

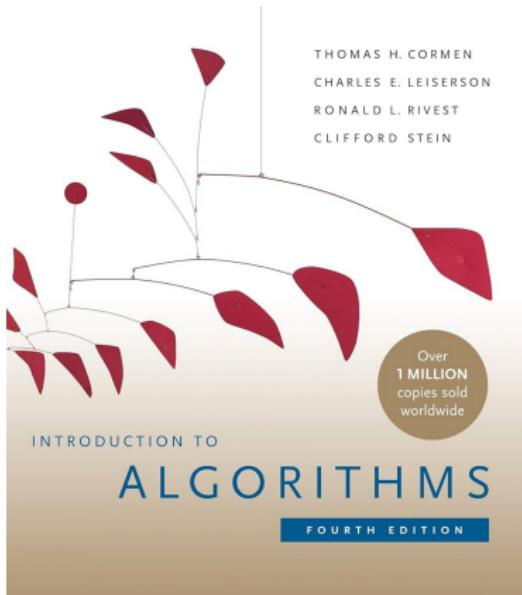
Introduction to Algorithms

Lecture 2: Asymptotic Notation

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Massachusetts Institute of Technology

February 11, 2025

Introduction to Algorithms



Content has been extracted from *Introduction to Algorithms*, Fourth Edition, by Cormen, Leiserson, Rivest, and Stein. MIT Press. 2022.

Visit <https://mitpress.mit.edu/9780262046305/introduction-to-algorithms/>.

Original slides from *Introduction to Algorithms* 6.046J/18.401J, Fall 2005 Class by Prof. Charles Leiserson and Prof. Erik Demaine. MIT OpenCourseWare Initiative available at <https://ocw.mit.edu/courses/6-046j-introduction-to-algorithms-sma-5503-fall-2005/>.

Plan

Asymptotic Notation

Solving recurrences

Substitution method

Recursion tree

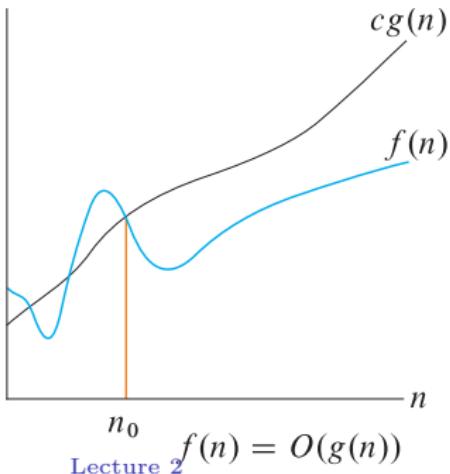
The master method

Asymptotic Notation

We write $f(n) = O(g(n))$ if there exist such constants $c > 0$, $n_0 > 0$ such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0$.

O -notation (upper bounds)

$O(g(n)) = \{f(n) : \text{there exist constants } c > 0, n_0 > 0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0\}$



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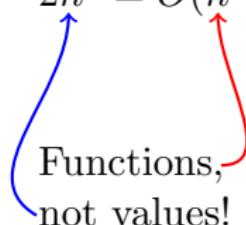
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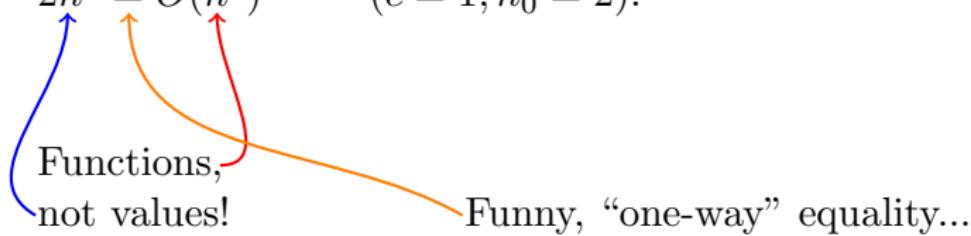


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Example:

$$2n^2 \in O(n^3)$$

(*Logicians*: $\lambda n.2n^2 \in O(\lambda n.n^3)$, but it's convenient to be sloppy,
as long as we understand what's really going on.)

Macro substitution

Convention:

A set in a formula represents an anonymous function in the set.

Example:

$$f(n) = n^3 + O(n^2)$$

means

$$f(n) = n^3 + h(n)$$

for some $h(n) \in O(n^2)$.

