Industrial applications and design case study

Struct Multidisc Optim 26, 235–248 (2004)
DOI 10.1007/s00158-003-0337-0

Design for six sigma through robust optimization

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Abstract The current push in industry is focused on ensuring not only that a product performs as desired but also that the product *consistently* performs as desired. To ensure consistency in product performance, "quality" is measured, improved, and controlled. Most quality initiatives have originated and been implemented in the product manufacturing stages. More recently, however, it has been observed that much of a product's performance and quality is determined by early design decisions, by the design choices made early in the product design cycle. Consequently, quality pushes have made their way into the design cycle, and "*design for quality*" is the primary objective. *How is this objective measured and met*?

The most recent quality philosophy, also originating in a manufacturing setting, is six sigma. The concepts of six sigma quality can be defined in an engineering design context through relation to the concepts of design reliability and robustness – probabilistic design approaches. Within this context, design quality is measured with respect to probability of constraint satisfaction and sensitivity of performance objectives, both of which can be related to a design "sigma level". In this paper, we define six sigma in an engineering design context and present an implementation of *design for six sigma* – a robust optimization formulation that incorporates approaches from structural reliability and robust design with the concepts and philosophy of six sigma. This formulation is demonstrated using a complex automotive application: vehicle side impact crash simulation. Results presented illustrate the tradeoff between performance and quality when optimizing for six sigma reliability and robustness.

Key words six sigma, sigma level, robustness, reliability, probability of failure

1 Introduction

Most real-world engineering problems involve at least an element of uncertainty (if not substantial uncertainty) – uncertainty in loading conditions, in material characteristics, in analysis/simulation model accuracy, in geometric properties, in manufacturing precision, in actual product usage, etc. Many optimization strategies, however, do not incorporate this uncertainty into a mathematical formulation. Optimization algorithms tend to push a design toward one or more constraints until the constraints are active, leaving the designer with a "high-risk" design, one for which even slight uncertainties in the problem formulation or changes in the operating environment could produce failed or unsafe designs. In addition, optimization algorithms tend to search for "peak" solutions, ones for which even slight changes in design variables and uncontrollable, uncertain parameters can result in substantial performance degradation. In this case the "optimal" performance is misleading: worst-case performance could potentially be much less than desirable and failed designs could occur.

Traditionally, engineering problems have been formulated to handle uncertainty only through crude safety factor methods that often lead to overdesigned products and do not offer insight into the effects of individual uncertainties and the actual margin of safety. In recent years, however, probabilistic design analysis and optimization methods, often also called *quality engineering methods*, have been developed to address uncertainty and randomness through statistical modeling and probabilistic analysis. These probabilistic methods have been developed

Received: 30 August 2001
Revised manuscript received: 27 September 2002
Published online: 16 January 2004

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to convert deterministic problem formulations into probabilistic formulations to model and assess the effects of known uncertainties. Until very recently, however, the computational expense of probabilistic *analysis* of a single design point, in terms of the number of function evaluations necessary to accurately capture performance variation and estimate probability of failure, has made the application of these methods impractical for all but academic investigations or very critical cases (Thanedar and Kodiyalam 1991). Consequently, probabilistic *optimization* has been considered prohibitively expensive, particularly for complex, multidisciplinary engineering design problems. With the steady increases in computing power, large-scale parallel processing capabilities, and availability of probabilistic analysis and optimization tools and systems, however, the combination of these technologies can facilitate effective probabilistic analysis and optimization for complex design problems, allowing for the identification of designs that qualify as not only feasible but as consistently feasible in the face of uncertainty.

