Introduction to Algorithms
Lecture 4: Quicksort

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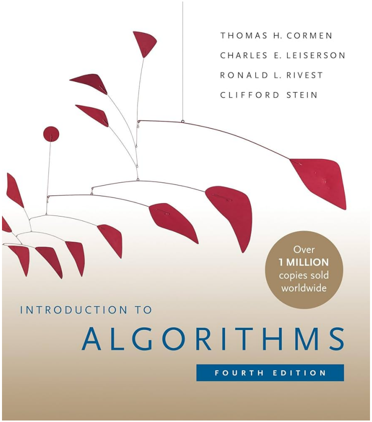
Introduction to Algorithms



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Plan

Description of Quicksort

Divide & Conquer

Partitioning

Worst-case Analysis

Intuition

Randomized Quicksort

Analysis

Expected Running Time

Quicksort

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- Divide-and-conquer algorithm.
- Sorts 'in place' (like insertion sort, but not like merge sort).
- Very practical (with tuning).

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Quicksort an n-element array:

1. **Divide:** Partition the array into two subarrays around a **pivot** x such that elements in lower subarray $\leq x \leq$ elements in upper subarray.

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Key:

Linear-time partitioning subroutine.

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Partitioning Subroutine

procedure PARTITION(A, p, r)
  $x \leftarrow A[r]$
  $i \leftarrow p - 1$
  for $j \leftarrow p$ **to** $r - 1$ **do**
    if $A[j] \leq x$ **then**
      $i \leftarrow i + 1$
      exchange $A[i] \leftrightarrow A[j]$
    end if
  end for
  exchange $A[i+1] \leftrightarrow A[r]$
  return $i + 1$
end procedure

Partitioning Subroutine

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            exchange A[i] ↔ A[j]
        end if
    end for
    exchange A[i + 1] ↔ A[r]
    return i + 1
end procedure
```

Running time:

$O(n)$ for n elements.

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    return i + 1
end procedure
```

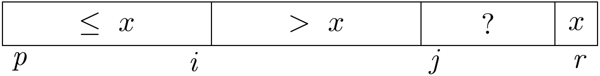
Running time:

$O(n)$ for n elements.

Invariants:

$\leq x$	$> x$	?	x
p … i		j	r

Loop Invariants: Definition



At the beginning of each iteration of the loop (lines 4–9), for any array index k:

1. **Low side:** If $p \leq k \leq i$, then $A[k] \leq x$.
2. **High side:** If $i + 1 \leq k \leq j - 1$, then $A[k] \geq x$.
3. **Pivot:** If $k = r$, then $A[k] = x$.

These conditions define the partitioning into three regions:

- Elements $\leq x$ (low side).
- Elements $> x$ (high side).
- The pivot element.

Initialization and Maintenance

Initialization:

- Before first iteration: $i = p - 1$, $j = p$.
- No values yet examined, so invariants hold trivially.
- Line 2 ensures pivot condition (3) holds.

Maintenance:

- If $A[j] > x$: only increment j, preserving high side property.
- If $A[j] \leq x$: increment i, swap $A[i]$ and $A[j]$, then increment j.
- Swapping ensures $A[i] \leq x$ and $A[i+1] \ldots A[j-1] > x$.

Termination and Correctness

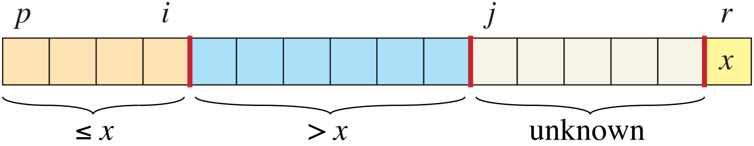
Termination:

- Loop ends when $j = r$.
- The unexamined subarray $A[j \dots r-1]$ is empty.
- All entries are in one of the three invariant regions.