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for any $f(n) \in O(n)$:

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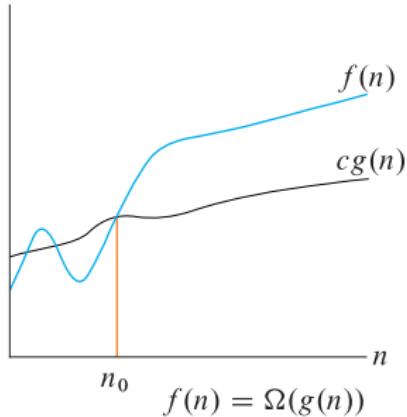
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O -notation is an upper-bound notation. It makes no sense to say $f(n)$ is at least $O(n^2)$

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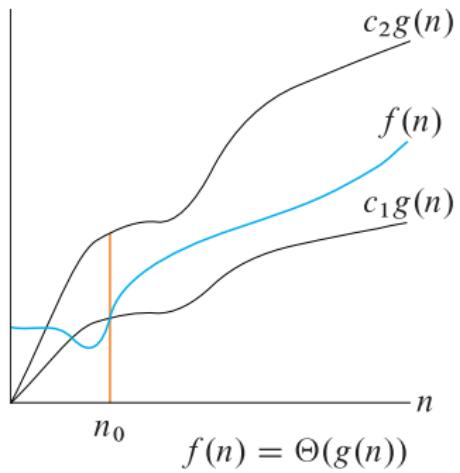
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Example:

$$\sqrt{n} = \Omega(\lg n) \quad (c = 1, n_0 = 16)$$

Θ -notation (tight bounds)

$$\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$$



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Example:

$$\frac{1}{2}n^2 - 2n = \Theta(n^2)$$

o -notation and ω -notation

O -notation and Ω -notation are like \leq and \geq .

o -notation and ω -notation are like $<$ and $>$.

$$o(g(n)) = \{f(n) : \text{for any constant } c > 0, \\ \text{there is a constant } n_0 > 0 \\ \text{such that } 0 \leq f(n) \leq cg(n) \\ \text{for all } n \geq n_0\}$$

Example:

$$2n^2 = o(n^3) \quad (n_0 = \frac{2}{c})$$

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Example:

$$\sqrt{n} = \omega(\lg n) \quad (n_0 = 1 + \frac{1}{c})$$

Plan

Asymptotic Notation

Solving recurrences

Substitution method

Recursion tree

The master method

Solving recurrences

- ▶ The analysis of merge-sort from **Lecture 1** required us to solve a recurrence.
- ▶ Recurrences are like solving integrals, differential equations, etc.
 - ▶ Learn a few tricks.
- ▶ **Lecture 3:** Applications of recurrences to divide-and-conquer algorithms.

Substitution method

The method is based on guessing a possible solution and then verifying it using mathematical induction. It is divided into the following steps:

- 1. Guess a solution:** Propose a general form of the solution $T(n)$, based on the structure of the problem.
- 2. Substitute into the recurrence:** Replace the conjectured solution in the recurrence equation to check if it holds.
- 3. Adjust if necessary:** If the conjecture is not valid, modify it by adding constants or additional terms.
- 4. Prove by induction:** Use mathematical induction to demonstrate that the conjecture is correct.

Substitution method

The most general method:

1. **Guess** the form of the solution.
2. **Verify** by induction.
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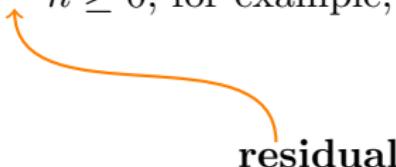
$$T(n) = 4T\left(\frac{n}{2}\right) + n$$

- ▶ Assume that $T(1) = \Theta(1)$.
- ▶ Guess $O(n^3)$. (*Prove O and Ω separately.*)
- ▶ Assume that $T(k) \leq ck^3$ for $k < n$.
- ▶ Prove $T(n) \leq cn^3$ by induction.

Example of substitution

$$\begin{aligned} T(n) &= 4T\left(\frac{n}{2}\right) + n \\ &\leq 4c\left(\frac{n}{2}\right)^3 + n \\ &= \left(\frac{c}{2}\right)n^3 + n \\ &= cn^3 - \left(\left(\frac{c}{2}\right)n^3 - n\right) \leftarrow \text{desired} - \text{residual} \\ &\leq cn^3 \leftarrow \text{desired} \end{aligned}$$

whenever $\left(\frac{c}{2}\right)n^3 - n \geq 0$, for example, if $c \geq 2$ and $n \geq 1$.



Example (continued)

- ▶ We must also handle the initial conditions, that is, ground the induction with base cases.
- ▶ **Base:** $T(n) = \Theta(1)$ for all $n \leq n_0$, where n_0 is a suitable constant.
- ▶ For $1 \leq n < n_0$, we have “ $\Theta(1)$ ” $\leq cn^3$, if we pick c big enough.

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This bound is not tight!

A tighter upper bound?

We shall prove that $T(n) = O(n^2)$.

