Test Report: Library of ODE Solvers (LODES)

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1   Revision History

Date	Version	Notes
December 18, 2017	1.0	Initial draft.

2 Symbols, Abbreviations and Acronyms

2.1 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the numeral analysis and ordinary differential equation literature and with existing documentation for solving ordinary differential equations. The symbols are listed in alphabetical order.

symbol	unit	type	description
dy/dx	-	$\mathbb{R}$	Rate of change of y with respect to x
$\varepsilon_{\text{relative}}$	-	$\mathbb{R}$	The relative error
F	-	$\mathbb{R}^3 \to \mathbb{R}$	Order function applied to the Runge Kutta Method
$f(x,y)$	-	$\mathbb{R}^2 \to \mathbb{R}$	Explicit form of the ODE function containing (x,y)
$f_x(x,y)$	-	$\mathbb{R}^2 \to \mathbb{R}$	Explicit form of the derivative of $f(x,y)$ with respect to x
$f_y(x,y)$	-	$\mathbb{R}^2 \to \mathbb{R}$	Explicit form of the derivative of $f(x,y)$ with respect to y
h	-	$\mathbb{R} \,.\, h > 0$	Step-size from $x_{(0)}$ to the next point $x_{(1)}$, where $x_{(1)} = x_{(0)} + h$
K_1, K_2, K_3, K_4	$\mathbb{R}$	-	Intermediary variables used in the Runge-Kutta method
n	-	$\mathbb{R}$	Reference recursion step
o	-	$\mathbb{R}$	Solution variable
T	-	-	Test
x_0	-	$\mathbb{R}$	Initial value x
x_k	-	$\mathbb{R}$	Final value x
x_n	-	$\mathbb{R}$	Intermediate n^{th} value x
y_0	-	$\mathbb{R}$	Initial value y
y_k	-	$\mathbb{R}$	Final value y
y_n	-	$\mathbb{R}$	Intermediate n^{th} value y
y'	-	$\mathbb{R} \to \mathbb{R}$	Implicit form of the first order ODE $= f(x,y)$
$y^{(n)}$	-	$\mathbb{R} \to \mathbb{R}$	Implicit form of the ODE to the nth order
y	-	$1 \text{ x } \mathbb{R}^k$	The array containing all intermediate y_n values

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3   Introduction

This document serves as the Test Report for the Library of ODE Solvers (LODES) which lays out in detail the results of the execution of the LODES Test Plan. The requirements for the software are described in the LODES Commonality Analysis. The aim of the Test Report is to document and show that LODES produces accurate and valid (in the scope of detailed in the Commonality Analysis and Test Plan) results.

The Overview (Section 4) discusses how testing slightly deviates from the Test Plan in its execution.

The following sections describe the evaluation of the software against the Functional Requirements (Section 5), Non-Functional Requirements (Section 6). It will also compare LODES to existing software in Comparison to Existing Implementation (Section 7).

In the sections after, Unit Testing (Section 8), Changes Due to Testing (Section 9), Automated Testing (Section 10), Trace to Requirements (Section 11), Trace to Modules (Section 12, and Code Coverage Metrics (Section 13) are discussed.

4   Overview

The Test Report documents the results of the tests carried out to verify the behaviour of the LODES software. The Test Plan suggests that it should carry out testing such that that the test report shall describe and analyze the numerical differences of the hand-executed solution to the numerical analysis methods versus the solutions obtained from LODES. This Test Report carries out the tests, with the same intent of verification, and instead of only comparing against the hand-executed solution, it compares the results with the MATLAB generated solutions as well.

5   Functional Requirements Evaluation

5.1   Calculation Tests

5.1.1   Simple Cases: T-1, T-5, T-9, T-13

Input: $f(x,y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$

Test Plan Test Case: T-1
Expected output: The value of $y_k = 3$. This test passes.

```
>> lodes(1, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:        0      2

The values of y are:        1      3


ans =

      0      2
```

Figure 1: T-1 result.

Test Plan Test Case: T-5
Expected output: The value of $y_k = 5$. This test passes.

```
>> lodes(2, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:        0      2

The values of y are:        1      5


ans =

      0      2
```

Figure 2: T-5 result.

Test Plan Test Case: T-9
Expected output: The value of $y_k = 5$. This test passes.

```
>> lodes(3, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:        0       2

The values of y are:        1       5


ans =

     0     2
```

Figure 3: T-9 result.

Test Plan Test Case: T-13
Expected output: The value of $y_k = 7$. This test passes.

```
>> lodes(4, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:        0       2

The values of y are:        1       7


ans =|

     0     2
```

Figure 4: T-13 result.

5.1.2   Simple-Iterative Cases: T-2, T-6, T-10, T-14

Input: $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

Test Plan Test Case: T-2
Expected output: The value of $y_k = 5.0625$. This test passes.

```
>> lodes(1, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:             0    0.5000    1.0000    1.5000    2.0000

The values of y are:     1.0000    1.5000    2.2500    3.3750    5.0625
```

Figure 5: T-2 result.

Test Plan Test Case: T-6

Expected output: The value of $y_k = 6.9729$. This test passes.

```
>> lodes(2, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:             0      0.5000      1.0000      1.5000      2.0000

The values of y are:     1.0000      1.6250      2.6406      4.2910      6.9729
```

Figure 6: T-6 result.

Test Plan Test Case: T-10

Expected output: The value of $y_k = 6.9729$. This test passes.

```
>> lodes(3, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:             0      0.5000      1.0000      1.5000      2.0000

The values of y are:     1.0000      1.6250      2.6406      4.2910      6.9729
```

Figure 7: T-10 result.

Test Plan Test Case: T-14

Expected output: The value of $y_k = 7.38340$. This test passes.

```
>> lodes(4, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:             0      0.5000      1.0000      1.5000      2.0000

The values of y are:     1.0000      1.6484      2.7173      4.4794      7.3840
```

Figure 8: T-14 result.

5.1.3 Linear Trigonometric Cases: T-3, T-7, T-11, T-15

Input: $f(x, y) = y, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

Test Plan Test Case: T-3

Expected output: The value of $y_k = -4$. This test passes.

```
>> lodes(1, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0      5

The values of y are:      1     -4
```

Figure 9: T-3 result.

Test Plan Test Case: T-7
Expected output: The value of $y_k = -43.8973$. This test passes.

```
>> lodes(2, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0      5

The values of y are:    1.0000  -43.8973
```

Figure 10: T-7 result.

Test Plan Test Case: T-11
Expected output: The value of $y_k = -43.8973$. This test passes.

```
>> lodes(3, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0      5

The values of y are:    1.0000  -43.8973
```

Figure 11: T-11 result.

Test Plan Test Case: T-15
Expected output: The value of $y_k = -1702.8$. This test passes.

```
>> lodes(4, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0      5

The values of y are:    1.0e+03 *

    0.0010   -1.7028
```

Figure 12: T-15 result.

5.1.4   Linear Trigonometric Iterative Cases: T-4, T-8, T-12, T-16

Input: $f(x, y) = sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

Test Plan Test Case: T-4
Expected output: The value of $y_k = -0.6695$. This test passes.

```
>> lodes(1, 'sin(x) - y^2', 0, 1, 5, 1, 1, 1)
The values of x are:      0     1     2     3     4     5

The values of y are:    1.0000          0    0.8415    1.0427    0.0966    -0.6695
```

Figure 13: T-4 result.

Test Plan Test Case: T-8
Expected output: The value of $y_k = -1.1012$. This test passes.

```
>> lodes(2, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are:      0     1     2     3     4     5

The values of y are:    1.0000    0.9207    0.9541    0.5700    0.0251    -1.1012
```

Figure 14: T-8 result.

Test Plan Test Case: T-12
Expected output: The value of $y_k = -1.1012$. This test passes.

```
>> lodes(3, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are:      0     1     2     3     4     5

The values of y are:    1.0000    0.9207    0.9541    0.5700    0.0251    -1.1012
```

Figure 15: T-12 result.

Test Plan Test Case: T-16
Expected output: The value of $y_k = -1.0284$. This test passes.

```
>> lodes(4, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are:      0     1     2     3     4     5

The values of y are:    1.0000    0.7863    0.9434    0.7180    0.1408    -1.0284
```

Figure 16: T-16 result.

6   Nonfunctional Requirements Evaluation

6.1   Speed Test Plan Test Case: T-17

Input: $f(x, y) = sin(x) - y^2, h = 1e - 3, x_0 = 0, y_0 = 1, x_k = 5$

Test Plan Test Case: T-17

Expected output: The execution time difference of LODES is not greater than 4 times that of MATLAB. This test fails as the execution time of LODES is 2,000 times greater than MATLAB's.

