

Calculating expectation

$$\begin{split} E[T(n)] &= E\bigg[\sum_{k=0}^{n-1} X_k \big(T(k) + T(n-k-1) + \Theta(n)\big)\bigg] \\ &= \sum_{k=0}^{n-1} E\big[X_k \big(T(k) + T(n-k-1) + \Theta(n)\big)\big] \\ &= \sum_{k=0}^{n-1} E\big[X_k\big] \cdot E\big[T(k) + T(n-k-1) + \Theta(n)\big] \\ &= \frac{1}{n} \sum_{k=0}^{n-1} E\big[T(k)\big] + \frac{1}{n} \sum_{k=0}^{n-1} E\big[T(n-k-1)\big] + \frac{1}{n} \sum_{k=0}^{n-1} \Theta(n) \end{split}$$

Linearity of expectation; $E[X_k] = 1/n$.