Image Processing HW5

User Blast

April 8, 2023

Contents

1 Problem Statement

Show that the Fourier transforms of

(a) $f(ax)$ is $\frac{1}{a}F(\frac{u}{a})$, where a is any nonzero real number.

(b) $f(x - x_0)$ is $F(u)\exp(-j2\pi u x_0)$.

2 Answer

2.1 A

$$F(f(ax)) = \int_{-\infty}^{\infty} f(ax) \exp\left(-jwx\right) dx$$

let

$$ax = t \Rightarrow dx = \frac{dt}{a} \text{ and } w = 2\pi u$$

$$F(f(ax)) = F(f(t)) = \int_{-\infty}^{\infty} f(t) \exp(-jw\frac{t}{a})\,\frac{dt}{a} = \frac{1}{a}\int_{-\infty}^{\infty} f(t) \exp(-j\frac{w}{a}t)\,dt = \frac{1}{a}F(\frac{u}{a})$$

2.2 B

$$F(f(x - x_0)) = \int_{-\infty}^{\infty} f(x - x_0) \exp(-jwx)\,dx$$

let

$$x - x_0 = t \Rightarrow dx = dt \text{ and } w = 2\pi u$$

$$F(f(x-x_0)) = F(f(t)) = \int_{-\infty}^{\infty} f(t) \exp(-jw(t + x0))\,dt = \int_{-\infty}^{\infty} f(t) \exp(-jwt) \exp(-jwx_0)\,dt$$

$$= \exp(-jwx_0) \int_{-\infty}^{\infty} f(t) \exp(-jwt)\,dt = F(x) \exp(-j2\pi ux_0)$$