Figure 17: T-17 LODES rk() result.

Figure 18: T-17 MATLAB ode45() result.

7   Comparison to Existing Implementation

This section compares the calculation of LODES to MATLAB's ode45() function.

Test Plan Test Case: T-18

The following input parameters were tested (as above):

- Test 1 (7.1): $f(x, y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$

- Test 2 (7.2): $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

- Test 3 (7.3): $f(x, y) = sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

- Test 4 (7.4): $f(x, y) = sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

This phase of testing is automated and the results are shown in Section 15. The following equation was implemented to compare the results labeled as **norm**:

$$\epsilon_{\text{relative}} = \text{norm} = \frac{\|\text{Result}_{\text{MATLAB}} - \text{Result}_{\text{LODES}}\|}{\|\text{Result}_{\text{MATLAB}}\|}$$

Output/Result: The $\epsilon_{\text{relative}}$ vs. h plots. The intermediate y values (Result) are compared according to the following formula of the relative error norm percentage -

$$\epsilon_{\text{relative}} = 100 * \frac{\text{Result}_{\text{MATLAB}} - \text{Result}_{\text{LODES}}}{\text{Result}_{\text{MATLAB}}}$$

7.1 Test 1: $f(x, y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$
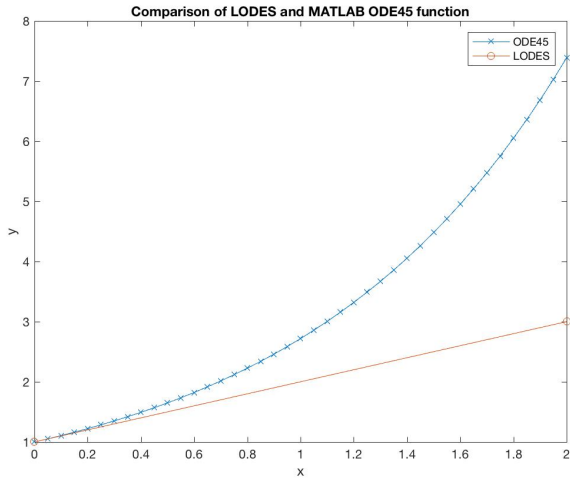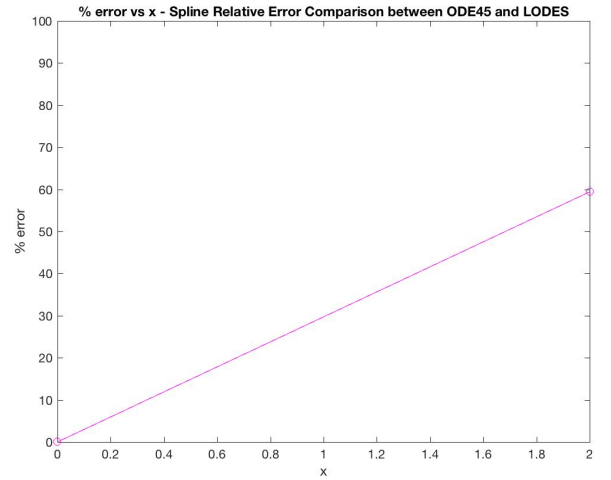
(a) Euler vs. ODE45   (b) Relative Error Norm - Euler

Figure 19: Euler's Method Analysis Graphs
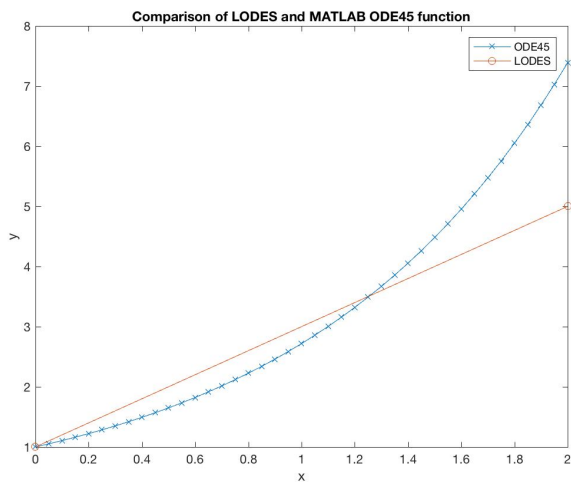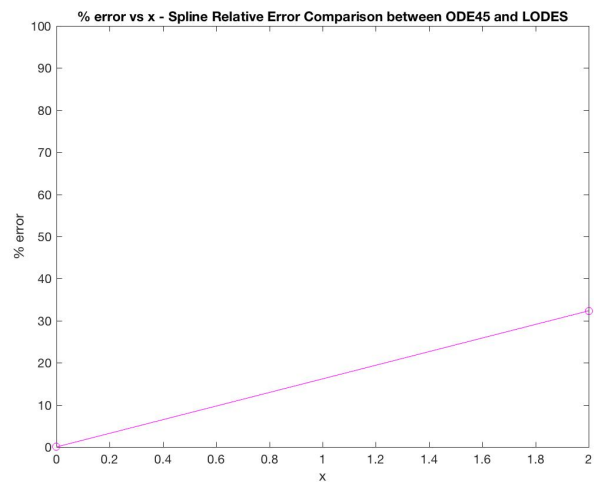
(a) Trapezoid vs. ODE45   (b) Relative Error Norm - Trapezoid

Figure 20: Trapezoidal Method Analysis Graphs
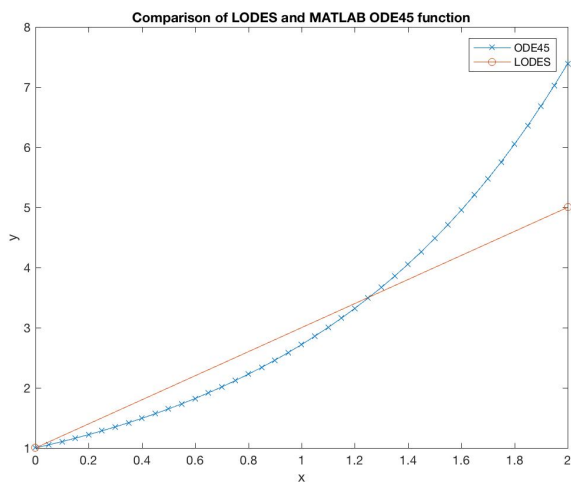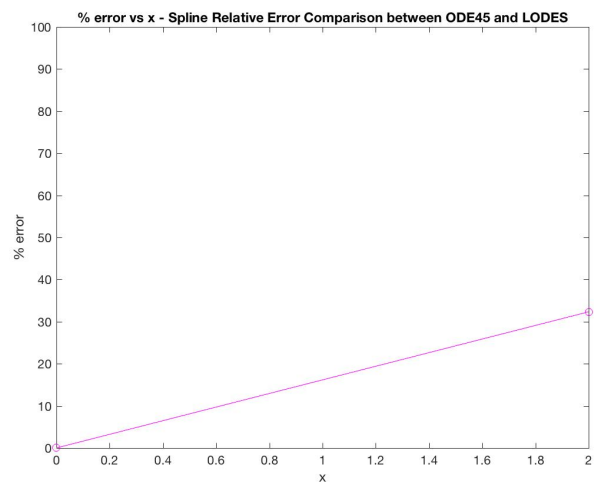
(a) Heun vs. ODE45   (b) Relative Error Norm - Heun

Figure 21: Heun's Method Analysis Graphs
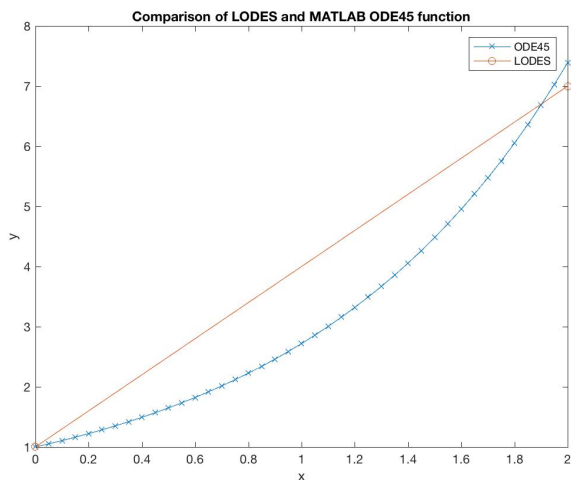
(a) Runge-Kutta 4 vs. ODE45
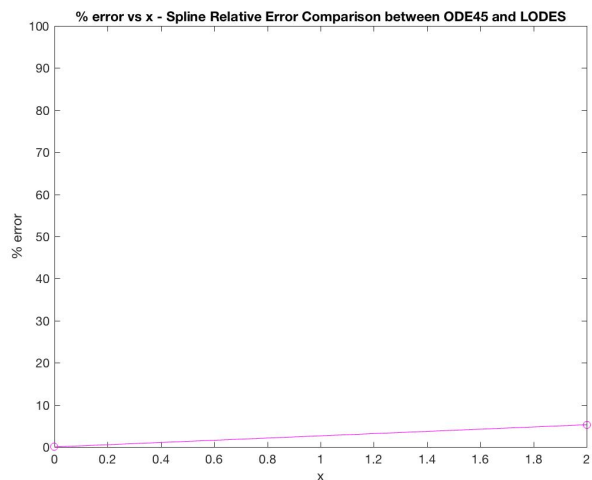
(b) Relative Error Norm - Runge-Kutta 4

