

ADCP

Coordinate Transformation

Formulas and Calculations



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TELEDYNE
RD INSTRUMENTS

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NOTES



ADCP Coordinate Transformation

1 Introduction

This booklet defines various formulas and calculations used by the ADCP. For more information on ADCP calculations, see our *Principles of Operation: A Practical Primer*.

1.1 How to Contact Teledyne RD Instruments

If you have technical issues or questions involving a specific application or deployment with your instrument, contact our Field Service group:

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2 Measuring Three-dimensional Velocities

Based on the principles of the Doppler effect, one beam of an ADCP can only detect changes in distance between energy-reflecting scatterers and the transducer. This means the ADCP can only determine water velocities when the scatterers within the beam are moving toward or away from the transducer face. The ADCP does not detect scatterers moving perpendicular to the beam because there is no change in distance between the scatterer and the transducer. By sensing only the scatterer velocity along one beam (Figure 1, page 3), that beam can determine the relative velocity of the scatterers along the beam using the formula:

$$V_{RELATIVE} = V_{COS}\theta$$

Equation 1

Where: V = velocity of the scatterers
 θ = angle between the scatterers direction of travel and the beam.

Obviously, we cannot measure water currents with just one beam. We need multiple beams to determine east, north, and vertical velocity components. With multiple beams pointing in different directions, each beam senses a different velocity component. For example, if one beam points east and another points north, the ADCP can measure the E/W and N/S current components. While the beams do not directly measure the east and north components, common trigonometry can convert the individual beam velocities into the east and north components. Adding a third beam lets us compute vertical velocities. We need only three beams to find the water velocity components, but we add a fourth beam to compute a second vertical velocity component. The difference between the two vertical components produces an error velocity that helps determine whether the water currents are horizontally homogeneous and whether all four beams are operating properly.

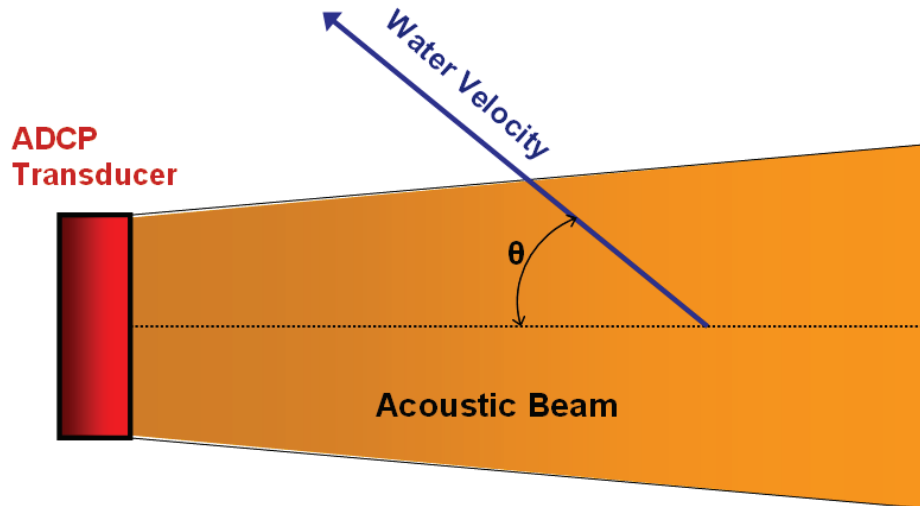


Figure 1. Measuring the Water Velocity Component along One Beam

3 Water-Mass Velocity / Radial Velocity Definition

The ADCP determines water-mass velocity by measuring the change in the round-trip time from the ADCP to the scattering mass. Time changes are the result of scatterer motion. The ADCP measures the Doppler frequency shift for each depth cell and each beam. The ADCP then computes the component of relative flow velocity (V , in mm/s) in the direction of each acoustic beam from this measured Doppler frequency shift (F_D) for each depth cell using the equation:

$$V = F_D \frac{C}{2F_S} 1000$$

Equation 2

Where:

C = Velocity of sound in water at the transducer face in m/s (see “[Speed of Sound](#),” page 4).

F_S = Transmitted acoustic frequency (76.8, 153.6, 307.2, 614.4, or 1228.8 kHz, based on ADCP model 75, 150, 300, 600, or 1200 kHz respectively).



NOTE. The sense of each beam’s radial velocity is positive for flow towards the transducer.

4 Speed of Sound

The speed of sound in water affects ADCP calculations used to determine velocity and depth data.

4.1 Correcting Velocity Data for Speed of Sound

To determine the true velocity data, the ADCP needs to know the speed of sound in water. You can tell the ADCP the speed of sound, or the ADCP can compute the speed of sound from temperature, salinity, and depth data. Table 1 lists several values for speed of sound (in m/s) for various temperature and salinity readings at shallow depths. If the ADCP is not given a speed of sound, it uses a default value of 1500 m/s. If the ADCP uses an incorrect speed of sound value, you can correct velocity data for the actual speed of sound by using the speed of sound measured at the transducer. To correct ADCP velocity data for the actual speed of sound, use the formula:

$$V_{CORRECTED} = V_{UNCORRECTED} \left(\frac{C}{C_A} \right)$$

Equation 3

Where C is the real sound speed at the transducer, and C_A is the speed of sound used by the ADCP. You can calculate the speed of sound in m/s, c , using the equation (Urick, 1983):

$$c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + (1.34 - 0.01T)(S - 35) + 0.016D$$

Equation 4

Where: D = Depth, in meters
 S = Salinity, in parts per thousand
 T = Temperature, in °C

Be aware that variation of sound speed with depth does NOT affect the estimate of the horizontal current. This is because sound speed variation has two effects that counteract each other. The effect of a changing speed of sound is to refract or bend the sound beams, but the bend added to the change of speed of sound will compensate for each other and therefore preserve the value of the horizontal current. Indeed, at higher sound speed, the Beam slant angle is closer to the horizontal while at lower sound speed it is closer to the vertical, therefore geometrically conserving the same horizontal component (Figure 2, page 6).

All that is necessary to obtain horizontal current at any depth is the sound speed at the transducer.

Table 1: Speed of Sound versus Temperature and Salinity

| Temp (°C) | Speed of Sound (m/s) | | | | | |
|--------------|----------------------|---------|---------|---------|---------|---------|
| | Salinity (ppt) | | | | | |
| | 0 | 20 | 25 | 30 | 35 | 40 |
| 0 | 1402.3 | 1429.10 | 1435.80 | 1442.50 | 1449.20 | 1455.90 |
| 1 | 1407.2 | 1433.80 | 1440.45 | 1447.10 | 1453.75 | 1460.40 |
| 2 | 1411.9 | 1438.38 | 1444.98 | 1451.58 | 1458.18 | 1464.78 |
| 3 | 1416.6 | 1442.86 | 1449.41 | 1455.96 | 1462.51 | 1469.06 |
| 4 | 1421.2 | 1447.24 | 1453.74 | 1460.24 | 1466.74 | 1473.24 |
| 5 | 1425.7 | 1451.51 | 1457.96 | 1464.41 | 1470.86 | 1477.31 |
| 6 | 1430.0 | 1455.68 | 1462.08 | 1468.48 | 1474.88 | 1481.28 |
| 7 | 1434.3 | 1459.75 | 1466.10 | 1472.45 | 1478.80 | 1485.15 |
| 8 | 1438.5 | 1463.73 | 1470.03 | 1476.33 | 1482.63 | 1488.93 |
| 9 | 1442.6 | 1467.61 | 1473.86 | 1480.11 | 1486.36 | 1492.61 |
| 10 | 1446.5 | 1471.39 | 1477.59 | 1483.79 | 1489.99 | 1496.19 |
| 11 | 1450.4 | 1475.08 | 1481.23 | 1487.38 | 1493.53 | 1499.68 |
| 12 | 1454.2 | 1478.68 | 1484.78 | 1490.88 | 1496.98 | 1503.08 |
| 13 | 1457.9 | 1482.19 | 1488.24 | 1494.29 | 1500.34 | 1506.39 |
| 14 | 1461.6 | 1485.62 | 1491.62 | 1497.62 | 1503.62 | 1509.62 |
| 15 | 1465.1 | 1488.95 | 1494.90 | 1500.85 | 1506.80 | 1512.75 |
| 16 | 1468.6 | 1492.21 | 1498.11 | 1504.01 | 1509.91 | 1515.81 |
| 17 | 1471.9 | 1495.38 | 1501.23 | 1507.08 | 1512.93 | 1518.78 |
| 18 | 1475.2 | 1498.47 | 1504.27 | 1510.07 | 1515.87 | 1521.67 |
| 19 | 1478.4 | 1501.48 | 1507.23 | 1512.98 | 1518.73 | 1524.48 |
| 20 | 1481.6 | 1504.42 | 1510.12 | 1515.82 | 1521.52 | 1527.22 |
| 21 | 1484.6 | 1507.28 | 1512.93 | 1518.58 | 1524.23 | 1529.88 |
| 22 | 1487.6 | 1510.07 | 1515.67 | 1521.27 | 1526.87 | 1532.47 |
| 23 | 1490.5 | 1512.78 | 1518.33 | 1523.88 | 1529.43 | 1534.98 |
| 24 | 1493.4 | 1515.43 | 1520.93 | 1526.43 | 1531.93 | 1537.43 |
| 25 | 1496.2 | 1518.01 | 1523.46 | 1528.91 | 1534.36 | 1539.81 |
| 26 | 1498.9 | 1520.52 | 1525.92 | 1531.32 | 1536.72 | 1542.12 |
| 27 | 1501.5 | 1522.96 | 1528.31 | 1533.66 | 1539.01 | 1544.36 |
| 28 | 1504.1 | 1525.35 | 1530.65 | 1535.95 | 1541.25 | 1546.55 |
| 29 | 1506.6 | 1527.67 | 1532.92 | 1538.17 | 1543.42 | 1548.67 |
| 30 | 1509.1 | 1529.93 | 1535.13 | 1540.33 | 1545.53 | 1550.73 |
| 31 | 1511.5 | 1532.13 | 1537.28 | 1542.43 | 1547.58 | 1552.73 |
| 32 | 1513.8 | 1534.28 | 1539.38 | 1544.48 | 1549.58 | 1554.68 |
| 33 | 1516.1 | 1536.38 | 1541.43 | 1546.48 | 1551.53 | 1556.58 |
| 34 | 1518.4 | 1538.42 | 1543.42 | 1548.42 | 1553.42 | 1558.42 |
| 35 | 1520.6 | 1540.41 | 1545.36 | 1550.31 | 1555.26 | 1560.21 |
| 36 | 1522.7 | 1542.35 | 1547.25 | 1552.15 | 1557.05 | 1561.95 |
| 37 | 1524.8 | 1544.24 | 1549.09 | 1553.94 | 1558.79 | 1563.64 |
| 38 | 1526.8 | 1546.09 | 1550.89 | 1555.69 | 1560.49 | 1565.29 |
| 39 | 1528.9 | 1547.90 | 1552.65 | 1557.40 | 1562.15 | 1566.90 |
| 40 | 1530.8 | 1549.66 | 1554.36 | 1559.06 | 1563.76 | 1568.46 |
| 41 | 1532.7 | 1551.38 | 1556.03 | 1560.68 | 1565.33 | 1569.98 |
| 42 | 1534.6 | 1553.07 | 1557.67 | 1562.27 | 1566.87 | 1571.47 |
| 43 | 1536.5 | 1554.71 | 1559.26 | 1563.81 | 1568.36 | 1572.91 |
| 44 | 1538.3 | 1556.32 | 1560.82 | 1565.32 | 1569.82 | 1574.32 |
| 45 | 1540.1 | 1557.90 | 1562.35 | 1566.80 | 1571.25 | 1575.70 |

