MTMW14
Project 1: The ocean recharge oscillator model

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Link to the GitHub repository for all the code for this project, including a juypter notebook file in the style of this report: https://github.com/ao825208/MTMW14-Project-1.git

Introduction

The ocean recharge oscillator model (ROM) is a reduced coupled model for the El Nino Southern Oscillation (ENSO) built by Fei-Fei Jin (1997). ENSO is considered the most prominent mode of ocean-atmosphere variability, and it is important to understand the interaction between surface wind stress and thermocline depth along the equator. ROM can be described using two ordinary differential equations:

$$\frac{dh_W}{dt} = -rh_W - \alpha bT_E - \alpha\xi_1 \tag{1}$$

$$\frac{dT_E}{dt} = RT_E + \gamma h_W - e_n(h_W + bT_E)^3 + \gamma\xi_1 + \xi_2 \tag{2}$$

The prognostic variables are defined h_W (west Pacific Ocean thermocline depth) and T_E (east Pacific SST anomaly). This project will focus on recreating Jin's ROM model in Python, exploring how an appropriate time scheme method forecasts an ensemble, and determine key numerical analysis properties such as stability and oscillatory behaviour. The time scheme I have chosen to focus on is the Runge-Kutta scheme, which will be applied to each task in the assignment.

Runge-Kutta time scheme:

The fourth-order accurate Runge-Kutta time scheme can be defined as:

$$q^{n+1} = q^n + \Delta t\frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{3}$$

With each k being defined:

$$k_1 = f(q^n, n\Delta t) \tag{4}$$

$$k_2 = f\left[q^n + \frac{k_1\Delta t}{2}, \left(n + \frac{1}{2}\right)\Delta t\right] \tag{5}$$

$$k_3 = f\left[q^n + \frac{k_2\Delta t}{2}, \left(n + \frac{1}{2}\right)\Delta t\right] \tag{6}$$

$$k_4 = f[q^n + k_3\Delta t, (n + 1)\Delta t] \tag{7}$$

Each of these parameters k will approximate the function f at the time step Δt. Each k will be evaluated successfully where f will be determined four times for a single time step. However, because f is being constantly re-evaluated, the same k parameter can not be used again, and so it is important to ensure a long time step is used for the scheme to hold a stable result.

Task A: *The neutral linear (deterministic) ROM.*

I will generate a ROM model using Runge-Kutta excluding nonlinearity and external forcing (e.g., $\xi_1, \xi_2, e_n = 0$). The coupling parameter will be set to its critical value, $\mu = \mu_c = \frac{2}{3}$, and the model will be run over a time period $\tau_c = \frac{2\pi}{\omega_c}$ (approximately 41 months). This should produce a single stable oscillation.

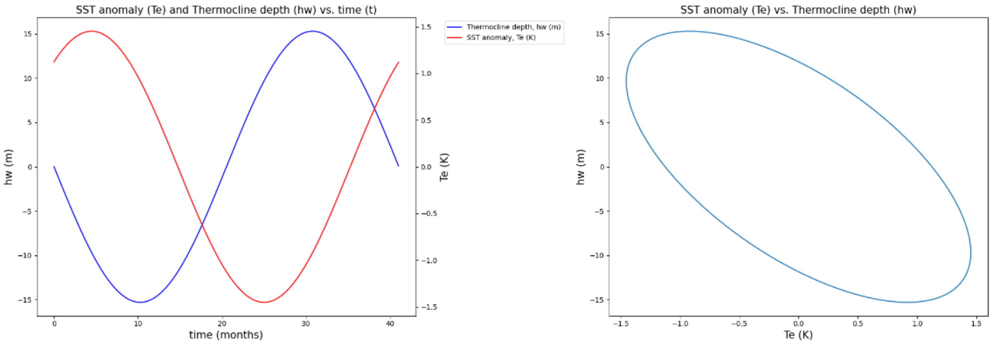


Figure 1: *SST anomaly (T_E) and thermocline depth (h_W) time series plot (left), SST anomaly (T_E) vs. thermocline depth (h_W) phase plot (right) for exactly one periodic oscillation (41 months).*¶

It is important for Runge-Kutta to be stable, so accurate values of variables are produced. By increasing the number of time steps and periodic oscillations, we can prove Runge-Kutta is stable.