Figure 22: Runge-Kutta 4 Method Analysis Graphs

7.2 Test 2: $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$
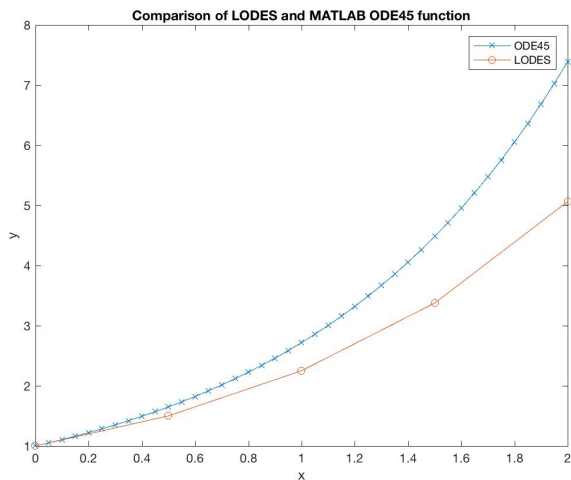
(a) Euler vs. ODE45
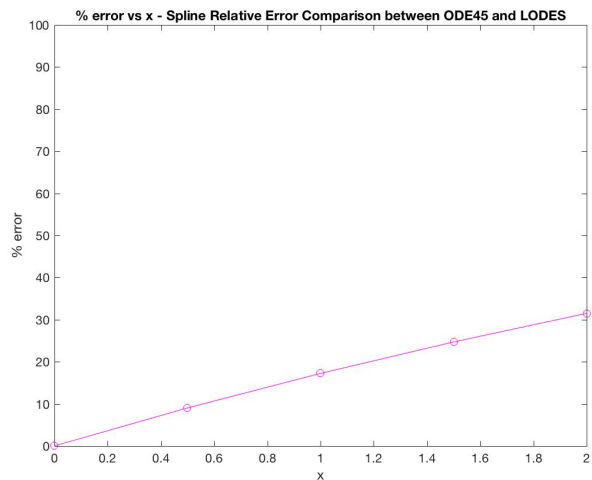
(b) Relative Error Norm - Euler

Figure 23: Euler's Method Analysis Graphs
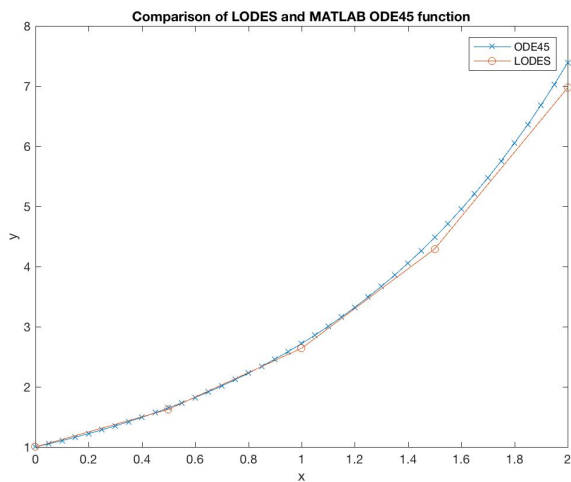
(a) Trapezoid vs. ODE45
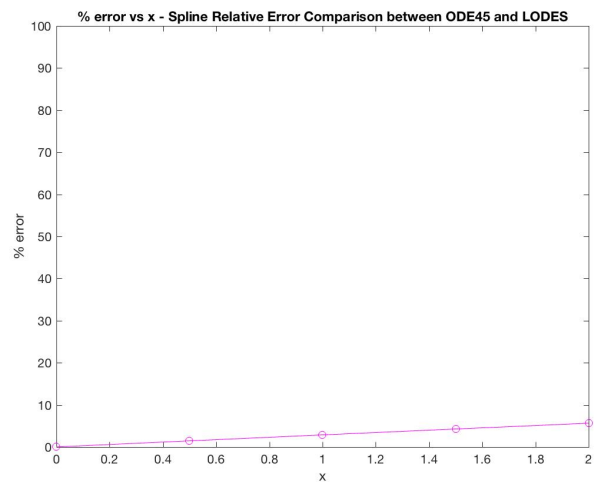
(b) Relative Error Norm - Trapezoid

Figure 24: Trapezoidal Method Analysis Graphs
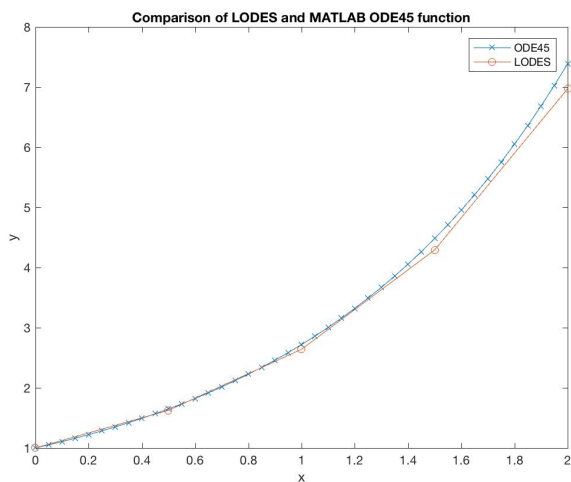
(a) Heun vs. ODE45
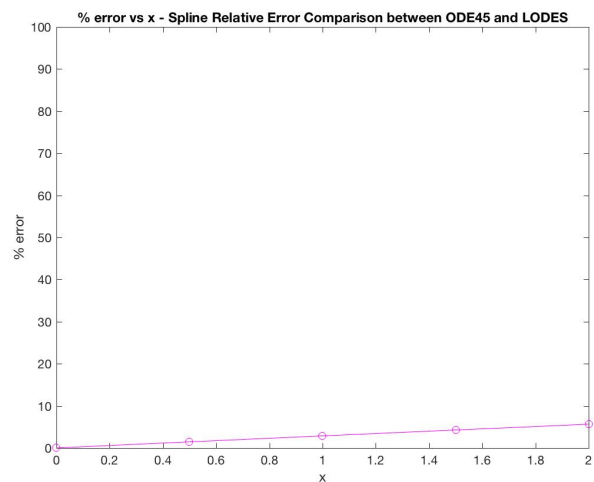
(b) Relative Error Norm - Heun

Figure 25: Heun's Method Analysis Graphs
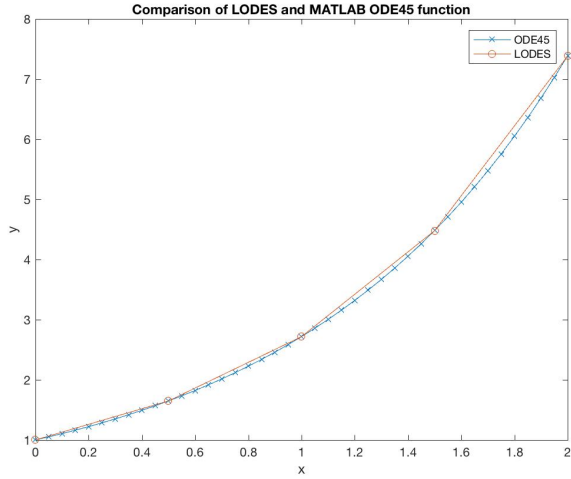
(a) Runge-Kutta 4 vs. ODE45
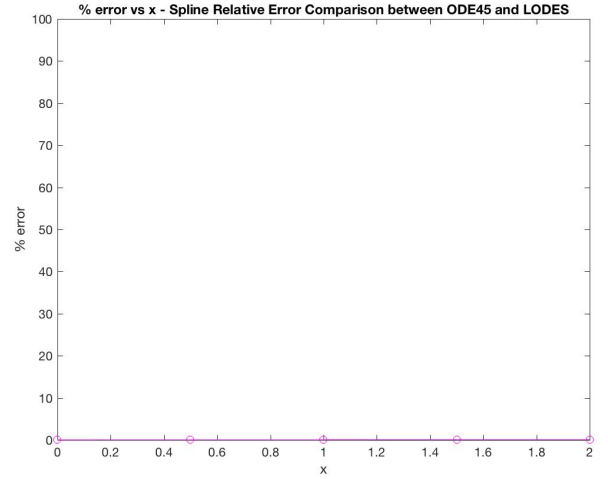
(b) Relative Error Norm - Runge-Kutta 4

