

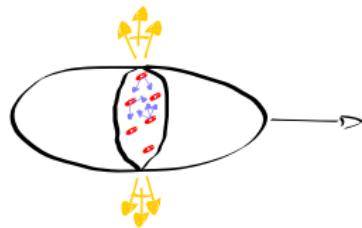
Transverse Coherent Stability in Synchrotrons

IBPT Meeting, KIT – 14. September 2021
Adrian Oeftiger (a.oeftiger@gsi.de)

Intro

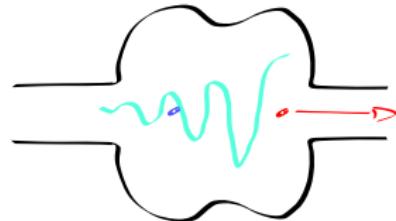
My area of research: collective effects in beam dynamics

direct space charge (SC)



- direct interaction of particles with beam self-field
- modifies and broadens betatron resonances

beam coupling impedance



- indirect interaction among particles via surrounding environment
- responsible for most intensity-dependent instabilities

Overview

Today:

- A. Space charge modelling for FAIR SIS100 (my current topic)
- B. Transverse single-bunch instabilities (instructive)
 - I. Head-tail instability
 - II. Transverse mode coupling instability
- C. Space charge impact on instabilities (my previous topic)

Not today:

- detailed theory, dispersion relations, derivations

A. Space Charge Modelling for FAIR SIS100

About FAIR



Facility for Antiproton and Ion Research:

- under construction at GSI, Germany
- key: heavy ion synchrotron SIS100
- operation close to *space charge limit*

supersymmetry	$S = 6$
circumference	1083.6 m
particles	from $A = 1$ (protons) to $A = 238$ (U^{28+})
injection energy	200 MeV/u
extraction energy	≤ 2.7 GeV/u (U^{28+})
intensity	$\leq 5 \times 10^{11} U^{28+}$ /cycle
max. SC tune shift	$\Delta Q_y^{SC} = -0.3$

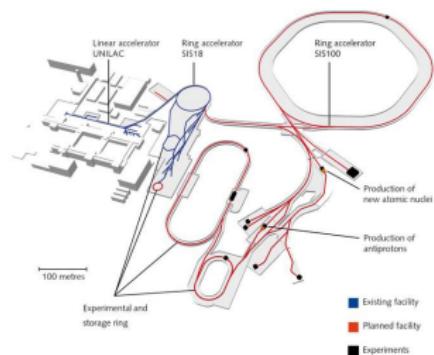
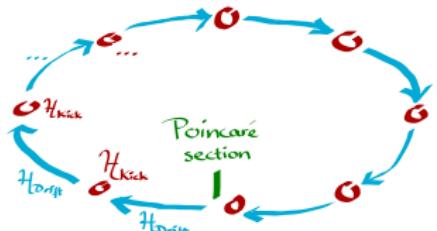
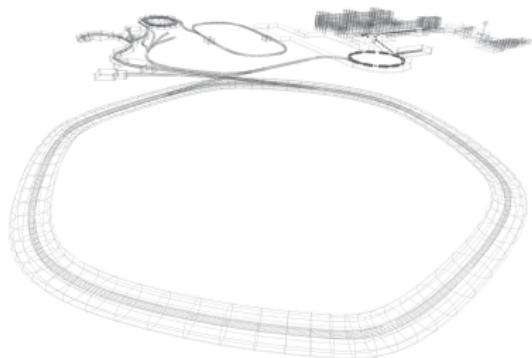


Figure: FAIR facility



Figure: SIS100 construction site
June 2021, youtube drone video ↗



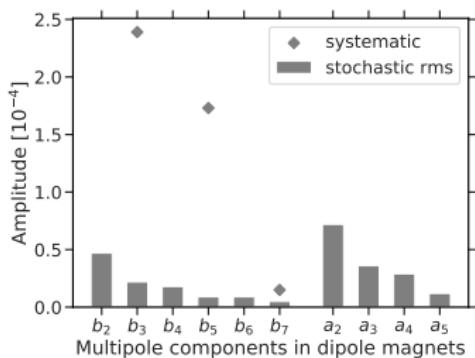
Developed detailed SIS100 simulation model:

- code base:
PyHEADTAIL (collective effects) +
SixTrackLib (single-particle
tracking)
- *non-linear* 3D space charge fields:
 - self-consistent particle-in-cell
 - frozen field map models
- full lattice in thin-lens treatment,
6D symplectic tracking
- GPU accelerated: fast frozen
model computes 1 s storage time
(160000 turns) in ≈ 15 min

Magnet Bench Measurements

Field error model for dipole + quadrupole magnets:

- cold bench measurements with magnetometer coils
 - ⇒ multipole expansion for magnets in series production
- plug into tracking simulation: statistical distribution of field errors for each magnet according to ensemble error model



b_n : normal
 a_n : skew
 $n = 2$: quadrupole
 $n = 3$: sextupole
...

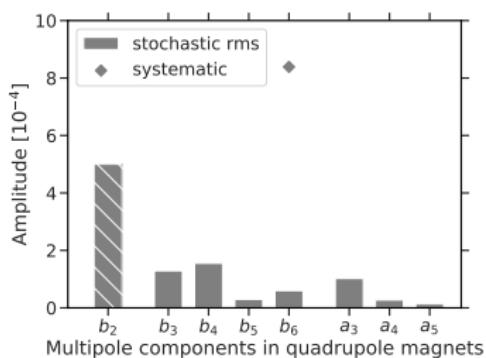


Figure: SIS100 field error model at U²⁸⁺ injection energy

Space Charge Simulations

Long-term tracking simulations with 3D space charge:

- cover the length of bunch accumulation: 160 000 turns
- strong space charge for U²⁸⁺ bunches:
up to $\Delta Q_y^{SC} = -0.3$

