Test Report: Library of ODE Solvers (LODES)

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1 Revision History

Date	Version	Notes
December 17, 2017	1.0	Initial draft.

2 Symbols, Abbreviations and Acronyms

2.1 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the numeral analysis and ordinary differential equation literature and with existing documentation for solving ordinary differential equations. The symbols are listed in alphabetical order.

symbol	unit	type	description
dy/dx	-	$\mathbb{R}$	Rate of change of y with respect to x
$\varepsilon_{\text{relative}}$	-	$\mathbb{R}$	The relative error
F	-	$\mathbb{R}^3 \to \mathbb{R}$	Order function applied to the Runge Kutta Method
$f(x, y)$	-	$\mathbb{R}^2 \to \mathbb{R}$	Explicit form of the ODE function containing (x, y)
$f_x(x, y)$	-	$\mathbb{R}^2 \to \mathbb{R}$	Explicit form of the derivative of $f(x, y)$ with respect to x
$f_y(x, y)$	-	$\mathbb{R}^2 \to \mathbb{R}$	Explicit form of the derivative of $f(x, y)$ with respect to y
h	-	$\mathbb{R}$. $h > 0$	Step-size from $x_{(0)}$ to the next point $x_{(1)}$, where $x_{(1)} = x_{(0)} + h$
K_1, K_2, K_3, K_4	$\mathbb{R}$	-	Intermediary variables used in the Runge-Kutta method
n	-	$\mathbb{R}$	Reference recursion step
o	-	$\mathbb{R}$	Solution variable
T	-	-	Test
x_0	-	$\mathbb{R}$	Initial value x
x_{k}	-	$\mathbb{R}$	Final value x
x_{n}	-	$\mathbb{R}$	Intermediate n^{th} value x
y_0	-	$\mathbb{R}$	Initial value y
y_{k}	-	$\mathbb{R}$	Final value y
y_{n}	-	$\mathbb{R}$	Intermediate n^{th} value y
y'	-	$\mathbb{R} \to \mathbb{R}$	Implicit form of the first order ODE $= f(x, y)$
$y^{(\text{n})}$	-	$\mathbb{R} \to \mathbb{R}$	Implicit form of the ODE to the n^{th} order
y	-	1 x $\mathbb{R}^k$	The array containing all intermediate y_{n} values

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3 Introduction

This document serves as the Test Report for the Library of ODE Solvers (LODES) which lays out in detail the results of the execution of the LODES Test Plan. The requirements for the software are described in the LODES Software Requirements Specification. The aim of the Test Report is to document and show that LODES produces accurate and valid (in the scope of detailed in the SRS and Test Plan) results.

The Overview (Section 4) discusses how testing slightly deviates from the Test Plan in its execution.

The following sections describe the evaluation of the software against the Functional Requirements (Section 5), Non-Functional Requirements (Section 6). It will also compare LODES to existing software in Comparison to Existing Implementation (Section 7).

In the sections after, Unit Testing (Section 8), Changes Due to Testing (Section 9), Automated Testing (Section 10), Trace to Requirements (Section 11), Trace to Modules (Section 12, and Code Coverage Metrics (Section 13) are discussed.

4 Overview

The Test Report documents the results of the tests carried out to verify the behaviour of the LODES software. The Test Plan suggests that it should carry out testing such that that the test report shall describe and analyze the numerical differences of the hand-executed solution to the numerical analysis methods versus the solutions obtained from LODES. This Test Report carries out the tests, with the same intent of verification, and instead of only comparing against the hand-executed solution, it compares the results with the MATLAB generated solutions as well.

5 Functional Requirements Evaluation

5.1 Calculation Tests

5.1.1 Simple Cases: T-1, T-5, T-9, T-13

Input: $f(x, y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$

Test Plan Test Case: T-1
Expected output: The value of $y_k = 3$. This test passes.

```
>> lodes(1, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:      0     2

The values of y are:      1     3


ans =

     0     2
```

Figure 1: T-1 result.

Test Plan Test Case: T-5
Expected output: The value of $y_k = 5$. This test passes.

```
>> lodes(2, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:      0     2

The values of y are:      1     5


ans =

     0     2
```

Figure 2: T-5 result.

Test Plan Test Case: T-9
Expected output: The value of $y_k = 5$. This test passes.

```
>> lodes(3, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:      0     2

The values of y are:      1     5


ans =

     0     2
```

Figure 3: T-9 result.

Test Plan Test Case: T-13
Expected output: The value of $y_k = 7$. This test passes.

```
>> lodes(4, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:      0     2

The values of y are:      1     7


ans =

     0     2
```

Figure 4: T-13 result.

5.1.2 Simple-Iterative Cases: T-2, T-6, T-10, T-14

Input: $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

Test Plan Test Case: T-2
Expected output: The value of $y_k = 5.0625$. This test passes.

```
>> lodes(1, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:          0    0.5000    1.0000    1.5000    2.0000

The values of y are:     1.0000    1.5000    2.2500    3.3750    5.0625
```

Figure 5: T-2 result.

Test Plan Test Case: T-6
Expected output: The value of $y_k = 6.9729$. This test passes.

```
>> lodes(2, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:          0    0.5000    1.0000    1.5000    2.0000

The values of y are:     1.0000    1.6250    2.6406    4.2910    6.9729
```

Figure 6: T-6 result.

Test Plan Test Case: T-10
Expected output: The value of $y_k = 6.9729$. This test passes.

```
>> lodes(3, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:          0    0.5000    1.0000    1.5000    2.0000

The values of y are:     1.0000    1.6250    2.6406    4.2910    6.9729
```

Figure 7: T-10 result.

Test Plan Test Case: T-14
Expected output: The value of $y_k = 7.38340$. This test passes.

```
>> lodes(4, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:          0    0.5000    1.0000    1.5000    2.0000

The values of y are:     1.0000    1.6484    2.7173    4.4794    7.3840
```

Figure 8: T-14 result.

5.1.3 Linear Trigonometric Cases: T-3, T-7, T-11, T-15

Input: $f(x, y) = y, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

Test Plan Test Case: T-3
Expected output: The value of $y_k = -4$. This test passes.

```
>> lodes(1, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0     5

The values of y are:      1    -4
```

Figure 9: T-3 result.

Test Plan Test Case: T-7
Expected output: The value of $y_k = -43.8973$. This test passes.

```
>> lodes(2, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0     5

The values of y are:     1.0000  -43.8973
```

Figure 10: T-7 result.

Test Plan Test Case: T-11
Expected output: The value of $y_k = -43.8973$. This test passes.

```
>> lodes(3, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0     5

The values of y are:     1.0000  -43.8973
```

Figure 11: T-11 result.

Test Plan Test Case: T-15
Expected output: The value of $y_k = -1702.8$. This test passes.

```
>> lodes(4, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0     5

The values of y are:    1.0e+03 *

    0.0010   -1.7028
```

Figure 12: T-15 result.

5.1.4 Linear Trigonometric Iterative Cases: T-4, T-8, T-12, T-16

Input: $f(x, y) = sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

Test Plan Test Case: T-4
Expected output: The value of $y_k = -0.6695$. This test passes.

```
>> lodes(1, 'sin(x) - y^2', 0, 1, 5, 1, 1, 1)
The values of x are:      0     1     2     3     4     5

The values of y are:     1.0000         0    0.8415    1.0427    0.0966   -0.6695
```

Figure 13: T-4 result.

Test Plan Test Case: T-8
Expected output: The value of $y_k = -1.1012$. This test passes.

```
>> lodes(2, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are:      0     1     2     3     4     5

The values of y are:     1.0000    0.9207    0.9541    0.5700    0.0251   -1.1012
```

Figure 14: T-8 result.

Test Plan Test Case: T-12
Expected output: The value of $y_k = -1.1012$. This test passes.

```
>> lodes(3, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are:      0     1     2     3     4     5

The values of y are:     1.0000    0.9207    0.9541    0.5700    0.0251   -1.1012
```

Figure 15: T-12 result.

Test Plan Test Case: T-16
Expected output: The value of $y_k = -1.0284$. This test passes.

```
>> lodes(4, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are:      0     1     2     3     4     5

The values of y are:     1.0000    0.7863    0.9434    0.7180    0.1408   -1.0284
```

Figure 16: T-16 result.

6 Nonfunctional Requirements Evaluation

6.1 Speed Test Plan Test Case: T-17

Input: $f(x, y) = sin(x) - y^2, h = 1e - 3, x_0 = 0, y_0 = 1, x_k = 5$

Test Plan Test Case: T-17
Expected output: The execution time difference of LODES is not greater than 4 times that of MATLAB. This test fails as the execution time of LODES is 2,000 times greater than MATLAB's.

Profile Summary
Generated 15-Dec-2017 23:08:41 using performance time.

Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
lodes	1	472.941 s	0.003 s	
rk	1	472.766 s	2.209 s	

Figure 17: T-17 LODES rk() result.

Profile Summary
Generated 15-Dec-2017 23:29:45 using performance time.

Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
testode45	1	0.273 s	0.034 s	
ode45	1	0.239 s	0.110 s	

Figure 18: T-17 MATLAB ode45() result.

