

Test Report: Library of ODE Solvers (LODES)

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1 Revision History

Date		Version	Notes
December 2017	18,	1.0	Initial draft.

2 Symbols, Abbreviations and Acronyms

2.1 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the numeral analysis and ordinary differential equation literature and with existing documentation for solving ordinary differential equations. The symbols are listed in alphabetical order.

symbol	unit	type	description
dy/dx	-	\mathbb{R}	Rate of change of y with respect to x
$\varepsilon_{\text{relative}}$	-	\mathbb{R}	The relative error
F	-	$\mathbb{R}^3 \rightarrow \mathbb{R}$	Order function applied to the Runge Kutta Method
$f(x, y)$	-	$\mathbb{R}^2 \rightarrow \mathbb{R}$	Explicit form of the ODE function containing (x, y)
$f_x(x, y)$	-	$\mathbb{R}^2 \rightarrow \mathbb{R}$	Explicit form of the derivative of $f(x, y)$ with respect to x
$f_y(x, y)$	-	$\mathbb{R}^2 \rightarrow \mathbb{R}$	Explicit form of the derivative of $f(x, y)$ with respect to y
h	-	$\mathbb{R} . h > 0$	Step-size from $x_{(0)}$ to the next point $x_{(1)}$, where $x_{(1)} = x_{(0)} + h$
K_1, K_2, K_3, K_4	\mathbb{R}	-	Intermediary variables used in the Runge-Kutta method
n	-	\mathbb{R}	Reference recursion step
o	-	\mathbb{R}	Solution variable
T	-	-	Test
x_0	-	\mathbb{R}	Initial value x
x_k	-	\mathbb{R}	Final value x
x_n	-	\mathbb{R}	Intermediate n^{th} value x
y_0	-	\mathbb{R}	Initial value y
y_k	-	\mathbb{R}	Final value y
y_n	-	\mathbb{R}	Intermediate n^{th} value y
y'	-	$\mathbb{R} \rightarrow \mathbb{R}$	Implicit form of the first order ODE = $f(x, y)$
$y^{(n)}$	-	$\mathbb{R} \rightarrow \mathbb{R}$	Implicit form of the ODE to the n^{th} order
y	-	$1 \times \mathbb{R}^k$	The array containing all intermediate y_n values

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3 Introduction

This document serves as the Test Report for the Library of ODE Solvers (LODES) which lays out in detail the results of the execution of the LODES Test Plan. The requirements for the software are described in the LODES Software Requirements Specification. The aim of the Test Report is to document and show that LODES produces accurate and valid (in the scope of detailed in the SRS and Test Plan) results.

The Overview (Section 4) discusses how testing slightly deviates from the Test Plan in its execution.

The following sections describe the evaluation of the software against the Functional Requirements (Section 5), Non-Functional Requirements (Section 6). It will also compare LODES to existing software in Comparison to Existing Implementation (Section 7).

In the sections after, Unit Testing (Section 8), Changes Due to Testing (Section 9), Automated Testing (Section 10), Trace to Requirements (Section 11), Trace to Modules (Section 12, and Code Coverage Metrics (Section 13) are discussed.

4 Overview

The Test Report documents the results of the tests carried out to verify the behaviour of the LODES software. The Test Plan suggests that it should carry out testing such that that the test report shall describe and analyze the numerical differences of the hand-executed solution to the numerical analysis methods versus the solutions obtained from LODES. This Test Report carries out the tests, with the same intent of verification, and instead of only comparing against the hand-executed solution, it compares the results with the MATLAB generated solutions as well.

5 Functional Requirements Evaluation

5.1 Calculation Tests

5.1.1 Simple Cases: T-1, T-5, T-9, T-13

Input: $f(x, y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$

Test Plan Test Case: T-1

Expected output: The value of $y_k = 3$. This test passes.

```
>> lodes(1, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:      0      2

The values of y are:      1      3

ans =

      0      2
```

Figure 1: T-1 result.

Test Plan Test Case: T-5

Expected output: The value of $y_k = 5$. This test passes.

```
>> lodes(2, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:      0      2

The values of y are:      1      5

ans =

      0      2
```

Figure 2: T-5 result.

Test Plan Test Case: T-9

Expected output: The value of $y_k = 5$. This test passes.


```
>> lodes(3, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:      0      2

The values of y are:      1      5

ans =

      0      2
```

Figure 3: T-9 result.

Test Plan Test Case: T-13

Expected output: The value of $y_k = 7$. This test passes.

```
>> lodes(4, 'y', 0, 1, 2, 2, 1, 1)
The values of x are:      0      2

The values of y are:      1      7

ans =|

      0      2
```

Figure 4: T-13 result.

5.1.2 Simple-Iterative Cases: T-2, T-6, T-10, T-14

Input: $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

Test Plan Test Case: T-2

Expected output: The value of $y_k = 5.0625$. This test passes.

```
>> lodes(1, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:      0      0.5000      1.0000      1.5000      2.0000

The values of y are:      1.0000      1.5000      2.2500      3.3750      5.0625
```

Figure 5: T-2 result.

Test Plan Test Case: T-6

Expected output: The value of $y_k = 6.9729$. This test passes.

```
>> lodes(2, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:      0      0.5000      1.0000      1.5000      2.0000
The values of y are:      1.0000      1.6250      2.6406      4.2910      6.9729
```

Figure 6: T-6 result.

Test Plan Test Case: T-10

Expected output: The value of $y_k = 6.9729$. This test passes.

```
>> lodes(3, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:      0      0.5000      1.0000      1.5000      2.0000
The values of y are:      1.0000      1.6250      2.6406      4.2910      6.9729
```

Figure 7: T-10 result.

Test Plan Test Case: T-14

Expected output: The value of $y_k = 7.38340$. This test passes.

```
>> lodes(4, 'y', 0, 1, 2, 0.5, 1, 1)
The values of x are:      0      0.5000      1.0000      1.5000      2.0000
The values of y are:      1.0000      1.6484      2.7173      4.4794      7.3840
```

Figure 8: T-14 result.

5.1.3 Linear Trigonometric Cases: T-3, T-7, T-11, T-15

Input: $f(x, y) = y, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

Test Plan Test Case: T-3

Expected output: The value of $y_k = -4$. This test passes.

```
>> lodes(1, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0      5

The values of y are:      1     -4
```

Figure 9: T-3 result.

Test Plan Test Case: T-7

Expected output: The value of $y_k = -43.8973$. This test passes.

```
>> lodes(2, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0      5

The values of y are:      1.0000 -43.8973
```

Figure 10: T-7 result.

Test Plan Test Case: T-11

Expected output: The value of $y_k = -43.8973$. This test passes.

```
>> lodes(3, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0      5

The values of y are:      1.0000 -43.8973
```

Figure 11: T-11 result.

Test Plan Test Case: T-15

Expected output: The value of $y_k = -1702.8$. This test passes.

```
>> lodes(4, 'sin(x) - y^2', 0, 1, 5, 5, 1, 1)
The values of x are:      0      5

The values of y are:      1.0e+03 *
                        0.0010  -1.7028
```

Figure 12: T-15 result.

5.1.4 Linear Trigonometric Iterative Cases: T-4, T-8, T-12, T-16

Input: $f(x, y) = \sin(x) - y^2$, $h = 1$, $x_0 = 0$, $y_0 = 1$, $x_k = 5$

Test Plan Test Case: T-4

Expected output: The value of $y_k = -0.6695$. This test passes.

```
>> lodes(1, 'sin(x) - y^2', 0, 1, 5, 1, 1, 1)
The values of x are:      0      1      2      3      4      5

The values of y are:      1.0000      0      0.8415      1.0427      0.0966      -0.6695
```

Figure 13: T-4 result.

Test Plan Test Case: T-8

Expected output: The value of $y_k = -1.1012$. This test passes.

```
>> lodes(2, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are:      0      1      2      3      4      5

The values of y are:      1.0000      0.9207      0.9541      0.5700      0.0251      -1.1012
```

Figure 14: T-8 result.

Test Plan Test Case: T-12

Expected output: The value of $y_k = -1.1012$. This test passes.

```
>> lodes(3, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are:      0      1      2      3      4      5

The values of y are:      1.0000      0.9207      0.9541      0.5700      0.0251      -1.1012
```

Figure 15: T-12 result.

Test Plan Test Case: T-16

Expected output: The value of $y_k = -1.0284$. This test passes.

```
>> lodes(4, 'sin(x) - y^2', 0, 1, 5, 1, 0, 1)
The values of x are:      0      1      2      3      4      5

The values of y are:      1.0000      0.7863      0.9434      0.7180      0.1408      -1.0284
```

Figure 16: T-16 result.