We can explain this result using the horizontal component of the sound velocity. The horizontal velocity measurement needs only the horizontal component of the sound transmitted by the ADCP. The speed of sound in the water next to the transducer controls the horizontal component of the

acoustic wavelength. As sound travels through the water, the horizontal wavelength remains constant as it passes from one stratified layer to the next (Figure 2). Because the horizontal wavelength is constant, the measured velocity is constant. Continuity of the wavelength parallel to a layer is the basic principle of Snell’s law. This explains why we only need to correct velocity for speed of sound at the transducer.

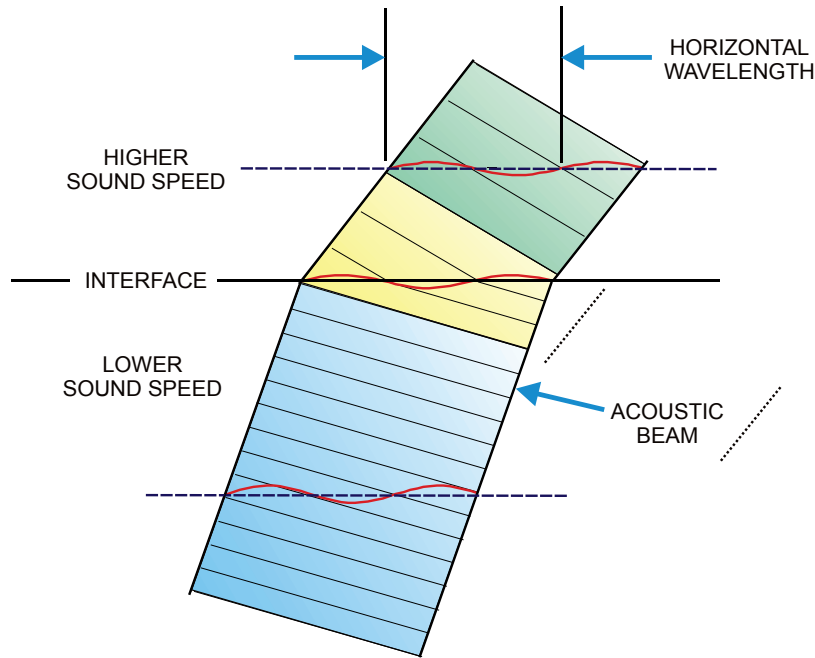


Figure 2. Conservation of Horizontal Wavelength

Nor does the sound speed away from the transducer affect the estimate of the *vertical* component of the platform velocity relative to the bottom, or relative to the water if the vertical current is negligible. The reason is that the entire Doppler shift occurs at the transducer and is therefore unaffected by refraction.

In contrast to horizontal velocity and platform velocity, Doppler measurement of vertical currents is affected slightly by the sound speed profile. Use the following formula (instead of Equation 3 on page 4) to correct the vertical current in the n^{th} range cell, using either the uncorrected bottom velocity or the platform velocity (from another instrument) to remove the transducer motion.

$$W_{Current} = W_{Corrected} + W_{Platform} = \frac{1}{K_n} \left[\frac{C_n}{C_A} W_{Uncorrected} + W_{Platform} \right]$$

Equation 5

Where:

$$W_{Platform} = -\left(\frac{C_O}{C_A}\right)W_{BOTTOM}$$

Equation 6

$$K_n = \frac{\cos \theta_n}{\cos \theta_0} = \sqrt{1 + \left[1 - \left(\frac{C_n}{C_0}\right)^2\right] \tan^2 \theta_0}$$

Equation 7

- C_0 = speed of sound at the transducer
- C_A = speed of sound used by the ADCP
- C_n = speed of sound at the n^{th} depth cell
- θ_0 = beam angle to the vertical (20° or 30°)

For example, below a thermocline where the temperature of seawater drops from 20°C to 15°C, the speed of sound drops about 1 percent, resulting in a -1.3 percent vertical current correction (for $\theta_0 = 30^\circ$). Since vertical currents are usually small, the error in absolute terms caused by neglecting to make this correction is usually tiny.

4.2 Correcting Depth Cell Depth for Speed of Sound

To calculate the depth cell positions, the ADCP uses the same sound speed C_A that it uses to convert Doppler shift to velocity (see “[Correcting Velocity Data for Speed of Sound](#),” page 4). This sound speed can be found in bytes 15-16 of the variable leader (see the WorkHorse Commands and Output Data Format guide). Also, it uses the nominal beam angle to the vertical (20° or 30°) to convert slant range to vertical distance. The nominal (uncorrected) depth of the center of the n^{th} depth cell for the downward-looking orientation is:

$$ED + dis1 + (n-1)*WS$$

Equation 8

Where:

- ED = transducer depth (known, or from bytes 17-18 of variable leader - see the WorkHorse Commands and Output Data Format guide)
- dis1 = vertical distance to the center of the first depth cell (from bytes 33-34 of the fixed leader)
- WS = vertical size (spacing) of the range cells (from bytes 13-14 of the fixed leader)

(For the upward-looking orientation, simply negate the last two terms.) If the sound speed profile is known, you can correct the distortion of the depth cell locations caused by the incorrectly assumed sound speed using the following procedure:

Determine the average sound speed C_1 between the transducer and first depth cell, and calculate the correction factor K_1 using Equation 7 on page 7. Correct $dis1$ by multiplying it by $K_1*(C_1/C_A)$.

Determine the average sound speed C_n between the $(n-1)^{st}$ and the n^{th} depth cell, and calculate the correction factor K_n using Equation 7 on page 7. Calculate the depth cell spacing as $K_n*(C_n/C_A)*WS$. Repeat for n from 2 to the number of depth cells WN .

Calculate the location of each depth cell as the cumulative corrected spacing from the first depth cell, added to the corrected depth of that cell from step a.

5 Velocity Data Processing

This section describes various processing steps that may be needed to convert raw radial beam velocity data to the desired form. Most of these steps are optionally performed by the ADCP, but may alternatively be performed by a data acquisition system (DAS) program such as *VmDas* or in a post-processing program such as *WinADCP*. Read this section if you want to understand what the ADCP has done or is able to do or if you are designing a DAS or post-processing program to process the raw velocity data.

You may want to consider the following velocity processing steps, which are described more fully elsewhere. Except where indicated, they apply both to water velocity and bottom track velocity.

Adjustment for incorrect sound speed. This step has already been discussed in “Correcting Velocity Data for Speed of Sound,” page 4. All measured velocity components are proportional to the speed of sound at the transducer, so this step may be performed out of sequence.

Depth cell mapping. Compensate for severe tilts by re-mapping range cells in different beams to the appropriate depth cells according to their recalculated depth. (See “Depth Cell Mapping,” page 9).

Screening. Check the reasonableness of the velocity components for each depth cell and flag bad data. (See “Screening,” page 10).

Transformation of beam radial velocities to orthogonal components. Taking into account the orientations of the beams, calculate the velocity components in a coordinate system fixed to the instrument. Also, use the redundant information among the four beams to calculate the “error velocity”. (See “Beam-to-Instrument Transformation,” page 11).

Three-beam solutions. In cases where exactly one beam has bad data, it is possible to calculate the velocity components using the remaining three beams (see “[Three-Beam Solutions](#),” page 14).

Rotation of velocity components to a different orthogonal system. Use available pitch, roll, and heading information to resolve the velocities in the ship or earth coordinate system. Resolving the velocity components in a stable coordinate system will prevent platform attitude fluctuations from causing bias during subsequent averaging (see “[Coordinate Systems](#),” page 16). *VmDas* will collect in Beam coordinates and rotate them to Earth coordinates. The raw Beam coordinate system frame velocities will be logged and therefore available to the user for any type of processing. *WinADCP* can accept 2 types of coordinate frame data: Beam and Earth coordinate system frames data. *WinADCP* will rotate Beam coordinate system frame data to Earth coordinate system frame for display using the internally logged attitude sensor data, i.e. Tilts and Heading.