7 Comparison to Existing Implementation

This section compares the calculation of LODES to MATLAB's ode45() function.

Test Plan Test Case: T-18
The following input parameters were tested (as above):

- Test 1 (7.1): $f(x, y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$
- Test 2 (7.2): $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

- Test 3 (7.3): $f(x, y) = sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$
- Test 4 (7.4): $f(x, y) = sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

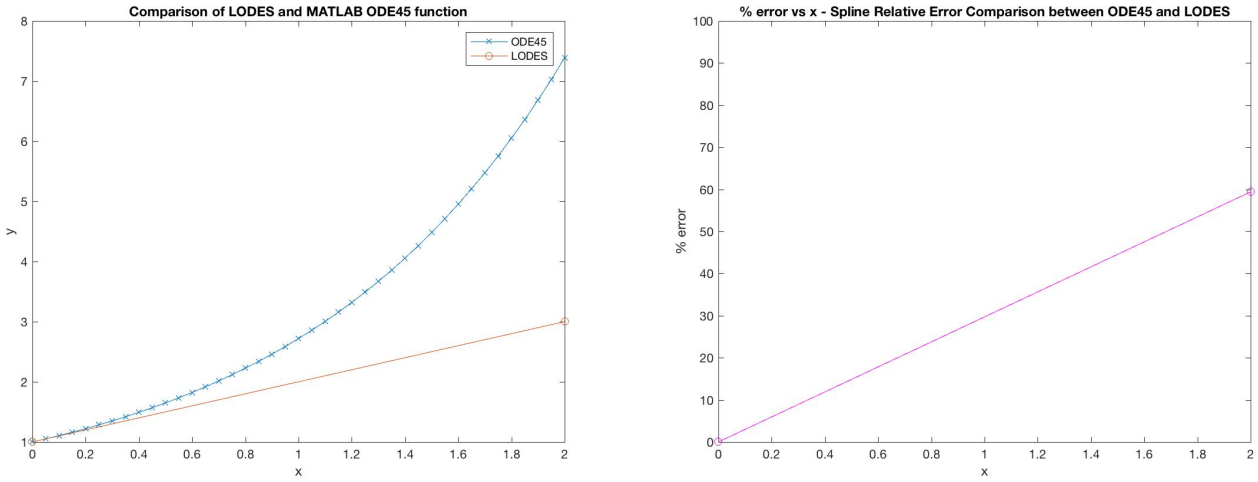
This phase of testing is automated and the results are shown in Section 15. The following equation was implemented to compare the results labeled as **norm**:

$$\epsilon_{\text{relative}} = \text{norm} = \frac{\|\text{Result}_{\text{MATLAB}} - \text{Result}_{\text{LODES}}\|}{\|\text{Result}_{\text{MATLAB}}\|}$$

Output/Result: The $\epsilon_{\text{relative}}$ vs. h plots. The intermediate y values (Result) are compared according to the following formula of the relative error norm percentage -

$$\epsilon_{\text{relative}} = 100 * \frac{\text{Result}_{\text{MATLAB}} - \text{Result}_{\text{LODES}}}{\text{Result}_{\text{MATLAB}}}$$

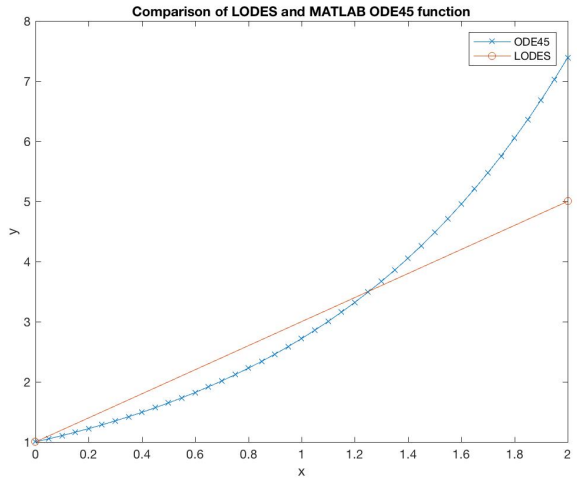
7.1 Test 1: $f(x, y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$



