

$$\frac{\theta_B}{K_D} + \theta_B = \frac{1}{K_D}$$

$$\theta_B\left(\frac{1}{K_D} + 1\right) = \frac{1}{K_D}$$

$$\boxed{\theta_B = \frac{1/K_D}{1/K_D + 1}}$$

Solving X^*:

$$\frac{d[X^*]}{dt} = -\frac{d[X]}{dt} = \frac{V_1[X]}{K_1 + [X]} - \frac{V_2[X^*]}{K_2 + [X^*]} \qquad \text{where } V_1 = \gamma_1[R^*]$$

$$= \frac{V_1(X_T - [X^*])}{K_1 + (X_T - [X^*])} - \frac{V_2[X^*]}{K_2 + [X^*]}$$

$$V_1/V_2 = \frac{\gamma_1\theta_B R_T}{V_2}$$

$$0 = \frac{\frac{V_1}{V_2}(X_T - [X^*])}{K_1 + (X_T - [X^*])} - \frac{[X^*]}{K_2 + [X^*]}$$

$$= \frac{\frac{\gamma_1 \theta_B R_T}{V_2}(X_T - [X^*])}{K_1 + (X_T - [X^*])} - \frac{[X^*]}{K_2 + [X^*]}$$

$x^* = [X^*]/X_T$

$$0 = \frac{\frac{\gamma_1\theta_B R_T X_T}{V_2} - \frac{\gamma_1\theta_B R_T [X^*]}{V_2}}{K_1 + (X_T - [X^*])} - \frac{[X^*]}{K_2+[X^*]}$$

$$\frac{\gamma_1 R_T}{V_2} = 5$$

$$\frac{\gamma_3 X_T}{V_4} = 10$$

$\div X_T$

$$0 = \frac{\frac{\gamma_1\theta_B R_T}{V_2} - \frac{\gamma_1\theta_B R_T x^*}{V_2}}{\kappa_1 + (1-x^*)} - \frac{x^*}{\kappa_2 + x^*}$$

$$0 = \frac{\left(\frac{V_1}{V_2}\right)(1-x^*)}{\kappa_1+(1-x^*)} - \frac{x^*}{\kappa_2+x^*}$$

$$\frac{x^*}{\kappa_2+x^*} = \left(\frac{V_1}{V_2}\right)\frac{(1-x^*)}{\kappa_1+(1-x^*)}$$

$$\frac{x^*}{\kappa_2+x^*} = \frac{\gamma_1\theta_B R_T}{V_2}\,\frac{(1-x^*)}{\kappa_1+(1-x^*)}$$

$$x^* = 5\theta_B \frac{(1-x^*)(\kappa_2+x^*)}{\kappa_1+(1-x^*)}$$

$$x^* = 5\theta_B \frac{[\kappa_2 + x^* - x^*\kappa_2 + x^{*2}]}{\kappa_1+(1-x^*)}$$

$$x^*(\kappa_1+(1-x^*)) = 5\theta_B[\kappa_2 + x^* - x^*\kappa_2 + x^{*2}]$$

$$x^*\kappa_1 + x^* - x^{*2} = 5\theta_B[\kappa_2+x^* - \kappa_2 x^* + x^{*2}]$$

$\downarrow$ Plugged into Wolfram Alpha

$$\boxed{x^* = \frac{[(\kappa + 5(\kappa-1)\theta_B + 1)^2 + 20\kappa\,\theta_B(5\theta_B - 1)]^{1/2} - \kappa - 5\theta_B\kappa + 5\theta_B - 1}{(-2+10\theta_B)}}$$

all Kappas = κ

Solving y^*:

(a) $$\frac{d[Y^*]}{dt} = -\frac{d[Y]}{} = \frac{V_3[Y]}{K_3+[Y]} - \frac{V_4[Y^*]}{K_4+[Y^*]}$$

