Introduction to Algorithms
Lecture 2: Asymptotic Notation

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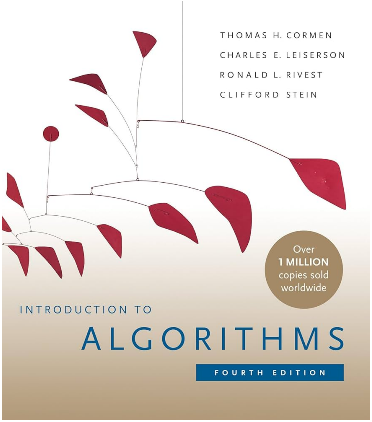
Introduction to Algorithms



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Visit `https://mitpress.mit.edu/9780262046305/introduction-to-algorithms/`.
Original slides from *Introduction to Algorithms 6.046J/18.401J*, Fall 2005 Class by Prof. Charles Leiserson and Prof. Erik Demaine. MIT OpenCourseWare Initiative available at
`https://ocw.mit.edu/courses/6-046j-introduction-to-algorithms-sma-5503-fall-2005/`.

Plan

Asymptotic Notation

Solving recurrences

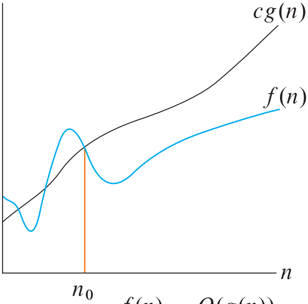
- Substitution method
- Recursion tree
- The master method

Asymptotic Notation

We write $f(n) = O(g(n))$ if there exist such constants $c > 0$, $n_0 > 0$ such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0$.

O-notation (upper bounds)

$$O(g(n)) = \{f(n) : \text{there exist constants } c > 0, n_0 > 0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0\}$$



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Example

$2n^2 = O(n^3) \qquad (c = 1, n_0 = 2).$

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Funny, “one-way” equality...

Set Definition of O-notation

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Example:

$2n^2 \in O(n^3)$

(*Logicians:* $\lambda n.2n^2 \in O(\lambda n.n^3)$, but it's convenient to be sloppy, as long as we understand what's really going on.)

Macro substitution

Convention:
A set in a formula represents an anonymous function in the set.

Example:

$f(n) = n^3 + O(n^2)$
means
$f(n) = n^3 + h(n)$
for some $h(n) \in O(n^2)$.

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A set in a formula represents an anonymous function in the set.

Example:

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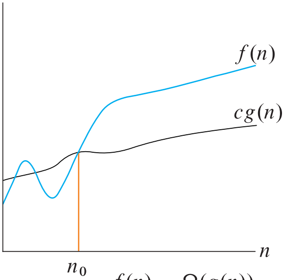
means
for any $f(n) \in O(n)$:
  $n^2 + f(n) = h(n)$
  for some $h(n) \in O(n^2)$.

Ω-notation (lower bounds)

O-notation is an upper-bound notation. It makes no sense to say $f(n)$ is at least $O(n^2)$

Ω-notation (lower bounds)

$$\Omega(g(n)) = \{f(n) : \text{there exist constants } c > 0, n_0 > 0 \text{ such that } 0 \leq cg(n) \leq f(n) \text{ for all } n \geq n_0\}$$



$f(n) = \Omega(g(n))$

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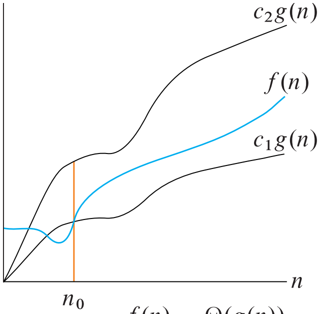
Example:

$\sqrt{n} = \Omega(\lg n) \qquad (c = 1, n_0 = 16)$

Θ-notation (tight bounds)

$$\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$$



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Θ-notation (tight bounds)

$$\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$$

Example:

$\frac{1}{2}n^2 - 2n = \Theta(n^2)$

o-notation and ω-notation

O-notation and Ω-notation are like $\leq$ and $\geq$.
o-notation and ω-notation are like $<$ and $>$.

$$o(g(n)) = \{f(n) : \text{for any constant } c > 0, \text{ there is a constant } n_0 > 0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0\}$$

Example:

$2n^2 = o(n^3) \qquad (n_0 = \frac{2}{c})$

o-notation and ω-notation

O-notation and Ω-notation are like $\leq$ and $\geq$.
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$$\begin{aligned} \omega(g(n)) = \{f(n) : & \text{ for any constant } c > 0, \\ & \text{ there is a constant } n_0 > 0 \\ & \text{ such that } 0 \leq cg(n) \leq f(n) \\ & \text{ for all } n \geq n_0\} \end{aligned}$$

Example:

$\sqrt{n} = \omega(\lg n) \qquad (n_0 = 1 + \frac{1}{c})$

Plan

Asymptotic Notation

Solving recurrences
- Substitution method
- Recursion tree
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Solving recurrences

- The analysis of merge-sort from ***Lecture 1*** required us to solve a recurrence.
- Recurrences are like solving integrals, differential equations, etc.
  - Learn a few tricks.
- ***Lecture 3:*** Applications of recurrences to divide-and-conquer algorithms.

Substitution method

The method is based on guessing a possible solution and then verifying it using mathematical induction. It is divided into the following steps:

1. **Guess a solution:** Propose a general form of the solution $T(n)$, based on the structure of the problem.
2. **Substitute into the recurrence:** Replace the conjectured solution in the recurrence equation to check if it holds.
3. **Adjust if necessary:** If the conjecture is not valid, modify it by adding constants or additional terms.
4. **Prove by induction:** Use mathematical induction to demonstrate that the conjecture is correct.

Substitution method

The most general method:

1. **Guess** the form of the solution.
2. **Verify** by induction.
3. **Solve** for constants.

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Example:

$T(n) = 4T(\frac{n}{2}) + n$

- Assume that $T(1) = \Theta(1)$.
- Guess $O(n^3)$. *(Prove O and Ω separately.)*
- Assume that $T(k) \leq ck^3$ for $k < n$.
- Prove $T(n) \leq cn^3$ by induction.

Example of substitution

$$\begin{aligned} T(n) &= 4T\left(\frac{n}{2}\right) + n \\ &\le 4c\left(\frac{n}{2}\right)^3 + n \\ &= \left(\frac{c}{2}\right)n^3 + n \\ &= cn^3 - \left(\left(\frac{c}{2}\right)n^3 - n\right) \longleftarrow \textbf{ desired } - \textbf{ residual} \\ &\le cn^3 \longleftarrow \textbf{ desired} \end{aligned}$$

whenever $\left(\frac{c}{2}\right)n^3 - n \ge 0$, for example, if $c \ge 2$ and $n \ge 1$.

residual

Example (continued)

- We must also handle the initial conditions, that is, ground the induction with base cases.
- **Base:** $T(n) = \Theta(1)$ for all $n \leq n_0$, where n_0 is a suitable constant.
- For $1 \leq n < n_0$, we have "$\Theta(1)$" $\leq cn^3$, if we pick c big enough.

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This bound is not tight!

¿A tighter upper bound?

We shall prove that $T(n) = O(n^2)$.

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Assume that $T(k) \leq ck^2$ for $k < n$:

$$\begin{aligned} T(n) &= 4T\left(\frac{n}{2}\right) + n \\ &\leq 4c\left(\frac{n}{2}\right)^2 + n \\ &= cn^2 + n \\ &= O(n^2) \end{aligned}$$

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Wrong! We must prove the I.H.