6 Nonfunctional Requirements Evaluation

6.1 Speed Test Plan Test Case: T-17

Input: $f(x, y) = \sin(x) - y^2$, $h = 1e - 3$, $x_0 = 0$, $y_0 = 1$, $x_k = 5$

Test Plan Test Case: T-17

Expected output: The execution time difference of LODES is not greater than 4 times that of MATLAB. This test fails as the execution time of LODES is 2,000 times greater than MATLAB's.

Profile Summary

Generated 15-Dec-2017 23:08:41 using performance time.

Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
lodes	1	472.941 s	0.003 s	
rk	1	472.766 s	2.209 s	

Figure 17: T-17 LODES rk() result.

Profile Summary

Generated 15-Dec-2017 23:29:45 using performance time.

Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
testode45	1	0.273 s	0.034 s	
ode45	1	0.239 s	0.110 s	

Figure 18: T-17 MATLAB ode45() result.

7 Comparison to Existing Implementation

This section compares the calculation of LODES to MATLAB's ode45() function.

Test Plan Test Case: T-18

The following input parameters were tested (as above):

- Test 1 (7.1): $f(x, y) = y$, $h = 2$, $x_0 = 0$, $y_0 = 1$, $x_k = 2$
- Test 2 (7.2): $f(x, y) = y$, $h = 0.5$, $x_0 = 0$, $y_0 = 1$, $x_k = 2$

- Test 3 (7.3): $f(x, y) = \sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$
- Test 4 (7.4): $f(x, y) = \sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

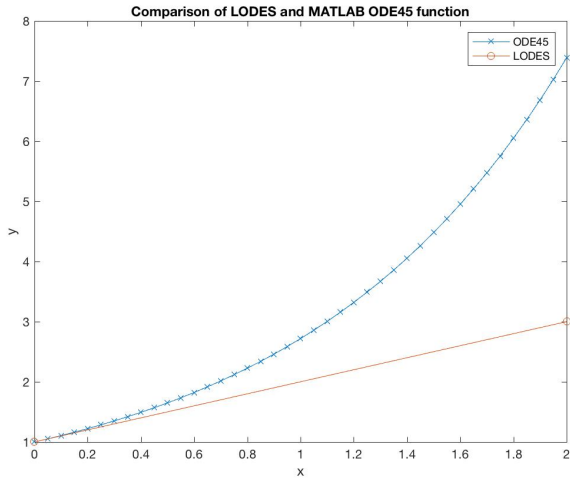
This phase of testing is automated and the results are shown in Section 15. The following equation was implemented to compare the results labeled as **norm**:

$$\epsilon_{\text{relative}} = \text{norm} = \frac{\|\text{Result}_{\text{MATLAB}} - \text{Result}_{\text{LODES}}\|}{\|\text{Result}_{\text{MATLAB}}\|}$$

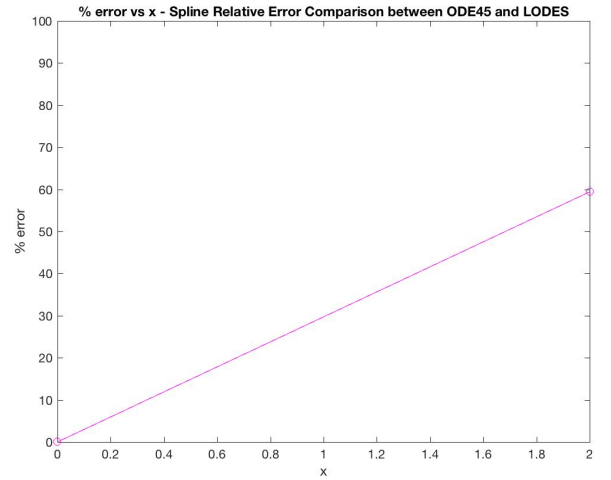
Output/Result: The $\epsilon_{\text{relative}}$ vs. h plots. The intermediate y values (Result) are compared according to the following formula of the relative error norm percentage -

$$\epsilon_{\text{relative}} = 100 * \frac{\text{Result}_{\text{MATLAB}} - \text{Result}_{\text{LODES}}}{\text{Result}_{\text{MATLAB}}}$$

