Test Report: Library of ODE Solvers (LODES)

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1 Revision History

Date		Version	Notes
December 2017	18,	1.0	Initial draft.

2 Symbols, Abbreviations and Acronyms

2.1 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the numeral analysis and ordinary differential equation literature and with existing documentation for solving ordinary differential equations. The symbols are listed in alphabetical order.

symbol	unit	type	description
dy/dx	-	\mathbb{R}	Rate of change of y with respect to x
$\varepsilon_{ m relative}$	-	\mathbb{R}	The relative error
F	-	$\mathbb{R}^3 \to \mathbb{R}$	Order function applied to the Runge Kutta Method
f(x,y)	-	$\mathbb{R}^2 \to \mathbb{R}$	Explicit form of the ODE function containing (x, y)
$f_x(x,y)$	-	$\mathbb{R}^2 \to \mathbb{R}$	Explicit form of the derivative of $f(x, y)$ with respect to x
$f_y(x,y)$	-	$\mathbb{R}^2 \to \mathbb{R}$	Explicit form of the derivative of $f(x, y)$ with respect to y
h	-	$\mathbb{R} . h > 0$	Step-size from $x_{(0)}$ to the next point $x_{(1)}$, where $x_{(1)} = x_{(0)} + h$
K_1, K_2, K_3, K_4	\mathbb{R}	-	Intermediary variables used in the Runge-Kutta method
n	-	\mathbb{R}	Reference recursion step
0	-	\mathbb{R}	Solution variable
T	-	-	Test
x_0	-	\mathbb{R}	Initial value x
$x_{ m k}$	-	\mathbb{R}	Final value x
$x_{ m n}$	-	\mathbb{R}	Intermediate n^{th} value x
y_0	-	\mathbb{R}	Initial value y
$y_{ m k}$	-	\mathbb{R}	Final value y
$y_{ m n}$	-	\mathbb{R}	Intermediate n^{th} value y
y'	- ,	$\mathbb{R} o \mathbb{R}$	Implicit form of the first order ODE = $f(x, y)$
$y^{(\mathrm{n})}$	-	$\mathbb{R} o \mathbb{R}$	Implicit form of the ODE to the n^{th} order
y	-	$1 \times \mathbb{R}^k$	The array containing all intermediate y_n values

Contents

1	Revision History				
2	Symbols, Abbreviations and Acronyms 2.1 Table of Symbols	ii ii			
3	Introduction	1			
4	Overview	1			
5	Functional Requirements Evaluation 5.1 Calculation Tests	2 2 2 3 4 5			
6	Nonfunctional Requirements Evaluation 6.1 Speed Test Plan Test Case: T-17	7 7			
7	Comparison to Existing Implementation 7.1 Test 1: $f(x,y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$	7 8 10 12 14			
8	Unit Testing	16			
9	Changes Due to Testing	16			
10	Automated Testing	16			
11	Trace to Requirements	17			
12	Trace to Modules	17			
13	Code Coverage Metrics	18			
14	Code Inspection Checklist	20			
15	5 Appendix A - Test Output				

List of Tables

$\frac{1}{2}$	Requirements Traceability Matrix
2	Design Traceability Matrix
List	of Figures
1	T-1 result
$\frac{2}{3}$	T-5 result
3 4	T-13 result
4 5	T-13 result
6	T-6 result
7	T-10 result
8	T-14 result
9	T-3 result
10	T-7 result
11	T-11 result
12	T-15 result
13	T-4 result
14	T-8 result
15	T-12 result
16	T-16 result
17	T-17 LODES rk() result
18	T-17 MATLAB ode45() result
19	Euler's Method Analysis Graphs
20	Trapezoidal Method Analysis Graphs
21	Heun's Method Analysis Graphs
22	Runge-Kutta 4 Method Analysis Graphs
23	Euler's Method Analysis Graphs
24	Trapezoidal Method Analysis Graphs
25	Heun's Method Analysis Graphs
26	Runge-Kutta 4 Method Analysis Graphs
27	Euler's Method Analysis Graphs
28	Trapezoidal Method Analysis Graphs
29	Heun's Method Analysis Graphs
30	Runge-Kutta 4 Method Analysis Graphs
31	Euler's Method Analysis Graphs
32	Trapezoidal Method Analysis Graphs
33	Heun's Method Analysis Graphs
34	Runge-Kutta 4 Method Analysis Graphs
35	Code Coverage Tool result

36	Code Review Checklist Output	20
00	Code review Checkinst Output	20

3 Introduction

This document serves as the Test Report for the Library of ODE Solvers (LODES) which lays out in detail the results of the execution of the LODES Test Plan. The requirements for the software are described in the LODES Software Requirements Specification. The aim of the Test Report is to document and show that LODES produces accurate and valid (in the scope of detailed in the SRS and Test Plan) results.

The Overview (Section 4) discusses how testing slightly deviates from the Test Plan in its execution.

The following sections describe the evaluation of the software against the Functional Requirements (Section 5), Non-Functional Requirements (Section 6). It will also compare LODES to existing software in Comparison to Existing Implementation (Section 7).

In the sections after, Unit Testing (Section 8), Changes Due to Testing (Section 9), Automated Testing (Section 10), Trace to Requirements (Section 11), Trace to Modules (Section 12, and Code Coverage Metrics (Section 13) are discussed.

4 Overview

The Test Report documents the results of the tests carried out to verify the behaviour of the LODES software. The Test Plan suggests that it should carry out testing such that that the test report shall describe and analyze the numerical differences of the hand-executed solution to the numerical analysis methods versus the solutions obtained from LODES. This Test Report carries out the tests, with the same intent of verification, and instead of only comparing against the hand-executed solution, it compares the results with the MATLAB generated solutions as well.

5 Functional Requirements Evaluation

5.1 Calculation Tests

5.1.1 Simple Cases: T-1, T-5, T-9, T-13

Input: $f(x,y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$

Test Plan Test Case: T-1

Expected output: The value of $y_k = 3$. This test passes.

>> lodes(1, 'y', 0, 1, 2, 2, 1, 1)
The values of x are: 0 2
The values of y are: 1 3
ans = 0 2

Figure 1: T-1 result.

Test Plan Test Case: T-5

Expected output: The value of $y_k = 5$. This test passes.

