MODEL REGRESI LAIN

Tim Dosen MK. Analisis Regresi
Politeknik Statistika STIS

BERBAGAI MODEL REGRESI LAIN

1. Model Log-Log (Double log)
2. Model Semi Log
   - Model log-lin
   - Model lin-log

MODEL LOG-LOG (DOUBLE LOG)

- Misalkan persamaan model regresi linier sederhana

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, i = 1, \ldots, n$$

  X= Harga mobil per unit, Y=Kuantitas yang diminta

- Berapa permintaan jika harga mobil = 0 rupiah?
- Apa mungkin suatu komoditas berharga 0 rupiah?
- Apa logis bila harga mobil per unit = 0, maka permintaan akan sebesar b_0?
- Diperlukan model lain!

MODEL LOG-LOG (DOUBLE LOG)

- Dikenal juga dengan model double log, model log linier atau model konstan elastisitas
- Menurut suatu teori ekonomi, hubungan antara kuantitas yang diminta dan harga suatu komoditas mempunyai bentuk sebagai berikut (berbentuk model regresi eksponensial):

$$Y = \beta_0 X^{\beta_1} exp^{\varepsilon}$$

  Y = kuantitas; X = harga; β_0, β_1=parameter; ε= error
- Model tidak linier dalam variabel dalam bentuk multiplikatif → estimasi rumit
- Untuk mempermudah, model ditransformasi

HASIL TRANSFORMASI LOGARITMA

- Transformasi dilakukan pada dua sisi → Model Log-Log (Double Log)
- Hasilnya menjadi

$$\ln Y = \ln \beta_0 + \beta_1 \ln X + \varepsilon$$

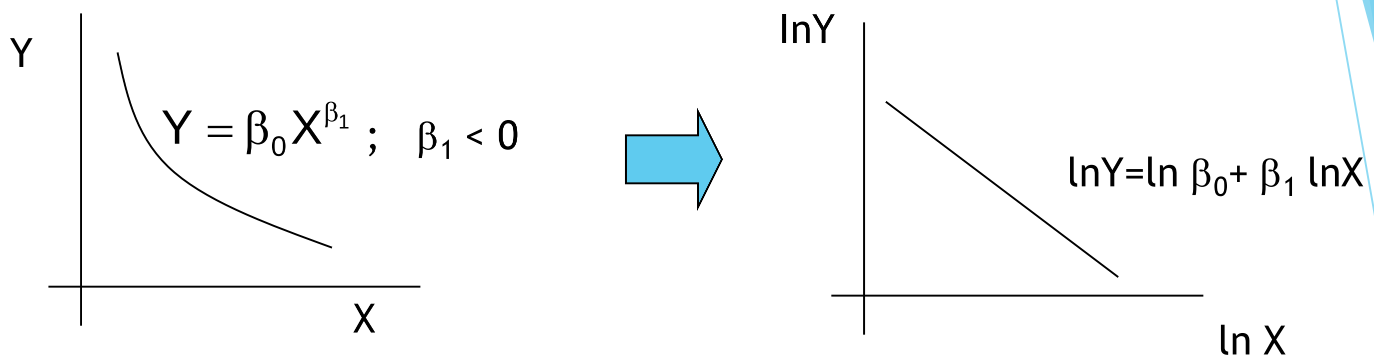
- Model dpt ditulis:

$$Y^* = \beta_0^* + \beta_1^* X^* + \varepsilon^*$$

dimana: $Y^* = \ln Y$ $X^* = \ln X$ $\beta_0^* = \ln \beta_0$ $\beta_1^* = \beta_1$ $\varepsilon^* = \varepsilon$

- Model menjadi model regresi linier → β_0^* dan β_1^* dapat ditaksir dengan OLS.

