1.1

2. second, linear

6. second, nonlinear

12. u No, v Yes

23. $\frac{dP}{dt} = \frac{d\frac{c_1e^t}{1+c_1e^t}}{dt}$

$$= \frac{c_1e^t(1+c_1e^t) - c_1e^tc_1e^t}{(1+c_1e^t)^2}$$

$$= \frac{c_1e^t}{(1+c_1e^t)^2}$$

$$= \frac{c_1e^t}{1+c_1e^t}\left(1-\frac{c_1e^t}{1+c_1e^t}\right)$$

$$= P(1-P)$$

Yes

1.2

4. $\frac{1}{2} = \frac{1}{4+C}$

$C = -2$

$y = \frac{1}{x^2-2} \Rightarrow \frac{1}{(x-\sqrt{2})(x+\sqrt{2})}$

$(\sqrt{2}, \infty)$

6.

$-4 = \frac{1}{\frac{1}{4}+C}$

$\frac{1}{4}+C = \frac{1}{-4}$

$C = -\frac{1}{2}$

$y = \frac{1}{x^2-\frac{1}{2}} \Rightarrow \left(\frac{1}{\sqrt{2}}, \infty\right)$

12. $c_1 e + c_2 e^{-1} = 0$

$c_1 e = \frac{-c_2}{e}$

$-c_1 e^2 = c_2$

$y' = c_1 e^x - c_2 e^{-x}$

$e = c_1 e - \frac{c_2}{e}$

$$e^2 = c_1 e^2 - c_2$$

$$e^2 = -2c_2$$

$$\frac{e^2}{-2} = c_2$$

$$c_1 = \frac{-c_2}{e^2}$$

$$c_1 = \frac{1}{2}$$

$$y = \frac{1}{2}e^x - \frac{1}{2}e^{2-x}$$

$$(-\infty, \infty)$$

1-3

2. $\frac{dP}{dt} = B - D$

where B is the birth rate, and D is the death rate

$$B = bP(t)$$

$$D = dP(t)$$

$$\frac{dP}{dt} = (b-d)P$$

5. $T_0 \approx 170$

$T_m \approx 75$

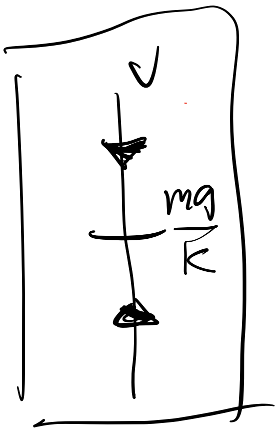
$\frac{dT}{dt} \approx -1$ when $T = 85$

$$\frac{dT}{dt} \approx -1 = k(85-75) \Rightarrow k \approx -0.1$$

2-1

160.

$$\frac{dv}{dt} = g - \frac{k}{m}v = 0 \Rightarrow v = \frac{gm}{k}$$



According to the phase portrait,

$\frac{mg}{k}$ is an attractor, when $t \to \infty$

$v \to \frac{mg}{k}$

2-2

2 $\quad y' = (x+1)^2 \Rightarrow \int y' dx = \quad y = \frac{(x+1)^3}{3} + c \qquad (-\infty, \infty)$

16 $\quad \frac{dQ}{Q-70} = k\,dt$

$\ln(Q-70) = kt + C$

$\pm\, Q - 70 = e^{kt} + e^{c}$

$Q = \pm e^{kt} + 70 + C \qquad (-\infty, \infty)$

160-b $\quad x\frac{dy}{dx} = y^2 - y$

$$\frac{dy}{y(y-1)} = \frac{dx}{x}$$

$$\left(\frac{1}{(y-1)} - \frac{1}{y}\right)dy = \frac{1}{x}dx$$

$$\ln\frac{y-1}{y} = \ln(x) + C$$

$$\ln\frac{y-1}{xy} = C$$

$$\frac{y-1}{xy} = e^{C} = C$$

$$1-\frac{1}{y} = xC \Rightarrow \frac{1}{1-xC} = y \Rightarrow \frac{1}{1-x0} = 0$$

No, IVP solution, for a deferrable and pass (0,0) => $y=0$

2-3

(a

$$\frac{dy}{dx} + \frac{2}{x}y = \frac{3}{x}$$

$$e^{\int\frac{2}{x}dx} = e^{2\ln x} = x^2$$

$$x^{-2} + x^{-2}\int 3x\,dx$$

$$y = \left(\frac{C}{x^2}\right) + \frac{3}{2}$$

$\uparrow$ Transient term

$(0, \infty)$

2b

$$\frac{dx}{dy} = \frac{x}{y} + 2y$$

$$x' - \frac{x}{y} = 2y$$

$$e^{-\int\frac{1}{y}dy} = e^{-\ln y} = \frac{1}{y}$$

$$x = Cy + y\int\frac{1}{y}\,2y\,dy$$

$$= cy + 2y^2$$

$$1 = 5c + 50 \Rightarrow c = \frac{-49}{5}$$

$$x = -\frac{49}{5}y + 2y^2$$

$$x = 2\left(y^2 - \frac{49}{10}y\right)$$

$$= 2\left(y^2 - \frac{49}{10}y + \frac{2401}{400}\right) - \frac{2401}{200}$$

$$\Rightarrow x + \frac{2401}{200} = 2\left(y - \frac{49}{20}\right)^2$$

$$\pm\sqrt{\frac{x + \frac{2401}{200}}{2}} + \frac{49}{20} = y \quad \left(-\frac{2401}{200}, \infty\right)$$