>> lodes(2, 'y', 0, 1, 2, 2, 1, 1)
The values of x are: 0 2

The values of y are: 1 5

ans = 0 2

Figure 2: T-5 result.

Test Plan Test Case: T-9

Expected output: The value of $y_k = 5$. This test passes.

```
>> lodes(3, 'y', 0, 1, 2, 2, 1, 1)
The values of x are: 0 2
The values of y are: 1 5

ans = 0 2
```

Figure 3: T-9 result.

Expected output: The value of $y_k = 7$. This test passes.

Figure 4: T-13 result.

5.1.2 Simple-Iterative Cases: T-2, T-6, T-10, T-14

Input: $f(x,y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

Test Plan Test Case: T-2

Expected output: The value of $y_k = 5.0625$. This test passes.

```
>> lodes(1, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are: 0 0.5000 1.0000 1.5000 2.0000
The values of y are: 1.0000 1.5000 2.2500 3.3750 5.0625
```

Figure 5: T-2 result.

Expected output: The value of $y_k = 6.9729$. This test passes.

>> lodes(2, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are: 0 0.5000 1.0000 1.5000 2.0000
The values of y are: 1.0000 1.6250 2.6406 4.2910 6.9729

Figure 6: T-6 result.

Test Plan Test Case: T-10

Expected output: The value of $y_k = 6.9729$. This test passes.

>> lodes(3, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are: 0 0.5000 1.0000 1.5000 2.0000
The values of y are: 1.0000 1.6250 2.6406 4.2910 6.9729

Figure 7: T-10 result.

Test Plan Test Case: T-14

Expected output: The value of $y_k = 7.38340$. This test passes.

>> lodes(4, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are: 0 0.5000 1.0000 1.5000 2.0000
The values of y are: 1.0000 1.6484 2.7173 4.4794 7.3840

Figure 8: T-14 result.

5.1.3 Linear Trigonometric Cases: T-3, T-7, T-11, T-15

Input: $f(x,y) = y, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

Test Plan Test Case: T-3

Expected output: The value of $y_k = -4$. This test passes.

```
>> lodes(1, '\sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are: 0 5
The values of y are: 1 -4
```

Figure 9: T-3 result.

Expected output: The value of $y_k = -43.8973$. This test passes.

```
>> lodes(2, '\sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are: 0 5
The values of y are: 1.0000 -43.8973
```

Figure 10: T-7 result.

Test Plan Test Case: T-11

Expected output: The value of $y_k = -43.8973$. This test passes.

```
>> lodes(3, '\sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are: 0 5
The values of y are: 1.0000 -43.8973
```

Figure 11: T-11 result.

Test Plan Test Case: T-15

Expected output: The value of $y_k = -1702.8$. This test passes.

```
>> lodes(4, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are: 0 5
The values of y are: 1.0e+03 *
0.0010 -1.7028
```

Figure 12: T-15 result.

5.1.4 Linear Trigonometric Iterative Cases: T-4, T-8, T-12, T-16

Input:
$$f(x,y) = sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$$

Expected output: The value of $y_k = -0.6695$. This test passes.

```
>> lodes(1, '\sin(x) - y^2', 0, 1, 5, 1, 1, 1)
The values of x are: 0 1 2 3 4 5
The values of y are: 1.0000 0 0.8415 1.0427 0.0966 -0.6695
```

Figure 13: T-4 result.

Test Plan Test Case: T-8

Expected output: The value of $y_k = -1.1012$. This test passes.

```
>> lodes(2, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are: 0 1 2 3 4 5
The values of y are: 1.0000 0.9207 0.9541 0.5700 0.0251 -1.1012
```

Figure 14: T-8 result.

Test Plan Test Case: T-12

Expected output: The value of $y_k = -1.1012$. This test passes.

```
>> lodes(3, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are: 0 1 2 3 4 5
The values of y are: 1.0000 0.9207 0.9541 0.5700 0.0251 -1.1012
```

Figure 15: T-12 result.

Test Plan Test Case: T-16

Expected output: The value of $y_k = -1.0284$. This test passes.

```
>> lodes(4, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are: 0 1 2 3 4 5

The values of y are: 1.0000 0.7863 0.9434 0.7180 0.1408 -1.0284
```

Figure 16: T-16 result.

6 Nonfunctional Requirements Evaluation

6.1 Speed Test Plan Test Case: T-17

Input:
$$f(x,y) = sin(x) - y^2, h = 1e - 3, x_0 = 0, y_0 = 1, x_k = 5$$

Test Plan Test Case: T-17

Expected output: The execution time difference of LODES is not greater than 4 times that of MATLAB. This test fails as the execution time of LODES is 2,000 times greater than MATLAB's.

Profile Summary

Generated 15-Dec-2017 23:08:41 using performance time.

<u>Function Name</u>	<u>Calls</u>	Total Time	Self Time*	Total Time Plot (dark band = self time)
<u>lodes</u>	1	472.941 s	0.003 s	
<u>rk</u>	1	472.766 s	2.209 s	

Figure 17: T-17 LODES rk() result.

Profile Summary

Generated 15-Dec-2017 23:29:45 using performance time.

<u>Function Name</u>	<u>Calls</u>	Total Time	Self Time*	Total Time Plot (dark band = self time)
testode45	1	0.273 s	0.034 s	
ode45	1	0.239 s	0.110 s	

Figure 18: T-17 MATLAB ode45() result.

7 Comparison to Existing Implementation

This section compares the calculation of LODES to MATLAB's ode45() function.

