In[ ]:=
```
X = {{0, 1}, {1, 0}}
MatrixForm[X]
```

Out[53]=
$$\{\{0, 1\}, \{1, 0\}\}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

In[ ]:=
```
Y = {{0, -i}, {i, 0}}
MatrixForm[Y]
```

Out[55]=
$$\{\{0, -i\}, \{i, 0\}\}$$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

In[59]:=
```
M1 = MatrixExp[-i * π / 2 * (X * Cos[ArcCos[-1 / 4]] + Y * Sin[ArcCos[-1 / 4]])]
MatrixForm[Simplify[M1]]
```

Out[59]=
$$\left\{\left\{0, \frac{1}{4}\left(i - \sqrt{15}\right)\right\}, \left\{\frac{1}{4}\left(i + \sqrt{15}\right), 0\right\}\right\}$$

$$\begin{pmatrix} 0 & \frac{1}{4}\left(i - \sqrt{15}\right) \\ \frac{1}{4}\left(i + \sqrt{15}\right) & 0 \end{pmatrix}$$

In[61]:=
```
M2 = MatrixExp[-i * π * (X * Cos[3 * ArcCos[-1 / 4]] + Y * Sin[3 * ArcCos[-1 / 4]])]
MatrixForm[Simplify[M2]]
```

Out[61]=
$$\left\{\left\{-1, \frac{1}{2}\left(\text{Cos}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right] - i\,\text{Sin}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right]\right) + \frac{1}{2}\left(-\text{Cos}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right] + i\,\text{Sin}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right]\right)\right\}, \left\{\frac{1}{2}\left(-\text{Cos}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right] - i\,\text{Sin}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right]\right) + \frac{1}{2}\left(\text{Cos}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right] + i\,\text{Sin}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right]\right), -1\right\}\right\}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

In[63]:=
```
MX = MatrixExp[-i * θ / 2 * X]
MatrixForm[Simplify[MX]]
```

Out[63]=
$$\left\{\left\{\text{Cos}\left[\frac{\theta}{2}\right], -i\,\text{Sin}\left[\frac{\theta}{2}\right]\right\}, \left\{-i\,\text{Sin}\left[\frac{\theta}{2}\right], \text{Cos}\left[\frac{\theta}{2}\right]\right\}\right\}$$

Out[64]//MatrixForm=
$$\begin{pmatrix} \text{Cos}\left[\frac{\theta}{2}\right] & -i\,\text{Sin}\left[\frac{\theta}{2}\right] \\ -i\,\text{Sin}\left[\frac{\theta}{2}\right] & \text{Cos}\left[\frac{\theta}{2}\right] \end{pmatrix}$$

In[65]:=
```
BB1 = M1.M2.M1.MX
MatrixForm[Simplify[BB1]]
```

Out[65]=
$$\left\{\left\{\frac{1}{16}\left(-i+\sqrt{15}\right)\left(i+\sqrt{15}\right)\text{Cos}\left[\frac{\theta}{2}\right] - \frac{1}{16}\,i\left(i-\sqrt{15}\right)^2\text{Sin}\left[\frac{\theta}{2}\right]\left(\frac{1}{2}\left(-\text{Cos}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right] - i\,\text{Sin}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right]\right) + \frac{1}{2}\left(\text{Cos}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right] + i\,\text{Sin}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right]\right)\right),\right.$$
$$-\frac{1}{16}\,i\left(-i+\sqrt{15}\right)\left(i+\sqrt{15}\right)\text{Sin}\left[\frac{\theta}{2}\right] + \frac{1}{16}\left(i-\sqrt{15}\right)^2\text{Cos}\left[\frac{\theta}{2}\right]\left(\frac{1}{2}\left(-\text{Cos}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right] - i\,\text{Sin}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right]\right) + \frac{1}{2}\left(\text{Cos}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right] + i\,\text{Sin}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right]\right)\right)\Big\},$$
$$\left\{-\frac{1}{16}\,i\left(-i-\sqrt{15}\right)\left(i-\sqrt{15}\right)\text{Sin}\left[\frac{\theta}{2}\right] + \frac{1}{16}\left(i+\sqrt{15}\right)^2\text{Cos}\left[\frac{\theta}{2}\right]\left(\frac{1}{2}\left(\text{Cos}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right] - i\,\text{Sin}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right]\right) + \frac{1}{2}\left(-\text{Cos}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right] + i\,\text{Sin}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right]\right)\right),\right.$$
$$\left.\frac{1}{16}\left(-i-\sqrt{15}\right)\left(i-\sqrt{15}\right)\text{Cos}\left[\frac{\theta}{2}\right] - \frac{1}{16}\,i\left(i+\sqrt{15}\right)^2\text{Sin}\left[\frac{\theta}{2}\right]\left(\frac{1}{2}\left(\text{Cos}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right] - i\,\text{Sin}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right]\right) + \frac{1}{2}\left(-\text{Cos}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right] + i\,\text{Sin}\left[3\,\text{ArcCos}\left[-\frac{1}{4}\right]\right]\right)\right)\right\}\right\}$$

Out[66]//MatrixForm=
$$\begin{pmatrix} \text{Cos}\left[\frac{\theta}{2}\right] & -i\,\text{Sin}\left[\frac{\theta}{2}\right] \\ -i\,\text{Sin}\left[\frac{\theta}{2}\right] & \text{Cos}\left[\frac{\theta}{2}\right] \end{pmatrix}$$