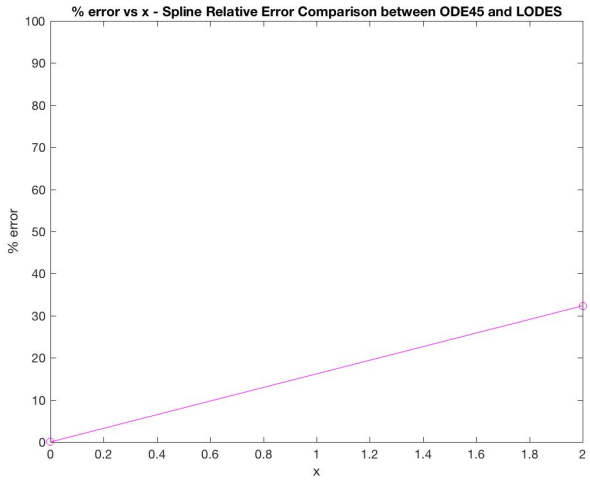
(a) Euler vs. ODE45

(b) Relative Error Norm - Euler

Figure 19: Euler's Method Analysis Graphs

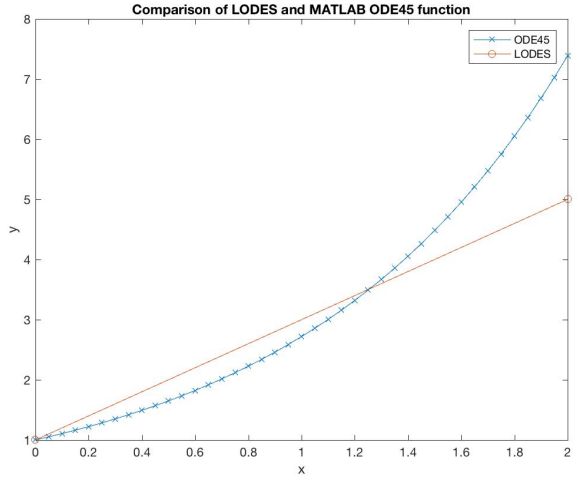


(a) Trapezoid vs. ODE45

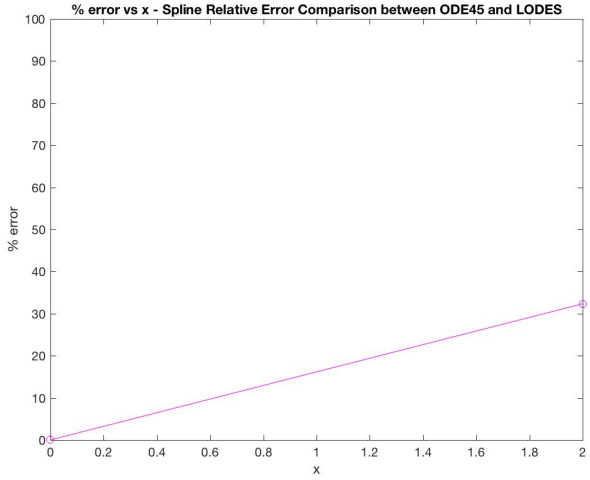


(b) Relative Error Norm - Trapezoid

Figure 20: Trapezoidal Method Analysis Graphs

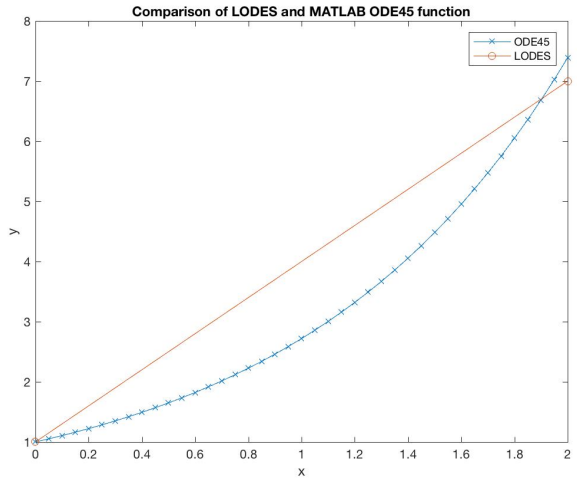


(a) Heun vs. ODE45

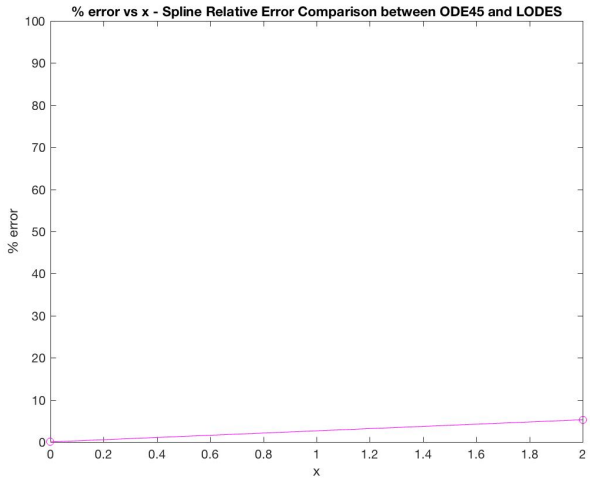


(b) Relative Error Norm - Heun

Figure 21: Heun's Method Analysis Graphs



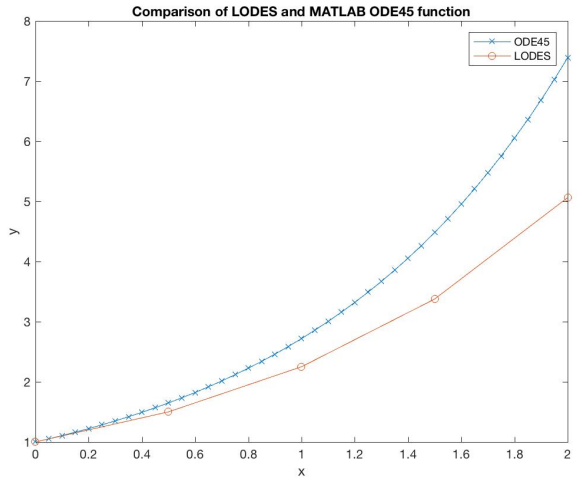
(a) Runge-Kutta 4 vs. ODE45



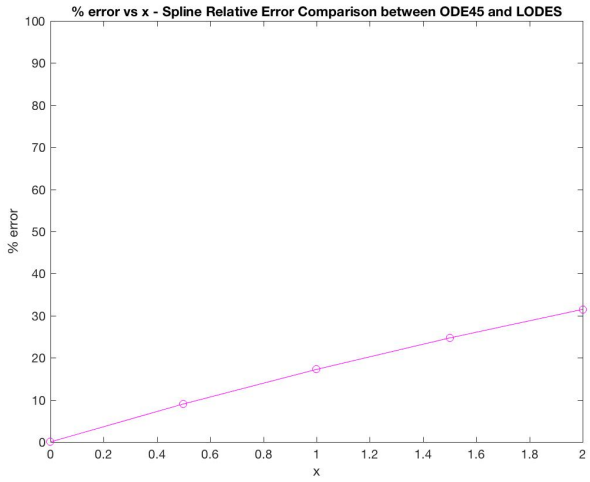
(b) Relative Error Norm - Runge-Kutta 4

Figure 22: Runge-Kutta 4 Method Analysis Graphs

7.2 Test 2: $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

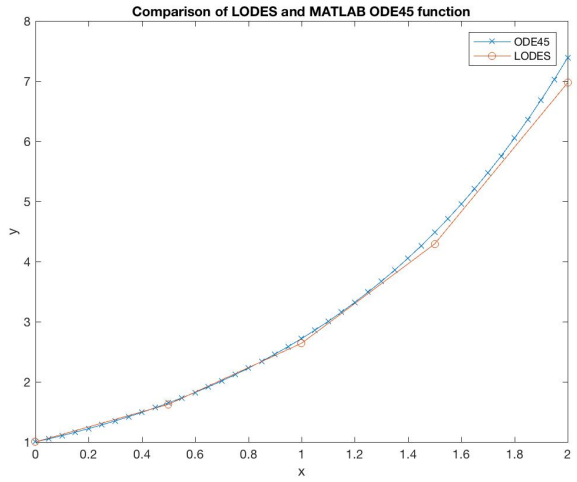


(a) Euler vs. ODE45

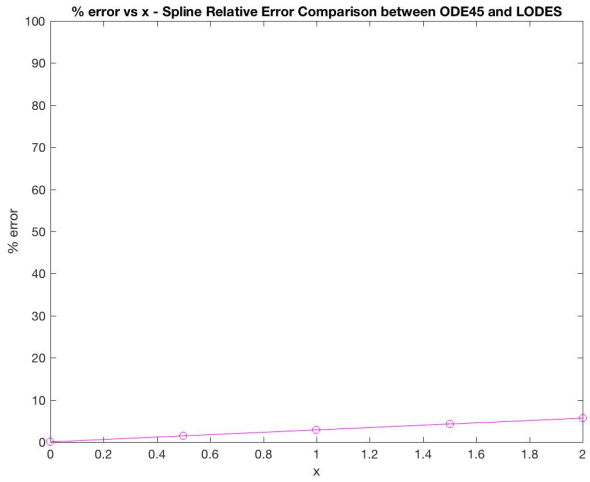


(b) Relative Error Norm - Euler

Figure 23: Euler's Method Analysis Graphs

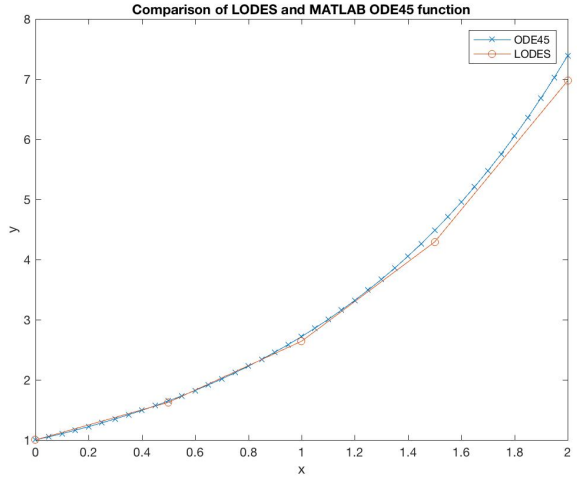


(a) Trapezoid vs. ODE45

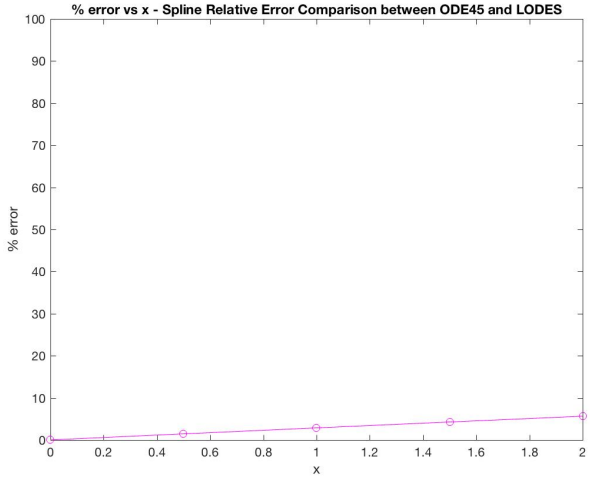


(b) Relative Error Norm - Trapezoid

Figure 24: Trapezoidal Method Analysis Graphs

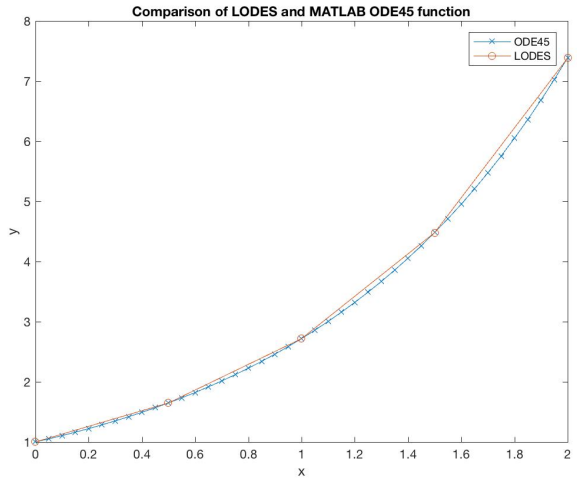


(a) Heun vs. ODE45

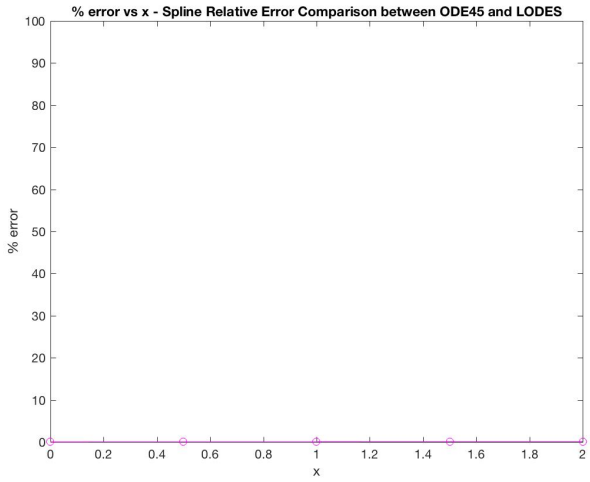


(b) Relative Error Norm - Heun

Figure 25: Heun's Method Analysis Graphs



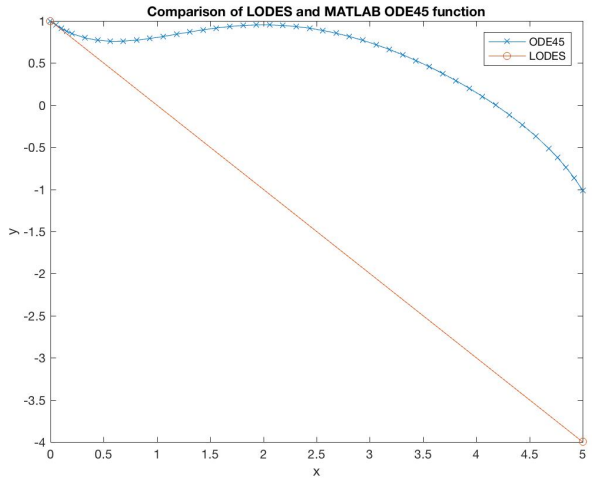
(a) Runge-Kutta 4 vs. ODE45



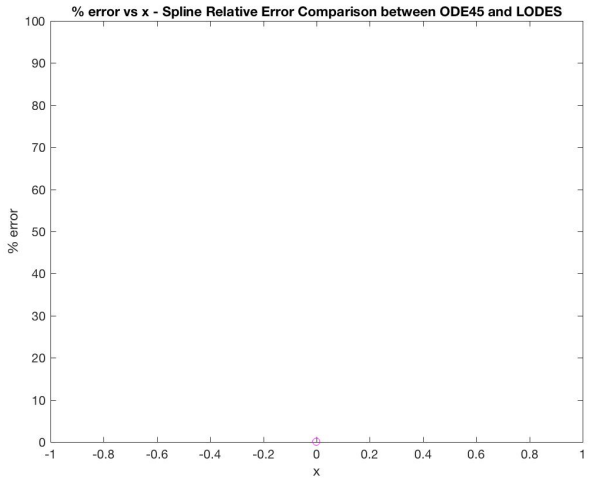
(b) Relative Error Norm - Runge-Kutta 4

Figure 26: Runge-Kutta 4 Method Analysis Graphs

