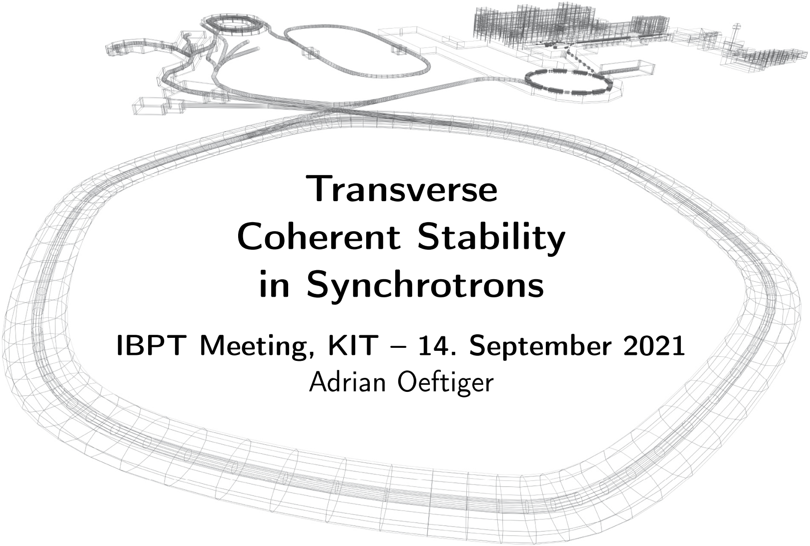


Transverse Coherent Stability in Synchrotrons

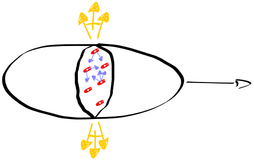
IBPT Meeting, KIT – 14. September 2021

Adrian Oeftiger

Intro

My area of research: collective effects in beam dynamics

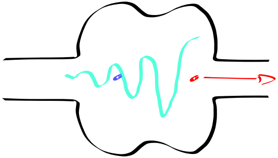
direct space charge (SC)



→ direct interaction of particles with beam self-field

→ modifies and broadens betatron resonances

beam coupling impedance



→ indirect interaction among particles via surrounding environment

→ responsible for most intensity-dependent instabilities

Overview

Today:

A. Space charge modelling for FAIR SIS100 (my current topic)
B. Transverse single-bunch instabilities (instructive)
   I. Head-tail instability
   II. Transverse mode coupling instability
C. Space charge impact on instabilities (my previous topic)

Not today:

- detailed theory, dispersion relations, derivations

A. Space Charge Modelling for FAIR SIS100

Facility for **A**ntiproton and **I**on **R**esearch:

- under construction at GSI, Germany
- key: heavy ion synchrotron SIS100

⟶ operation close to *space charge limit*

supersymmetry	$S = 6$
circumference	1083.6 m
particles	from $A = 1$ (protons) to $A = 238$ (U^{28+})
injection energy	200 MeV/u
extraction energy	≤2.7 GeV/u (U^{28+})
intensity	$\leq 5 \times 10^{11}\, U^{28+}$/cycle
max. SC tune shift	$\Delta Q_y^{SC} = -0.3$

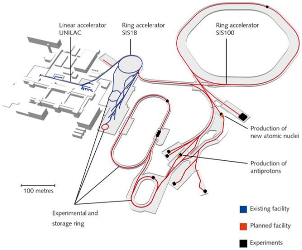
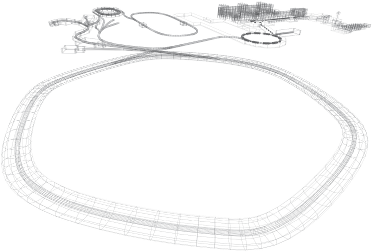
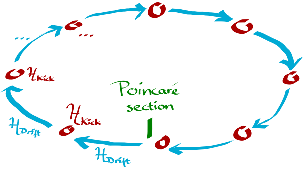


Figure: FAIR facility

Figure: SIS100 construction site June 2021, youtube drone video ↗

FAIR SIS100





Developed detailed SIS100 simulation model:

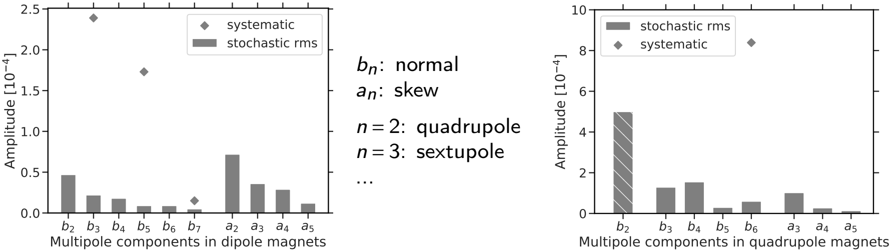
- code base:
  `PyHEADTAIL` (collective effects) + `SixTrackLib` (single-particle tracking)
- *non-linear* 3D space charge fields:
  - self-consistent particle-in-cell
  - frozen field map models
- full lattice in thin-lens treatment, 6D symplectic tracking
- GPU accelerated: fast frozen model computes 1 s storage time (160 000 turns) in ≈ 15 min

Magnet Bench Measurements

Field error model for dipole + quadrupole magnets:

- cold bench measurements with magnetometer coils

  $\Longrightarrow$ multipole expansion for magnets in series production
- plug into tracking simulation: statistical distribution of field errors for each magnet according to ensemble error model



b_n: normal
a_n: skew

$n = 2$: quadrupole
$n = 3$: sextupole
...

Figure: SIS100 field error model at U^{28+} injection energy

Space Charge Simulations

Long-term tracking simulations with 3D space charge:

- cover the length of bunch accumulation: 160 000 turns
- strong space charge for U^{28+} bunches: up to $\Delta Q_y^{SC} = -0.3$

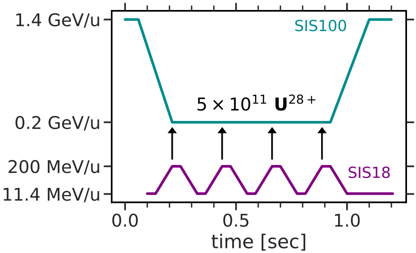


Figure: heavy ion accumulation in SIS100

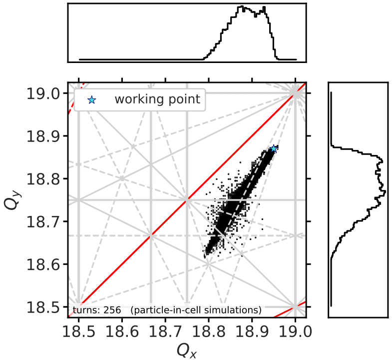
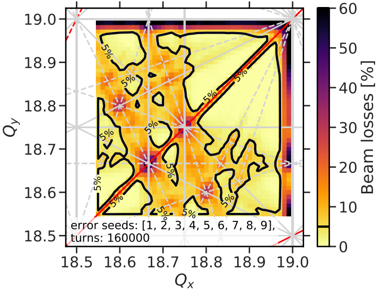


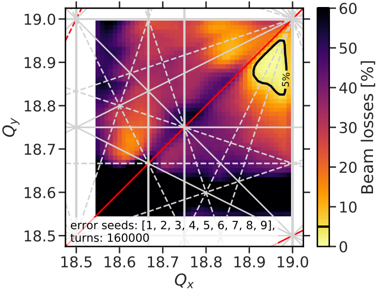
Figure: incoherent tune footprint

Beam loss predictions for heavy-ion fast extraction tune quadrant:



(a) no space charge, just magnet errors

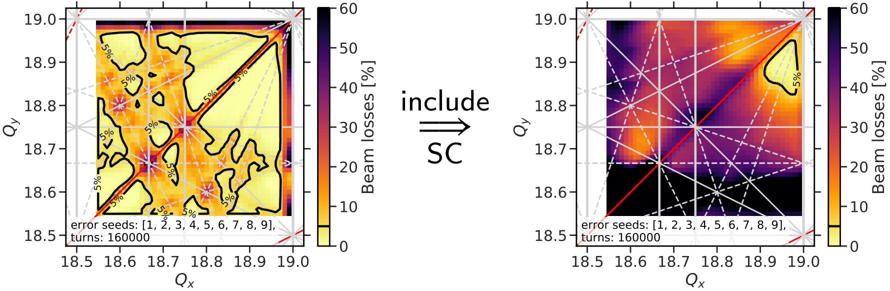
include
$\Longrightarrow$
SC



(b) space charge + magnet errors

Results

Beam loss predictions for heavy-ion fast extraction tune quadrant:



(a) no space charge, just magnet errors

(b) space charge + magnet errors

Powerful prediction model being used to

- identify ideal working point regions
- assess magnet quality (series production of quads just started!)
- develop intensity upgrades: space charge mitigation aspects

B. Coherent Stability vs. Impedances

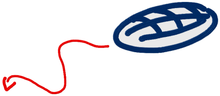
About Instabilities

"Beam distribution" $\doteq$ set of particles with 6D phase space coordinates

$$\zeta = (x, x', y, y, z, \delta)$$

"Collective instability" $\doteq$ some statistical moment of beam distribution $f(\zeta)$ grows *exponentially* in amplitude

1^{st} order: centroid



e.g. $\langle x \rangle = \int d^6\zeta \; x \cdot f(\zeta)$

2^{nd} order: envelope

e.g. $\sigma_x = \int d^6\zeta \; \left(x - \langle x \rangle\right)^2 \cdot f(\zeta)$

Typical instabilities...

Instabilities can be classified by

- longitudinal plane / *transverse plane*
- *single-bunch (short-range wake)* /
  coupled-bunch and/or multi-turn (long-range wake)

Among the most relevant transverse single-bunch instabilities:

- (slow) head-tail instability
- transverse mode coupling instability
  (TMCI or sometimes "fast" head-tail instability)

Part I: Head-tail Instability

Head-tail Instability in CPS I

CERN Proton Synchrotron in the 90s:

- preparing for LHC operation: higher intensities $N \approx 2 \times 10^{12}$ ppb
- observation of horizontal head-tail instabilities at injection [1]

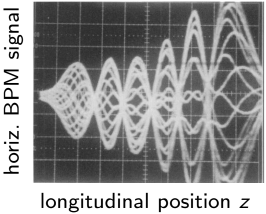


Figure: horizontal difference signal along bunch, consecutive turns

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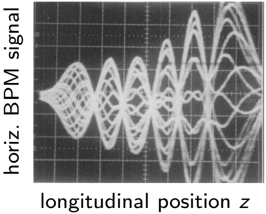


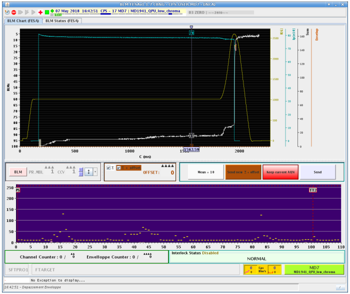
Figure: horizontal difference signal along bunch, consecutive turns

$\Longrightarrow$ stabilisation by "sharing" of Landau damping via strong coupling [2]:

- provide strong skew component a_2 in lattice: skew quadrupoles
- working point close to coupling line $Q_x \approx Q_y$
- Landau damped in vertical plane $\rightsquigarrow$ "share" with horizontal plane

CERN Proton Synchrotron in 2018:

- standard operation: natural chromaticity $Q'_{x,y} \doteq \frac{dQ_{x,y}}{d\delta} \approx -Q_{x,y}$
  - ⟶ strong coupling suppresses horizontal head-tail instability
  - ⟶ future: strong space charge, vertical integer resonance problematic!



(a) Stable beam!

Figure: MD1941, 7.5.2018 – preparation of low-chroma cycle

Head-tail Instability in CPS II

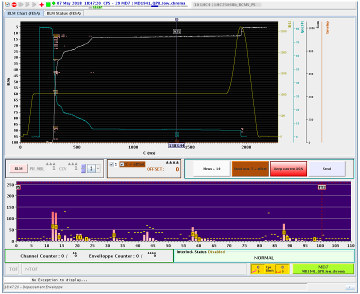
CERN Proton Synchrotron in 2018:

- future operation: correct vertical chromaticity $Q'_y \rightsquigarrow 0$
  - $\longrightarrow$ required to reduce integer resonance stop-band extent

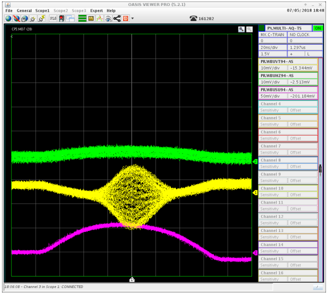
Figure: MD1941, 7.5.2018 – preparation of low-chroma cycle

CERN Proton Synchrotron in 2018:

- future operation: correct vertical chromaticity $Q'_y \rightsquigarrow 0$
  - ⟶ required to reduce integer resonance stop-band extent
  - ⟶ upgrade with new high-frequency transverse feedback for stabilisation



(b) Beam loss due to instability!