ILUSTRASI GEOMETRIS



- Bila harga komoditi (X) sangat mahal, maka permintaan akan minimal, yaitu $\exp(\beta_0)$, dan bila harga sangat murah, maka permintaan maksimal.
- Harga tidak akan pernah mencapai nilai nol, sehingga permasalahan dalam regresi linier dapat teratasi dengan fungsi ini.
- Model Log-Log ini tidak dapat dibentuk dari data yang mempunyai nilai = 0

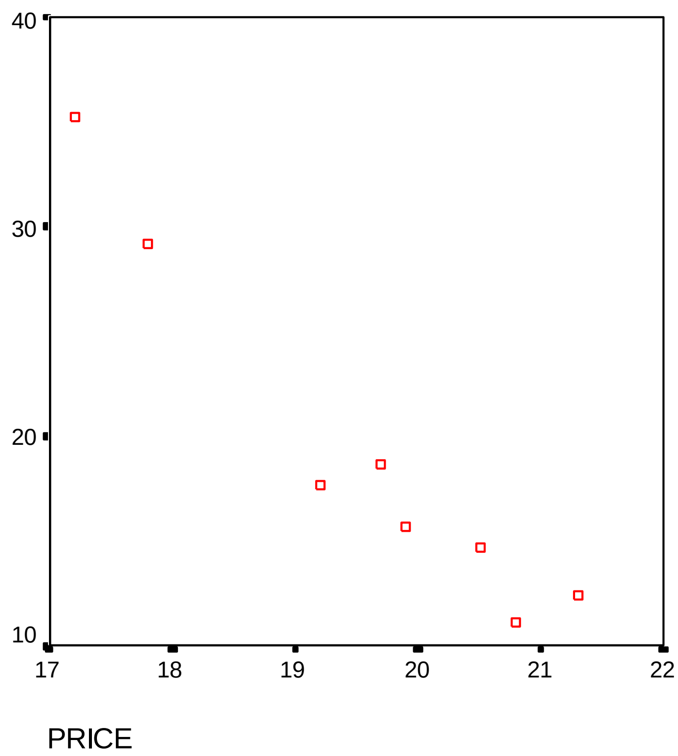
MODEL LOG-LOG vs MODEL LINIER

- Slope β_1 dalam model log-log menyatakan elastisitas Y terhadap X yaitu ukuran persentase perubahan dalam Y bila diketahui perubahan persentase X
- Dengan kata lain, bila Y menyatakan kuantitas yang diminta dan X menyatakan harga komoditas per unit maka β_1 menyatakan elastisitas harga pada permintaan
- β_0 juga bisa diinterpretasikan dengan mengembalikan model ke bentuk semula melalui $\exp(\beta_0^*)$

CONTOH

- Berikut data tentang harga dan penjualan softdrink

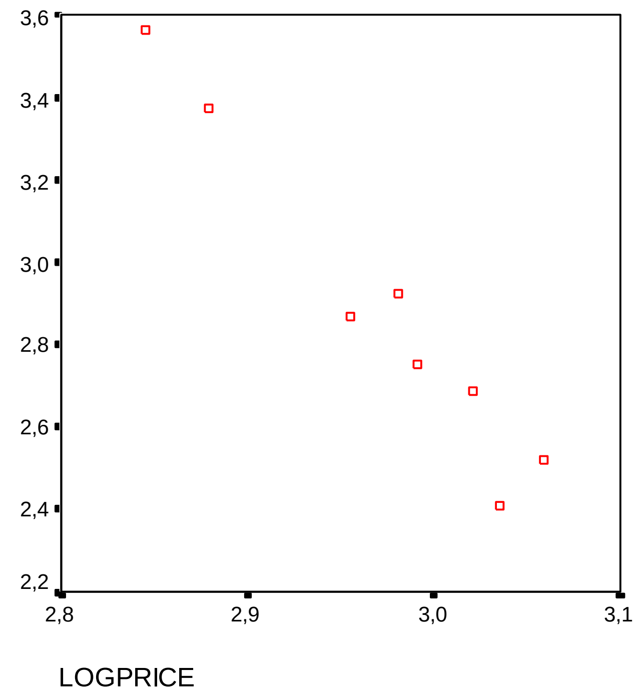
harga	penjualan
19.2	17.6
20.5	14.7
19.7	18.6
21.3	12.4
20.8	11.1
19.9	15.7
17.8	29.2
17.2	35.2



CONTOH

- Berikut data setelah dilakukan transformasi

log harga	log penjualan
2,95	2,87
3,02	2,69
2,98	2,92
3,06	2,52
3,03	2,41
2,99	2,75
2,88	3,37
2,84	3,56



CONTOH

Misalkan harga softdrink (X) dan penjualan softdrink (Y).

