Test Plan for the Library of ODE Solvers (LODES)

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1 Revision History

Date	Version	Notes
October 24, 2017	1.0	Initial draft.

2 Symbols, Abbreviations and Acronyms

The following table lists the symbols, abbreviations and acronyms used in the Test Plan. The software library's Commonality Analysis (CA) tables provide supplementary items in addition to the ones listed below.

symbol	description
CA	Commonality Analysis
IDE	Integrated Development Environment
IVP	Initial Value Problem
ODE	Ordinary Differential Equation
LODES	Library of ODE Solvers
SRS	Software Requirements Specification
T	Test
O	Output

Table 1: Symbols, Abbreviations, and Acronyms used in the Test Plan

Contents

List of Tables

3   General Information

The following section provides an overview of the Test Plan for the Library of Ordinary Differential Equation (ODE) Solvers.
This section explains the purpose of this document, the scope of the system, and an overview of the following sections.

3.1   Purpose

The main purpose of this document (the Test Plan) is to describe the verification and validation process that will be used to test the functionality of LODES. This document closely follows the requirements and governs the subsequent testing activities. This document is intended to be used as a reference for all testing and will be used to increase confidence in the software implementation.

This document will be used as a guide and starting point for the Test Report. The test cases listed in this document will be executed and the output will be analyzed to uncover errors, increase confidence and correctness in the software.

3.2   Scope

The scope of the testing is limited to the Library of ODE Solvers. Given the appropriate inputs, each program in LODES is intended to find the solution to an Initial Value Problem (IVP).

Due to time and cost constraints, the scope of testing is limited to automated unit and manual system verification and validation activities.

Static testing will be briefly described and will be left to the developer and verifier to perform with due diligence.

3.3   Overview of Document

The following sections provide more detail about the testing of LODES. Information about the testing process is provided and the software specifications that were discussed in the Commonality Analysis are stated. The evaluation process that will be followed during testing is outlined and test cases for both the system testing and unit testing are provided.

4   Plan

This section provides a description of the software that is being tested, the team that will perform the testing, the approach to automated testing, the tools to be used for verification, and the non-testing based verification.

4.1 Software Description

The software being tested is the Library of ODE Solvers. Given the ODE, initial values of x and y, and the final value of x, the programs calculate the final value of y through the use of numerical methods.

4.2 Test Team

The test team that will execute the test cases, write and review the Test Report consists of:

- Paul Aoanan

- To be determined (The test report will be reviewed by an independent individual)

4.3 Automated Testing Approach

Automated unit testing will be implemented for LODES as described in Section 4.4. A combination of the Unit Testing Framework and MATLAB's Test Class will be used for automated testing.

4.4 Verification Tools

The verification tools to be used will be the following:

1. Unit Testing Framework
   A Unit Testing Framework designed in MATLAB that will compare MATLAB's own functional programs with LODES' running the same inputs will be implemented.

   The following algorithm will be implemented to compare the results:

$$\epsilon_{\text{relative}} = \frac{\text{Result}_{\text{MATLAB}} - \text{Result}_{\text{LODES}}}{\text{Result}_{\text{MATLAB}}}$$

2. MATLAB's Test Class
   MATLAB's Test Class (matlab.unittest.testcase) will be used for providing the framework for automated system testing. The documentation can be referenced in MATH-WORKS.

3. Static Analyer
   The program's IDE (MATLAB) will be used as a Static Analyzer tool for program debugging and for checking syntax errors.

4. Continuous Integration
   The source code and the project repository is located in GitHub at: `https://github.com/aoananp/cas741/`. It provides the Build Server functionality to fully maintain and document the software through its lifecycle. As well, it provides the compare functionality for future regression testing and analysis of code updates.

5. Code Coverage Tool
   Due to the commercial nature of MATLAB, only commercial code coverage tools are viable for use due to the maturity, increased confidence, and detailed documentation that they offer. Other coverage tools may be considered, but no code coverage tool will be considered in the scope of this test plan due to budget constraints.

4.5   Non-Testing Based Verification

LODES will undergo the following non-testing based verification activities:

Code Inspection
   LODES will undergo an initial desk review by an independent body. The code will be perused for syntax errors and correct program calls. This code inspection activity provides the initial sanity check for the developer, the reviewer, and the software.

Code Walkthrough
   LODES will undergo code walkthrough by the developer and an independent body. They will jointly review the code and reference the Commonality Analysis for algorithm adherence. This activity will also involve logic analysis, loop and recursion boundary tracing (using by-hand test cases), passing of variables and references, and if the code is programmatically correct. The code walkthrough will be executed with the guidance laid out in Section 7.1.