7.1 Test 1: $f(x, y) = y, h = 2, x_0 = 0, y_0 = 1, x_k = 2$

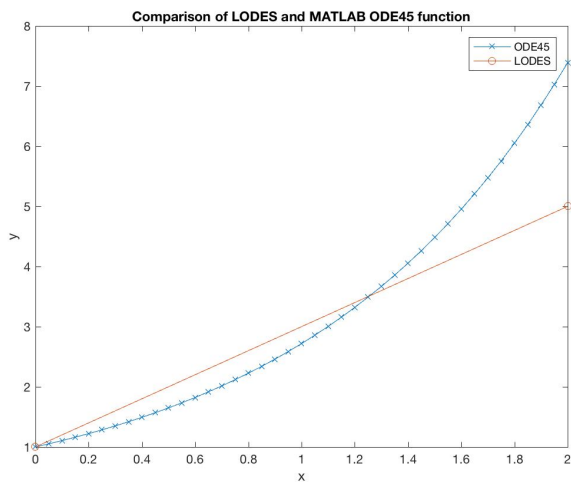


(a) Euler vs. ODE45

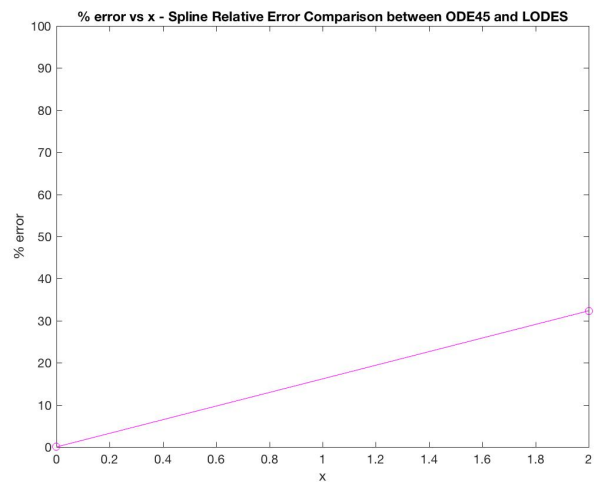


(b) Relative Error Norm - Euler

Figure 19: Euler's Method Analysis Graphs

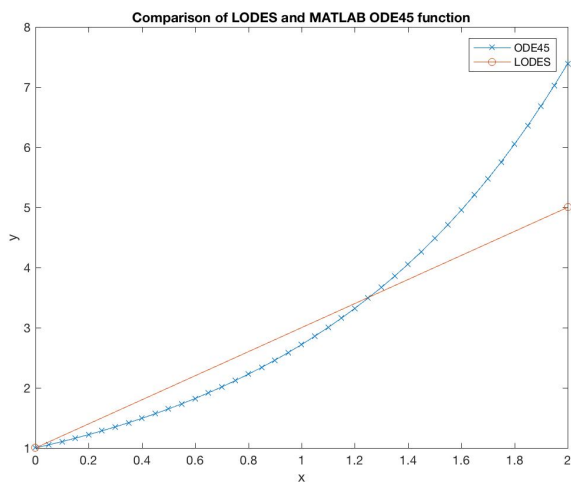


(a) Trapezoid vs. ODE45

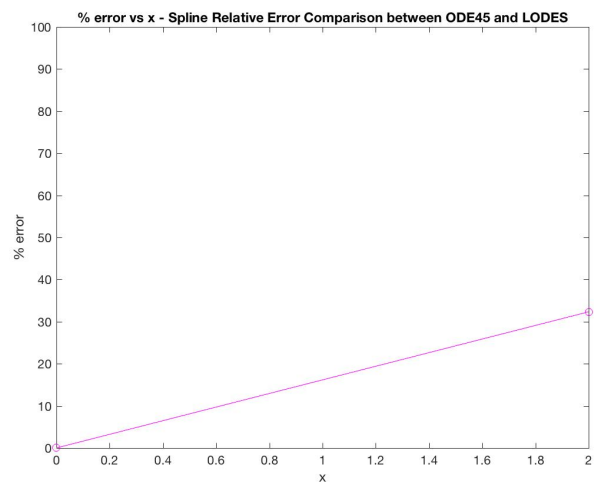


(b) Relative Error Norm - Trapezoid

Figure 20: Trapezoidal Method Analysis Graphs

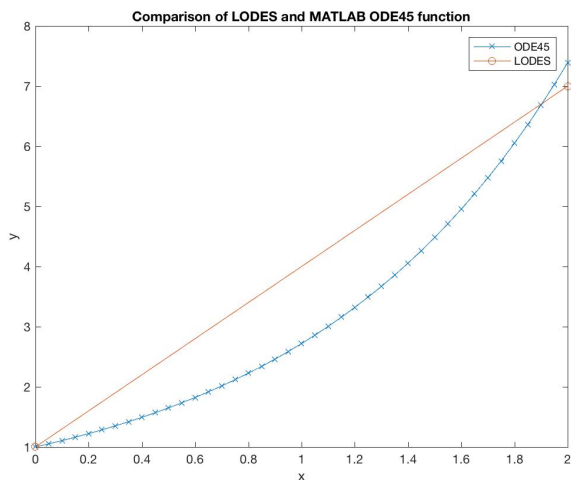


(a) Heun vs. ODE45

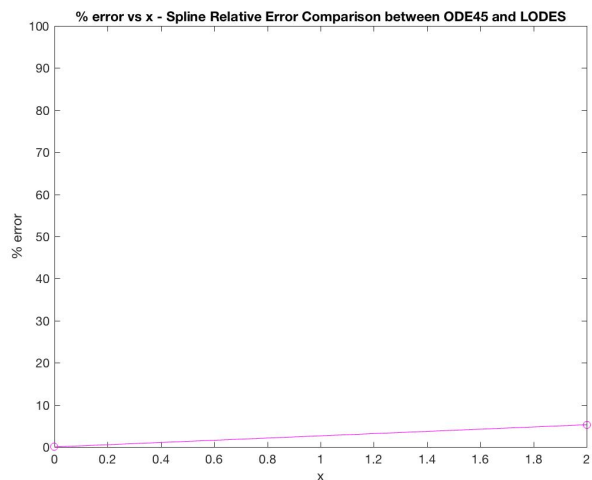


(b) Relative Error Norm - Heun

Figure 21: Heun's Method Analysis Graphs



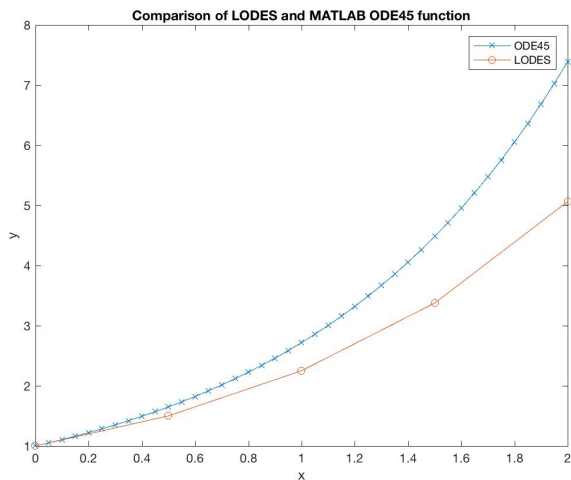
(a) Runge-Kutta 4 vs. ODE45



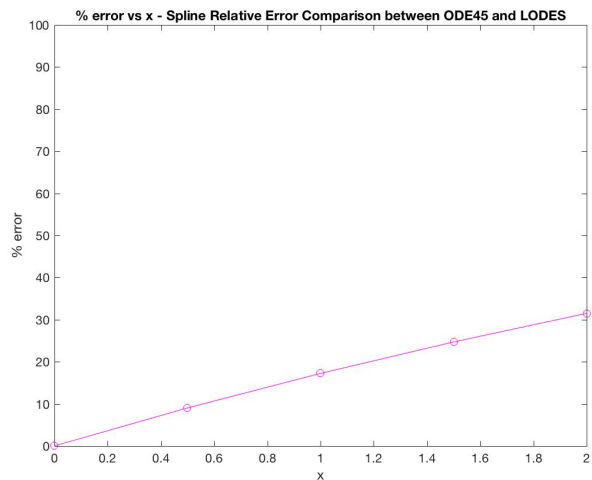
(b) Relative Error Norm - Runge-Kutta 4

Figure 22: Runge-Kutta 4 Method Analysis Graphs

7.2 Test 2: $f(x, y) = y, h = 0.5, x_0 = 0, y_0 = 1, x_k = 2$

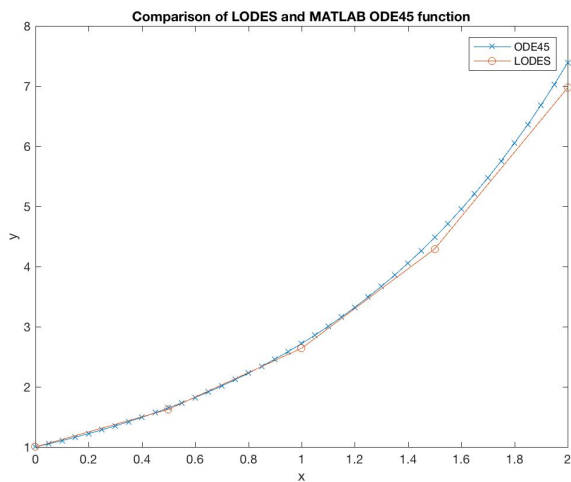


(a) Euler vs. ODE45

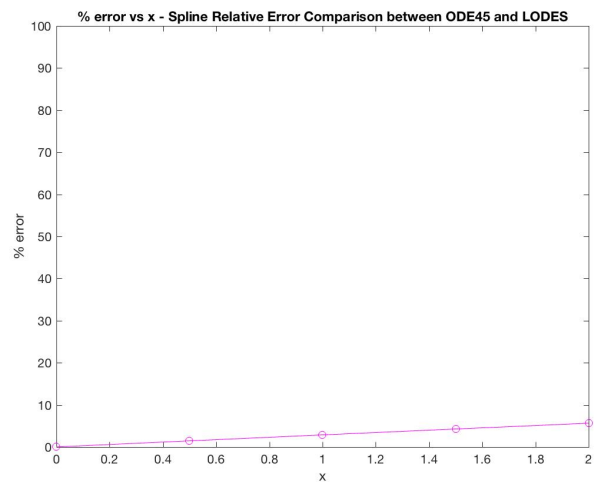


(b) Relative Error Norm - Euler

Figure 23: Euler's Method Analysis Graphs

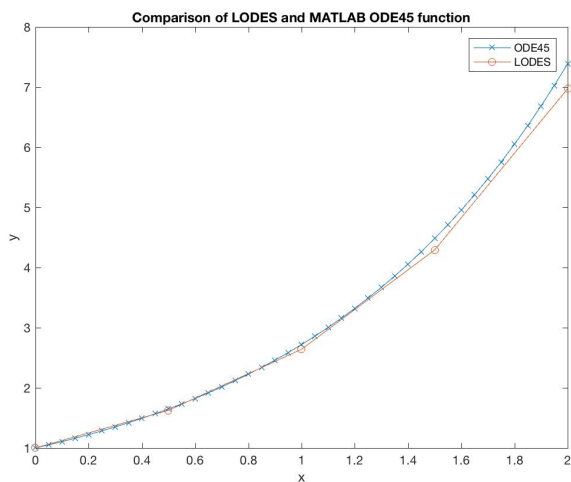


(a) Trapezoid vs. ODE45

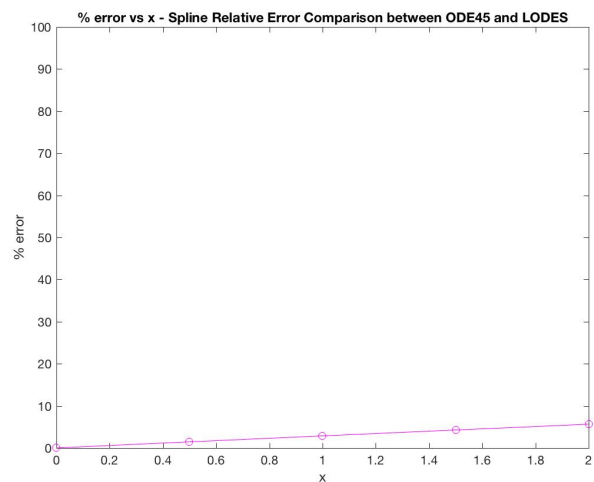


(b) Relative Error Norm - Trapezoid

Figure 24: Trapezoidal Method Analysis Graphs

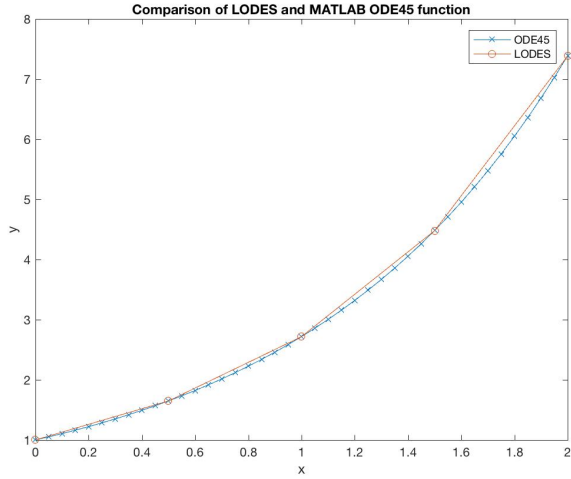


(a) Heun vs. ODE45

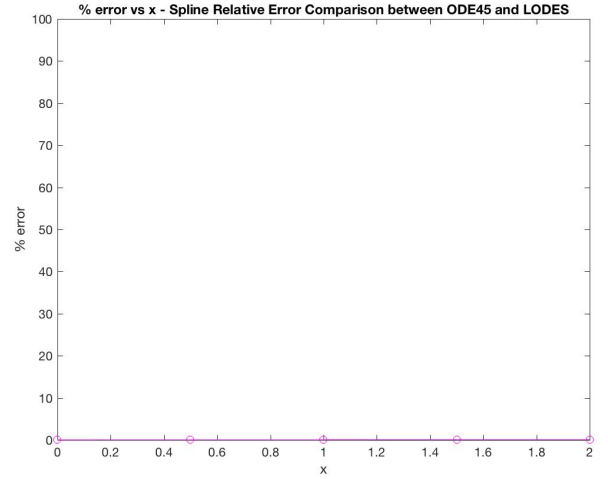


(b) Relative Error Norm - Heun

Figure 25: Heun's Method Analysis Graphs



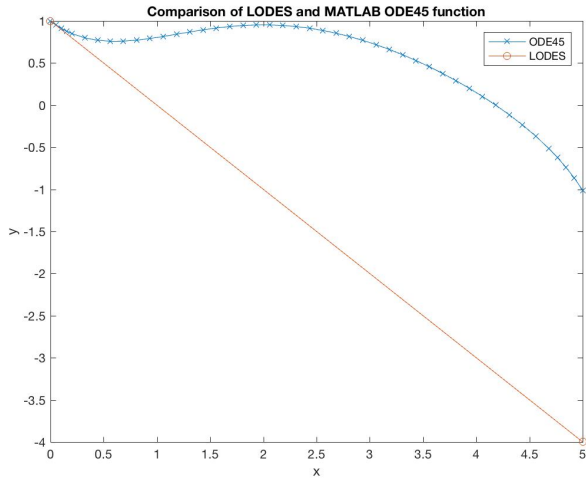
(a) Runge-Kutta 4 vs. ODE45



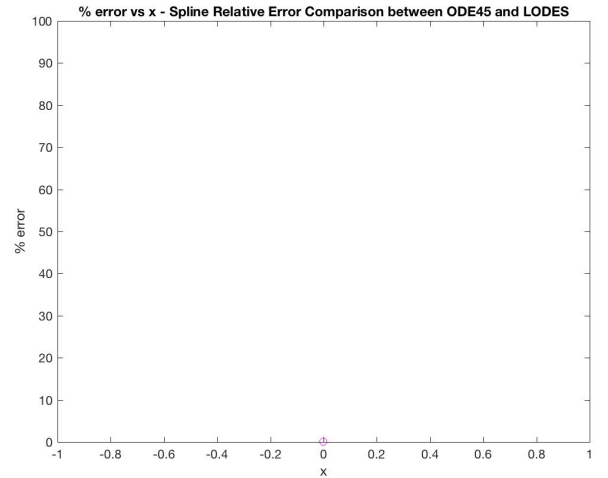
(b) Relative Error Norm - Runge-Kutta 4

Figure 26: Runge-Kutta 4 Method Analysis Graphs

7.3 Test 3: $f(x, y) = \sin(x) - y^2, h = 5, x_0 = 0, y_0 = 1, x_k = 5$

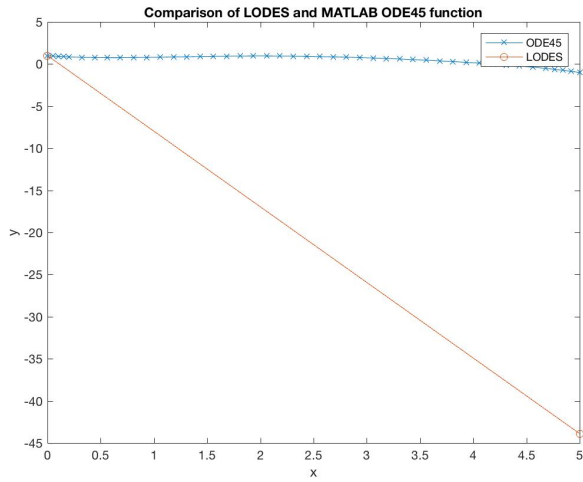


(a) Euler vs. ODE45

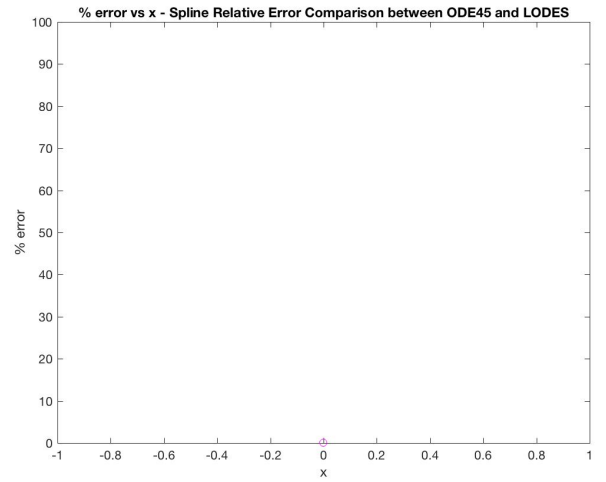


(b) Relative Error Norm - Euler

Figure 27: Euler's Method Analysis Graphs

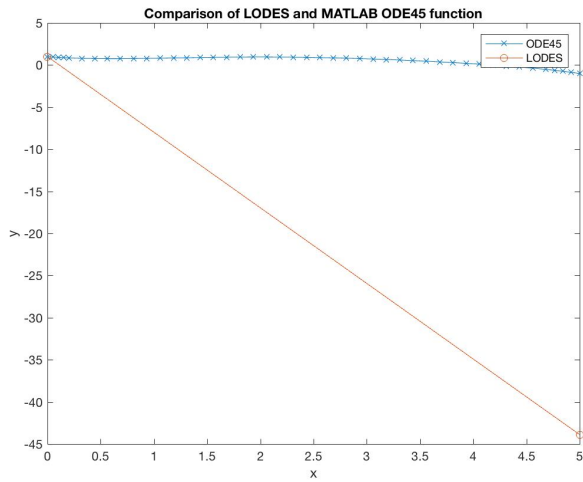


(a) Trapezoid vs. ODE45

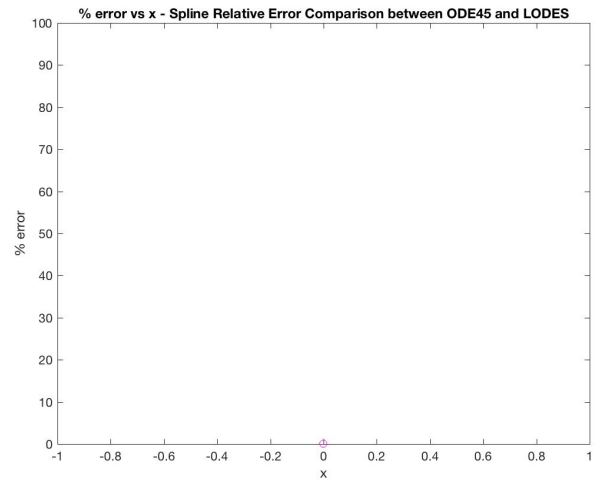


(b) Relative Error Norm - Trapezoid