Test Plan Test Case: T-18

The following input parameters were tested (as above):

• Test 1 (7.1):
$$f(x,y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$$

• Test 2 (7.2):
$$f(x,y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$$

• Test 3 (7.3):
$$f(x,y) = \sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$$

• Test 4 (7.4):
$$f(x,y) = \sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$$

This phase of testing is automated and the results are shown in Section 15. The following equation was implemented to compare the results labeled as **norm**:

$$\epsilon_{\text{relative}} = \text{norm} = \frac{\|\text{Result}_{\text{MATLAB}} - \text{Result}_{\text{LODES}}\|}{\|\text{Result}_{\text{MATLAB}}\|}$$

Output/Result: The $\epsilon_{\text{relative}}$ vs. h plots. The intermediate y values (Result) are compared according to the following formula of the relative error norm percentage -

$$\epsilon_{\text{relative}} = 100 * \frac{\text{Result}_{\text{MATLAB}} - \text{Result}_{\text{LODES}}}{\text{Result}_{\text{MATLAB}}}$$

7.1 Test 1:
$$f(x,y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$$

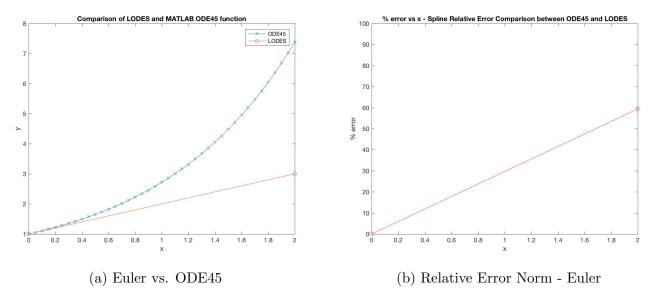


Figure 19: Euler's Method Analysis Graphs

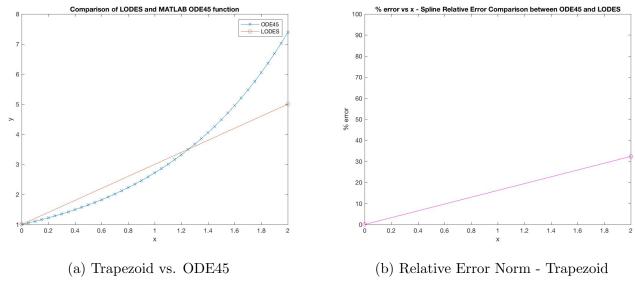


Figure 20: Trapezoidal Method Analysis Graphs

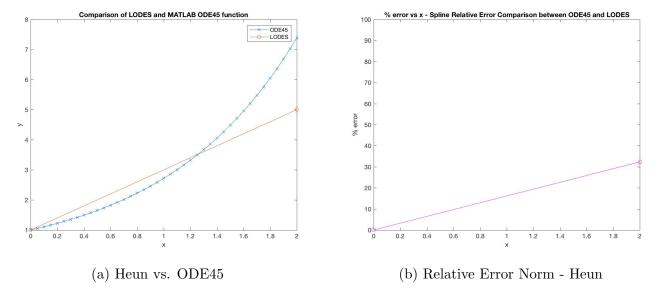


Figure 21: Heun's Method Analysis Graphs

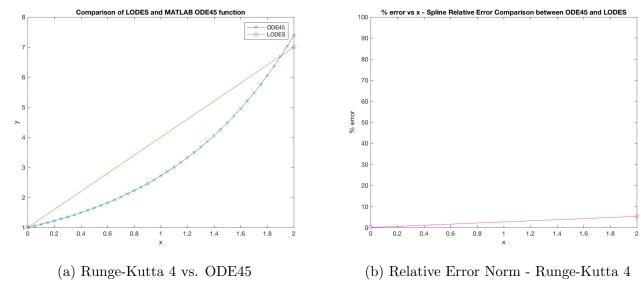


Figure 22: Runge-Kutta 4 Method Analysis Graphs

7.2 Test 2:
$$f(x,y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$$

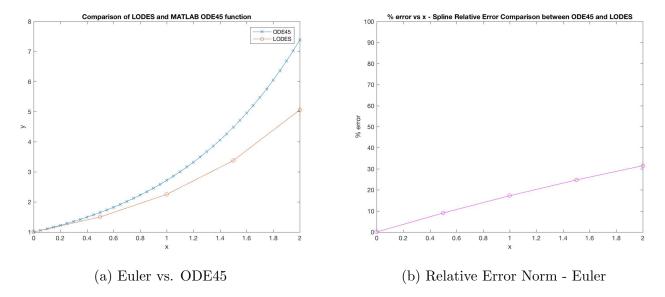


Figure 23: Euler's Method Analysis Graphs