The probabilistic/statistical "quality engineering" design methods developed in recent decades have come from several different communities: structural reliability (Thanedar and Kodiyalam 1991; Hasofer and Lind 1974; Rackwitz and Fiessler 1978; Hohenbichler and Rackwitz 1981; Madsen et al. 1986; Thoft-Christensen and Baker 1982; Cornell 1969; Melchers 1999), Taguchi robust quality engineering (Byrne and Taguchi 1987; Ross 1996; Phadke 1989) and, more recently, "six sigma" quality engineering (Harry 1997; Harry and Schroeder 2000; Pande et al. 2000). Each of these classes of approaches has a specific focus in probabilistic analysis and/or optimization. Structural reliability analysis is geared toward assessing the probability of failure of a design with respect to specified structural and/or performance constraints and the evaluated variation of these constraint functions. Thus a deterministic design problem is converted to a reliability analysis problem by converting deterministic constraints into probabilistic constraints:

$$g(\mathbf{X}) \leq 0 \tag{1}$$

becomes

$$P_f = P(g(\mathbf{X}) > 0) \leq P^U, \tag{2}$$

where $\mathbf{X}$ is the set of design parameters, one or more of which are uncertain or vary randomly, P_f is the probability of failure for the constraint or *limit state function* g, and P^U is the required upper bound for the probability of failure (Melchers 1999). Reliability-based optimization thus focuses on the effects of input uncertainties, modeled through random variables with assigned probability distributions and properties, upon satisfaction of probabilistic constraints (Thanedar and Kodiyalam 1991; Belegundu 1988; Choi et al. 1996; Chen et al. 1997; Xiao et al. 1999; Koch and Kodiyalam 1999, Koch et al. 2000).

With Taguchi-based quality engineering methods, the focus is on performance objectives, which are expanded from deterministic "minimize" or "maximize" objectives to include both mean performance and performance variation. With Taguchi methods, the goals are to drive mean performance toward a target, "mean on target", and to "minimize variance" of performance (Phadke 1989). Taguchi methods employ metrics such as signal-to-noise ratio and loss function to achieve these goals (Ross 1996; Phadke 1986). However, within Taguchi-based methods, constraints are not formulated, as is typically done with optimization formulations. Taguchi methods employ design of experiments (DOE) (Montgomery 1996) to evaluate potential designs. The best alternative with respect to the chosen objective metrics is selected from among those evaluated. Numerical optimization is not generally performed between evaluated design points.

The rapidly growing current push in industry with respect to managing uncertainty and seeking "quality" products is *design for six sigma* or DFSS. At the heart of DFSS methods is DOE and other statistical analysis methods used to capture performance variation, with a focus on maintaining ± 6-sigma (± 6 standard deviations) of performance variation within design specification limits (Harry 1997; Harry and Schroeder 2000; Pande et al. 2000). Thus the focus here is again on constraints, as with the probabilistic constraints in reliability-based design. With DFSS arising from manufacturing quality control initiatives, DOE with a minimum number of physical experiments is often critical. Like Taguchi methods, optimization is again generally not performed since physical experiments are conducted; optimization is usually not even mentioned in the few available DFSS references.

There is significant overlap between these different schools of probabilistic approaches for uncertainty assessment. Designing for a quality level of 6-sigma with respect to design specification limits is the equivalent of designing for a reliability level of 99.9999998% (probability of failure of 0.0000002%) with respect to defined limit states functions. However, each of these different types of probabilistic approaches focuses on only part of a design or optimization problem (constraints or objective only), and optimization is often not included. With much of engineering design today performed using computer simulation models, and with the level of uncertainty prevalent in both these models and the engineering problems themselves, the application of probabilistic analysis and optimization is not only feasible but critical for identifying designs that perform consistently in the face of this uncertainty. What is needed is a complete formulation that incorporates concepts from each of these schools of probabilistic analysis and design to facilitate comprehensive probabilistic optimization.

In this paper, we present a six sigma-based robust design optimization formulation that combines approaches from structural reliability and robust design with the concepts and philosophy of six sigma to facilitate comprehensive probabilistic optimization. Variability is incorporated within all elements of this probabilistic optimization formulation – input design variable bound formulation, output constraint formulation, and robust objective formulation. An overview of the background and concepts of six sigma and relations to reliability and robustness is presented in the next section. In Sect. 3, we detail the six sigma-based robust optimization formulation, as implemented within a commercial software design environment(Koch et al. 2002). In Sect. 4, this formulation is demonstrated using a complex automotive application – vehicle side impact crash simulation. A discussion of results and current investigations is provided in closing in Sect. 5.