Averaging. Often the average velocity over a record of many pings is desired. The ADCP can average an arbitrary number of pings into an ensemble, and the user can average ensembles together. Care should be taken to properly handle bad data. For current profiles with bad data and variable platform velocity, it has been found advantageous to average the velocity profile relative to a reference layer with relatively few bad data points and then add the average reference layer velocity back in (see WL-command). Note: This reference layer is not related to the water-mass reference layer controlled by the BK and BL commands. *VmDas* can also process the data to average simultaneously over short and long selectable average times.

Addition of platform velocity to water velocity. To measure currents, the platform velocity must be added to the water velocity measurement, which is relative to the platform on which the instrument is mounted. If the ADCP recorded bottom track data, the platform velocity is found by negating all components of the bottom velocity. The ADCP cannot perform this step, but *VmDas* and *WinADCP* can. Alternatively, the platform velocity measurement may come from a separate navigation instrument such as GPS (Global Positioning System).

Profile inversion. The ADCP always stores the data from the depth cell closest to the transducer first. In some cases, it may be desirable to reverse the order of the depth cells, so that the farthest depth cell is first.

5.1 Depth Cell Mapping

Under conditions of severe pitch and roll, measurements from each beam at the same range may come from different depths, resulting in a smearing out (loss of spatial resolution) of the average velocity profile. If high-quality ping-synchronized pitch and roll data is available (from a vertical gyro, for

instance) and if each ping is recorded in a separate ensemble, this effect can be countered by re-mapping each beam's range cell from the actual to the nominal depth. Note that the internal tilt sensors are affected by lateral accelerations, so wave-frequency tilts may not be sufficiently well measured by them for depth cell mapping to be of advantage.

Depth cell mapping is enabled using the last bit of the EX command. When this bit is set, the ADCP implements depth cell mapping by choosing the range cell in each beam calculated to lie closest to the nominal position of the depth cell (no interpolation is done).

5.2 Screening

The ADCP performs four kinds of screening on velocity data: the correlation test, the fish rejection algorithm, the error velocity test, and the percent good test.

The correlation test compares the correlation level of each of the four beams to the threshold set by the BC or WC command. If less, that beam's velocity is marked bad. Low correlation may be caused by poor signal-to-noise ratio or by excess speed when the ambiguity velocity is low (see BZ, BM, WZ, and WM commands).

As described more fully in [“Fish Rejection Algorithm,” page 21](#), the fish rejection algorithm compares the echo intensity measurements among the beams and may mark one or all beams as bad (see also the WA command). This algorithm does not apply to bottom track velocities.

The error velocity test compares the error velocity (see [“Beam-to-Instrument Transformation,” page 11](#)) to the threshold set by the BE or WE command. If greater, all four velocities are marked bad.

If after the first three screening tests, only one beam velocity in a particular depth cell is marked bad, a three-beam solution is still possible if enabled (see [“Three-Beam Solutions,” page 14](#)). In all other cases of bad data, none of the velocity components will be included from the ping in the ensemble average for the particular range cell or bottom-track velocity.

The percent good test is enabled only for radial beam velocity output (EX00xxx command). After ensemble averaging, the percentage of good values in the average is compared to the threshold set by the BC or WC commands: If less, the velocity is marked bad. This test operates independently for each beam and for each depth cell.

5.3 Beam-to-Instrument Transformation

The vector of beam velocities is transformed to the vector of velocity components in a coordinate system fixed to the instrument (with error velocity appended) through multiplication by the instrument transformation matrix. The instrument coordinate system is described in “[Coordinate Systems](#),” [page 16](#). The instrument transformation matrix is displayed by the PS3 command. Unless the ADCP has been calibrated to compensate for small beam misalignments, the instrument transformation matrix has the following nominal value:

| component | beam 1 | beam 2 | beam 3 | beam 4 |
|-----------|--------|--------|--------|--------|
| X | c*a | -c*a | 0 | 0 |
| Y | 0 | 0 | -c*a | c*a |
| Z | b | b | b | b |
| e | d | d | -d | -d |

Where: $c = +1$ for a convex transducer head, -1 for concave
 $a = 1 / [2 \sin(\theta)] = 1.4619$ for $\theta = 20^\circ$, 1.0000 for $\theta = 30^\circ$
 $b = 1 / [4 \cos(\theta)] = 0.2660$ for $\theta = 20^\circ$, 0.2887 for $\theta = 30^\circ$
 $d = a / \sqrt{2} = 1.0337$ for $\theta = 20^\circ$, 0.7071 for $\theta = 30^\circ$

The first three rows are the generalized inverse of the beam directional matrix representing the components of each beam in the instrument coordinate system (see PS3 command for WorkHorse systems – Contact TRDI’s Field Service for other systems (see “[How to Contact Teledyne RD Instruments](#),” [page 1](#)). The last row representing the error velocity is orthogonal to the other three rows and has been normalized so that its magnitude (root-mean-square) matches the mean of the magnitudes of the first two rows. This normalization has been chosen so that in horizontally homogeneous flows, the variance of the error velocity will indicate the portion of the variance of each of the nominally-horizontal components (X and Y) attributable to instrument noise (short-term error).

The above table is equivalent to the matrix equation below:

$$\begin{bmatrix} X \\ Y \\ Z \\ e \end{bmatrix} = \begin{bmatrix} ca(b_1 - b_2) \\ ca(b_4 - b_3) \\ b(b_1 + b_2 + b_3 + b_4) \\ d(b_1 + b_2 - b_3 - b_4) \end{bmatrix}$$

Equation 9

The velocity will be left in instrument coordinates if the EX01xxx command is selected. Usually, an additional rotation is desired, which is accomplished by multiplying the instrument transformation matrix on the left by the rotation matrix M described in “[Rotation of Velocity Components to](#)

Ship or Earth Coordinates,” page 18 before using it to transform the velocity components of each depth cell and the bottom track velocity.

H-ADCP Systems

Unless the HADCP has been calibrated to compensate for small beam misalignments, the instrument transformation matrix has the following nominal value:

| component | beam 1 | beam 2 | beam 3 | beam 4 |
|-----------|--------|--------|--------|--------|
| X | -a | a | 0 | 0 |
| Y | -b | -b | -c | 0 |
| Z | 0 | 0 | 0 | 0 |
| E | e1 | e1 | -e2 | 0 |

Where a, b, c, d, e1 and e2 are defined in the table below:

| Beam Angle | a | b | c | e1 | e2 |
|------------|---------------------------|---|---------------------------------|--|--|
| θ | $\frac{1}{2 \sin \theta}$ | $\frac{\cos \theta}{1 + 2 \cos^2 \theta}$ | $\frac{1}{1 + 2 \cos^2 \theta}$ | $\frac{1}{2 \sin \theta \sqrt{1 + 2 \cos^2 \theta}}$ | $\frac{1}{\tan \theta \sqrt{1 + 2 \cos^2 \theta}}$ |
| 30deg | 1 | 0.34641 | 0.40000 | 0.63246 | 1.09545 |
| 25deg | 1.18310 | 0.34294 | 0.37839 | 0.72776 | 1.31916 |
| 20deg | 1.46190 | 0.33972 | 0.36153 | 0.87900 | 1.65198 |

The instrument transformation matrix table above is equivalent to the matrix equation below:

$$\begin{bmatrix} X \\ Y \\ e \end{bmatrix} = \begin{bmatrix} a(b_2 - b_1) \\ -b(b_1 + b_2) - cb_3 \\ e_1(b_1 + b_2) - e_2b_3 \end{bmatrix}$$

Equation 10

Error velocity results from the comparison of two independent estimates of the Y velocity component, beam 3 and the average of beams 1 and 2 projected onto the Y axis. It is scaled so that the expected variance of the error velocity is the same as that of the X velocity component when the water velocity is steady and uniform, resulting in the formula:

$$\text{Error} = e = e_1(b_1 + b_2) - e_2b_3$$

Equation 11

Channel Master Systems

Unless the Channel Master has been calibrated to compensate for small beam misalignments, the instrument transformation matrix has the following nominal value:

| component | beam 1 | beam 2 |
|-----------|--------|--------|
| X | -a | a |
| Y | -b | -b |

Where

$$a = \frac{1}{2 \sin \theta} = 1.4619022 \text{ for } \theta = 20^\circ$$

and

$$b = \frac{1}{2 \cos \theta} = 0.5320889 \text{ for } \theta = 20^\circ$$

The instrument transformation matrix table above is equivalent to the matrix equation below:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \frac{-1}{2 \sin \theta} & \frac{1}{2 \sin \theta} \\ \frac{-1}{2 \cos \theta} & \frac{-1}{2 \cos \theta} \end{bmatrix} \times \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

Equation 12



NOTE. Scatterers moving in a positive direction along the X axis (i.e. left to right when pointing away) will give a positive Doppler on beam 2 (i.e. moving toward the transducer) and a negative Doppler on beam 1 (i.e. moving away from the transducer). This applies for a default (i.e. right-bank) orientation, where the positive X direction is downstream.

5.4 Three-Beam Solutions

If exactly one beam has been marked bad in the screening step due to low correlation or fish detection, and if enabled by the EXxxx1x command, then a three-beam solution is calculated by the ADCP. This is accomplished by replacing the bad radial beam velocity with a value calculated from the last row of the instrument transformation matrix so as to force the error velocity to zero. Indeed, the actual error velocity cannot be computed in this case, because there is no longer any redundant information. The X, Y, and Z components in the instrument coordinates are then calculated in the usual way using the first three rows of the instrument transformation matrix.