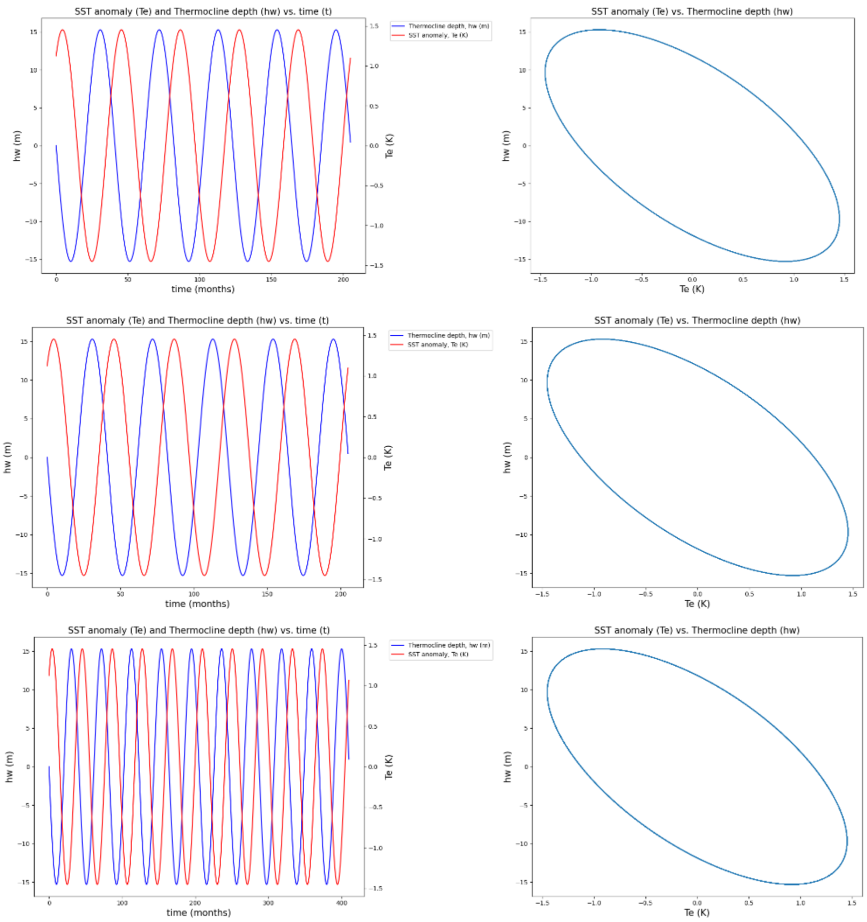


Figure 2: *Time series plot (left), phase plot (right) stability test for increasing time steps, nt.*

As shown in Figure 2, there is no damping or growth to the linear model when nt is increased. Therefore, the Runge-Kutta scheme is stable.

Task B: *Sub-critical and super-critical settings of the coupled parameter.*

I will demonstrate what will happen if the model is ran with the coupling parameter set as the sub-critical ($\mu < \frac{2}{3}$) and super-critical ($\mu > \frac{2}{3}$) values.

