**SWARM ROBOTICS - ROB-GY 6333**

**Graph**

* Each **vertex** or **node** is a robot
* Each **edge** models the interaction between two robots
* The **graph** describes the connectivity properties of the system
* Many properties about the dynamics of the system can be inferred from the graph (without looking at differential equation)

**Undirected Graph**

An **undirected graph** G= (V, E) is composed of:

A black text on a white background

AI-generated content may be incorrect.

A math equation with black text

AI-generated content may be incorrect.



A diagram of a triangle with circles and lines

AI-generated content may be incorrect.

A group of math symbols

AI-generated content may be incorrect.

**Directed Graph**

**A white background with black text

AI-generated content may be incorrect.**

**A math equation with black text

AI-generated content may be incorrect.**

A diagram of a triangle with circles and lines

AI-generated content may be incorrect.

A group of letters and numbers

AI-generated content may be incorrect.

**Neighbor**

**A close up of words

AI-generated content may be incorrect.**

A close-up of a white background

AI-generated content may be incorrect.

A black text on a white background

AI-generated content may be incorrect.

**Degree**

**A close-up of a text

AI-generated content may be incorrect.**

**A black line with white text

AI-generated content may be incorrect.**

A close up of text

AI-generated content may be incorrect.

A diagram of a mathematical equation

AI-generated content may be incorrect.

**Path**

****

A **cycle\*\*** is a sequence of vertices, such that ***only*** the first and last vertex are equal.

A diagram of a path

AI-generated content may be incorrect. A diagram of a path

AI-generated content may be incorrect.

\*Walk, Trail, vs. Path: Walks can repeat both vertices and edges. Trails can repeat vertices but not edges.

\*\*Circuit vs Cycle. A circuit is any walk where the first and last vertex are equal. Note that a cycle is a trail, but not a path!

**Connectedness**

* A directed graph is **strongly connected** if there exists a **directed path** from any vertex to any other vertex
* A directed graph is **weakly connected** if there exists a path from any vertex to any other vertex when the graph is viewed as undirected

A diagram of connected lines

AI-generated content may be incorrect.A diagram of a weak connection

AI-generated content may be incorrect.

* A **tree** is a connected graph without any cycles.

A black line with circles

AI-generated content may be incorrect.

* A **forest** is a graph without any cycles (Not necessarily connected!)

A black circles with a white background

AI-generated content may be incorrect.

**A graph with lines and dots

AI-generated content may be incorrect.Special Graphs**

A diagram of a graph

AI-generated content may be incorrect.

**Adjacency Matrix A**

**A black text on a white background

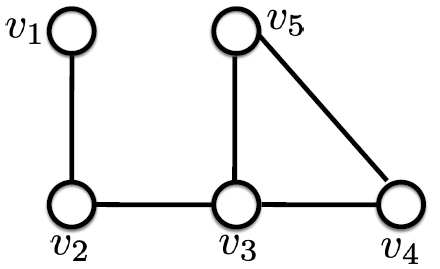
AI-generated content may be incorrect.**

A black text on a white background

AI-generated content may be incorrect.

Example:

A number and a mathematical equation

AI-generated content may be incorrect.

**Degree Matrix**

**A number and a mathematical equation

AI-generated content may be incorrect.**The degree matrix Δ ∈ ℝ(*n*×*n)* is a diagonal matrix with the node degrees as diagonal elements

**A math equation with black text

AI-generated content may be incorrect.**

**A diagram of a connection

AI-generated content may be incorrect.**

**Incidence (Only undirected graphs)**

****

**A black text on a white background

AI-generated content may be incorrect.**

**A diagram of a line with arrows and a red line

AI-generated content may be incorrect.**

**A number of numbers and digits

AI-generated content may be incorrect.**

**Laplacian Matrix L**

The Laplacian matrix L is the difference between the degree matrix and the adjacency matrix

A number of numbers and symbols

AI-generated content may be incorrect.A diagram of a triangle with circles and lines

AI-generated content may be incorrect.A math equations with lines and symbols

AI-generated content may be incorrect.

A number and a mathematical equation

AI-generated content may be incorrect.

A number with numbers on it

AI-generated content may be incorrect.

A screenshot of a math book

AI-generated content may be incorrect.

A white background with black and red text

AI-generated content may be incorrect.

A white background with black text

AI-generated content may be incorrect.

**Control Theory**

**A black and white image of a sign

AI-generated content may be incorrect.**

**A white background with black text

AI-generated content may be incorrect.**

**A white background with black text

AI-generated content may be incorrect.**

**A black text on a white background

AI-generated content may be incorrect.**

**A math equations and formulas

AI-generated content may be incorrect.**

**Real Symmetric Matrices**

**A mathematical symbols and symbols

AI-generated content may be incorrect.**

**A black text on a white background

AI-generated content may be incorrect.**



A math equation with black text

AI-generated content may be incorrect.

Jordan Normal:

A mathematical equations and formulas

AI-generated content may be incorrect.

A white text with black text

AI-generated content may be incorrect.

**Weighted Directed Graphs**

**A diagram of mathematical equations

AI-generated content may be incorrect.**

**A close up of black text

AI-generated content may be incorrect.**

**Connectivity**

**A white background with black text

AI-generated content may be incorrect.**

**Consensus**

**A close-up of a word

AI-generated content may be incorrect.**

Initial states is x\_0

A math equations and formulas

AI-generated content may be incorrect.