For example, let's assume our standard WorkHorse ADCP has marked only Beam 4 bad.

We then need to use a three-beam solution.

As aforementioned, since we only have 3 beams available we cannot compute the error velocity as we do not have redundancy in the data anymore. Therefore, we can zero the error velocity:

$$e = 0$$

Equation 13

Now if we replace the error velocity in the last row of the transformation matrix (1) (see Equation 9, page 11) described in previous chapter, we obtain:

$$b_4 = b_1 + b_2 - b_3$$

Equation 14

Now that we have expressed the bad beam in function of the other beams themselves expressed as a function of Beam 4, let's replace Beam 4 expression above (5) in the transformation matrix (see Equation 9, page 11) described in previous chapter. We then obtain the three beam solution transformation matrix for a WorkHorse ADCP with Beam 4 bad:

$$\begin{bmatrix} X \\ Y \\ Z \\ e \end{bmatrix} = \begin{bmatrix} ca(b_1 - b_2) \\ ca(b_1 + b_2 - 2b_3) \\ 2b(b_1 + b_2) \\ 0 \end{bmatrix}$$

Equation 15

H-ADCP Systems – Two Beams Solution:

As the HADCP measures velocity in the XY-plane using three beam measurements, we can still provide a two dimensional measurement of the velocity if exactly one beam has been marked bad in the screening step due to low correlation or fish detection, and if enabled by the EXxxx1x command. Then, a two-beam solution is calculated by the HADCP.

The actual error velocity cannot be computed in this case, because there is no longer any redundant information in the data.

The method is similar to the three beam solution described above:

- Step1: Zero the error velocity
- Step2: Replace error velocity in the HADCP transformation matrix (see Equation 10, page 12).
- Step3: Use the new equation resulting from Step 2 to express the marked bad beam as a function of the others
- Step4: Replace the marked bad beam by the new equation resulting from Step 3 in the remaining Beams equations in the HADCP transformation matrix (see Equation 10, page 12).

For example:

If Beam 1 only is marked bad the resulting two-beam solution will result in the following transformation matrix:

$$\begin{bmatrix} X \\ Y \\ e \end{bmatrix} = \begin{bmatrix} (2a)b_2 - \left(\frac{ae_2}{e_1}\right)b_3 \\ -b_3 \\ 0 \end{bmatrix}$$

Equation 16

If Beam 3 only is marked bad the resulting two-beam solution will result in the following transformation matrix:

$$\begin{bmatrix} X \\ Y \\ e \end{bmatrix} = \begin{bmatrix} a(b_2 - b_1) \\ \left(-\frac{e_1}{e_2}\right)(b_1 + b_2) \\ 0 \end{bmatrix}$$

Equation 17

5.5 Coordinate Systems

The ADCP can produce velocity measurements in any of the following four sets of coordinate axes. Except for the first, they are all right-handed orthogonal systems.

Radial Beam Coordinates. (BM1, BM2, BM3, BM4) Radial Beam Coordinates are selected by the EX00xxx command. These are the “raw” velocity measurements measured independently by each transducer, in units of millimeters per second. The sense is positive when the motion is towards the transducer. These axes are not orthogonal.

Instrument Coordinates. (X, Y, Z) Instrument Coordinates are selected by the EX01xxx command. This set of axes is always oriented the same relative to the transducer head. When you look at the face of the transducer head, the transducers are labeled clockwise in the order 3-1-4-2 (see [Figure 3, page 17](#)). The X-axis lies in the direction from transducer 1 towards transducer 2 and the Y-axis lies in the direction from transducer 4 towards transducer 3. The Z-axis lies along the axis of symmetry of the four beams, pointing away from the water towards the pressure case. The internal compass is mounted so that when the X-Y plane is level, the compass measures the orientation of the Y-axis relative to magnetic north.

Ship Coordinates (or Righted Instrument Coordinates). (S, F, M) Ship Coordinates are selected by the EX10xxx command. We use the names starboard, forward, and mast, although these axes are more commonly called the pitch, roll, and yaw axes, respectively. Assuming that Beam 3 is aligned with the keel on the forward side of the ADCP, for the downward-looking orientation, these axes are identical to the instrument axes:

$$S = X, F = Y, M = Z$$

For the upward-looking orientation, these axes are rotated 180° about the Y-axis:

$$S = -X, F = Y, M = -Z$$

The M (mast) axis points in a direction that is closer to upward than downward.

The importance of the ship axes is that the attitude angles (pitch, roll, and heading) measure the orientation of the ship axes relative to the earth axes, regardless of up/down orientation. The sense of internal sensors Tilt 1 (pitch) and Tilt 2 (roll) is positive for counterclockwise tilts respectively about the S and F axes, using the right-hand rule ([Figure 3, page 17](#)).

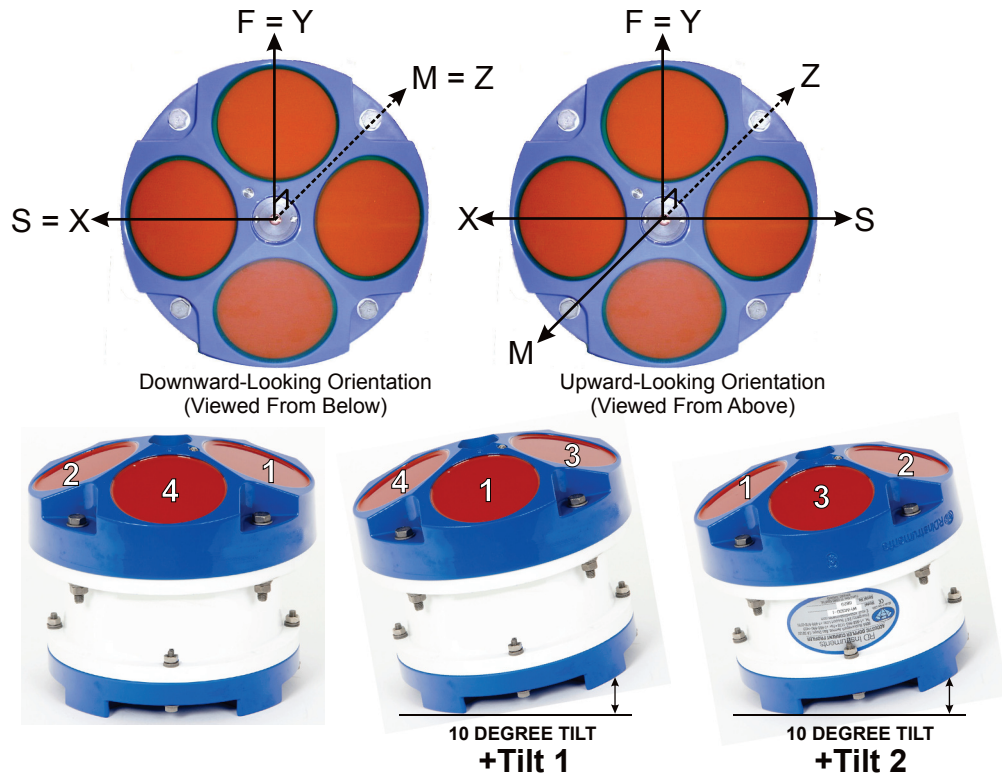


Figure 3. WorkHorse Transducer Beam Axes and Tilt Signs

If Beam 3 is not aligned with the keel, then we must distinguish the attitude angles of the ADCP from those of the ship, so we need two different coordinate systems. In this special case, it makes sense to use the term “righted instrument axes” to describe the ADCP orientation, leaving the term “ship axes” to describe the orientation of the ship itself. The EA-command can be used to compensate the misalignment in some cases (see end of “Rotation of Velocity Components to Ship or Earth Coordinates,” page 18).

The velocities can be resolved in a leveled version of the ship coordinates, sometimes called “horizontal” coordinates (compensating for pitch and roll), by using the EX101xx command.

For the HADCP, the Y-axis is pointing away from Beam 3, and the X-axis is from Beam 2 to Beam 1 as shown in the figure below:

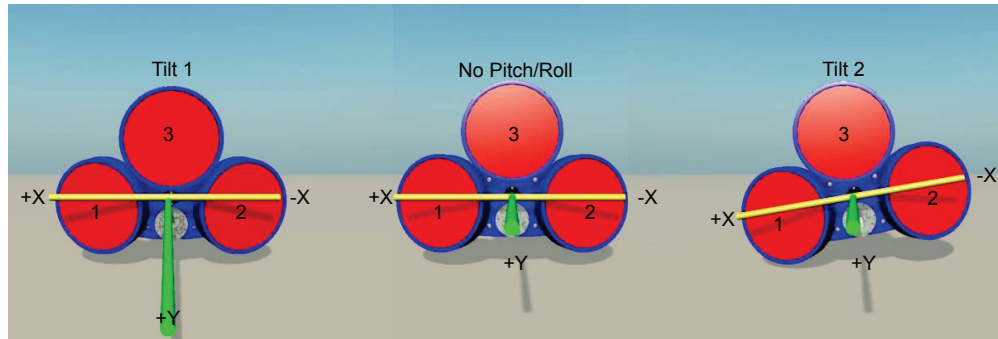


Figure 4. Horizontal ADCP Beam axes

Pitch (Tilt 1) is measured around the X-axis.

Roll (Tilt 2) is measured around the Y-axis.

Earth Axes, also known as Geographic or Geodetic Coordinates. (E, N, U) Earth Axes are selected with command EX11xxx. These axes are named east, north, and up. Strictly speaking, these terms refer to true orientations, although magnetic orientations are often used instead. This is the most commonly used coordinate system because it provides a stable reference frame for ensemble averaging.

