



Figure 3.2 Graphic examples of the O , Ω , and Θ notations. In each part, the value of n_0 shown is the minimum possible value, but any greater value also works. **(a)** O -notation gives an upper bound for a function to within a constant factor. We write $f(n) = O(g(n))$ if there are positive constants n_0 and c such that at and to the right of n_0 , the value of $f(n)$ always lies on or below $cg(n)$. **(b)** Ω -notation gives a lower bound for a function to within a constant factor. We write $f(n) = \Omega(g(n))$ if there are positive constants n_0 and c such that at and to the right of n_0 , the value of $f(n)$ always lies on or above $cg(n)$. **(c)** Θ -notation bounds a function to within constant factors. We write $f(n) = \Theta(g(n))$ if there exist positive constants n_0 , c_1 , and c_2 such that at and to the right of n_0 , the value of $f(n)$ always lies between $c_1g(n)$ and $c_2g(n)$ inclusive.

O -notation

As we saw in Section 3.1, O -notation describes an *asymptotic upper bound*. We use O -notation to give an upper bound on a function, to within a constant factor.

Here is the formal definition of O -notation. For a given function $g(n)$, we denote by $O(g(n))$ (pronounced “big-oh of g of n ” or sometimes just “oh of g of n ”) the *set of functions*

$$O(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0\}^1$$

A function $f(n)$ belongs to the set $O(g(n))$ if there exists a positive constant c such that $f(n) \leq cg(n)$ for sufficiently large n . Figure 3.2(a) shows the intuition behind O -notation. For all values n at and to the right of n_0 , the value of the function $f(n)$ is on or below $cg(n)$.

The definition of $O(g(n))$ requires that every function $f(n)$ in the set $O(g(n))$ be *asymptotically nonnegative*: $f(n)$ must be nonnegative whenever n is sufficiently large. (An *asymptotically positive* function is one that is positive for all

¹ Within set notation, a colon means “such that.”