

Test Report: Library of ODE Solvers (LODES)

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1 Revision History

Date	Version	Notes
December 16, 2017	1.0	Initial draft.

2 Symbols, Abbreviations and Acronyms

2.1 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the numeral analysis and ordinary differential equation literature and with existing documentation for solving ordinary differential equations. The symbols are listed in alphabetical order.

symbol	unit	type	description
dy/dx	-	\mathbb{R}	Rate of change of y with respect to x
$\varepsilon_{\text{relative}}$	-	\mathbb{R}	The relative error
F	-	$\mathbb{R}^3 \rightarrow \mathbb{R}$	Order function applied to the Runge Kutta Method
$f(x, y)$	-	$\mathbb{R}^2 \rightarrow \mathbb{R}$	Explicit form of the ODE function containing (x, y)
$f_x(x, y)$	-	$\mathbb{R}^2 \rightarrow \mathbb{R}$	Explicit form of the derivative of $f(x, y)$ with respect to x
$f_y(x, y)$	-	$\mathbb{R}^2 \rightarrow \mathbb{R}$	Explicit form of the derivative of $f(x, y)$ with respect to y
h	-	$\mathbb{R} . h > 0$	Step-size from $x_{(0)}$ to the next point $x_{(1)}$, where $x_{(1)} = x_{(0)} + h$
K_1, K_2, K_3, K_4	\mathbb{R}	-	Intermediary variables used in the Runge-Kutta method
n	-	\mathbb{R}	Reference recursion step
o	-	\mathbb{R}	Solution variable
T	-	-	Test
x_0	-	\mathbb{R}	Initial value x
x_k	-	\mathbb{R}	Final value x
x_n	-	\mathbb{R}	Intermediate n^{th} value x
y_0	-	\mathbb{R}	Initial value y
y_k	-	\mathbb{R}	Final value y
y_n	-	\mathbb{R}	Intermediate n^{th} value y
y'	-	$\mathbb{R} \rightarrow \mathbb{R}$	Implicit form of the first order ODE = $f(x, y)$
$y^{(n)}$	-	$\mathbb{R} \rightarrow \mathbb{R}$	Implicit form of the ODE to the n^{th} order
y	-	$1 \times \mathbb{R}^k$	The array containing all intermediate y_n values

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3 Introduction

This document serves as the Test Report for the Library of ODE Solvers (LODES) which lays out in detail the results of the execution of the LODES Test Plan. The requirements for the software are described in the LODES Software Requirements Specification. The aim of the Test Report is to document and show that LODES produces accurate and valid (in the scope of detailed in the SRS and Test Plan) results.

The Overview (Section 4) discusses how testing slightly deviates from the Test Plan in its execution.

The following sections describe the evaluation of the software against the Functional Requirements (Section 5), Non-Functional Requirements (Section 6). It will also compare LODES to existing software in Comparison to Existing Implementation (Section 7).

In the sections after, Unit Testing (Section 8), Changes Due to Testing (Section 9), Automated Testing (Section 10), Trace to Requirements (Section 11), Trace to Modules (Section 12, and Code Coverage Metrics (Section 13) are discussed.

4 Overview

The Test Report documents the results of the tests carried out to verify the behaviour of the LODES software. The Test Plan suggests that it should carry out testing such that that the test report shall describe and analyze the numerical differences of the hand-executed solution to the numerical analysis methods versus the solutions obtained from LODES. This Test Report carries out the tests, with the same intent of verification, and instead of only comparing against the hand-executed solution, it compares the results with the MATLAB generated solutions as well.

5 Functional Requirements Evaluation

5.1 Calculation Tests

5.1.1 Simple Cases: T-1, T-5, T-9, T-13

Input: $f(x, y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$

Test Plan Test Case: T-1

Expected output: The value of $y_k = 3$. This test passes.

```
>> lodes(1, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:      0      2

The values of y are:      1      3

ans =

      0      2
```

Figure 1: T-1 result.

Test Plan Test Case: T-5

Expected output: The value of $y_k = 5$. This test passes.

```
>> lodes(2, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:      0      2

The values of y are:      1      5

ans =

      0      2
```

Figure 2: T-5 result.

Test Plan Test Case: T-9

Expected output: The value of $y_k = 5$. This test passes.

```
>> lodes(3, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:      0      2

The values of y are:      1      5

ans =

      0      2
```

Figure 3: T-9 result.

Test Plan Test Case: T-13

Expected output: The value of $y_k = 7$. This test passes.

```
>> lodes(4, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:      0      2

The values of y are:      1      7

ans =|

      0      2
```

Figure 4: T-13 result.

5.1.2 Simple-Iterative Cases: T-2, T-6, T-10, T-14

Input: $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

Test Plan Test Case: T-2

Expected output: The value of $y_k = 5.0625$. This test passes.

```
>> lodes(1, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:      0      0.5000      1.0000      1.5000      2.0000

The values of y are:      1.0000      1.5000      2.2500      3.3750      5.0625
```

Figure 5: T-2 result.

Test Plan Test Case: T-6

Expected output: The value of $y_k = 6.9729$. This test passes.

```
>> lodes(2, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:      0      0.5000      1.0000      1.5000      2.0000
The values of y are:      1.0000      1.6250      2.6406      4.2910      6.9729
```

Figure 6: T-6 result.

Test Plan Test Case: T-10

Expected output: The value of $y_k = 6.9729$. This test passes.

```
>> lodes(3, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:      0      0.5000      1.0000      1.5000      2.0000
The values of y are:      1.0000      1.6250      2.6406      4.2910      6.9729
```

Figure 7: T-10 result.

Test Plan Test Case: T-14

Expected output: The value of $y_k = 7.38340$. This test passes.

```
>> lodes(4, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:      0      0.5000      1.0000      1.5000      2.0000
The values of y are:      1.0000      1.6484      2.7173      4.4794      7.3840
```

Figure 8: T-14 result.

5.1.3 Linear Trigonometric Cases: T-3, T-7, T-11, T-15

Input: $f(x, y) = y, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

Test Plan Test Case: T-3

Expected output: The value of $y_k = -4$. This test passes.

```
>> lodes(1, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0      5

The values of y are:      1     -4
```

Figure 9: T-3 result.

Test Plan Test Case: T-7

Expected output: The value of $y_k = -43.8973$. This test passes.

```
>> lodes(2, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0      5

The values of y are:      1.0000 -43.8973
```

Figure 10: T-7 result.

Test Plan Test Case: T-11

Expected output: The value of $y_k = -43.8973$. This test passes.

```
>> lodes(3, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0      5

The values of y are:      1.0000 -43.8973
```

Figure 11: T-11 result.

Test Plan Test Case: T-15

Expected output: The value of $y_k = -1702.8$. This test passes.

```
>> lodes(4, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0      5

The values of y are:      1.0e+03 *
                        0.0010  -1.7028
```

Figure 12: T-15 result.

5.1.4 Linear Trigonometric Iterative Cases: T-4, T-8, T-12, T-16

Input: $f(x, y) = \sin(x) - y^2$, $h = 1$, $x_0 = 0$, $y_0 = 1$, $x_k = 5$

Test Plan Test Case: T-4

Expected output: The value of $y_k = -0.6695$. This test passes.

```
>> lodes(1, 'sin(x) - y^2', 0, 1, 5, 1, 1, 1)
The values of x are:      0      1      2      3      4      5

The values of y are:      1.0000      0      0.8415      1.0427      0.0966      -0.6695
```

Figure 13: T-4 result.

Test Plan Test Case: T-8

Expected output: The value of $y_k = -1.1012$. This test passes.

```
>> lodes(2, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are:      0      1      2      3      4      5

The values of y are:      1.0000      0.9207      0.9541      0.5700      0.0251      -1.1012
```

Figure 14: T-8 result.