5.6 Rotation of Velocity Components to Ship or Earth Coordinates

Unless the first two bits of the EX command are 00 (beam coordinates) or 01 (instrument coordinates), another transformation is needed to rotate the velocities from instrument coordinates to the desired coordinates. This transformation can be considered to be the net result of three successive rotations in this order:

- R about the F axis,
- P about the leveled S axis, and then
- H about the U axis,

Where R, P, and H are the roll, pitch, and heading angles as traditionally defined so as to represent the orientation of a set of gimbals.

The rotation matrix M is thus:

$$M = \begin{bmatrix} CH & SH & 0 \\ -SH & CH & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & CP & -SP \\ 0 & SP & CP \end{bmatrix} \begin{bmatrix} CR & 0 & SR \\ 0 & 1 & 0 \\ -SR & 0 & CR \end{bmatrix}$$

$$= \begin{bmatrix} (CH * CR + SH * SP * SR) & (SH * CP) & (CH * SR - SH * SP * CR) \\ (-SH * CR + CH * SP * SR) & (CH * CP) & (-SH * SR - CH * SP * CR) \\ (-CP * SR) & (SP) & (CP * CR) \end{bmatrix}$$

Equation 18

Where we have used the abbreviations CH = cos(H), SH = sin(H), CP = cos(P), SP=sin(P), CR=cos(R), and SR = sin(R).

Use the EZ command to determine which source the ADCP uses for the measured attitude angles. The pitch, roll, and heading angles to be used in the rotation matrix differ somewhat from the sensor measurements as follows:

Since the roll describes the ship axes rather than the instrument axes, in the case of upward-looking orientation, 180 degrees must be added to the measured roll before it is used to calculate M. This is equivalent to negating the first and third columns of M. R is set to 0 if the “use tilt” bit of the EX command is not set.

The internal tilt sensors do not measure exactly the same pitch as a set of gimbals would (the roll is the same). Only in the case of the internal pitch sensor being selected (EZxxx1xxx), the measured pitch is modified using the following algorithm.

$$P = \arctan[\tan(\text{Tilt1}) * \cos(\text{Tilt2})]$$

Equation 19

Where: Tilt1 is the measured pitch from the internal sensor, and
Tilt2 is the measured roll from the internal sensor

The raw pitch (Tilt 1) is recorded in the variable leader. P is set to 0 if the “use tilt” bit of the EX command is not set.

The measured heading is modified by the values EA and EB set by the EA and EB commands as shown in the WorkHorse Commands and Output Data Format guide.

H' is the raw heading

H' + EB is the true heading recorded in the variable leader

H = H' + EB + EA is the heading used in M (for earth coordinates)

H = EA is used in M to give ship coordinates (EX10xxx command)

The EA command allows the ship's true heading to be recorded in the variable leader even when the Y-axis of the ADCP is not aligned to the ship's keel. However, this only works for internal pitch and roll sensors. The ADCP does not provide for misalignment between the ADCP and the axes of an external gyro, except in the case where the external gyro provides only heading information (see EA command). In general, an additional rotation is needed from the instrument axes to the axes of the external gyro (usually the ship axes), as described by three alignment angles, before the attitude angles from the gyro can be used to further rotate the velocity to earth coordinates.

6 Water-Mass Layer Processing

Refer to [Figure 5A](#). Using the BL-command, the user enters the water-mass layer boundaries in decimeters (meters x 10) as a NEAR WATER-MASS LAYER DEPTH and a FAR WATER-MASS LAYER DEPTH. User also enters the minimum size for the water-mass layer as MINIMUM WATER-MASS LAYER SIZE in decimeters. The BK-command controls the conditions under which a speed log reference-layer ping will be sent.

The user-defined water-mass layer is used unless the layer comes within 15% of the water boundary (sea floor for down-looking systems; surface for up-looking systems). As the user-defined water-mass layer comes within 15% of the boundary ([Figure 5B](#)), the layer compresses in size until the MINIMUM WATER-MASS LAYER SIZE is reached. At that point ([Figure 5C](#)), the water-mass layer retains the MINIMUM WATER-MASS LAYER SIZE and begins to move its NEAR/FAR WATER-MASS LAYER DEPTH toward the transducer face until it can go no further.

If the boundary begins to move away from the transducers, this process reverses until the original user-defined settings are reached (e.g., [Figure 5C](#) to [B](#) to [A](#)).

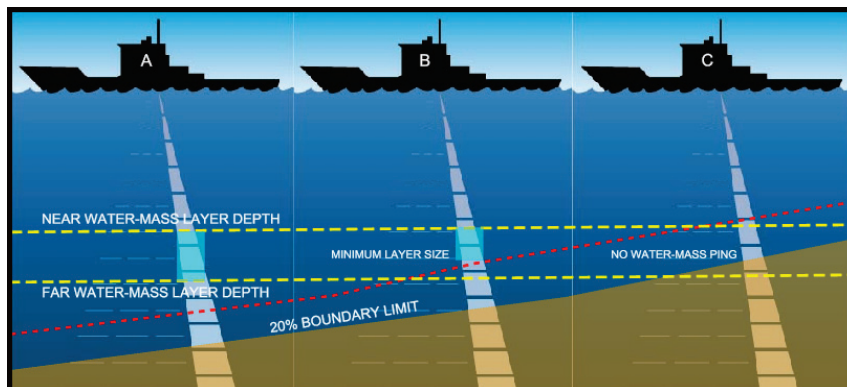


Figure 5. Water-Mass Layer Processing

7 Fish Rejection Algorithm

This section explains how the WA-command reduces the effects of fish swimming within the ADCP transducer beams.

7.1 History Behind the WA Command

ADCPs typically have four beams mounted to one transducer head in a JANUS configuration. The beams are mounted at 30° (sometimes 20°) to the horizontal. Because of this configuration, it is possible that an ADCP will “see” fish within one or more beams when profiling.

All four ADCP beams have side lobes. Each side lobe is shaped like a cone (see [Figure 6, page 23](#)). Part of this cone looks down a neighboring beam’s main lobe. Therefore, when acoustic energy is propagating back to the ADCP along a beam’s main lobe, part of this energy is seen by the other beams’ side lobes. The amount of energy seen depends on the relative amplitude of the side lobe and the echo intensity of the surrounding water.

When only one beam sees a fish, we can expect to see a *larger* error velocity in the affected depth cell. This is true as long as the echo intensity of the fish is not much greater than that of the surrounding water. When the fish echo is much greater than the surrounding water, the strong echo from the “fishy” beam is heard by the two neighboring beams as well as the opposite beam. This causes the ADCP to measure about the SAME velocity in all four beams, which gives a low error velocity.

For buoy-mounted ADCPs, fish tend to “school” at some depth below the ADCP. In this situation, the fish seem to remain “still” relative to the ADCP. If an actual water current is present near the fish, the fish velocity (in this case, a near-zero velocity) will dominate the Doppler measurement. Another situation occurs when the fish are not in all four beams. This is a non-homogeneous situation that can cause a measurement error.

On vessel-mounted ADCPs, a forward-facing beam hears the Doppler of the vessel to which it is attached. A strong echo from a fish causes the other three beams to hear the same Doppler, and the ADCP outputs an apparent vertical velocity. If a side-facing (port/starboard) beam hears a strong fish echo, the ADCP outputs a low velocity measurement (no apparent vertical or error velocity).

7.2 Method to Reduce Fish Effects

The echo intensity measurement from each beam is a good indicator of the relative strength of the returning acoustic energy. If we knew what the normal echo strength was for a given depth cell, we could detect fish in the profile data. For a large set of single-ping profile data, we could observe

these typical echo intensity values, and then go back through the profile data to remove the “fishy-looking” depth cells.

ADCPs typically sum together many pings of data before storing or transmitting the average. To screen-out fish from the data before accumulation, the ADCP could compare the echo data from all four beams. If the four echo intensities do not match on a given depth cell, the ADCP could reject the data. Indeed, an exact match will never occur, so we must allow some acceptable difference.

Each beam has a quiescent value that is an offset relative to zero. This value is typically 30-40 counts (one count is about 0.5 dB). The largest side lobe is down 35 dB (70 counts) or more from the beam's main lobe. For each depth cell of data, we can define an algorithm to remove “fishy” data by repeating the following cycle for each depth cell of data. Let’s assume we start with depth cell $x = 1$ and $WA = 50$ counts. NOTE: We could probably use a count value of 40 and still not throw out too much “good” data.

Algorithm

Compare depth cell x from each beam to find the highest and the two lowest depth cell x echo levels.

If the difference between the highest and the lowest depth cell x value is more than 50 counts, mark the velocity in the beam with the lowest depth cell x value as bad. In effect, we are now using a 3-beam solution for the depth cell x layer.

If the difference between the highest and second lowest depth cell x value is also more than 50 counts, mark the velocities in depth cell x of all four beams as bad.

If the above step was true (a fish detected), also mark depth cell $x+1$ velocities as bad. This is because the ADCP samples echo intensity near the end of the depth cell.

Increment x and repeat for the next depth cell until all depth cells have been tested for the current ping.

As an example, use the Algorithm and [Figure 6, page 23](#) to determine the outcome of depth cell 13 when $WA = 50$ counts.

This algorithm properly accounts for the failure of one beam. (Remember, the ADCP needs only three beams to compute velocity in three planes. The ADCP uses the fourth beam as a data-quality check.) If the ADCP is deployed for an extended time, there is a chance a beam could fail.