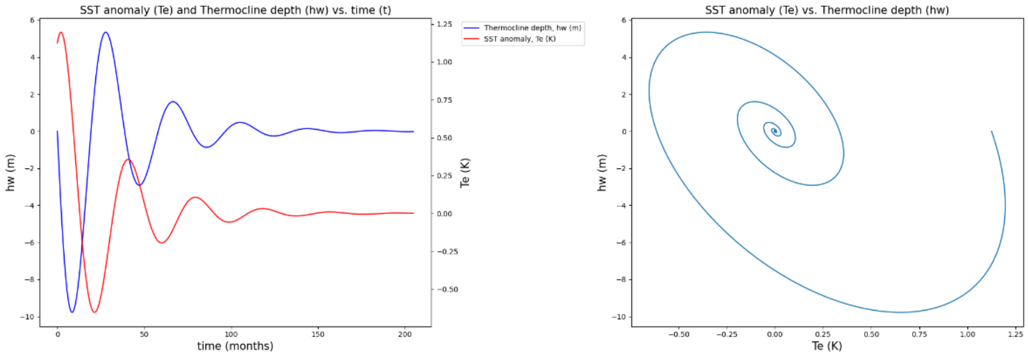


Figure 3: *Time series plot (left), phase plot (right), sub-critical model (μ = 0.6).*

In Figure 3, by setting the sub-critical value $\mu = 0.6$, the thermocline depth and SST anomaly appear to converge to 0, generating a damping oscillation for both the time series and phase trajectory plot, where the growth rate is decaying and has become negative.

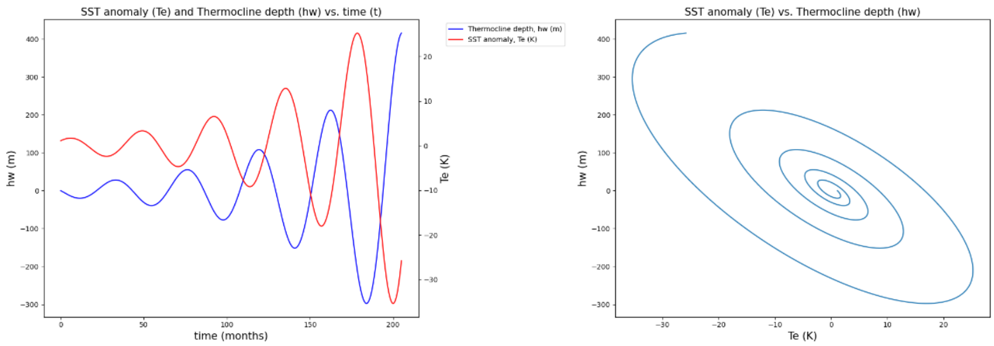


Figure 4: *Time series plot (left), phase plot (right), super-critical model (μ = 0.7).*

In Figure 4, by setting the super-critical value $\mu = 0.7$, thermocline depth and SST anomaly solutions converge to infinity. Linear growth rate is positive, and the system gives an unstable oscillatory mode. These values are not ideal for μ as it will generate some unrealistic values for modelling ENSO.

Task C: *Extending ROM to include the impact of non-linearity.*

Non-linearity can be turned on by setting $e_n = 0.1$.

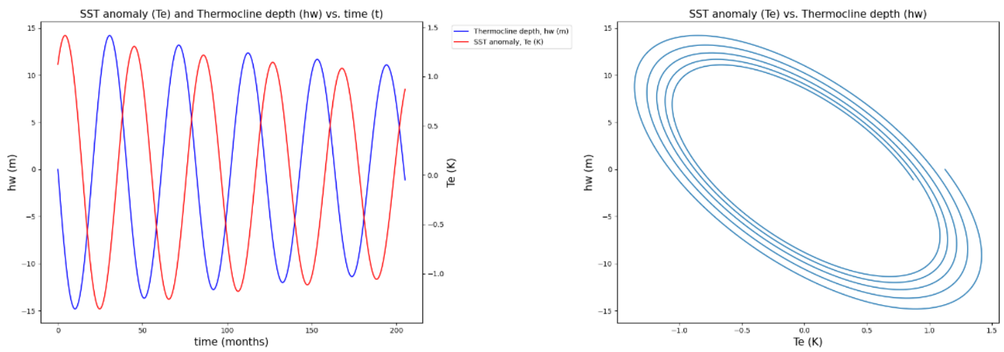


Figure 5: *Time series plot (left), phase plot (right), with non-linearity variable (e_n = 0.1) turned on.*

In figure 5, the system dampens but decays at a slower rate compared to the sub-critical case. The solution is stable overall, but if we increased time, solutions will tend to 0.