Test Plan Test Case: T-12

Expected output: The value of $y_k = -1.1012$. This test passes.

```
>> lodes(3, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are:      0      1      2      3      4      5

The values of y are:      1.0000      0.9207      0.9541      0.5700      0.0251      -1.1012
```

Figure 15: T-12 result.

Test Plan Test Case: T-16

Expected output: The value of $y_k = -1.0284$. This test passes.

```
>> lodes(4, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are:      0      1      2      3      4      5

The values of y are:      1.0000      0.7863      0.9434      0.7180      0.1408      -1.0284
```

Figure 16: T-16 result.

6 Nonfunctional Requirements Evaluation

6.1 Speed Test Plan Test Case: T-17

Input: $f(x, y) = \sin(x) - y^2$, $h = 1e - 3$, $x_0 = 0$, $y_0 = 1$, $x_k = 5$

Test Plan Test Case: T-17

Expected output: The execution time difference of LODES is not greater than 4 times that of MATLAB. This test fails as the execution time of LODES is 2,000 times greater than MATLAB's.

Profile Summary

Generated 15-Dec-2017 23:08:41 using performance time.

Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
lodes	1	472.941 s	0.003 s	
rk	1	472.766 s	2.209 s	

Figure 17: T-17 LODES rk() result.

Profile Summary

Generated 15-Dec-2017 23:29:45 using performance time.

Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
testode45	1	0.273 s	0.034 s	
ode45	1	0.239 s	0.110 s	

Figure 18: T-17 MATLAB ode45() result.

7 Comparison to Existing Implementation

This section compares the calculation of LODES to MATLAB's ode45() function.

Test Plan Test Case: T-18

The following input parameters were tested (as above):

- Test 1 (7.1): $f(x, y) = y$, $h = 2$, $x_0 = 0$, $y_0 = 1$, $x_k = 2$
- Test 2 (7.2): $f(x, y) = y$, $h = 0.5$, $x_0 = 0$, $y_0 = 1$, $x_k = 2$

- Test 3 (7.3): $f(x, y) = \sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$
- Test 4 (7.4): $f(x, y) = \sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

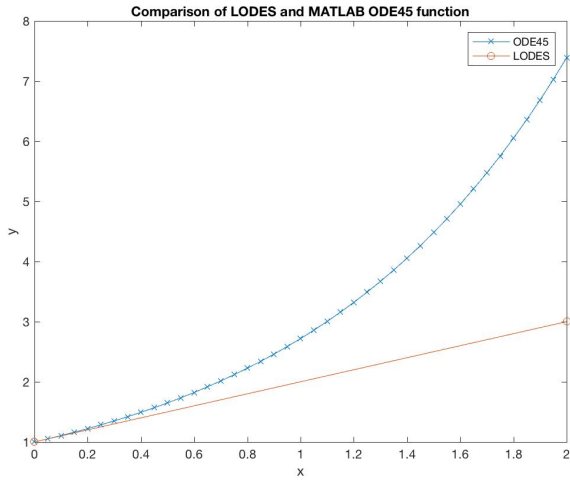
This phase of testing is automated and the results are shown in Section 14. The following equation was implemented to compare the results labeled as **norm**:

$$\epsilon_{\text{relative}} = \text{norm} = \frac{\|\text{Result}_{\text{MATLAB}} - \text{Result}_{\text{LODES}}\|}{\|\text{Result}_{\text{MATLAB}}\|}$$

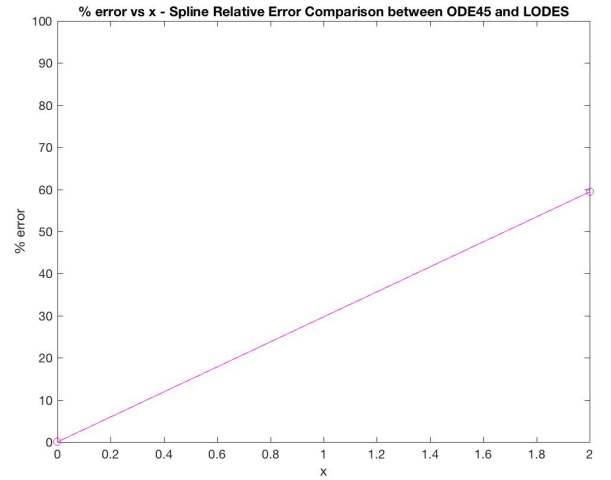
Output/Result: The $\epsilon_{\text{relative}}$ vs. h plots. The intermediate y values (Result) are compared according to the following formula of the relative error norm percentage -

$$\epsilon_{\text{relative}} = 100 * \frac{\text{Result}_{\text{MATLAB}} - \text{Result}_{\text{LODES}}}{\text{Result}_{\text{MATLAB}}}$$