Using the Algorithm and the data supplied for Depth cell 13 in [Figure 6, page 23](#) we have:

| STEP | CASE 1 | CASE 2 |
|------|--------|--------|
|------|--------|--------|

| STEP | CASE 1 | CASE 2 |
|--------|--|---|
| A | Highest = Beam 4 (81) 2 ND Low = Beam 1 (39) Lowest = Beam 2 (30) | Highest = Beam 4 (81) 2 ND Low = Beam 2 (30) Lowest = Beam 1 (29) |
| B | Is 81 (Bm 4) - 30 (Bm 2) > 50? Yes (51), so Beam 2 is bad at this point. | Is 81 (Bm 4) - 29 (Bm 1) > 50? Yes (52), so Beam 1 is bad at this point. |
| C | Is 81 (Bm 4) - 39 (Bm 1) > 50? No (42), so Step B applies. That is, the ADCP marks the velocities in Beam 2 velocities as bad and uses a 3-beam solution for Depth cell 13. | Is 81 (Bm 4) - 30 (Bm 2) > 50? Yes (51), so Step B no longer applies. The ADCP marks the velocities in Depth cell 13 of all four beams as bad. |
| D | Since Step C was false, this step is not applicable | Since Step C was true, the ADCP also marks the velocities in Depth cell 14 of all four beams as bad. |
| Result | Throw out beam 2 data and do a 3-beam solution. | Throw out all beam data. Depth cells 13 and 14 are bad. |

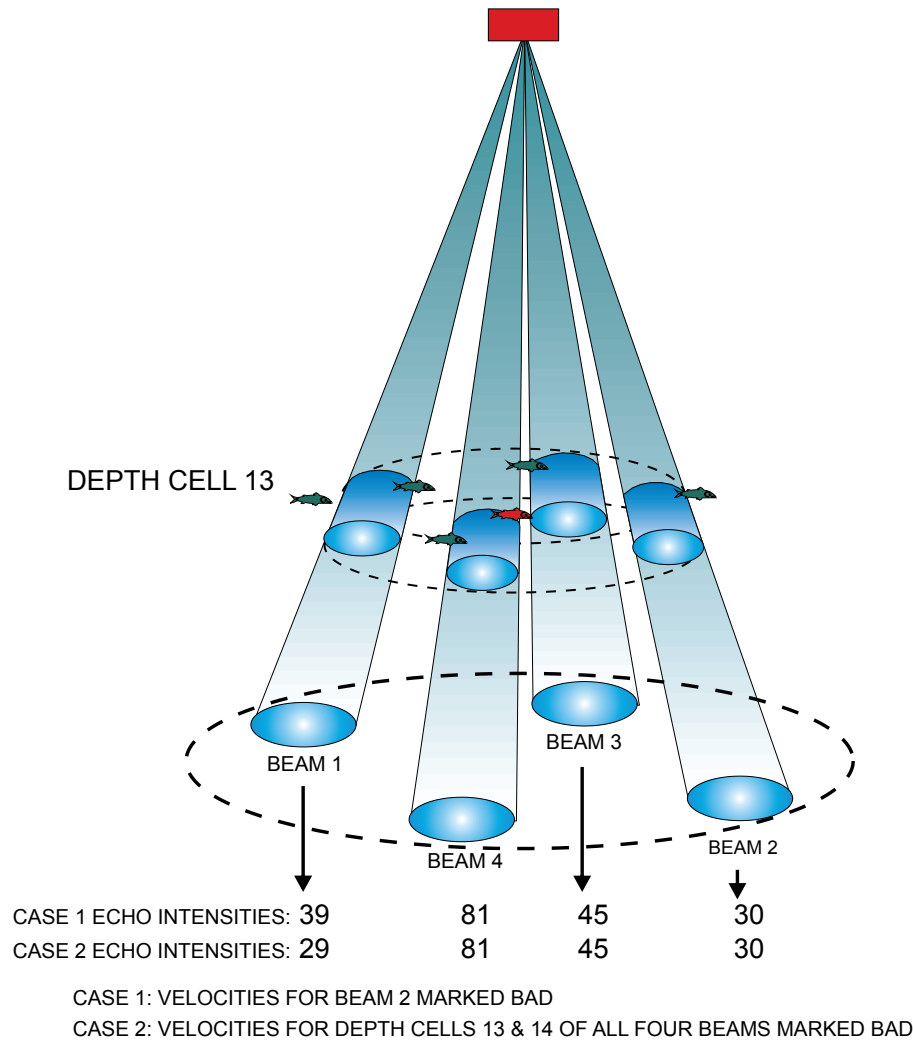


Figure 6. Example of Fish Rejection Algorithm

8 Standard Deviation

One limitation of the ADCP is the velocity data inaccuracy caused by internal and external biasing errors and random errors. The ADCP measures a 128-segment profile of water currents in a time duration equal to the two-way propagation time to the maximum range of interest. This “single-ping” time interval is typically only a fraction of a second. Each single-ping depth cell velocity measurement is characterized by a long-term bias error and a short-term random error. Programming the ADCP to average the results of multiple pings can reduce the short-term random error, but because the long-term bias is about 0.2 cm/s, there is little benefit to reducing standard deviation below this level. An easy way to adjust standard deviation to the desired value is to increase or decrease the pings per ensemble setting (WP command).

Modeling the horizontal standard deviation of velocity is a complicated process that is beyond the scope of this manual. However, [Figure 7, page 25](#) through [Figure 12, page 30](#) show the single-ping standard deviation for all ADCP frequencies based on depth cell size, profiling range, and the velocity of the ADCP relative to the water current. Our *PlanADCP* program can predict the horizontal standard deviation of water profile velocity automatically. If you need detailed information about determining standard deviation, call Teledyne RD Instruments (see [“How to Contact Teledyne RD Instruments,” page 1](#)).