Figure 28: Trapezoidal Method Analysis Graphs

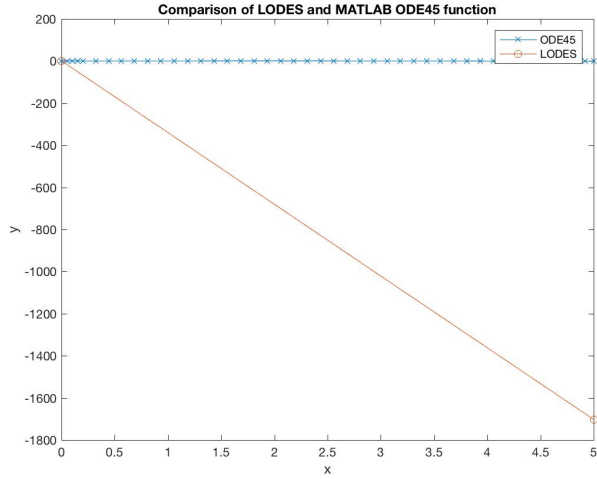


(a) Heun vs. ODE45

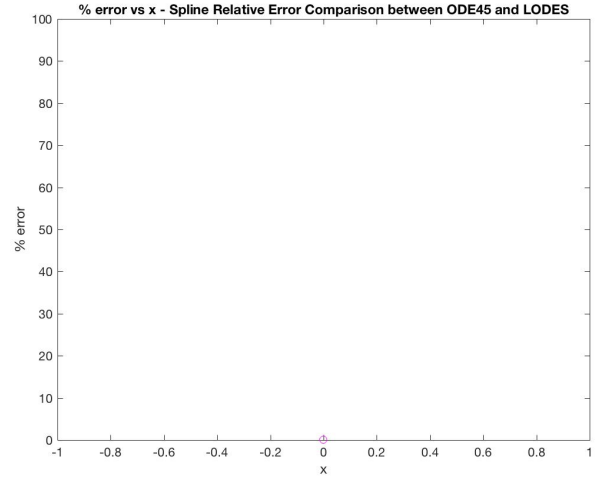


(b) Relative Error Norm - Heun

Figure 29: Heun's Method Analysis Graphs



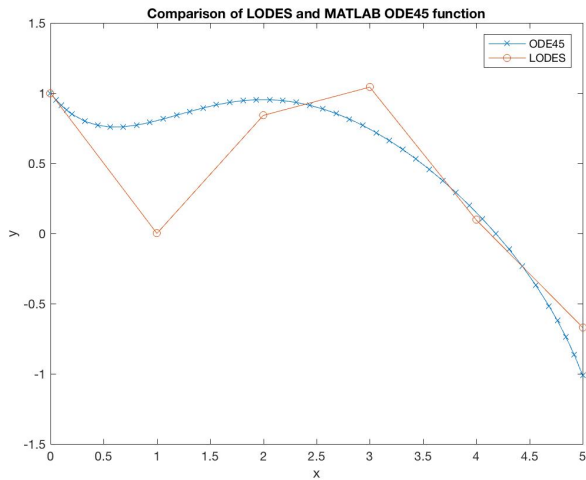
(a) Runge-Kutta 4 vs. ODE45



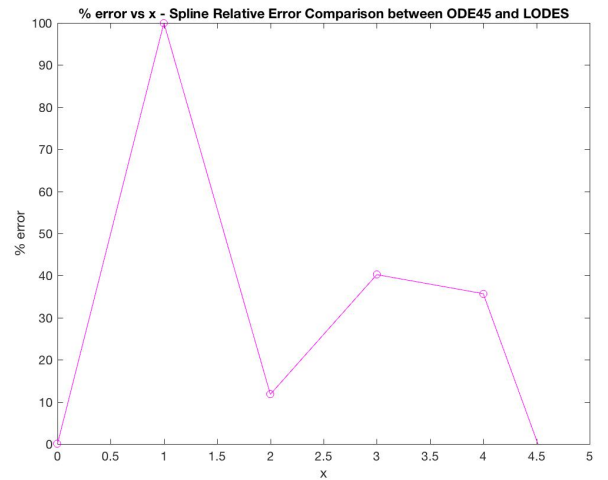
(b) Relative Error Norm - Runge-Kutta 4

Figure 30: Runge-Kutta 4 Method Analysis Graphs

7.4 Test 4: $f(x, y) = \sin(x) - y^2, h = 1, x_0 = 0, y_0 = 1, x_k = 5$

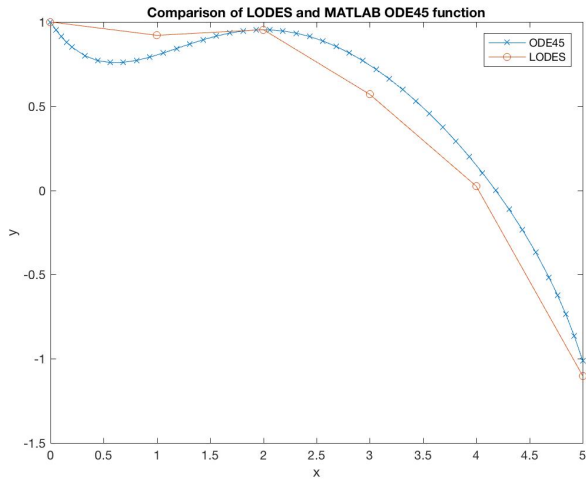


(a) Euler vs. ODE45

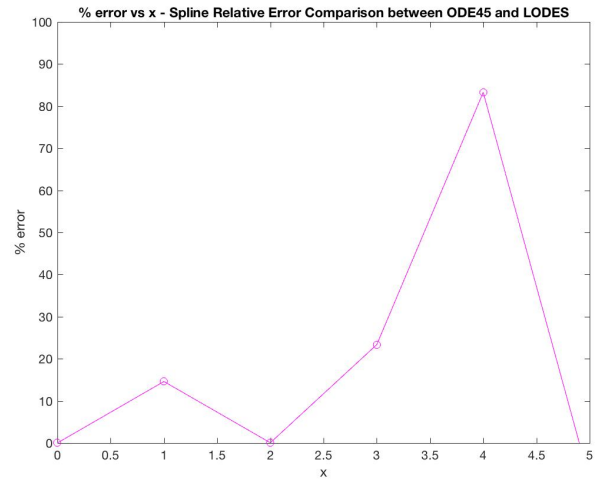


(b) Relative Error Norm - Euler

Figure 31: Euler's Method Analysis Graphs

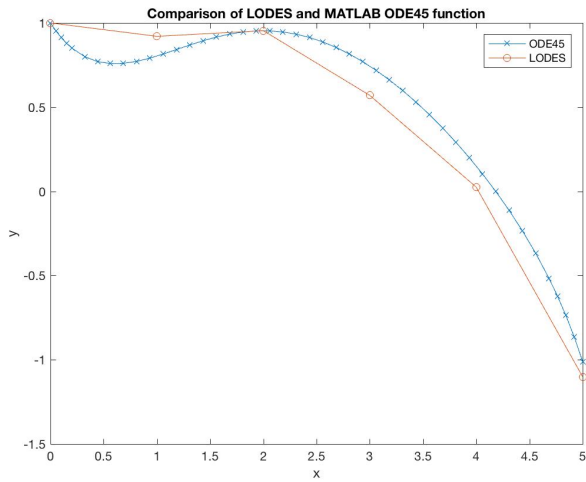


(a) Trapezoid vs. ODE45

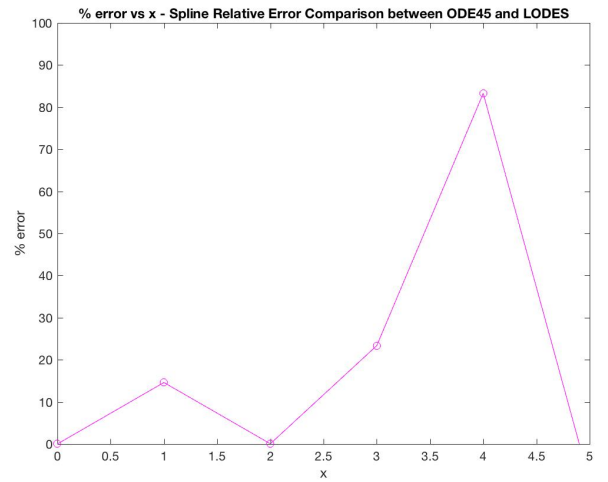


(b) Relative Error Norm - Trapezoid

Figure 32: Trapezoidal Method Analysis Graphs

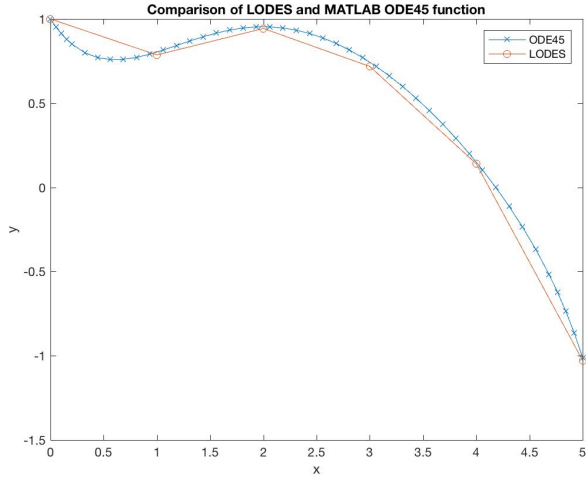


(a) Heun vs. ODE45

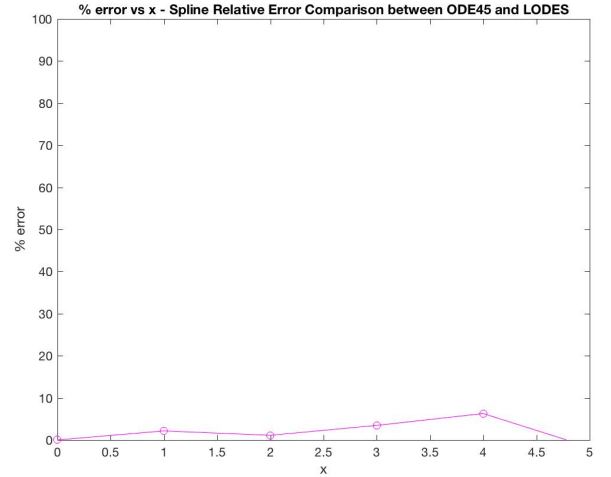


(b) Relative Error Norm - Heun

Figure 33: Heun's Method Analysis Graphs



(a) Runge-Kutta 4 vs. ODE45



(b) Relative Error Norm - Runge-Kutta 4

Figure 34: Runge-Kutta 4 Method Analysis Graphs

8 Unit Testing

Unit testing was performed and the results and execution are the similar on the level of Functional Requirement as described in Section 5 since the library is only of one level.

Further unit testing can be explored in the future.

Unit testing of the modules implemented by MATLAB and the Operating System (OS) has been deemed unnecessary due to proof of use. The parser functionality tests have been embedded in the functional tests of the modules of LODES in Section 5.

9 Changes Due to Testing

None.

10 Automated Testing

Automated testing has been performed in Section 7 and These were implemented in MATLAB's Unit Testing Framework.

The ϵ used is based on MATLAB's precision. Note that the tests were forced to produce a failure result for MATLAB to output the actual vs. expected results table. This provides the reader of this Test Report the ability to directly compare the outputs produced by MATLAB's ODE Solver, ODE45(), to that of LODES'.