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&= cn^2 + n \\
&= \cancel{O(n^2)} \text{ } \textbf{Wrong!} \text{ We must prove the I.H.} \\
&= cn^2 - (-n) \text{ [} \textbf{desired} - \textbf{residual} \text{]} \\
&\leq cn^2 \text{ for } \textbf{no} \text{ choice of } c > 0. \text{ Lose!}
\end{aligned}
$$

¡A tighter upper bound!

IDEA:

- Strengthen the inductive hypothesis.
- **Subtract** a low-order term.
- *Inductive hypothesis:* $T(k) \leq c_1 k^2 - c_2 k$ for $k < n$.

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\begin{aligned}
T(n) &= 4T\left(\frac{n}{2}\right) + n \\
&= 4\left(c_1\left(\frac{n}{2}\right)^2 - c_2\left(\frac{n}{2}\right)\right) + n \\
&= c_1 n^2 - 2c_2 n + n \\
&= c_1 n^2 - c_2 n - c_2 n + n \\
&= c_1 n^2 - c_2 n - (c_2 n - n) \\
&\leq c_1 n^2 - c_2 n \text{ if } c_2 \geq 1.
\end{aligned}
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Pick c_1 big enough to handle the initial conditions.

Recursion-tree method

- A recursion tree models the costs (time) of a recursive execution of an algorithm.
- The recursion-tree method can be unreliable, just like any method that uses ellipses (...).
- The recursion-tree method promotes intuition, however.
- The recursion-tree method is good for generating guesses for the substitution method.

Steps of the recurrence-tree method

1. **Expand the recurrence** over multiple levels until a general pattern emerges.
2. **Determine the cost at each level**, which usually depends on the number of subproblems and their size.
3. **Calculate the depth of the tree**, which is the total number of levels until reaching base cases.
4. **Sum the costs of all levels** to obtain the overall cost.

Example of recursion tree

Solve $T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + n^2$

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$$T(n/4) \qquad\qquad T(n/2)$$

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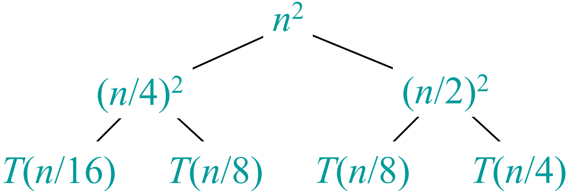

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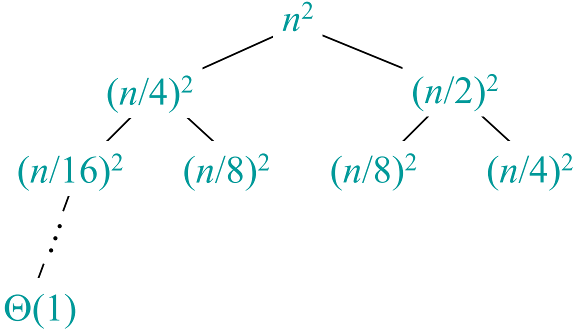

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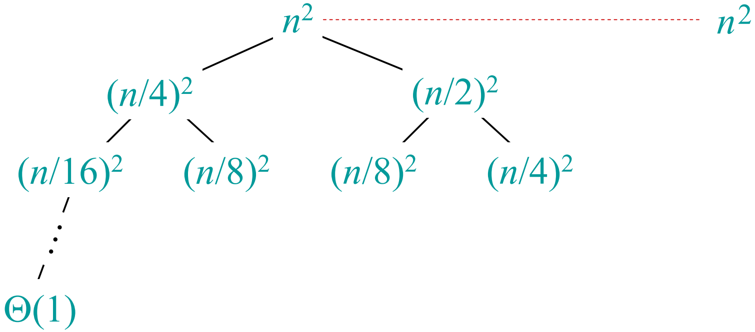