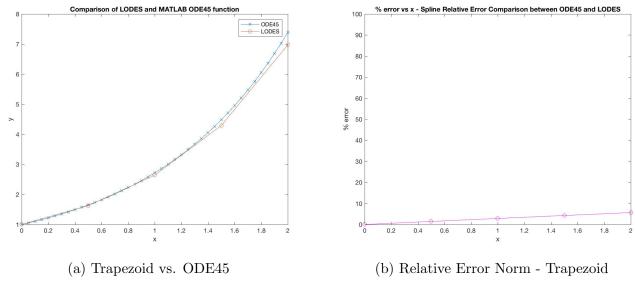


Figure 24: Trapezoidal Method Analysis Graphs

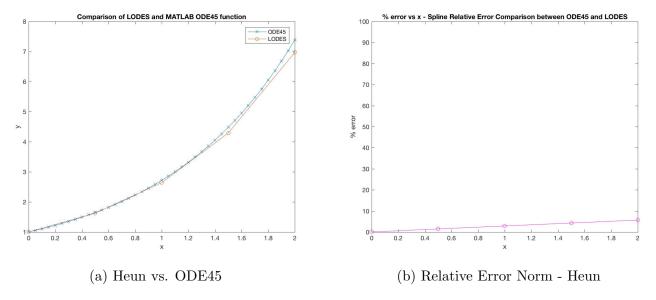


Figure 25: Heun's Method Analysis Graphs

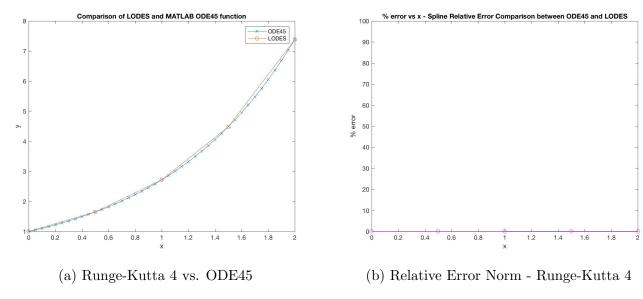


Figure 26: Runge-Kutta 4 Method Analysis Graphs

7.3 Test 3:
$$f(x,y) = sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$$

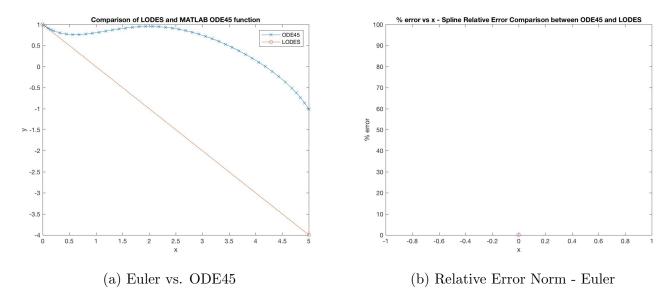


Figure 27: Euler's Method Analysis Graphs

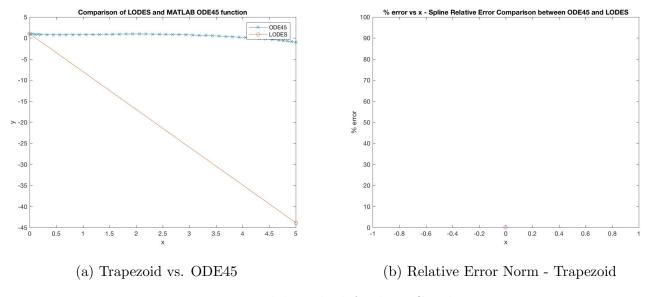


Figure 28: Trapezoidal Method Analysis Graphs

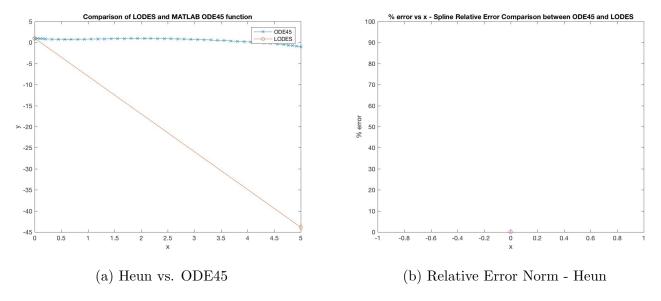


Figure 29: Heun's Method Analysis Graphs

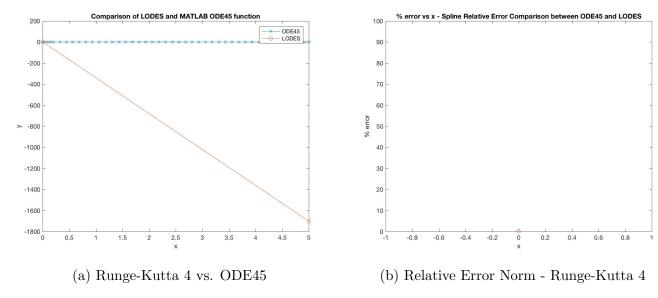


Figure 30: Runge-Kutta 4 Method Analysis Graphs

7.4 Test 4:
$$f(x,y) = \sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$$

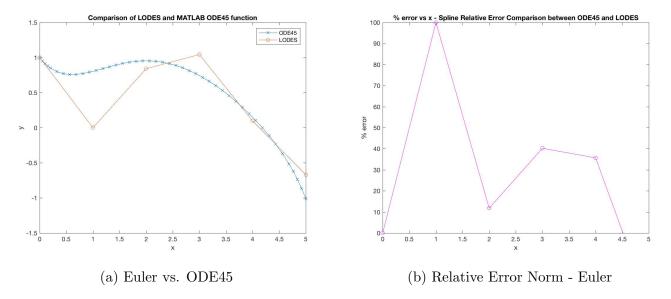


Figure 31: Euler's Method Analysis Graphs

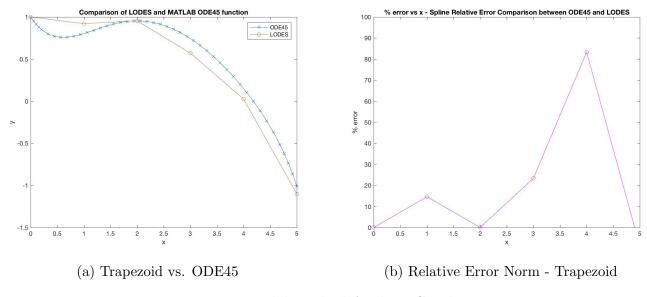


Figure 32: Trapezoidal Method Analysis Graphs

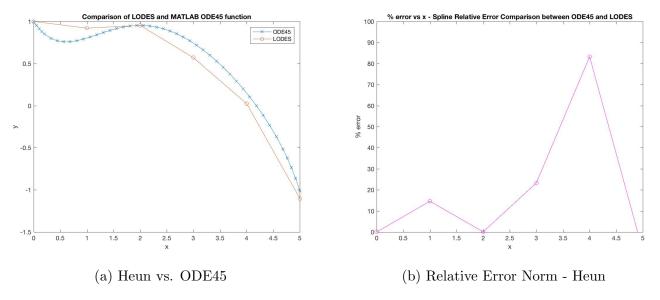


Figure 33: Heun's Method Analysis Graphs

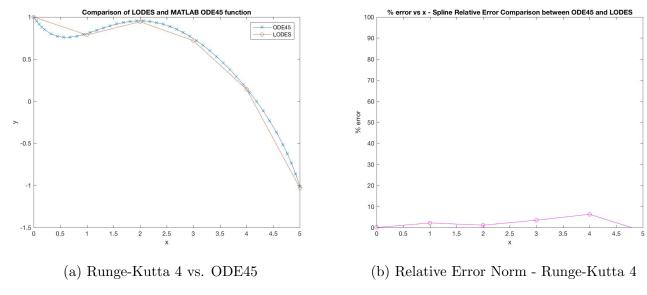


Figure 34: Runge-Kutta 4 Method Analysis Graphs

8 Unit Testing

Unit testing was performed and the results and execution are the similar on the level of Functional Requirement as described in Section 5 since the library is only of one level.

Further unit testing can be explored in the future.

Unit testing of the modules implemented by MATLAB and the Operating System (OS) has been deemed unnecessary due to proof of use. The parser functionality tests have been embedded in the functional tests of the modules of LODES in Section 5.

9 Changes Due to Testing

None.