- Model Regresi Linier: Est. Y = 131,004 – 5,713 X
  s.e. (12,266) (0,626)
  R^2 = 0,9328 MSE = 5,634
- Model Log-Log: Est. ln Y = 18,357 – 5,208 ln X
  s.e. (1,385) (0,469)
  R^2 = 0,9535 MSE = 0,009

- Manakah model yang lebih baik?

ANALISIS

- Slope dan intercept kedua bentuk model berbeda.
- Interpretasinya:
  - **Model regresi linier**

    Bila harga softdrink naik sebesar 1 satuan, maka penjualan komoditi tsb akan turun 5,7 ≈ 6 unit.

  - **Model log-log**

    Setiap kenaikan harga softdrink sebesar 1%, jumlah penjualan akan turun 5,2 %.

CONTOH (Gujarati & Porter p. 161)

EXAMPLE 6.3
Expenditure on Durable Goods in Relation to Total Personal Consumption Expenditure

Table 6.3 presents data on total personal consumption expenditure (PCEXP), expenditure on durable goods (EXPDUR), expenditure on nondurable goods (EXPNONDUR), and expenditure on services (EXPSERVICES), all measured in 2000 billions of dollars.[13]

Suppose we wish to find the elasticity of expenditure on durable goods with respect to total personal consumption expenditure. Plotting the log of expenditure on durable goods against the log of total personal consumption expenditure, you will see that the relationship between the two variables is linear. Hence, the double-log model may be appropriate. The regression results are as follows:

$$\begin{aligned} \widehat{\ln \text{EXDUR}_t} &= -7.5417 + 1.6266 \ln \text{PCEX}_t \\ \text{se} &= (0.7161) \quad (0.0800) \\ t &= (-10.5309)^* \quad (20.3152)^* \qquad r^2 = 0.9695 \end{aligned} \tag{6.5.5}$$

where * indicates that the p value is extremely small.

CONTOH (Gujarati & Porter p. 161)

TABLE 6.3
Total Personal Expenditure and Categories (Billions of chained [2000] dollars; quarterly data at seasonally adjusted annual rates)

Sources: Department of Commerce, Bureau of Economic Analysis. *Economic Report of the President*, 2007, Table B-17, p. 347.

Year or quarter	EXPSERVICES	EXPDUR	EXPNONDUR	PCEXP
2003-I	4,143.3	971.4	2,072.5	7,184.9
2003-II	4,161.3	1,009.8	2,084.2	7,249.3
2003-III	4,190.7	1,049.6	2,123.0	7,352.9
2003-IV	4,220.2	1,051.4	2,132.5	7,394.3
2004-I	4,268.2	1,067.0	2,155.3	7,479.8
2004-II	4,308.4	1,071.4	2,164.3	7,534.4
2004-III	4,341.5	1,093.9	2,184.0	7,607.1
2004-IV	4,377.4	1,110.3	2,213.1	7,687.1
2005-I	4,395.3	1,116.8	2,241.5	7,739.4
2005-II	4,420.0	1,150.8	2,268.4	7,819.8
2005-III	4,454.5	1,175.9	2,287.6	7,895.3
2005-IV	4,476.7	1,137.9	2,309.6	7,910.2
2006-I	4,494.5	1,190.5	2,342.8	8,003.8
2006-II	4,535.4	1,190.3	2,351.1	8,055.0
2006-III	4,566.6	1,208.8	2,360.1	8,111.2

As these results show, the elasticity of EXPDUR with respect to PCEX is about 1.63, suggesting that if total personal expenditure goes up by 1 percent, on average, the expenditure on durable goods goes up by about 1.63 percent.

MODEL SEMI-LOG

- Prinsip model sama dengan model log-log, yaitu melakukan transformasi logaritma terhadap data.
- Bedanya, pada model semi-log data yang ditransformasi hanya salah satu dari Y atau X.