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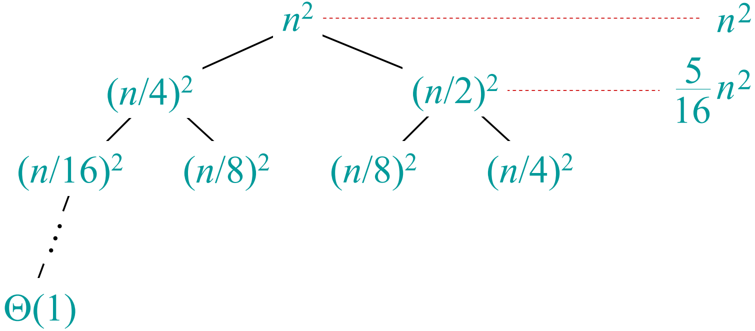

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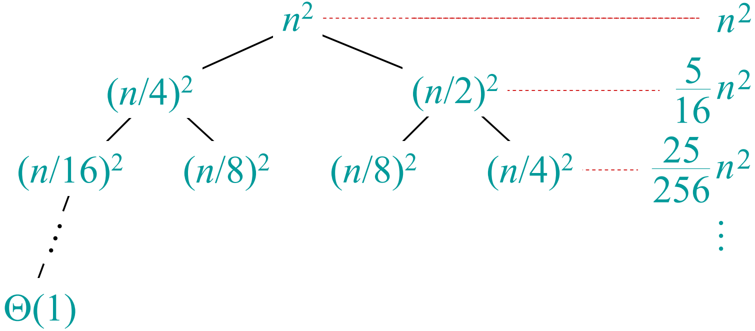

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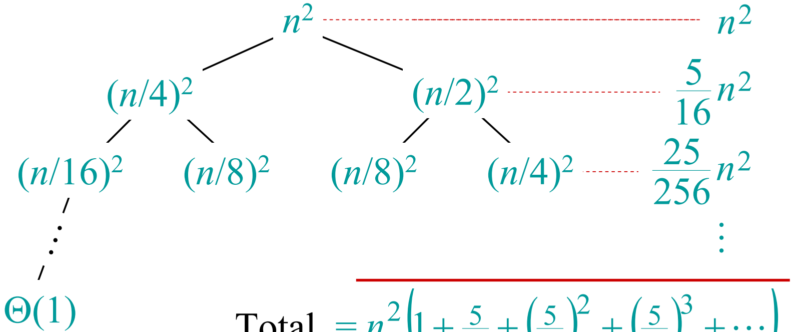

$$\text{Total } = n^2\left(1 + \frac{5}{16} + \left(\frac{5}{16}\right)^2 + \left(\frac{5}{16}\right)^3 + \cdots\right)$$

$$= \Theta(n^2)$$ *geometric series*

The master method

The master method applies to recurrences of the form:

$$T(n) = aT\left(\frac{n}{b}\right) + f(n),$$

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Note

asymptotically positive means $f(n) > 0$ for $n \geq n_0$.

Three common cases

Compare $f(n)$ with $n^{\log_b a}$:

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Case 1 $f(n) < n^{\log_b a}$

Case 2 $f(n) = n^{\log_b a}$

Case 3 $f(n) > n^{\log_b a}$

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Compare $f(n)$ with $n^{\log_b a}$:

1 $f(n) = O\left(n^{\log_b a - \varepsilon}\right)$ for some constant $\varepsilon > 0$.

- $f(n)$ grows polynomially slower than $n^{\log_b a}$ (by an n^{ε} factor, polynomially smaller).
- **Solution:**

$$T(n) = \Theta(n^{\log_b a}).$$

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   - **Solution:**
   $$T(n) = \Theta(n^{\log_b a}).$$
2. $f(n) = \Theta(n^{\log_b a} \lg^k n)$ for some constant $k \geq 0$.
   - $f(n)$ and $n^{\log_b a}$ grow at similar rates, up to poly log factor.
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3. $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some constant $\varepsilon > 0$.
   - $f(n)$ grows polynomially faster than $n^{\log_b a}$ (by an n^{ε} factor, polynomially faster),
     and $f(n)$ satisfies the ***regularity condition*** that $af\left(\frac{n}{b}\right) \leq cf(n)$ for some constant $c < 1$.
   - **Solution:**
     $$T(n) = \Theta(f(n)).$$

Examples

Ex.

$$T(n) = 4T\left(\frac{n}{2}\right) + n$$

$$a = 4, b = 2 \implies n^{\log_b a} = n^2 \; ; \; f(n) = n.$$

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CASE 1:

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$$\textsc{Case } 1{:} f(n) = O(n^{2-\varepsilon}) \text{ for } \varepsilon = 1.$$

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CASE 2:

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Examples

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Case 3:

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Ex.