7.1 Test 1: $f(x, y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$

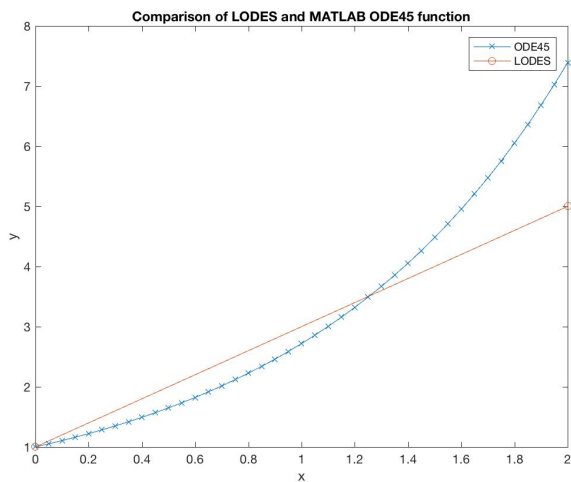


(a) Euler vs. ODE45

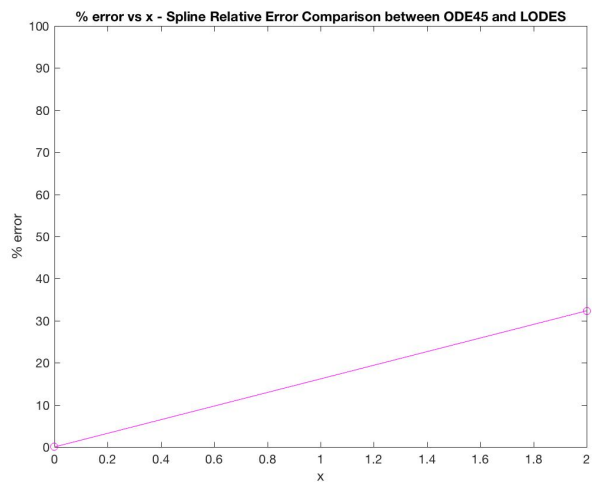


(b) Relative Error Norm - Euler

Figure 19: Euler's Method Analysis Graphs

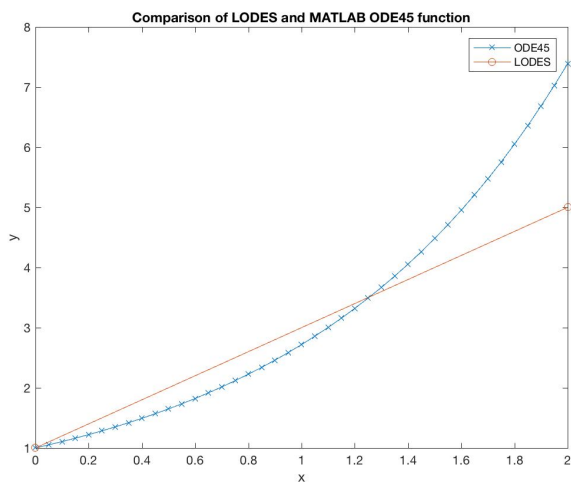


(a) Trapezoid vs. ODE45

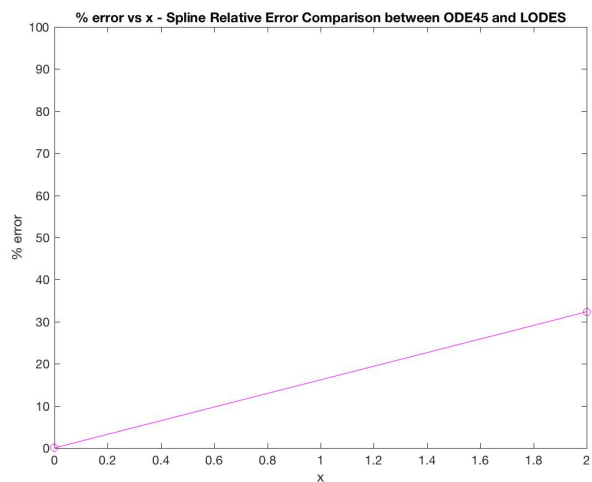


(b) Relative Error Norm - Trapezoid

Figure 20: Trapezoidal Method Analysis Graphs

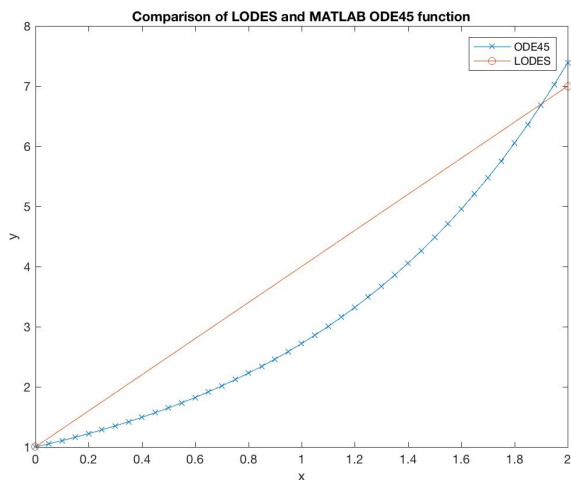


(a) Heun vs. ODE45

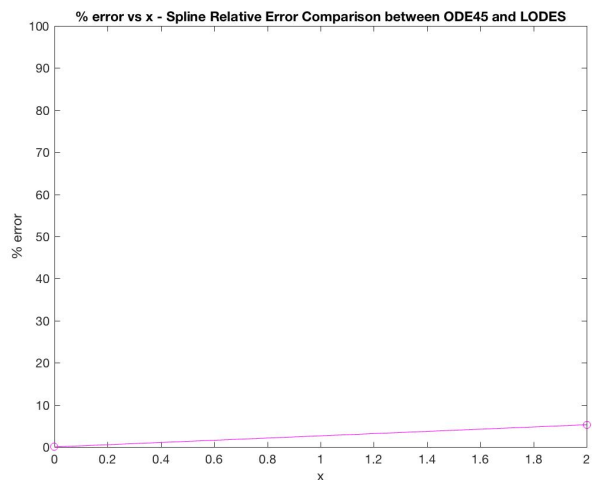


(b) Relative Error Norm - Heun

Figure 21: Heun's Method Analysis Graphs



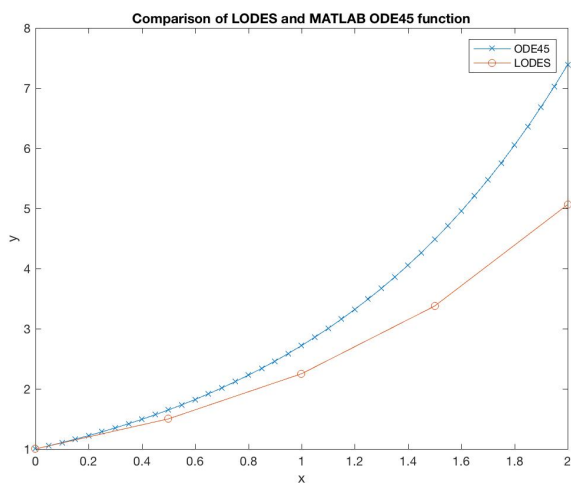
(a) Runge-Kutta 4 vs. ODE45



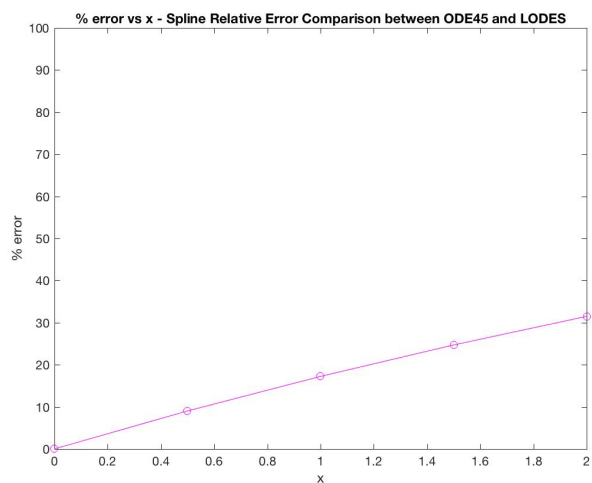
(b) Relative Error Norm - Runge-Kutta 4

Figure 22: Runge-Kutta 4 Method Analysis Graphs

7.2 Test 2: $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

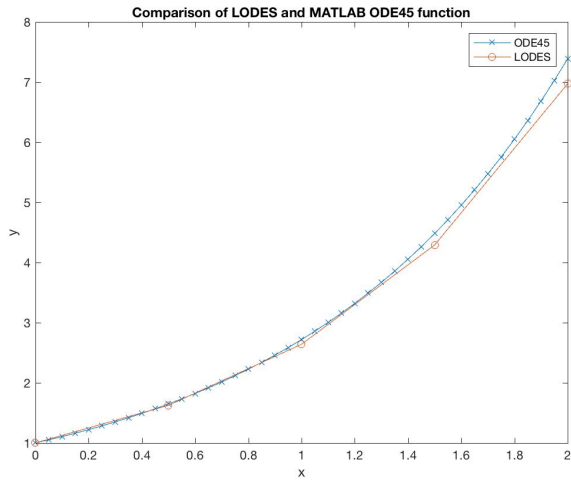


(a) Euler vs. ODE45

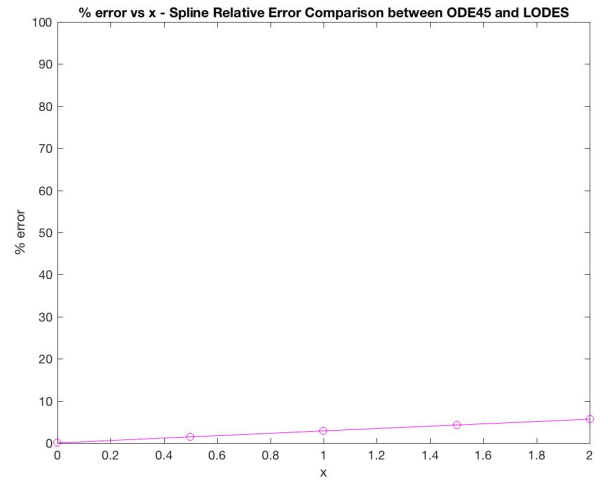


(b) Relative Error Norm - Euler

Figure 23: Euler's Method Analysis Graphs

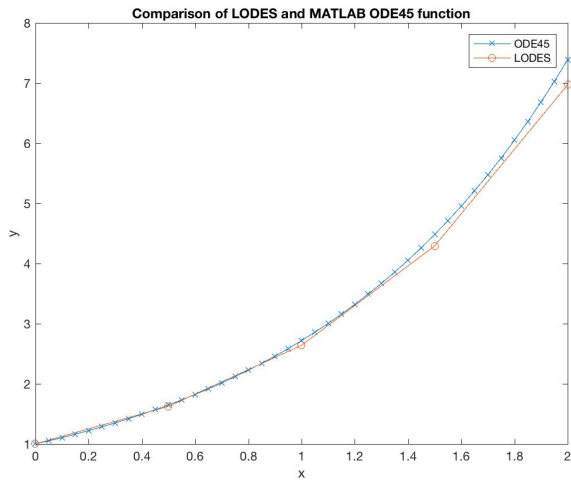


(a) Trapezoid vs. ODE45

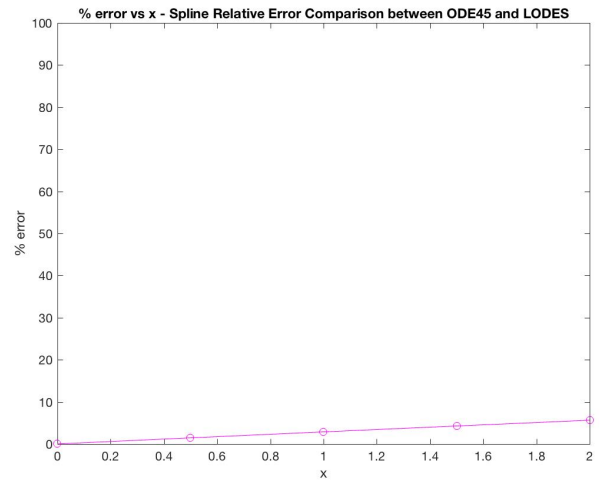


(b) Relative Error Norm - Trapezoid

Figure 24: Trapezoidal Method Analysis Graphs

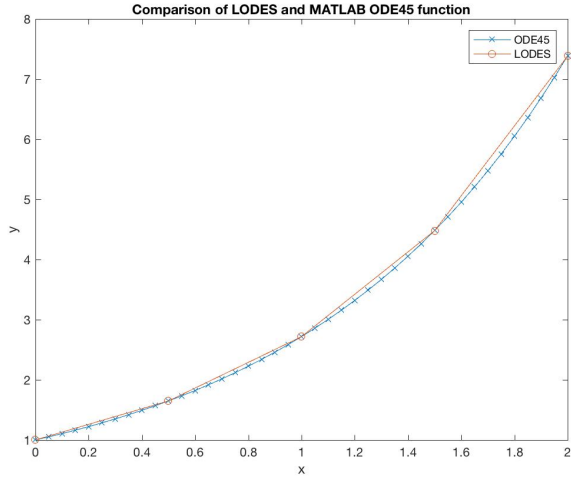


(a) Heun vs. ODE45

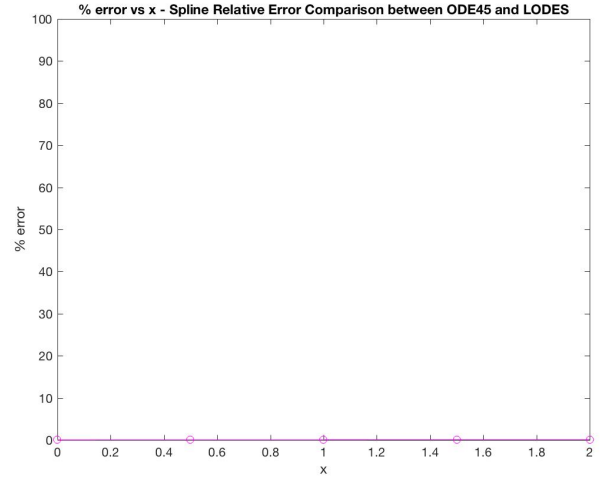


(b) Relative Error Norm - Heun

