

# Multi-objective Optimization by Learning Space Partitions

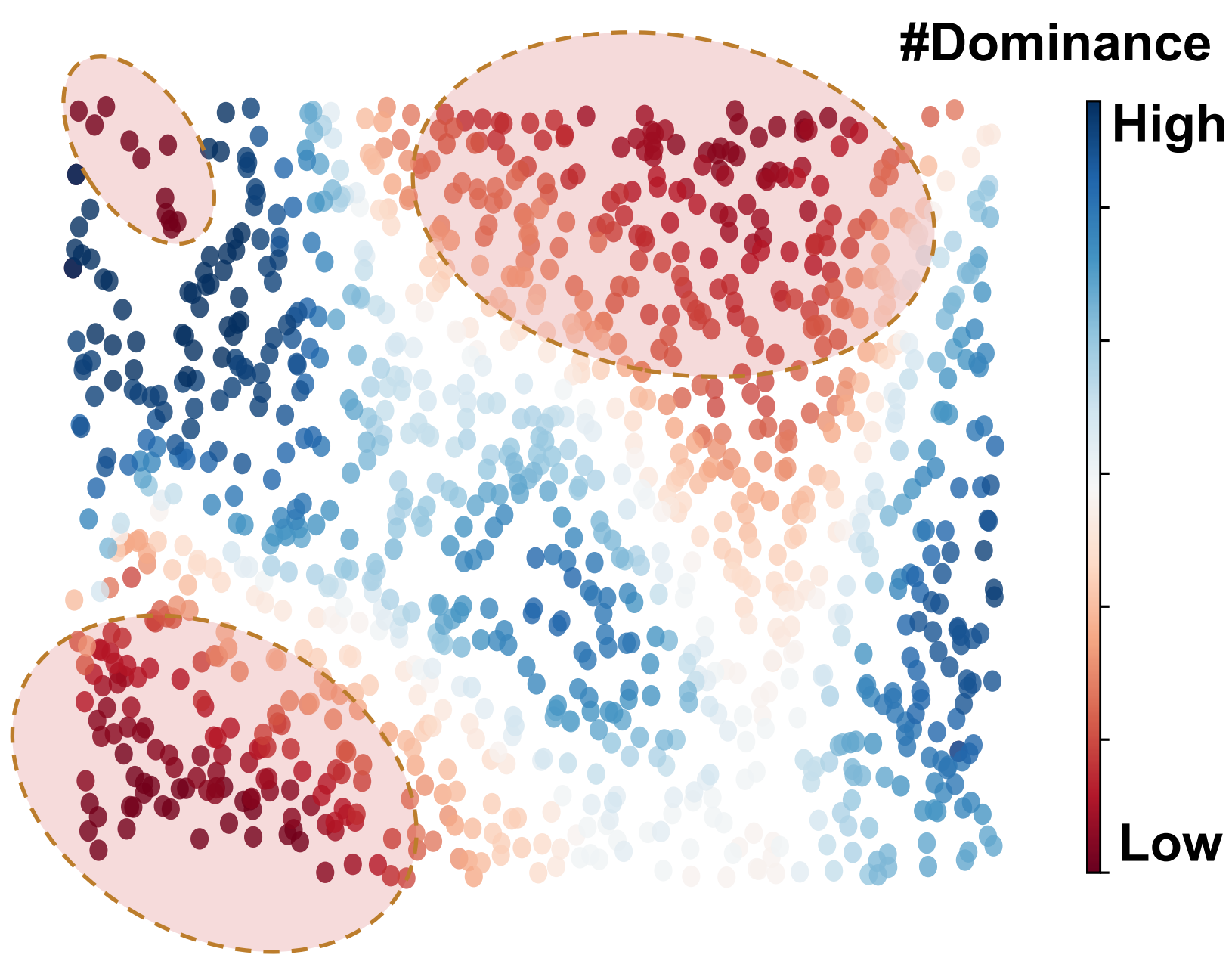
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## Motivation

Problem: Branin-Curran

$$f^{(1)}(x_1, x_2) = \left(15x_2 - \frac{5.1(15x_1 - 5)x^2}{4\pi^2} + \frac{75x_1 - 25}{\pi} - 5\right)^2 + \left(10 - \frac{10}{8\pi}\right) * \cos(15x_1 - 5)$$

$$f^{(2)}(x_1, x_2) = [1 - \exp(\frac{-1}{2x_2})] \frac{2300x_1^3 + 1900x_1^2 + 2092x_1 + 60}{100x_1^3 + 500x_1^2 + 4x_1 + 20}, \text{ where } (x_1, x_2) \in [0, 1]$$