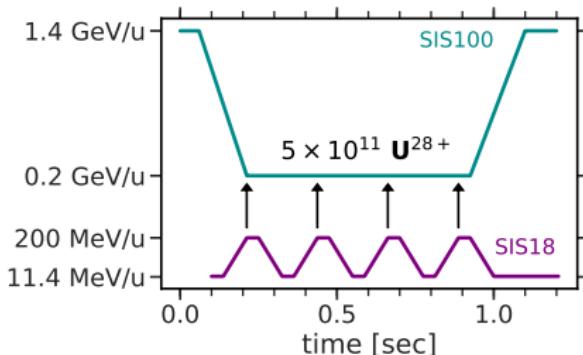


Figure: heavy ion accumulation in SIS100

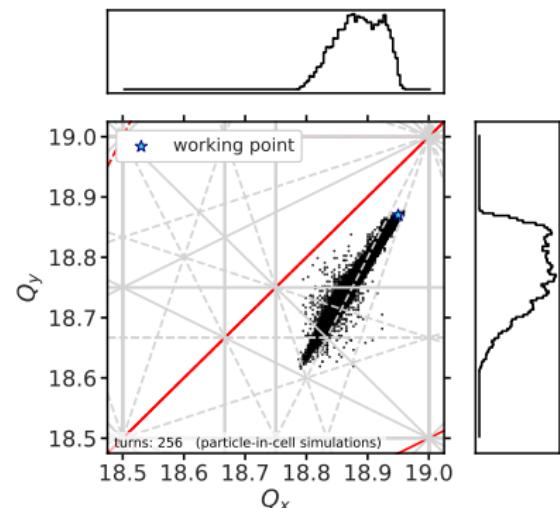
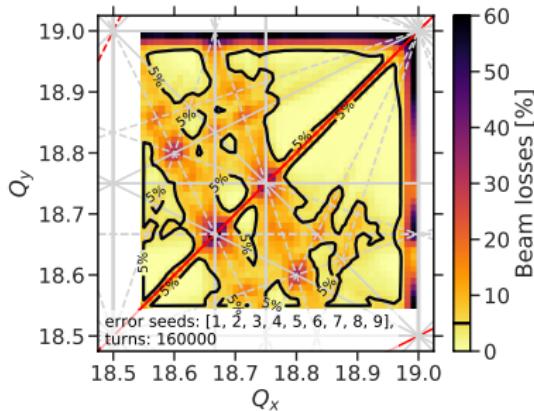


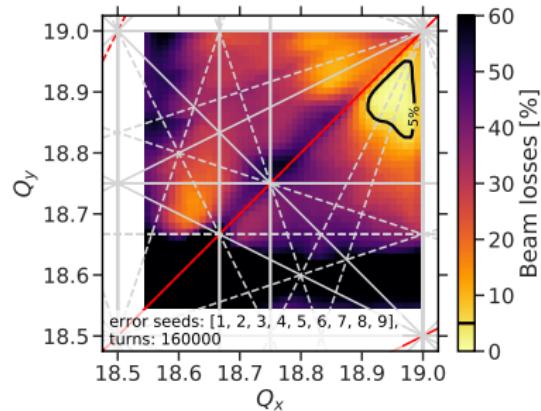
Figure: incoherent tune footprint

Results

Beam loss predictions for heavy-ion fast extraction tune quadrant:



include
↔
SC

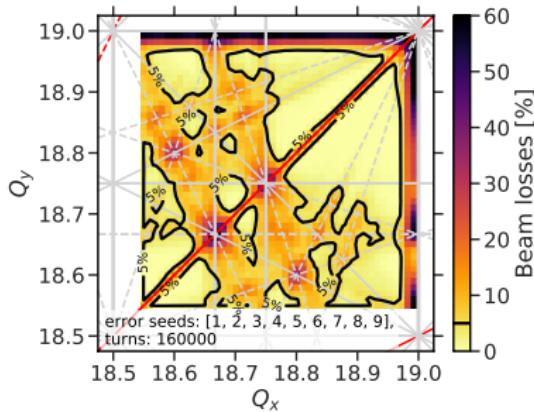


(a) no space charge, just magnet errors

(b) space charge + magnet errors

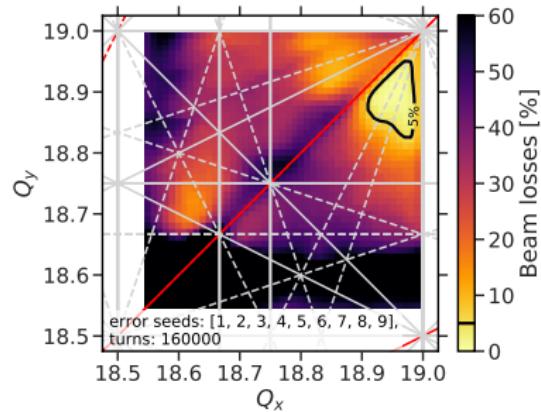
Results

Beam loss predictions for heavy-ion fast extraction tune quadrant:



include
↔
SC

(a) no space charge, just magnet errors



(b) space charge + magnet errors

Powerful prediction model being used to

- identify ideal working point regions
- assess magnet quality (series production of quads just started!)
- develop intensity upgrades: space charge mitigation aspects

B. Coherent Stability vs. Impedances

About Instabilities

"Beam distribution" \doteq set of particles with 6D phase space coordinates

$$\zeta = (x, x', y, y', z, \delta)$$

"Collective instability" \doteq some statistical moment of beam distribution
 $f(\zeta)$ grows exponentially in amplitude

1st order: centroid



$$\text{e.g. } \langle x \rangle = \int d^6\zeta \ x \cdot f(\zeta)$$

2nd order: envelope



$$\text{e.g. } \sigma_x = \int d^6\zeta \ (x - \langle x \rangle)^2 \cdot f(\zeta)$$

Typical instabilities...