Figure 25: Heun's Method Analysis Graphs



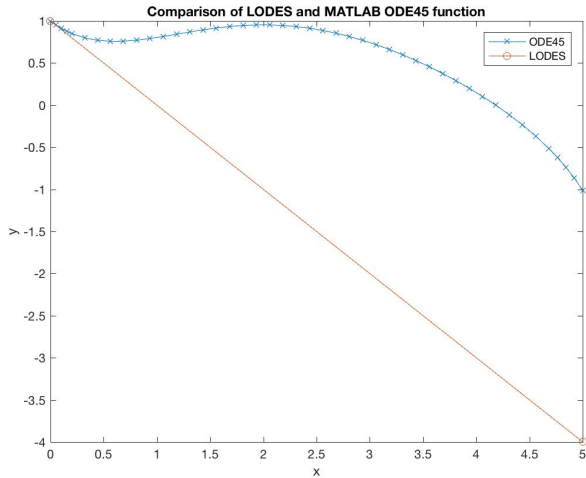
(a) Runge-Kutta 4 vs. ODE45



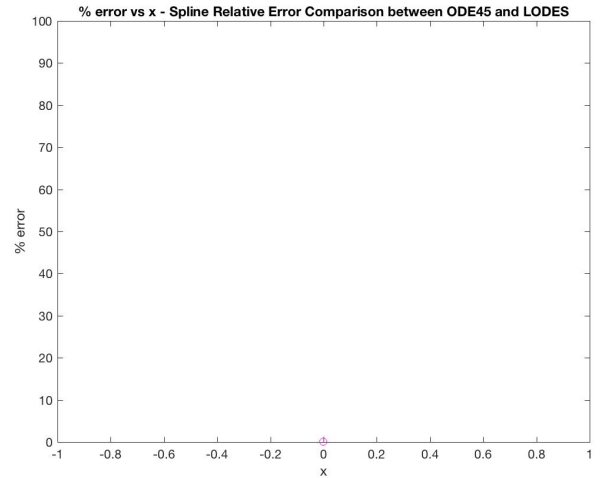
(b) Relative Error Norm - Runge-Kutta 4

Figure 26: Runge-Kutta 4 Method Analysis Graphs

7.3 Test 3: $f(x, y) = \sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

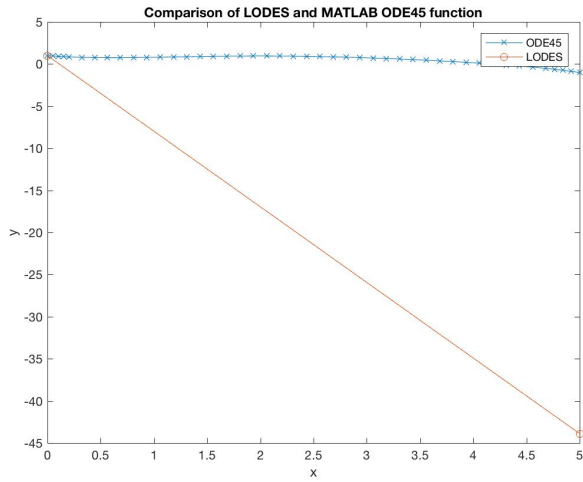


(a) Euler vs. ODE45

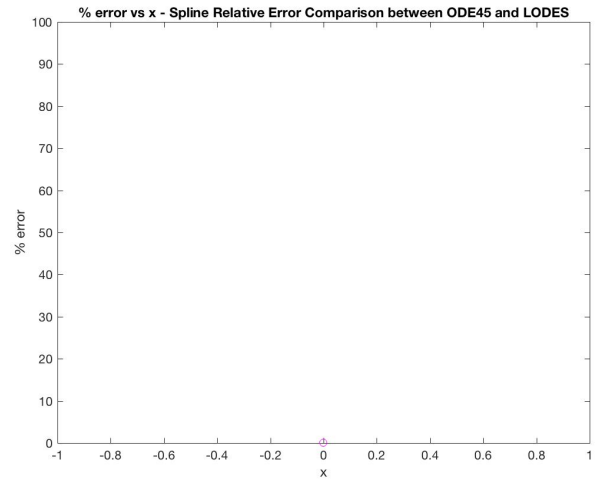


(b) Relative Error Norm - Euler

Figure 27: Euler's Method Analysis Graphs

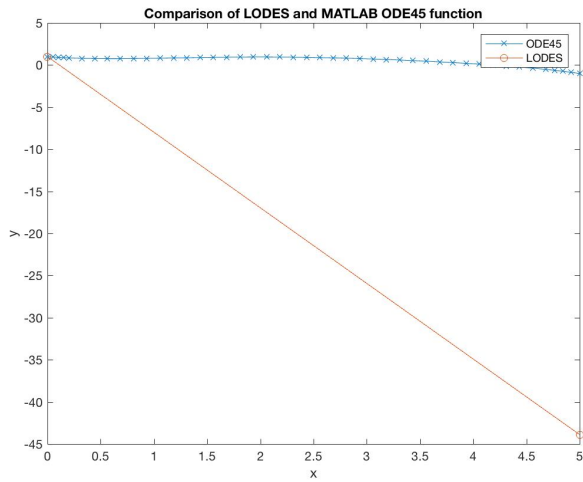


(a) Trapezoid vs. ODE45

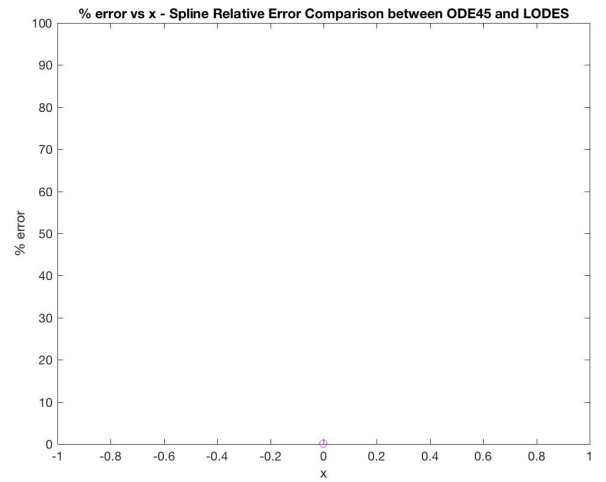


(b) Relative Error Norm - Trapezoid

Figure 28: Trapezoidal Method Analysis Graphs

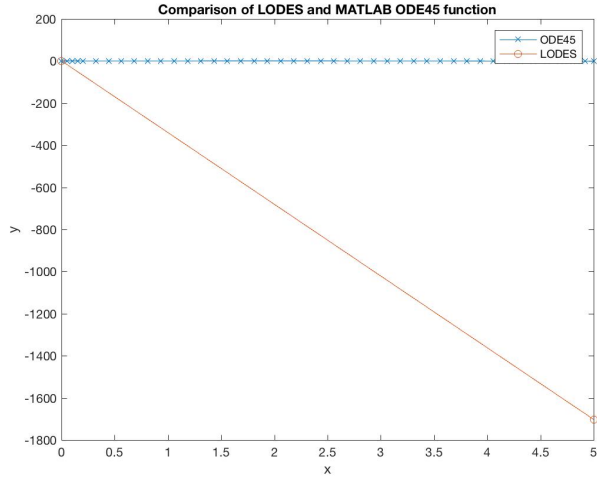


(a) Heun vs. ODE45

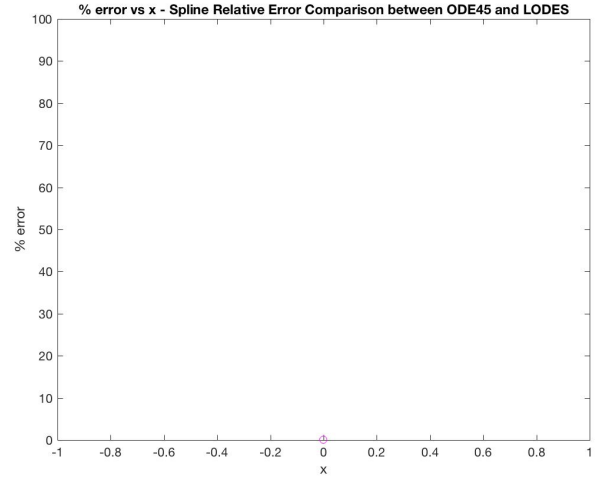


(b) Relative Error Norm - Heun

Figure 29: Heun's Method Analysis Graphs



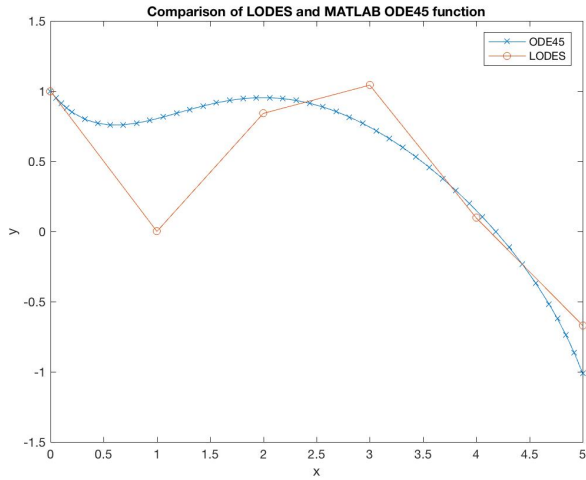
(a) Runge-Kutta 4 vs. ODE45



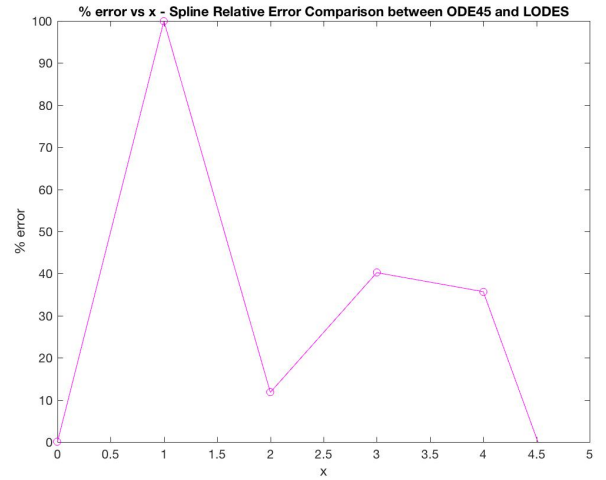
(b) Relative Error Norm - Runge-Kutta 4

Figure 30: Runge-Kutta 4 Method Analysis Graphs

7.4 Test 4: $f(x, y) = \sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

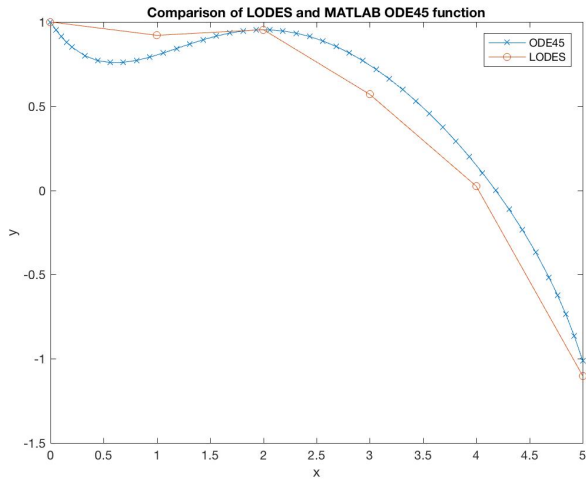


(a) Euler vs. ODE45

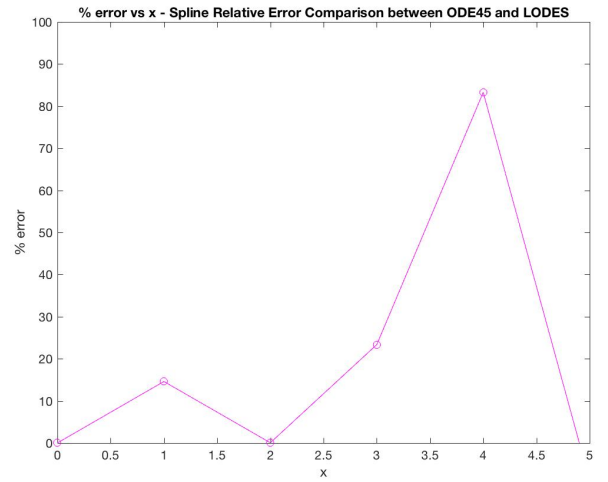


(b) Relative Error Norm - Euler

Figure 31: Euler's Method Analysis Graphs

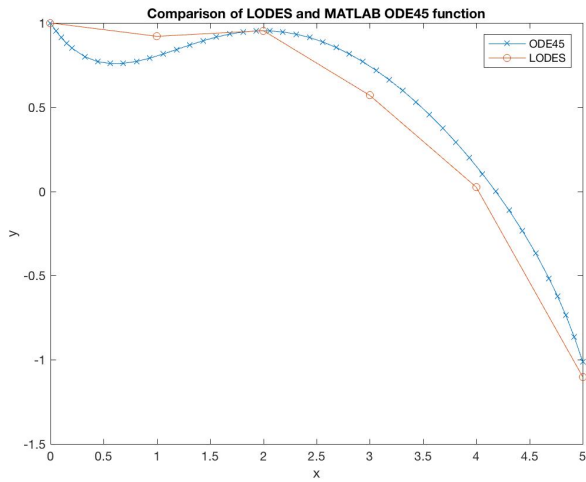


(a) Trapezoid vs. ODE45

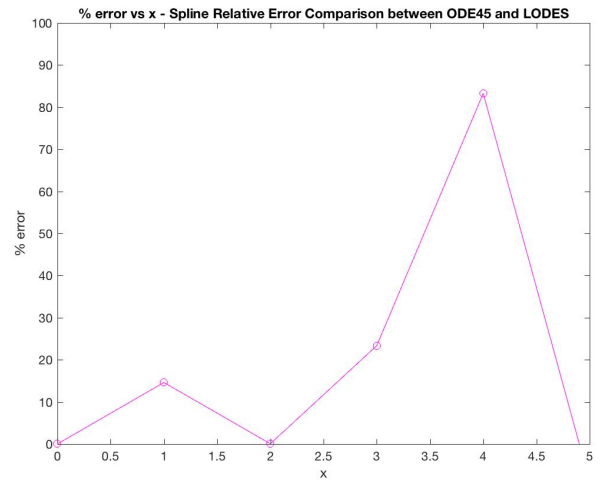


