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Homework-4
FEG904 - Fixed Income Securities
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Problem 1
In this exercise I will ask you to explore the properties of returns on Treasury bond indices of different maturities. You will also have to estimate the relation between realized rates of return (holding period return) and initial yields to maturity.

a. For each "return index", RI, compute the monthly realized returns: RET'=ΔRI'/RI. (The index is a price, so calculating the return is the same as calculating the return of a stock)

   Compute the standard deviation of returns. Plot the standard deviations of returns as a function of maturity. Please interpret this result, is this as expected? Why?

b. Compute the first percentile of the distribution of returns (you can use the PERCENTILE function in excel). This can be thought of as a monthly VAR, with 99% confidence (A non-parametric VAR to be more specific). Plot the monthly VAR as a function of maturity. Please interpret this result.

c. Regress returns on the lagged monthly yield (y). The model is as follows:

$$\widetilde{RET'} = a + b \times \widetilde{y} + e$$

   You must convert the yields in the file to monthly yields as follows: Let y=RED.YIELD/200 and then let $\widetilde{y} = (1 + y)^{(1/6)} - 1$. The yields presented in the file are annualized semi-annual yields so you must divide them by 2 to get the semi-annual yield and divide by 100 to get them in percentage and then turn them into monthly yields.

   Report intercepts (a), slope coefficients (b) and R-squares as a function of maturity (to do this you can use the INTERCEPT, SLOPE and RSQ functions). Is the initial yield to maturity a good predictor of future bond returns? If the answer is yes, then intercepts should be close to zero, slopes close to one and the R-squares high. Why would fulfilling these conditions imply that initial yields are good predictors of future returns? How does the result change with maturity?

Solution:

a. Figure 1 shows the positive relationship between the standard deviation of returns and the length of maturity.

   This may be explained by the fact that the duration of the security is positively correlated to its maturity, while a higher duration means that the price will be more volatile in case of a change in the market interest rate.

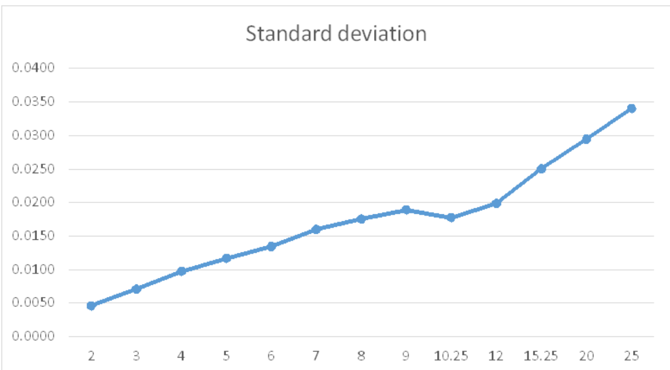


Figure 1: The standard deviations of returns as a functionof maturity

b. As is shown in Figure 2, the monthly VAR with 99% confidence is positively related the maturity. This can be resulted by a higher standard deviation of returns shown in (a), which comes with a higher interest risk. A higher VAR there implies that the potential loss due to the movements of interest rate is bigger.

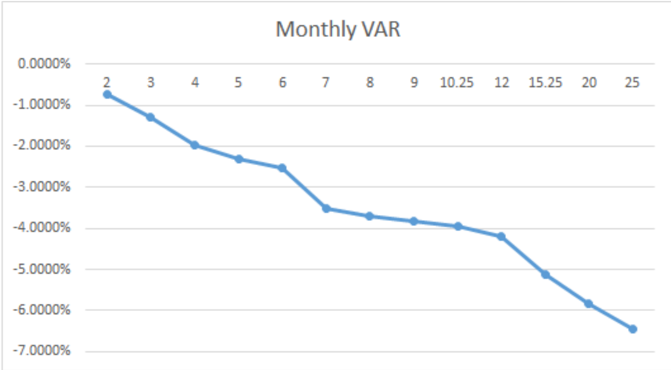
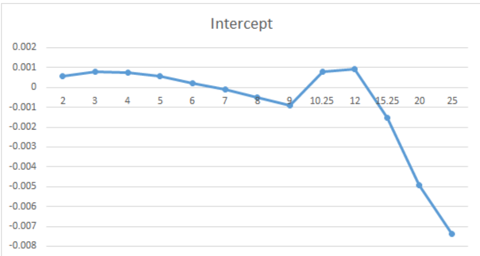
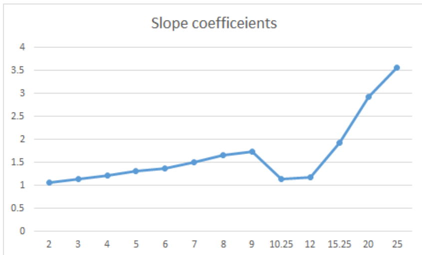


