The set of restrictions  $F_1, F_2, \ldots, F_n$  is the set of dependencies that can be checked efficiently. We now must ask whether testing only the restrictions is sufficient. Let  $F' = F_1 \cup F_2 \cup \cdots \cup F_n$ . F' is a set of functional dependencies on schema R, but, in general,  $F' \neq F$ . However, even if  $F' \neq F$ , it may be that  $F'^+ = F^+$ . If the latter is true, then every dependency in F is logically implied by F', and, if we verify that F' is satisfied, we have verified that F is satisfied. We say that a decomposition having the property  $F'^+ = F^+$  is a dependency-preserving decomposition.

Figure 7.10 shows an algorithm for testing dependency preservation. The input is a set  $D = \{R_1, R_2, \dots, R_n\}$  of decomposed relation schemas, and a set F of functional dependencies. This algorithm is expensive since it requires computation of  $F^+$ . Instead of applying the algorithm of Figure 7.10, we consider two alternatives.

First, note that if each member of F can be tested on one of the relations of the decomposition, then the decomposition is dependency preserving. This is an easy way to show dependency preservation; however, it does not always work. There are cases where, even though the decomposition is dependency preserving, there is a dependency in F that cannot be tested in any one relation in the decomposition. Thus, this alternative test can be used only as a sufficient condition that is easy to check; if it fails we cannot conclude that the decomposition is not dependency preserving; instead we will have to apply the general test.

We now give a second alternative test for dependency preservation that avoids computing  $F^+$ . We explain the intuition behind the test after presenting the test. The test applies the following procedure to each  $\alpha \to \beta$  in F.

```
result = \alpha

repeat

for each R_i in the decomposition

t = (result \cap R_i)^+ \cap R_i

result = result \cup t

until (result does not change)
```

The attribute closure here is under the set of functional dependencies F. If *result* contains all attributes in  $\beta$ , then the functional dependency  $\alpha \to \beta$  is preserved. The decomposition is dependency preserving if and only if the procedure shows that all the dependencies in F are preserved.

The two key ideas behind the preceding test are as follows:

• The first idea is to test each functional dependency  $\alpha \to \beta$  in F to see if it is preserved in F' (where F' is as defined in Figure 7.10). To do so, we compute the closure of  $\alpha$  under F'; the dependency is preserved exactly when the closure includes  $\beta$ . The decomposition is dependency preserving if (and only if) all the dependencies in F are found to be preserved.