Model Semi Log terdiri atas dua jenis, yaitu:

- Model Log-Lin
- Model Lin-Log

MODEL LOG-LIN (GROWTH MODEL)

- Ekonom, pebisnis dan pemerintah kadang tertarik untuk mengetahui tingkat pertumbuhan variabel ekonomi tertentu seperti GNP, jumlah uang beredar dll
- Misalkan kita ingin mengetahui tingkat pertumbuhan pengeluaran konsumsi. Kita dapat menggunakan formula compound interest dari teori ekonomi:

$$Y_t = Y_0(1+r)^t$$

  Dimana r= laju pertumbuhan dari Y

- Jika ditransformasi akan menjadi

$$\ln Y = \ln Y_0 + t\ln(1+r)$$

MODEL LOG-LIN (GROWTH MODEL)

- Anggap $\beta_0 = \ln Y_0$ dan $\beta_1 = \ln(1+r)$
- Sehingga persamaan dapat ditulis menjadi

$$\ln Y = \beta_0 + \beta_1 t$$

- Persamaan sudah berbentuk linier sehingga kita dapat mengestimasi β_0 dan β_1 dengan OLS
- Interpretasi:

β_1 merupakan rasio antara perubahan relatif Y terhadap perubahan absolut X, dituliskan sebagai berikut :

$$\beta_1 = \frac{\text{perubahan relatif dalam Y}}{\text{perubahan absolut dalam X}}$$

MODEL LOG-LIN (GROWTH MODEL)

Penggunaan:

- Variabel X menyatakan unit waktu (tahun, bulan, dan seterusnya)
- Y dapat menyatakan pengangguran, penduduk, keuntungan, penjualan, GNP, dan sebagainya.
- β_1 merupakan suatu **ukuran pertumbuhan *(growth rate)*** bila $\beta_1 > 0$, atau β_1 merupakan suatu **ukuran penyusutan *(decay)*** bila $\beta_1 < 0$
- Oleh karenanya, model ini disebut juga **model pertumbuhan**.

CONTOH

- Misalkan berdasarkan data pertumbuhan Produk Nasional Bruto (PNB) atas dasar harga konstan (pertumbuhan riil) tahun 1986 – 2004 di suatu negara, diperoleh persamaan:

  ln PNB = 6,9636 + 0,0796 Tahun

  s.e. (0,0151) (0,0017)

  R^2 = 0,9756

- Analisis:

  Persamaan tersebut menyatakan bahwa $\hat{\beta}_1$ = 0,0796. Artinya, setiap tahunnya PNB naik/tumbuh 7,96 % pada periode 1986 – 2004.

CONTOH (Gujarati $ Porter p. 163)

EXAMPLE 6.4

The Rate of Growth Expenditure on Services

To illustrate the growth model (6.6.6), consider the data on expenditure on services given in Table 6.3. The regression results over time (t) are as follows:

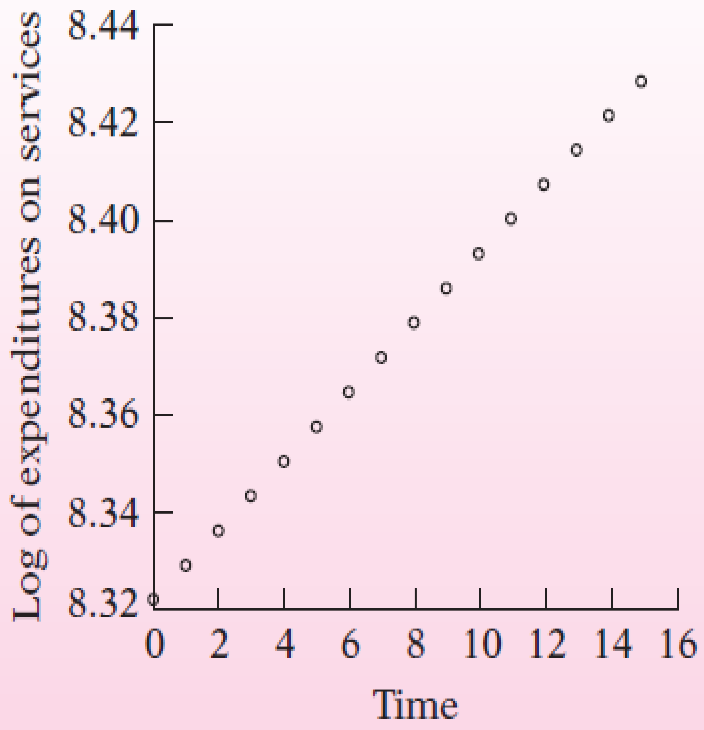
$$
\begin{aligned}
\widehat{\ln \text{EXS}_t} &= \quad 8.3226 \quad + \quad 0.00705t \\
\text{se} &= \quad (0.0016) \qquad (0.00018) \qquad r^2 = 0.9919 \\
t &= (5201.625)^* \quad (39.1667)^*
\end{aligned}
\tag{6.6.8}
$$

Note: EXS stands for expenditure on services and * denotes that the p value is extremely small.