**Observation:**

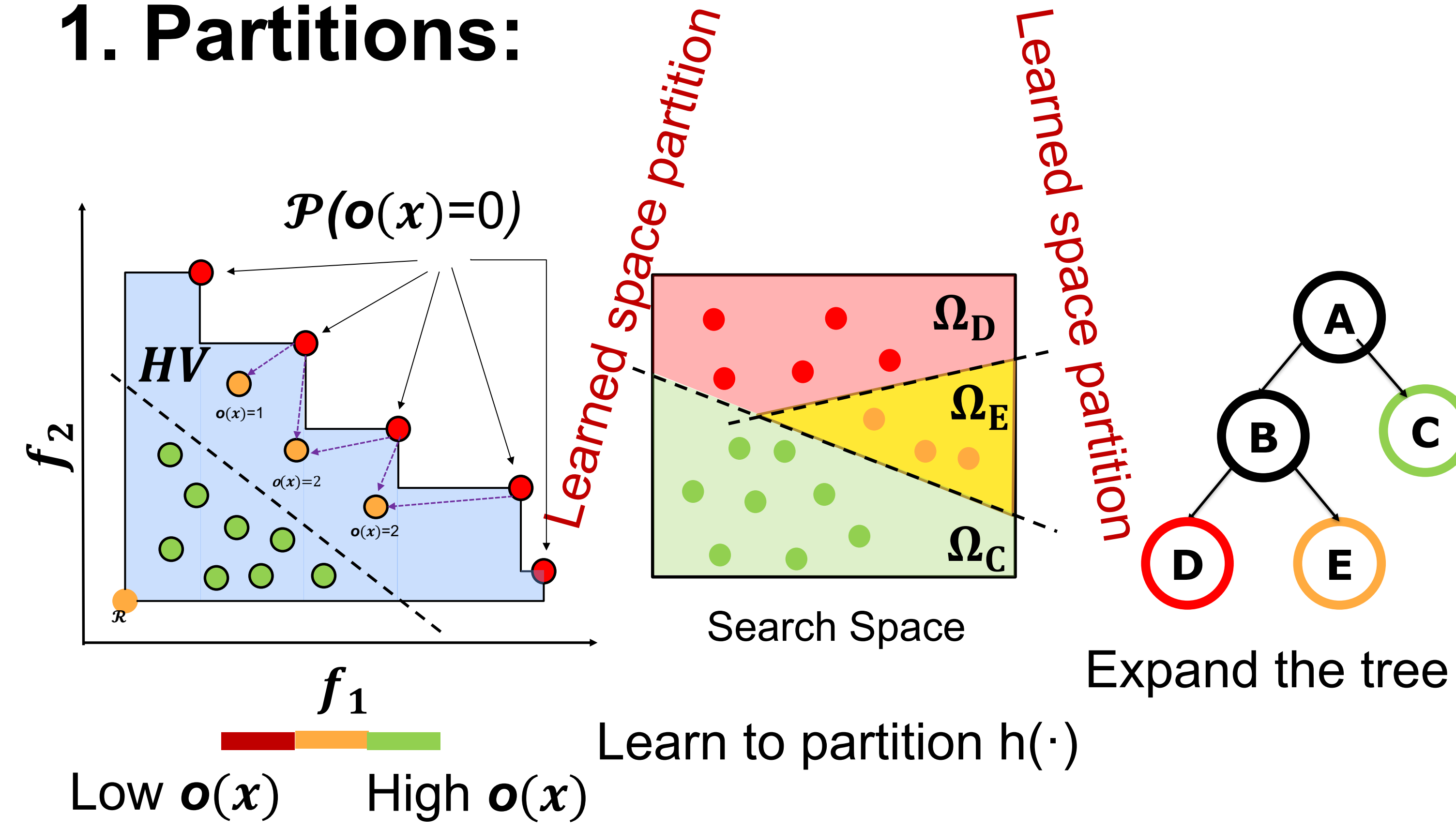
- The good samples are gathering in small regions (shaded areas).