Figure 2: The monthly VAR as a function of maturity

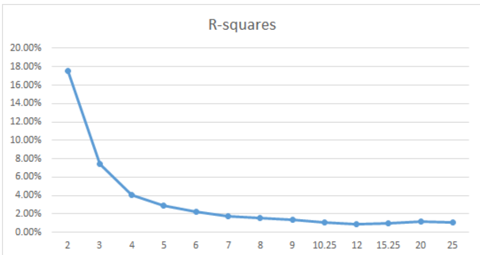
c. Overall, the initial yield to maturity is not a good predictor of future bond returns with a R-squares lower than 18% for all maturities. Also, combining the plot of intercepts, slope coefficients and R-squares in Figure ??, we can find that the higher the maturity, the lower the ability to predict the future bond returns for initial yield, which means the predicting ability decreases as maturity increases.



(a) Intercepts



(b) Slope coefficients



(c) R-squares

Figure 3: The intercepts(a), slope coefficients(b) and R-squares(c) as a function of mat

Problem 2

This problem asks you to evaluate the properties of the durations of U.S. Treasury bond indices. The problem also asks you to compute statistical durations based on the time-series properties of returns and yields.

1. Compute averages and standard deviations of the durations reported in the file. Plot the average durations as a function of maturity.

2. Compute annualized durations using the formula for a par bond. For using the formula, you must use y = RED.YIELD/200 and n be the number of years until maturity multiplied by 2 (Since

our formulas use semi-annual values). Plot the average durations as a function of maturity. How is this different from part a)? What do you think is the reason for this difference? (hint: use the properties of duration, and remember these are portfolio, so we are using the yield which is a weighted average of the yield of each component of the index)

Note: Maturity is found at the top of the column, for example “BOFA ML US TRSY OTR 1.5-2.5Y ($) - TOT RETURN IND” is an index with bonds from 1.5 to 2.5 years in maturity. Assume that the maturity of the index is the mid-point. In this case 2 years.

3. Regress the rates of return on the indices on the changes in the semiannual yield change for the corresponding maturity:

$$RET = a + b \times \Delta y + e$$

Plot the slope coefficients, divided by two, as a function of maturity (you can do this easily with the SLOPE function). And interpret your results.

Solution:

a. Table 1 shows the averages and standard deviations of the durations. And the relationship between the averages of duration and maturity is shown in Figure 4.

Table 1: Averages and standard deviations of the durations

Maturity (Year)	2	3	4	5	6	7	8	9	10.25	12	15.25	20	25
Duration averages	1.7761	2.6996	3.5852	4.2769	5.0822	5.7740	6.4892	7.2041	7.3234	8.0085	9.6459	11.6866	13.7333
Standard deviations	0.0712	0.0878	0.1494	0.2216	0.2886	0.3136	0.3927	0.4729	0.6722	0.7134	0.7780	1.3975	1.8633

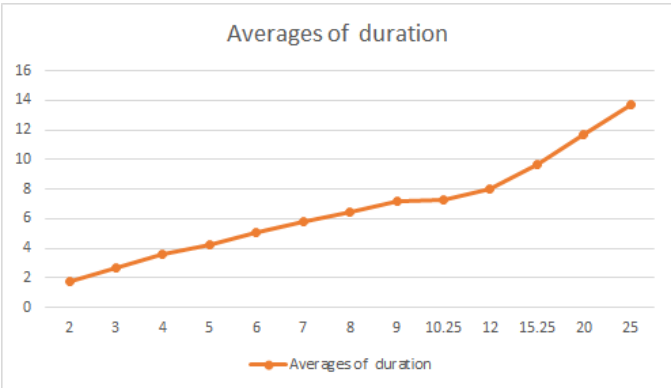


Figure 4: The average durations as a function of maturity

b. Annualized durations computed using formula is shown in Figure 5, according to the following formula:

$$D^* = \frac{1}{y}[1 - \frac{1}{(1+y)^n}]$$

We can draw the computed durations and the original durations in the same plot. As shown in Figure 6. The computed durations are slightly higher than the original durations and the difference increases as maturity increases.