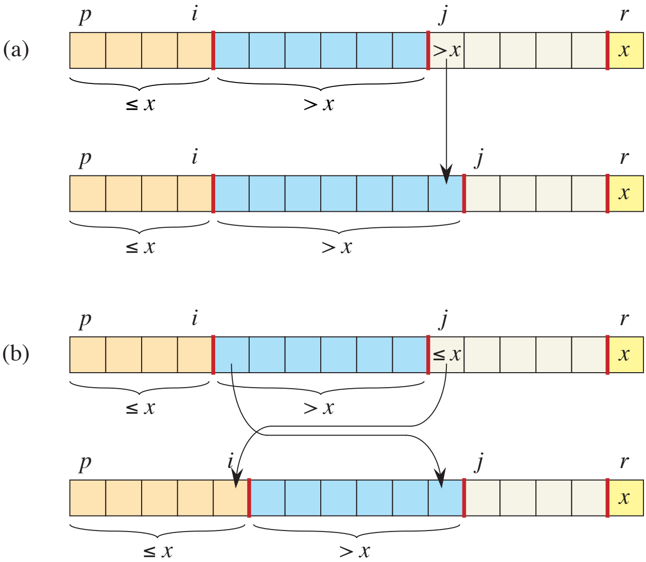
Correctness:

- Array is partitioned into:
  1. Elements $\leq x$ (low side).
  2. Elements $> x$ (high side).
- Pivot is placed immediately after the low side.

Regions in the Partition Procedure

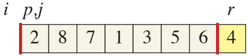


Handling Cases During Partition



Example of partitioning

(a)



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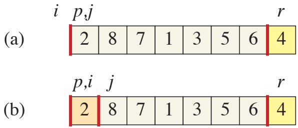

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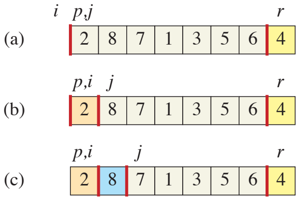

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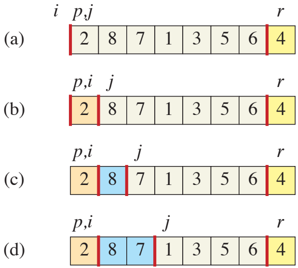

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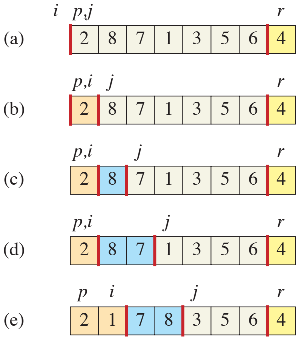

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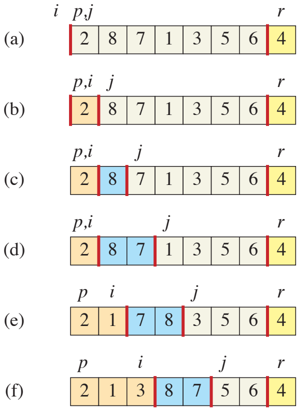

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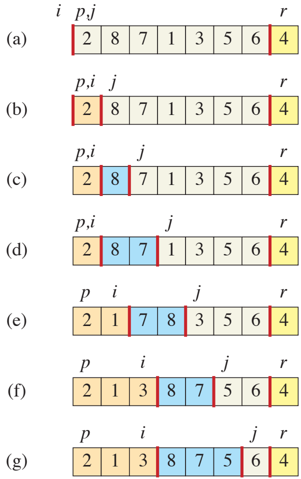

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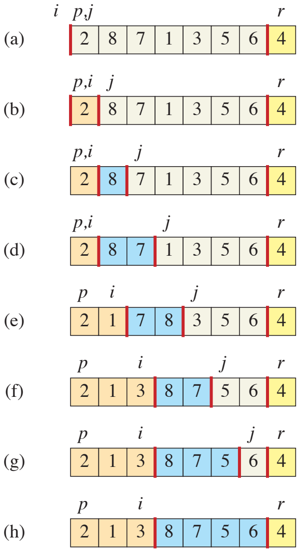

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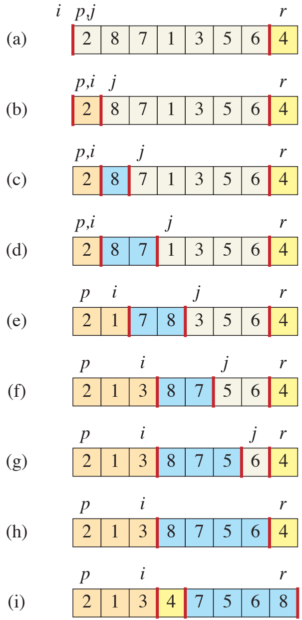

