



Substitution method

$$\begin{aligned} E[T(n)] &\leq \frac{2}{n} \sum_{k=2}^{n-1} ak \lg k + \Theta(n) \\ &= \frac{2a}{n} \left(\frac{1}{2} n^2 \lg n - \frac{1}{8} n^2 \right) + \Theta(n) \\ &= an \lg n - \left(\frac{an}{4} - \Theta(n) \right) \\ &\leq an \lg n, \end{aligned}$$

if a is chosen large enough so that $an/4$ dominates the $\Theta(n)$.