$V_3 = \gamma_3[X^*]$ $\quad \frac{\gamma_3 X_T}{V_4} = 10$ $\quad [Y] = Y_T - [Y^*]$

$$0 = \frac{V_3(Y_T - [Y^*])}{K_3 + (Y_T - [Y^*])} - \frac{V_4[Y^*]}{K_4+[Y^*]}$$

$\div$ by V_4

$$0 = \frac{V_3(Y_T - [Y^*])}{K_3 + (Y_T - [Y^*])} - \frac{V_4[Y^*]}{K_4 + [Y^*]}$$

$\div$ by V_4

$$0 = \left(\frac{V_3}{V_4}\right)\frac{(Y_T - [Y^*])}{K_3 + (Y_T - [Y^*])} - \frac{[Y^*]}{K_4 + [Y^*]}$$

$\div$ by Y_T

$$0 = \left(\frac{V_3}{V_4}\right)\frac{(1 - y^*)}{K_3 + (1 - y^*)} - \frac{y^*}{K_4 + y^*}$$

Plug in V_3

$$\frac{y^*}{K_4 + y^*} = \frac{\gamma_3 X^* X_T}{V_4}\frac{(1-y^*)}{K_3 + (1-y^*)}$$

$$\frac{y^*}{K_4 + y^*} = \frac{10x^*(1-y^*)}{K_3 + (1-y^*)}$$

$$y^* = \frac{10x^*(1-y^*)(K_4 + y^*)}{K_3 + (1-y^*)}$$

$$y^* = \frac{10x^*\left[K_4 + y^* - K_4y^* - y^{*2}\right]}{K_3 + (1-y^*)}$$

$$y^*\left[K_3 + 1 - y^*\right] = 10x^*\left[K_4 + y^* - K_4y^* - y^{*2}\right]$$

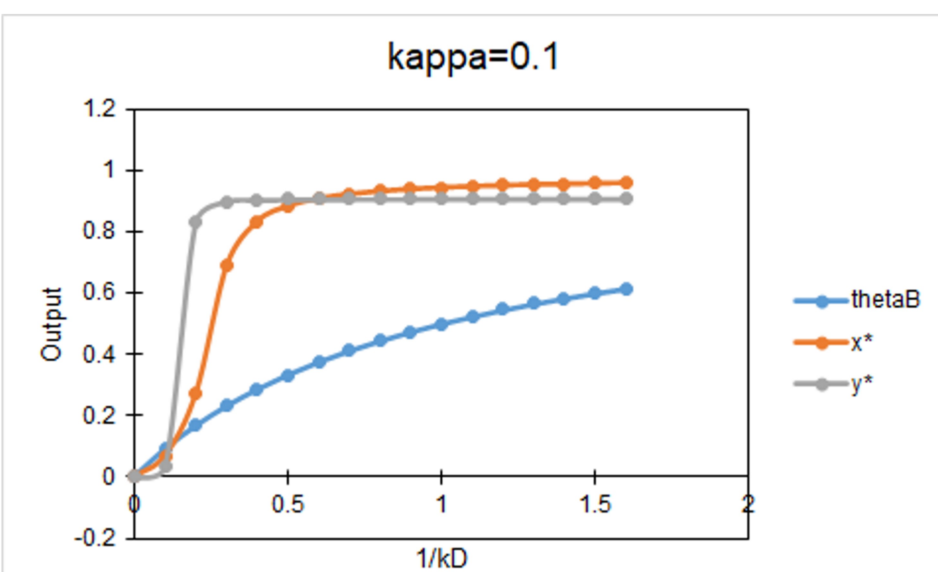
$$K_3y^* + y^* - y^{*2} = 10x^*K_4 + 10x^*y^* - 10x^*K_4y^* - 10x^*y^{*2}$$