2
Six sigma design quality: reliability and robustness

Six sigma is a quality philosophy at the highest level, relating to all processes, and a quality measure at the lowest level. The term “sigma” refers to standard deviation, σ. Standard deviation or variance, σ^2, is a measure of dispersion of a set of data around the mean value, μ, of this data. This property can be used both to describe the known variability of factors that influence a system (product or process) and as a measure of performance variability, and thus quality. Performance variation can be characterized as a number of standard deviations from the mean performance, as shown in Fig. 1. The areas under the normal distribution in Fig. 1 associated with each σ-level relate directly to the probability of performance falling in that particular range (for example, $\pm 1\sigma$ is equivalent to a probability of 0.683). These probabilities are displayed in Table 1 as percent variation and number of defective parts per million parts.

Quality can be measured using any of the variability metrics in Table 1 – “sigma level”, percent variation or probability (equivalent to reliability), or number of defects per million parts – by comparing the associated performance specification limits and the measured performance variation. In Fig. 1, the lower and upper specification limits that define the desired performance range are shown to coincide with $\pm 3\sigma$ from the mean. The design associated with this level of performance variance would be considered a “3σ” design. *Is this design of acceptable quality?* Traditionally, if $\pm 3\sigma$ worth of performance variation was identified to lie within the set specification limits, that was considered acceptable variation; in this case, 99.73% of the variation is within specification limits, or the probability of meeting the requirements defined by these limits is 99.73%. In engineering terms, this probability was deemed acceptable.

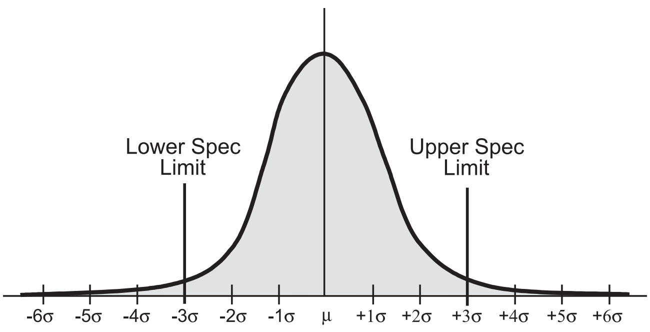


Fig. 1 Normal distribution, 3-σ design

Table 1 Sigma level as percent variation and defects per million

Sigma level	Percent variation	Defects per million (short term)	Defects per million (long term – 1.5 sigma shift)
$\pm 1\sigma$	68.26	317 400	697 700
$\pm 2\sigma$	95.46	45 400	308 733
$\pm 3\sigma$	99.73	2700	66 803
$\pm 4\sigma$	99.9937	63	6200
$\pm 5\sigma$	99.999943	0.57	233
$\pm 6\sigma$	99.9999998	0.002	3.4

More recently, however, the 3σ quality level has been viewed as insufficient quality, initially from a manufacturing perspective and currently extending to an engineering design perspective. Motorola, in defining “six sigma quality” (Harry and Schroeder 2000), translated the sigma quality level to the number of defective *parts per million* (ppm) parts being manufactured. In this case, as can be seen in Table 1, $\pm 3\sigma$ corresponds to 2700 ppm defective. This number was deemed unacceptable. Furthermore, Motorola and others observed that even at some observed variation level, mean performance could not be maintained over time. If a part is to be manufactured to some nominal specification, plus/minus some specification limit, the mean performance will change and the distribution thus shift. A good example of this is tool wear. If a process is set up to manufacture a part to a nominal dimension of 10 in. with a tolerance of ± 0.10 in., even if the process meets this nominal dimension on average, the cutting tool will wear with time, and the average part dimension will shift to, say, 10.05 in. This will cause the distribution of performance variation to shift while the specification limits remain fixed, and thus the area of the distribution outside one of the specification limits will increase.