**Intuition:**

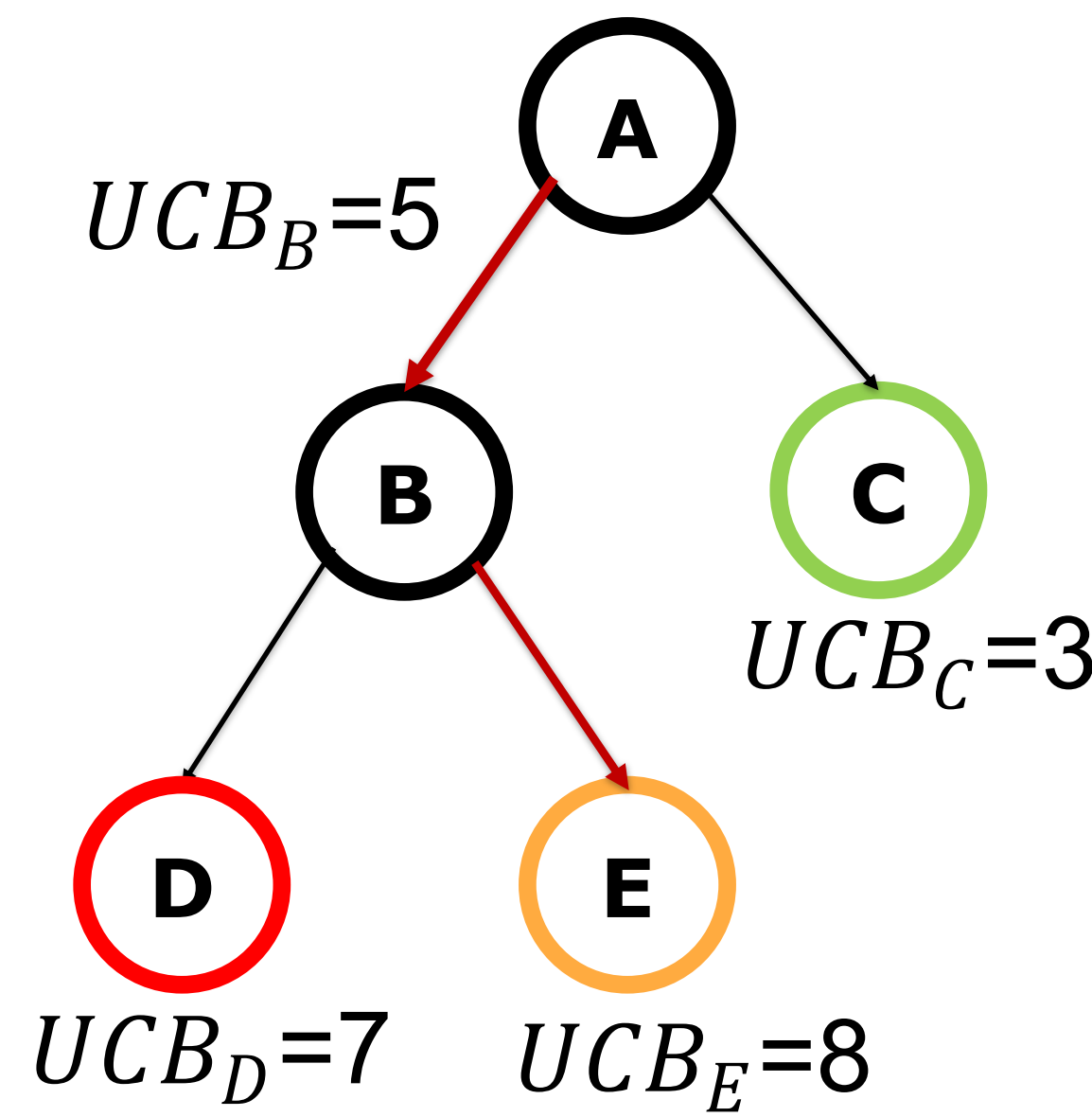
- The search space can be learned to partition by samples.