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Examples

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$$T(n) = 4T\left(\frac{n}{2}\right) + n^3$$

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$$\text{Case } 3\text{:} f(n) = \Omega(n^{2+\varepsilon}) \text{ for } \varepsilon = 1.$$

$$\textbf{and } 4\left(\frac{n}{2}\right)^3 \le cn^3 \text{ (reg. cond.) for } c = \frac{1}{2}.$$

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$$\textbf{and } 4\left(\frac{n}{2}\right)^3 \le cn^3 \text{ (reg. cond.) for } c = \frac{1}{2}.$$

$$\therefore T(n) = \Theta(n^3).$$

Examples

Ex.

$$T(n) = 4T\left(\frac{n}{2}\right) + \frac{n^2}{\lg n}$$

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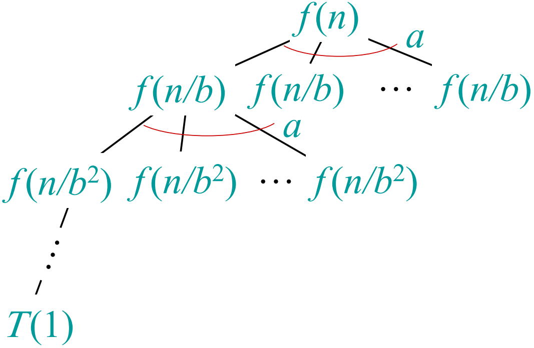
- $f(n) = \frac{n^2}{\lg n}$. Have $f(n) = o(n)$, so that $f(n)$ grows more slowly than n, it doesn't grow polynomially slower.
- In terms of the master theorem, have $f(n) = n^2 \lg^{-1} n$, so that $k = -1$.
- Master theorem holds only for $k \geq 0$, so case 2 does not apply.
- Master method does not apply.

Intuition behind of master theorem

Recursion tree:



Intuition behind of master theorem

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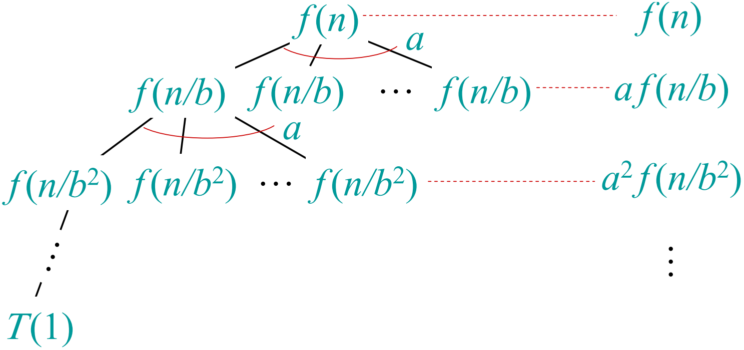

Intuition behind of master theorem

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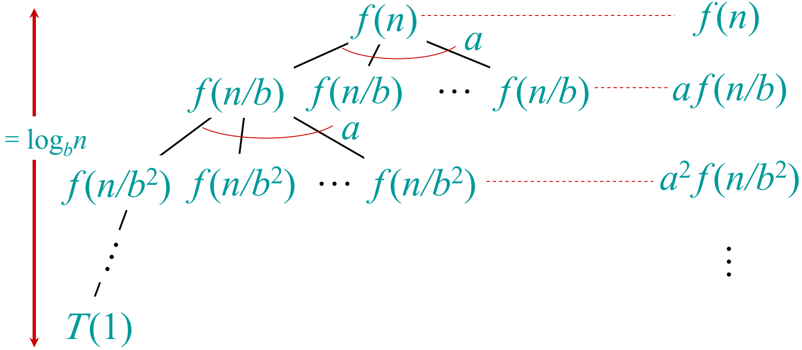

