## APPENDIX C CALCULATIONS AND FORMULAS

## C-O. INTRODUCTION

This appendix defines various calculations and formulas used by the ADCP. The ADCP measures range-segmented, horizontal profiles of water-mass velocity, spectral width, and echo intensity. This appendix describes measurement and data processing considerations for these parameters. For more information on ADCP calculations, see RDI's Principles of Operation: A Practical Primer.

## C-1. WATER-MASS VELOCITY - RADIAL VELOCITY DEFINITION

The ADCP determines water-mass velocity by measuring the Doppler frequency shift (first moment) of the backscattered water-mass echo. When you select beam-coordinate output velocities (0-command), the ADCP supplies the measured Doppler frequency shift as output for each depth cell (bin) and each beam. We can compute the component of relative flow velocity ( $V$, in $\mathrm{cm} / \mathrm{s}$ ) in the direction of each acoustic beam from this measured Doppler frequency shift ( $F_{D}$ ) for each depth cell using the equation:

$$
V=F_{D} \frac{C}{2 F_{S}}
$$

where: $\quad F_{S}=$ Transmitted acoustic frequency (76.8, 153.6, 307.2, 614.4, or 1228.8 kHz , dependent on ADCP model 75, $150,300,600$, or 1200 kHz respectively).
$C=$ Velocity of sound in water at the transducer face (in $\mathrm{cm} / \mathrm{s}$ ) calculated from the expression:
$C=1449.2+4.6 \mathrm{~T}-0.055 \mathrm{~T}^{2}+0.00029 \mathrm{~T}^{3}+$ $(1.34-0.01 T)(S-35)+0.016 D$
where: $\quad D=$ Depth, in meters
$S=$ Salinity, in parts per thousand
$\mathrm{T}=$ Temperature, in ${ }^{\circ} \mathrm{C}$
NOTE: The ADCP measures water temperature at its transducer face. You can calculate $C$ from this measured water temperature using nominal values for depth and salinity. Table C-1 lists several values for $C$ (in $\mathrm{cm} / \mathrm{s}$ ) for various temperature and salinity readings at shallow depths.