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Assume that $T(k) \leq ck^2$ for $k < n$:

$$\begin{aligned} T(n) &= 4T\left(\frac{n}{2}\right) + n \\ &\leq 4c\left(\frac{n}{2}\right)^2 + n \\ &= cn^2 + n \\ &= O(n^2) \end{aligned}$$

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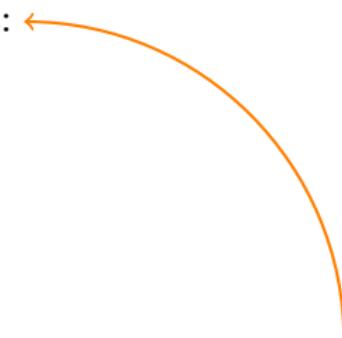
$$= cn^2 + n$$

~~$= O(n^2)$~~ **Wrong!** We must prove the I.H.

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|A tighter upper bound!

IDEA:

- ▶ Strengthen the inductive hypothesis.
- ▶ **Subtract** a low-order term.
- ▶ *Inductive hypothesis:* $T(k) \leq c_1 k^2 - c_2 k$ for $k < n$.

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$$\begin{aligned} T(n) &= 4T\left(\frac{n}{2}\right) + n \\ &= 4\left(c_1\left(\frac{n}{2}\right)^2 - c_2\left(\frac{n}{2}\right)\right) + n \\ &= c_1n^2 - 2c_2n + n \\ &= c_1n^2 - c_2n - c_2n + n \\ &= c_1n^2 - c_2n - (c_2n - n) \\ &\leq c_1n^2 - c_2n \text{ if } c_2 \geq 1. \end{aligned}$$

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Pick c_1 big enough to handle the initial conditions.

Recursion-tree method

- ▶ A recursion tree models the costs (time) of a recursive execution of an algorithm.
- ▶ The recursion-tree method can be unreliable, just like any method that uses ellipses (...).
- ▶ The recursion-tree method promotes intuition, however.
- ▶ The recursion-tree method is good for generating guesses for the substitution method.

Steps of the recurrence-tree method

1. **Expand the recurrence** over multiple levels until a general pattern emerges.
2. **Determine the cost at each level**, which usually depends on the number of subproblems and their size.
3. **Calculate the depth of the tree**, which is the total number of levels until reaching base cases.
4. **Sum the costs of all levels** to obtain the overall cost.