(c) Vertical rigid bunch mode (yellow)!

Figure: MD1941, 7.5.2018 – preparation of low-chroma cycle

Intermezzo...

Head-tail instability in bunched beam:

- growth rate scales with intensity, $\tau^{-1} \propto N$
- requires transverse to longitudinal coupling

  $\longrightarrow$ always present for finite chromaticity $Q'_{x,y} \doteq \frac{dQ_{x,y}}{d\delta}$

- can be mitigated via
  1. appropriate shift in chromaticity (evidently near-zero Q'_y worse than $Q'_y \approx -Q_y$)
  2. strong coupling: transverse sharing of Landau damping (skew quadrupoles, close tunes $Q_x \approx Q_y$, requires asymmetric $\tau_{x,y}$!)
  3. transverse feedback of appropriate bandwidth

...let's understand better...

Stored particles experience synchrotron like a large RLC circuit:

- impedance $Z(\omega) \leftrightarrow$ voltage drop w.r.t. motion of charged particles
- wakefield $W(t) \leftrightarrow$ electromagnetic field induced by passing particles

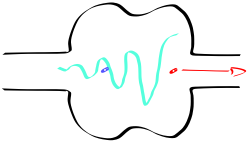


Figure: leading particle acting on trailing particle via longitudinal wake.

- impedance and wakefields split into longitudinal & transverse planes
  → related to each other via Panofsky-Wenzel theorem
  (implied by 3D Maxwell equations)

Wakefields and Impedances

Stored particles experience synchrotron like a large RLC circuit:

- impedance $Z(\omega) \leftrightarrow$ voltage drop w.r.t. motion of charged particles
- wakefield $W(t) \leftrightarrow$ electromagnetic field induced by passing particles

impedance = Fourier { wakefield }

$$Z_\perp(\omega) = \frac{iC}{\beta} \int_{-\infty}^{+\infty} dt \; e^{-i\omega t} \; W_\perp(t)$$

- impedance and wakefields split into longitudinal & transverse planes

  $\longrightarrow$ related to each other via Panofsky-Wenzel theorem (implied by 3D Maxwell equations)

Typical Wake Sources

Typical impedance models of accelerators comprise:

- resistive wall of vacuum chamber
- indirect space charge
  (induced mirror current in perfectly conducting vacuum chamber)
- broad-band impedance $Q \approx 1$ (bellows, BPMs, collimators, ...)
- narrow-band impedance $Q \gg 1$ (rf cavities, kickers, septa, ...)

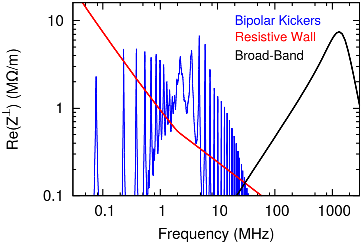


Figure: impedance model for FAIR SIS100, resistive part vs. frequency [3]

Two-particle Model: Results

Powerful analytical toy model with 2 particles: Refs. [4, 5, 6]

2 degrees of freedom $\implies$ 2 “coherent” head-tail modes:

- “+”: in-phase, rigid bunch mode $m = 0$
- “−”: anti-phase, head-tail mode $|m| = 1$

$\implies$ equal growth rates of opposite sign for constant wake W_0! (unstable vs. damped)

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$\Longrightarrow$ equal growth rates of opposite sign for constant wake W_0! (unstable vs. damped)

Growth rates of head-tail modes given by eigenvalues $\lambda_\pm \propto \exp(t/\tau)$:

$$\tau_\pm^{-1} = \mp \frac{N \frac{Q'}{Q} \tilde{z}}{p_0 \eta} \left(\frac{W_0}{C} \right)$$

for N (weak!) intensity, Q'/Q relative chromaticity, $\tilde{z}$ longitudinal amplitude, p_0 momentum, η slip factor, C circumference.