Pseudocode for Quicksort

```
procedure QUICKSORT(A, p, r)
    if p < r then
        q ← PARTITION(A,p, r)
        QUICKSORT(A, p, q − 1)
        QUICKSORT(A, q + 1, r)
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end procedure
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Initial call:

QUICKSORT(A, 1, n)

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- In practice, there are better partitioning algorithms for when duplicate input elements may exist.
- Let $T(n)$ = worst-case running time on an array of n elements.

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=& \Theta(n^2)
\end{aligned}
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Arithmetic Series!

Worst-case Recursion Tree

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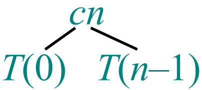

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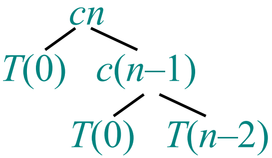

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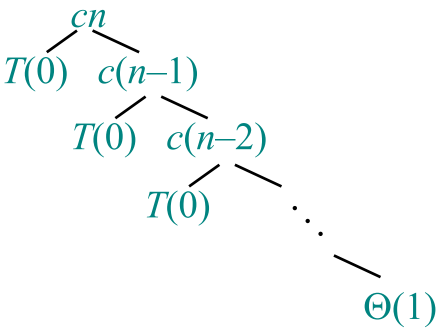

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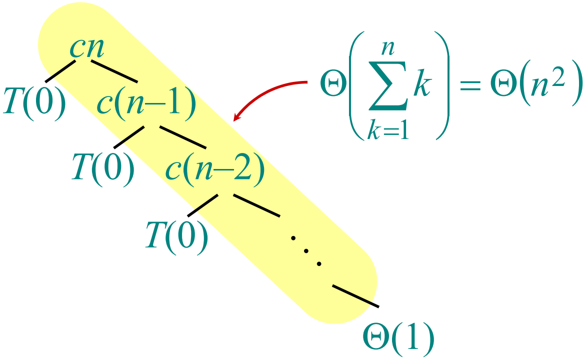

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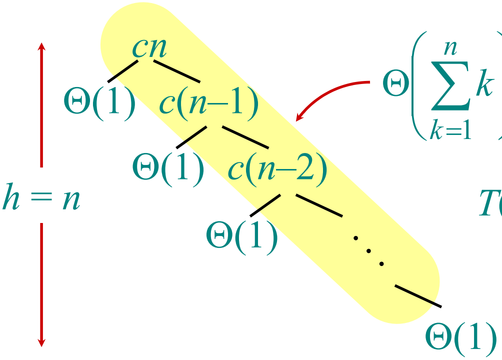

$$\Theta\left(\sum_{k=1}^{n} k\right) = \Theta(n^2)$$

$$T(n) = \Theta(n) + \Theta(n^2)$$
$$= \Theta(n^2)$$

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Best-case Performance

For intuition only!

If we're lucky, PARTITION splits the array evenly:

Best-case Performance

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If we're lucky, PARTITION splits the array evenly:

$$\begin{aligned} T(n) &= 2T\left(\frac{n}{2}\right) + \Theta(n) \\ &= \Theta(n \lg n) \end{aligned}$$

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- Which case is this? Case 2.

What if the split is always $\frac{1}{10} : \frac{9}{10}$?

$$T(n) = T\left(\frac{1}{10}n\right) + T\left(\frac{9}{10}n\right) + \Theta(n)$$

What is the solution to this recurrence?

Performance of “Almost-best” Case

$$T(n)$$

Performance of “Almost-best” Case

$$cn$$

$$T\left(\tfrac{1}{10}n\right) \qquad\qquad T\left(\tfrac{9}{10}n\right)$$

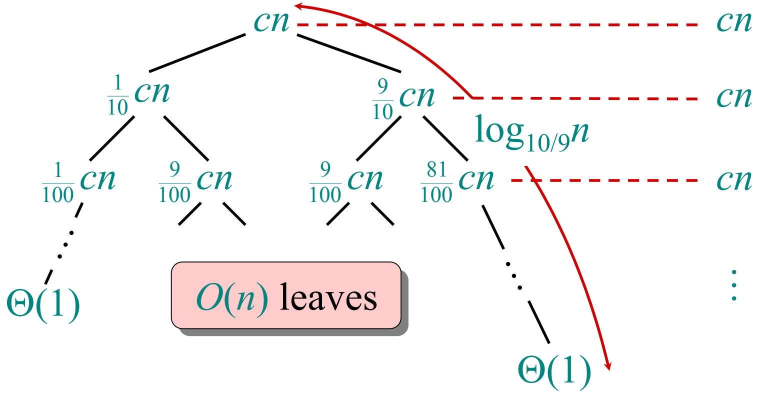
Performance of “Almost-best” Case

$$cn$$

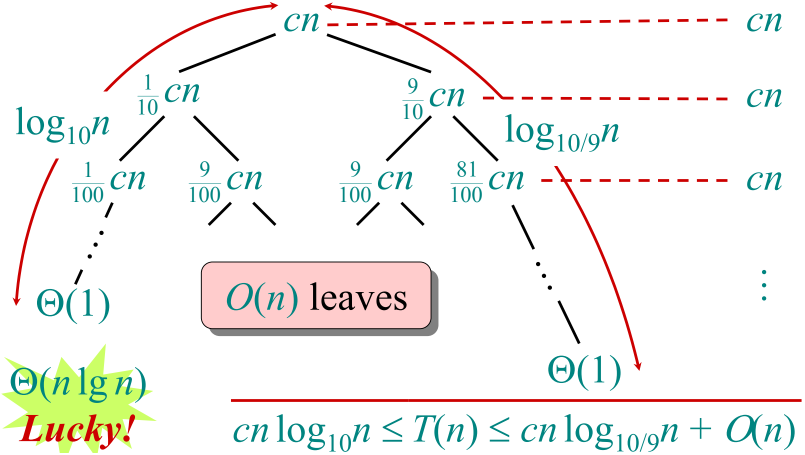
$$\frac{1}{10}cn \qquad \frac{9}{10}cn$$