11 Trace to Requirements

The following table shows the traceability mapping for the test cases laid out in this Test Report to the requirements described in the Commonality Analysis.

Table 1: Requirements Traceability Matrix

Test Number	CA Requirements
T1	IM1, O1, O2, O3, O4, O5
T2	IM1, O1, O2, O3, O4, O5
T3	IM1, O1, O2, O3, O4, O5
T4	IM1, O1, O2, O3, O4, O5
T5	IM2, O1, O2, O3, O4, O5
T6	IM2, O1, O2, O3, O4, O5
T7	IM2, O1, O2, O3, O4, O5
T8	IM2, O1, O2, O3, O4, O5
T9	IM3, O1, O2, O3, O4, O5
T10	IM3, O1, O2, O3, O4, O5
T11	IM3, O1, O2, O3, O4, O5
T12	IM3, O1, O2, O3, O4, O5
T13	IM4, O1, O2, O3, O4, O5
T14	IM4, O1, O2, O3, O4, O5
T15	IM4, O1, O2, O3, O4, O5
T16	IM4, O1, O2, O3, O4, O5
T17	NFR1
T18	NFR2

12 Trace to Modules

The following table shows the traceability mapping for the test cases laid out in this Test Report to the modules in the Module Guide (MG).

Table 2: Design Traceability Matrix

Test Number	MG Modules
T1	M1, M2, M3, M4, M5
T2	M1, M2, M3, M4, M5
T3	M1, M2, M3, M4, M5
T4	M1, M2, M3, M4, M5
T5	M1, M2, M3, M4, M6
T6	M1, M2, M3, M4, M6
T7	M1, M2, M3, M4, M6
T8	M1, M2, M3, M4, M6
T9	M1, M2, M3, M4, M7
T10	M1, M2, M3, M4, M7
T11	M1, M2, M3, M4, M7
T12	M1, M2, M3, M4, M7
T13	M1, M2, M3, M4, M8
T14	M1, M2, M3, M4, M8
T15	M1, M2, M3, M4, M8
T16	M1, M2, M3, M4, M8
T17	-
T18	-

13 Code Coverage Metrics

The following Code Coverage figure was obtained from MATLAB's Code Coverage Report Tool. Running the test program with in-boundary conditions (excluding extraneous scenarios), the code coverage tool produces the following results:

Profiler Coverage Report

Run the Coverage Report after you run the Profiler to identify how much of a file ran when it was profiled ([Learn More](#)).

[Rerun This Report](#)

[Run Report on Current Folder](#)

Report for folder /Users/paulaoanan/School/cas741/Project/cas741/src








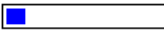
lodetest.m	 Coverage :86.5% Total time: 13.1 seconds Total lines: 52
rk.m	 Coverage :70.0% Total time: 1.6 seconds Total lines: 30