Intuition behind of master theorem

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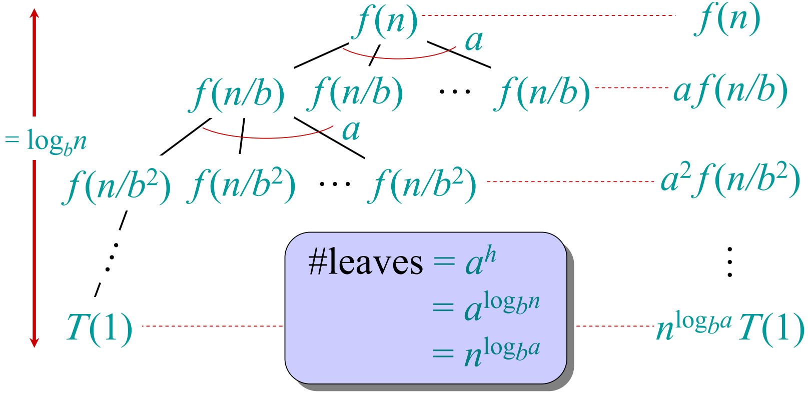
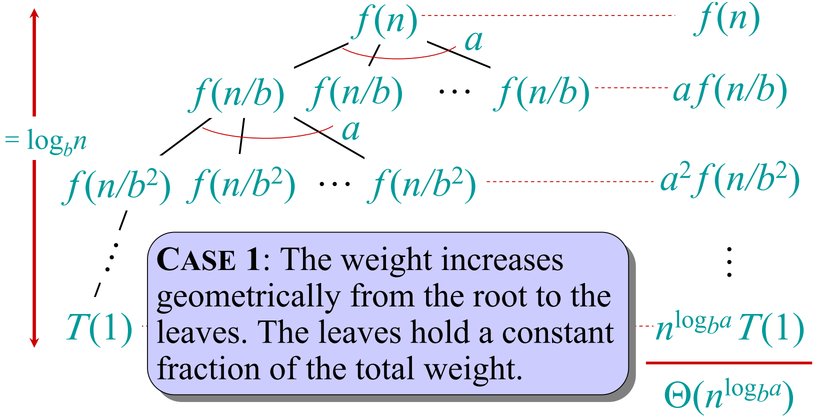


Intuition behind of master theorem

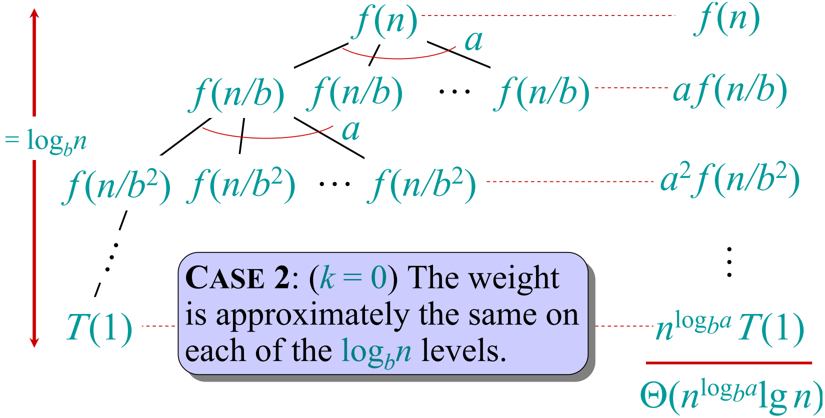
Recursion tree:



CASE 1: The weight increases geometrically from the root to the leaves. The leaves hold a constant fraction of the total weight.