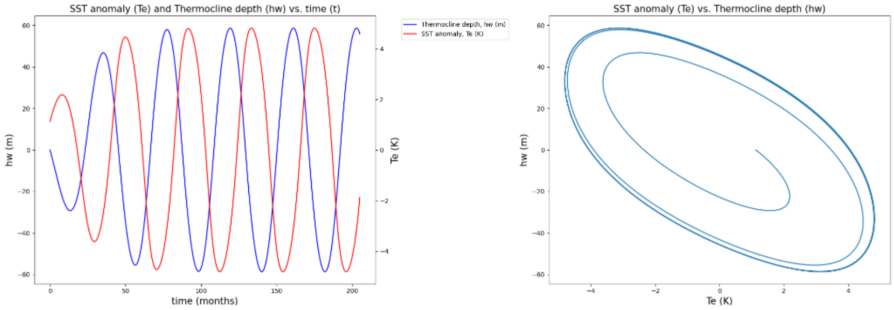


Figure 6: *Time series plot (left), phase plot (right), non-linearity turned on and considering supercritical values of μ.*

Figure 6 shows that variables grow to stable oscillations when we increase μ beyond its critical value with non-linearity switched on. The growth rate has been limited and the amplitude of the oscillations has returned to appropriate values.

Task D: *Test the self-excitation hypotheses.*

The coupling parameter μ can be modified to vary on an annual cycle, using the following equation:

$$\mu = \mu_0 \left(1 + \mu_{ann} \cos\left(\frac{2\pi t}{\tau} - \frac{5\pi}{6}\right)\right) \tag{8}$$

These new parameters will be set as: $\mu_0 = 0.75$, $\mu_{ann} = 0.2$ and $\tau = 12$ (months). To make sure units of time stay consistent, τ will be non-dimensionalised with t.

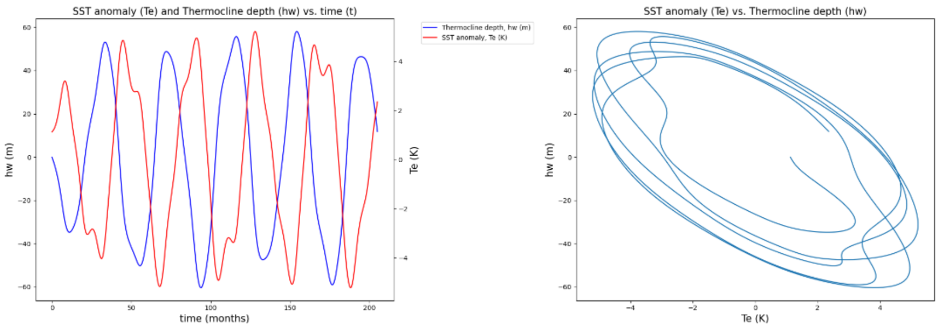


Figure 7: *Time series plot (left), phase plot (right), non-linearity turned on and showing the variation in the coupling parameter considered with an annual cycle.*

Figure 7 shows that some stochastic noise has generated from the new coupling parameter function added to the scheme. The model tends grows to a stable oscillatory solution after a couple periods. For either variable T_E or h_W, noise is quite minimal, however when the model reaches a peak or a trough, there is more noise and uncertainty.

Task E: *Testing the stochastic initiation hypotheses by adding wind forcing to linear ROM.*

Wind stress forcing can be added to the model using the equation:

$$\xi_1 = f_{ann} \cos\left(\frac{2\pi t}{\tau}\right) + f_{ran} W \frac{\tau_{cor}}{\Delta t} \tag{9}$$

W is a random number picked within the range -1 and 1, which represents the white noise process the stochastic wind stress forcing experiences. The following parameters will be set: $e_n = 0$, $\mu_0 =$

0.75, $\mu_{ann} = 0.75$, $f_{ann} = 0.02$, $f_{ran} = 0.2$ and $\tau_{cor} = \frac{1}{30}$ (one day). For simplicity, the time step Δt will also be set to one day as to make τ_{cor} consistent with t.

