

		j	0	1	2	3	4	5	6
i	x_i	y_j		B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0	↑	↑	↑	↖	←	↖
2	B		0	↖	←	←	↑	↖	←
3	C		0	↑	↑	↖	←	↑	↑
4	B		0	↑	↑	↑	↑	↖	←
5	D		0	↑	↖	↑	↑	↑	↑
6	A		0	↑	↑	↑	↖	↑	↖
7	B		0	↑	↑	↑	↑	↑	↑

Figure 14.8 The c and b tables computed by LCS-LENGTH on the sequences $X = \langle A, B, C, B, D, A, B \rangle$ and $Y = \langle B, D, C, A, B, A \rangle$. The square in row i and column j contains the value of $c[i, j]$ and the appropriate arrow for the value of $b[i, j]$. The entry 4 in $c[7, 6]$ —the lower right-hand corner of the table—is the length of an LCS $\langle B, C, B, A \rangle$ of X and Y . For $i, j > 0$, entry $c[i, j]$ depends only on whether $x_i = y_j$ and the values in entries $c[i - 1, j]$, $c[i, j - 1]$, and $c[i - 1, j - 1]$, which are computed before $c[i, j]$. To reconstruct the elements of an LCS, follow the $b[i, j]$ arrows from the lower right-hand corner, as shown by the sequence shaded blue. Each “↖” on the shaded-blue sequence corresponds to an entry (highlighted) for which $x_i = y_j$ is a member of an LCS.

The initial call is PRINT-LCS(b, X, m, n). For the b table in Figure 14.8, this procedure prints $BCBA$. The procedure takes $O(m + n)$ time, since it decrements at least one of i and j in each recursive call.

Improving the code

Once you have developed an algorithm, you will often find that you can improve on the time or space it uses. Some changes can simplify the code and improve constant factors but otherwise yield no asymptotic improvement in performance. Others can yield substantial asymptotic savings in time and space.

In the LCS algorithm, for example, you can eliminate the b table altogether. Each $c[i, j]$ entry depends on only three other c table entries: $c[i - 1, j - 1]$, $c[i - 1, j]$, and $c[i, j - 1]$. Given the value of $c[i, j]$, you can determine in $O(1)$ time which of these three values was used to compute $c[i, j]$, without inspecting table b . Thus, you can reconstruct an LCS in $O(m + n)$ time using a procedure similar to PRINT-LCS. (Exercise 14.4-2 asks you to give the pseudocode.) Although this method saves $\Theta(mn)$ space, the auxiliary space requirement for computing