Table C-1. Speed of Sound versus Temperature and Salinity

| Temp.$\left({ }^{\circ} \mathrm{C}\right)$ | Speed of Sound ( $\mathrm{cm} / \mathrm{s}$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 20 | $\begin{gathered} \text { Salin } \\ 25 \end{gathered}$ | $\begin{gathered} \text { (ppt) } \\ 30 \end{gathered}$ | 35 | 40 |
| 0 | 140230 | 142910 | 143580 | 144250 | 144920 | 145590 |
| 1 | 140720 | 143380 | 144045 | 144710 | 145375 | 146040 |
| 2 | 141198 | 143838 | 144498 | 145158 | 145818 | 146478 |
| 3 | 141666 | 144286 | 144941 | 145596 | 146251 | 146906 |
| 4 | 142124 | 144724 | 145374 | 146024 | 146674 | 147324 |
| 5 | 142571 | 145151 | 145796 | 146441 | 147086 | 147731 |
| 6 | 143008 | 145568 | 146208 | 146848 | 147488 | 148128 |
| 7 | 143435 | 145975 | 146610 | 147245 | 147880 | 148515 |
| 8 | 143853 | 146373 | 147003 | 147633 | 148263 | 148893 |
| 9 | 144261 | 146761 | 147386 | 148011 | 148636 | 149261 |
| 10 | 144659 | 147139 | 147759 | 148379 | 148999 | 149619 |
| 11 | 145048 | 147508 | 148123 | 148738 | 149353 | 149968 |
| 12 | 145428 | 147868 | 148478 | 149088 | 149698 | 150308 |
| 13 | 145799 | 148219 | 148824 | 149429 | 150034 | 150639 |
| 14 | 146162 | 148562 | 149162 | 149762 | 150362 | 150962 |
| 15 | 146515 | 148895 | 149490 | 150085 | 150680 | 151275 |
| 16 | 146861 | 149221 | 149811 | 150401 | 150991 | 151581 |
| 17 | 147198 | 149538 | 150123 | 150708 | 151293 | 151878 |
| 18 | 147527 | 149847 | 150427 | 151007 | 151587 | 152167 |
| 19 | 147848 | 150148 | 150723 | 151298 | 151873 | 152448 |
| 20 | 148162 | 150442 | 151012 | 151582 | 152152 | 152722 |
| 21 | 148468 | 150728 | 151293 | 151858 | 152423 | 152988 |
| 22 | 148767 | 151007 | 151567 | 152127 | 152687 | 153247 |
| 23 | 149058 | 151278 | 151833 | 152388 | 152943 | 153498 |
| 24 | 149343 | 151543 | 152093 | 152643 | 153193 | 153743 |
| 25 | 149621 | 151801 | 152346 | 152891 | 153436 | 153981 |
| 26 | 149892 | 152052 | 152592 | 153132 | 153672 | 154212 |
| 27 | 150156 | 152296 | 152831 | 153366 | 153901 | 154436 |
| 28 | 150415 | 152535 | 153065 | 153595 | 154125 | 154655 |
| 29 | 150667 | 152767 | 153292 | 153817 | 154342 | 154867 |
| 30 | 150913 | 152993 | 153513 | 154033 | 154553 | 155073 |
| 31 | 151153 | 153213 | 153728 | 154243 | 154758 | 155273 |
| 32 | 151388 | 153428 | 153938 | 154448 | 154958 | 155468 |
| 33 | 151618 | 153638 | 154143 | 154648 | 155153 | 155658 |
| 34 | 151842 | 153842 | 154342 | 154842 | 155342 | 155842 |
| 35 | 152061 | 154041 | 154536 | 155031 | 155526 | 156021 |
| 36 | 152275 | 154235 | 154725 | 155215 | 155705 | 156195 |
| 37 | 152484 | 154424 | 154909 | 155394 | 155879 | 156364 |
| 38 | 152689 | 154609 | 155089 | 155569 | 156049 | 156529 |
| 39 | 152890 | 154790 | 155265 | 155740 | 156215 | 156690 |
| 40 | 153086 | 154966 | 155436 | 155906 | 156376 | 156846 |
| 41 | 153278 | 155138 | 155603 | 156068 | 156533 | 156998 |
| 42 | 153467 | 155307 | 155767 | 156227 | 156687 | 157147 |
| 43 | 153651 | 155471 | 155926 | 156381 | 156836 | 157291 |
| 44 | 153832 | 155632 | 156082 | 156532 | 156982 | 157432 |
| 45 | 154010 | 155790 | 156235 | 156680 | 157125 | 157570 |

## C-2. STANDARD DEVIATION FORMULA

This section presents the formula used to estimate the standard deviation (accuracy) of ADCP velocity calculations.

The ADCP measures a complete 128-segment profile of water currents in a time duration equal to the two-way propagation time to the maximum range of interest. This "single-ping" interval is typically only a fraction of a second. Each single-ping, depth cell velocity measurement has a long-term bias error and a short-term random error. You can reduce the short-term random error by programming the ADCP to average the results of multiple pings. The size of this random error is related to acoustic frequency, transmit pulse length, and number of pings per ensemble. We calculate the short-term velocity measurement precision for a four-beam $A D C P$ as follows.

$$
\text { Standard Deviation }(\mathrm{cm} / \mathrm{s})=\frac{1.6 \times 10^{7}}{\mathrm{~F} \times \mathrm{I} \times \mathrm{B} \times \sqrt{\mathrm{P}}}
$$

Where: $\quad F=A D C P$ acoustic frequency in Hz
$I=$ Transmit pulse length in meters
$B=$ Beam angle coefficient ( 1 for $30^{\circ}$; 0.684 for $20^{\circ}$ )
$P=$ Number of pings per ensemble
For example:
A $150-\mathrm{kHz}$ ADCP (actual acoustic frequency $=153,600 \mathrm{~Hz}$ ) is used with a transmit pulse length of 8 meters and a pings per ensemble value of 170. What is the standard deviation in $\mathrm{cm} / \mathrm{s}$ ?