7.3 Test 3: $f(x, y) = sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

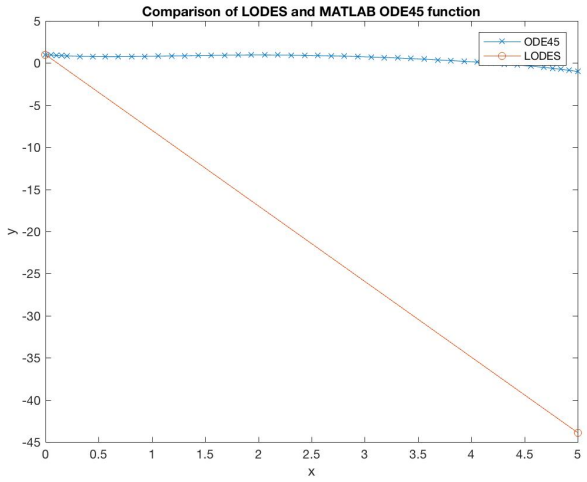


(a) Euler vs. ODE45

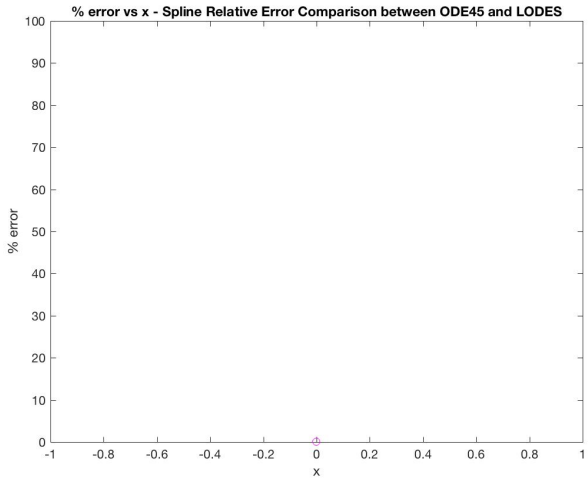


(b) Relative Error Norm - Euler

Figure 27: Euler's Method Analysis Graphs

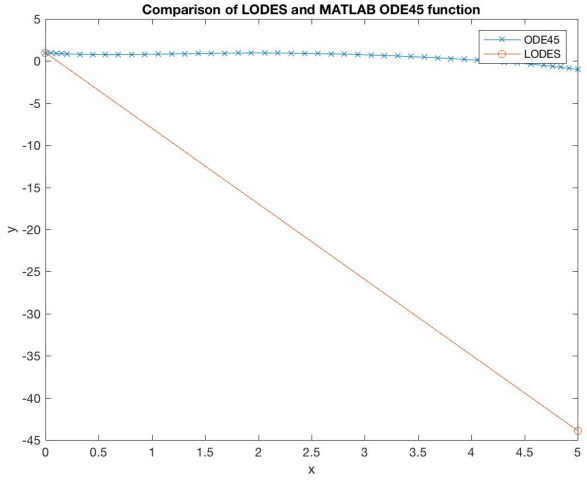


(a) Trapezoid vs. ODE45

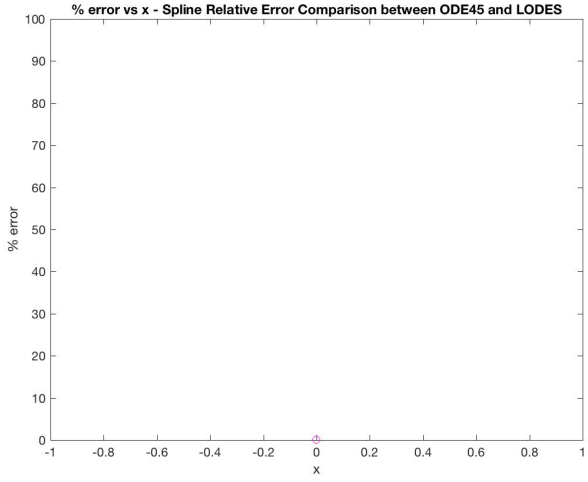


(b) Relative Error Norm - Trapezoid

Figure 28: Trapezoidal Method Analysis Graphs

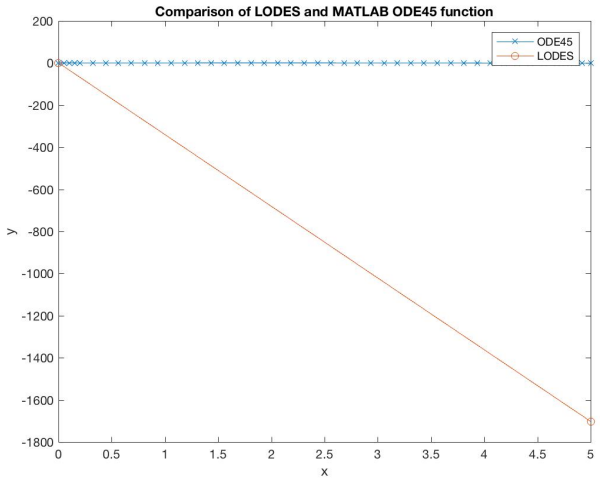


(a) Heun vs. ODE45

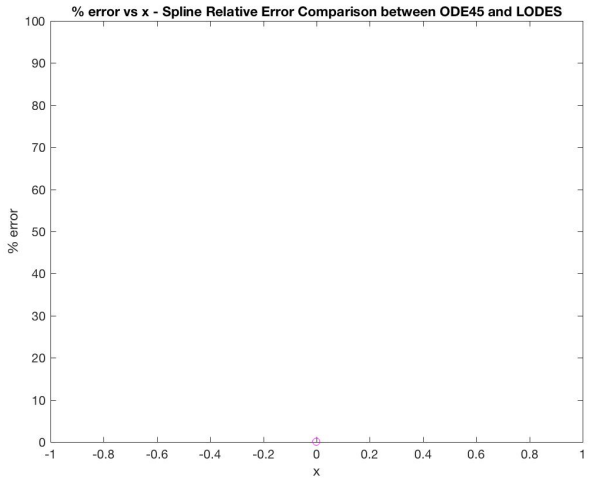


(b) Relative Error Norm - Heun

Figure 29: Heun's Method Analysis Graphs



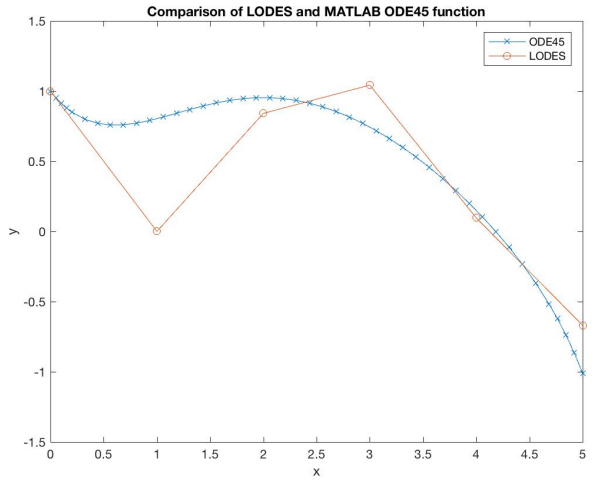
(a) Runge-Kutta 4 vs. ODE45



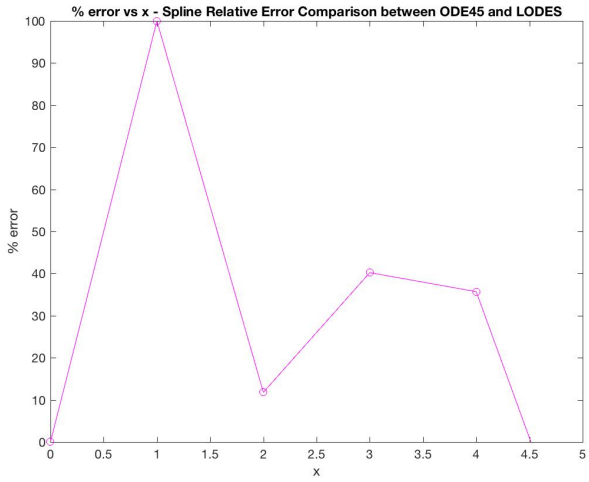
(b) Relative Error Norm - Runge-Kutta 4

Figure 30: Runge-Kutta 4 Method Analysis Graphs

7.4 Test 4: $f(x, y) = sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

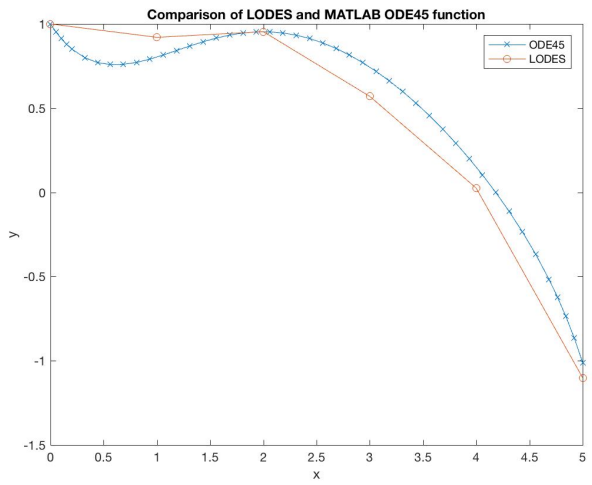


(a) Euler vs. ODE45

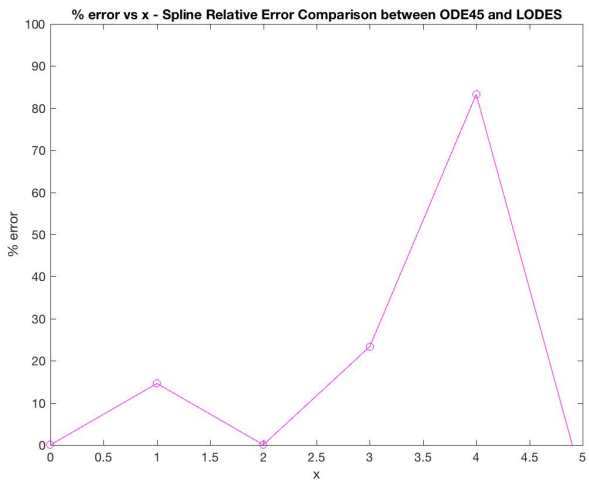


(b) Relative Error Norm - Euler

Figure 31: Euler's Method Analysis Graphs

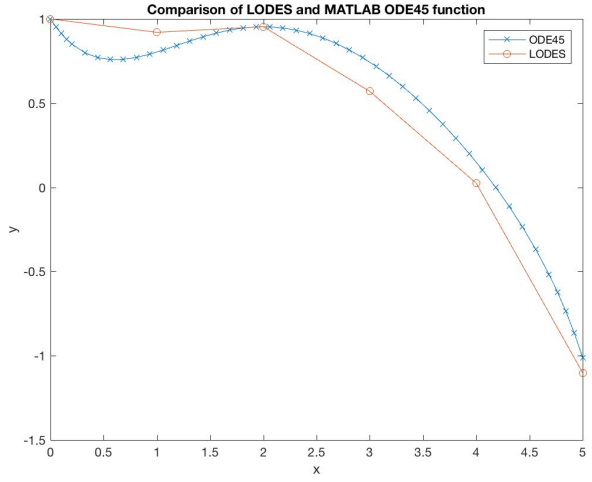


(a) Trapezoid vs. ODE45

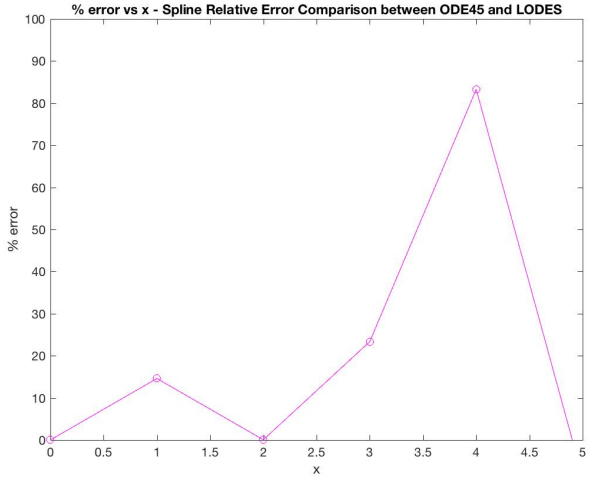


(b) Relative Error Norm - Trapezoid

Figure 32: Trapezoidal Method Analysis Graphs

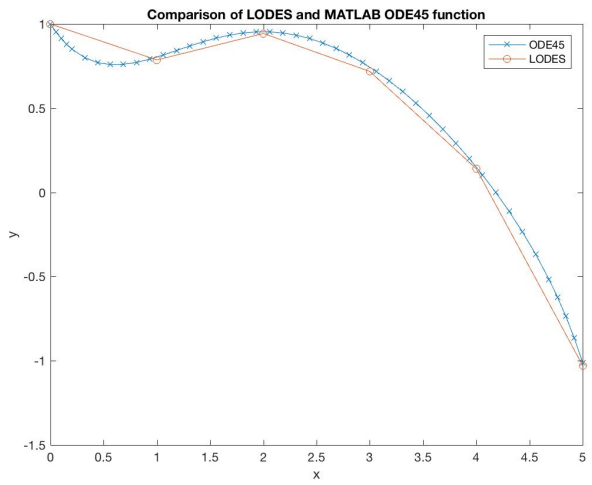


(a) Heun vs. ODE45

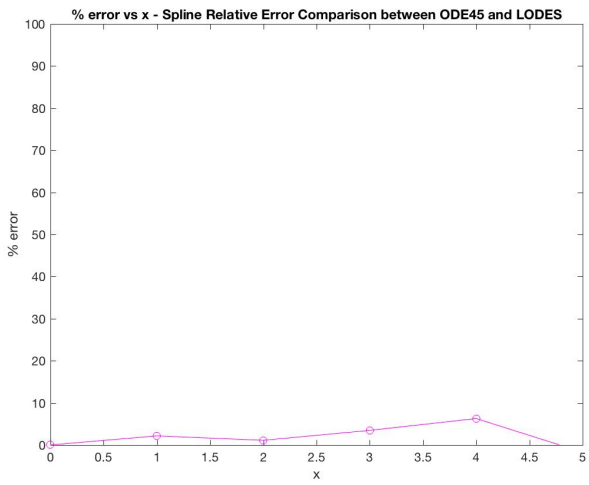


(b) Relative Error Norm - Heun

Figure 33: Heun's Method Analysis Graphs



(a) Runge-Kutta 4 vs. ODE45



(b) Relative Error Norm - Runge-Kutta 4