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2 degrees of freedom $\Longrightarrow$ 2 "coherent" head-tail modes:

- "+": in-phase, rigid bunch mode $m = 0$
- "−": anti-phase, head-tail mode $|m| = 1$

important relationship

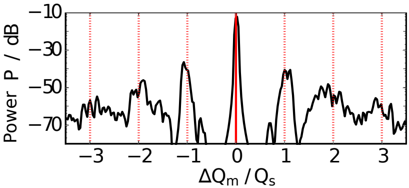
$$\tau_{\pm}^{-1} \sim \mp \frac{Q'/Q}{\eta}$$

	negative chroma $Q'/Q > 0$	positive chroma $Q'/Q < 0$
below transition $\eta < 0$	damped "+", unstable "−"	unstable "+", damped "−"
above transition $\eta > 0$	unstable "+", damped "−"	damped "+", unstable "−"

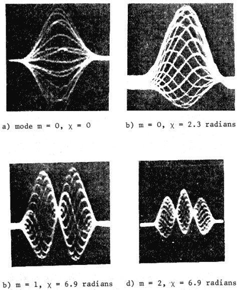
$\Longrightarrow$ always one unstable mode!

Fortunately, situation is a bit better for real bunches:

- spectrum of infinitely many head-tail modes
  - ⟶ instability conditions for rigid mode $m = 0$ as in 2-particle model
  - ⟶ role of "–" mode distributed among all non-rigid head-tail modes



(a) simulated centroid spectrum with head-tail modes [7]



(b) head-tail modes $|m| = 0, 1, 2$ measured in CERN PS [8]

Head-tail Modes in Bunches

Fortunately, situation is a bit better for real bunches:

- spectrum of infinitely many head-tail modes
  - $\longrightarrow$ instability conditions for rigid mode $m = 0$ as in 2-particle model
  - $\longrightarrow$ role of "–" mode distributed among all non-rigid head-tail modes
  - $\Longrightarrow$ $\sum_m 1/\tau_m = 0$, i.e. all non-rigid modes much weaker than $m = 0$

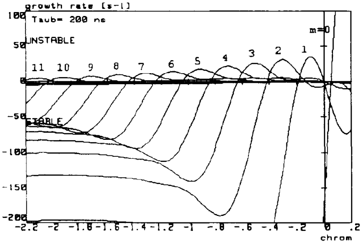


FIGURE 5: Instability growth rates for various modes (m = 0–11) versus chromaticity.

Figure: PS diagram for head-tail modes below transition $\eta < 0$ [1]

Head-tail Instabilities in LHC

CERN LHC operates above transition, $\eta > 0$.

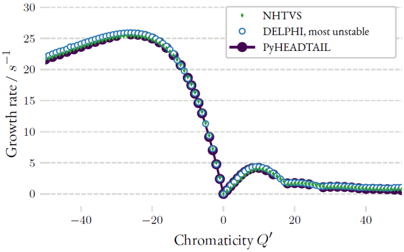


Figure: growth rates versus chromaticity [9]

Counter-measures:

- chromaticity adjustment to $Q'/Q > 0$ (sextupole magnets)
- transverse feedback
- Landau octupole magnets (increase transverse tune spread)
- impedance mitigation (e.g. Molybdenum coating of graphite collimators)

Observations from LHC in 2010:

- first ramp with a single-bunch of $N \approx 1 \times 10^{11}$ ppb
- 1-node $m = -1$ head-tail instability appeared around $E_{kin} = 1.8\,\mathrm{TeV}$
  - ⟶ no transverse feedback on, no Landau octupoles
  - ⟶ injection plateau stable conditions!

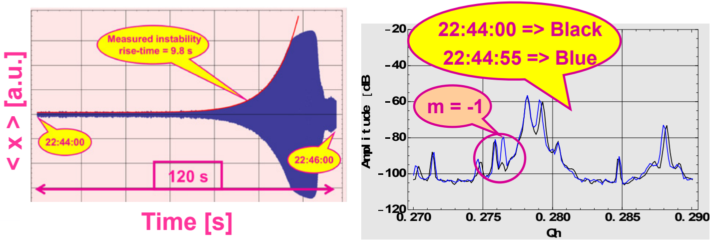


Figure: Dedicated instability measurement 2 days after first instability in LHC [10, 11]

Part II: Transverse Mode Coupling Instability

Overview TMCI

TMCI major limitation, poses a strict upper intensity threshold:

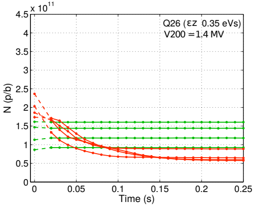
- "the ultimate transverse instability": single-bunch, $Q'/Q = 0$ [12]
- particularly for machines with short bunches

  $\Longrightarrow$ lepton machines are often affected (SPEAR II, PETRA, PEP, LEP, ...)