10 Automated Testing

Automated testing has been performed in Section 7. These were implemented in MATLAB's Unit Testing Framework.

11 Trace to Requirements

The following table shows the traceability mapping for the test cases laid out in this Test Report to the requirements described in the Commonality Analysis.

Table 1: Requirements Traceability Matrix

Test Number	CA Requirements
T1	IM1, O1, O2, O3, O4, O5
T2	IM1, O1, O2, O3, O4, O5
Т3	IM1, O1, O2, O3, O4, O5
T4	IM1, O1, O2, O3, O4, O5
T5	IM2, O1, O2, O3, O4, O5
Т6	IM2, O1, O2, O3, O4, O5
T7	IM2, O1, O2, O3, O4, O5
Т8	IM2, O1, O2, O3, O4, O5
Т9	IM3, O1, O2, O3, O4, O5
T10	IM3, O1, O2, O3, O4, O5
T11	IM3, O1, O2, O3, O4, O5
T12	IM3, O1, O2, O3, O4, O5
T13	IM4, O1, O2, O3, O4, O5
T14	IM4, O1, O2, O3, O4, O5
T15	IM4, O1, O2, O3, O4, O5
T16	IM4, O1, O2, O3, O4, O5
T17	NFR1
T18	NFR2

12 Trace to Modules

The following table shows the traceability mapping for the test cases laid out in this Test Report to the modules in the Module Guide (MG).

Table 2: Design Traceability Matrix

Test Number	MG Modules
T1	M1, M2, M3, M4, M5
T2	M1, M2, M3, M4, M5
Т3	M1, M2, M3, M4, M5
T4	M1, M2, M3, M4, M5
T5	M1, M2, M3, M4, M6
Т6	M1, M2, M3, M4, M6
T7	M1, M2, M3, M4, M6
Т8	M1, M2, M3, M4, M6
Т9	M1, M2, M3, M4, M7
T10	M1, M2, M3, M4, M7
T11	M1, M2, M3, M4, M7
T12	M1, M2, M3, M4, M7
T13	M1, M2, M3, M4, M8
T14	M1, M2, M3, M4, M8
T15	M1, M2, M3, M4, M8
T16	M1, M2, M3, M4, M8
T17	-
T18	-

13 Code Coverage Metrics

The following Code Coverage figure was obtained from MATLAB's Code Coverage Report Tool. Running the test program with in-boundary conditions (excluding extraneous scenarios), the code coverage tool produces the following results:

Profiler Coverage Report

Run the Coverage Report after you run the Profiler to identify how much of a file ran when it was profiled (Learn More).

Rerun This Report Run Report on Current Folder

Report for folder /Users/paulaoanan/School/cas741/Project/cas741/src



Figure 35: Code Coverage Tool result.

Code Inspection Checklist 14

The figure below shows the completed Code Checklist.

7.1Code Inspection Checklist

- Are there defects in the code?
- 2. Does the code adhere to MATLAB coding standards and guidelines (syntax, function calls, etc.)? Pass
- 3. Does the code sensibly apply the ODE solvers as defined in the Commonality Analysis?

- 4. Does the code contain spelling errors?
- 5. Do the recursion/loops have the correct terminating cases according to the formulas defined in the Commonality Analysis? Pass
- 6. Does the code have the correct inputs and outputs? [What does this mean? Do you mean whether it agrees with the requirements? —SS It shall agree with the inputs and outputs listed in the functional requirements listed above through code walkthrough. -PA Pass
- 7. Do the datatypes in the code adhere to ones defined in the requirements and specifications? Pass
- 8. Does the code satisfy the ranges and assumptions defined in the Commonality Analysis?

This isn't actually a code walkthrough procedure. This is a code inspection checklist. A walkthrough is when a group of reviewers "play computer" and go through the code and its algorithms by hand. —SS [Noted - that was the intent of the section :-). The section has been renamed. —PA

Performed by: Shown Ruppert

Figure 36: Code Review Checklist Output.

15 Appendix A - Test Output

The raw output of automated testing can be found here: Automated Testing Output

```
Running testClasses
yeuler =
     1
           3
ytestarr =
    1.0000
               7.3891
norm =
    0.5886
errorarray =
         0
               0.5940
```

Verification failed in testClasses/testLODESSolution1.

 $Framework\ Diagnostic:$

```
<a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:</pre>
  bold">verifyEqual </a> failed.
—> The values are not equal using "isequaln".
—> The error was not within relative tolerance.
---> Failure table:
      <strong>Index</strong><strong>
                                 Actual </strong > strong >
                                                         Expected
                                      </strong><strong>
                                                        RelativeError
        </strong><strong>
                             Error
           </strong>strong> RelativeTolerance </strong>
      </strong> <strong> ----- </strong> <strong>
         2
                     7.3890570168536 -4.3890570168536 -0.593994200727192
            2.22044604925031e-16
Actual double:
            3
Expected double:
     Stack Information:
In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
  testing/testClasses.m',15,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
```

testing/testClasses.m (testClasses.testLODESSolution1) at 15

ytrap =

22

1 5

ytestarr =

23

Verification failed in testClasses/testLODESSolution1.

Framework Diagnostic:

```
<a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:bold">verifyEqual</a> failed.
```

- —> The values are not equal using "isequaln".
- —> The error was not within relative tolerance.
- ---> Failure table:

```
<strong>Index</strong><strong> Actual</strong><strong> Expected
  </strong><strong> Error </strong><strong> RelativeTrong>
  </strong>
<strong> <strong
```

```
24
```

yheun =

1

ytestarr =

norm =

1.0000

0.3204

5

```
_____</strong>
                                        <strong>.....</strong>
       2
                         7.3890570168536
                                            -2.3890570168536
                                                               -0.323323667878653
              2.22044604925031e-16
Actual double:
              5
Expected double:
      1.00000000000000000
                         7.389057016853605
Stack Information:
In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
  testing/testClasses.m',17,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
  testing/testClasses.m (testClasses.testLODESSolution1) at 17</a>
         7.3891
```

```
errorarray = 0 0.3233
```

Verification failed in testClasses/testLODESSolution1.

```
Framework Diagnostic:
```

```
<a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:bold">verifyEqual</a> failed .
```