Symbolic Execution
   Generally, symbolic execution will be performed using the boundary input conditions. Test conditions will be analyzed and executed by-hand. Generally, inputs in, on, and around the boundary conditions are chosen.

5   System Test Description

System testing will be executed to provide increased confidence that LODES will achieve the goals defined in the Commonality Analysis. It uses a "black box" approach wherein it tests the system as a whole through the use of input and output analysis.

5.1   Tests for Faulty Input

5.1.1   Input

The input will be based on the Assumptions table in the Commonality Analysis. Each test will correspond to an entry from the assumptions item whilst altering a specific input variable to a non-permissible value. The list of inputs is in order with the entries in the table.

Table 2: Faulty Input Test Cases

Number	Input	Expected Outcome
01	ODE Function Call $\notin$ {euler741, trap741, heun741, rk4741} $\cup$ MATLAB functions	error: undefined function call
02	$f(x,y) = y'' + y' + x + 2$	success: false
03	$f(x,y) = y' + 1$	success: false
04	$f(x,y) = (dx/dy) + 1$	success: false
05	$f(x,y) = (y+1)/x$	success: false
06	$f(x,y) = y/(x-3)$	success: false
07	Boundary Value Problem	success: false
08	$h = -1$	success: false
09	$h = 0$	success: false
10	$x_0 = i$	success: false
11	$x_0 = [0,1]$	success: false
12	$y_0 = i$	success: false
13	$y_0 = [0,1]$	success: false
14	$x_k = i$	success: false
15	$x_k = [0,1]$	success: false

5.2 Tests for Functional Requirements

5.2.1 Calculation Tests

Euler's Method

1. **T-1: Simple Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x,y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$

   Output: $y_k = 3$, success = true

   How test will be performed: Automated system test

2. **T-2: Simple-Iterative Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x,y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

   Output: $y_k = 5.0625$, success = true

   How test will be performed: Automated system test

3. **T-3: Non-linear Case**

   Type: Functional, Automated, System

Initial State: Not applicable

Input: $f(x, y) = sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

Output: $y_k = -4$, success $=$ true

How test will be performed: Automated system test

4. **T-4: Non-linear Iterative Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

   Output: $y_k = -0.669532$, success $=$ true

   How test will be performed: Automated system test

Trapezoid Method

1. **T-5: Simple Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$

   Output: $y_k = 3$, success $=$ true

   How test will be performed: Automated system test

2. **T-6: Simple-Iterative Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

   Output: $y_k = 5.0625$, success $=$ true

   How test will be performed: Automated system test

3. **T-7: Non-linear Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

   Output: $y_k = -4$, success $=$ true

   How test will be performed: Automated system test

4. **T-8: Non-linear Iterative Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

   Output: $y_k = -0.6695$, success = true

   How test will be performed: Automated system test

Heun's Method

1. **T-9: Simple Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$

   Output: $y_k = 5$, success = true

   How test will be performed: Automated system test

2. **T-10: Simple-Iterative Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

   Output: $y_k = 6.972900$, success = true

   How test will be performed: Automated system test

3. **T-11: Non-linear Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

   Output: $y_k = -43.897311$, success = true

   How test will be performed: Automated system test

4. **T-12: Non-linear Iterative Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

   Output: $y_k = -1.101197$, success = true

   How test will be performed: Automated system test

Runge-Kutta Method

1. **T-13: Simple Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$

   Output: $y_k = 7$, success = true

   How test will be performed: Automated system test

2. **T-14: Simple-Iterative Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

   Output: $y_k = 7.383970$, success = true

   How test will be performed: Automated system test

3. **T-15: Non-linear Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

   Output: $y_k = -1702.845129$, success = true

   How test will be performed: Automated system test

4. **T-16: Non-linear Iterative Case**

   Type: Functional, Automated, System

   Initial State: Not applicable

   Input: $f(x, y) = sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

   Output: $y_k = -1.028367$, success = true

   How test will be performed: Automated system test

5.3 Tests for Nonfunctional Requirements

5.3.1 Performance Requirements

Speed

1. **T-17: Speed Benchmark**

   Type: Non-Functional, Automated, Performance

   Initial State: Not Applicable

Input/Condition: $f(x, y) = sin(x) - y^2, h = 1E - 5, x_0 = 0, y_0 = 1, x_k = 5$

Output/Result: $\sigma_{\text{LODES}} \leq 4 * \sigma_{\text{MATLAB}}$ (LODES' runtimes shall be no more than four (4) times that of MATLAB's.)