The interpretation of Equation 6.6.8 is that over the quarterly period 2003-I to 2006-III, expenditures on services increased at the (quarterly) rate of 0.705 percent. Roughly, this is equal to an annual growth rate of 2.82 percent. Since 8.3226 = log of EXS at the beginning of the study period, by taking its antilog we obtain 4115.96 (billion dollars) as the beginning value of EXS (i.e., the value at the beginning of 2003). The regression line obtained in Eq. (6.6.8) is sketched in Figure 6.4.

CONTOH (Gujarati $ Porter p. 163)

FIGURE 6.4



MODEL LIN-LOG

- Model ini telah digunakan dalam model pengeluaran Engel
- $Y = \beta_0 + \beta_1 \ln X + \varepsilon$

  β_1 merupakan ukuran rasio antara perubahan absolut Y terhadap perubahan relatif X, dituliskan sebagai berikut :

$$\beta_1 = \frac{\text{perubahan absolut dalam Y}}{\text{perubahan relatif dalam X}}$$

- Digunakan pada situasi dimana perubahan relatif pada X akan mengakibatkan perubahan absolut pada Y.
- Misal: Perusahaan mempunyai target omset, maka kita dapat melihat kenaikan keuntungan.

ILUSTRASI

- Misal diperoleh persamaan yang menunjukkan hubungan antara laba (dalam juta) dan omset:

  Laba = 1040,1105 + 24,9879 Ln Omset

  s.e. (18,8574) (2,0740)

  R^2 = 0,9236

- Interpretasi:

  Setiap Omset naik 1% maka laba akan naik sebesar 0,24 juta rupiah.

- Bagaimana jika perusahaan menargetkan tahun depan omset naik 5%?

CONTOH (Gujarati $ Porter p. 165)

EXAMPLE 6.5

As an illustration of the lin–log model, let us revisit our example on food expenditure in India, Example 3.2. There we fitted a linear-in-variables model as a first approximation. But if we plot the data we obtain the plot in Figure 6.5. As this figure suggests, food expenditure increases more slowly as total expenditure increases, perhaps giving credence to Engel's law. The results of fitting the lin–log model to the data are as follows:

$$\begin{aligned} \widehat{\text{FoodExp}}_i &= -1283.912 \quad + 257.2700 \ln \text{TotalExp}_i \\ t &= \quad (-4.3848)^* \qquad (5.6625)^* \qquad r^2 = 0.3769 \end{aligned} \tag{6.6.14}$$

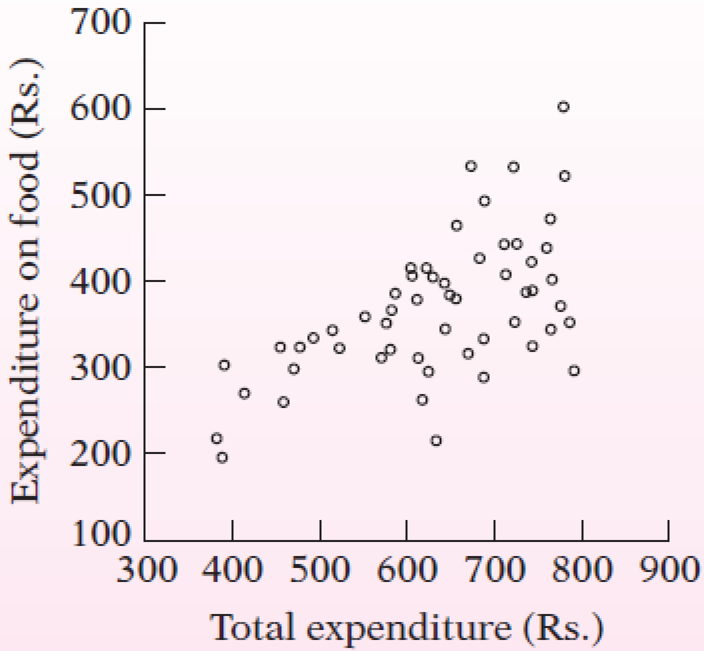
Note: * denotes an extremely small *p value*.

