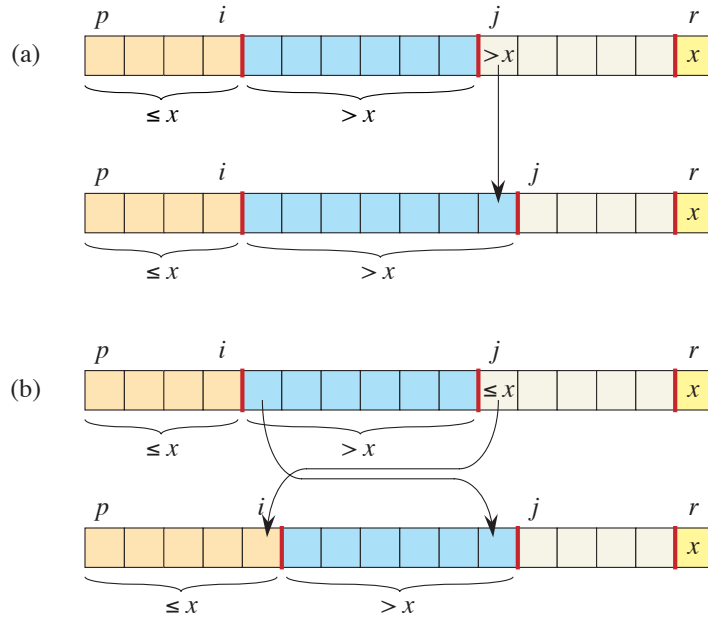


**Figure 7.2** The four regions maintained by the procedure PARTITION on a subarray  $A[p:r]$ . The tan values in  $A[p:i]$  are all less than or equal to  $x$ , the blue values in  $A[i+1:j-1]$  are all greater than  $x$ , the white values in  $A[j:r-1]$  have unknown relationships to  $x$ , and  $A[r] = x$ .



**Figure 7.3** The two cases for one iteration of procedure PARTITION. **(a)** If  $A[j] > x$ , the only action is to increment  $j$ , which maintains the loop invariant. **(b)** If  $A[j] \leq x$ , index  $i$  is incremented,  $A[i]$  and  $A[j]$  are swapped, and then  $j$  is incremented. Again, the loop invariant is maintained.

Exercise 7.1-3 asks you to show that the running time of PARTITION on a subarray  $A[p:r]$  of  $n = r - p + 1$  elements is  $\Theta(n)$ .

## Exercises

### 7.1-1

Using Figure 7.1 as a model, illustrate the operation of PARTITION on the array  $A = \langle 13, 19, 9, 5, 12, 8, 7, 4, 21, 2, 6, 11 \rangle$ .