Figure 26: Runge-Kutta 4 Method Analysis Graphs

7.3 Test 3: $f(x, y) = sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$
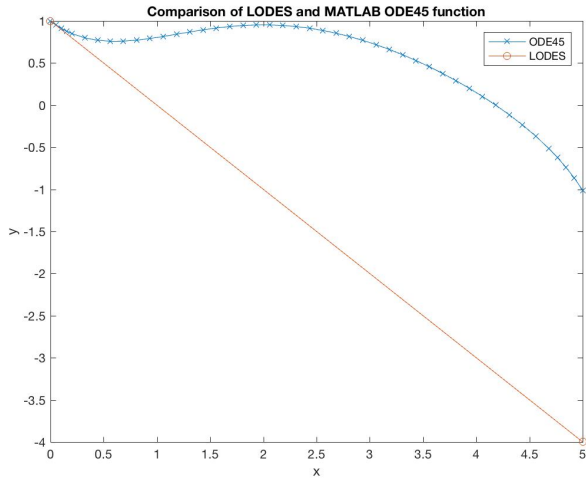
(a) Euler vs. ODE45
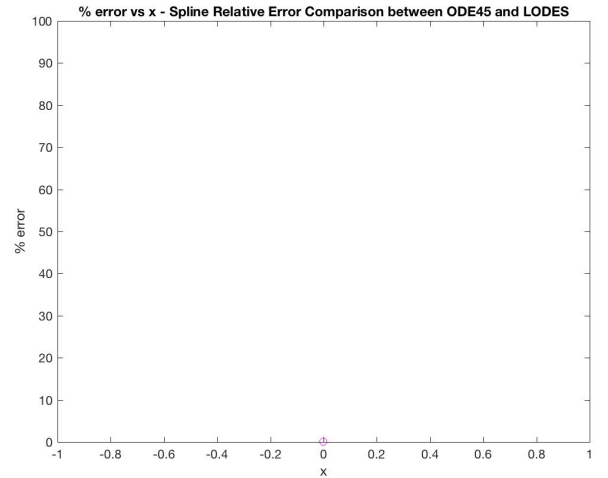
(b) Relative Error Norm - Euler

Figure 27: Euler's Method Analysis Graphs
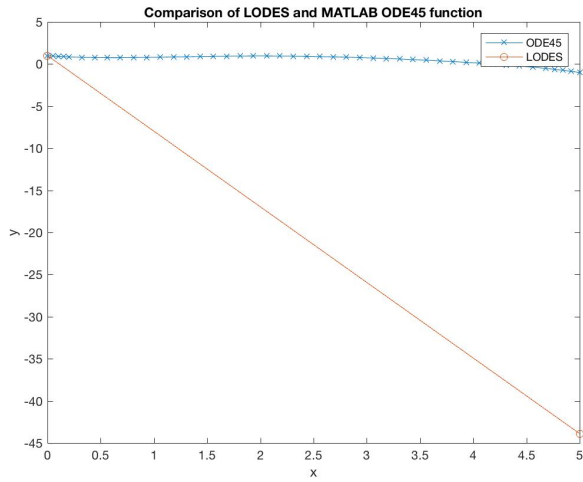
(a) Trapezoid vs. ODE45
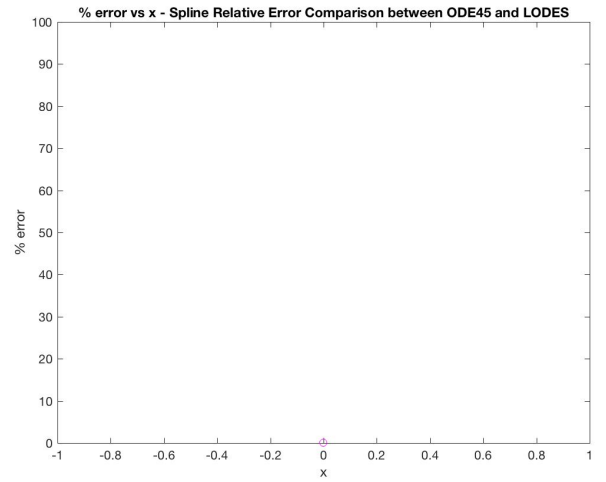
(b) Relative Error Norm - Trapezoid

Figure 28: Trapezoidal Method Analysis Graphs
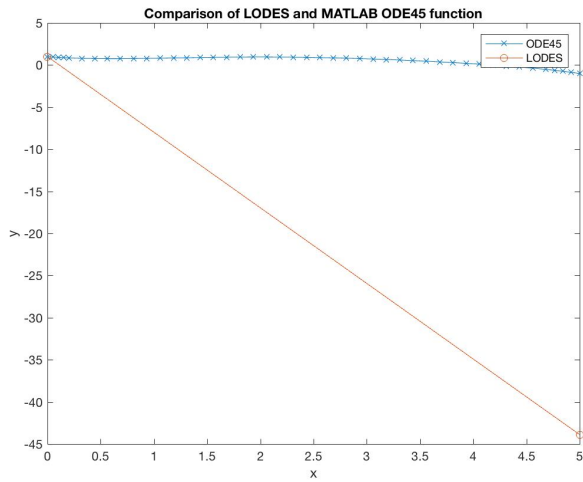
(a) Heun vs. ODE45
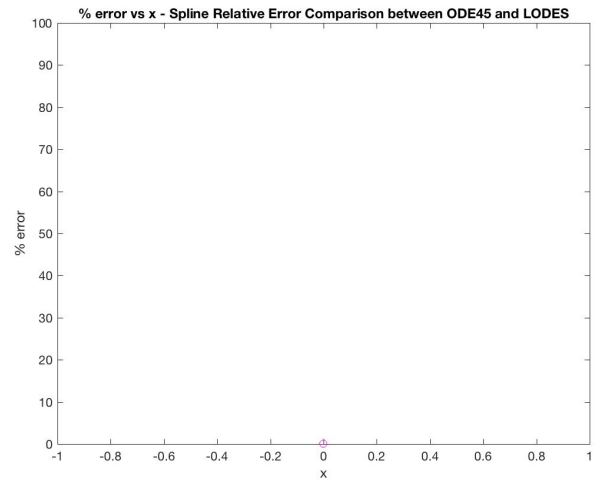
(b) Relative Error Norm - Heun

Figure 29: Heun's Method Analysis Graphs
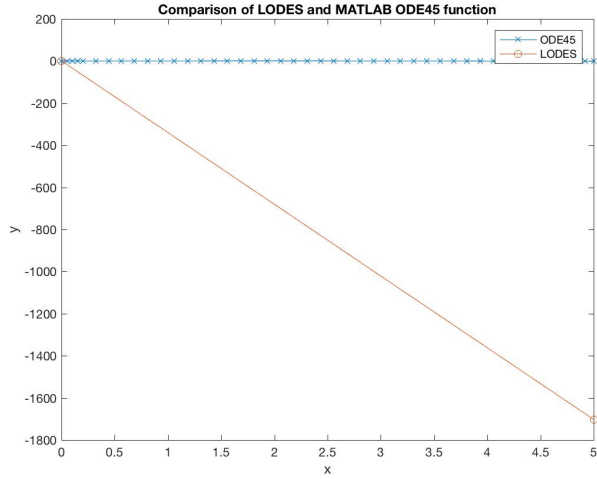
(a) Runge-Kutta 4 vs. ODE45
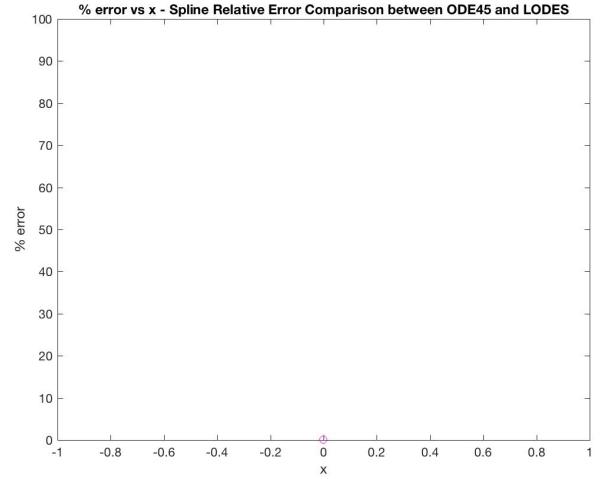
(b) Relative Error Norm - Runge-Kutta 4