How test will be performed: Using MATLAB's Run and Time functionality, the execution time of a program will be measured and compared through program calls to the respective MATLAB and LODES functions.

5.3.2 Results Analysis

Benchmark Results

1. **T-18: MATLAB Benchmark**

   Type: Non-Functional, Automated, Precision

   Initial State: Not Applicable

   Input/Condition: $f(x, y) = sin(x) - y^2, h = (\text{VARIABLE}), x_0 = 0, y_0 = 1, x_k = 5$

   Output/Result: The $\epsilon_{\text{relative}}$ vs. h plot. The y_k values will be compared according to the following formula -

   $$\epsilon_{\text{relative}} = \frac{\text{Result}_{\text{MATLAB}} - \text{Result}_{\text{LODES}}}{\text{Result}_{\text{MATLAB}}}$$

   How test will be performed: The h (step-size) values will be varied across the range (0, 1000]. $\epsilon_{\text{relative}}$ will be plotted against h.

5.4 Traceability Between Test Cases and Requirements

The following table shows the traceability mapping for the test cases laid out in this Test Plan to the requirements described in the Commonality Analysis.

Table 3: Requirements Traceability Matrix

Test Number	CA Requirements
T1	IM1, O1, O2, O3, O4, O5
T2	IM1, O1, O2, O3, O4, O5
T3	IM1, O1, O2, O3, O4, O5
T4	IM1, O1, O2, O3, O4, O5
T5	IM2, O1, O2, O3, O4, O5
T6	IM2, O1, O2, O3, O4, O5
T7	IM2, O1, O2, O3, O4, O5
T8	IM2, O1, O2, O3, O4, O5
T9	IM3, O1, O2, O3, O4, O5
T10	IM3, O1, O2, O3, O4, O5
T11	IM3, O1, O2, O3, O4, O5
T12	IM3, O1, O2, O3, O4, O5
T13	IM4, O1, O2, O3, O4, O5
T14	IM4, O1, O2, O3, O4, O5
T15	IM4, O1, O2, O3, O4, O5
T16	IM4, O1, O2, O3, O4, O5
T17	NFR1
T18	Future NFR2

6 Unit Testing Plan

6.0.1 ODE String Parser

Parser Functionality

1. **T-19: Simple**

   Type: Functional, Manual, Unit

   Initial State: Not Applicable

   Input/Condition: $f(x,y) = ax^2 + by^2 - cx - dy - 1$

   Output/Result: Machine-interpreted $f(x,y)$

   How test will be performed: $f(x,y)$ will be passed into the parser function and the output shall be a machine-interpreted function.

2. **T-20: Trigonometric**

   Type: Functional, Manual, Unit

   Initial State: Not Applicable

   Input/Condition: $f(x,y) = sin^2(x) - y^3$

   Output/Result: Machine-interpreted $f(x,y)$

How test will be performed: $f(x, y)$ will be passed into the parser function and the output shall be a machine-interpreted function.

Function Iteration

1. **T-21: Program Iteration**

   Type: Functional, Manual, Unit

   Initial State: Not Applicable

   Input/Condition: $f(x, y) = y, h = .1, x_0 = 0, y_0 = 1, x_k = 1$

   Output/Result: y_k through each time step

   How test will be performed: Each iteration of the loops or recursions in the code will be observed in run-time. Each variable will be monitored prior to loop or function re-entry. This will be done through all four methods (Euler, Trapezoid, Heun, and Runge-Kutta).

7 Appendix

7.1 Walkthrough Questionnaire

1. Are there defects in the code?

2. Does the code adhere to MATLAB coding standards and guidelines (syntax, function calls, etc.)?

3. Does the code sensibly apply the ODE solvers as defined in the Commonality Analysis?

4. Does the code contain spelling errors?

5. Do the recursion/loops have the correct terminating cases according to the formulas defined in the Commonality Analysis?

6. Does the code have the correct inputs and outputs?

7. Do the datatypes in the code adhere to ones defined in the requirements and specifications?

8. Does the code satisfy the ranges and assumptions defined in the Commonality Analysis?

7.2 Symbolic Parameters

The definition of the test cases will call for SYMBOLIC_CONSTANTS. Their values are defined in this section for easy maintenance.

symbol	unit	description
ϵ	none	The measure of the difference/error between results obtained with LODES and MATLAB.
σ	seconds	The measure of time a program executes.

7.3 Usability Survey Questions

No usability study will be performed for LODES.