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```

 $F^+ = F$ 
apply the reflexivity rule /* Generates all trivial dependencies */
repeat
  for each functional dependency  $f$  in  $F^+$ 
    apply the augmentation rule on  $f$ 
    add the resulting functional dependencies to  $F^+$ 
  for each pair of functional dependencies  $f_1$  and  $f_2$  in  $F^+$ 
    if  $f_1$  and  $f_2$  can be combined using transitivity
      add the resulting functional dependency to  $F^+$ 
until  $F^+$  does not change any further

```

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**Figure 7.7** A procedure to compute  $F^+$ .

- $AG \rightarrow I$ . Since  $A \rightarrow C$  and  $CG \rightarrow I$ , the pseudotransitivity rule implies that  $AG \rightarrow I$  holds.

Another way of finding that  $AG \rightarrow I$  holds is as follows: We use the augmentation rule on  $A \rightarrow C$  to infer  $AG \rightarrow CG$ . Applying the transitivity rule to this dependency and  $CG \rightarrow I$ , we infer  $AG \rightarrow I$ .

Figure 7.7 shows a procedure that demonstrates formally how to use Armstrong's axioms to compute  $F^+$ . In this procedure, when a functional dependency is added to  $F^+$ , it may be already present, and in that case there is no change to  $F^+$ . We shall see an alternative way of computing  $F^+$  in Section 7.4.2.

The left-hand and right-hand sides of a functional dependency are both subsets of  $R$ . Since a set of size  $n$  has  $2^n$  subsets, there are a total of  $2^n \times 2^n = 2^{2n}$  possible functional dependencies, where  $n$  is the number of attributes in  $R$ . Each iteration of the repeat loop of the procedure, except the last iteration, adds at least one functional dependency to  $F^+$ . Thus, the procedure is guaranteed to terminate, though it may be very lengthy.

#### 7.4.2 Closure of Attribute Sets

We say that an attribute  $B$  is **functionally determined** by  $\alpha$  if  $\alpha \rightarrow B$ . To test whether a set  $\alpha$  is a superkey, we must devise an algorithm for computing the set of attributes functionally determined by  $\alpha$ . One way of doing this is to compute  $F^+$ , take all functional dependencies with  $\alpha$  as the left-hand side, and take the union of the right-hand sides of all such dependencies. However, doing so can be expensive, since  $F^+$  can be large.

An efficient algorithm for computing the set of attributes functionally determined by  $\alpha$  is useful not only for testing whether  $\alpha$  is a superkey, but also for several other tasks, as we shall see later in this section.