(b) Relative Error Norm - Trapezoid

Figure 32: Trapezoidal Method Analysis Graphs

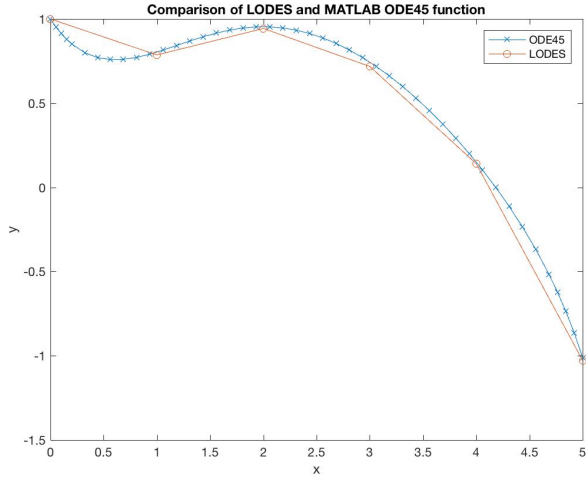


(a) Heun vs. ODE45

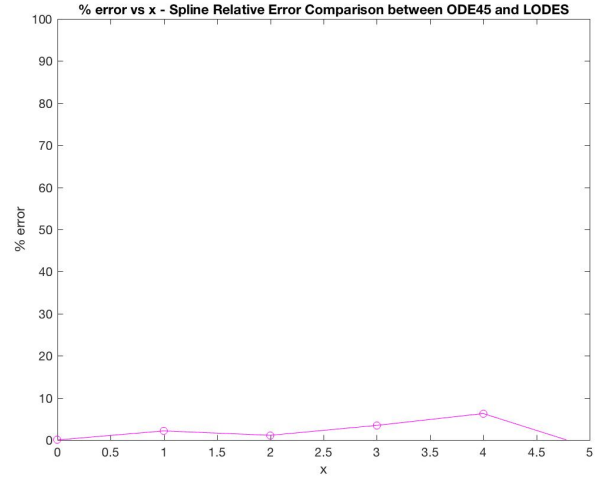


(b) Relative Error Norm - Heun

Figure 33: Heun's Method Analysis Graphs



(a) Runge-Kutta 4 vs. ODE45



(b) Relative Error Norm - Runge-Kutta 4

Figure 34: Runge-Kutta 4 Method Analysis Graphs

8 Unit Testing

Unit testing was performed and the results and execution are the similar on the level of Functional Requirement as described in Section 5 since the library is only of one level.

Further unit testing can be explored in the future.

9 Changes Due to Testing

None.

10 Automated Testing

Automated testing has been performed in Section 7. These were implemented in MATLAB's Unit Testing Framework.

11 Trace to Requirements

12 Trace to Modules

13 Code Coverage Metrics

14 Appendix A - Test Output

Running testClasses

yeuler =

1 3

ytestarr =

1.0000 7.3891

norm =

0.5886

errorarray =

0 0.5940

Verification failed in testClasses/testLODESSolution1.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".
 —> The error was not within relative tolerance.
 —> Failure table:

Index	Actual	Expected
Error	RelativeTolerance	RelativeError
-----	-----	-----
-----	-----	-----
-----	-----	-----
2	3	7.3890570168536
		-4.3890570168536
		-0.593994200727192
		2.22044604925031e-16

Actual double:
 1 3
 Expected double:
 1.0000000000000000 7.389057016853605

Stack Information:

In

ytrap =
 1 5
 ytestarr =

1.0000 7.3891

norm =

0.3204

errorarray =

0 0.3233

Verification failed in testClasses/testLODESSolution1.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	Expected
	Error 	RelativeError
	RelativeTolerance 	
-----	-----	-----

-----	-----	

2	5	7.3890570168536	-2.3890570168536	-0.323323667878653
2.22044604925031e-16				

Actual double:

1	5
---	---

Expected double:

1.0000000000000000	7.389057016853605
--------------------	-------------------

Stack Information:

In [---](matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m',17,1))

20

yheun =

1	5
---	---

ytestarr =

1.0000	7.3891
--------	--------

norm =

0.3204

errorarray =

0 0.3233

Verification failed in testClasses/testLODESSolution1.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	Expected
	Error	RelativeError
	RelativeTolerance	
-----	-----	-----

-----	-----	
2	5	7.3890570168536
		-2.3890570168536
		-0.323323667878653
		2.22044604925031e-16

Actual double:

1 5

Expected double:

1.0000000000000000 7.389057016853605

Stack Information:

In ["/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m",19,1](/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m) testClasses.testLODESSolution1) at 19

yrk =

1 7

ytestarr =

1.0000 7.3891

22

norm =

0.0522

errorarray =

0 0.0527

Verification failed in testClasses/testLODESSolution1.