Example of recursion tree

Solve $T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + n^2$

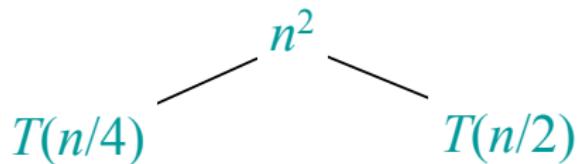
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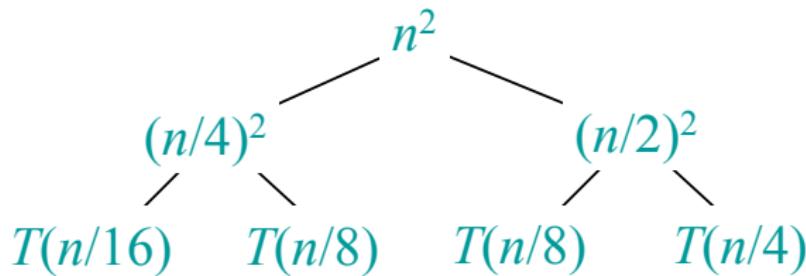
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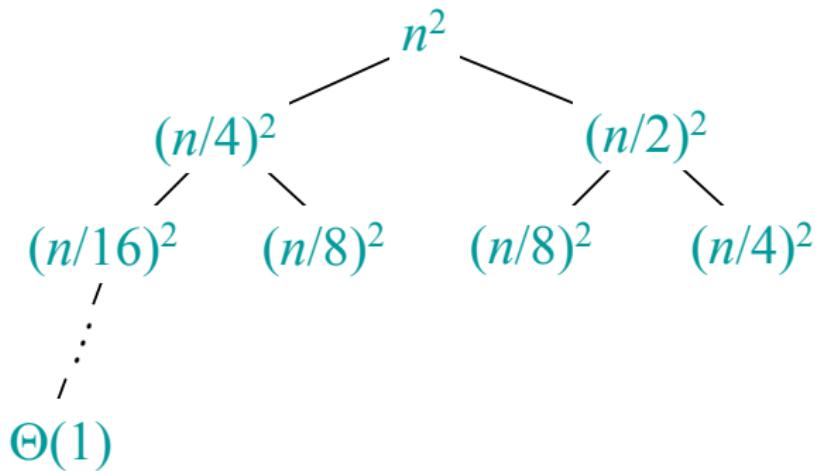
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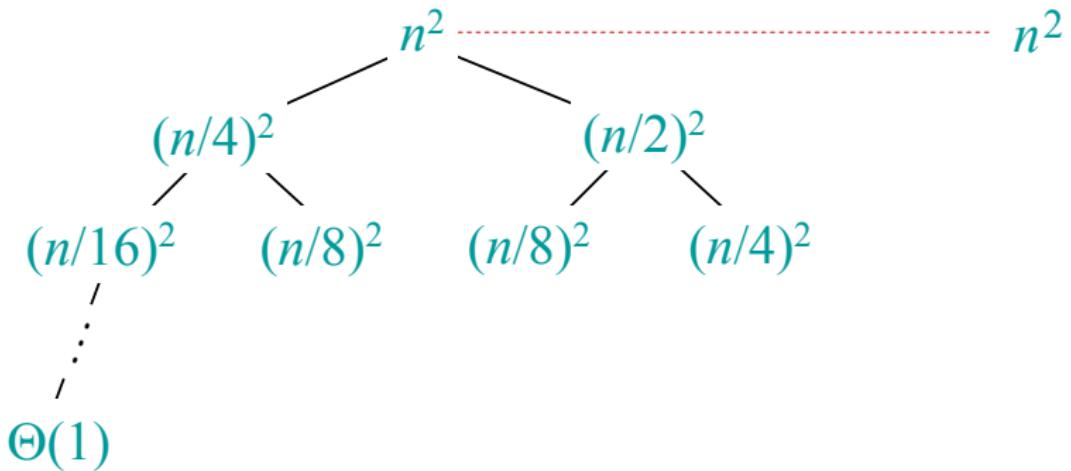
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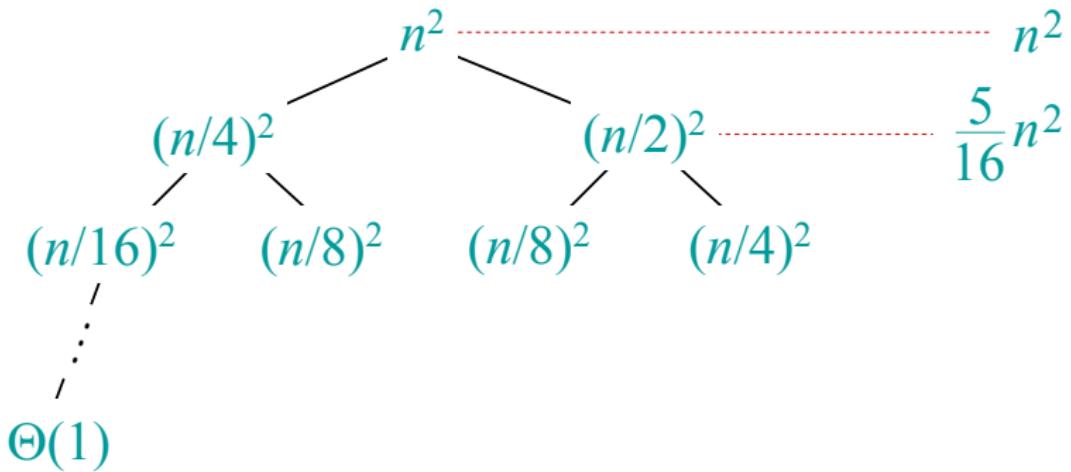
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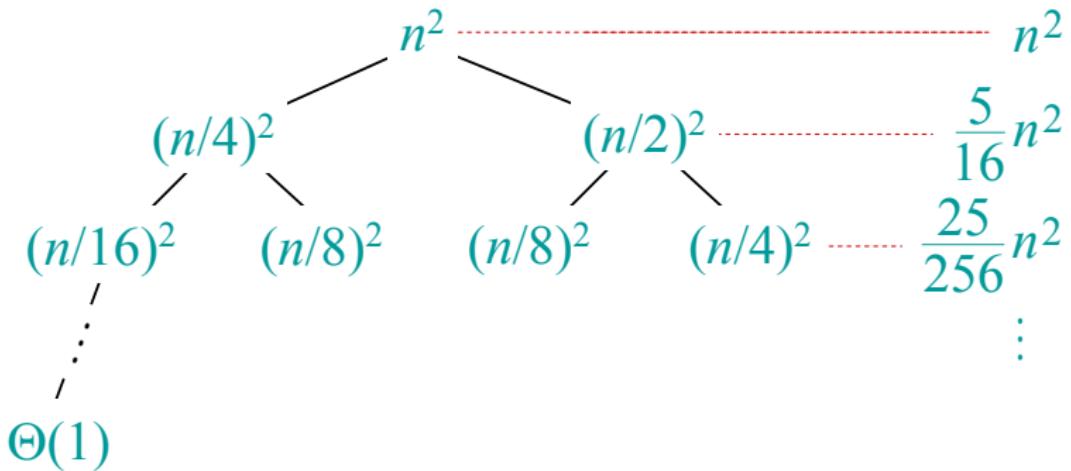
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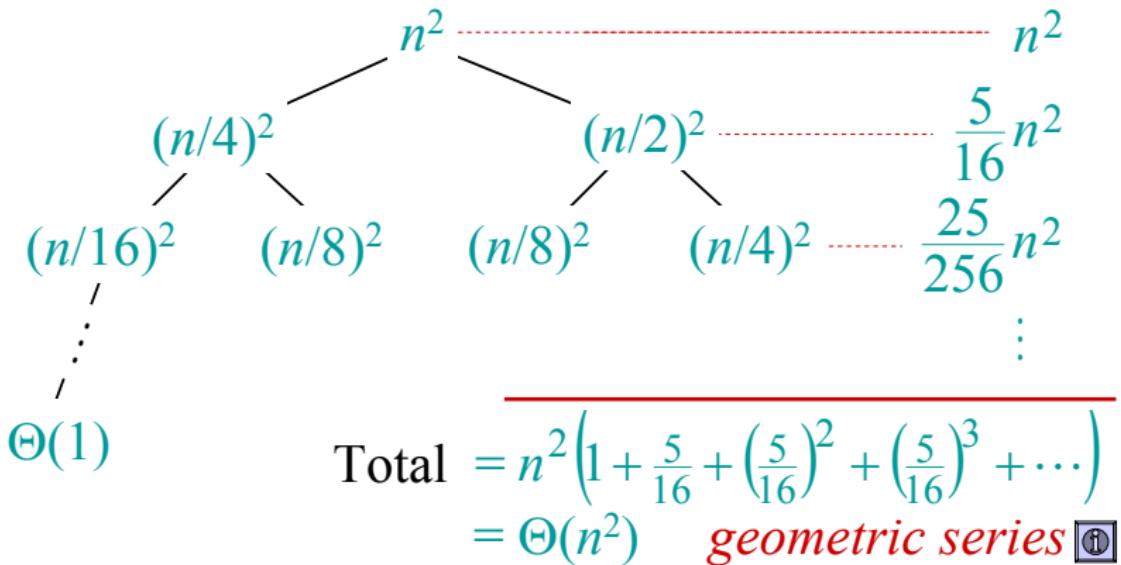
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$$T(n) = aT\left(\frac{n}{b}\right) + f(n),$$