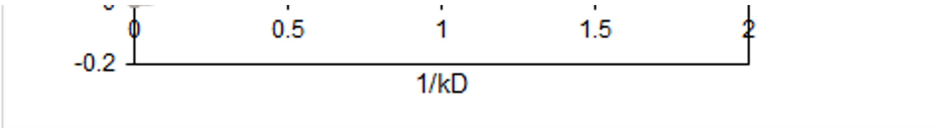
$$(10x^* - 1)y^{*2} + \left[K_3 + 1 - 10x^* + 10x^*K_4\right]y^* - 10x^*K_4 = 0$$

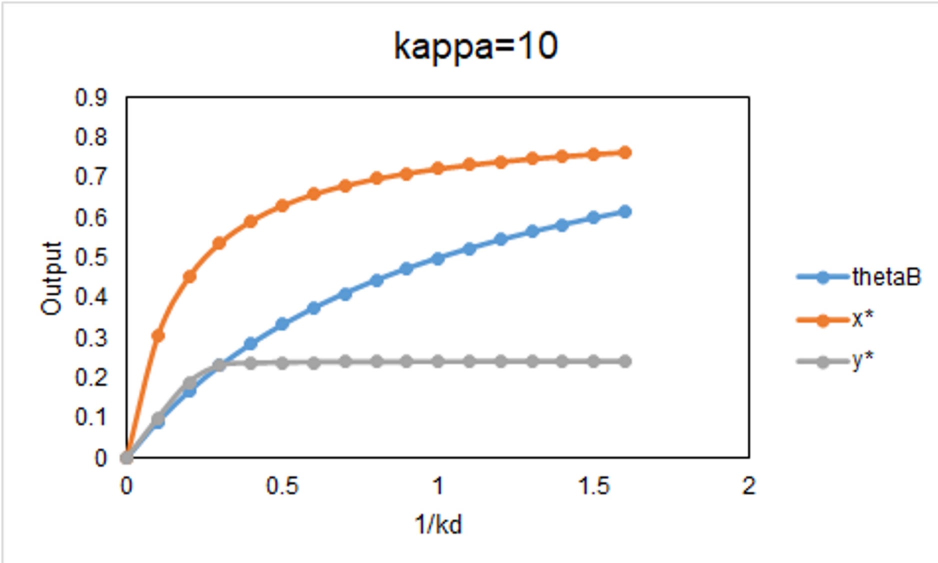
$\downarrow$ Plugged into Wolfram Alpha

$$\boxed{y^* = \frac{\sqrt{\left[K + 10Kx^* - 10x^* + 1\right]^2 + (10x^* - 1)(-10x^*K)} + -K - 10Kx^* + 10x^* - 1}{2(10x^* - 1)}}$$

b)







c) $\text{output} = \dfrac{\text{input}^{n_H}}{C_{1/2} + \text{input}^{n_H}}$ $\Rightarrow$ values from plots in part b

For θ_B:

	input	output
$\kappa=1$	0.1	0.090909
$\kappa=10$	0.1	0.090909

$\boxed{n_H = 1.3}$

estimated from graph:
$C_{1/2} \approx 0.48$

For x^*:

0.1	0.069666	$C_{1/2} \approx 0.25$	$n_H = 1.7$
0.1	0.304557	$C_{1/2} \approx 0.15$	$n_H = 1.2$

For y^*:

0.1	0.03767	$C_{1/2} \approx 0.12$	$n_H = 2.3$
0.1	0.100982	$C_{1/2} \approx 0.12$	$n_H = 1.9$

d)

For $\kappa = 0.1$

θ_B: $\% = \dfrac{0.130435 - 0.090909}{0.090909} = 43\%$

x^*: $\% = \dfrac{0.140502 - 0.069666}{0.069666} = 102\%$

y^*: $\% = \dfrac{0.560836 - 0.03767}{0.03767} = 139\%$

For $\kappa = 10$

θ_B: 43%

X^*: $\dfrac{0.38973 - 0.304557}{0.304557} = 27.97\%$

y^*: $\dfrac{0.148177 - 0.100982}{0.100982} = 46.7\%$

e) For $\kappa = 0.1$, there are large changes in output, and for $\kappa = 10$ the changes are smaller. Your parameters that you choose will be reflective of the system you want.
If you want to amplify ... will choose a smaller κ, whereas if you want

e) For K=0.1, there are large changes in output, and for K=10 the changes are smaller. Your parameters that you choose will be reflective of the system you want. If you want to amplify signal, you'll choose a smaller K, wheras if you want the latter, you will choose larger K values.