For a $30^{\circ}$ beam angle (most common) :
$\frac{1.6 \times 10^{7}}{\mathrm{~F} \times \mathrm{I} \times \mathrm{B} \times \sqrt{\mathrm{P}}}=\frac{16,000,000}{153,600 \times 8 \times 1 \times \sqrt{170}} \approx 1.0 \mathrm{~cm} / \mathrm{s}$

For a $20^{\circ}$ beam angle:

$$
\frac{1.6 \times 10^{7}}{\mathrm{~F} \times \mathrm{I} \times \mathrm{B} \times \sqrt{\mathrm{P}}}=\frac{16,000,000}{153,600 \times 8 \times 0.684 \times \sqrt{170}} \approx 1.46 \mathrm{~cm} / \mathrm{s}
$$

## C-3. VELOCITY DATA PROCESSING

This section defines the ADCP coordinate systems, presents the velocity processing algorithm, and outlines the algorithm's derivation. When the ADCP is in the radial beam-coordinate velocity output mode (set by 0 -command), the raw ADCP velocity data is a measurement of the four-beam, radial velocity components. To convert this Doppler frequency data from the ADCP's transducercoordinate system to earth-referenced east, north, and vertical velocity profiles, we must:
a. Compensate for ADCP pitch and roll to map the tilted, slant-beam Doppler frequency data into cells at the same depth.
b. Compensate for speed of sound variations.
c. Calculate the three orthogonal velocity components for each depth cell in the ADCP coordinates.
d. Check the reasonableness of depth cell velocity components for each depth cell in the ADCP coordinates.
e. Resolve the $A D C P$ velocity components for each depth cell into east, north, and vertical components.

C-3.1. COORDINATE SYSTEMS. We define three coordinate systems for the ADCP earth, horizontal, and instrument.

Earth-coordinate System. This system has its $+\mathrm{X} E$ axis pointing east, $+\mathrm{Y} E$ axis pointing north, and $+Z E$ axis pointing up. The $E$ indicates an earth-coordinate system component. This is a right-handed coordinate system. The ADCP collects data in the instrument-coordinate system, but can convert the collected data to earth coordinates. This is explained below in the description of the instrument-coordinate system.

Horizontal-coordinate System. This system differs from the earth-coordinate system by a rotation of HH (heading) degrees about the ZE-axis. This system has the components $\mathrm{XH}, \mathrm{YH}$, and ZH .

Instrument-coordinate System. We define the instrument-coordinate system such that without any roll, pitch, or heading change, the instrument's $\mathrm{X}, \mathrm{Y}$, and Z components are the same as XE, YE, and ZE in the earth-coordinate system. See Figure C-1. The ADCP's X and Y planes pass through the centers of the four transducers. The X -axis runs through transducer beams 1 and 2; the Y-axis runs through beams 3 and 4. When you look at the faces of the transducer head, the transducers are clockwise in the order 3-1-4-2. This means that for upward-facing ADCPs beam 1 is in the +X direction, and beam 3 is in the +Y direction. For downward-facing ADCPs, beam 2 is in the $+X$ direction, and beam 3 remains in the $+Y$ direction. The directions fore, aft, port, and starboard refer to the instrument-coordinate system and are in the $+\mathrm{Y},-\mathrm{Y},-\mathrm{X}$, and +X directions, respectively.

The ADCP collects data in the instrument-coordinate system, but can convert the data to earth coordinates. In Direct-Reading and Vessel-Mounted models, this conversion is usually done by an external software program (e.g., DAS). We can use the tilt sensors and compass readings to define a conversion routine to express any instrument-coordinate vector in earth coordinates. The conversion depends on how the tilt sensors are fixed to the ADCP. The general procedure is to first convert the vectors to a horizontal plane using a combination of rotations about the roll and pitch axes. Heading is then corrected
with a rotation about the Z -axis. In general, the actual magnitude of the rotation angles will not be those recorded by the sensors.

If we assume the ADCP platform is nonaccelerating and the tilt sensor pivots are frictionless, we can use the sensor readings to measure roll and pitch. We define the roll, pitch, and heading measures as follows.

Pitch - Pitch angles rotate about the ADCP's X-axis. For upward-facing units, the ADCP outputs a positive pitch value in instrument coordinates when beam 4 is higher than beam 3. For downward-facing ADCPs, a positive pitch occurs when beam 3 is higher than beam 4. See Figure C-1 for information on how to convert pitch data to the earth-coordinate system.