Figure 30: Runge-Kutta 4 Method Analysis Graphs

7.4 Test 4: $f(x, y) = sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$
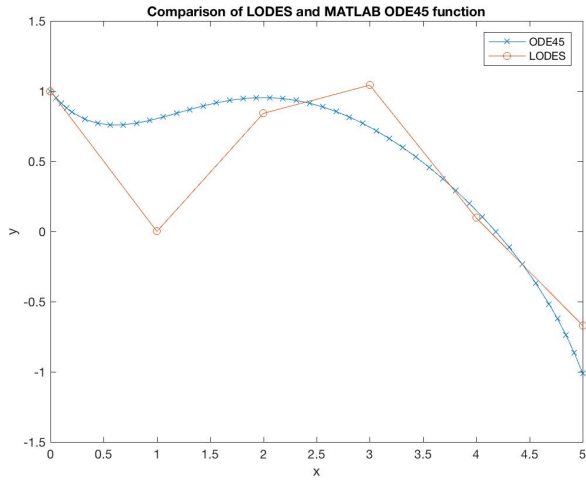
(a) Euler vs. ODE45
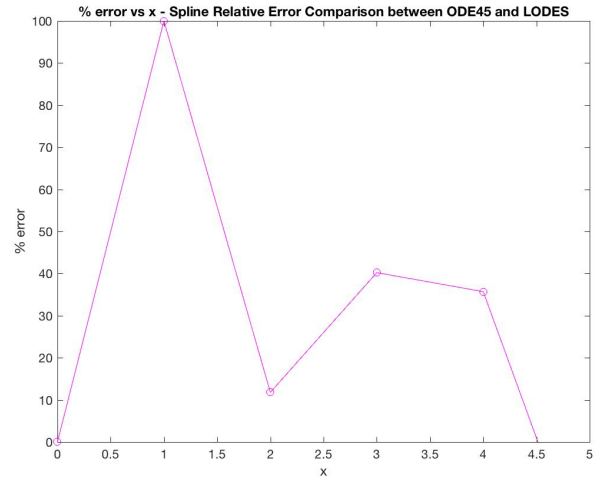
(b) Relative Error Norm - Euler

Figure 31: Euler's Method Analysis Graphs
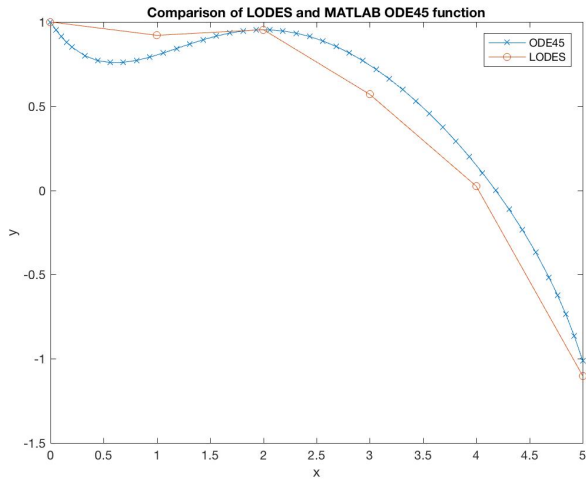
(a) Trapezoid vs. ODE45
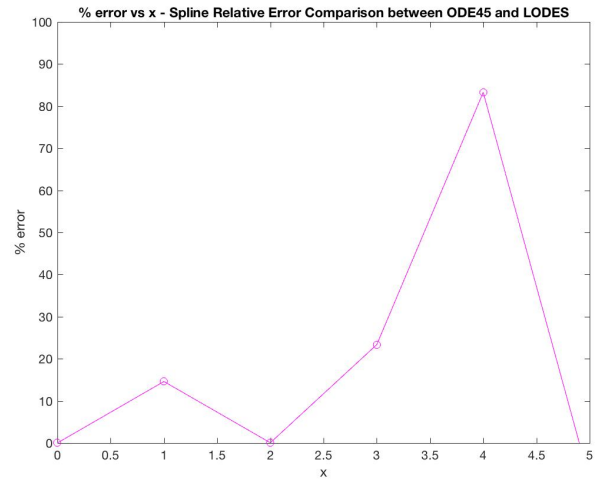
(b) Relative Error Norm - Trapezoid

Figure 32: Trapezoidal Method Analysis Graphs
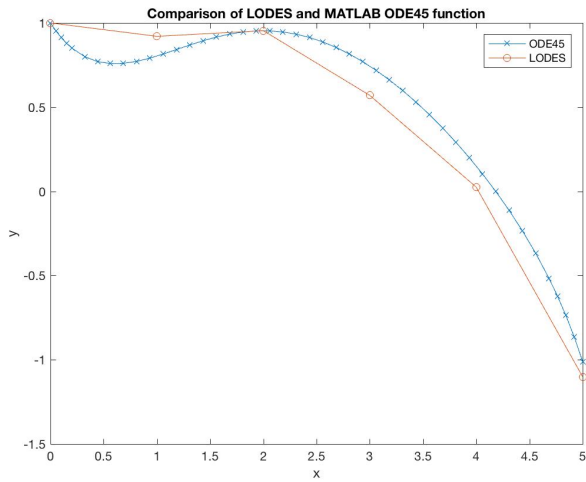
(a) Heun vs. ODE45
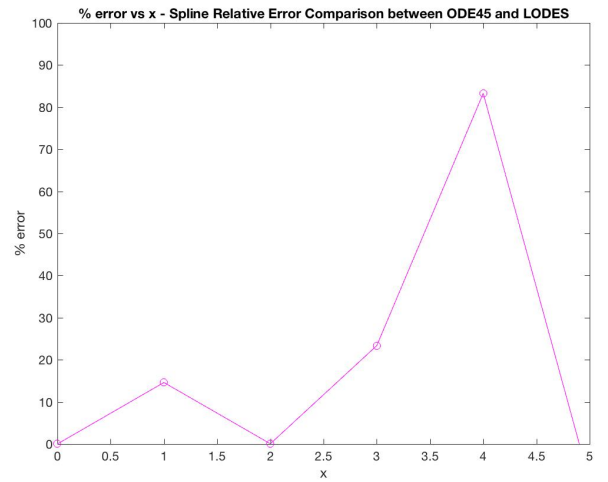
(b) Relative Error Norm - Heun

Figure 33: Heun's Method Analysis Graphs
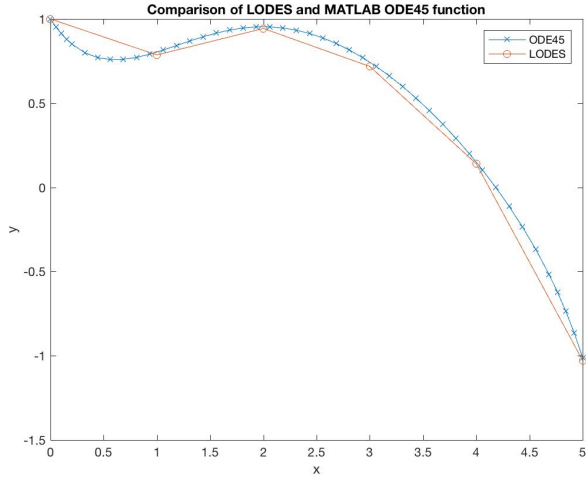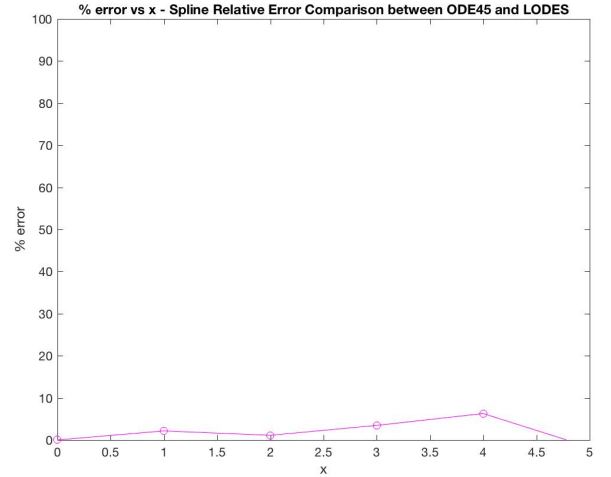
(a) Runge-Kutta 4 vs. ODE45   (b) Relative Error Norm - Runge-Kutta 4

Figure 34: Runge-Kutta 4 Method Analysis Graphs

8   Unit Testing

Unit testing was performed and the results and execution are the similar on the level of
Functional Requirement as described in Section 5 since the library is only of one level.