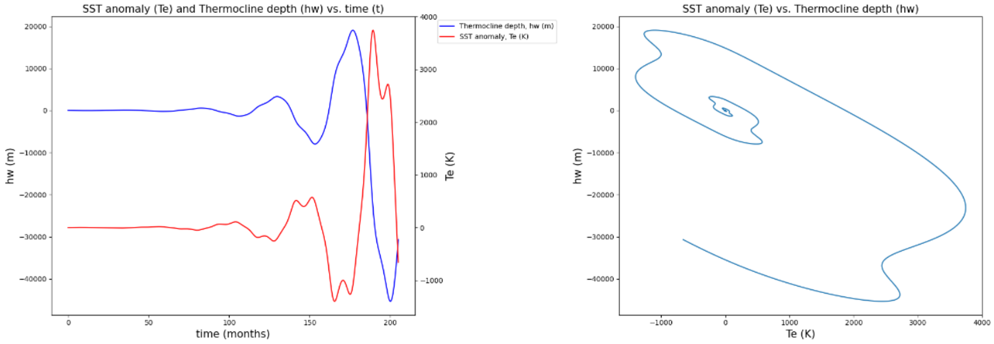
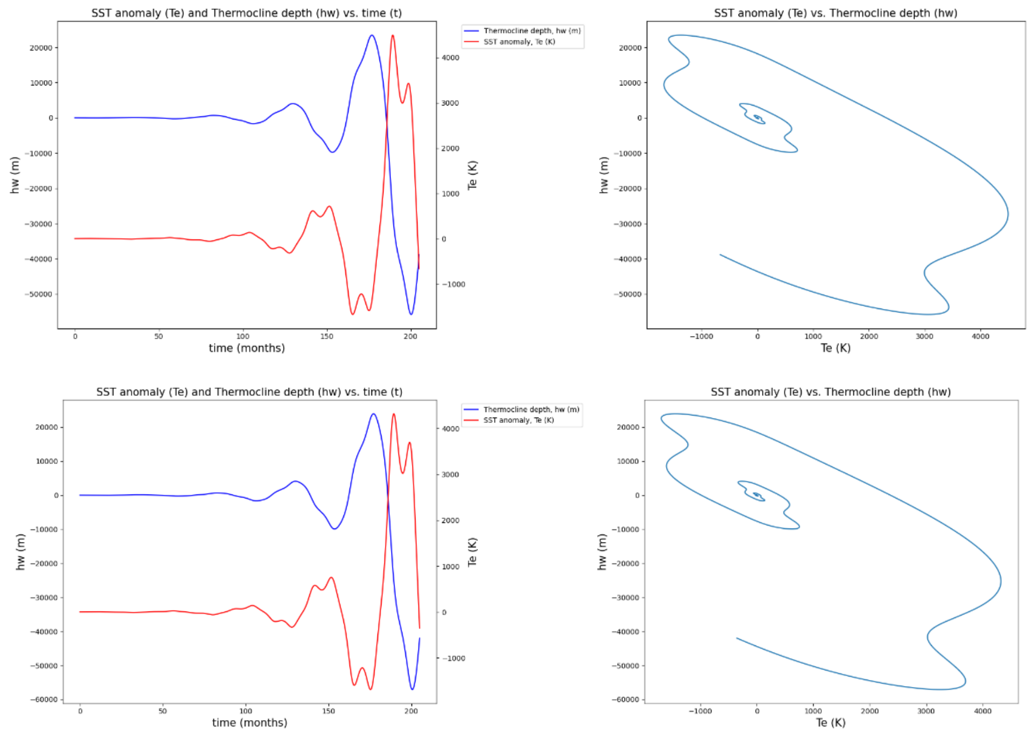


Figure 8: *Time series plot (left), phase plot (right), adding noisy wind forcing to the linear model.*

In Figure 8, the growth rate of the oscillations is very small and changes in the size of values is minimal. At the fourth periodic oscillation, extraordinary change begins to occur, and the amplitude grows to extreme values. The phase plot mimics this rapid growth by showing the solution converge to infinity after its initial periodic oscillations.



