

A tighter upper bound!

IDEA: Strengthen the inductive hypothesis.

• Subtract a low-order term.

Inductive hypothesis: $T(k) \le c_1 k^2 - c_2 k$ for $k \le n$.

$$T(n) = 4T(n/2) + n$$

$$= 4(c_1(n/2)^2 - c_2(n/2)) + n$$

$$= c_1n^2 - 2c_2n + n$$

$$= c_1n^2 - c_2n - (c_2n - n)$$

$$\leq c_1n^2 - c_2n \quad \text{if } c_2 \geq 1.$$

Pick c_1 big enough to handle the initial conditions.