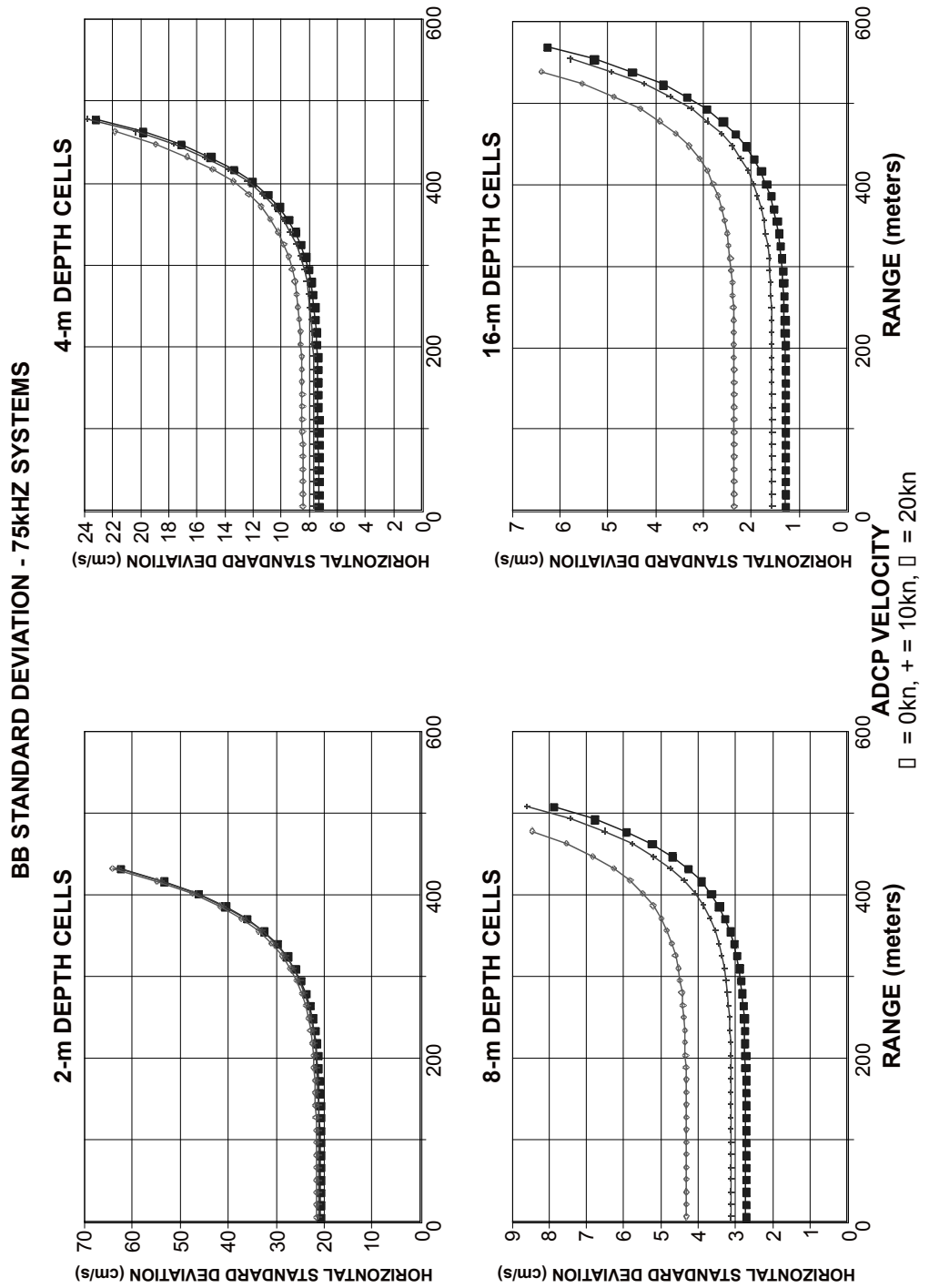


Figure 7. Single-Ping Standard Deviation Charts for 75-kHz Systems

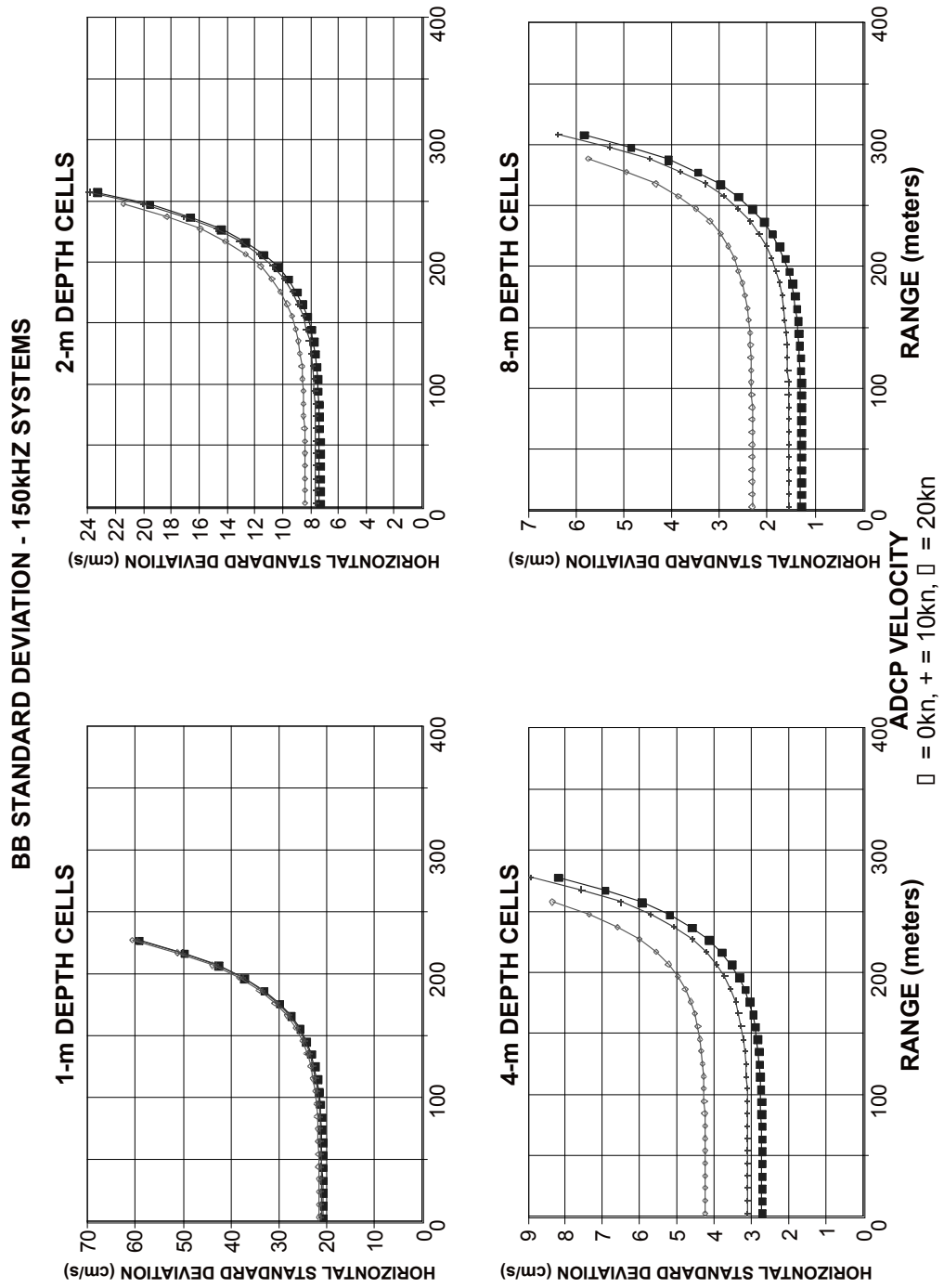


Figure 8. Single-Ping Standard Deviation Charts for 150-kHz Systems

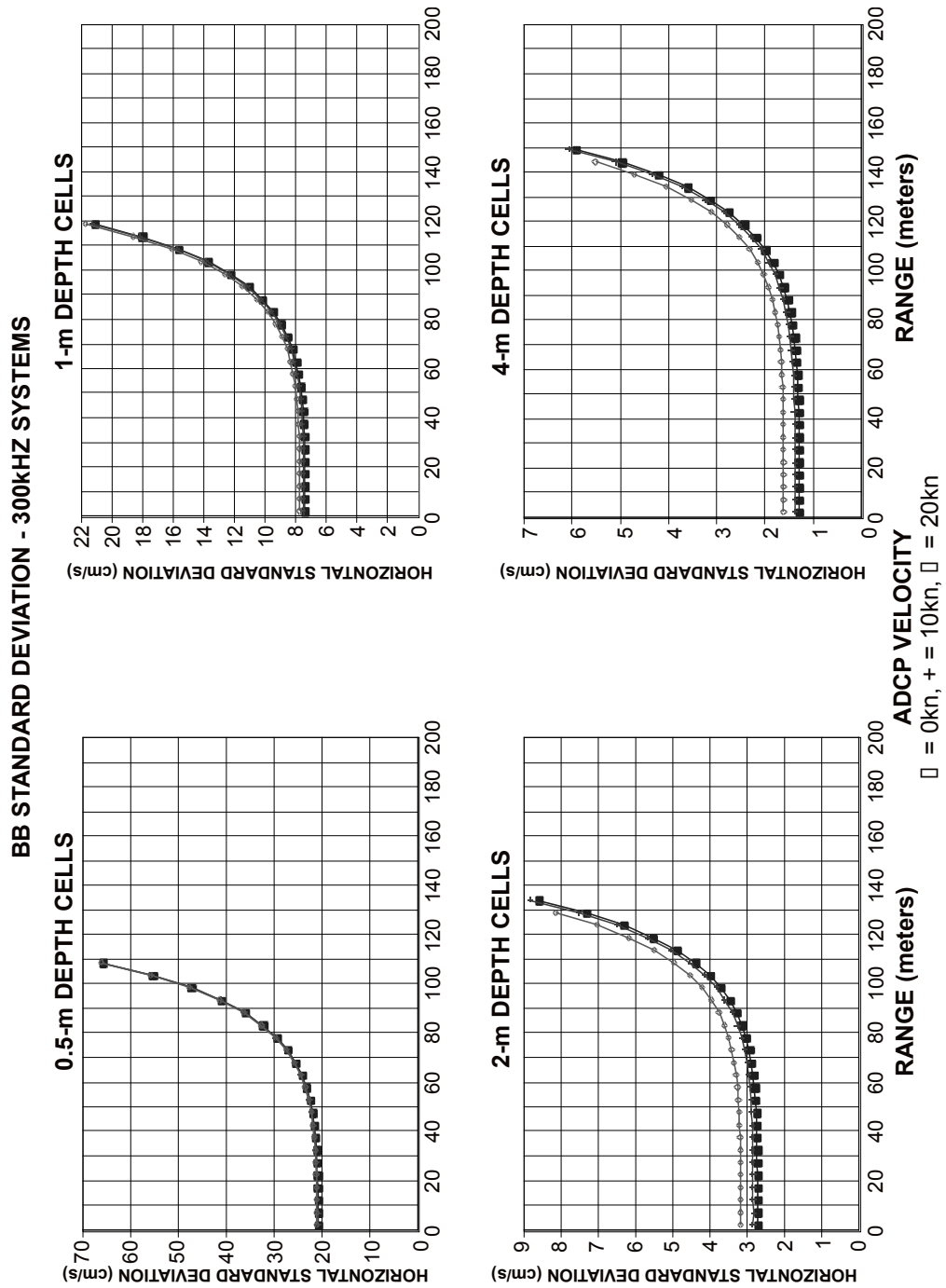


Figure 9. Single-Ping Standard Deviation Charts for 300-kHz Systems

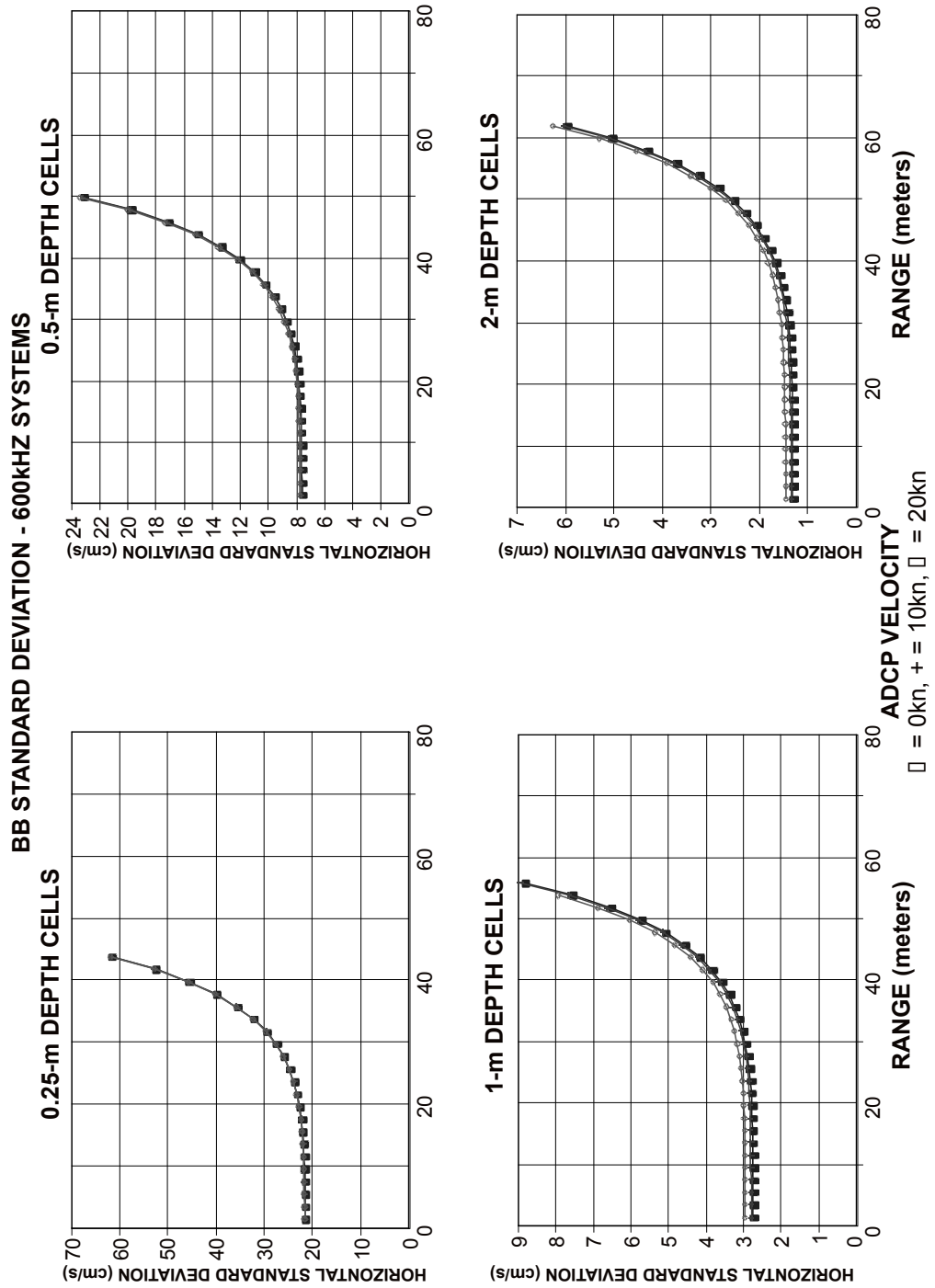