Further unit testing can be explored in the future.

Unit testing of the modules implemented by MATLAB and the Operating System (OS)
has been deemed unnecessary due to proof of use. The parser functionality tests have been
embedded in the functional tests of the modules of LODES in Section 5.

9   Changes Due to Testing

None.

10   Automated Testing

Automated testing has been performed in Section 7 and These were implemented in MAT-
LAB's Unit Testing Framework.

The ε used is based on MATLAB's precision. Note that the tests were forced to produce
a failure result for MATLAB to output the actual vs. expected results table. This provides
the reader of this Test Report the ability to directly compare the outputs produced by
MATLAB's ODE Solver, ODE45(), to that of LODES'.

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11 Trace to Requirements

The following table shows the traceability mapping for the test cases laid out in this Test Report to the requirements described in the Commonality Analysis.

Table 1: Requirements Traceability Matrix

Test Number	CA Requirements
T1	IM1, O1, O2, O3, O4, O5
T2	IM1, O1, O2, O3, O4, O5
T3	IM1, O1, O2, O3, O4, O5
T4	IM1, O1, O2, O3, O4, O5
T5	IM2, O1, O2, O3, O4, O5
T6	IM2, O1, O2, O3, O4, O5
T7	IM2, O1, O2, O3, O4, O5
T8	IM2, O1, O2, O3, O4, O5
T9	IM3, O1, O2, O3, O4, O5
T10	IM3, O1, O2, O3, O4, O5
T11	IM3, O1, O2, O3, O4, O5
T12	IM3, O1, O2, O3, O4, O5
T13	IM4, O1, O2, O3, O4, O5
T14	IM4, O1, O2, O3, O4, O5
T15	IM4, O1, O2, O3, O4, O5
T16	IM4, O1, O2, O3, O4, O5
T17	NFR1
T18	NFR2

12 Trace to Modules

The following table shows the traceability mapping for the test cases laid out in this Test Report to the modules in the Module Guide (MG).

Table 2: Design Traceability Matrix

Test Number	MG Modules
T1	M1, M2, M3, M4, M5
T2	M1, M2, M3, M4, M5
T3	M1, M2, M3, M4, M5
T4	M1, M2, M3, M4, M5
T5	M1, M2, M3, M4, M6
T6	M1, M2, M3, M4, M6
T7	M1, M2, M3, M4, M6
T8	M1, M2, M3, M4, M6
T9	M1, M2, M3, M4, M7
T10	M1, M2, M3, M4, M7
T11	M1, M2, M3, M4, M7
T12	M1, M2, M3, M4, M7
T13	M1, M2, M3, M4, M8
T14	M1, M2, M3, M4, M8
T15	M1, M2, M3, M4, M8
T16	M1, M2, M3, M4, M8
T17	-
T18	-

13 Code Coverage Metrics

The following Code Coverage figure was obtained from MATLAB's Code Coverage Report Tool. Running the test program with in-boundary conditions (excluding extraneous scenarios), the code coverage tool produces the following results:

Figure 35: Code Coverage Tool result.

14 Code Inspection Checklist

The figure below shows the completed Code Checklist.

7.1 Code Inspection Checklist

1. Are there defects in the code? Pass

2. Does the code adhere to MATLAB coding standards and guidelines (syntax, function calls, etc.)? Pass

3. Does the code sensibly apply the ODE solvers as defined in the Commonality Analysis? Pass

4. Does the code contain spelling errors? Pass

5. Do the recursion/loops have the correct terminating cases according to the formulas defined in the Commonality Analysis? Pass

6. Does the code have the correct inputs and outputs? [What does this mean? Do you mean whether it agrees with the requirements? —SS] [It shall agree with the inputs and outputs listed in the functional requirements listed above through code walkthrough. —PA] Pass

7. Do the datatypes in the code adhere to ones defined in the requirements and specifications? Pass

8. Does the code satisfy the ranges and assumptions defined in the Commonality Analysis? Pass.

[This isn't actually a code walkthrough procedure. This is a code inspection checklist. A walkthrough is when a group of reviewers "play computer" and go through the code and its algorithms by hand. —SS] [Noted - that was the intent of the section :-). The section has been renamed. —PA]

Performed by: Shawn Ruppert

Figure 36: Code Review Checklist Output.

15   Appendix A - Test Output

The raw output of automated testing can be found here: Automated Testing Output

Running testClasses

yeuler =

    1   3

ytestarr =

   1.0000   7.3891

norm =

   0.5886

errorarray =

      0   0.5940

==

\<strong\>Verification failed in testClasses/testLODESSolution1.\</strong\>

_____

Framework Diagnostic:
_____

verifyEqual failed.
---> The values are not equal using "isequaln".
---> The error was not within relative tolerance.
---> Failure table:
Index   Actual   Expected   Error   RelativeError   RelativeTolerance
_____   _____   _____   _____   _____   _____

  2   3   7.3890570168536   -4.3890570168536   -0.593994200727192   2.22044604925031e-16

Actual double:
     1   3
Expected double:
     1.000000000000000   7.389057016853605

_____

Stack Information:
_____

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution1) at 15
==

ytrap =

     1   5

ytestarr =

   1.0000   7.3891

norm =

   0.3204

errorarray =

        0   0.3233

===

Verification failed in testClasses/testLODESSolution1.

   _____

   Framework Diagnostic:
   _____

   <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font−weight:
      bold">verifyEqual failed.
   ——> The values are not equal using "isequaln".
   ——> The error was not within relative tolerance.
   ——> Failure table:
        Index   Actual   Expected
           Error   RelativeError
              RelativeTolerance
        _____   _____   _____
           _____   <strong>

```
                  ---------------- </strong>    <strong>------------------- </strong>

        2         5          7.3890570168536    −2.3890570168536    −0.323323667878653
                2.22044604925031e−16
```

Actual double:
```
        1        5
```
Expected double:
```
        1.000000000000000    7.389057016853605
```

────────────────────

Stack Information:
────────────────────

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
    testing/testClasses.m',17,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
    testing/testClasses.m (testClasses.testLODESSolution1) at 17

═══

yheun =

```
    1        5
```

ytestarr =

```
    1.0000    7.3891
```

norm =

```
    0.3204
```

errorarray =

    0   0.3233

━━

Verification failed in testClasses/testLODESSolution1.

    ─────────────────────
    Framework Diagnostic:
    ─────────────────────
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font−weight:
        bold">verifyEqual failed.
    ──> The values are not equal using "isequaln".
    ──> The error was not within relative tolerance.
    ──> Failure table:
        Index   Actual   Expected
           Error   RelativeError
           RelativeTolerance
        _____   _____   _____
           _____
         _____   _____

        2   5   7.3890570168536   −2.3890570168536   −0.323323667878653
              2.22044604925031e−16

    Actual double:
            1   5
    Expected double:
            1.000000000000000   7.389057016853605

———————————————
Stack Information:
———————————————

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
    testing/testClasses.m',19,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
    testing/testClasses.m (testClasses.testLODESSolution1) at 19

==

yrk =

    1   7

ytestarr =

    1.0000   7.3891

norm =

    0.0522

errorarray =

        0   0.0527

==

Verification failed in testClasses/testLODESSolution1.

———————————————

Framework Diagnostic:

<a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
   bold">verifyEqual failed.
---> The values are not equal using "isequaln".
---> The error was not within relative tolerance.
---> Failure table:
        Index   Actual   Expected
           Error
        RelativeError   RelativeTolerance
        _____   _____   _____
           _____
        _____   _____

        2   7   7.3890570168536   -0.389057016853605
              -0.0526531350301141   2.22044604925031e-16

Actual double:
          1   7
Expected double:
        1.000000000000000   7.389057016853605

Stack Information:

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m',21,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
   testing/testClasses.m (testClasses.testLODESSolution1) at 21
===

.
yeuler =

```
    1.0000      1.5000      2.2500      3.3750      5.0625


ytestarr =

    1.0000      1.6487      2.7183      4.4817      7.3891


norm =

    0.2832


errorarray =

         0      0.0902      0.1723      0.2469      0.3149
```

==

Verification failed in testClasses/testLODESSolution2.