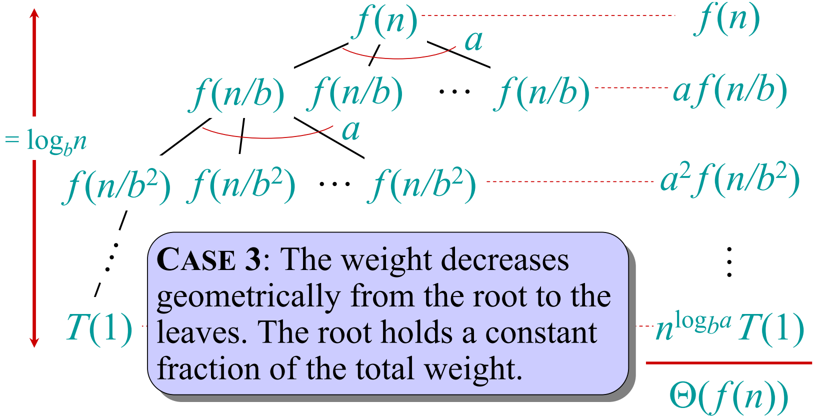
Intuition behind of master theorem

Recursion tree:



Intuition behind of master theorem

Recursion tree:



CASE 3: The weight decreases geometrically from the root to the leaves. The root holds a constant fraction of the total weight.

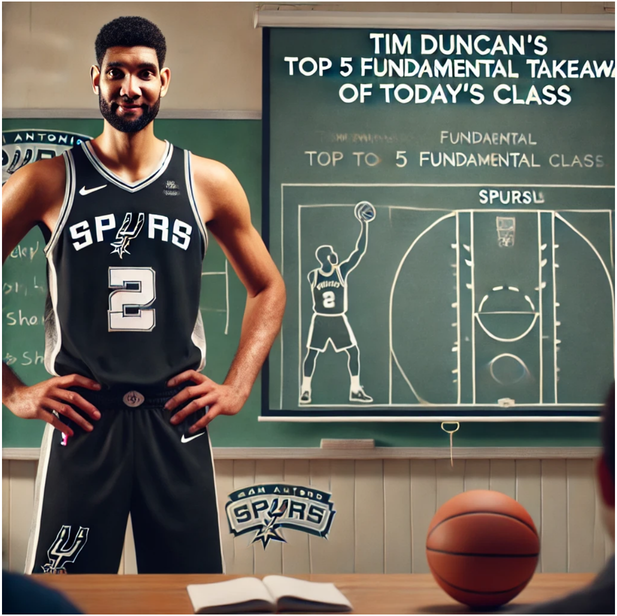
Appendix: geometric series

$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+a}}{1 - x} \text{ for } x \neq 1$$

$$1 + x + x^2 + \cdots = \frac{1}{1 - x} \text{ for } |x| < 1$$

End of Lecture 2.

TDT5FTOTC


TIM DUNCAN'S
TOP 5 FUNDAMENTAL TAKEAW
OF TODAY'S CLASS
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Top 5 Fundamental Takeaways

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3. **Recursion Tree Intuition:** A recursion tree models the breakdown of recursive calls, where the total complexity is derived by summing work across all levels.
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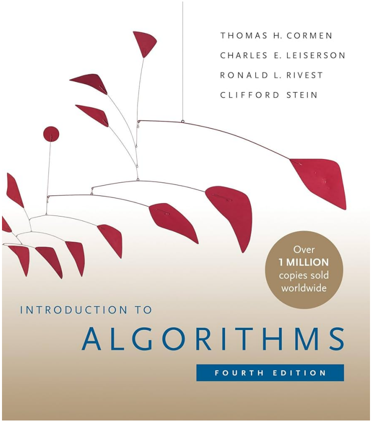
1 **O, Ω, and Θ** notations describe upper, lower, and tight bounds on algorithm growth (o and ω represent strict bounds).

Introduction to Algorithms



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