Figure 34: Runge-Kutta 4 Method Analysis Graphs

8 Unit Testing

Unit testing was performed and the results and execution are the similar on the level of Functional Requirement as described in Section 5 since the library is only of one level.

Further unit testing can be explored in the future.

Unit testing of the modules implemented by MATLAB and the Operating System (OS) has been deemed unnecessary due to proof of use. The parser functionality tests have been embedded in the functional tests of the modules of LODES in Section 5.

9 Changes Due to Testing

None.

10 Automated Testing

Automated testing has been performed in Section 7. These were implemented in MATLAB's Unit Testing Framework.

11 Trace to Requirements

The following table shows the traceability mapping for the test cases laid out in this Test Report to the requirements described in the Commonality Analysis.

Table 1: Requirements Traceability Matrix

Test Number	**CA Requirements**
T1	IM1, O1, O2, O3, O4, O5
T2	IM1, O1, O2, O3, O4, O5
T3	IM1, O1, O2, O3, O4, O5
T4	IM1, O1, O2, O3, O4, O5
T5	IM2, O1, O2, O3, O4, O5
T6	IM2, O1, O2, O3, O4, O5
T7	IM2, O1, O2, O3, O4, O5
T8	IM2, O1, O2, O3, O4, O5
T9	IM3, O1, O2, O3, O4, O5
T10	IM3, O1, O2, O3, O4, O5
T11	IM3, O1, O2, O3, O4, O5
T12	IM3, O1, O2, O3, O4, O5
T13	IM4, O1, O2, O3, O4, O5
T14	IM4, O1, O2, O3, O4, O5
T15	IM4, O1, O2, O3, O4, O5
T16	IM4, O1, O2, O3, O4, O5
T17	NFR1
T18	NFR2

12 Trace to Modules

The following table shows the traceability mapping for the test cases laid out in this Test Report to the modules in the Module Guide (MG).