─────────────────────────
Framework Diagnostic:
─────────────────────────

verifyEqual failed.
——> The values are not equal using "isequaln".
——> The error was not within relative tolerance.
——> Failure table:
        Index   Actual   Expected
           Error
        RelativeError   RelativeTolerance

_____   _____   _____   _____   _____   _____

2	1.5	1.6487215144387	−0.148721514438696	−0.0902041449306423	2.22044604925031e−16
3	2.25	2.71828199730153	−0.468281997301532	−0.172271308777529	2.22044604925031e−16
4	3.375	4.48169001126243	−1.10669001126243	−0.246935867603812	2.22044604925031e−16
5	5.0625	7.3890570168536	−2.3265570168536	−0.314865213727136	2.22044604925031e−16

Actual double:
    1.000000000000000   1.500000000000000   2.250000000000000   3.375000000000000
        5.062500000000000
Expected double:
    1.000000000000000   1.648721514438696   2.718281997301532   4.481690011262426
        7.389057016853605

_____

Stack Information:
_____

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution2) at 32

==

ytrap =

    1.0000   1.6250   2.6406   4.2910   6.9729

ytestarr =

   1.0000   1.6487   2.7183   4.4817   7.3891

norm =

   0.0502

errorarray =

        0   0.0144   0.0286   0.0425   0.0563

==

Verification failed in testClasses/testLODESSolution2.

    ————————————————
    Framework Diagnostic:
    ————————————————
    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font−weight:
      bold">verifyEqual failed.
    ——> The values are not equal using "isequaln".
    ——> The error was not within relative tolerance.
    ——> Failure table:
         Index   Actual
           Expected   Error
               RelativeError   RelativeTolerance

_____   _____   _____   _____   _____   _____

2	1.625	1.6487215144387	−0.0237215144386962	−0.0143878236748625	2.22044604925031e−16
3	2.640625	2.71828199730153	−0.0776569973015317	−0.028568410 9958506	2.22044604925031e−16
4	4.291015625	4.48169001126243	−0.190674386262426	−0.042545197 4106339	2.22044604925031e−16
5	6.972900390625	7.3890570168536	−0.416156626228605	−0.0563206678848734	2.22044604925031e−16

Actual double:
  1.000000000000000 1.625000000000000 2.640625000000000 4.291015625000000
    6.972900390625000
Expected double:
  1.000000000000000 1.648721514438696 2.718281997301532 4.481690011262426
    7.389057016853605

_____

Stack Information:
_____

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
    testing/testClasses.m',34,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
    testing/testClasses.m (testClasses.testLODESSolution2) at 34
==

yheun =

  1.0000   1.6250   2.6406   4.2910   6.9729

ytestarr =

    1.0000   1.6487   2.7183   4.4817   7.3891

norm =

    0.0502

errorarray =

       0   0.0144   0.0286   0.0425   0.0563

==

Verification failed in testClasses/testLODESSolution2.

————————————————
Framework Diagnostic:
————————————————

verifyEqual failed.
——> The values are not equal using "isequaln".
——> The error was not within relative tolerance.
——> Failure table:
      Index   Actual
     Expected   Error
       RelativeError   RelativeTolerance

_____   _____   _____   _____   _____   _____

2	1.625	1.6487215144387	−0.0237215144386962	−0.0143878236748625	2.22044604925031e−16
3	2.640625	2.71828199730153	−0.0776569973015317	−0.0285684109958506	2.22044604925031e−16
4	4.291015625	4.48169001126243	−0.190674386262426	−0.0425451974106339	2.22044604925031e−16
5	6.972900390625	7.3890570168536	−0.416156626228605	−0.0563206678848734	2.22044604925031e−16

Actual double:
     1.000000000000000   1.625000000000000   2.640625000000000   4.291015625000000
          6.972900390625000
Expected double:
     1.000000000000000   1.648721514438696   2.718281997301532   4.481690011262426
          7.389057016853605

_____

Stack Information:
_____

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution2) at 36
===

yrk =

    1.0000   1.6484   2.7173   4.4794   7.3840

ytestarr =

    1.0000   1.6487   2.7183   4.4817   7.3891

norm =

    6.1253e−04

errorarray =

    1.0e−03 *

        0   0.1723   0.3443   0.5165   0.6884

===
Verification failed in testClasses/testLODESSolution2.

     ————————————————
     Framework Diagnostic:
     ————————————————
     <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font−weight:
        bold">verifyEqual failed.
     —> The values are not equal using "isequaln".
     —> The error was not within relative tolerance.
     —> Failure table:
         Index   Actual
           Expected   Error

| RelativeError | | RelativeTolerance | </strong |

_____ _____ _____ _____ _____ _____

2	1.6484375	1.6487215144387	-0.00028401443869619
	$-0.00017226343940 3762$	$2.22044604925031e{-}16$	
3	2.71734619140625	2.71828199730153	-0.000935805895281749
	$-0.00034426372841 7704$	$2.22044604925031e{-}16$	
4	4.47937536239624	4.48169001126243	$-0.0023146488 6618582$
	-0.000516467863767717	$2.22044604925031e{-}16$	
5	7.38397032395005	7.3890570168536	-0.00508669290355268
	$-0.00068840893932 0743$	$2.22044604925031e{-}16$	

Actual double:
> 1.000000000000000   1.648437500000000   2.717346191406250   4.479375362396240
>   7.383970323950052

Expected double:
> 1.000000000000000   1.648721514438696   2.718281997301532   4.481690011262426
>   7.389057016853605

_____

Stack Information:

_____

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution2) at 38

==

.

yeuler =

$$1 \qquad -4$$

ytestarr =

$$1.0000 \qquad -1.0108$$

norm =

$$2.1023$$

errorarray =

$$0 \qquad -2.9573$$

==

Verification failed in testClasses/testLODESSolution3.

————————————————————

Framework Diagnostic:

————————————————————

verifyEqual failed.
——> The values are not equal using "isequaln".
——> The error was not within relative tolerance.
——> Failure table:
      Index   Actual   Expected
           Error

RelativeError RelativeTolerance
_____ _____
_____ _____ <
strong>_____ _____

		RelativeError	RelativeTolerance
2	−4	−1.01079006197098	−2.98920993802902
		2.95730048255542	2.22044604925031e−16

Actual double:
     1   −4
Expected double:
    1.000000000000000   −1.010790061970984

_____

Stack Information:
_____

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
testing/testClasses.m',49,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
testing/testClasses.m (testClasses.testLODESSolution3) at 49
===

ytrap =

   1.0000   −43.8973

ytestarr =

   1.0000   −1.0108

norm =

```
      30.1622


errorarray =

      0    −42.4287



==========================================================================
<strong>Verification failed in testClasses/testLODESSolution3.</strong>


      _____

      Framework Diagnostic:
      _____

      <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font−weight:
         bold">verifyEqual</a> failed.
      −−> The values are not equal using "isequaln".
      −−> The error was not within relative tolerance.
      −−> Failure table:
             <strong>Index</strong><strong>        Actual       </strong><strong>
                Expected      </strong><strong>        Error      </strong><strong>
                RelativeError  </strong><strong>     RelativeTolerance  </strong>
             <strong>_____ </strong>    <strong>_____ </strong>    <strong>
                _____ </strong>    <strong>_____ </strong>    <
                strong>_____ </strong>    <strong>_____ </strong>

             2        −43.8973106866578    −1.01079006197098    −42.8865206246869
                42.4287121907991    2.22044604925031e−16


   Actual double:
         1.000000000000000 −43.897310686657846
```

Expected double:
    1.000000000000000   −1.010790061970984

————————————
Stack Information:
————————————

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
    testing/testClasses.m',51,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
    testing/testClasses.m (testClasses.testLODESSolution3) at 51

==

yheun =

    1.0000   −43.8973

ytestarr =

    1.0000   −1.0108

norm =

    30.1622

errorarray =

        0   −42.4287

==

Verification failed in testClasses/testLODESSolution3.

———————————————

Framework Diagnostic:

———————————————

<a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font−weight:
  bold">verifyEqual failed.
——> The values are not equal using "isequaln".
——> The error was not within relative tolerance.
——> Failure table:
      Index   Actual
        Expected   Error
        RelativeError   RelativeTolerance
      _____   _____
        _____   _____   <
        strong>_____   _____

      2   −43.8973106866578   −1.01079006197098   −42.8865206246869
          42.4287121907991   2.22044604925031e−16

Actual double:
      1.000000000000000   −43.897310686657846
Expected double:
      1.000000000000000   −1.010790061970984

———————————————

Stack Information:

———————————————

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
  testing/testClasses.m',53,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
  testing/testClasses.m (testClasses.testLODESSolution3) at 53

yrk =

   1.0 e+03 *

    0.0010   −1.7028

ytestarr =

    1.0000   −1.0108

norm =

   1.1969 e+03

errorarray =

   1.0 e+03 *

      0   −1.6837

Verification failed in testClasses/testLODESSolution3.