Instabilities can be classified by

- longitudinal plane / *transverse plane*
- *single-bunch (short-range wake) / coupled-bunch and/or multi-turn (long-range wake)*

Among the most relevant transverse single-bunch instabilities:

- (slow) head-tail instability
- transverse mode coupling instability
(TMCI or sometimes “fast” head-tail instability)

Part I: Head-tail Instability

Head-tail Instability in CPS I

CERN Proton Synchrotron in the 90s:

- preparing for LHC operation: higher intensities $N \approx 2 \times 10^{12}$ ppb
- observation of horizontal head-tail instabilities at injection [1]

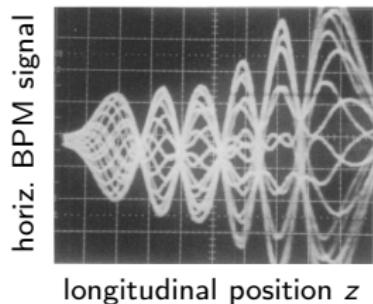


Figure: horizontal difference signal along bunch, consecutive turns

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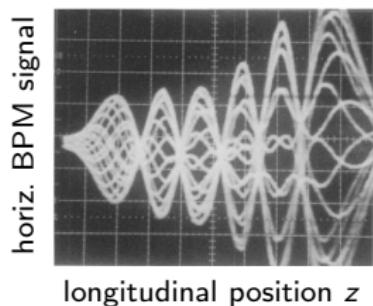


Figure: horizontal difference signal along bunch, consecutive turns

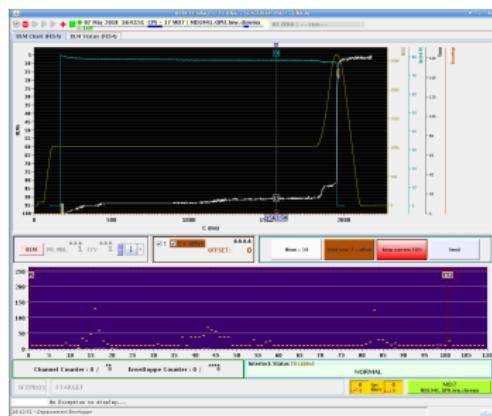
⇒ stabilisation by “sharing” of Landau damping via strong coupling [2]:

- provide strong skew component a_2 in lattice: skew quadrupoles
- working point close to coupling line $Q_x \approx Q_y$
- Landau damped in vertical plane \rightsquigarrow “share” with horizontal plane

Head-tail Instability in CPS II

CERN Proton Synchrotron in 2018:

- standard operation: natural chromaticity $Q'_{x,y} \doteq \frac{dQ_{x,y}}{d\delta} \approx -Q_{x,y}$
 - strong coupling suppresses horizontal head-tail instability
 - future: strong space charge, vertical integer resonance problematic!



(a) Stable beam!

Figure: MD1941, 7.5.2018 – preparation of low-chroma cycle

CERN Proton Synchrotron in 2018:

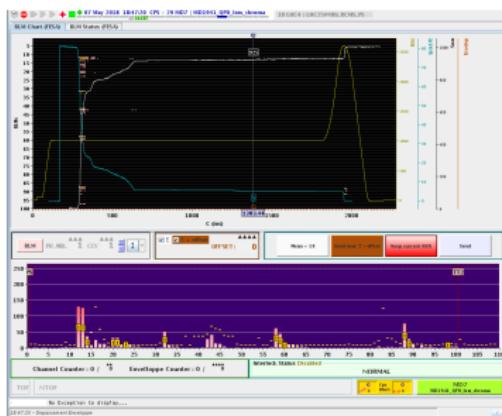
- future operation: correct vertical chromaticity $Q'_y \rightsquigarrow 0$
 - required to reduce integer resonance stop-band extent

Figure: MD1941, 7.5.2018 – preparation of low-chroma cycle

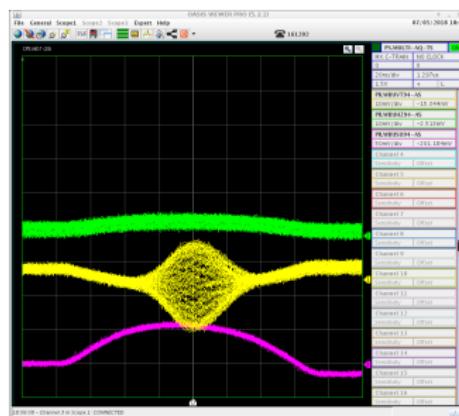
Head-tail Instability in CPS II

CERN Proton Synchrotron in 2018:

- future operation: correct vertical chromaticity $Q'_y \sim 0$
 - required to reduce integer resonance stop-band extent
 - upgrade with new high-frequency transverse feedback for stabilisation



(b) Beam loss due to **instability!**



(c) Vertical rigid bunch mode (yellow)!

Figure: MD1941, 7.5.2018 – preparation of low-chroma cycle



Head-tail instability in bunched beam:

- growth rate scales with intensity, $\tau^{-1} \propto N$
- requires transverse to longitudinal coupling
 - always present for finite chromaticity
$$Q'_{x,y} \doteq \frac{dQ_{x,y}}{d\delta}$$
- can be mitigated via
 1. appropriate shift in chromaticity
(evidently near-zero Q'_y worse than $Q'_y \approx -Q_y$)
 2. strong coupling: transverse sharing of Landau damping (skew quadrupoles, close tunes $Q_x \approx Q_y$, requires asymmetric $\tau_{x,y}!$)
 3. transverse feedback of appropriate bandwidth

...let's understand better...

Stored particles experience synchrotron like a large RLC circuit:

- impedance $Z(\omega) \leftrightarrow$ voltage drop w.r.t. motion of charged particles
- wakefield $W(t) \leftrightarrow$ electromagnetic field induced by passing particles

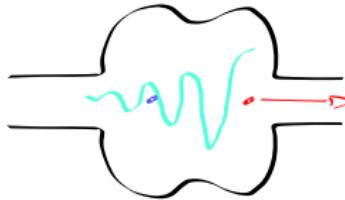


Figure: leading particle acting on trailing particle via longitudinal wake.

- impedance and wakefields split into longitudinal & transverse planes
→ related to each other via Panofsky-Wenzel theorem
(implied by 3D Maxwell equations)

Wakefields and Impedances

Stored particles experience synchrotron like a large RLC circuit:

- impedance $Z(\omega) \leftrightarrow$ voltage drop w.r.t. motion of charged particles
- wakefield $W(t) \leftrightarrow$ electromagnetic field induced by passing particles



impedance = Fourier { wakefield }

$$Z_{\perp}(\omega) = \frac{iC}{\beta} \int_{-\infty}^{+\infty} dt e^{-i\omega t} W_{\perp}(t)$$

- impedance and wakefields split into longitudinal & transverse planes
→ related to each other via Panofsky-Wenzel theorem
(implied by 3D Maxwell equations)

Typical Wake Sources

Typical impedance models of accelerators comprise:

- resistive wall of vacuum chamber
- indirect space charge
(induced mirror current in perfectly conducting vacuum chamber)
- broad-band impedance $Q \approx 1$ (bellows, BPMs, collimators, ...)
- narrow-band impedance $Q \gg 1$ (rf cavities, kickers, septa, ...)