(a)

(b)

Figure 9: *Time series plot (left), phase plot (right), turning off random forcing (a), turning off annual forcing (b).*

Setting $f_{ran} = 0$, evaluates greater values of T_E and h_W where solutions tend to be unstable, so annual forcing has a detrimental effect on the model with linearity. The same issue arises when we remove annual forcing and concentration on the random forcing. Therefore, a linear model is not appropriate for measuring ROM.

Task F: *Testing non-linearity and stochastic forcing together.*

Considering non-linearity ($e_n = 0.1$), we produce the following system.

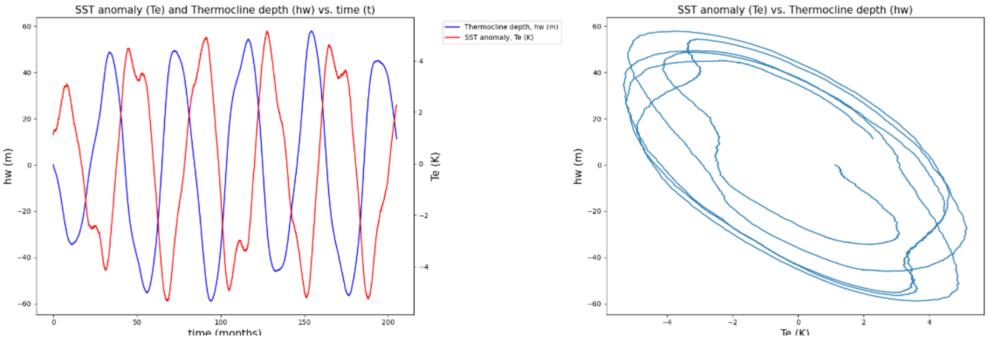


Figure 10: *Time series plot (left), phase plot (right), with non-linearity turned on for the wind forcing system.*

The time series graph in Figure 10 shows more noise than the linear model and oscillations tend to be more stable, alongside the phase plot as well.

Task G: *Testing whether chaotic behaviour can be triggered through addition of initial condition uncertainty.*

To design the ensemble, I will add minor perturbations to the initial conditions that stay within the amplitude of the oscillations for Task F. This will prevent the ensemble simulating chaotic behaviour at the initial conditions. The perturbations for T_E will be set as a random number within the range $-1 < T_{pert} < 1$, and h_W to be within $-2.5 < h_W < 2.5$. I will simulate and ensemble size of 10 different initial conditions and show how they vary with the models plots.