Framework Diagnostic:

[verifyEqual](matlab:helpPopup matlab.unittest.TestCase.verifyEqual) failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	Expected
Error		
RelativeError	RelativeTolerance	
-----	-----	-----
2	7.3890570168536	-0.389057016853605
-0.0526531350301141	2.22044604925031e-16	

Actual double:

1 7

Expected double:

1.0000000000000000 7.389057016853605

Stack Information:

In

.
yeuler =

1.0000 1.5000 2.2500 3.3750 5.0625

ytestarr =

1.0000 1.6487 2.7183 4.4817 7.3891

norm =

0.2832

errorarray =

0 0.0902 0.1723 0.2469 0.3149

24

Verification failed in testClasses/testLODESSolution2.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index</th><th>Actual</th><th>Expected</th></tr><tr><th></th><th>Error</th><th></th></tr><tr><th>RelativeError</th><th>RelativeTolerance</th></tr></thead><tbody><tr><td>-----</td><td>-----</td><td></td></tr><tr><td>-----</td><td>-----</td><td></td></tr></tbody></table>
--

**>----- -----
strong>**

2	1.5	1.6487215144387	-0.148721514438696
		-0.0902041449306423	2.22044604925031e-16
3	2.25	2.71828199730153	-0.468281997301532
		-0.172271308777529	2.22044604925031e-16
4	3.375	4.48169001126243	-1.10669001126243
		-0.246935867603812	2.22044604925031e-16
5	5.0625	7.3890570168536	-2.3265570168536
		-0.314865213727136	2.22044604925031e-16

Actual double:

1.0000000000000000	1.5000000000000000	2.2500000000000000	3.3750000000000000
5.0625000000000000			

Expected double:

1.0000000000000000	1.648721514438696	2.718281997301532	4.481690011262426
7.389057016853605			

Stack Information:

In >/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution2) at 32

ytrap =

1.0000	1.6250	2.6406	4.2910	6.9729
--------	--------	--------	--------	--------

ytestarr =

1.0000 1.6487 2.7183 4.4817 7.3891

norm =

0.0502

errorarray =

0 0.0144 0.0286 0.0425 0.0563

26

Verification failed in testClasses/testLODESSolution2.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index</th><th>Actual</th></th><th></th></tr><tr><th>Expected</th></th><th>Error</th></th><th></th></tr><tr><th>RelativeError</th></th><th>RelativeTolerance</th></th><th></th></tr></thead><tbody><tr><td>-----</td></td><td>-----</td></td><td></td></tr><tr><td>-----</td></td><td>-----</td></td><td></td></tr><tr><td>-----</td></td><td>-----</td></td><td></td></tr></tbody></table>
--

strong>

2	1.625	1.6487215144387	-0.0237215144386962
	-0.0143878236748625	2.22044604925031e-16	
3	2.640625	2.71828199730153	-0.0776569973015317
	-0.0285684109958506	2.22044604925031e-16	
4	4.291015625	4.48169001126243	-0.190674386262426
	-0.0425451974106339	2.22044604925031e-16	
5	6.972900390625	7.3890570168536	-0.416156626228605
	-0.0563206678848734	2.22044604925031e-16	

Actual double:

1.0000000000000000	1.6250000000000000	2.6406250000000000	4.2910156250000000
6.972900390625000			

Expected double:

1.0000000000000000	1.648721514438696	2.718281997301532	4.481690011262426
7.389057016853605			

Stack Information:

In

yheun =

1.0000	1.6250	2.6406	4.2910	6.9729
--------	--------	--------	--------	--------

ytestarr =

1.0000	1.6487	2.7183	4.4817	7.3891
--------	--------	--------	--------	--------

norm =

0.0502

errorarray =

0	0.0144	0.0286	0.0425	0.0563
---	--------	--------	--------	--------

Verification failed in testClasses/testLODESSolution2.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	
Expected	Error	
RelativeError	RelativeTolerance	
-----	-----	
-----	-----	
-----	-----	
-----	-----	

2	1.625	1.6487215144387	-0.0237215144386962
	-0.0143878236748625	2.22044604925031e-16	
3	2.640625	2.71828199730153	-0.0776569973015317
	-0.0285684109958506	2.22044604925031e-16	
4	4.291015625	4.48169001126243	-0.190674386262426
	-0.0425451974106339	2.22044604925031e-16	
5	6.972900390625	7.3890570168536	-0.416156626228605
	-0.0563206678848734	2.22044604925031e-16	

Actual double:

1.0000000000000000	1.6250000000000000	2.6406250000000000	4.2910156250000000
6.972900390625000			

Expected double:

1.0000000000000000	1.648721514438696	2.718281997301532	4.481690011262426
7.389057016853605			

Stack Information:

In [---

---](/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m)

yrk =

1.0000	1.6484	2.7173	4.4794	7.3840
--------	--------	--------	--------	--------

ytestarr =

norm =

errorarray =

0 0.1723 0.3443 0.5165 0.6884

Framework Diagnostic :

Index	Actual	Expected	Error	RelativeError	RelativeTolerance
-----	-----	-----	-----	-----	-----

**>----- ----- **

2	1.6484375	1.6487215144387	-0.00028401443869619
	-0.000172263439403762	2.22044604925031e-16	
3	2.71734619140625	2.71828199730153	-0.000935805895281749
	-0.000344263728417704	2.22044604925031e-16	
4	4.47937536239624	4.48169001126243	-0.00231464886618582
	-0.000516467863767717	2.22044604925031e-16	
5	7.38397032395005	7.3890570168536	-0.00508669290355268
	-0.000688408939320743	2.22044604925031e-16	

Actual double:

1.0000000000000000	1.6484375000000000	2.717346191406250	4.479375362396240
	7.383970323950052		

Expected double:

1.0000000000000000	1.648721514438696	2.718281997301532	4.481690011262426
	7.389057016853605		

Stack Information:

In

.
yeuler =

1 -4

ytestarr =

1.0000 -1.0108

norm =

2.1023

errorarray =

0 -2.9573

32

Verification failed in testClasses/testLODESSolution3.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index 	Actual Error	Expected
RelativeError	RelativeTolerance	
-----	-----	
-----	-----	<
-----	-----	

2 -4 -1.01079006197098 -2.98920993802902
2.95730048255542 2.22044604925031e-16

Actual double:

1 -4

Expected double:

1.0000000000000000 -1.010790061970984

Stack Information:

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution3) at 49

33

ytrap =

1.0000 -43.8973

ytestarr =

1.0000 -1.0108

norm =

30.1622

```

errorarray =

    0    -42.4287

```

Verification failed in testClasses/testLODESSolution3.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	
Expected	Error	
RelativeError	RelativeTolerance	
-----	-----	
-----	-----	
strong>-----	-----	
2	-43.8973106866578	-1.01079006197098
42.4287121907991	2.22044604925031e-16	-42.8865206246869

Actual double:

1.0000000000000000 -43.897310686657846

Expected double:

1.0000000000000000 -1.010790061970984

Stack Information:

In [/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m \(testClasses.testLODESSolution3\) at 51](/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m)

yheun =

1.0000 -43.8973

ytestarr =

1.0000 -1.0108

norm =

30.1622

errorarray =

0 -42.4287

Verification failed in testClasses/testLODESSolution3.

Framework Diagnostic:

[verifyEqual](matlab:helpPopup matlab.unittest.TestCase.verifyEqual) failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	Error	RelativeTolerance
Expected			
RelativeError			
-----	-----	-----	<
-----	-----	-----	<
strong>-----	strong>-----	strong>-----	strong>-----
2	-43.8973106866578	-1.01079006197098	-42.8865206246869
42.4287121907991	2.22044604925031e-16		

Actual double:

1.0000000000000000 -43.897310686657846

Expected double:

1.0000000000000000 -1.010790061970984

Stack Information:

In

yrk =

1.0e+03 *

0.0010 -1.7028

ytestarr =

1.0000 -1.0108

norm =

1.1969e+03

errorarray =

1.0e+03 *

0 -1.6837

Verification failed in testClasses/testLODESSolution3.