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Note

asymptotically positive means $f(n) > 0$ for $n \geq n_0$.

Three common cases

Compare $f(n)$ with $n^{\log_b a}$:

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Case 2 $f(n) = n^{\log_b a}$

Case 3 $f(n) > n^{\log_b a}$

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Compare $f(n)$ with $n^{\log_b a}$:

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- ▶ $f(n)$ grows polynomially slower than $n^{\log_b a}$ (by an n^ε factor, polynomially smaller).

▶ **Solution:**

$$T(n) = \Theta(n^{\log_b a}).$$

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2 $f(n) = \Theta(n^{\log_b a} \lg^k n)$ for some constant $k \geq 0$.

- ▶ $f(n)$ and $n^{\log_b a}$ grow at similar rates, up to poly log factor.

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3 $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some constant $\varepsilon > 0$.

- ▶ $f(n)$ grows polynomially faster than $n^{\log_b a}$ (by an n^ε factor, polynomially faster),
and $f(n)$ satisfies the ***regularity condition*** that
 $af\left(\frac{n}{b}\right) \leq cf(n)$ for some constant $c < 1$.

▶ **Solution:**

$$T(n) = \Theta(f(n)).$$

Examples

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$$T(n) = 4T\left(\frac{n}{2}\right) + n$$
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$$a = 4, b = 2 \implies n^{\log_b a} = n^2 ; f(n) = \frac{n^2}{\lg n}.$$

Examples

Ex.

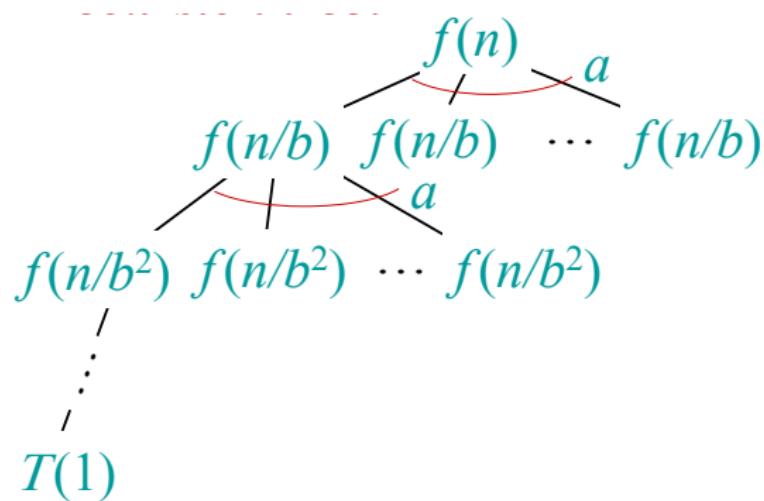
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- ▶ $f(n) = \frac{n^2}{\lg n}$. Have $f(n) = o(n)$, so that $f(n)$ grows more slowly than n , it doesn't grow polynomially slower.
- ▶ In terms of the master theorem, have $f(n) = n^2 \lg^{-1} n$, so that $k = -1$.
- ▶ Master theorem holds only for $k \geq 0$, so case 2 does not apply.
- ▶ Master method does not apply.