Roll - Roll angles rotate about the ADCP's Y-axis. For upward-facing units, the ADCP outputs a positive roll value in instrument coordinates when beam 1 is higher than beam 2. For downward-facing ADCPs, a positive roll occurs when beam 2 is higher than beam 1. See Figure C-1 for information on how to convert roll data to the earth-coordinate system.

Heading - Heading angles rotate about the ADCP's Z-axis. The compass reading is in degrees with the +YE direction equal to $000^{\circ}$ and the +XE direction equal to $+090^{\circ}$.


| TILT SIGNS | FOR A UNIT FACING:UP |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| USING THIS COORD SYSTEM: | INSTR | EARTH | INSTA | EARTH |
| TILT 1 (PITCH) <br> Beam 3 HIGHER THAN BEAM 4 BEAM 4 HIGHER THAN BEAM 3 | + | + | + | + |
| TILT 2 (ROLL) <br> BEAM 1 HIGHER THAN BEAM 2 BEAM 2 HIGHER THAN BEAM 1 | + | + | - | $\pm$ |

NOTE: THE ADCP ALWAYS OUTPUTS TILT ANGLES IN INSTRUMENT COORDINATES; YOU MUST MANUALLY CONVERT TILT SIGNS TO THE EARTH-COORDINATE SYSTEM.


+TILT 1 (INSTR)
-TILT 1 (EARTH)

+TILT 2 (INSTR)
-TILT 2 (EARTH)

Figure C-1. Transducer Beam Axes and Tilt Signs

C-3.2. VELOCITY PROCESSING ALGORITHM. The following algorithm calculates the east (X), north (Y), vertical (Z), and error (E) velocity components from the raw ADCP Doppler frequency data. We describe the algorithm using FORTRAN-like variables and arithmetic.

The algorithm input has NBINS of range-gated Doppler frequency in four arrays (D1, D2, D3, D4) containing frequency estimates from the four transducer beams. Heading is from the flux-gate compass. Pitch and roll data are from the tilt sensors mounted inside the transducer head. The algorithm produces three, resolved-velocity arrays each with NBINS of velocity data at depths corresponding to those measured with zero pitch and roll.

In this algorithm, steps 1 and 2 are done once for each ping. Step 3 begins a loop that ends in step 6. The loop executes NBINS times for each ping.

Step 1 - Determine Rotation Angles from Sensor Readings. For each new set of pitch, roll, heading, and Doppler frequency data, find the rotation angles for the conversion matrix RR, PP, and HH. There are three cases that vary depending on how the tilt sensors are fixed to the ADCP.

Case 1 - Pitch and Roll Axes Fixed to $A D C P$. The sensor axes are fixed to the ADCP axes and rotate with the ADCP. Use this set of equations with pendulum attitude sensors. The rotation angles are:
$\mathrm{RR}=\mathrm{ROLL}$
PP $=\operatorname{ARCSIN}(S I N(P I T C H) ~ * ~ C O S(R O L L) ~ / ~ K A) ~$
$\mathrm{HH}=$ HEADING
where $K A=\operatorname{SQRT}\left(1-(\operatorname{SIN}(\text { PITCH }) * \operatorname{SIN}(\text { ROLL }))^{2}\right)$
Case 2 - Roll Axis Fixed to $A D C P$. The roll axis is fixed to the $A D C P$, and the pitch axis in gimballed inside the roll axis.

$$
\begin{aligned}
& \mathrm{PP}=\mathrm{PITCH} \\
& \mathrm{RR}=\text { ROLL } \\
& \mathrm{HH}=\mathrm{HEADING}
\end{aligned}
$$

Case 3 - Pitch Axis Fixed to System. The pitch axis is fixed to the ADCP, and the roll axis is gimballed inside the pitch axis.

$$
\begin{aligned}
& \text { PP }= \text { PITCH } \\
& \mathrm{RR}= \text { ROLL } \\
& H H= \text { HEADING }+\mathrm{DD} \\
& \text { where } \quad \mathrm{DD}=\operatorname{ARCSIN}(\operatorname{SIN}(\text { PITCH }) * \operatorname{SIN}(\text { ROLL }) / \mathrm{KB}) \\
& \mathrm{KB}=\operatorname{SQRT}\left(\operatorname{COS}(\mathrm{PITCH})^{2}+(\operatorname{SIN}(\text { PITCH }) * \operatorname{SIN}(\text { ROLL }))^{2}\right)
\end{aligned}
$$

Notice in case 1, an adjusted reading of the pitch sensor was used for the pitch rotation angle, while case 3 requires an adjustment to heading.