Problem 2

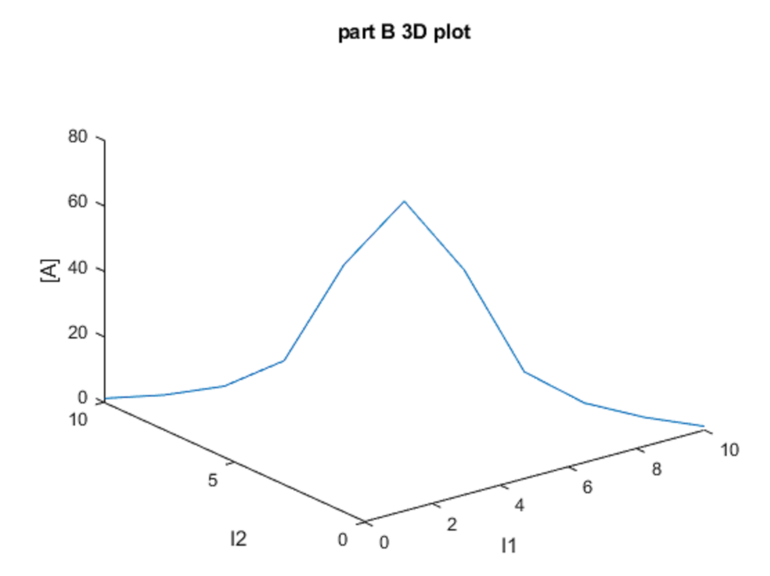
a) See attached code

[A] = 1.1097

[B] = 49.4451

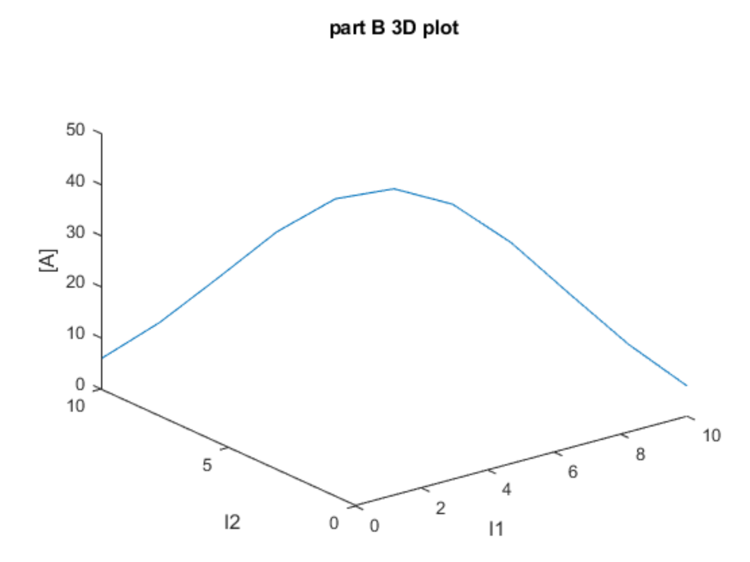
[C] = 49.4451

b) See attached code



c) AND logic gate

d) See attached code



This gate is probably called a fuzzy operator because the maximum is reached more gradually rather than abruptly like in part B. It also doesn't reach as high of values, it's more of a hill shape than mountain shape (part B).

e) In this case, the AND gate allows for quick and large responses when inhibitors are present, so it allows the system to operate at some maximum easily.