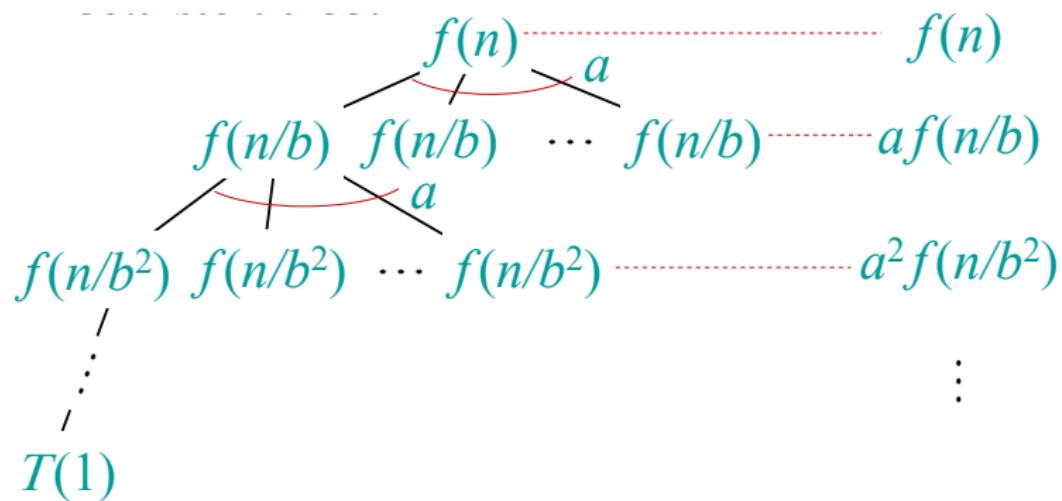
Intuition behind of master theorem

Recursion tree:



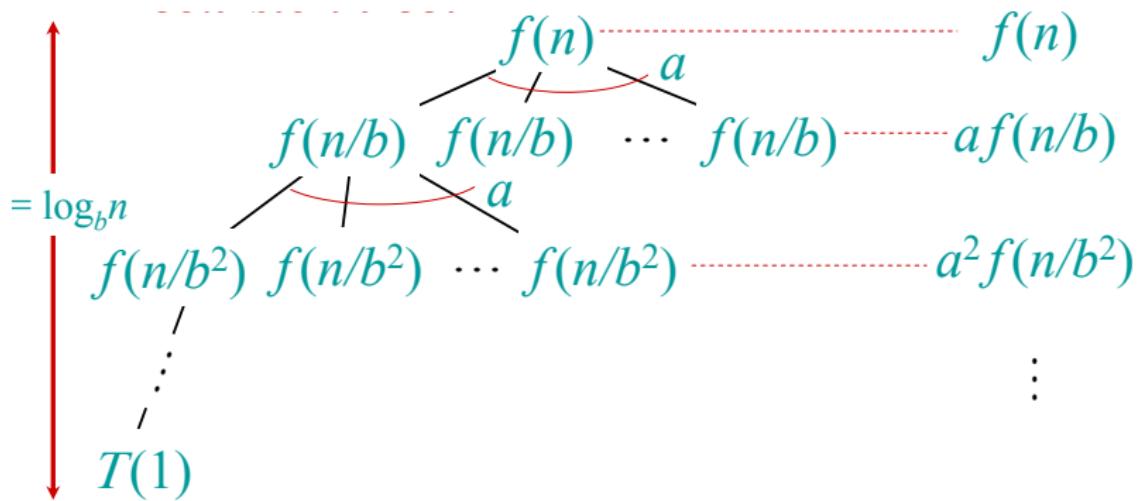
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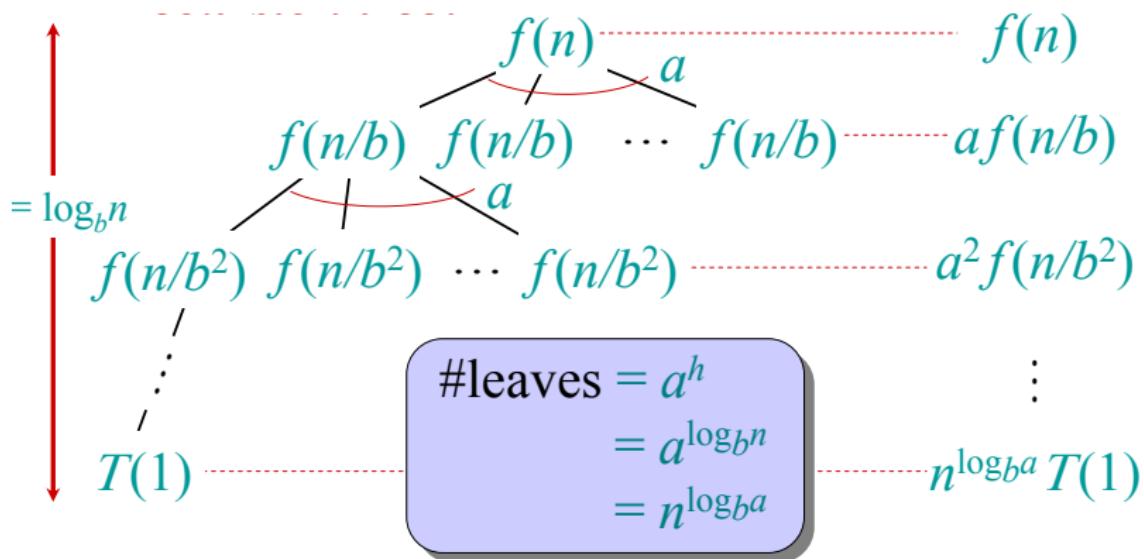
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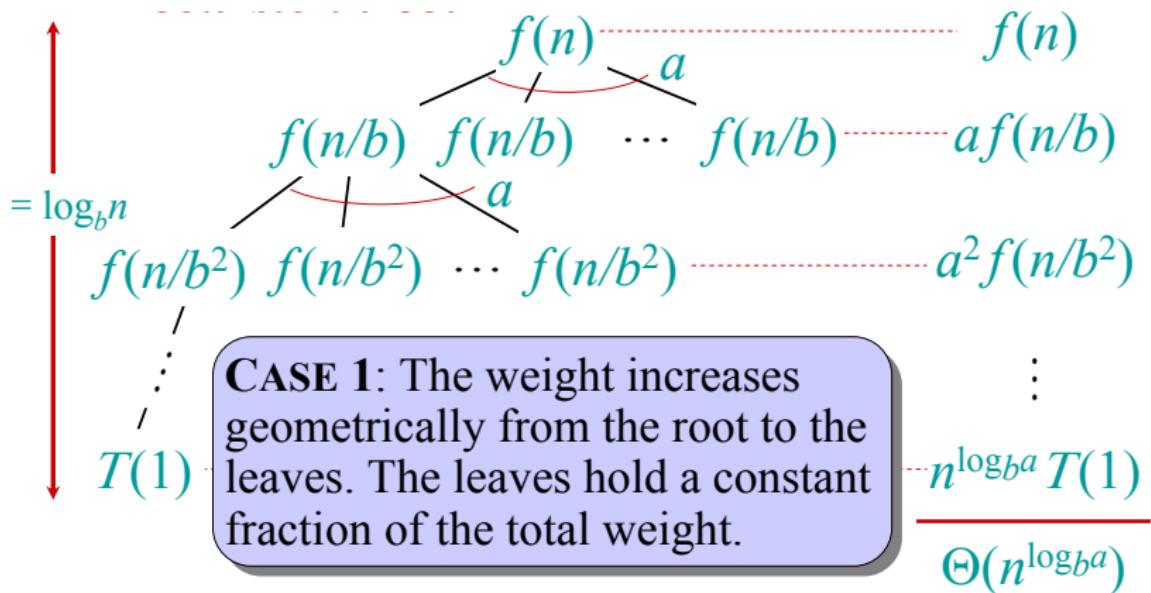
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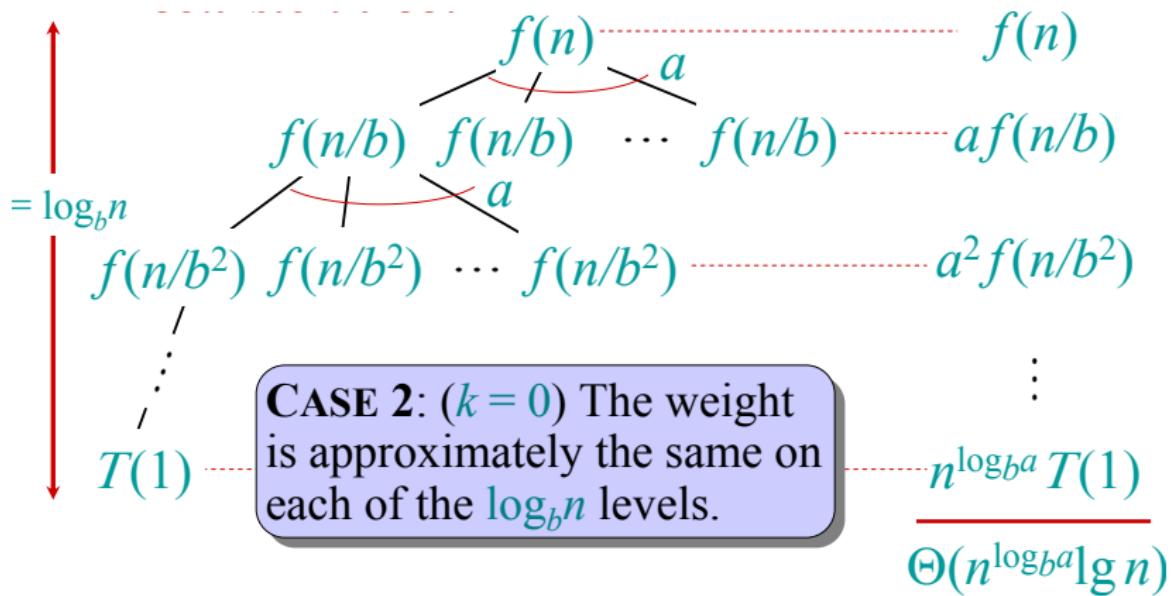
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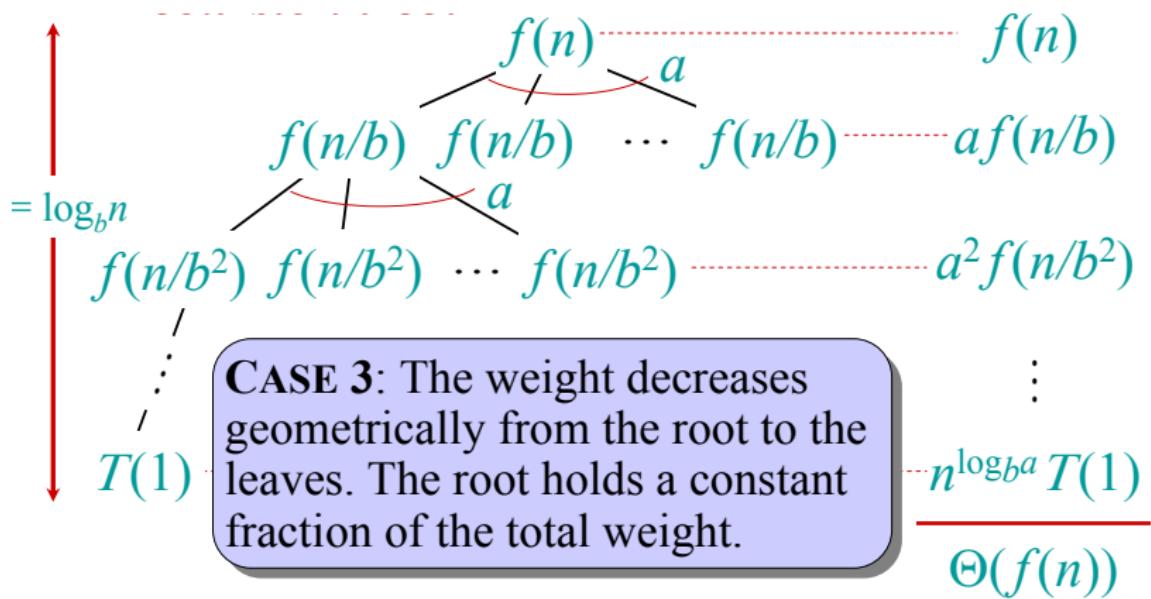
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Appendix: geometric series