Step 2 - Calculate Trigonometric Functions and Scaling Factors. Find the Z component of the conversion matrix $M(I, J)$ that expresses in earth coordinates a vector initially acquired in instrument coordinates. For the three different cases, the components are:

$$
\begin{array}{lll}
\mathrm{CP}=\cos (\mathrm{PP}) & \mathrm{CR}=\cos (\mathrm{RR}) & \mathrm{CH}=\cos (\mathrm{HH}) \\
\mathrm{SP}=\sin (\mathrm{PP}) & \mathrm{CR}=\sin (\mathrm{RR}) & \mathrm{SH}=\sin (\mathrm{sH}) \\
\mathrm{S} 30=\sin (30) \\
\text { (30 }
\end{array}
$$

Case 1 - Tilt Sensors Fixed to $A D C P$.

$$
\begin{aligned}
& M(3,1)=-S R * C P \\
& M(3,2)=S P \\
& M(3,3)=C P * C R
\end{aligned}
$$

Case 2 - Roll Axis Fixed, Pitch Axis Gimballed Inside Roll.
$M(3,1)=-S R * C P$
$M(3,2)=S P$
$M(3,3)=C P * C R$
Case 3 - Pitch Axis Fixed, Roll Axis Gimballed Inside Pitch.

$$
\begin{aligned}
& M(3,1)=-S R \\
& M(3,2)=C R * S P \\
& M(3,3)=C P * C R
\end{aligned}
$$

Form the scale factors for the depth index:

$$
\begin{aligned}
& \mathrm{SC}(1)=\mathrm{C} 30 /(\mathrm{M}(3,3) * \mathrm{C} 30+\operatorname{ZSG}(1) * \mathrm{M}(3,1) * \mathrm{~S} 30) \\
& \mathrm{SC}(2)=\mathrm{C} 30 /(\mathrm{M}(3,3) * \mathrm{C} 30+\mathrm{ZSG}(2) * \mathrm{M}(3,1) * \mathrm{~S} 30) \\
& \mathrm{SC}(3)=\mathrm{C} 30 /(\mathrm{M}(3,3) * \mathrm{C} 30+\mathrm{ZSG}(3) * \mathrm{M}(3,2) * \mathrm{~S} 30) \\
& \mathrm{SC}(4)=\mathrm{C} 30 /(\mathrm{M}(3,3) * \mathrm{C} 30+\mathrm{ZSG}(4) * \mathrm{M}(3,2) * \mathrm{~S} 30)
\end{aligned}
$$

where ZSG is a sign that varies depending on the orientation and beam pattern of the transducers. The transducers can face up or down and have concave or convex beam patterns. ZSG is given in the following table.

|  | Up <br> Convex | Up <br> Concave | Down <br> Convex | Down <br> Concave |
| :--- | :---: | :---: | :---: | :---: |
| ZSG(1) | + | - | + | - |
| ZSG(2) | - | + | - | + |
| ZSG(3) | + | - | - | + |
| ZSG(4) | - | + | + | - |

Form the transducer to instrument-coordinate system scaling constants and the error velocity constant:

```
VXS =C / ( }\mp@subsup{F}{\textrm{t}}{*}*4*\operatorname{sin}(30)
VYS =C / ( }\mp@subsup{F}{\textrm{t}}{*}*4*\operatorname{sin}(30)
VZS = C / ( }\mp@subsup{F}{\textrm{t}}{*}*8*\operatorname{cos(30))
VES = C / ( }\mp@subsup{\textrm{F}}{\textrm{t}}{*}*8
```

where $C=$ speed of sound in the profiled water (in $\mathrm{cm} / \mathrm{s}$ ); see $\mathbb{T} \mathrm{C}-1$
$\mathrm{F}_{\mathrm{t}}=$ is the transmitted sonar frequency ( Hz ).

