their coordinates directly in the *Origin*() field of the edge; there is no strict need for a separate type of vertex record. Even more important is to realize that in many applications the faces of the subdivision carry no interesting meaning (think of the network of rivers or roads that we looked at before). If that is the case, we can completely forget about the face records, and the *IncidentFace*() field of half-edges. As we will see, the algorithm of the next section doesn't need these fields (and is actually simpler to implement if we don't need to update them). Some implementations of doubly-connected edge lists may also insist that the graph formed by the vertices and edges of the subdivision be connected. This can always be achieved by introducing dummy edges, and has two advantages. Firstly, a simple graph transversal can be used to visit all half-edges, and secondly, the *InnerComponents*() list for faces is not necessary.

2.3 Computing the Overlay of Two Subdivisions

Now that we have designed a good representation of a subdivision, we can tackle the general map overlay problem. We define the overlay of two subdivisions $\mathcal{S}_1$ and $\mathcal{S}_2$ to be the subdivision $\mathcal{O}(\mathcal{S}_1,\mathcal{S}_2)$ such that there is a face f in $\mathcal{O}(\mathcal{S}_1,\mathcal{S}_2)$ if and only if there are faces f_1 in $\mathcal{S}_1$ and f_2 in $\mathcal{S}_2$ such that f is a maximal connected subset of $f_1 \cap f_2$. This sounds more complicated than it is: what it means is that the overlay is the subdivision of the plane induced by the edges from $\mathcal{S}_1$ and $\mathcal{S}_2$. Figure 2.4 illustrates this. The general map overlay problem

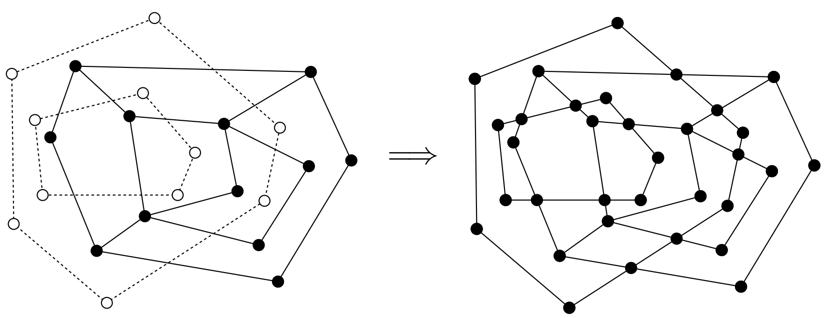

Figure 2.4
Overlaying two subdivisions

is to compute a doubly-connected edge list for $\mathcal{O}(\mathcal{S}_1,\mathcal{S}_2)$, given the doubly-connected edge lists of $\mathcal{S}_1$ and $\mathcal{S}_2$. We require that each face in $\mathcal{O}(\mathcal{S}_1,\mathcal{S}_2)$ be labeled with the labels of the faces in $\mathcal{S}_1$ and $\mathcal{S}_2$ that contain it. This way we have access to the attribute information stored for these faces. In an overlay of a vegetation map and a precipitation map this would mean that we know for each region in the overlay the type of vegetation and the amount of precipitation.

Let's first see how much information from the doubly-connected edge lists for $\mathcal{S}_1$ and $\mathcal{S}_2$ we can re-use in the doubly-connected edge list for $\mathcal{O}(\mathcal{S}_1,\mathcal{S}_2)$. Consider the network of edges and vertices of $\mathcal{S}_1$. This network is cut into pieces by the edges of $\mathcal{S}_2$. These pieces are for a large part re-usable; only the edges that have been cut by the edges of $\mathcal{S}_2$ should be renewed. But does this also