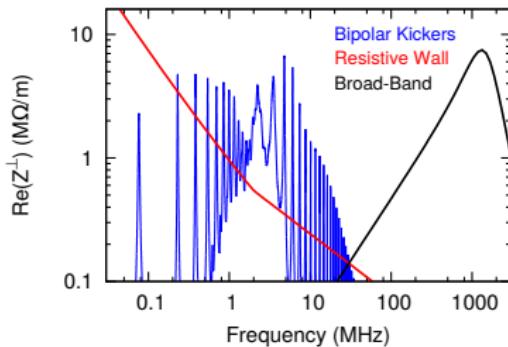


Figure: impedance model for FAIR SIS100, resistive part vs. frequency [3]

Powerful analytical toy model with 2 particles: Refs. [4, 5, 6]

2 degrees of freedom \implies 2 “coherent” head-tail modes:

- “+”: in-phase, rigid bunch mode $m = 0$
 - “−”: anti-phase, head-tail mode $|m| = 1$
- \implies equal growth rates of opposite sign for constant wake W_0 !
(unstable vs. damped)

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- \Rightarrow equal growth rates of opposite sign for constant wake W_0 !
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Growth rates of head-tail modes given by eigenvalues $\lambda_{\pm} \propto \exp(t/\tau)$:

$$\tau_{\pm}^{-1} = \mp \frac{N \frac{Q'}{Q} \tilde{z}}{p_0 \eta} \left(\frac{W_0}{C} \right)$$

for N (weak!) intensity, Q'/Q relative chromaticity, \tilde{z} longitudinal amplitude,
 p_0 momentum, η slip factor, C circumference.

Two-particle Model: Results



Powerful analytical toy model with 2 particles: Refs. [4, 5, 6]

2 degrees of freedom \Rightarrow 2 “coherent” head-tail modes:

- “+”: in-phase, rigid bunch mode $m=0$
- “−”: anti-phase, head-tail mode $|m|=1$

important relationship

$$\tau_{\pm}^{-1} \sim \mp \frac{Q'/Q}{\eta}$$

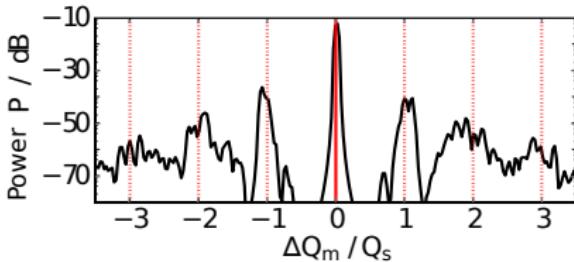
	negative chroma $Q'/Q < 0$	positive chroma $Q'/Q > 0$
below transition $\eta < 0$	damped “+”, unstable “−”	unstable “+”, damped “−”
above transition $\eta > 0$	unstable “+”, damped “−”	damped “+”, unstable “−”

\Rightarrow always one unstable mode!

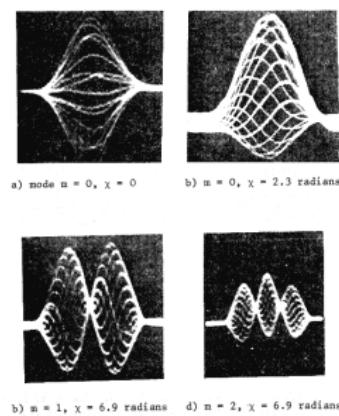
Head-tail Modes in Bunches

Fortunately, situation is a bit better for real bunches:

- spectrum of infinitely many head-tail modes
 - instability conditions for rigid mode $m=0$ as in 2-particle model
 - role of “-” mode distributed among all non-rigid head-tail modes



(a) simulated centroid spectrum with head-tail modes [7]



(b) head-tail modes $|m|=0,1,2$ measured in CERN PS [8]

Head-tail Modes in Bunches

Fortunately, situation is a bit better for real bunches:

- spectrum of infinitely many head-tail modes
 - instability conditions for rigid mode $m=0$ as in 2-particle model
 - role of “-” mode distributed among all non-rigid head-tail modes
 - ⇒ $\sum_m 1/\tau_m = 0$, i.e. all non-rigid modes much weaker than $m=0$

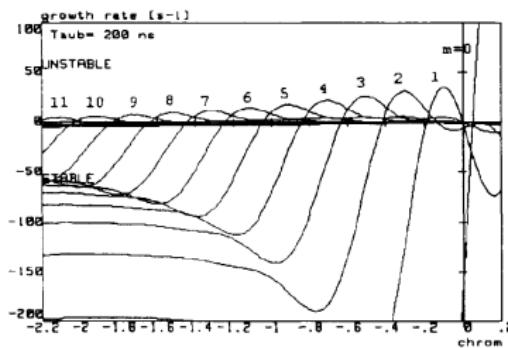


FIGURE 5: Instability growth rates for various modes ($m = 0-11$) versus chromaticity.

Figure: PS diagram for head-tail modes below transition $\eta < 0$ [1]

Head-tail Instabilities in LHC

CERN LHC operates above transition, $\eta > 0$.

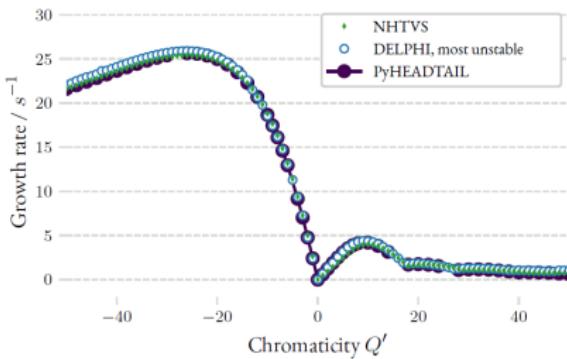


Figure: growth rates versus chromaticity [9]

Counter-measures:

- chromaticity adjustment to $Q'/Q > 0$ (sextupole magnets)
- transverse feedback
- Landau octupole magnets (increase transverse tune spread)
- impedance mitigation (e.g. Molybdenum coating of graphite collimators)

First Head-tail Instability in LHC

Observations from LHC in 2010:

- first ramp with a single-bunch of $N \approx 1 \times 10^{11}$ ppb
- 1-node $m = -1$ head-tail instability appeared around $E_{kin} = 1.8$ TeV
 - no transverse feedback on, no Landau octupoles
 - injection plateau stable conditions!