Table 2: Design Traceability Matrix

Test Number	**MG Modules**
T1	M1, M2, M3, M4, M5
T2	M1, M2, M3, M4, M5
T3	M1, M2, M3, M4, M5
T4	M1, M2, M3, M4, M5
T5	M1, M2, M3, M4, M6
T6	M1, M2, M3, M4, M6
T7	M1, M2, M3, M4, M6
T8	M1, M2, M3, M4, M6
T9	M1, M2, M3, M4, M7
T10	M1, M2, M3, M4, M7
T11	M1, M2, M3, M4, M7
T12	M1, M2, M3, M4, M7
T13	M1, M2, M3, M4, M8
T14	M1, M2, M3, M4, M8
T15	M1, M2, M3, M4, M8
T16	M1, M2, M3, M4, M8
T17	-
T18	-

13 Code Coverage Metrics

The following Code Coverage figure was obtained from MATLAB's Code Coverage Report Tool. Running the test program with in-boundary conditions (excluding extraneous scenarios), the code coverage tool produces the following results:

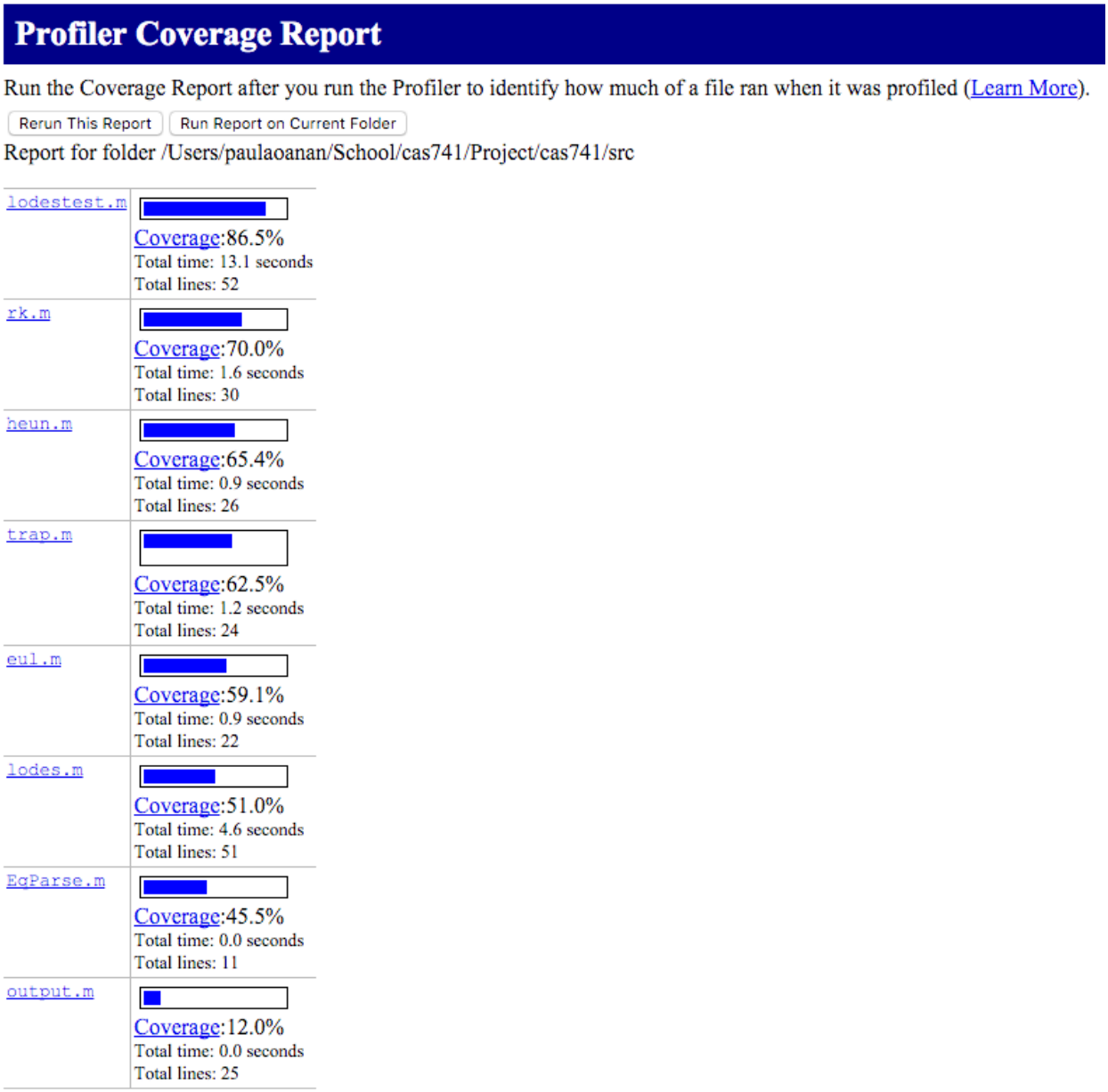


Figure 35: Code Coverage Tool result.

14 Code Inspection Checklist

This is a placeholder for the completed code inspection checklist.

15 Appendix A - Test Output

```
Running testClasses

yeuler =

     1     3


ytestarr =

    1.0000    7.3891


norm =

    0.5886


errorarray =

         0    0.5940

================================================================================
<strong>Verification failed in testClasses/testLODESSolution1.</strong>

    ----------
    Framework Diagnostic:
    ----------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
```

```
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
            <strong>Index</strong><strong>    Actual</strong><strong>        Expected
               </strong><strong>          Error       </strong><strong>       RelativeError
                  </strong><strong>     RelativeTolerance  </strong>
            <strong>_____</strong>    <strong>______</strong>    <strong>_______________
               </strong>    <strong>________________</strong>    <strong>
                ________________</strong>    <strong>____________________</strong>

            2         3          7.3890570168536     -4.3890570168536     -0.593994200727192
                    2.22044604925031e-16

    Actual double:
             1     3
    Expected double:
           1.000000000000000    7.389057016853605

    ____________________
    Stack Information:
    ____________________
    In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
       testing/testClasses.m',15,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
       testing/testClasses.m (testClasses.testLODESSolution1) at 15</a>
================================================================================

ytrap =

     1     5


ytestarr =
```

```
    1.0000    7.3891

norm =

    0.3204

errorarray =

         0    0.3233

================================================================================
<strong>Verification failed in testClasses/testLODESSolution1.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
            <strong>Index</strong><strong>    Actual</strong><strong>       Expected
               </strong><strong>         Error      </strong><strong>      RelativeError
                  </strong><strong>     RelativeTolerance  </strong>
            <strong>_____</strong>    <strong>______</strong>    <strong>______________
               </strong>    <strong>________________</strong>    <strong>
                ________________</strong>    <strong>__________________</strong>
```

```
            2          5          7.3890570168536     -2.3890570168536     -0.323323667878653
                  2.22044604925031e-16

    Actual double:
             1     5
    Expected double:
          1.000000000000000    7.389057016853605

    ---------------------
    Stack Information:
    ---------------------
    In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
       testing/testClasses.m',17,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
       testing/testClasses.m (testClasses.testLODESSolution1) at 17</a>
================================================================================

yheun =

     1     5


ytestarr =

    1.0000    7.3891


norm =

    0.3204


errorarray =
```

```
     0     0.3233

================================================================================
<strong>Verification failed in testClasses/testLODESSolution1.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
            <strong>Index</strong><strong>    Actual</strong><strong>        Expected
               </strong><strong>          Error      </strong><strong>      RelativeError
                  </strong><strong>     RelativeTolerance  </strong>
            <strong>_____</strong>    <strong>______</strong>    <strong>______________
               </strong>    <strong>________________</strong>    <strong>
                ________________</strong>    <strong>__________________</strong>

            2         5         7.3890570168536     -2.3890570168536     -0.323323667878653
                    2.22044604925031e-16

    Actual double:
             1     5
    Expected double:
           1.000000000000000   7.389057016853605

    ----------------------
    Stack Information:
```

```
---------------------
In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m',19,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m (testClasses.testLODESSolution1) at 19</a>
==========================================================================

yrk =

     1     7


ytestarr =

    1.0000    7.3891


norm =

    0.0522


errorarray =

         0    0.0527


==========================================================================
<strong>Verification failed in testClasses/testLODESSolution1.</strong>

---------------------
Framework Diagnostic:
---------------------
```

<a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
bold">verifyEqual failed.
--> The values are not equal using "isequaln".
--> The error was not within relative tolerance.
--> Failure table:
Index Actual Expected
 Error
RelativeError RelativeTolerance
_____ ______ _____
 _____
_____ _____

2 7 7.3890570168536 -0.389057016853605
-0.0526531350301141 2.22044604925031e-16

Actual double:
1 7
Expected double:
1.000000000000000 7.389057016853605

Stack Information:

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
testing/testClasses.m',21,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
testing/testClasses.m (testClasses.testLODESSolution1) at 21