$$T\left(\frac{1}{100}n\right) \quad T\left(\frac{9}{100}n\right) \qquad T\left(\frac{9}{100}n\right) \quad T\left(\frac{81}{100}n\right)$$

Performance of “Almost-best” Case



Performance of "Almost-best" Case



More Intuition

Suppose we alternate lucky, unlucky, lucky, unlucky, lucky, ...

$$L(n) = 2U\left(\frac{n}{2}\right) + \Theta(n) \qquad \textit{lucky}$$

$$U(n) = L(n-1) + \Theta(n) \qquad \textit{unlucky}$$

Solving:

$$\begin{aligned} L(n) &= 2\left(L\left(\frac{n}{2} - 1\right) + \Theta(\frac{n}{2})\right) + \Theta(n) \\ &= 2L\left(\frac{n}{2} - 1\right) + \Theta(n) \\ &= \Theta(n \lg n) \qquad \textbf{Lucky!} \end{aligned}$$

How can we make sure we are usually lucky?

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IDEA:
Partition around a **random** element.

- Running time is independent of the input order.
- No assumptions need to be made about the input distribution.
- No specific input elicits the worst-case behavior.
- The worst case is determined only by the output of a random-number generator.

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Randomized Quicksort Analysis

Let $T(n)$ = the random variable for the running time of randomized quicksort on an input of size n, assuming random numbers are independent.

For $k = 0, 1, \ldots, n-1$, define the **indicator random variable**.

$$X_k = \begin{cases} 1 & \text{if PARTITION generates a } k : n-k-1 \text{ split,} \\ 0 & \text{otherwise.} \end{cases}$$

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$$X_k = \begin{cases} 1 & \text{if PARTITION generates a } k : n-k-1 \text{ split,} \\ 0 & \text{otherwise.} \end{cases}$$

$E[X_k] = 0 \cdot Pr\{X_k = 0\} + 1 \cdot Pr\{X_k = 1\} = Pr\{X_k = 1\} = \frac{1}{n}$,
since all splits are equally likely, assuming element are distinct.

Analysis (Cont.)

$$T(n) = \begin{cases} T(0) + T(n-1) + \Theta(n) & \text{if } 0 : n-1 \text{ split,} \\ T(1) + T(n-2) + \Theta(n) & \text{if } 1 : n-2 \text{ split,} \\ \quad\vdots & \\ T(n-1) + T(0) + \Theta(n) & \text{if } n-1 : 0 \text{ split.} \end{cases}$$

$$= \sum_{k=0}^{n-1} X_k (T(k) + T(n-k-1) + \Theta(n))$$

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Calculating expectation

Take expectations of both sides.

$$E[T(n)] = E\left[\sum_{k=0}^{n-1} X_k(T(k) + T(n-k-1) + \Theta(n))\right]$$

Calculating expectation