heun.m	 Coverage :65.4% Total time: 0.9 seconds Total lines: 26
trap.m	 Coverage :62.5% Total time: 1.2 seconds Total lines: 24
eul.m	 Coverage :59.1% Total time: 0.9 seconds Total lines: 22
lodes.m	 Coverage :51.0% Total time: 4.6 seconds Total lines: 51
EgParse.m	 Coverage :45.5% Total time: 0.0 seconds Total lines: 11
output.m	 Coverage :12.0% Total time: 0.0 seconds Total lines: 25

Figure 35: Code Coverage Tool result.

14 Code Inspection Checklist

The figure below shows the completed Code Checklist.

7.1 Code Inspection Checklist

1. Are there defects in the code? *Pass*
2. Does the code adhere to MATLAB coding standards and guidelines (syntax, function calls, etc.)? *Pass*
3. Does the code sensibly apply the ODE solvers as defined in the Commonality Analysis? *Pass*
4. Does the code contain spelling errors? *Pass*
5. Do the recursion/loops have the correct terminating cases according to the formulas defined in the Commonality Analysis? *Pass*
6. Does the code have the correct inputs and outputs? [What does this mean? Do you mean whether it agrees with the requirements? —SS] [It shall agree with the inputs and outputs listed in the functional requirements listed above through code walkthrough. —PA] *Pass*
7. Do the datatypes in the code adhere to ones defined in the requirements and specifications? *Pass*
8. Does the code satisfy the ranges and assumptions defined in the Commonality Analysis? *Pass*.

[This isn't actually a code walkthrough procedure. This is a code inspection checklist. A walkthrough is when a group of reviewers "play computer" and go through the code and its algorithms by hand. —SS] [Noted - that was the intent of the section :-). The section has been renamed. —PA]

Performed by: Shawn Ruppert

Figure 36: Code Review Checklist Output.

15 Appendix A - Test Output

The raw output of automated testing can be found here: [Automated Testing Output](#)

Running testClasses

yeuler =

1 3

ytestarr =

1.0000 7.3891

norm =

0.5886

errorarray =

0 0.5940

Verification failed in testClasses/testLODESSolution1.