This shift was observed by Motorola and others to be approximately 1.5σ and was used to define “long-term sigma quality” as opposed to “short-term sigma quality”. This explains the last column in Table 1. While the defects per million parts for the short term correspond directly to the percent variation for a given sigma level associated with the standard normal distribution, the defects per million parts for the long term correspond to a 1.5σ shift in the mean. In this case, 3σ quality leads to 66 803 defects per million, which is certainly undesirable. Consequently, a quality goal of $\pm 6\sigma$ was defined; hence “six sigma quality” came to define the desired level of acceptable performance variation. With this quality goal, the level of defects per million parts, as shown in Fig. 1, is

0.002 for short-term quality and 3.4 for long-term quality; both are acceptable quality levels.

The focus on achieving six sigma quality is commonly referred to as *design for six sigma* (DFSS): striving to maintain six standard deviations ($\mu \pm 6\sigma$) of performance variation within the defined acceptable limits, as illustrated in Fig. 2. In Fig. 2a and b, the mean, μ, and lower specification limit (LSL) and upper specification limit (USL) on performance variation are held fixed. Figure 2a represents the 3σ design of Fig. 1; $\pm 3\sigma$ worth of performance variation is within the defined specification limits. To achieve a 6σ *design*, one for which the probability that the performance will remain within the set limits is essentially 100%, the performance variation must be reduced (reduced σ_y), as shown in Fig. 2b. Note that while 6σ is a measure of design quality, *six sigma* is a quality improvement process and philosophy; a six sigma process does not imply or guarantee a 6σ design.

In an engineering design context, these concepts of six sigma quality, measurement, and improvement can be related to two probabilistic design measures: *reliability and robustness*. The two goals in *designing* for quality are: (1) striving to maintain performance within acceptable limits, consistently (*reliability*), and (2) striving to reduce performance variation and thus increase *robustness*. Reliability is defined as the probability of satisfying constraints; conversely, the probability of failure, probability of *not* satisfying constraints, is often measured. The reliability can be directly related to the sigma level: a short-term sigma level of $\pm 3\sigma$ is equivalent to a reliability of 99.73%. The term "robustness", on the other hand, refers simply to the amount of performance variability. In the robust engineering design context, robustness is defined as the sensitivity of performance parameters to fluctuations in uncertain design parameters. This sensitivity is captured through performance variability estimation. The fundamental motive underlying robust design is to improve the quality of a product or process by not only striving to achieve performance targets or goals ("mean on target") but also by minimizing performance variation (Chen et al. 1996).

In implementing a six sigma approach in engineering design, to *design for six sigma*, performance variability is measured and, if needed, improved with respect to the two mentioned quality goals by *shifting* the performance distribution relative to the constraints or specification limits (thus improving reliability) and by *shrink-*

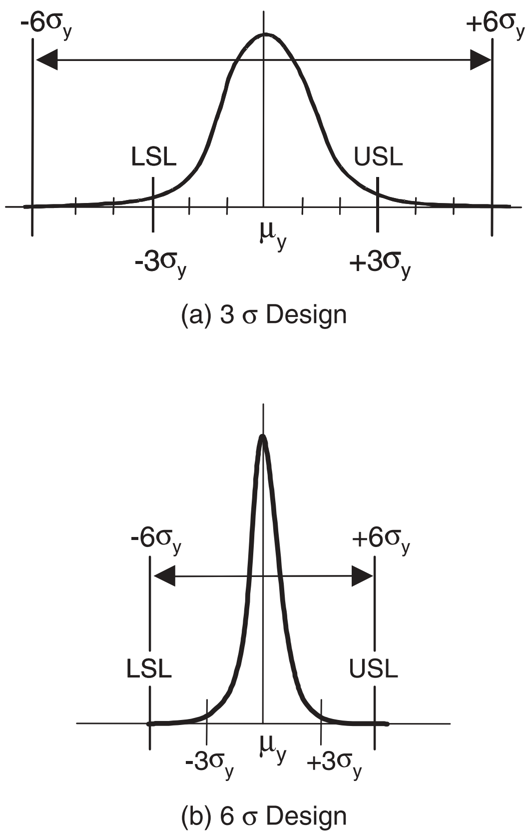


Fig. 2 Design for six σ (LSL, USL: lower, upper specification limits)