Overview TMCI

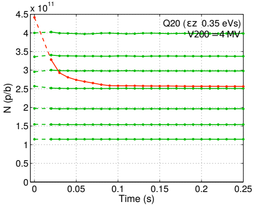
TMCI major limitation, poses a strict upper intensity threshold:

- "the ultimate transverse instability": single-bunch, $Q'/Q = 0$ [12]
- particularly for machines with short bunches
  - $\Longrightarrow$ lepton machines are often affected (SPEAR II, PETRA, PEP, LEP, ...)
- only one hadron machine with TMCI signature: CERN SPS (2003)
  - $\longrightarrow$ still subject of dispute in the community (effect of space charge!)
  - $\longrightarrow$ development of alternative optics over the course of several PhD theses successful: instability threshold shifted higher



(a) original Q26 optics ($\gamma_t = 22.8$)

faster $\overset{\Longrightarrow}{Q_s}$



(b) Q20 optics ($\gamma_t = 18$)

Figure: Measurements of intensity threshold from fast vertical instability in SPS [13]

Two-particle Model Revisited

Two-particle model illustrates mode coupling:

- zero chromaticity $Q'/Q = 0$
- $m = 0$ moves down, $|m| = 1$ moves up in frequency

$\Longrightarrow$ merging of modes = onset of TMCI

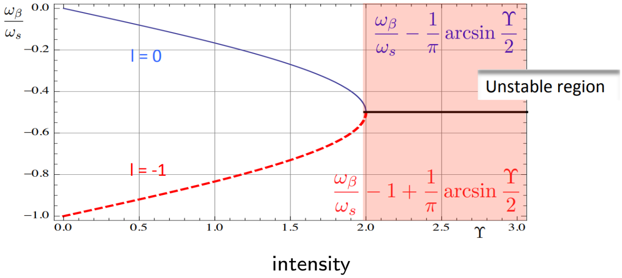


Figure: merging of modes $m = 0$ and $|m| = 1$ and TMCI threshold $\Upsilon = 2$ [14]

Two-particle Model Revisited

Two-particle model illustrates mode coupling:

- zero chromaticity $Q'/Q = 0$
- $m = 0$ moves down, $|m| = 1$ moves up in frequency

$\Longrightarrow$ merging of modes = onset of TMCI

Stable eigenmodes below threshold intensity:

$$N \leq N_{thr} = \frac{8}{\pi e^2} \frac{p_0 \omega_s}{\left(\frac{W_0}{C}\right) \beta_y}$$

for $\omega_s/2\pi$ synchrotron frequency, β_y betatron function at impedance location.

Two-particle Model

PyHEADTAIL simulation example: TMCI jupyter notebook ↗

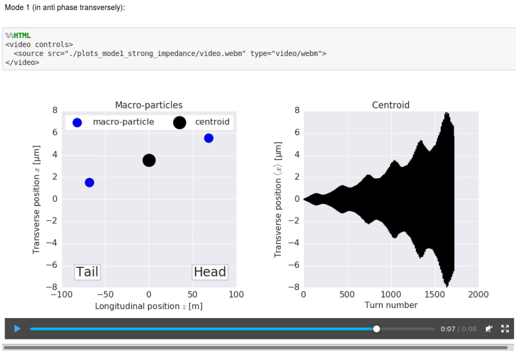


Figure: click me! ↗

TMCI in LHC

Since the SPS experience, the intensity upgrade for LHC is carefully analysed and adjusted to remain below TMCI threshold:

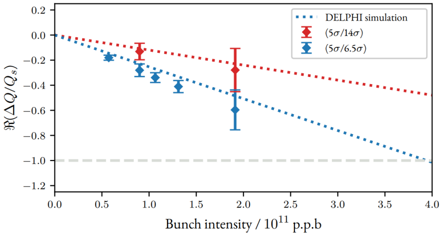


Figure: tune shift measurements in LHC [9]
(blue: nominal LHC collimator configuration, red: HL-LHC-like impedance)

C. Impact of Space Charge on Coherent Stability

Selected results from [15]