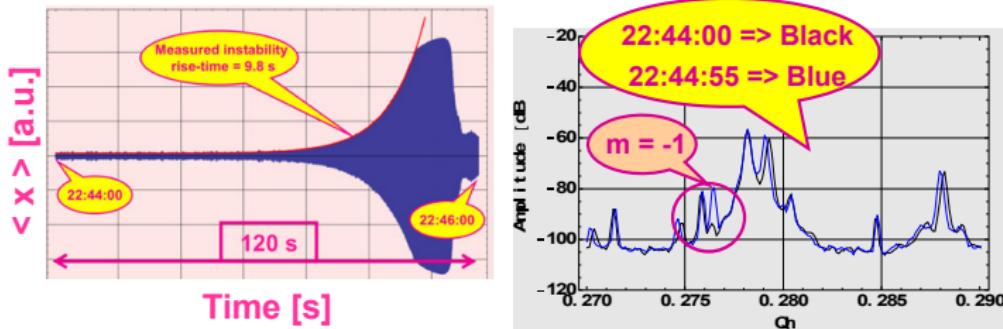


Figure: Dedicated instability measurement 2 days after first instability in LHC [10, 11]

Part II: Transverse Mode Coupling Instability

Overview TMCI

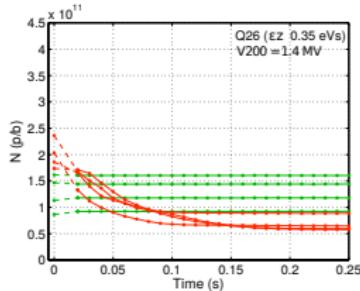
TMCI major limitation, poses a strict upper intensity threshold:

- “the ultimate transverse instability”: single-bunch, $Q'/Q = 0$ [12]
- particularly for machines with short bunches
- ⇒ lepton machines are often affected (SPEAR II, PETRA, PEP, LEP, ...)

Overview TMCI

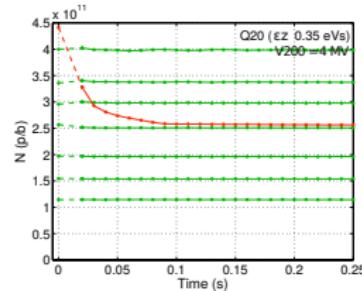
TMCI major limitation, poses a strict upper intensity threshold:

- “the ultimate transverse instability”: single-bunch, $Q'/Q = 0$ [12]
- particularly for machines with short bunches
 - ⇒ lepton machines are often affected (SPEAR II, PETRA, PEP, LEP, ...)
- only one hadron machine with TMCI signature: CERN SPS (2003)
 - still subject of dispute in the community (effect of space charge!)
 - development of alternative optics over the course of several PhD theses successful: instability threshold shifted higher



(a) original Q26 optics ($\gamma_t = 22.8$)

faster
⇒
 Q_s



(b) Q20 optics ($\gamma_t = 18$)

Figure: Measurements of intensity threshold from fast vertical instability in SPS [13]

Two-particle Model Revisited

Two-particle model illustrates mode coupling:

- zero chromaticity $Q'/Q = 0$
- $m=0$ moves down, $|m|=1$ moves up in frequency
- ⇒ merging of modes = onset of TMCI

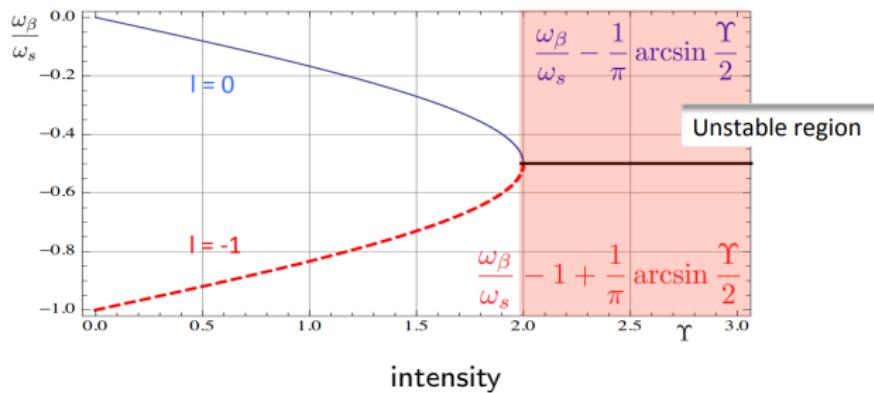


Figure: merging of modes $m=0$ and $|m|=1$ and TMCI threshold $\Upsilon = 2$ [14]

Two-particle model illustrates mode coupling:

- zero chromaticity $Q'/Q = 0$
- $m=0$ moves down, $|m|=1$ moves up in frequency
- ⇒ merging of modes = onset of TMCI

Stable eigenmodes below threshold intensity:

$$N \leq N_{thr} = \frac{8}{\pi e^2} \frac{p_0 \omega_s}{\left(\frac{W_0}{C}\right) \beta_y}$$

for $\omega_s/2\pi$ synchrotron frequency, β_y betatron function at impedance location.

