DSC 40A - Homework 6

Due: Wednesday, Feb 28 at 11:59pm

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. Homeworks are due to Gradescope by 11:59pm on the due date. You can use a slip day to extend the deadline by 24 hours.

Homework will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should **always explain and justify** your conclusions, using sound reasoning. Your goal should be to convince the reader of your assertions. If a question does not require explanation, it will be explicitly stated.

Homeworks should be written up and turned in by each student individually. You may talk to other students in the class about the problems and discuss solution strategies, but you should not share any written communication and you should not check answers with classmates. You can tell someone how to do a homework problem, but you cannot show them how to do it.

For each problem you submit, you should **cite your sources** by including a list of names of other students with whom you discussed the problem. Instructors do not need to be cited.

This homework will be graded out of 50 points. The point value of each problem or sub-problem is indicated by the number of avocados shown.

Problem 1. Reflection and Feedback Form

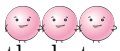
 Make sure to fill out this Reflection and Feedback Form, linked here for two points on this homework! This form is primarily for your benefit; research shows that reflecting and summarizing knowledge helps you understand and remember it.

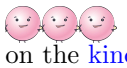
Problem 2. Baseball

a)  Suppose that your baseball team has 12 players but you only have 8 matching team hats. How many ways are there for you to select who gets a hat and who doesn't?

b)  Suppose that your baseball team has 12 players and you have 16 jerseys, numbered 1 through 16. You will give out one jersey to each player, and four will be leftover. How many ways are there for you to assign jerseys to players?

c)  As before, your baseball team has 12 players and you have 16 jerseys, numbered 1 through 16. You will give out one jersey to each player, and four will be leftover. How many ways are there for you to assign jerseys to players if you must give out jersey number 7?

d)  Suppose that at the batting cage, where a machine pitches the ball, each time you swing the bat, your probability of hitting the ball is 3/5. If you swing 9 times, what is the probability that you hit the ball exactly 7 times?

e)  Suppose that when a pitcher is throwing the ball, your probability of hitting the ball depends on the kind of pitch. Your probability of hitting the ball is

- $\frac{1}{2}$ for a fastball,
- $\frac{1}{3}$ for a breaking ball, and
- $\frac{1}{4}$ for a changeup

Suppose that at practice, a pitcher throws you three fastballs, three breaking balls, and three changeups. What is the probability that you miss one breaking ball and one changeup, but hit all 7 other balls?

Problem 3. NBA Draft

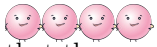
In an NBA draft, there are two categories of players to be selected from: college players and international players. The probability that a college player is selected is 0.7, while the probability that an international player is selected is 0.3.

Answer the following sub questions based on information above. Calculations by hand or by code are both acceptable as long as sufficient work is shown.

a)  Suppose that 100 players declared for the draft this year. There are 60 players selected from each year's draft. How many draft classes of 60 students can be selected from the 100 players who declared for the draft?

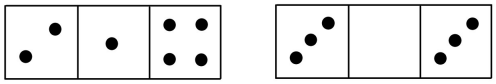
b)  How many draft classes of 60 students can be selected from the 100 players who declared for the draft if order does matter?

c)  If the probability that a draft pick from the college players will be a starter in their rookie year is 0.5, and the probability for an international player to achieve the same is 0.2, what is the overall probability that the draft pick will not be a starter in their rookie year?

d)  After the draft, it's found that the likelihood of a player being a first-round pick given that they are a college player is 0.8, and the likelihood of being a first-round pick given that they are an international player is 0.4. During the draft, a player is announced as a first-round pick. What is the probability that this player is international?

Problem 4. From Dominoes to Trominoes

A *tromino* is a rectangular tile divided into three sections. On each section, there is some number of dots between 0 and 6, inclusive. Two example trominoes are shown below.



A complete set of trominoes consists of every possible combination of dots on each section.

a)  Draw a picture of all the trominoes that have at least one 1, at least one 3, and only 1s and 3s (no other numbers). No explanation needed.

b)  Draw a picture of all the trominoes that have a 1, a 3, and a 6. No explanation needed.

c)  How many trominoes are in a complete set?

Hint: The answers to parts (a) and (b) should help you.

d)  In class, we calculated that the number of dominoes in a complete set is 28. How does your answer to part (c) relate to the number 28? Explain why this makes sense.

Problem 5. Book Club

A book club includes p people, and there are b books that the book club is considering reading this month. Before deciding on which book to read this month, the book club president asks each person which of the b books they have already read.

A *book club description* is a description of who, among p people, has already read each of b books. For example, if a book club has $p = 3$ people (Ben, Tunan, Pallavi) and there are $b = 2$ books (Book 1, Book 2), one possible book club description is as follows:

- Ben has read Book 1.
- Tunan has read Book 1 and Book 2.
- Pallavi has read neither Book 1 nor Book 2.

a)  How many book club descriptions are possible? Give your answer as a formula involving p and b, with explanation of where the formula comes from.

b)  How many book club descriptions are such that nobody has read Book 1? Give your answer as a formula involving p and b, with explanation of where the formula comes from.

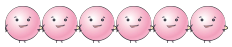
c)  How many book club descriptions are such that there is at least one book that nobody has read? Give your answer as a formula involving p and b, with explanation of where the formula comes from.

Problem 6. Hockey-stick identity

You've learned in lectures that:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

which means the number of combinations of picking k elements out of n unique elements.

a)  Prove Pascal's rule:

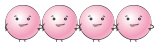
$$\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$$

You may assume $n > k$. Then based on the meaning of $\binom{n}{k}$, give intuitive explanation for this rule.

Hint: You may find the following properties useful:

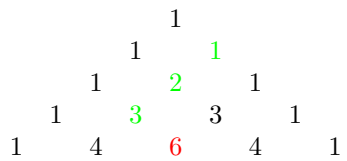
- $(n+1) - (k+1) = n - k$
- $(n+1-(k+1))! = (n+1-(k+1))(n-(k+1))!$

In general, there is no restriction on the relative sizes of n and k. If $n < k$, the value of the binomial coefficient is zero and the identity remains valid.

b)  Use Pascal's rule, show Hockey-stick identity:

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n-1}{k} + \binom{n-2}{k} + \ldots + \binom{k}{k}$$

The Hockey-stick name comes from the visualized pattern in Pascal's triangle, in which each element's value equals the sum of its left-upper and right-upper neighbors:



$$\binom{4}{2} = \binom{3}{1} + \binom{2}{1} + \binom{1}{1}$$