- --> The values are not equal using "isequaln".
- —> The error was not within relative tolerance.
- ---> Failure table:

Actual double:

1 5

Expected double:

yrk =

1 7

ytestarr =

1.0000 7.3891

norm =

0.0522

errorarray =

0.0527

 $<\!\!\mathrm{strong}\!>\!\!\mathrm{Verification}\ \ \, \mathrm{failed}\ \ \, \mathrm{in}\ \ \, \mathrm{testClasses/testLODESSolution} 1.<\!/\!\,\mathrm{strong}\!>$

Framework Diagnostic:

```
<\!a\ href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual"\ style="font-weight:helpPopup matlab.unittest".TestCase.verifyEqual"\ style="font-weight:helpPopup matlab.unittest".TestCase.verifyEqual" style="font-weight:helpPopup matlab.unittest".TestCase.verifyEqual style="font-wei
          bold">verifyEqual </a> failed.
—> The values are not equal using "isequaln".
--> The error was not within relative tolerance.
---> Failure table:
                          <strong>Index</strong><strong>
                                                                                                                                        Actual </strong><strong>
                                                                                                                                                                                                                                             Expected
                                    </strong><strong>
                                                                                                                            Error </strong>strong>
                                    RelativeError </strong> RelativeTolerance </strong>
                          </strong> <strong> ----- </strong> <strong>
                                      2
                                                                                         7.3890570168536 -0.389057016853605
                                     -0.0526531350301141 2.22044604925031e-16
 Actual double:
 Expected double:
```

Stack Information:

In /Users/paulaoanan/School/cas741/Project/cas741/src/ testing/testClasses.m (testClasses.testLODESSolution1) at 21

veuler =

28

1.0000 1.5000 2.2500 3.3750 5.0625ytestarr = 1.0000 1.6487 2.7183 4.4817 7.3891norm = 0.2832errorarray =

Verification failed in testClasses/testLODESSolution2.

0.2469

E 1 D: +:

0

Framework Diagnostic:

0.0902

verifyEqual failed.

0.3149

—> The values are not equal using "isequaln".

0.1723

- —> The error was not within relative tolerance.
- ---> Failure table:

```
<strong>____</strong> <strong> <strong> <strong>
        strong>
      2
                1.5 	 1.6487215144387 	 -0.148721514438696
         -0.0902041449306423
                           2.22044604925031e-16
      3
                     2.71828199730153 -0.468281997301532
               2.25
         -0.172271308777529
                          2.22044604925031e-16
              3.375
                     4.48169001126243
                                   -1.10669001126243
         -0.246935867603812
                          2.22044604925031e-16
      5
             5.0625
                   7.3890570168536
                                   -2.3265570168536
                          2.22044604925031e-16
         -0.314865213727136
Actual double:
                                     2.2500000000000000
                                                     3.3750000000000000
     1.0000000000000000
                     1.5000000000000000
          5.0625000000000000
Expected double:
     1.00000000000000000
                     1.648721514438696
                                     2.718281997301532 4.481690011262426
          7.389057016853605
Stack Information:
```

In /Users/paulaoanan/School/cas741/Project/cas741/src/ testing/testClasses.m (testClasses.testLODESSolution2) at 32

ytrap =

1.00001.62502.6406 4.29106.9729

```
30
```

ytestarr =

1.0000 1.6487 2.7183 7.38914.4817norm =0.0502errorarray = 0 0.01440.02860.04250.0563Verification failed in testClasses/testLODESSolution2. Framework Diagnostic: verifyEqual failed. --> The values are not equal using "isequaln". —> The error was not within relative tolerance. ---> Failure table: Index Actual Expected Error RelativeError strong> RelativeTolerance

```
_____</strong> <strong> ____</strong>
         strong>_____</strong> <strong>____</
         strong>
      2
                      1.625
                               1.6487215144387
                                                -0.0237215144386962
         -0.0143878236748625
                             2.22044604925031e-16
       3
                    2.640625
                              2.71828199730153
                                                -0.0776569973015317
                             2.22044604925031e-16
         -0.0285684109958506
                 4.291015625
                              4.48169001126243
                                                 -0.190674386262426
         -0.0425451974106339
                             2.22044604925031e-16
                               7.3890570168536
       5
              6.972900390625
                                                 -0.416156626228605
         -0.0563206678848734
                             2.22044604925031e-16
Actual double:
                                         2.6406250000000000
      1.0000000000000000
                       1.6250000000000000
                                                           4.291015625000000
           6.972900390625000
Expected double:
      1.00000000000000000
                       1.648721514438696
                                         2.718281997301532
                                                          4.481690011262426
           7.389057016853605
```

In /Users/paulaoanan/School/cas741/Project/cas741/src/ testing/testClasses.m (testClasses.testLODESSolution2) at 34

yheun =

1.0000 1.62502.6406 4.2910 6.9729

```
ytestarr =
    1.0000    1.6487    2.7183    4.4817    7.3891

norm =
    0.0502
errorarray =
    0    0.0144    0.0286    0.0425    0.0563
```

The second secon

 $<\!\!\mathrm{strong}\!>\!\!\mathrm{Verification}\ \ \, \mathrm{failed}\ \ \, \mathrm{in}\ \ \, \mathrm{testClasses/testLODESSolution2.} <\!\!/\,\mathrm{strong}\!>$

Framework Diagnostic:

32

---> Failure table:

verifyEqual failed.

^{--&}gt; The values are not equal using "isequaln".