Head-tail Spectrum vs. Space Charge

Space charge modifies head-tail spectrum in bunched beam

⟶ rigid mode $m = 0$ unaffected

⟶ negative modes $m < 0$ are pushed lower

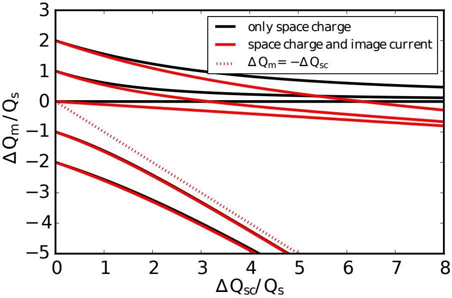
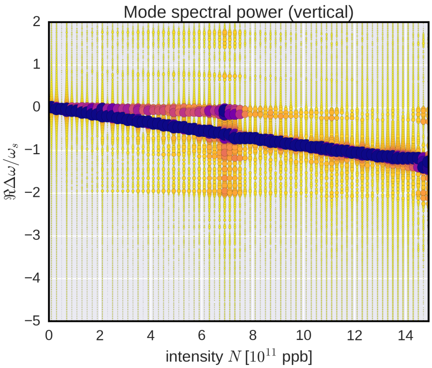


Figure: Head-tail mode tunes vs. space charge strength [7]

TMCI vs. Space Charge

Simulations for HL-LHC injection plateau at $Q'_{x,y} = 0$:

wakefield only, no space charge



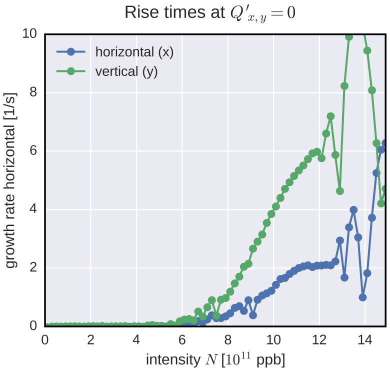
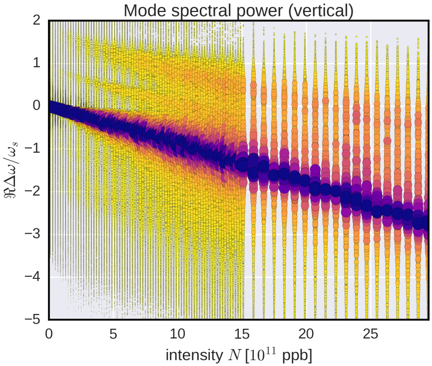


Figure: vertical TMCI above $N_{\mathrm{TMCI,th}} \approx 6 \times 10^{11}$ ppb

TMCI vs. Space Charge

Simulations for HL-LHC injection plateau at $Q'_{x,y} = 0$:

wakefield + space charge



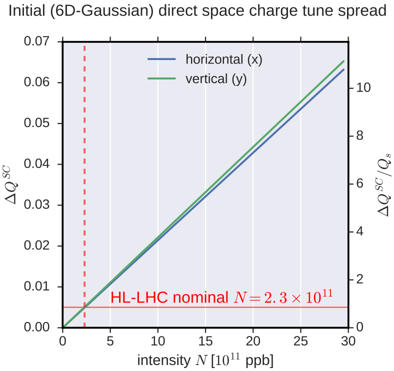


Figure: stable over 50 kturns, no mode coupling

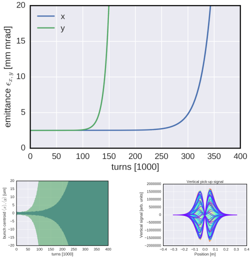
Head-tail Instability vs. Space Charge

Simulations for HL-LHC injection plateau at $Q'_{x,y} = 5$ and $N < N_{\mathrm{TMCI,th}}$:

Head-tail Instability vs. Space Charge

Simulations for HL-LHC injection plateau at $Q'_{x,y} = 5$ and $N < N_{\mathrm{TMCI,th}}$:

wakefield

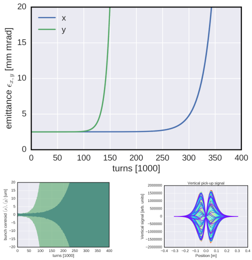


(a) 20 000 turns rise time

Head-tail Instability vs. Space Charge

Simulations for HL-LHC injection plateau at $Q'_{x,y} = 5$ and $N < N_{\mathrm{TMCI,th}}$:

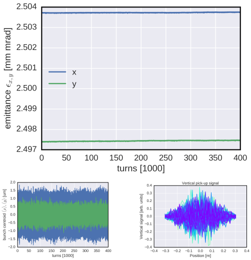
wakefield



(a) 20 000 turns rise time

+

space charge

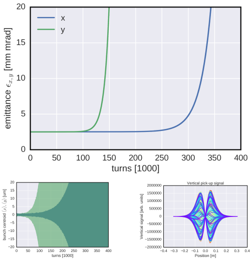


(b) stable

Head-tail Instability vs. Space Charge

Simulations for HL-LHC injection plateau at $Q'_{x,y} = 5$ and $N < N_{\mathrm{TMCI,th}}$:

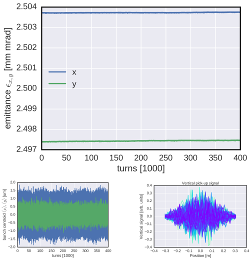
wakefield



(a) 20 000 turns rise time

+

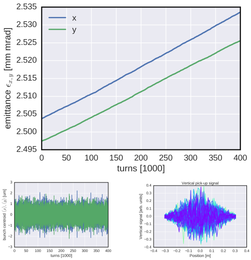
space charge



(b) stable

=

wake + space charge



(c) stable!

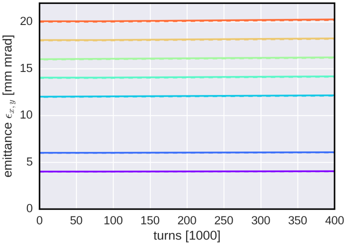
$\longrightarrow$ no Landau octupole magnets, no transverse feedback!

$\Longrightarrow$ space charge non-linearity stabilises head-tail instability

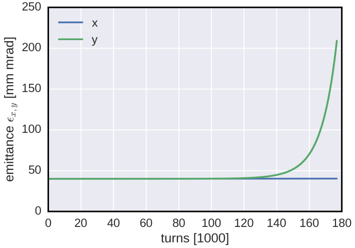
Stability Limit

Weaken space charge ($\propto N/(\sigma_x + \sigma_y)$) at constant wake strength ($\propto N$)!
$\Longrightarrow$ find stability limit by varying beam size via $\epsilon_{x,y}$

weaker SC $\longrightarrow$



weaker $\Longrightarrow$ SC



- space charge decreases with $\Delta Q^{SC}_{x,y} \propto \frac{1}{\gamma^2 \epsilon_{x,y}}$
- stable until $\epsilon_{x,y} = 20\,\text{mm mrad} \Leftrightarrow E_{kin} = 1.27\,\text{TeV}$
- head-tail instability returns with rise time as without SC (20 000 turns), $\epsilon_{x,y} = 40\,\text{mm mrad} \Leftrightarrow E_{kin} = 1.8\,\text{TeV}$

First Head-tail Instability in LHC

Observations from LHC in 2010:

- first ramp with a single-bunch of $N \approx 1 \times 10^{11}$ ppb
- 1-node $m = -1$ head-tail instability appeared around $E_{kin} = 1.8\,\text{TeV}$
  - $\longrightarrow$ no transverse feedback on, no Landau octupoles
  - $\longrightarrow$ injection plateau stable conditions!
  - $\Longrightarrow$ hint at role of space charge in LHC!

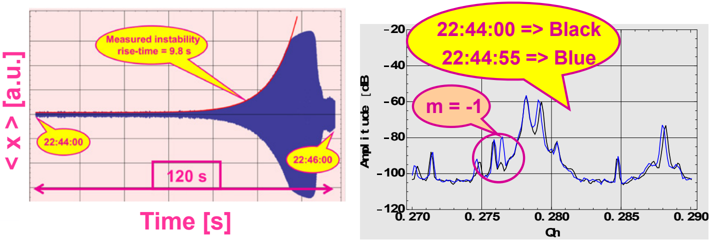


Figure: Dedicated instability measurement 2 days after first instability in LHC [10, 11]

Conclusions

Summary & Outlook

Summary on head-tail modes:

- transverse dipole eigenmodes of bunches along longitudinal plane
- head-tail instability:
  - driven by machine impedance / wakefields
  - at finite chromaticity $Q'/Q \neq 0$
  - $\longrightarrow$ mitigation: chromaticity shift, Landau octupoles (tune spread), transverse feedback, strong coupling (for asymmetric growth rates)

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  - reason for intensity limitation in many large storage rings
  - only way to mitigate: shift threshold (e.g. via faster Q_s)

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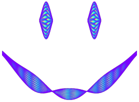
Current research questions:

- role of space charge in TMCI (SPS!)
- detailed mechanism for Landau damping of non-rigid head-tail modes by nonlinear space charge

Thank you for your attention!



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