Step 3 - Depth Cell Correction for Pitch and Roll. The pitch and roll information maps the tilted, range-gated, Doppler frequency estimates into depth cells (bins) at the same depth. The following loop executes from 1 to the number of bins (NBINS).

```
REPEAT for IB = 1 to NBINS
    J1 = IB*SC(1) + 0.5
    J2 = IB*SC(2) + 0.5
    J3 = IB*SC(3) + 0.5
    J4 = IB*SC(4) + 0.5
```

where J1, J2, J3, and J4 are integers of the constant depth indices for Doppler frequency data in the $\mathrm{IB}^{\text {th }}$ bin of the input arrays D1, D2, D3, D4.

Now check that all indices are $>0$ and $\leq$ NBINS, and that the data in D1(J1), D2(J2), D3(J3), and D4(J4) are valid. The latter test normally uses the STATUS nibble from the ADCP with the Doppler frequency data. If any of the four estimates are bad, skip to the end of the loop.

Step 4 - ADCP Coordinate Velocity Components. Find the VX, VY, VZ, and VE instrument-coordinate velocities for the $I^{\text {th }}$ bin. This conversion depends on transducer orientation (up/down) and beam pattern (concave/convex). The four cases are (Note: D1-D4 are in Hz; Table 4-3 shows scaling factors):

```
UPWARD CONVEX
    vx = vXS * (-D1(J1) + D2(J2))
    VY = VYS * (-D3(J3) + D4(J4))
    VZ = VZS * (-D1(J1) - D2(J2) - D3(J3) - D4(J4))
    VE = VES * (+D1(J1) + D2(J2) - D3(J3) - D4(J4))
DOWNWARD CONVEX
    VX = VXS * (+D1(J1) - D2(J2))
    VY = VYS * (-D3(J3) + D4(J4))
    VZ = VZS * (+D1(J1) + D2(J2) + D3(J3) + D4(J4))
    VE = VES * (+D1(J1) + D2(J2) - D3(J3) - D4(J4))
UPWARD CONCAVE
    VX = vXS * (+D1(J1) - D2(J2))
    VY = VYS * (+D3(J3) - D4(J4))
    VZ = VZS * (-D1(J1) - D2(J2) - D3(J3) - D4(J4))
    VE = VES * (+D1(J1) + D2(J2) - D3(J3) - D4(J4))
DOWNWARD CONCAVE
    VX = VXS * (-D1(J1) + D2(J2))
    VY = VYS * (+D3(J3) - D4(J4))
    VZ = VZS * (+D1(J1) + D2(J2) + D3(J3) + D4(J4))
    VE = VES * (+D1(J1) + D2(J2) - D3(J3) - D4(J4))
```

Check whether the absolute value of VE (error velocity) exceeds a preselected value (e.g., three times the expected velocity standard deviation). VE checks the validity of the calculation of the three orthogonal velocity components VX, VY, and VZ. This is possible because the ADCP needs only three beams to calculate the three velocity components. The fourth beam provides redundant data and allows computation of a "data reasonableness" velocity component.

Step 5 - Convert to Earth Coordinates. The conversion to earth coordinates (shown by the letter E added to the variable name) is done by multiplying the velocity vector by the conversion matrix. For the three cases described above, the results are:

Case 1 - Tilt Sensors Fixed to $A D C P$.

$$
\begin{aligned}
& \mathrm{VXE}=\mathrm{VX} *(\mathrm{CH} * \mathrm{CR}+\mathrm{SH} * \mathrm{SR} * \mathrm{SP})+\mathrm{VY} * \mathrm{SH} * \mathrm{CP}+\mathrm{VZ} *(\mathrm{CH} * \mathrm{SR}-\mathrm{SH} * \mathrm{CR} * \mathrm{SP}) \\
& \mathrm{VYE}=-\mathrm{VX} *(\mathrm{SH} * \mathrm{CR}-\mathrm{CH} * \mathrm{SR} * \mathrm{SP})+\mathrm{VY} * \mathrm{CH} * \mathrm{CP}-\mathrm{VZ} *(\mathrm{SH} * \mathrm{SR}+\mathrm{CH} * \mathrm{SP} * \mathrm{CR}) \\
& \mathrm{VZE}=-\mathrm{VX} * \mathrm{SR} * \mathrm{CP}+\mathrm{VY} * \mathrm{SP}+\mathrm{VZ} * \mathrm{CP} * \mathrm{CR}
\end{aligned}
$$

Case 2 - Roll Axis Fixed, Pitch Axis Gimballed Inside Roll. Same as the case 1 transformation.

