



Hairy recurrence

$$E[T(n)] = \frac{2}{n} \sum_{k=2}^{n-1} E[T(k)] + \Theta(n)$$

(The $k = 0, 1$ terms can be absorbed in the $\Theta(n)$.)

Prove: $E[T(n)] \leq an \lg n$ for constant $a > 0$.

- Choose a large enough so that $an \lg n$ dominates $E[T(n)]$ for sufficiently small $n \geq 2$.

Use fact: $\sum_{k=2}^{n-1} k \lg k \leq \frac{1}{2} n^2 \lg n - \frac{1}{8} n^2$ (exercise).