Linearity of expectation.

$$
\begin{aligned}
E[T(n)] &= E\left[\sum_{k=0}^{n-1} X_k(T(k) + T(n-k-1) + \Theta(n))\right] \\
&= \sum_{k=0}^{n-1} E\left[X_k(T(k) + T(n-k-1) + \Theta(n))\right]
\end{aligned}
$$

Calculating expectation

Independence of X_k from other random choices.

$$\begin{aligned}
E[T(n)] &= E\left[\sum_{k=0}^{n-1} X_k(T(k) + T(n-k-1) + \Theta(n))\right] \\
&= \sum_{k=0}^{n-1} E\left[X_k(T(k) + T(n-k-1) + \Theta(n))\right] \\
&= \sum_{k=0}^{n-1} E[X_k] \cdot E[T(k) + T(n-k-1) + \Theta(n)]
\end{aligned}$$

Calculating expectation

$E[X_k] = \frac{1}{n}$.

$$
\begin{aligned}
E[T(n)] &= E\left[\sum_{k=0}^{n-1} X_k(T(k) + T(n-k-1) + \Theta(n))\right] \\
&= \sum_{k=0}^{n-1} E\left[X_k(T(k) + T(n-k-1) + \Theta(n))\right] \\
&= \sum_{k=0}^{n-1} E[X_k] \cdot E[T(k) + T(n-k-1) + \Theta(n)] \\
&= \frac{1}{n}\sum_{k=0}^{n-1} E[T(k) + T(n-k-1) + \Theta(n)]
\end{aligned}
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E[T(n)] &= E\left[\sum_{k=0}^{n-1} X_k(T(k) + T(n-k-1) + \Theta(n))\right] \\
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&= \sum_{k=0}^{n-1} E[X_k] \cdot E[T(k) + T(n-k-1) + \Theta(n)] \\
&= \frac{1}{n}\sum_{k=0}^{n-1} E[T(k)] + \frac{1}{n}\sum_{k=0}^{n-1} E[T(n-k-1)] + \frac{1}{n}\sum_{k=0}^{n-1} \Theta(n)
\end{aligned}
$$

Calculating expectation

Summations have identical terms.

$$
\begin{aligned}
E[T(n)] &= E\left[\sum_{k=0}^{n-1} X_k(T(k) + T(n-k-1) + \Theta(n))\right] \\
&= \sum_{k=0}^{n-1} E\left[X_k(T(k) + T(n-k-1) + \Theta(n))\right] \\
&= \sum_{k=0}^{n-1} E[X_k] \cdot E[T(k) + T(n-k-1) + \Theta(n)] \\
&= \frac{1}{n}\sum_{k=0}^{n-1} E[T(k)] + \frac{1}{n}\sum_{k=0}^{n-1} E[T(n-k-1)] + \frac{1}{n}\sum_{k=0}^{n-1} \Theta(n) \\
&= \frac{2}{n}\sum_{k=0}^{n-1} E[T(k)] + \frac{1}{n}\sum_{k=0}^{n-1} \Theta(n)
\end{aligned}
$$

Calculating expectation

$n \cdot \Theta(n) = \Theta(n^2)$.

$$
\begin{aligned}
E[T(n)] &= E\left[\sum_{k=0}^{n-1} X_k(T(k) + T(n-k-1) + \Theta(n))\right] \\
&= \sum_{k=0}^{n-1} E\left[X_k(T(k) + T(n-k-1) + \Theta(n))\right] \\
&= \sum_{k=0}^{n-1} E[X_k] \cdot E[T(k) + T(n-k-1) + \Theta(n)] \\
&= \frac{1}{n}\sum_{k=0}^{n-1} E[T(k)] + \frac{1}{n}\sum_{k=0}^{n-1} E[T(n-k-1)] + \frac{1}{n}\sum_{k=0}^{n-1} \Theta(n) \\
&= \frac{2}{n}\sum_{k=0}^{n-1} E[T(k)] + \frac{1}{n}\Theta(n^2)
\end{aligned}
$$

Calculating expectation