Figure 10. Single-Ping Standard Deviation Charts for 600-kHz Systems

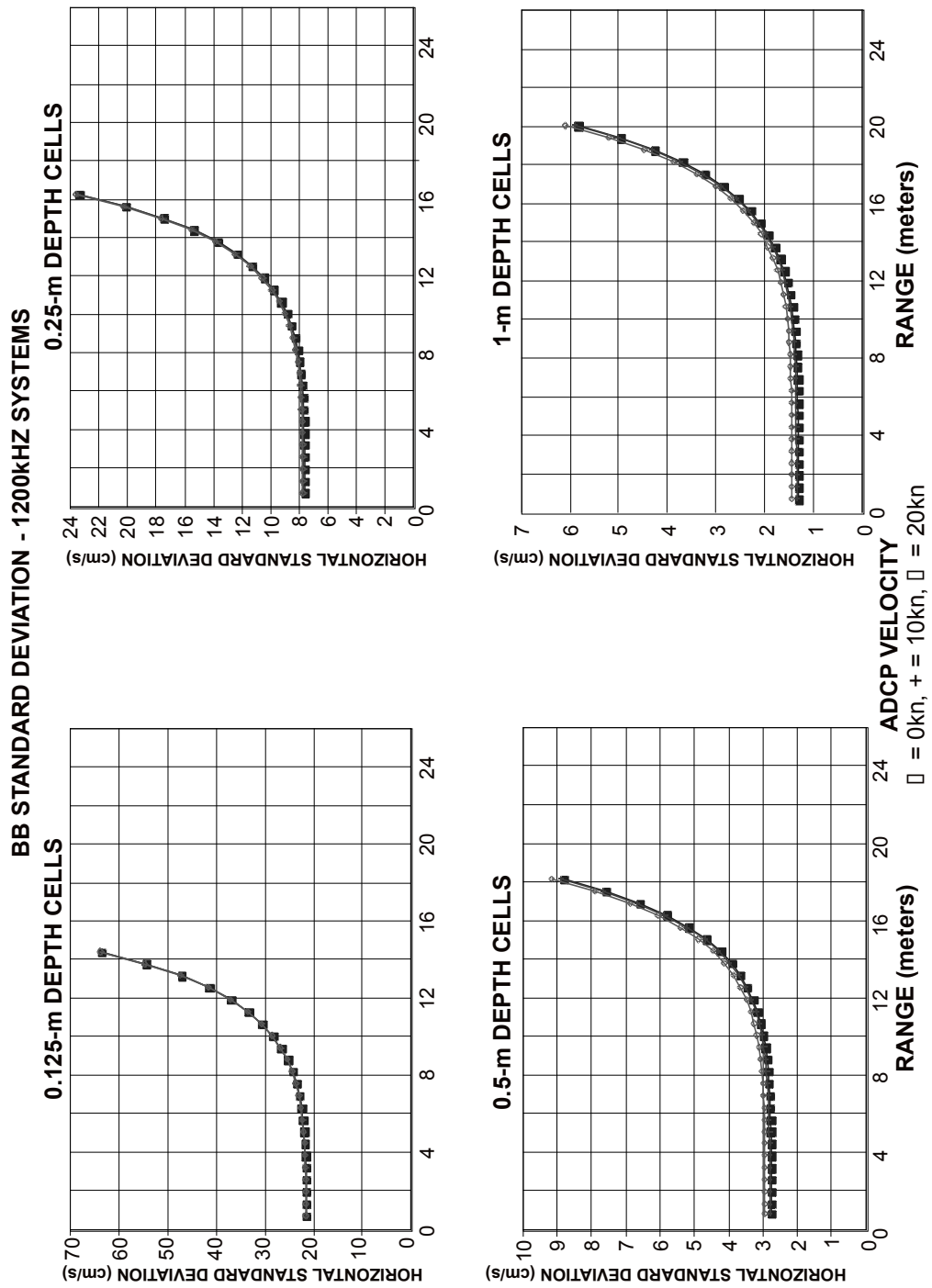


Figure 11. Single-Ping Standard Deviation Charts for 1200-kHz Systems

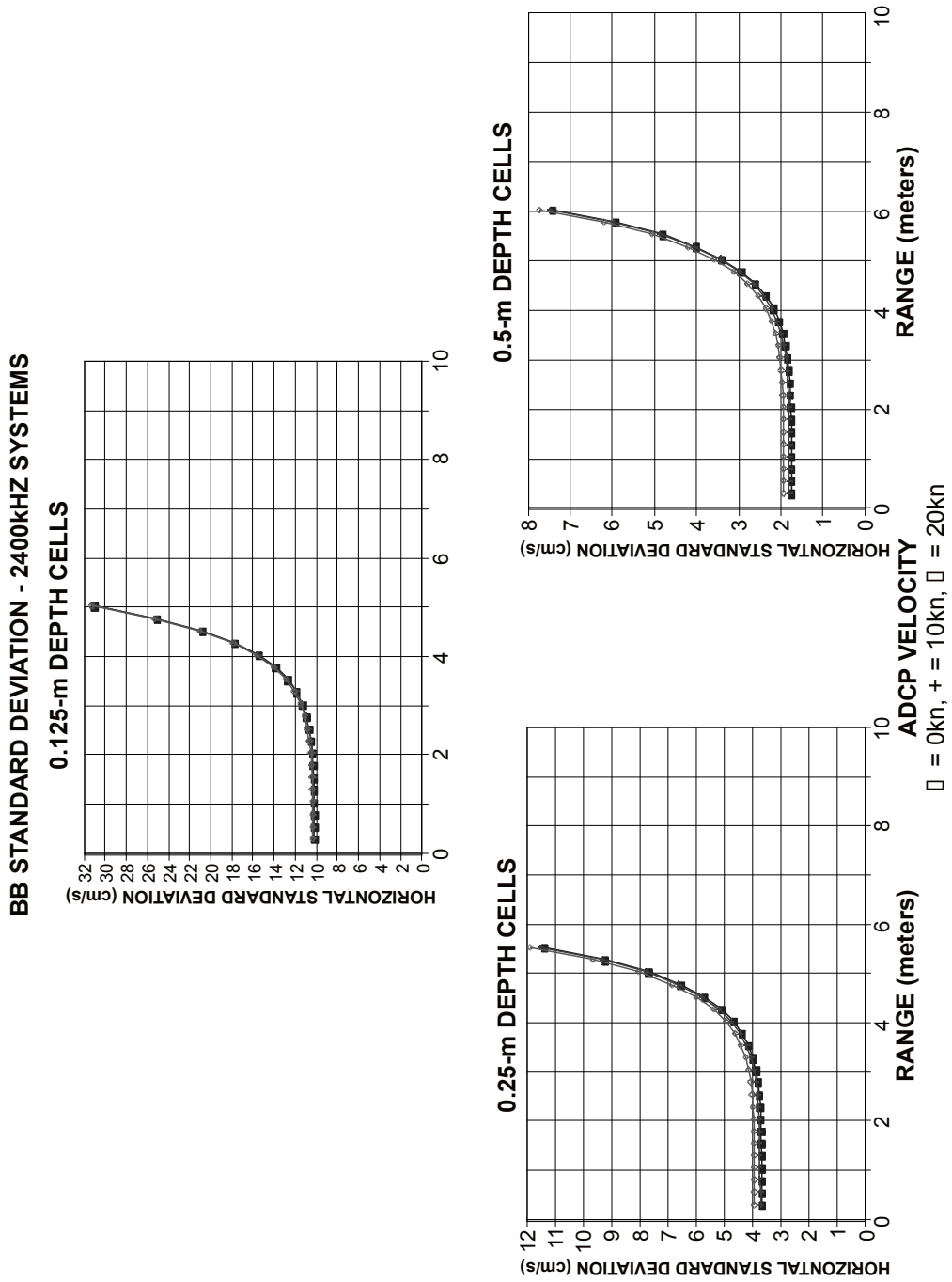


Figure 12. Single-Ping Standard Deviation Charts for 2400-kHz Systems

NOTES

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