————————————
Framework Diagnostic:
————————————

verifyEqual failed.
—-> The values are not equal using "isequaln".
—-> The error was not within relative tolerance.
—-> Failure table:
          Index   Actual   Expected   Error   RelativeError   RelativeTolerance
          _____   _____   _____   _____   _____   _____

       2   −1702.84512882598   −1.01079006197098   −1701.83433876401   1683.66746250505   2.22044604925031e−16

Actual double:
   1.0e+03 *

   0.001000000000000 −1.702845128825983
Expected double:
   1.000000000000000 −1.010790061970984

_____

Stack Information:
_____

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution3) at 55
==

.
yeuler =

```
    1.0000              0      0.8415      1.0427      0.0966     −0.6695


ytestarr =

    1.0000      0.8035      0.9538      0.7436      0.1502     −1.0108


norm =

    0.4569


errorarray =

         0      1.0000      0.1178      0.4022      0.3570     −0.3376
```

==

Verification failed in testClasses/testLODESSolution4.

    ──────────────────────
    Framework Diagnostic:
    ──────────────────────

    <a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font−weight:
        bold">verifyEqual failed.
    ──> The values are not equal using "isequaln".
    ──> The error was not within relative tolerance.
    ──> Failure table:
            Index   Actual
                Expected   Error   ×
            strong>   RelativeError   <strong>   RelativeTolerance </

```
          strong>
          <strong>_____</strong>    <strong>_____ </strong>    <strong>
          _____ </strong>    <strong>_____ </strong>    <
          strong>_____ </strong>    <strong>_____ </
          strong>


     2                          0      0.803498260367072      −0.803498260367072
                            −1      2.22044604925031e−16
     3         0.841470984807897         0.95380754323836      −0.112336558430464
          −0.117776965832184      2.22044604925031e−16
     4          1.04269499336001         0.743638851851789        0.299056141508218
          0.402152389918193      2.22044604925031e−16
     5        0.0966021522418493        0.150235602951416      −0.0536334507095666
          −0.356995609934823      2.22044604925031e−16
     6         −0.669532318883836         −1.01079006197098        0.341257743087148
          −0.337614857848636      2.22044604925031e−16
```

Actual double:
```
     1.000000000000000                       0     0.841470984807897      1.042694993360007
          0.096602152241849     −0.669532318883836
```
Expected double:
```
     1.000000000000000     0.803498260367072     0.953807543238360     0.743638851851789
          0.150235602951416     −1.010790061970984
```

Stack Information:

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
    testing/testClasses.m',66,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
    testing/testClasses.m (testClasses.testLODESSolution4) at 66

===

The identifier was:

MATLAB: class : InvalidHandleThere was an error! The message was:
Invalid or deleted object.
ytrap =

    1.0000   0.9207   0.9541   0.5700   0.0251   −1.1012

ytestarr =

    1.0000   0.8035   0.9538   0.7436   0.1502   −1.0108

norm =

    0.1277

errorarray =

         0   0.1459   0.0003   0.2335   0.8328   −0.0894

==

Verification failed in testClasses/testLODESSolution4.

————————————————
Framework Diagnostic :
————————————————

<a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font−weight:
bold">verifyEqual failed .
—-> The values are not equal using "isequaln".
—-> The error was not within relative tolerance.

—-> Failure table:
Index   Actual
          Expected   Error   <
strong>   RelativeError   RelativeTolerance

_____   _____
_____   _____   <
strong>_____   _____ </
strong>

Index	Actual	Expected	Error	RelativeError	RelativeTolerance
2	0.920735492403948	0.803498260367072	0.117237232036877	0.14590850760936	$2.22044604925031e{-}16$
3	0.954130968258594	0.95380754323836	0.000323425020233414	0.000339088343897265	$2.22044604925031e{-}16$
4	0.56999267801283	0.743638851851789	-0.173646173838959	-0.233508743399502	$2.22044604925031e{-}16$
5	0.0251222646962669	0.150235602951416	-0.125113338255149	-0.832780884139753	$2.22044604925031e{-}16$
6	-1.10119664749119	-1.01079006197098	-0.0904065855202025	0.0894415061263214	$2.22044604925031e{-}16$

Actual double:
    1.000000000000000   0.920735492403948   0.954130968258594   0.569992678012830
        0.025122264696267 -1.101196647491187
Expected double:
    1.000000000000000   0.803498260367072   0.953807543238360   0.743638851851789
        0.150235602951416 -1.010790061970984

_____

Stack Information:
_____

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution4) at 68

===

The identifier was:
MATLAB: class : InvalidHandleThere was an error! The message was:
Invalid or deleted object.
yheun =

    1.0000   0.9207   0.9541   0.5700   0.0251   −1.1012

ytestarr =

    1.0000   0.8035   0.9538   0.7436   0.1502   −1.0108

norm =

    0.1277

errorarray =

        0   0.1459   0.0003   0.2335   0.8328   −0.0894

===

Verification failed in testClasses/testLODESSolution4.

————————————————

Framework Diagnostic:

—————————————————

<a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font−weight:
bold">verifyEqual failed.
—> The values are not equal using "isequaln".
—> The error was not within relative tolerance.
—> Failure table:
Index   Actual
            Expected   Error   <
strong>   RelativeError   RelativeTolerance

_____   _____
_____   _____   <
strong>_____   _____ </
strong>

Index	Actual	Expected	Error	RelativeError	RelativeTolerance
2	0.920735492403948	0.803498260367072	0.117237232036877	0.14590850760936	2.22044604925031e−16
3	0.954130968258594	0.95380754323836	0.000323425020233414	0.00033908343897265	2.22044604925031e−16
4	0.56999267801283	0.743638851851789	−0.173646173838959	−0.233508743399502	2.22044604925031e−16
5	0.0251222646962669	0.150235602951416	−0.125113338255149	−0.832780884139753	2.22044604925031e−16
6	−1.10119664749119	−1.01079006197098	−0.0904065855202025	0.0894415061263214	2.22044604925031e−16

Actual double:
  1.000000000000000   0.920735492403948   0.954130968258594   0.569992678012830
        0.025122264696267   −1.101196647491187
Expected double:
  1.000000000000000   0.803498260367072   0.953807543238360   0.743638851851789
        0.150235602951416   −1.010790061970984

Stack Information:

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution4) at 70

===

yrk =

    1.0000   0.7863   0.9434   0.7180   0.1408   −1.0284

ytestarr =

    1.0000   0.8035   0.9538   0.7436   0.1502   −1.0108

norm =

    0.0187

errorarray =

        0   0.0214   0.0109   0.0345   0.0626   −0.0174

===

Verification failed in testClasses/testLODESSolution4.

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Framework Diagnostic:

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verifyEqual failed.
—–> The values are not equal using "isequaln".
—–> The error was not within relative tolerance.
—–> Failure table:
        Index   Actual   Expected   Error   RelativeError   RelativeTolerance
        _____   _____   _____   _____   _____   _____

Index	Actual	Expected	Error	RelativeError	RelativeTolerance
2	0.78631839551037	0.803498260367072	−0.0171798648567016	−0.0213813342282199	2.22044604925031e−16
3	0.943371788846183	0.95380754323836	−0.0104357543921774	−0.0109411531353023	2.22044604925031e−16
4	0.717989268823369	0.743638851851789	−0.0256495830284202	−0.0344919889063735	2.22044604925031e−16
5	0.14083244087226	0.150235602951416	−0.00940316207915609	−0.0625894388176213	2.22044604925031e−16
6	−1.02836702616913	−1.01079006197098	−0.0175769641981469	0.0173893322257965	2.22044604925031e−16

Actual double:
    1.000000000000000   0.786318395510370   0.943371788846183   0.717989268823369   0.140832440872260   −1.028367026169131
Expected double:

$$1.000000000000000 \quad 0.803498260367072 \quad 0.953807543238360 \quad 0.743638851851789$$
$$0.150235602951416 \quad -1.010790061970984$$

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Stack Information:

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In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution4) at 72

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Done testClasses

Failure Summary:

Name	Failed	Incomplete	Reason(s)
testClasses/testLODESSolution1	X		Failed by verification.
testClasses/testLODESSolution2	X		Failed by verification.
testClasses/testLODESSolution3	X		Failed by verification.
testClasses/testLODESSolution4	X		Failed by verification.

result =

1 4 TestResult array with properties:

Name
Passed
Failed
Incomplete
Duration
Details

Totals:
  0 Passed, 4 Failed, 0 Incomplete.
  12.6715 seconds testing time.