^{—&}gt; The error was not within relative tolerance.

```
_____</strong> <strong> _____</strong>
         strong>_____</strong> <strong>____</
         strong>
       2
                      1.625
                               1.6487215144387
                                                -0.0237215144386962
          -0.0143878236748625
                             2.22044604925031e-16
       3
                    2.640625
                              2.71828199730153
                                                -0.0776569973015317
                             2.22044604925031e-16
          -0.0285684109958506
                 4.291015625
                              4.48169001126243
                                                 -0.190674386262426
          -0.0425451974106339
                             2.22044604925031e-16
                               7.3890570168536
       5
               6.972900390625
                                                 -0.416156626228605
          -0.0563206678848734
                             2.22044604925031e-16
Actual double:
                                         2.6406250000000000
                                                           4.291015625000000
      1.0000000000000000
                       1.6250000000000000
           6.972900390625000
Expected double:
      1.0000000000000000
                        1.648721514438696
                                         2.718281997301532
                                                           4.481690011262426
           7.389057016853605
```

In /Users/paulaoanan/School/cas741/Project/cas741/src/ testing/testClasses.m (testClasses.testLODESSolution2) at 36

yrk =

1.0000 1.6484 2.71734.47947.3840 ytestarr =

1.0000 1.6487

2.7183

4.4817

7.3891

norm =

6.1253e-04

errorarray =

1.0e - 03 *

 $0 \quad 0.1723$

 $0.3443 \qquad 0.5165$

0.6884

 $<\!\!\mathrm{strong}\!>\!\!\mathrm{Verification}\ \ \, \mathrm{failed}\ \ \, \mathrm{in}\ \ \, \mathrm{test}Classes/\mathrm{testLODESSolution}2.<\!/\mathrm{strong}\!>$

Framework Diagnostic:

---> Failure table:

 $<\!a\ href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:bold">verifyEqual failed .$

^{—&}gt; The values are not equal using "isequaln".

^{—&}gt; The error was not within relative tolerance.

```
RelativeError
                          </strong><strong>
                                              RelativeTolerance </strong
        >
      <strong>_____</strong>
         strong>
      2
                   1.6484375
                               1.6487215144387
                                               -0.00028401443869619
         -0.000172263439403762
                             2.22044604925031e-16
      3
              2.71734619140625
                              2.71828199730153
                                               -0.000935805895281749
         -0.000344263728417704
                             2.22044604925031e-16
             4.47937536239624
                              4.48169001126243
                                               -0.00231464886618582
         -0.000516467863767717
                             2.22044604925031e-16
      5
              7.38397032395005
                               7.3890570168536
                                               -0.00508669290355268
         -0.000688408939320743
                             2.22044604925031e-16
Actual double:
     1.00000000000000000
                      1.648437500000000
                                      2.717346191406250
                                                      4.479375362396240
          7.383970323950052
Expected double:
     1.0000000000000000
                      1.648721514438696
                                      2.718281997301532
                                                      4.481690011262426
          7.389057016853605
```

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution2) at 38

yeuler =

ytestarr =

1.0000 -1.0108

norm =

36

2.1023

errorarray =

0 -2.9573

Verification failed in testClasses/testLODESSolution3.

Framework Diagnostic:

verifyEqual failed .

--> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

---> Failure table:

Index

Actual
Error

Expected

```
test
test

ytrap =

1.0000

ytestarr =
```

```
norm =
```

RelativeError RelativeTolerance ____ _____ _____ strong>_____ ____ 2 -1.01079006197098 -2.989209938029022.22044604925031e-162.95730048255542Actual double: 1 -4Expected double: Stack Information: In /Users/paulaoanan/School/cas741/Project/cas741/src/ testing/testClasses.m (testClasses.testLODESSolution3) at 49 -43.8973

0 -43.8973

1.0000 -1.0108

errorarray =

0 -42.4287

Verification failed in testClasses/testLODESSolution3.

Framework Diagnostic:

verifyEqual failed .

- —> The values are not equal using "isequaln".
- —> The error was not within relative tolerance.
- ---> Failure table:

```
<strong>Index</strong><strong>
    Expected </strong><strong>
    RelativeError </strong><strong>
    RelativeTolerance </strong>
<strong>-----</strong> <strong>
-----</strong> <strong> <
```

Actual double:

```
Stack Information:
        In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
           testing/testClasses.m',51,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
           testing/testClasses.m (testClasses.testLODESSolution3) at 51</a>
    yheun =
        1.0000
                 -43.8973
39
    ytestarr =
        1.0000
                  -1.0108
    norm =
       30.1622
    errorarray =
                -42.4287
```

Expected double:

Verification failed in testClasses/testLODESSolution3.

Framework Diagnostic:

- verifyEqual failed.
- —> The values are not equal using "isequaln".
- —> The error was not within relative tolerance.
- ---> Failure table:

```
<strong>Index</strong><strong>
    Expected </strong><strong>
    RelativeError </strong><strong>
<strong> RelativeTolerance </strong>
<strong> <strong
```

Actual double:

Expected double:

Stack Information:

In < a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m', 53, 1)">/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution3) at 53

$$1.0e + 03 *$$

$$0.0010 -1.7028$$

ytestarr =

$$1.0000 -1.0108$$

norm =

$$1.1969e+03$$

errorarray =

$$1.0e + 03 *$$

$$0 -1.6837$$

 $<\!\!\mathrm{strong}\!>\!\!\mathrm{Verification} \ \ \, \mathrm{failed} \ \ \, \mathrm{in} \ \ \, \mathrm{testClasses/testLODESSolution} 3.<\!/\mathrm{strong}\!>$

Framework Diagnostic:

```
<a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:</pre>
  bold">verifyEqual </a> failed.
—> The values are not equal using "isequaln".
—> The error was not within relative tolerance.
---> Failure table:
      <strong>Index</strong><strong>
                                Actual
                                             </strong>strong>
                                               </strong>strong>
        Expected
                  </strong><strong>
                                       Error
        RelativeError </strong> RelativeTolerance </strong>
      <strong>
        _____</strong> <strong> _____</strong>
        2
             -1702.84512882598 -1.01079006197098
                                             -1701.83433876401
        1683.66746250505
                       2.22044604925031e-16
Actual double:
     1.0e + 03 *
     Expected double:
     Stack Information:
In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
  testing/testClasses.m',55,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
  testing/testClasses.m (testClasses.testLODESSolution3) at 55</a>
```

veuler =

1.0000 0 0.84151.0427 0.0966-0.6695ytestarr =1.0000 0.80350.95380.74360.1502-1.0108norm =0.4569errorarray = 0 1.0000 0.11780.40220.3570-0.3376

Verification failed in testClasses/testLODESSolution4.

