Exercise 1: directed and switching graphs

```
# header to start
%matplotlib notebook

import numpy as np
import matplotlib.pyplot as plt

import scipy.linalg

import pickle
```

You are given four examples of sets of four **directed** graphs:

- In the first example, contained in the directory Exercise1_Graph10a, all the graphs contain 10 vertices
- In the second example, contained in the directory Exercise1_Graph10b, all the graphs contain 10 vertices
- In the third example, contained in the directory Exercise1_Graph100a, all the graphs contain 100 vertices
- In the last example, contained in the directory Exercise1_Graph100b, all the graphs contain 100 vertices

The following code can be used to load each graph and compute its Laplacian.

```
# to load the list of edges from file, you can do the following
with open('Question1_Graph10/graph1.pickle', 'rb') as f:
    E1 = pickle.load(f)

# the laplacian can then be found using
L = getLaplacian(E, 10, directed=True)

# note that it is possible to find the eigenvalues of a function using the function
# https://docs.scipy.org/doc/numpy/reference/generated/numpy.linalg.eigvals.html
# np.linalg.eigvals()

# it is possible to sort an array of numbers using
# https://docs.scipy.org/doc/numpy/reference/generated/numpy.sort.html
# np.sort()

# it is possible to find the exponential of a matrix using
# https://docs.scipy.org/doc/scipy/reference/generated/scipy.linalg.expm.html#scipy.linalg.expm
# scipy.linalg.expm()
```

Question 1

For all the graphs in all the examples, which graph contains a path from vertex 1 to vertex 10? What is then the minimum path length? Explain how you computed the answer.

Question 2

For each of these graphs, which one contains a rooted-out branching? Why?

Question 3

Assume that for each of the four examples, we implement the consensus protocol where the graphs are switching amoung the four possible graphs, one after the other (after graph 6 we switch back to graph 1). Which of these examples will lead to a converging consensus protocol? Why?

Question 4

Simulate the consensus protocol for the four switching graphs examples and verify your answer to the previous question (use random initial conditions uniformly distributed between -10 and 10). Assume that the graphs are switching every 0.1 seconds in a first case and every 1 seconds in the second case. Plot the state of every vertex as a function of time in a graph (one graph per example). How is the convergence speed influenced by switching time?

```
Start coding or generate with AI.
```