$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x} \text{ for } x \neq 1$$

$$1 + x + x^2 + \cdots = \frac{1}{1 - x} \text{ for } |x| < 1$$

End of Lecture 2.

TDT5FTOTTC



Tim Duncan in Wikipedia.

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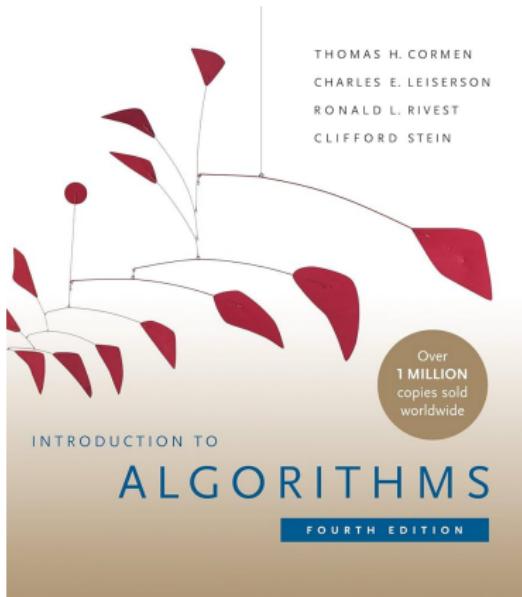
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Content has been extracted from *Introduction to Algorithms*, Fourth Edition, by Cormen, Leiserson, Rivest, and Stein. MIT Press. 2022.

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Original slides from *Introduction to Algorithms* 6.046J/18.401J, Fall 2005 Class by Prof. Charles Leiserson and Prof. Erik Demaine. MIT OpenCourseWare Initiative available at <https://ocw.mit.edu/courses/6-046j-introduction-to-algorithms-sma-5503-fall-2005/>.