## Methodology

### 1. Partitions:

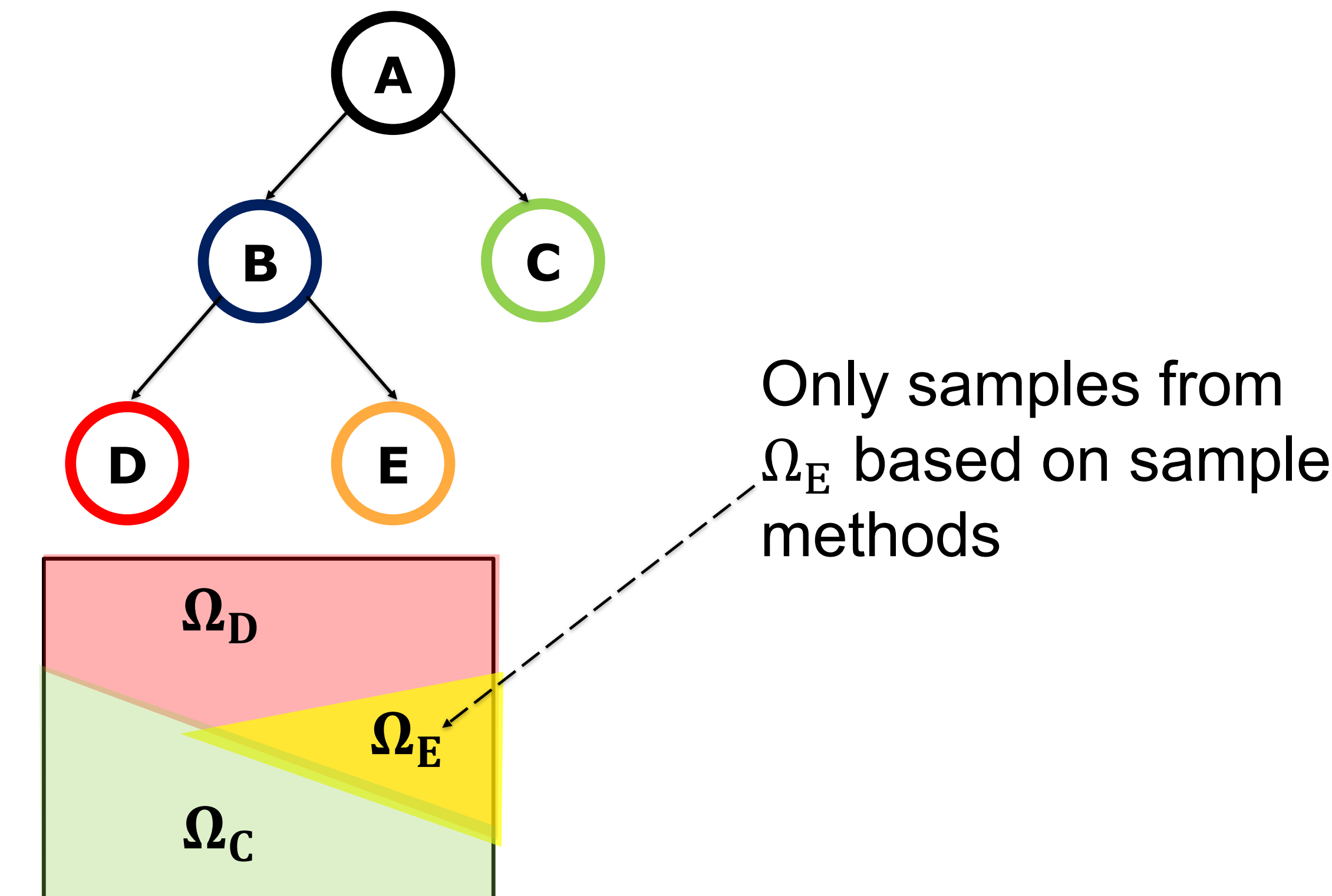


### 2. Select:



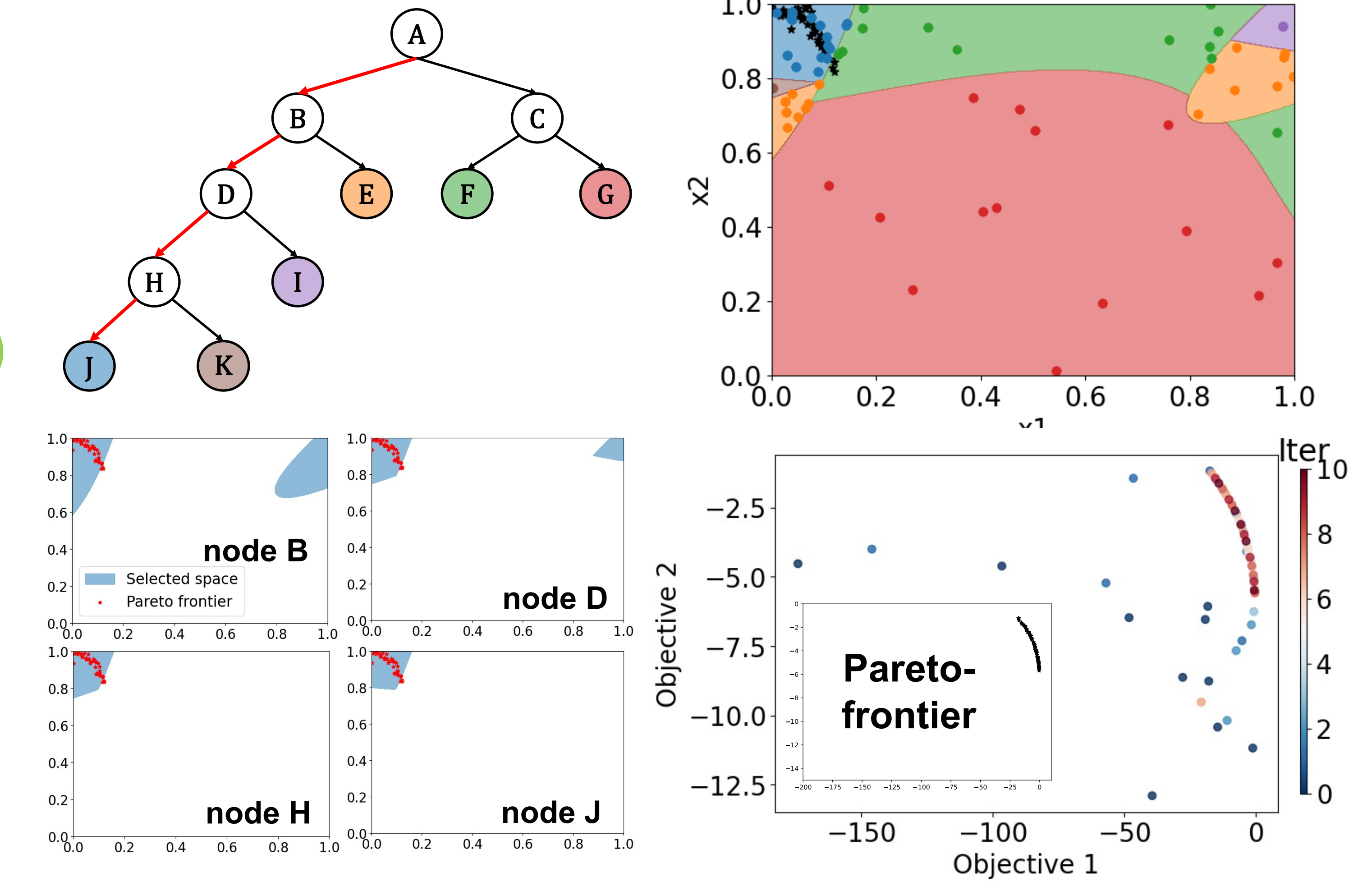
Select the leaf node with most UCB value to trade off the exploration and exploitation.

### 3. Sample:



## Visualization of LaMOO

Problem: Branin-Curran



## Learning Space Partitions

**Algorithm 1** LaMOO Pseudocode.

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1: Inputs: Initial  $D_0$  from uniform sampling, sample budget  $T$ .
2: for  $t = 0, \dots, T$  do
3:   Set  $\mathcal{L} \leftarrow \{\Omega_{\text{root}}\}$  (collections of regions to be split).
4:   while  $\mathcal{L} \neq \emptyset$  do
5:      $\Omega_j \leftarrow \text{pop\_first\_element}(\mathcal{L})$ ,  $D_{t,j} \leftarrow D_t \cap \Omega_j$ ,  $n_{t,j} \leftarrow |D_{t,j}|$ .
6:     Compute dominance number  $o_{t,j}$  of  $D_{t,j}$  using dominance numbers and train SVM model  $h(\cdot)$ .
7:     If  $(D_{t,j}, o_{t,j})$  is splittable by SVM, then  $\mathcal{L} \leftarrow \mathcal{L} \cup \text{Partition}(\Omega_j, h(\cdot))$ .
8:   end while
9:   for  $k = \text{root}$ ,  $k$  is not leaf node do
10:     $D_{t,k} \leftarrow D_t \cap \Omega_k$ ,  $v_{t,k} \leftarrow \text{HyperVolume}(D_{t,k})$ ,  $n_{t,k} \leftarrow |D_{t,k}|$ .
11:     $k \leftarrow \arg \max_{c \in \text{children}(k)} \text{UCB}_{t,c}$ , where  $\text{UCB}_{t,c} := v_{t,c} + 2C_p \sqrt{\frac{2 \log(n_{t,k})}{n_{t,c}}}$ 
12:  end for
13:   $D_{t+1} \leftarrow D_t \cup D_{\text{new}}$ , where  $D_{\text{new}}$  is drawn from  $\Omega_k$  based on qEHVI or CMA-ES.
14: end for