==========

.
yeuler =

1.0000 1.5000 2.2500 3.3750 5.0625

```
ytestarr =

    1.0000    1.6487    2.7183    4.4817    7.3891


norm =

    0.2832


errorarray =

         0    0.0902    0.1723    0.2469    0.3149
```



```
==============================================================================
<strong>Verification failed in testClasses/testLODESSolution2.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
            <strong>Index</strong><strong>    Actual</strong><strong>         Expected
                  </strong><strong>           Error       </strong><strong>
               RelativeError   </strong><strong>     RelativeTolerance  </strong>
            <strong>_____</strong>    <strong>______</strong>    <strong>
               ---------------</strong>    <strong>-----------------</strong>    <
```

```
        strong>-------------------</strong>    <strong>-------------------</
        strong>

    2          1.5     1.6487215144387     -0.148721514438696
       -0.0902041449306423     2.22044604925031e-16
    3         2.25     2.71828199730153     -0.468281997301532
       -0.172271308777529     2.22044604925031e-16
    4        3.375     4.48169001126243      -1.10669001126243
       -0.246935867603812     2.22044604925031e-16
    5       5.0625     7.3890570168536       -2.3265570168536
       -0.314865213727136     2.22044604925031e-16

Actual double:
       1.000000000000000   1.500000000000000   2.250000000000000   3.375000000000000
            5.062500000000000
Expected double:
       1.000000000000000   1.648721514438696   2.718281997301532   4.481690011262426
            7.389057016853605

  ----------------
  Stack Information:
  ----------------
  In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
     testing/testClasses.m',32,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
     testing/testClasses.m (testClasses.testLODESSolution2) at 32</a>
==============================================================================

ytrap =

    1.0000    1.6250    2.6406    4.2910    6.9729
```

```
ytestarr =

    1.0000    1.6487    2.7183    4.4817    7.3891


norm =

    0.0502


errorarray =

         0    0.0144    0.0286    0.0425    0.0563
```

```
<strong>Verification failed in testClasses/testLODESSolution2.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
            <strong>Index</strong><strong>          Actual     </strong><strong>
               Expected     </strong><strong>             Error        </strong><strong>
                     RelativeError    </strong><strong>     RelativeTolerance  </strong>
            <strong>_____</strong>    <strong>______________</strong>    <strong>
               ________________</strong>    <strong>__________________</strong>    <
               strong>__________________</strong>    <strong>__________________</
```

```
strong>

     2                  1.625      1.6487215144387     -0.0237215144386962
    -0.0143878236748625     2.22044604925031e-16
     3               2.640625     2.71828199730153     -0.0776569973015317
    -0.0285684109958506     2.22044604925031e-16
     4            4.291015625     4.48169001126243      -0.190674386262426
    -0.0425451974106339     2.22044604925031e-16
     5          6.972900390625      7.3890570168536      -0.416156626228605
    -0.0563206678848734     2.22044604925031e-16

Actual double:
       1.000000000000000   1.625000000000000   2.640625000000000   4.291015625000000
            6.972900390625000
Expected double:
       1.000000000000000   1.648721514438696   2.718281997301532   4.481690011262426
            7.389057016853605

_____________________
Stack Information:
_____________________
In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m',34,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m (testClasses.testLODESSolution2) at 34</a>
=================================================================================

yheun =

    1.0000    1.6250    2.6406    4.2910    6.9729


ytestarr =
```

```
    1.0000    1.6487    2.7183    4.4817    7.3891

norm =

    0.0502

errorarray =

         0    0.0144    0.0286    0.0425    0.0563

================================================================================
<strong>Verification failed in testClasses/testLODESSolution2.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
            <strong>Index</strong><strong>          Actual     </strong><strong>
               Expected     </strong><strong>            Error        </strong><strong>
                     RelativeError    </strong><strong>      RelativeTolerance  </strong>
            <strong>_____</strong>    <strong>_____________</strong>    <strong>
               _____________</strong>    <strong>__________________</strong>    <
               strong>__________________</strong>    <strong>__________________</
               strong>
```

```
    2                1.625      1.6487215144387     -0.0237215144386962
   -0.0143878236748625     2.22044604925031e-16
    3             2.640625      2.71828199730153     -0.0776569973015317
   -0.0285684109958506     2.22044604925031e-16
    4          4.291015625      4.48169001126243      -0.190674386262426
   -0.0425451974106339     2.22044604925031e-16
    5          6.972900390625      7.3890570168536      -0.416156626228605
   -0.0563206678848734     2.22044604925031e-16

Actual double:
       1.000000000000000   1.625000000000000   2.640625000000000   4.291015625000000
            6.972900390625000
Expected double:
       1.000000000000000   1.648721514438696   2.718281997301532   4.481690011262426
            7.389057016853605

_____________________

Stack Information:
_____________________

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m',36,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m (testClasses.testLODESSolution2) at 36</a>
================================================================================


yrk =

    1.0000    1.6484    2.7173    4.4794    7.3840


ytestarr =
```

```
    1.0000    1.6487    2.7183    4.4817    7.3891

norm =

    6.1253e-04


errorarray =

    1.0e-03 *

         0    0.1723    0.3443    0.5165    0.6884

================================================================================
<strong>Verification failed in testClasses/testLODESSolution2.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
            <strong>Index</strong><strong>          Actual      </strong><strong>
               Expected     </strong><strong>              Error         </strong><strong>
                     RelativeError     </strong><strong>     RelativeTolerance  </strong>
               >
            <strong>_____</strong>    <strong>________________</strong>    <strong>
               ________________</strong>    <strong>____________________</strong>    <
```

```
strong>--------------------</strong>    <strong>--------------------</
strong>

     2              1.6484375     1.6487215144387     -0.00028401443869619
    -0.000172263439403762    2.22044604925031e-16
     3        2.71734619140625    2.71828199730153    -0.000935805895281749
    -0.000344263728417704    2.22044604925031e-16
     4        4.47937536239624    4.48169001126243    -0.00231464886618582
    -0.000516467863767717    2.22044604925031e-16
     5        7.38397032395005    7.3890570168536     -0.00508669290355268
    -0.000688408939320743    2.22044604925031e-16

Actual double:
       1.000000000000000   1.648437500000000   2.717346191406250   4.479375362396240
            7.383970323950052
Expected double:
       1.000000000000000   1.648721514438696   2.718281997301532   4.481690011262426
            7.389057016853605

--------------------
Stack Information:
--------------------
In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m',38,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m (testClasses.testLODESSolution2) at 38</a>
==========================================================================

.
yeuler =

     1    -4
```

```
ytestarr =

    1.0000   -1.0108


norm =

    2.1023


errorarray =

         0   -2.9573

==========================================================================
<strong>Verification failed in testClasses/testLODESSolution3.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
            <strong>Index</strong><strong>    Actual</strong><strong>         Expected
                  </strong><strong>          Error      </strong><strong>
               RelativeError  </strong><strong>     RelativeTolerance  </strong>
            <strong>_____</strong>    <strong>______</strong>    <strong>
               ________________</strong>    <strong>________________</strong>    <
               strong>________________</strong>    <strong>__________________</strong>
```

```
            2          -4          -1.01079006197098      -2.98920993802902
               2.95730048255542      2.22044604925031e-16

    Actual double:
              1     -4
    Expected double:
           1.000000000000000   -1.010790061970984

    ---------------------
    Stack Information:
    ---------------------
    In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
       testing/testClasses.m',49,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
       testing/testClasses.m (testClasses.testLODESSolution3) at 49</a>
================================================================================

ytrap =

    1.0000   -43.8973


ytestarr =

    1.0000    -1.0108


norm =

   30.1622
```

```
errorarray =

         0   -42.4287
```

```
<strong>Verification failed in testClasses/testLODESSolution3.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
            <strong>Index</strong><strong>          Actual      </strong><strong>
               Expected     </strong><strong>          Error       </strong><strong>
               RelativeError  </strong><strong>     RelativeTolerance  </strong>
            <strong>_____</strong>    <strong>________________</strong>    <strong>
               ________________</strong>    <strong>________________</strong>    <
               strong>________________</strong>    <strong>__________________</strong>

            2         -43.8973106866578     -1.01079006197098     -42.8865206246869
               42.4287121907991     2.22044604925031e-16

    Actual double:
           1.000000000000000  -43.897310686657846
    Expected double:
           1.000000000000000   -1.010790061970984

    ------------------
```

```
    Stack Information:
    ------------------
    In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
       testing/testClasses.m',51,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
       testing/testClasses.m (testClasses.testLODESSolution3) at 51</a>
================================================================================

yheun =

    1.0000   -43.8973


ytestarr =

    1.0000    -1.0108


norm =

   30.1622


errorarray =

         0   -42.4287


================================================================================
<strong>Verification failed in testClasses/testLODESSolution3.</strong>

    ----------------------
    Framework Diagnostic:
```

```
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
            <strong>Index</strong><strong>          Actual      </strong><strong>
               Expected     </strong><strong>          Error      </strong><strong>
               RelativeError  </strong><strong>     RelativeTolerance  </strong>
            <strong>_____</strong>    <strong>________________</strong>    <strong>
               ________________</strong>    <strong>________________</strong>    <
               strong>________________</strong>    <strong>__________________</strong>

            2         -43.8973106866578     -1.01079006197098     -42.8865206246869
               42.4287121907991     2.22044604925031e-16

    Actual double:
           1.000000000000000  -43.897310686657846
    Expected double:
           1.000000000000000   -1.010790061970984

    ----------------------
    Stack Information:
    ----------------------
    In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
       testing/testClasses.m',53,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
       testing/testClasses.m (testClasses.testLODESSolution3) at 53</a>
================================================================================


yrk =

    1.0e+03 *
```

```
    0.0010   -1.7028

ytestarr =

    1.0000   -1.0108

norm =

   1.1969e+03

errorarray =

   1.0e+03 *

         0   -1.6837

================================================================================
<strong>Verification failed in testClasses/testLODESSolution3.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
```

```
<strong>Index</strong><strong>          Actual      </strong><strong>
   Expected      </strong><strong>          Error       </strong><strong>
   RelativeError  </strong><strong>     RelativeTolerance  </strong>
<strong>_____</strong>    <strong>_________________</strong>    <strong>
   _________________</strong>    <strong>_________________</strong>    <
   strong>_________________</strong>    <strong>___________________</strong>

 2         -1702.84512882598     -1.01079006197098     -1701.83433876401
   1683.66746250505     2.22044604925031e-16

Actual double:
       1.0e+03 *

       0.001000000000000   -1.702845128825983
Expected double:
       1.000000000000000   -1.010790061970984
```



```
Stack Information:

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m',55,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m (testClasses.testLODESSolution3) at 55</a>
```

```
.
yeuler =

    1.0000         0    0.8415    1.0427    0.0966   -0.6695


ytestarr =
```

```
    1.0000    0.8035    0.9538    0.7436    0.1502   -1.0108

norm =

    0.4569

errorarray =

         0    1.0000    0.1178    0.4022    0.3570   -0.3376

==============================================================================
<strong>Verification failed in testClasses/testLODESSolution4.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
            <strong>Index</strong><strong>          Actual      </strong><strong>
                    Expected     </strong><strong>           Error       </strong><
               strong>      RelativeError   </strong><strong>     RelativeTolerance  </
               strong>
            <strong>_____</strong>    <strong>__________________</strong>    <strong>
               _________________</strong>    <strong>__________________</strong>    <
               strong>__________________</strong>    <strong>___________________</
               strong>
```

```
    2                         0     0.803498260367072      -0.803498260367072
                           -1     2.22044604925031e-16
    3          0.841470984807897      0.95380754323836      -0.112336558430464
    -0.117776965832184     2.22044604925031e-16
    4           1.04269499336001     0.743638851851789       0.299056141508218
    0.402152389918193     2.22044604925031e-16
    5         0.0966021522418493     0.150235602951416     -0.0536334507095666
    -0.356995609934823     2.22044604925031e-16
    6         -0.669532318883836     -1.01079006197098       0.341257743087148
    -0.337614857848636     2.22044604925031e-16

Actual double:
       1.000000000000000                   0    0.841470984807897    1.042694993360007
            0.096602152241849   -0.669532318883836
Expected double:
       1.000000000000000    0.803498260367072    0.953807543238360    0.743638851851789
            0.150235602951416   -1.010790061970984

_____________________
Stack Information:
_____________________

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m',66,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m (testClasses.testLODESSolution4) at 66</a>
=========================================================================
The identifier was:
MATLAB:class:InvalidHandleThere was an error! The message was:
Invalid or deleted object.
ytrap =

    1.0000     0.9207     0.9541     0.5700     0.0251    -1.1012
```

```
ytestarr =

    1.0000    0.8035    0.9538    0.7436    0.1502   -1.0108


norm =

    0.1277


errorarray =

         0    0.1459    0.0003    0.2335    0.8328   -0.0894
```



```
==============================================================================
<strong>Verification failed in testClasses/testLODESSolution4.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
            <strong>Index</strong><strong>          Actual      </strong><strong>
                    Expected     </strong><strong>            Error         </strong><
               strong>       RelativeError     </strong><strong>     RelativeTolerance
               </strong>
```

