



# Recursive squaring

**Theorem:** 
$$\begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n .$$

**Algorithm:** Recursive squaring.

Time =  $\Theta(\lg n)$  .

*Proof of theorem.* (Induction on  $n$ .)

Base ( $n = 1$ ): 
$$\begin{bmatrix} F_2 & F_1 \\ F_1 & F_0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^1 .$$