	α	β	$R - \alpha - \beta$
t_1	$a_1 \ldots a_i$	$a_{i+1} \ldots a_j$	$a_{j+1} \ldots a_n$
t_2	$a_1 \ldots a_i$	$b_{i+1} \ldots b_j$	$b_{j+1} \ldots b_n$
t_3	$a_1 \ldots a_i$	$a_{i+1} \ldots a_j$	$b_{j+1} \ldots b_n$
t_4	$a_1 \ldots a_i$	$b_{i+1} \ldots b_j$	$a_{j+1} \ldots a_n$

Figure 7.13 Tabular representation of $\alpha \twoheadrightarrow \beta$.

7.6.1 Multivalued Dependencies

Functional dependencies rule out certain tuples from being in a relation. If $A \rightarrow B$, then we cannot have two tuples with the same A value but different B values. Multivalued dependencies, on the other hand, do not rule out the existence of certain tuples. Instead, they *require* that other tuples of a certain form be present in the relation. For this reason, functional dependencies sometimes are referred to as **equality-generating dependencies**, and multivalued dependencies are referred to as **tuple-generating dependencies**.

Let $r(R)$ be a relation schema and let $\alpha \subseteq R$ and $\beta \subseteq R$. The **multivalued dependency**

$$\alpha \twoheadrightarrow \beta$$

holds on R if, in any legal instance of relation $r(R)$, for all pairs of tuples t_1 and t_2 in r such that $t_1[\alpha] = t_2[\alpha]$, there exist tuples t_3 and t_4 in r such that

$$\begin{aligned}
&t_1[\alpha] = t_2[\alpha] = t_3[\alpha] = t_4[\alpha]\\
&t_3[\beta] = t_1[\beta]\\
&t_3[R - \beta] = t_2[R - \beta]\\
&t_4[\beta] = t_2[\beta]\\
&t_4[R - \beta] = t_1[R - \beta]
\end{aligned}$$

This definition is less complicated than it appears to be. Figure 7.13 gives a tabular picture of t_1, t_2, t_3, and t_4. Intuitively, the multivalued dependency $\alpha \twoheadrightarrow \beta$ says that the relationship between α and β is independent of the relationship between α and $R - \beta$. If the multivalued dependency $\alpha \twoheadrightarrow \beta$ is satisfied by all relations on schema R, then $\alpha \twoheadrightarrow \beta$ is a *trivial* multivalued dependency on schema R. Thus, $\alpha \twoheadrightarrow \beta$ is trivial if $\beta \subseteq \alpha$ or $\beta \cup \alpha = R$. This can be seen by looking at Figure 7.13 and considering the two special cases $\beta \subseteq \alpha$ and $\beta \cup \alpha = R$. In each case, the table reduces to just two columns and we see that t_1 and t_2 are able to serve in the roles of t_3 and t_4.

To illustrate the difference between functional and multivalued dependencies, we consider the schema r_2 again, and an example relation on that schema is shown in Figure 7.14. We must repeat the department name once for each address that an instructor has, and we must repeat the address for each department with which an instructor is associated. This repetition is unnecessary, since the relationship between an instructor