Framework Diagnostic:

- —> The values are not equal using "isequaln".
- —> The error was not within relative tolerance.
- ---> Failure table:

```
strong>
       <strong>_____</strong>
          _____</strong> <strong>_____</strong>
         strong>
       2
                                   0.803498260367072
                                                       -0.803498260367072
                                2.22044604925031e-16
                                                       -0.112336558430464
                0.841470984807897
                                    0.95380754323836
          -0.117776965832184
                             2.22044604925031e-16
                 1.04269499336001
                                   0.743638851851789
                                                       0.299056141508218
                            2.22044604925031e-16
         0.402152389918193
       5
               0.0966021522418493
                                  0.150235602951416
                                                      -0.0536334507095666
          -0.356995609934823
                             2.22044604925031e-16
       6
               -0.669532318883836
                                   -1.01079006197098
                                                       0.341257743087148
          -0.337614857848636
                             2.22044604925031e-16
Actual double:
      1.00000000000000000
                                          0.841470984807897 1.042694993360007
           0.096602152241849 -0.669532318883836
Expected double:
                        0.803498260367072
      1.0000000000000000
                                        0.953807543238360
                                                          0.743638851851789
           0.150235602951416 -1.010790061970984
Stack Information:
In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
  testing/testClasses.m',66,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
  testing/testClasses.m (testClasses.testLODESSolution4) at 66</a>
```

The identifier was:

 $1.0000 \qquad 0.9207 \qquad 0.9541 \qquad 0.5700 \qquad 0.0251 \qquad -1.1012$

ytestarr =

 $1.0000 \qquad 0.8035 \qquad 0.9538 \qquad 0.7436 \qquad 0.1502 \qquad -1.0108$

norm =

45

0.1277

errorarray =

 $0 \quad 0.1459 \quad 0.0003 \quad 0.2335 \quad 0.8328 \quad -0.0894$

Verification failed in testClasses/testLODESSolution4.

Framework Diagnostic:

verifyEqual failed .

^{--&}gt; The values are not equal using "isequaln".

^{--&}gt; The error was not within relative tolerance.

```
---> Failure table:
      <strong>Index</strong><strong>
Actual 
<strong>
               Expected </strong>strong> Error </strong>
                     RelativeError </strong>strong> RelativeTolerance
         strong>
         </strong>
      <strong>____</strong> <strong> <strong>
          _____</strong> <strong>_____</strong>
         strong>_____</strong> <strong>____</
         strong>
               0.920735492403948
       2
                                0.803498260367072
                                                       0.117237232036877
               0.14590850760936
                                 2.22044604925031e-16
                0.954130968258594
                                   0.95380754323836
                                                     0.000323425020233414
                              2.22044604925031e\!-\!16
         0.000339088343897265
                                  0.743638851851789
       4
                 0.56999267801283
                                                       -0.173646173838959
              -0.233508743399502
                                 2.22044604925031e-16
       5
              0.0251222646962669
                                0.150235602951416
                                                       -0.125113338255149
              -0.832780884139753
                                 2.22044604925031e-16
       6
                -1.10119664749119
                                -1.01079006197098
                                                      -0.0904065855202025
              0.0894415061263214
                                 2.22044604925031e-16
Actual double:
                       0.920735492403948
      1.0000000000000000
                                         0.954130968258594
                                                           0.569992678012830
           0.025122264696267 -1.101196647491187
Expected double:
      1.0000000000000000
                       0.803498260367072 0.953807543238360
                                                         0.743638851851789
           0.150235602951416 -1.010790061970984
```

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution4) at 68

The identifier was:

MATLAB: class: InvalidHandleThere was an error! The message was:

Invalid or deleted object.

yheun =

1.0000 0.9207 0.9541 0.5700 0.0251 -1.1012

ytestarr =

 $1.0000 \qquad 0.8035 \qquad 0.9538 \qquad 0.7436 \qquad 0.1502 \qquad -1.0108$

norm =

47

0.1277

errorarray =

 $0 \qquad 0.1459 \qquad 0.0003 \qquad 0.2335 \qquad 0.8328 \qquad -0.0894$

 $<\!\!\mathrm{strong}\!>\!\!\mathrm{Verification}\ \ \, \mathrm{failed}\ \ \, \mathrm{in}\ \ \, \mathrm{testClasses/testLODESSolution} 4.<\!/\!\,\mathrm{strong}\!>$

Framework Diagnostic:

- —> The values are not equal using "isequaln".
- —> The error was not within relative tolerance.
- --> Failure table:

Actual double:

Expected double:

verifyEqual failed.

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution4) at 70

yrk =

1.0000 0.7863

0.9434

0.7180

0.1408

-1.0284

ytestarr =

1.0000

0.8035

0.9538

0.7436

0.1502

-1.0108

norm =

49

0.0187

errorarray =

 $0 \quad 0.0214$

0.0109

0.0345

0.0626

-0.0174

Verification failed in testClasses/testLODESSolution4.

Framework Diagnostic:

- verifyEqual failed.
- --> The values are not equal using "isequaln".
- —> The error was not within relative tolerance.
- ---> Failure table:

```
0.78631839551037
                              0.803498260367072
                                                    -0.0171798648567016
   -0.0213813342282199
                          2.22044604925031e-16
         0.943371788846183
                               0.95380754323836
                                                    -0.0104357543921774
   -0.0109411531353023
                          2.22044604925031e-16
4
        0.717989268823369
                              0.743638851851789
                                                    -0.0256495830284202
   -0.0344919889063735
                          2.22044604925031e-16
5
          0.14083244087226
                              0.150235602951416
                                                   -0.00940316207915609
   -0.0625894388176213
                          2.22044604925031e-16
6
         -1.02836702616913 -1.01079006197098
                                                    -0.0175769641981469
  0.0173893322257965
                         2.22044604925031e-16
```

Actual double:

Expected double:

Stack Information:

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution4) at 72

Done testClasses

Failure Summary:

Name	Failed	Incomplete	Reason(s)
${\tt testClasses/testLODESSolution1}$	X		Failed by verification.
testClasses/testLODESSolution2	X		Failed by verification.
testClasses/testLODESSolution3	X		Failed by verification.
testClasses/testLODESSolution4	X		Failed by verification.

result =

1 4 TestResult array with properties:

Name

Passed

Failed

Incomplete

Duration

Details

Totals:

 $0\ \ Passed\ ,\ \ 4\ \ Failed\ ,\ \ 0\ \ Incomplete\ .$

12.6715 seconds testing time.