Two-particle Model

PyHEADTAIL simulation example: [TMCI jupyter notebook ↗](#)

Mode 1 (in anti phase transversely):

```
%HTML
<video controls>
  <source src="./plots_model_strong_impedance/video.webm" type="video/webm">
</video>
```

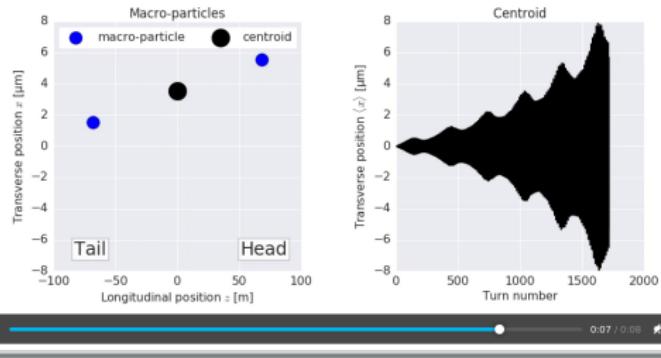


Figure: [click me! ↗](#)

TMCI in LHC

Since the SPS experience, the intensity upgrade for LHC is carefully analysed and adjusted to remain below TMCI threshold:

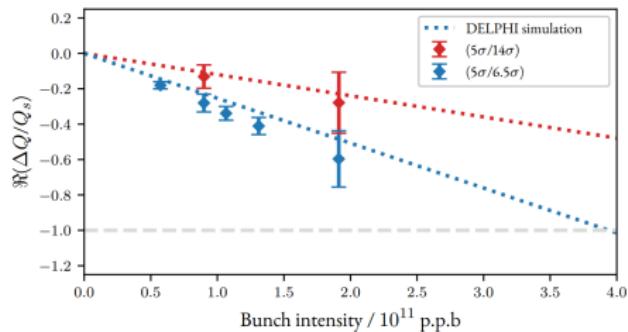


Figure: tune shift measurements in LHC [9]
 (blue: nominal LHC collimator configuration, red: HL-LHC-like impedance)

C. Impact of Space Charge on Coherent Stability

Selected results from [15]

Head-tail Spectrum vs. Space Charge

Space charge modifies head-tail spectrum in bunched beam

- rigid mode $m = 0$ unaffected
- negative modes $m < 0$ are pushed lower

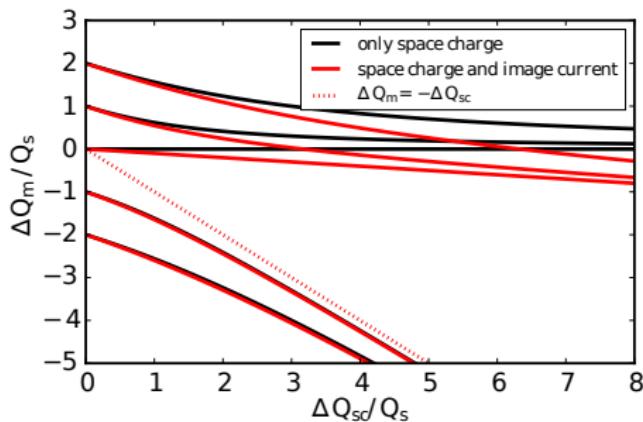


Figure: Head-tail mode tunes vs. space charge strength [7]

TMCI vs. Space Charge

Simulations for HL-LHC injection plateau at $Q'_{x,y} = 0$:

wakefield only, no space charge

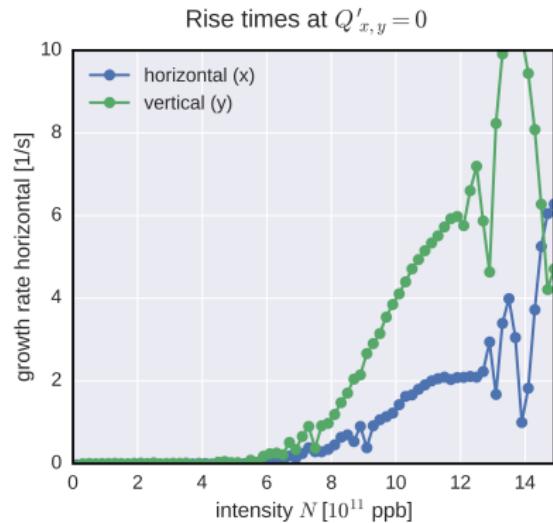
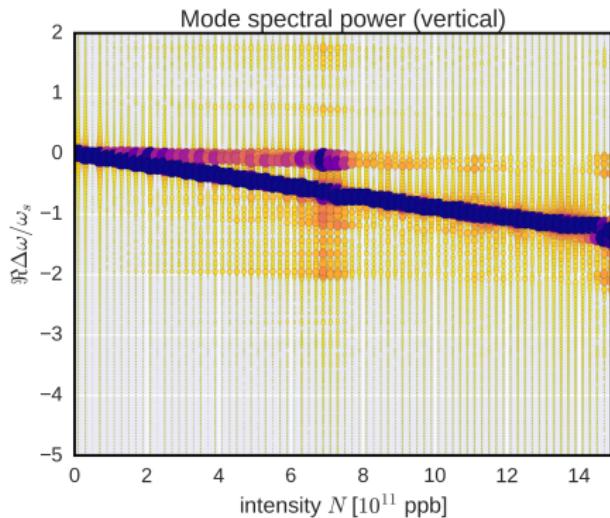


Figure: vertical TMCI above $N_{\text{TMCI,th}} \approx 6 \times 10^{11} \text{ ppb}$

TMCI vs. Space Charge

Simulations for HL-LHC injection plateau at $Q'_{x,y} = 0$:

wakefield + space charge

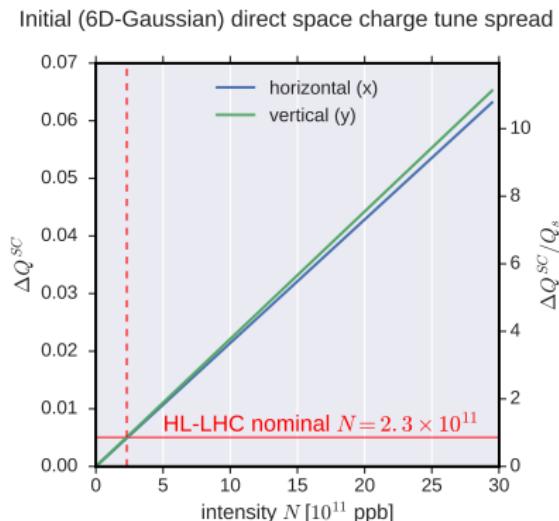
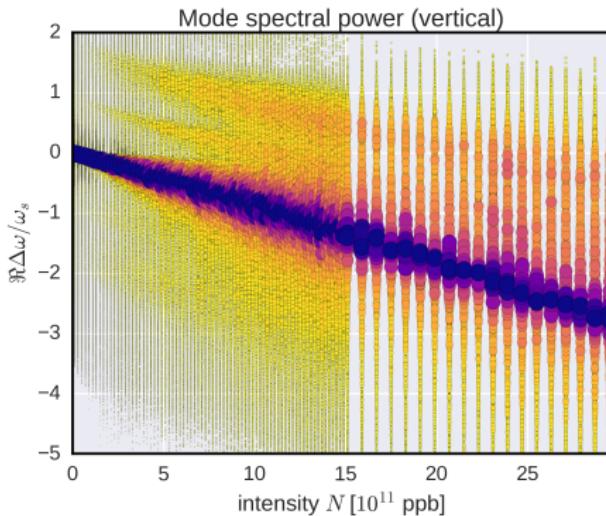


Figure: stable over 50 kturns, no mode coupling

Head-tail Instability vs. Space Charge

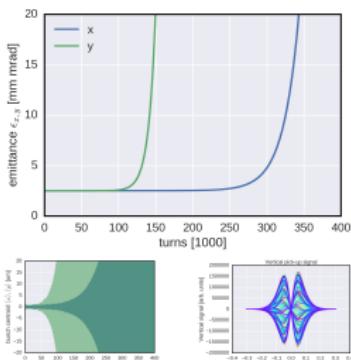


Simulations for HL-LHC injection plateau at $Q'_{x,y} = 5$ and $N < N_{\text{TMCI,th}}$:

Head-tail Instability vs. Space Charge

Simulations for HL-LHC injection plateau at $Q'_{x,y} = 5$ and $N < N_{\text{TMCI,th}}$:

wakefield

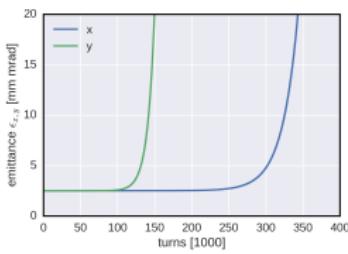


(a) 20000 turns rise time

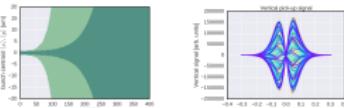
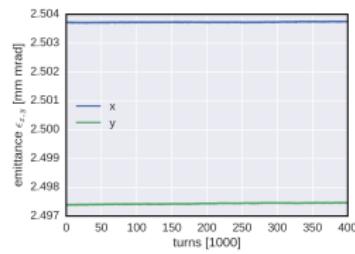
Head-tail Instability vs. Space Charge

Simulations for HL-LHC injection plateau at $Q'_{x,y} = 5$ and $N < N_{\text{TMCI,th}}$:

wakefield



space charge



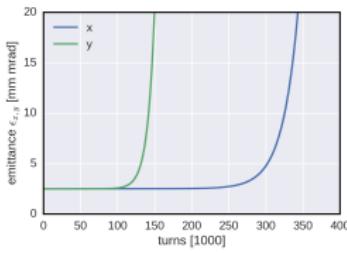
(a) 20000 turns rise time

(b) stable

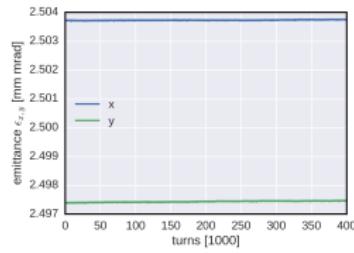
Head-tail Instability vs. Space Charge

Simulations for HL-LHC injection plateau at $Q'_{x,y} = 5$ and $N < N_{\text{TMCI,th}}$:

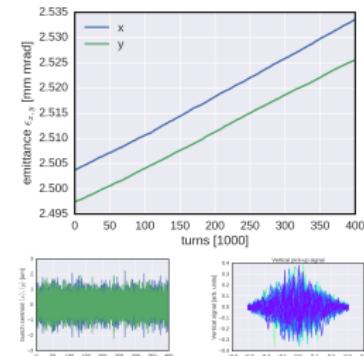
wakefield



space charge



wake + space charge



(a) 20000 turns rise time

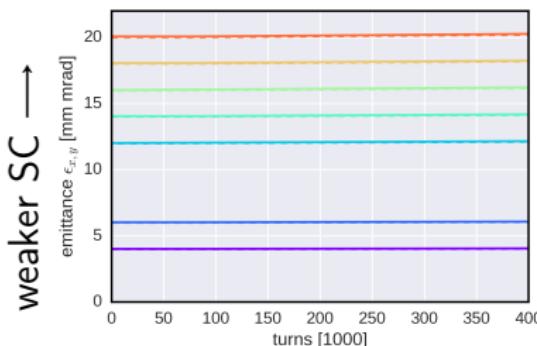
(b) stable

(c) stable!

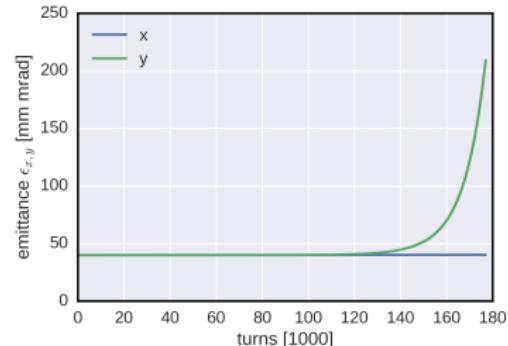
- no Landau octupole magnets, no transverse feedback!
- space charge non-linearity stabilises head-tail instability

Stability Limit

Weaken space charge ($\propto N/(\sigma_x + \sigma_y)$) at constant wake strength ($\propto N$)!
 \Rightarrow find stability limit by varying beam size via $\epsilon_{x,y}$



weaker
 \Rightarrow
 SC



- space charge decreases with $\Delta Q_{x,y}^{SC} \propto \frac{1}{\gamma^2 \epsilon_{x,y}}$
- **stable** until $\epsilon_{x,y} = 20 \text{ mm mrad} \Leftrightarrow E_{kin} = 1.27 \text{ TeV}$
- head-tail **instability** returns with rise time as without SC (20000 turns), $\epsilon_{x,y} = 40 \text{ mm mrad} \Leftrightarrow E_{kin} = 1.8 \text{ TeV}$

First Head-tail Instability in LHC

Observations from LHC in 2010:

- first ramp with a single-bunch of $N \approx 1 \times 10^{11}$ ppb
- 1-node $m = -1$ head-tail instability appeared around $E_{kin} = 1.8$ TeV
 - no transverse feedback on, no Landau octupoles
 - injection plateau stable conditions!
 - ⇒ hint at role of space charge in LHC!