Framework Diagnostic:

[verifyEqual](matlab:helpPopup matlab.unittest.TestCase.verifyEqual) failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	Expected
Error	RelativeTolerance	RelativeError
-----	-----	-----
-----	-----	-----
-----	-----	-----
2	3	7.3890570168536
		-4.3890570168536
		-0.593994200727192
		2.22044604925031e-16

Actual double:

1 3

Expected double:

1.0000000000000000 7.389057016853605

Stack Information:

In [>/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m \(testClasses.testLODESSolution1\) at 15](matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m',15,1))

ytrap =

1 5

ytestarr =

1.0000 7.3891

norm =

0.3204

errorarray =

0 0.3233

23

Verification failed in testClasses/testLODESSolution1.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	Expected
	Error 	RelativeError
	RelativeTolerance 	
-----	-----	-----

----- -----

2	5	7.3890570168536	-2.3890570168536	-0.323323667878653
2.22044604925031e-16				

Actual double:

1	5
Expected double:	
1.0000000000000000	7.389057016853605

Stack Information:

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution1) at 17

yheun =

1	5
---	---

ytestarr =

1.0000	7.3891
--------	--------

norm =

0.3204

errorarray =

0 0.3233

Verification failed in testClasses/testLODESSolution1.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	Expected
	Error 	RelativeError
	RelativeTolerance 	
-----	-----	-----

-----	-----	
2	5 7.3890570168536 -2.3890570168536	-0.323323667878653
	2.22044604925031e-16	

Actual double:

1 5

Expected double:

1.0000000000000000 7.389057016853605

Stack Information:

In [---

---](/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m)

yrk =

1 7

ytestarr =

1.0000 7.3891

norm =

0.0522

errorarray =

0 0.0527

Verification failed in testClasses/testLODESSolution1.

Framework Diagnostic:

[verifyEqual](matlab:helpPopup matlab.unittest.TestCase.verifyEqual) failed.

- > The values are not equal using "isequaln".
- > The error was not within relative tolerance.
- > Failure table:

Index	Actual	Expected
Error		
RelativeError	RelativeTolerance	
-----	-----	-----
2	7.3890570168536	-0.389057016853605
-0.0526531350301141	2.22044604925031e-16	

Actual double:
1 7
Expected double:
1.0000000000000000 7.389057016853605

Stack Information:

In </Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m> (testClasses.testLODESSolution1) at 21

.
yeuler =

1.0000 1.5000 2.2500 3.3750 5.0625

ytestarr =

1.0000 1.6487 2.7183 4.4817 7.3891

norm =

0.2832

errorarray =

0 0.0902 0.1723 0.2469 0.3149

Verification failed in testClasses/testLODESSolution2.

Framework Diagnostic:

[verifyEqual](matlab:helpPopup matlab.unittest.TestCase.verifyEqual) failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	Expected
Error	RelativeTolerance	
RelativeError	RelativeTolerance	

```

<strong>----- </strong>    <strong>----- </strong>    <strong>
----- </strong>    <strong>----- </strong>    <
strong>----- </strong>    <strong>----- </
strong>

```

```

2          1.5      1.6487215144387      -0.148721514438696
-0.0902041449306423      2.22044604925031e-16
3          2.25      2.71828199730153      -0.468281997301532
-0.172271308777529      2.22044604925031e-16
4          3.375      4.48169001126243      -1.10669001126243
-0.246935867603812      2.22044604925031e-16
5          5.0625      7.3890570168536      -2.3265570168536
-0.314865213727136      2.22044604925031e-16

```

Actual double:

```

1.0000000000000000      1.5000000000000000      2.2500000000000000      3.3750000000000000
5.0625000000000000

```

Expected double:

```

1.0000000000000000      1.648721514438696      2.718281997301532      4.481690011262426
7.389057016853605

```

Stack Information:

In <matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m',32,1)> at 32

ytrap =

```

1.0000      1.6250      2.6406      4.2910      6.9729

```

```

ytestarr =

    1.0000    1.6487    2.7183    4.4817    7.3891

```

```

norm =

    0.0502

```

```

errorarray =

    0    0.0144    0.0286    0.0425    0.0563

```

30

Verification failed in testClasses/testLODESSolution2.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	
Expected	Error	
RelativeError	RelativeTolerance	

```

<strong>----- </strong>    <strong>----- </strong>    <strong>
----- </strong>    <strong>----- </strong>    <
strong>----- </strong>    <strong>----- </
strong>

```

2	1.625	1.6487215144387	-0.0237215144386962
	-0.0143878236748625	2.22044604925031e-16	
3	2.640625	2.71828199730153	-0.0776569973015317
	-0.0285684109958506	2.22044604925031e-16	
4	4.291015625	4.48169001126243	-0.190674386262426
	-0.0425451974106339	2.22044604925031e-16	
5	6.972900390625	7.3890570168536	-0.416156626228605
	-0.0563206678848734	2.22044604925031e-16	

Actual double:

1.0000000000000000	1.6250000000000000	2.6406250000000000	4.2910156250000000
	6.972900390625000		

Expected double:

1.0000000000000000	1.648721514438696	2.718281997301532	4.481690011262426
	7.389057016853605		

Stack Information:

In >/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution2) at 34

yheun =

1.0000	1.6250	2.6406	4.2910	6.9729
--------	--------	--------	--------	--------

```

ytestarr =

    1.0000    1.6487    2.7183    4.4817    7.3891

```

```

norm =

    0.0502

```

```

errorarray =

    0    0.0144    0.0286    0.0425    0.0563

```

32

Verification failed in testClasses/testLODESSolution2.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	
Expected	Error	
RelativeError	RelativeTolerance	

```

<strong>----- </strong>    <strong>----- </strong>    <strong>
----- </strong>    <strong>----- </strong>    <
strong>----- </strong>    <strong>----- </
strong>

```

```

2          1.625          1.6487215144387          -0.0237215144386962
-0.0143878236748625      2.22044604925031e-16
3          2.640625          2.71828199730153          -0.0776569973015317
-0.0285684109958506      2.22044604925031e-16
4          4.291015625          4.48169001126243          -0.190674386262426
-0.0425451974106339      2.22044604925031e-16
5          6.972900390625          7.3890570168536          -0.416156626228605
-0.0563206678848734      2.22044604925031e-16

```

Actual double:

```

1.0000000000000000      1.6250000000000000      2.6406250000000000      4.2910156250000000
6.972900390625000

```

Expected double:

```

1.0000000000000000      1.648721514438696      2.718281997301532      4.481690011262426
7.389057016853605

```

Stack Information:

In

yrk =

```

1.0000      1.6484      2.7173      4.4794      7.3840

```

ytestarr =

1.0000 1.6487 2.7183 4.4817 7.3891

norm =

6.1253e-04

errorarray =

1.0e-03 *

0 0.1723 0.3443 0.5165 0.6884

Verification failed in testClasses/testLODESSolution2.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	
Expected	Error	

	RelativeError	RelativeTolerance
2	1.6484375	-0.00028401443869619
3	2.71734619140625	-0.000935805895281749
4	4.47937536239624	-0.00231464886618582
5	7.38397032395005	-0.00508669290355268

Actual double:

1.0000000000000000	1.6484375000000000	2.717346191406250	4.479375362396240
7.383970323950052			

Expected double:

1.0000000000000000	1.648721514438696	2.718281997301532	4.481690011262426
7.389057016853605			

Stack Information:

In [/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m \(testClasses.testLODESSolution2\) at 38](/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m)

.
yeuler =

1 -4

ytestarr =

1.0000 -1.0108

norm =

2.1023

errorarray =

0 -2.9573

Verification failed in testClasses/testLODESSolution3.

Framework Diagnostic:

[verifyEqual](matlab:helpPopup matlab.unittest.TestCase.verifyEqual) failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	Expected
	Error	

RelativeError	RelativeTolerance
2	-4
2.95730048255542	-1.01079006197098
	-2.98920993802902
	2.22044604925031e-16

Actual double:

1 -4

Expected double:

1.0000000000000000 -1.010790061970984

Stack Information:

In [/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m \(testClasses.testLODESSolution3\) at 49](/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m)

ytrap =

1.0000 -43.8973

ytestarr =

1.0000 -1.0108

norm =

30.1622

errorarray =

0 -42.4287

Verification failed in testClasses/testLODESSolution3.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	
Expected	Error	
RelativeError	RelativeTolerance	
-----	-----	
-----	-----	
strong>-----	strong>-----	strong>
2	-43.8973106866578	-1.01079006197098
42.4287121907991	2.22044604925031e-16	-42.8865206246869

Actual double:

1.0000000000000000 -43.897310686657846

Expected double:
1.0000000000000000 -1.010790061970984

Stack Information:

In [/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m \(testClasses.testLODESSolution3\) at 51](/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m)

yheun =

1.0000 -43.8973

39

ytestarr =

1.0000 -1.0108

norm =

30.1622

errorarray =

0 -42.4287

Verification failed in testClasses/testLODESSolution3.