Sum up.

$$
\begin{aligned}
E[T(n)] &= E\left[\sum_{k=0}^{n-1} X_k(T(k) + T(n-k-1) + \Theta(n))\right] \\
&= \sum_{k=0}^{n-1} E\left[X_k(T(k) + T(n-k-1) + \Theta(n))\right] \\
&= \sum_{k=0}^{n-1} E[X_k] \cdot E[T(k) + T(n-k-1) + \Theta(n)] \\
&= \frac{1}{n}\sum_{k=0}^{n-1} E[T(k)] + \frac{1}{n}\sum_{k=0}^{n-1} E[T(n-k-1)] + \frac{1}{n}\sum_{k=0}^{n-1} \Theta(n) \\
&= \frac{2}{n}\sum_{k=0}^{n-1} E[T(k)] + \Theta(n)
\end{aligned}
$$

Hairy recurrence

$$E[T(n)] = \frac{2}{n} \sum_{k=2}^{n-1} E[T(k)] + \Theta(n)$$

(The $k = 0$, 1 terms can be absorbed in the $\Theta(n)$.)

Prove:
$E[T(n)] \leq an \lg n$ for constant $a > 0$.

- Choose a large enough so that $an \lg n$ dominates $E[T(n)]$ for sufficiently small $n \geq 2$.

Use fact:
$\sum_{k=2}^{n-1} k \lg k \leq \frac{1}{2} n^2 \lg n - \frac{1}{8} n^2$ (exercise).

Substitution method

Substitute inductive hypothesis.

$$E[T(n)] \leq \frac{2}{n} \sum_{k=2}^{n-1} ak \lg k + \Theta(n)$$

Substitution method

Use fact.

$$
\begin{aligned}
E[T(n)] &\le \frac{2}{n}\sum_{k=2}^{n-1} ak\lg k + \Theta(n) \\
&\le \frac{2a}{n}\left(\frac{1}{2}n^2\lg n - \frac{1}{8}n^2\right) + \Theta(n)
\end{aligned}
$$

Substitution method

$$
\begin{aligned}
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&\le \frac{2a}{n}\left(\frac{1}{2}n^2\lg n - \frac{1}{8}n^2\right) + \Theta(n) \\
&= an\lg n - \left(\frac{an}{4} - \Theta(n)\right) \\
&\le an\lg n,
\end{aligned}
$$

if a is chosen large enough so that $\dfrac{an}{4}$ dominates the $\Theta(n)$.

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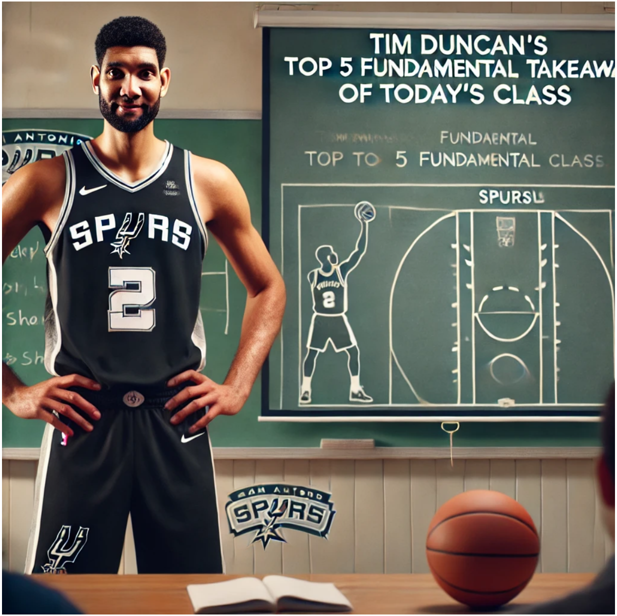
- Quicksort behaves well even with caching and virtual memory.

End of Lecture 4.

TDT5FTOTC


TIM DUNCAN'S
TOP 5 FUNDAMENTAL TAKEAW
OF TODAY'S CLASS
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TOP TO 5 FUNDAMENTAL CLASS
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Top 5 Fundamental Takeaways

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Top 5 Fundamental Takeaways

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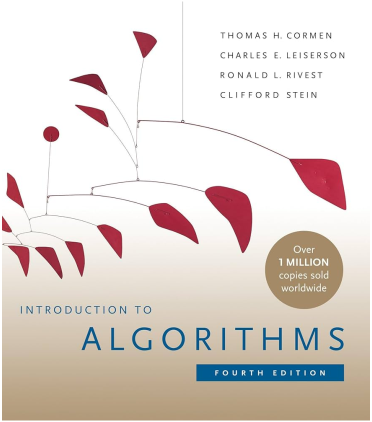
1 **Quicksort is Highly Efficient in Practice** – It outperforms merge sort in most cases and benefits from hardware optimizations.

Introduction to Algorithms



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