CONTOH (Gujarati $ Porter p. 165)

Observation	Food Expenditure	Total Expenditure	Observation	Food Expenditure	Total Expenditure
1	217.0000	382.0000	29	390.0000	655.0000
2	196.0000	388.0000	30	385.0000	662.0000
3	303.0000	391.0000	31	470.0000	663.0000
4	270.0000	415.0000	32	322.0000	677.0000
5	325.0000	456.0000	33	540.0000	680.0000
6	260.0000	460.0000	34	433.0000	690.0000
7	300.0000	472.0000	35	295.0000	695.0000
8	325.0000	478.0000	36	340.0000	695.0000
9	336.0000	494.0000	37	500.0000	695.0000
10	345.0000	516.0000	38	450.0000	720.0000
11	325.0000	525.0000	39	415.0000	721.0000
12	362.0000	554.0000	40	540.0000	730.0000
13	315.0000	575.0000	41	360.0000	731.0000
14	355.0000	579.0000	42	450.0000	733.0000
15	325.0000	585.0000	43	395.0000	745.0000
16	370.0000	586.0000	44	430.0000	751.0000
17	390.0000	590.0000	45	332.0000	752.0000
18	420.0000	608.0000	46	397.0000	752.0000
19	410.0000	610.0000	47	446.0000	769.0000
20	383.0000	616.0000	48	480.0000	773.0000
21	315.0000	618.0000	49	352.0000	773.0000
22	267.0000	623.0000	50	410.0000	775.0000
23	420.0000	627.0000	51	380.0000	785.0000
24	300.0000	630.0000	52	610.0000	788.0000
25	410.0000	635.0000	53	530.0000	790.0000
26	220.0000	640.0000	54	360.0000	795.0000
27	403.0000	648.0000	55	305.0000	801.0000
28	350.0000	650.0000			

CONTOH (Gujarati $ Porter p. 165)

FIGURE 6.5



Interpreted in the manner described earlier, the slope coefficient of about 257 means that an increase in the total food expenditure of 1 percent, on average, leads to about 2.57 rupees increase in the expenditure on food of the 55 families included in the sample. (*Note:* We have divided the estimated slope coefficient by 100.)

TUGAS RESPONSI

To study the relationship between investment rate (investment expenditure as a ratio of the GDP) and savings rate (savings as a ratio of GDP), Martin Feldstein and Charles Horioka obtained data for a sample of 21 countries. (See Table 6.8.) The investment rate for each country is the average rate for the period 1960–1974 and the savings rate is the average savings rate for the period 1960–1974. The variable Invrate represents the investment rate and the variable Savrate represents the savings rate.†

a. Plot the investment rate against the savings rate.

b. Based on this plot, do you think the following models might fit the data equally well?

$$\text{Invrate}_i = \beta_1 + \beta_2 \text{Savrate}_i + u_i$$
$$\ln \text{Invrate}_i = \alpha_1 + \alpha_2 \ln \text{Savrate}_i + u_i$$

Note : $u_i = error$

c. Estimate both of these models and obtain the usual statistics.

d. How would you interpret the slope coefficient in the linear model? In the log–linear model? Is there a difference in the interpretation of these coefficients?

e. Given the results of the two regression models, which model would you prefer? Why?

TUGAS RESPONSI

TABLE 6.8

	SAVRATE	INVRATE
Australia	0.250	0.270
Austria	0.285	0.282
Belgium	0.235	0.224
Canada	0.219	0.231
Denmark	0.202	0.224
Finland	0.288	0.305
France	0.254	0.260
Germany	0.271	0.264
Greece	0.219	0.248
Ireland	0.190	0.218
Italy	0.235	0.224
Japan	0.372	0.368
Luxembourg	0.313	0.277
Netherlands	0.273	0.266
New Zealand	0.232	0.249
Norway	0.278	0.299
Spain	0.235	0.241
Sweden	0.241	0.242
Switzerland	0.297	0.297
U.K.	0.184	0.192
U.S.	0.186	0.186

Note: SAVRATE = Savings as a ratio of GDP.
INVRATE = Investment expenditure as a ratio of GDP.