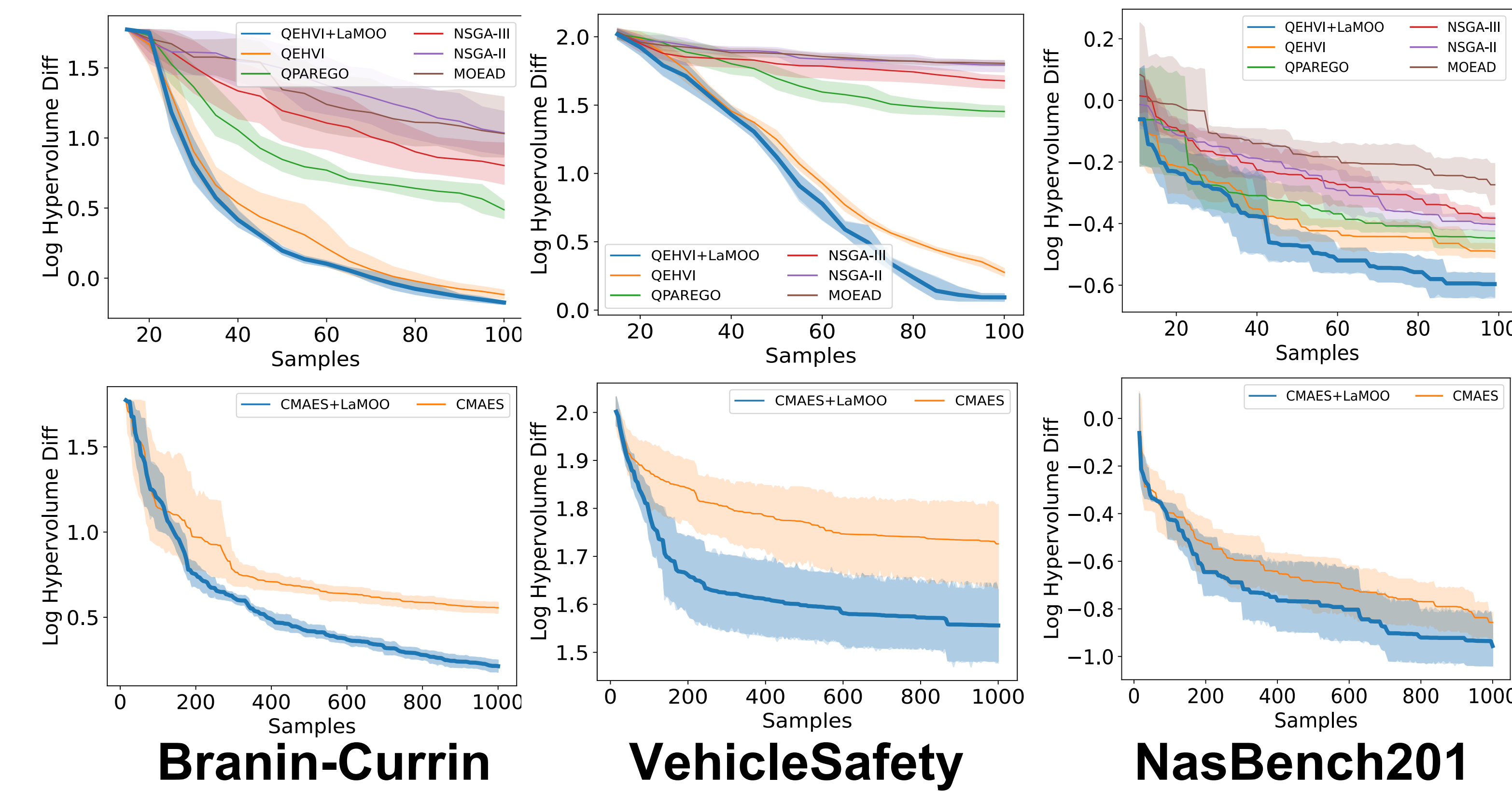
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**Learning Partitions:**

- Sort the dominance numbers of samples  $o(x)$  to get two groups of samples, i.e., good, bad.
- Leverage a SVM classifier to learn a boundary to partition the search space.

## Experiment Results

### 1. Small-scale Problems



### 2. Molecule Discovery