----- ---------- ---------- ---------- ---------- ----------

2	0.920735492403948	0.803498260367072	0.117237232036877	0.14590850760936	2.22044604925031e-16
3	0.954130968258594	0.95380754323836	0.000323425020233414	0.000339088343897265	2.22044604925031e-16
4	0.56999267801283	0.743638851851789	-0.173646173838959	-0.233508743399502	2.22044604925031e-16
5	0.0251222646962669	0.150235602951416	-0.125113338255149	-0.832780884139753	2.22044604925031e-16
6	-1.10119664749119	-1.01079006197098	-0.0904065855202025	0.0894415061263214	2.22044604925031e-16

Actual double:

1.000000000000000 0.920735492403948 0.954130968258594 0.569992678012830 0.025122264696267 -1.101196647491187

Expected double:

1.000000000000000 0.803498260367072 0.953807543238360 0.743638851851789 0.150235602951416 -1.010790061970984

Stack Information:

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution4) at 68

==========

The identifier was:

MATLAB:class:InvalidHandleThere was an error! The message was:

```
Invalid or deleted object.
yheun =

    1.0000    0.9207    0.9541    0.5700    0.0251   -1.1012


ytestarr =

    1.0000    0.8035    0.9538    0.7436    0.1502   -1.0108


norm =

    0.1277


errorarray =

         0    0.1459    0.0003    0.2335    0.8328   -0.0894


================================================================================
<strong>Verification failed in testClasses/testLODESSolution4.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
    --> The error was not within relative tolerance.
    --> Failure table:
```

Index Actual Expected Error RelativeError RelativeTolerance
_____ _____ _____ _____ _____ _____

2 0.920735492403948 0.803498260367072 0.117237232036877 0.14590850760936 2.22044604925031e-16
3 0.954130968258594 0.95380754323836 0.000323425020233414 0.000339088343897265 2.22044604925031e-16
4 0.56999267801283 0.743638851851789 -0.173646173838959 -0.233508743399502 2.22044604925031e-16
5 0.0251222646962669 0.150235602951416 -0.125113338255149 -0.832780884139753 2.22044604925031e-16
6 -1.10119664749119 -1.01079006197098 -0.0904065855202025 0.0894415061263214 2.22044604925031e-16

Actual double:
1.000000000000000 0.920735492403948 0.954130968258594 0.569992678012830 0.025122264696267 -1.101196647491187
Expected double:
1.000000000000000 0.803498260367072 0.953807543238360 0.743638851851789 0.150235602951416 -1.010790061970984

Stack Information:

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m',70,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/

```
testing/testClasses.m (testClasses.testLODESSolution4) at 70</a>
==============================================================================

yrk =

    1.0000    0.7863    0.9434    0.7180    0.1408   -1.0284


ytestarr =

    1.0000    0.8035    0.9538    0.7436    0.1502   -1.0108


norm =

    0.0187


errorarray =

         0    0.0214    0.0109    0.0345    0.0626   -0.0174


==============================================================================
<strong>Verification failed in testClasses/testLODESSolution4.</strong>

    ----------------------
    Framework Diagnostic:
    ----------------------
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
       bold">verifyEqual</a> failed.
    --> The values are not equal using "isequaln".
```

```
--> The error was not within relative tolerance.
--> Failure table:
        <strong>Index</strong><strong>          Actual      </strong><strong>
           Expected     </strong><strong>            Error        </strong><strong>
                RelativeError   </strong><strong>     RelativeTolerance  </strong>
        <strong>-----</strong>    <strong>-----------------</strong>    <strong>
           ----------------</strong>    <strong>-------------------</strong>    <
           strong>-------------------</strong>    <strong>-------------------</
           strong>

        2         0.78631839551037     0.803498260367072      -0.0171798648567016
           -0.0213813342282199     2.22044604925031e-16
        3         0.943371788846183      0.95380754323836      -0.0104357543921774
           -0.0109411531353023     2.22044604925031e-16
        4         0.717989268823369     0.743638851851789      -0.0256495830284202
           -0.0344919889063735     2.22044604925031e-16
        5         0.14083244087226     0.150235602951416     -0.00940316207915609
           -0.0625894388176213     2.22044604925031e-16
        6        -1.02836702616913     -1.01079006197098      -0.0175769641981469
           0.0173893322257965     2.22044604925031e-16

Actual double:
       1.000000000000000    0.786318395510370    0.943371788846183    0.717989268823369
            0.140832440872260   -1.028367026169131
Expected double:
       1.000000000000000    0.803498260367072    0.953807543238360    0.743638851851789
            0.150235602951416   -1.010790061970984

-------------------
Stack Information:
-------------------
```

```
In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m',72,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m (testClasses.testLODESSolution4) at 72</a>
==========================================================================

.
Done testClasses
----------

Failure Summary:

     Name                                Failed  Incomplete  Reason(s)
    ================================================================
     testClasses/testLODESSolution1      X                   Failed by verification.
    ----------------------------------------------------------------
     testClasses/testLODESSolution2      X                   Failed by verification.
    ----------------------------------------------------------------
     testClasses/testLODESSolution3      X                   Failed by verification.
    ----------------------------------------------------------------
     testClasses/testLODESSolution4      X                   Failed by verification.


result =

  1 4 <a href="matlab:helpPopup matlab.unittest.TestResult" style="font-weight:bold">
     TestResult</a> array with properties:

    Name
    Passed
    Failed
    Incomplete
    Duration
    Details
```

```
Totals:
   0 Passed, 4 Failed, 0 Incomplete.
   12.6715 seconds testing time.
```