Framework Diagnostic:

verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	
Expected	Error	
RelativeError	RelativeTolerance	
-----	-----	
-----	-----	
strong>-----	strong>-----	strong>
2	-43.8973106866578	-1.01079006197098
42.4287121907991	2.22044604925031e-16	-42.8865206246869

Actual double:

1.0000000000000000 -43.897310686657846

Expected double:

1.0000000000000000 -1.010790061970984

Stack Information:

In /Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution3) at 53

$$\begin{aligned} \text{yrk} = & \\ & 1.0\text{e}+03 * \\ & 0.0010 \quad -1.7028 \end{aligned}$$

$$\begin{aligned} \text{ytestarr} = & \\ & 1.0000 \quad -1.0108 \end{aligned}$$

$$\begin{aligned} \text{norm} = & \\ & 1.1969\text{e}+03 \end{aligned}$$

$$\begin{aligned} \text{errorarray} = & \\ & 1.0\text{e}+03 * \\ & 0 \quad -1.6837 \end{aligned}$$

Verification failed in testClasses/testLODESSolution3.

Framework Diagnostic:

[verifyEqual](matlab:helpPopup matlab.unittest.TestCase.verifyEqual) failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

Index	Actual	Error	RelativeTolerance
Expected			
RelativeError			
-----	-----	-----	-----
-----	-----	-----	-----
-----	-----	-----	-----
2	-1702.84512882598	-1.01079006197098	-1701.83433876401
1683.66746250505	2.22044604925031e-16		

Actual double:

1.0e+03 *

0.00100000000000000 -1.702845128825983

Expected double:

1.0000000000000000 -1.010790061970984

Stack Information:

In

.
yeuler =


```
1.0000      0      0.8415      1.0427      0.0966     -0.6695
```

```
ytestarr =
```

```
1.0000      0.8035      0.9538      0.7436      0.1502     -1.0108
```

```
norm =
```

```
0.4569
```

```
errorarray =
```

```
0      1.0000      0.1178      0.4022      0.3570     -0.3376
```

```
<strong>Verification failed in testClasses/testLODESSolution4.</strong>
```

```
Framework Diagnostic:
```

```
<a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
bold">verifyEqual</a> failed.
```

```
—> The values are not equal using "isequaln".
```

```
—> The error was not within relative tolerance.
```

```
—> Failure table:
```

Index	Actual	
Expected		Error
RelativeError		RelativeTolerance

```

    strong>
<strong>-----</strong>    <strong>-----</strong>    <strong>
-----</strong>    <strong>-----</strong>    <
strong>-----</strong>    <strong>-----</
strong>

```

```

2          0      0.803498260367072      -0.803498260367072
          -1      2.22044604925031e-16
3      0.841470984807897      0.95380754323836      -0.112336558430464
-0.117776965832184      2.22044604925031e-16
4      1.04269499336001      0.743638851851789      0.299056141508218
0.402152389918193      2.22044604925031e-16
5      0.0966021522418493      0.150235602951416      -0.0536334507095666
-0.356995609934823      2.22044604925031e-16
6      -0.669532318883836      -1.01079006197098      0.341257743087148
-0.337614857848636      2.22044604925031e-16

```

Actual double:

```

1.0000000000000000          0      0.841470984807897      1.042694993360007
0.096602152241849      -0.669532318883836

```

Expected double:

```

1.0000000000000000      0.803498260367072      0.953807543238360      0.743638851851789
0.150235602951416      -1.010790061970984

```

Stack Information:

In

The identifier was:

MATLAB: class:InvalidHandleThere was an error! The message was:
Invalid or deleted object.
ytrap =

1.0000 0.9207 0.9541 0.5700 0.0251 -1.1012

ytestarr =

1.0000 0.8035 0.9538 0.7436 0.1502 -1.0108

norm =

0.1277

errorarray =

0 0.1459 0.0003 0.2335 0.8328 -0.0894

Verification failed in testClasses/testLODESSolution4.

Framework Diagnostic:

<a href="matlab:helpPopup matlab.unittest.TestCase.verifyEqual" style="font-weight:
bold">verifyEqual failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

IndexExpected		ActualError		RelativeTolerance	
RelativeError		RelativeTolerance		RelativeTolerance	
-----		-----		-----	
-----		-----		-----	
-----		-----		-----	
-----		-----		-----	
2	0.920735492403948	0.803498260367072	0.117237232036877		
	0.14590850760936	2.22044604925031e-16			
3	0.954130968258594	0.95380754323836	0.000323425020233414		
	0.000339088343897265	2.22044604925031e-16			
4	0.56999267801283	0.743638851851789	-0.173646173838959		
	-0.233508743399502	2.22044604925031e-16			
5	0.0251222646962669	0.150235602951416	-0.125113338255149		
	-0.832780884139753	2.22044604925031e-16			
6	-1.10119664749119	-1.01079006197098	-0.0904065855202025		
	0.0894415061263214	2.22044604925031e-16			

Actual double:

1.0000000000000000 0.920735492403948 0.954130968258594 0.569992678012830
0.025122264696267 -1.101196647491187

Expected double:

1.0000000000000000 0.803498260367072 0.953807543238360 0.743638851851789
0.150235602951416 -1.010790061970984

Stack Information:

In

The identifier was:

MATLAB: class:InvalidHandleThere was an error! The message was:

Invalid or deleted object.

yheun =

1.0000	0.9207	0.9541	0.5700	0.0251	-1.1012
--------	--------	--------	--------	--------	---------

ytestarr =

1.0000	0.8035	0.9538	0.7436	0.1502	-1.0108
--------	--------	--------	--------	--------	---------

norm =

0.1277

errorarray =

0	0.1459	0.0003	0.2335	0.8328	-0.0894
---	--------	--------	--------	--------	---------

Verification failed in testClasses/testLODESSolution4.

Framework Diagnostic:

Stack Information:

In >/Users/paulaoanan/School/cas741/Project/cas741/src/testing/testClasses.m (testClasses.testLODESSolution4) at 70

yrk =

1.0000	0.7863	0.9434	0.7180	0.1408	-1.0284
--------	--------	--------	--------	--------	---------

ytestarr =

1.0000	0.8035	0.9538	0.7436	0.1502	-1.0108
--------	--------	--------	--------	--------	---------

norm =

0.0187

errorarray =

0	0.0214	0.0109	0.0345	0.0626	-0.0174
---	--------	--------	--------	--------	---------

Verification failed in testClasses/testLODESSolution4.

Framework Diagnostic:

[verifyEqual](matlab:helpPopup matlab.unittest.TestCase.verifyEqual) failed.

—> The values are not equal using "isequaln".

—> The error was not within relative tolerance.

—> Failure table:

	Index	Actual	
Expected	Error		
RelativeError	RelativeTolerance		
-----	-----		
-----	-----		
strong>-----	strong>-----	strong>-----	strong>-----
2	0.78631839551037	0.803498260367072	-0.0171798648567016
-0.0213813342282199	2.22044604925031e-16		
3	0.943371788846183	0.95380754323836	-0.0104357543921774
-0.0109411531353023	2.22044604925031e-16		
4	0.717989268823369	0.743638851851789	-0.0256495830284202
-0.0344919889063735	2.22044604925031e-16		
5	0.14083244087226	0.150235602951416	-0.00940316207915609
-0.0625894388176213	2.22044604925031e-16		
6	-1.02836702616913	-1.01079006197098	-0.0175769641981469
0.0173893322257965	2.22044604925031e-16		

Actual double:

```
1.0000000000000000  0.786318395510370  0.943371788846183  0.717989268823369
0.140832440872260  -1.028367026169131
```

Expected double:


```

1.0000000000000000    0.803498260367072    0.953807543238360    0.743638851851789
0.150235602951416    -1.010790061970984

```

Stack Information:

```

In <a href="matlab:opentoline('/Users/paulaoanan/School/cas741/Project/cas741/src/
testing/testClasses.m',72,1)">/Users/paulaoanan/School/cas741/Project/cas741/src/
testing/testClasses.m (testClasses.testLODESSolution4) at 72</a>

```

```

.
Done testClasses
-----

```

Failure Summary:

Name	Failed	Incomplete	Reason(s)
testClasses/testLODESSolution1	X		Failed by verification.
testClasses/testLODESSolution2	X		Failed by verification.
testClasses/testLODESSolution3	X		Failed by verification.
testClasses/testLODESSolution4	X		Failed by verification.

result =

```

1 4 <a href="matlab:helpPopup matlab.unittest.TestResult" style="font-weight:bold">
    TestResult</a> array with properties:

```

Name
Passed
Failed
Incomplete
Duration
Details

Totals:

0 Passed, 4 Failed, 0 Incomplete.
12.6715 seconds testing time.