ORIE 3300/5300: Optimization I (Linear Programming)

Recitation 4

Fall 2020
Last Updated: September 27, 2020
Due: Monday, October 5$^{\text{th}}$ at 12pm (noon) US Eastern Time

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This is the third, and final, recitation devoted to the Cornell Dairy Bar. Recall that previously, it had expanded from a single dairy bar serving just Cornell University to multiple facilities serving several regions. However, its planning horizon only focused on the upcoming semester, without regard to future semesters. A consequence of this is that the dairy bar did not explicitly manage inventories, and thus did not make use of the fact that demand for ice-cream fluctutates from semester to semester. By manufacturing excess ice-cream during a semester with low demand, the dairy bar will be able to rely on inventoried ice-cream to help meet high demand in future semester.

Ice-cream can be refrigerated for a long time before it is shipped to retailers, however it is still perishable. For that reason, the Cornell Dairy Bar's operating horizon is now divided into multiple quarters (`Quarters`). However, because demand is not known with certainty, the dairy bar will be updating projected demands quarterly, and resolving the linear program each quarter. This is what is known as a "rolling horizon" model, which is very common in applications. It means that at the beginning of a quarter, the planner solves a linear program that includes planning for multiple quarters ahead, but resolves at the beginning of the next quarter, as forecast demands, costs etc. may be updated (therefore, you will only need to write down and solve a **single** linear program for this lab).

The dairy bar's production plants have limited inventory capacity. Moreover, the capacity for each flavor (i.e., `f in Flavors`) varies from one quarter to the next, as some of the space is reserved for other items. The projected inventory capacity per production plant `p` is given by the parameter `inv_cap[p,q]`, giving the number of units of ice-cream that can be held from quarter `q` until the next one. In addition, ice-cream producted at one plant cannot be put into inventory at a different plant. Currently, the Dairy Bar has a number of containers in inventory (`current_inv[p, f]`) that can be used to fulfill demand in the first quarter of the planning horizon, or in quarters farther in the future.

Assume there are no explicit costs associated with containers in inventory, but there is a handling cost (`hand_cost`) for moving one unit of ice-cream into, or out of, inventory. This cost is shared among all production plants, all ice-cream flavors, and for every quarter. The Dairy Bar's management have decided that, at the end of the horizon of the linear programming model (i.e. at the end of the last quarter), the number of units of ice-cream in inventory should be exactly equal to the current inventory.

Before proceeding, download the files `lab4.mod` and `lab4.dat` from Canvas. As usual, you will have to complete `lab4.mod` for this lab.

Problem 1. Browsing through `lab4.mod`, you will notice that we have renamed some of the variables used in Lab 3 (i.e. `prod`, `ship`) and introduced a few new ones:

```
var inv {Plants, Flavors, Quarters};
var into {Plants, Flavors, Quarters};
var out_of {Plants, Flavors, Quarters};
```

The first of these variables is the inventory at the end of each quarter, the variable `into` represents the amount put into inventory during the quarter, and `out_of` represents the amount removed from inventory during the quarter.

(i) Given the above information and the per-unit handling cost `hand_cost`, what is the total handling cost (i.e. for all flavors and all plants) at a given quarter `q`?

(ii) Complete the objective function in `lab4.mod` so that it also takes into account the total handling cost *over all quarters*.

Problem 2 (Inventory constraints)**.** This part of the lab is asking you to encode various inventory constraints. In particular:

(i) recall that every production plant `p` has an inventory capacity `inv_cap[p, q]` for every quarter `q`, meaning that the total units of ice-cream in that plant *at the end of that quarter* should not exceed that number. Encode this in AMPL by filling in the constraint named `do_not_exceed_inventory_capacity` in `lab4.mod`.

(ii) in addition, the Dairy Bar management is asking that the inventory at the end of the last quarter is equal to the current inventory (`current_inv`) for each ice-cream flavor and production plant. Encode this in AMPL by filling in the constraint named `end_with_correct_amount_of_inventory`. (*Hint: the AMPL function* `last`*, used for accessing ordered sets, might be useful here.*)

Problem 3 (More inventory constraints)**.** The last part of the lab is about determining the amount of product handled during future quarters, as well as the inventory at the end of future quarters (recall that, during each quarter, product is shipped from plants to regions and new units are produced as well).

(i) Looking at `lab4.mod`, you will see that there are 2 constraints for each task (determining handling and inventory); the first of the two constraints (which is given) is about just the first quarter, while the second constraint (that you have to fill out) is meant to handle the future quarters. Why is the first quarter handled separately?

(ii) Using the constraints handling the first semester as a guide, fill in the other two constraints:

- `determine_amount_handled_in_future_quarters` to determine how much product is placed into and out of inventory during each future quarter, and
- `determine_inventory_at_end_of_future_quarters` to determine how much inventory remains at the end of each future quarter, after production and shipping are taken into account.

(*Hint: given a quarter* `q`*, you can access its preceding quarter using the syntax* `prev(q)`.)

To check for correctness, apply the completed model file to `lab4.dat` – the optimal value is approximately 28,790,635.