Case 3 - Pitch Axis Fixed, Roll Axis Gimballed Inside Pitch.

```
VXE = VX*CH*CR + VY*(CH*SR*SP + SH*CP) + VZ*(CH*SR*CP - SH*SP)
VYE = -VX*SH*CR - VY*(SH*SR*SP - CH*CP) - VZ*(SH*SR*CP + CH*SP)
VZE = -VX*SR + VY*CR*SP + VZ*CP*CR
```

Step 6 - Place Results in Output Array. The output arrays U, V, and W are indexed by the bin index IB. $U$ is the east(+)/west(-) component, $V$ is the north(+)/south(-) component, and W is the down(-)/up(+) component. From the directions of the earth-coordinate axes it can be seen that

$$
\begin{aligned}
& \mathrm{U}(\mathrm{IB})=\mathrm{VXE} \\
& \mathrm{~V}(\mathrm{IB})=\mathrm{VYE} \\
& \mathrm{~W}(\mathrm{IB})=\mathrm{VZE}
\end{aligned}
$$

This completes the processing for the $\mathrm{IB}^{\text {th }}$ bin. From here, loop back to process the remaining bins. When all the bins are processed, the velocity processing algorithm is complete.

## C-4. ECHO SPECTRAL WIDTH

The ADCP determines the echo spectral width by measuring the second moment of the backscattered water-mass echo for each depth cell (bin). It is a measurement of the expected statistical uncertainty in a single-ping, mean water-mass velocity measurement. See RDI's Principles of Operation: A Practical Primer for information on calculating spectral width.

## C-5. ECHO INTENSITY

The ADCP echo-intensity data are a measurement of the intensity of the backscattered echo for each depth cell. See RDI's Principles of Operation: A Practical Primer for information on calculating echo intensity.

## C-6. FISH REJECTION ALGORITHM

This section explains how the GF-command reduces the effects of fish swimming within the ADCP transducer beams.

C-6.1. HISTORY BEHIND THE CF-COMMAND. ADCPs typically have four beams mounted to one transducer head in a JANUS configuration. The beams are mounted at $30^{\circ}$ (sometimes $20^{\circ}$ ) to the horizontal. Because of this configuration, it is possible that an ADCP will "see" fish within one or more beams when profiling.

All four ADCP beams have side lobes. Each side lobe is shaped like a cone (see Figure C-2). Part of this cone looks down a neighboring beam's main lobe. Therefore, when acoustic energy is propagating back to the ADCP along a beam's main lobe, part of this energy is seen by the other beams' side lobes. The amount of energy seen depends on the relative amplitude of the side lobe and the echo intensity of the surrounding water.

When only one beam sees a fish, we can expect to see a larger error velocity in the affected bin. This is true as long as the echo intensity of the fish is not much greater than that of the surrounding water. When the fish echo is much greater than the surrounding water, the strong echo from the "fishy" beam is heard by the two neighboring beams as well as the opposite beam. This causes the ADCP to measure about the SAME velocity in all four beams, which gives a low error velocity.

For buoy-mounted ADCPs, fish tend to "school" at some depth below the ADCP. In this situation, the fish seem to remain "still" relative to the ADCP. If an actual water current is present near the fish, the fish velocity (in this case, a near-zero velocity) will dominate the doppler measurement. Another situation occurs when the fish are not in all four beams. This is a nonhomogeneous situation that can cause a measurement error.

On vessel-mounted ADCPs, a forward-facing beam hears the doppler of the vessel to which it is attached. A strong echo from a fish causes the other three beams to hear the same doppler, and the ADCP outputs an apparent vertical velocity. If a side-facing (port/starboard) beam hears a strong fish echo, the ADCP outputs a low velocity measurement (no apparent vertical or error velocity).

C-6.2. ONE METHOD TO REDUCE FISH EFFECTS. The echo intensity (AGC) measurement from each beam is a good indicator of the relative strength of the returning acoustic energy. If we knew what the normal echo strength was for a given bin, we could detect fish in the profile data. For a large set of sin-gle-ping profile data, we could observe these typical AGC values, and then go back through the profile data to remove the "fishy-looking" bins.