Here, the averages of duration computed using formula are based on the index yield which is the weighted average of yield of each components. So we derive the duration of the index portfolio assuming the portfolio to be a par bond rather than using the weighted average of original durations of each components in the index portfolio. However, since most of the bonds in the market tend to sell at discount, the “par bond assumption“ overestimates the coupon rate and therefore overestimates the duration of the index.

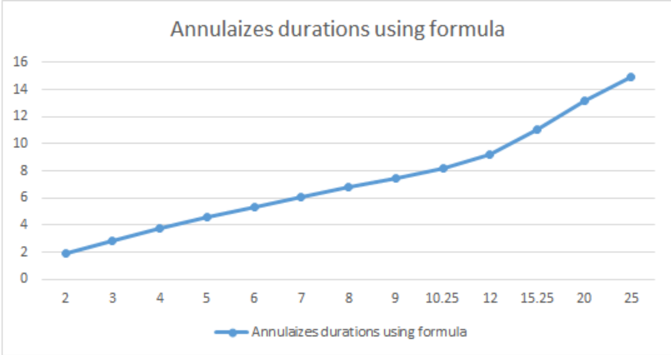


Figure 5: Annualized durations computed using formula

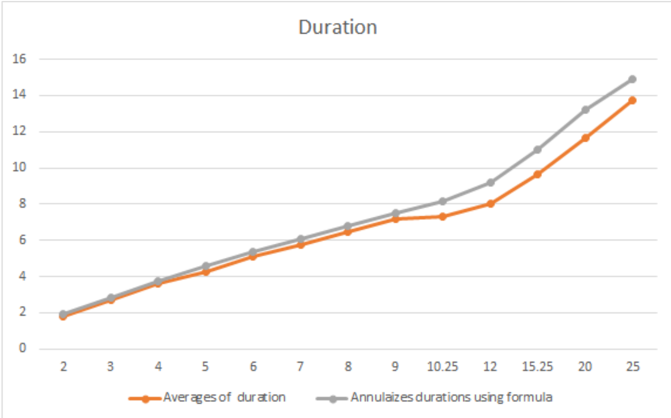


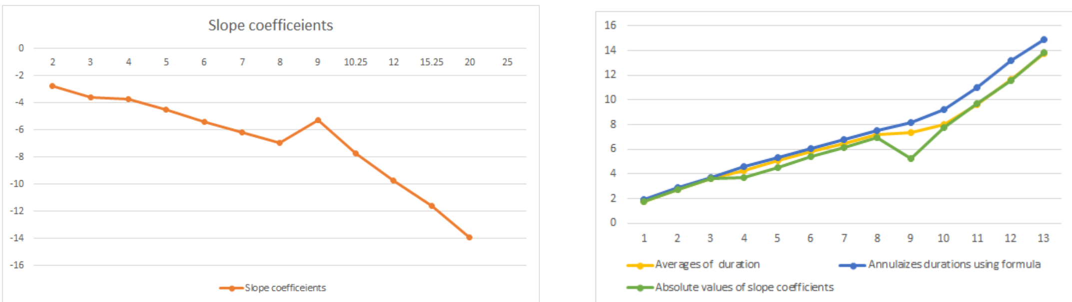
Figure 6: Annualized durations computed using formula

c. From Figure 7(a), we can find that the slope coefficients of the regressions are negative and decreases as maturity increases. The increase of the absolute values of the slope coefficients with maturity can be explained by the increased durations shown in (b). As the duration increases, the price will change more as interest rate changes, which also means that the change in the return will be bigger as the yield changes resulting in a bigger slope coefficient out of the regression.

Furthermore, according to the definition of the modified duration:

$$\frac{dP}{P} = -D^* \times dy$$

The minus slope coefficients -b in the regression can be interpreted as an estimation for the modified duration. By drawing the durations in (a) and (b), as well as the opposite numbers of the slope coefficients in Figure 7(b), we can see that the opposite numbers of the slope coefficients from regression can be seen as a good predictor for the index duration.



(a) (b)

Figure 7