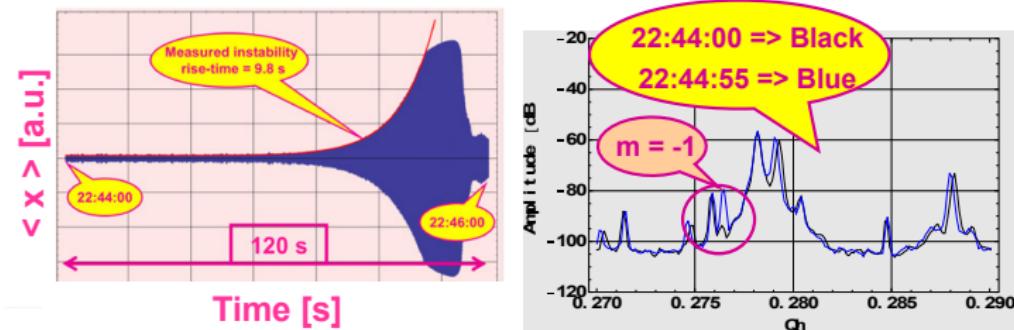


Figure: Dedicated instability measurement 2 days after first instability in LHC [10, 11]

Conclusions

Summary on head-tail modes:

- transverse dipole eigenmodes of bunches along longitudinal plane
- head-tail instability:
 - driven by machine impedance / wakefields
 - at finite chromaticity $Q'/Q \neq 0$
 - mitigation: chromaticity shift, Landau octupoles (tune spread), transverse feedback, strong coupling (for asymmetric growth rates)

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Current research questions:

- role of space charge in TMCI (SPS!)
- detailed mechanism for Landau damping of non-rigid head-tail modes by nonlinear space charge

Thank you for your attention!



References I

- [1] R Cappi. "Observation of high-order head-tail instabilities at the CERN-PS". In: *Part. Accel.* 50 (Mar. 1995), 117–124. 9 p. URL: <https://cds.cern.ch/record/279023>.
- [2] Elias Metral. "Coupled Landau damping of transverse coherent instabilities in particle accelerators". 1999. URL: <https://cds.cern.ch/record/1379849>.
- [3] Vladimir Kornilov. *Coupling Impedances in SIS100*. 2015. URL: https://wiki.gsi.de/pub/SIS100BD/BeamDynamicsReports/Impedances_SIS100.pdf.

References II

- [4] Y. Chin and K. Satoh. "Transverse Mode Coupling in a Bunched Beam". In: *IEEE Transactions on Nuclear Science* 30.4 (1983), pp. 2566–2568. DOI: 10.1109/TNS.1983.4332883.
- [5] Kohtaro Satoh and Yongho Chin. "Transverse mode coupling in a bunched beam". In: *Nuclear Instruments and Methods in Physics Research* 207.3 (1983), pp. 309–320. ISSN: 0167-5087. DOI: [https://doi.org/10.1016/0167-5087\(83\)90641-5](https://doi.org/10.1016/0167-5087(83)90641-5). URL: <https://www.sciencedirect.com/science/article/pii/0167508783906415>.
- [6] Alexander Wu Chao. "Physics of collective beam instabilities in high energy accelerators". In: *Wiley series in beam physics and accelerator technology* (1993).

References III

- [7] Rahul Singh. "Tune Measurement at GSI SIS-18: Methods and Applications". PhD thesis. Darmstadt: Technische Universität, 2014.
URL: <http://tuprints.ulb.tu-darmstadt.de/3976/>.
- [8] J. Gareyte and F. Sacherer. "Head-Tail Type Instabilities in the CERN PS and Booster". In: *9th International Conference on High-Energy Accelerators*. 1975, pp. 341–346.
- [9] David Amorim. "Study of the Transverse Mode Coupling Instability in the CERN Large Hadron Collider. Étude de l'instabilité de couplage des modes transverses dans le Grand Collisionneur de Hadrons du CERN". 2019. URL:
<https://cds.cern.ch/record/2707064>.

References IV

- [10] E Métral, B Salvant, and N Mounet. "Stabilization of the LHC single-bunch transverse instability at high-energy by Landau octupoles". In: *Proc. IPAC* (2011), pp. 04–09.
- [11] Elias Metral et al. "Measurement and interpretation of transverse beam instabilities in the CERN large hadron collider (LHC) and extrapolations to HL-LHC". In: (July 2016), TUAM2X01. 7 p. DOI: 10.18429/JACoW-HB2016-TUAM2X01. URL: <https://cds.cern.ch/record/2199121>.
- [12] Jacques Gareyte. "Transverse Mode Coupling Instabilities". In: *AIP Conf. Proc.* 592 (Oct. 2000), 260–278. 19 p. URL: <https://cds.cern.ch/record/477074>.

References V

- [13] Elias Metral. "Effect of space charge on CERN LHC and SPS transverse instabilities". In: *Space Charge Workshop, Darmstadt, Germany*. 2017. URL: <https://indico.gsi.de/event/5600/session/6/contribution/22/material/slides/0.pdf>.
- [14] Giovanni Rumolo. "Beam Instabilities II". In: *CERN Accelerator School, Trondheim, Norway*. 2013. URL: <https://cas.web.cern.ch/sites/cas.web.cern.ch/files/lectures/trondheim-2013/rumolo2.pdf>.
- [15] Adrian Oeftiger. "Single-Bunch Stability With Direct Space Charge". In: *Workshop on Impedances and Beam Instabilities* (Sept. 2017). URL: <https://cds.cern.ch/record/2290575>.