Framework Diagnostic:

<a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
bold">verifyEqual failed.
—> The values are not equal using "isequaln".
—> The error was not within relative tolerance.
—> Failure table:

Index	Actual	Expected	Error	RelativeError	RelativeTolerance
2	-1702.84512882598	-1.01079006197098	-1701.83433876401	1683.66746250505	2.22044604925031e-16

Actual double:

1.0e+03 *

0.00100000000000000 -1.702845128825983

Expected double:

1.0000000000000000 -1.010790061970984

Stack Information:

In [/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m \(testClasses.testLODESSolution3\) at 55](/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m)

.
yeuler =

1.0000 0 0.8415 1.0427 0.0966 -0.6695

ytestarr =

1.0000 0.8035 0.9538 0.7436 0.1502 -1.0108

norm =

0.4569

errorarray =

0 1.0000 0.1178 0.4022 0.3570 -0.3376

Verification failed in testClasses/testLODESSolution4.

39

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	
Expected	Error	
RelativeError	RelativeTolerance	
-----	-----	
-----	-----	
-----	-----	
-----	-----	

2	0	0.803498260367072	-0.803498260367072
	-1	2.22044604925031e-16	
3	0.841470984807897	0.95380754323836	-0.112336558430464
	-0.117776965832184	2.22044604925031e-16	
4	1.04269499336001	0.743638851851789	0.299056141508218
	0.402152389918193	2.22044604925031e-16	
5	0.0966021522418493	0.150235602951416	-0.0536334507095666
	-0.356995609934823	2.22044604925031e-16	
6	-0.669532318883836	-1.01079006197098	0.341257743087148
	-0.337614857848636	2.22044604925031e-16	

Actual double:

1.0000000000000000	0	0.841470984807897	1.042694993360007
0.096602152241849	-0.669532318883836		

Expected double:

1.0000000000000000	0.803498260367072	0.953807543238360	0.743638851851789
0.150235602951416	-1.010790061970984		

Stack Information:

In

The identifier was:

MATLAB: class:InvalidHandleThere was an error! The message was:

Invalid or deleted object.

ytrap =

1.0000	0.9207	0.9541	0.5700	0.0251	-1.1012
--------	--------	--------	--------	--------	---------

ytestarr =

1.0000 0.8035 0.9538 0.7436 0.1502 -1.0108

norm =

0.1277

errorarray =

0 0.1459 0.0003 0.2335 0.8328 -0.0894

41

Verification failed in testClasses/testLODESSolution4.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	
Expected	Error	
strong> RelativeError		RelativeTolerance
		

```

<strong>----- </strong>    <strong>----- </strong>    <strong>
----- </strong>    <strong>----- </strong>    <
strong>----- </strong>    <strong>----- </
strong>

```

```

2      0.920735492403948      0.803498260367072      0.117237232036877
      0.14590850760936      2.22044604925031e-16
3      0.954130968258594      0.95380754323836      0.000323425020233414
0.000339088343897265      2.22044604925031e-16
4      0.56999267801283      0.743638851851789      -0.173646173838959
      -0.233508743399502      2.22044604925031e-16
5      0.0251222646962669      0.150235602951416      -0.125113338255149
      -0.832780884139753      2.22044604925031e-16
6      -1.10119664749119      -1.01079006197098      -0.0904065855202025
      0.0894415061263214      2.22044604925031e-16

```

42

Actual double:

```

1.0000000000000000      0.920735492403948      0.954130968258594      0.569992678012830
0.025122264696267      -1.101196647491187

```

Expected double:

```

1.0000000000000000      0.803498260367072      0.953807543238360      0.743638851851789
0.150235602951416      -1.010790061970984

```

Stack Information:

In [---](matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m',68,1))

The identifier was:

MATLAB: class:InvalidHandleThere was an error! The message was:

Invalid or deleted object.

yheun =

1.0000	0.9207	0.9541	0.5700	0.0251	-1.1012
--------	--------	--------	--------	--------	---------

ytestarr =

1.0000	0.8035	0.9538	0.7436	0.1502	-1.0108
--------	--------	--------	--------	--------	---------

norm =

0.1277

errorarray =

0	0.1459	0.0003	0.2335	0.8328	-0.0894
---	--------	--------	--------	--------	---------

Verification failed in testClasses/testLODESSolution4.

Framework Diagnostic:

verifyEqual failed.
—> The values are not equal using "isequaln".
—> The error was not within relative tolerance.
—> Failure table:

Index		Actual			
Expected				Error	
RelativeError				RelativeTolerance	
					
-----		-----		-----	

-----				-----	

2	0.920735492403948	0.803498260367072		0.117237232036877	
	0.14590850760936	2.22044604925031e-16			
3	0.954130968258594	0.95380754323836		0.000323425020233414	
	0.000339088343897265	2.22044604925031e-16			
4	0.56999267801283	0.743638851851789		-0.173646173838959	
	-0.233508743399502	2.22044604925031e-16			
5	0.0251222646962669	0.150235602951416		-0.125113338255149	
	-0.832780884139753	2.22044604925031e-16			
6	-1.10119664749119	-1.01079006197098		-0.0904065855202025	
	0.0894415061263214	2.22044604925031e-16			

Actual double:

```
1.0000000000000000  0.920735492403948  0.954130968258594  0.569992678012830
0.025122264696267  -1.101196647491187
```

Expected double:

```
1.0000000000000000  0.803498260367072  0.953807543238360  0.743638851851789
0.150235602951416  -1.010790061970984
```

Stack Information:

In

testing/testClasses.m (testClasses.testLODESSolution4) at 70

yrk =

1.0000	0.7863	0.9434	0.7180	0.1408	-1.0284
--------	--------	--------	--------	--------	---------

ytestarr =

1.0000	0.8035	0.9538	0.7436	0.1502	-1.0108
--------	--------	--------	--------	--------	---------

norm =

0.0187

errorarray =

0	0.0214	0.0109	0.0345	0.0626	-0.0174
---	--------	--------	--------	--------	---------

Verification failed in testClasses/testLODESSolution4.

Framework Diagnostic:

[verifyEqual](matlab:helpPopup matlab.unittest.TestCase.verifyEqual) failed.
—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Expected	Actual	Error	RelativeTolerance
RelativeError	RelativeTolerance	RelativeError	RelativeTolerance	RelativeTolerance
-----	-----	-----	-----	-----
-----	-----	-----	-----	-----
-----	-----	-----	-----	-----
2	0.78631839551037	0.803498260367072	-0.0171798648567016	
	-0.0213813342282199	2.22044604925031e-16		
3	0.943371788846183	0.95380754323836	-0.0104357543921774	
	-0.0109411531353023	2.22044604925031e-16		
4	0.717989268823369	0.743638851851789	-0.0256495830284202	
	-0.0344919889063735	2.22044604925031e-16		
5	0.14083244087226	0.150235602951416	-0.00940316207915609	
	-0.0625894388176213	2.22044604925031e-16		
6	-1.02836702616913	-1.01079006197098	-0.0175769641981469	
	0.0173893322257965	2.22044604925031e-16		

Actual double:

```
1.0000000000000000  0.786318395510370  0.943371788846183  0.717989268823369
0.140832440872260  -1.028367026169131
```

Expected double:

```
1.0000000000000000  0.803498260367072  0.953807543238360  0.743638851851789
0.150235602951416  -1.010790061970984
```

Stack Information:

```
In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m',72,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution4) at 72</a>
```

```
.  
Done testClasses  
-----
```

Failure Summary:

Name	Failed	Incomplete	Reason(s)
testClasses/testLODESSolution1	X		Failed by verification.
testClasses/testLODESSolution2	X		Failed by verification.
testClasses/testLODESSolution3	X		Failed by verification.
testClasses/testLODESSolution4	X		Failed by verification.

```
result =
```

```
1 4 <a href="matlab:helpPopup matlab.unittest.TestResult" style="font-weight:bold">  
    TestResult</a> array with properties:
```

```
Name  
Passed  
Failed  
Incomplete  
Duration  
Details
```

Totals:

0 Passed, 4 Failed, 0 Incomplete.

12.6715 seconds testing time.