A math equations and formulas

AI-generated content may be incorrect.

A math equations on a white background

AI-generated content may be incorrect.

A group of math equations

AI-generated content may be incorrect. A black and red text on a white background

AI-generated content may be incorrect.

Because of how Laplacians are created, all its eigenvalues must be non-negative.

**Stability**

**A math equations on a white background

AI-generated content may be incorrect.**

**Symmetric Laplacian (Undirected Graph)**

**A white paper with black text and numbers

AI-generated content may be incorrect.**

A close up of words

AI-generated content may be incorrect.

1. **Intuition:** A directed graph might not arrive at a consensus
2. **Math:** Gershgorin’s Theorem tells us that the worst we can expect are zero eigenvalues, but it does not tell us the number of zero eigenvalues.
3. **Directed Graphs**
4. A white background with black text

   AI-generated content may be incorrect.

A red arrow pointing towards a circle

AI-generated content may be incorrect. A close-up of a white background

AI-generated content may be incorrect.

A screenshot of a math problem

AI-generated content may be incorrect.

**Rooted out-branching**

A math equations with red text

AI-generated content may be incorrect.A white background with black text

AI-generated content may be incorrect.

**Consensus Protocol on Digraphs**

A math equations and formulas

AI-generated content may be incorrect.

**Balanced Digraph**

**A white background with black text

AI-generated content may be incorrect.**

A white background with black text

AI-generated content may be incorrect.

A diagram of a graph

AI-generated content may be incorrect.A white text with black text

AI-generated content may be incorrect.

**Connected**

**A white text with black text

AI-generated content may be incorrect.**

Find local control rule to get agents to **agree on the same value** in a distributed manner

A close up of a text

AI-generated content may be incorrect. A table of equations with red text

AI-generated content may be incorrect.

A close up of a word

AI-generated content may be incorrect.

A white background with black text and numbers

AI-generated content may be incorrect.

* The rate of (exponential) convergence is defined by the second smaller (real part) eigenvalue of *L*, also known as algebraic connectivity or Fiedler eigenvalue

**Stability Review For Undirected Graphs**

**A math equations on a white background

AI-generated content may be incorrect.A diagram of a mathematical equation

AI-generated content may be incorrect.**

**Lypaunov Stability**

* the origin (*x*= 0) is stable if a trajectory starting close to it does not go away as time increases
* the origin (*x*= 0) is **asymptotically stable** if a trajectory starting sufficiently close to it converges to it as time increases

A close up of words

AI-generated content may be incorrect.

A white background with black text

AI-generated content may be incorrect.

**The consensus protocol with Lyapunov**

**A math equations and formulas

AI-generated content may be incorrect.**when graph is connected

**Switching Connected graphs**

**A black text on a white background

AI-generated content may be incorrect.A black and white math equation

AI-generated content may be incorrect.**

**Formation Specification**

Specification: set of desired relative distances between agents, The constraints that define the formation can always be written as relative distance constraints between agents (pair-wise geometrical constraints).

A math equation with black text

AI-generated content may be incorrect.

**Framework**

****

**A math equations on a white background

AI-generated content may be incorrect.**

**Rigidity Mapping**

**A math equations and formulas

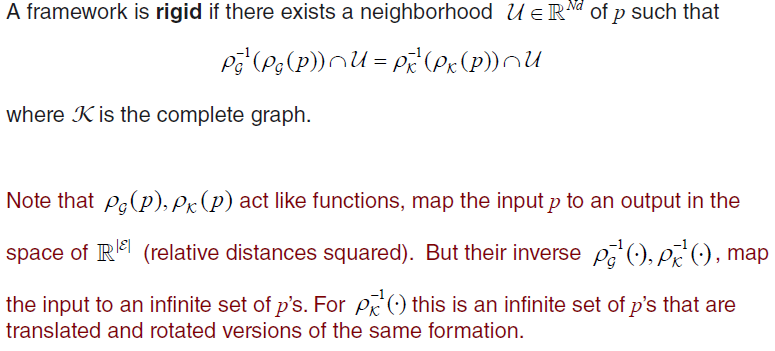
AI-generated content may be incorrect.**

****

**A math equations and formulas

AI-generated content may be incorrect.**

**1stDefinition of Rigidity**

****

**A black and white text

AI-generated content may be incorrect.**

**Equivalence and Congruence of Frameworks**

**A black text on a white background

AI-generated content may be incorrect.**

****

**Infinitesimal Rigidity**

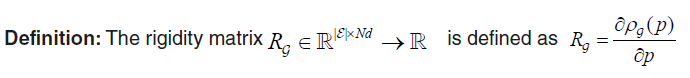
A close up of text

AI-generated content may be incorrect.

A black and red text

AI-generated content may be incorrect.

**Rigidity Matrix**

****

**A screenshot of a computer

AI-generated content may be incorrect.**

**A white background with black text

AI-generated content may be incorrect.**

**A black text on a white background

AI-generated content may be incorrect.**

**A black text on a white background

AI-generated content may be incorrect.**

**A math equations on a white background

AI-generated content may be incorrect.**

**A math equation with black text

AI-generated content may be incorrect.**