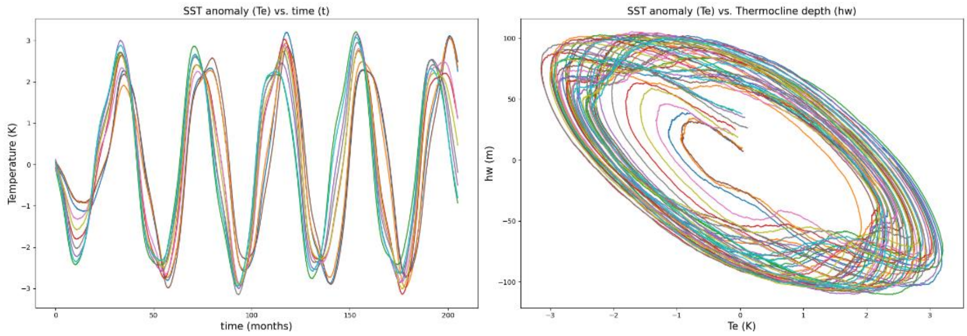
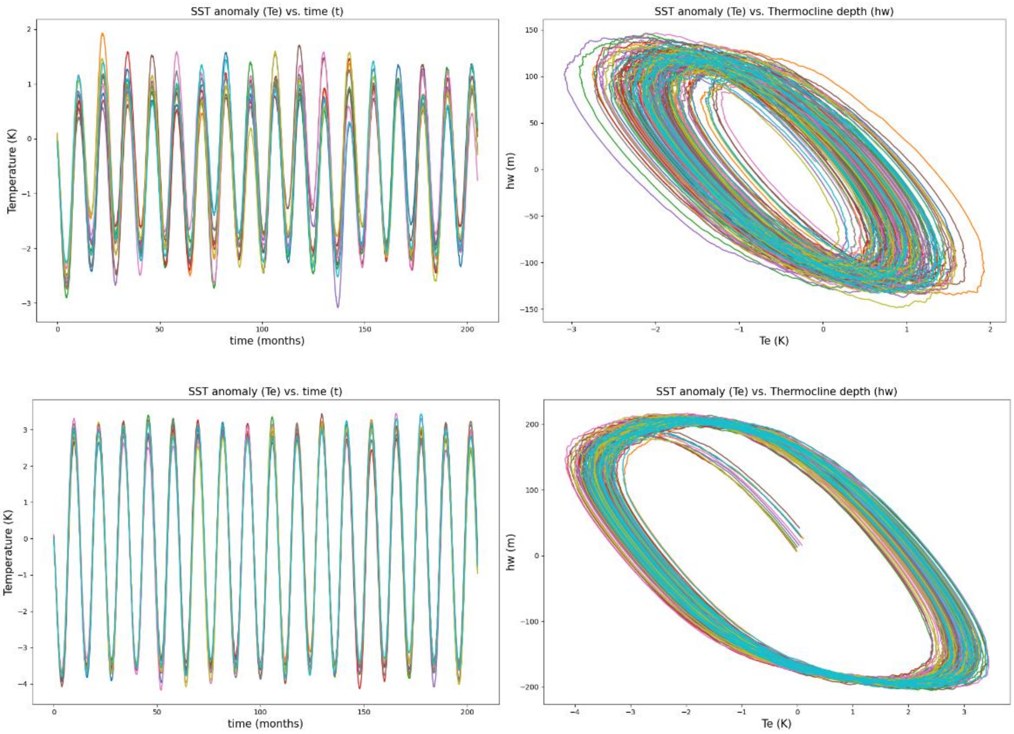


Figure 11: *Time series plot of SST (left), phase plot (right), with an ensemble size of 10.*

Figure 11 shows a plume diagram for SST anomalies on the left, and how the different initial conditions for T_E vary over time. There is slight disorder with the initial conditions differing at the beginning, however as time increases, the different ensemble members correlate closely together and do not simulate chaos. This ensemble resembles the observed ENSO signals found in Jin's paper, as it follows a steady stable trend for multiple periodic oscillations that do not exceed the boundaries of the solution.

If we were to increase stochastic forcing, can we simulate chaos?



(a)

(b)

Figure 12: *Time series plot (left), phase plot (right), increasing annual forcing (a), increasing random forcing (b).*

Figure 12 shows increasing the stochastic forcing does not make Jin's model chaotic. Although the variables are becoming noisier and less correlated, they stay within the stable boundaries of the model, and there is not huge uncertainty between ensemble members. Therefore, Jin's model is not chaotic.

It is likely that Jin's model is far too simple, hence why it won't simulate chaos. If we had an equation to define the second parameter ξ_2 which represents the heating in the system, this may produce a chaotic system if it were added to the Runge-Kutta time scheme. Further research into alternative time schemes may also be necessary in finding a chaotic system, as Runge-Kutta is conditionally stable for large time steps.

References:

Jin, F.-F. (1997a). An equatorial ocean recharge paradigm for ENSO: Part I: Conceptual model. J. Atmos. Sci., 54, 811–829.

Jin, F.-F. (1997b). An equatorial ocean recharge paradigm for ENSO: Part II: A stripped down coupled model. J. Atmos. Sci., 54, 830–847.