ADCPs typically sum together many pings of data before storing or transmitting the average. To screen-out fish from the data before accumulation, the ADCP could compare the echo data from all four beams. If the four echo intensities do not match on a given bin, the ADCP could reject the data. Of course, an exact match will never occur, so we must allow some acceptable difference.

Each beam has a quiescent value that is an offset relative to zero. This value is typically $30-40$ counts (one count is about 0.5 dB ). The largest side
lobe is down 35 dB ( 70 counts) or more from the beam's main lobe. For each bin of data, we can define an algorithm to remove "fishy" data by repeating the following cycle for each bin of data. Lets assume we start with bin $x=1$ and $\mathrm{CF}=25$ ( 50 counts). NOTE: We could probably use a count value of 40 and still not throw out too much "good" data.

## Algorithm 1

a. Compare bin x from each beam to find the highest and the lowest bin x echo levels.
b. If the difference between the highest and lowest bin $x$ value is more than 50 counts, mark the velocities in bin x of all four beams as bad.
c. If step $b$ was true (fish detected), also mark bin $x+1$ velocities as bad. This is because the ADCP samples AGC near the end of the bin.
d. Increment $x$ and repeat for the next bin until all bins have been tested for the current ping.

This algorithm does not account for the failure of one beam. (Remember, the ADCP needs only three beams to compute velocity in three planes. The ADCP uses the fourth beam as a data-quality check.) If the ADCP is deployed for an extended time, there is a chance a beam could fail. If this happens, Algorithm 1 may flag all the data as "fishy," causing all velocity data from the deployment to be lost. To avoid this situation, we modified Algorithm 1 with Algorithm 2. The CF-command uses Algorithm 2 and repeats the following cycle for each bin of data.

Algorithm 2
a. Compare bin $x$ from each beam to find the highest and the two lowest bin $x$ echo levels.
b. If the difference between the highest and the lowest bin $x$ value is more than 50 counts, mark the velocity in the beam with the lowest bin $x$ value as bad. In effect, we are now using a 3-beam solution for the bin $x$ layer.
c. If the difference between the highest and second lowest bin $x$ value is also more than 50 counts, mark the velocities in bin $x$ of all four beams as bad (i.e., beam OK, just bins affected).
d. If step c was true (a fish detected), also mark bin $x+1$ velocities as bad. This is because the ADCP samples AGC near the end of the bin.
e. Increment $x$ and repeat for the next bin until all bins have been tested for the current ping.

As an example, use Algorithm 2 and Figure C-2 to determine the outcome of Case 1 and Case 2 for bin 13 when $\mathrm{CF}=25$ ( 50 counts).

Using Algorithm 2 and the data supplied for Bin 13 in Figure C-2 we have:

| STEP | CASE 1 | CASE 2 |
| :---: | :---: | :---: |
| A | Highest $=$ Beam $4(81)$ $2^{\text {ND Low }}=$ Beam $2(39)$ Lowest $=$ Beam 1 (30) | Highest $=$ Beam 4 (81) $2^{\text {ND Low }}=$ Beam $2(29)$ Lowest $=$ Beam $1 \quad(30)$ |
| B | Is 81 ( Bm 4 ) - 30 ( $\mathrm{Bm} \mathrm{1)}>50$ ? Yes (51), so Beam 1 is bad at this point. | Is 81 ( Bm 4 ) - $29(\mathrm{Bm} 2)>50$ ? Yes (52), so Beam 2 is bad at this point. |
| C | Is 81 ( Bm 4 ) - 39 ( Bm 2$)>50$ ? <br> No (42), so Step B applies. <br> That is, the ADCP marks all Beam 1 velocities as bad and uses a 3-beam solution for Bin 13. | Is 81 ( Bm 4 ) - $30(\mathrm{Bm} 1)>50$ ? Yes (51), so Step B no longer applies. The ADCP marks the velocities in Bin 13 of all four beams as bad. |
| D | Since Step C was false, this step is not applicable. | Since Step C was true, the ADCP also marks the velocities in Bin 14 of all four beams as bad. |



