

Master theorem (reprise)

$$T(n) = a T(n/b) + f(n)$$

Case 1:
$$f(n) = O(n^{\log_b a - \varepsilon})$$
, constant $\varepsilon > 0$
 $\Rightarrow T(n) = \Theta(n^{\log_b a})$.

Case 2:
$$f(n) = \Theta(n^{\log_b a} \lg^k n)$$
, constant $k \ge 0$
 $\Rightarrow T(n) = \Theta(n^{\log_b a} \lg^{k+1} n)$.

Case 3: $f(n) = \Omega(n^{\log_b a + \varepsilon})$, constant $\varepsilon > 0$, and regularity condition

$$\Rightarrow T(n) = \Theta(f(n))$$
.

Merge sort:
$$a = 2$$
, $b = 2 \implies n^{\log_b a} = n^{\log_2 2} = n$
 \Rightarrow Case 2 $(k = 0) \Rightarrow T(n) = \Theta(n \lg n)$.