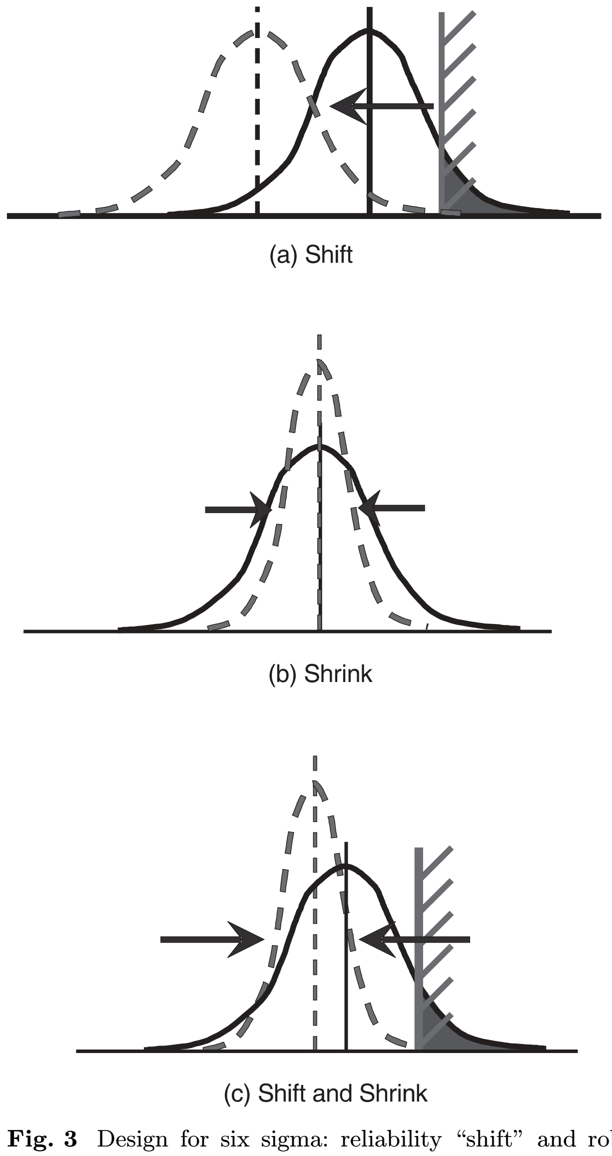


Fig. 3 Design for six sigma: reliability "shift" and robust "shrink"

ing the performance distribution to reduce the variability and sensitivity of the design (thus improving robustness). These concepts of "shift" and "shrink" are illustrated in Fig. 3. In Fig. 3a, the performance distribution is shifted until an acceptable level of reliability (area of distribution inside constraint) is achieved. In Fig. 3b, the spread of the performance distribution is shrunk to reduce the potential performance loss associated with the tails of the distribution and thus reduce the sensitivity and improve the robustness.

The ideal case in improving design quality in designing for six sigma is to both shift and shrink a performance distribution, as illustrated in Fig. 3c. As shown in Fig. 3a, while the loss of mean performance associated with the shifted distribution may be acceptable, given the increased reliability, the potential worst-case performance associated with the left tail may be undesirable. With the shrunken distribution of Fig. 3b, the level of reduced spread possible may not be sufficient to achieve an acceptable reliability level without shifting the mean performance. In Fig. 3c, with both shift and shrink, the reliability level is achieved, the robustness improved, and the worst-case performance not significantly affected. This is the desired goal in implementing DFSS.

The key elements necessary for implementing the six sigma quality goals discussed in this section are methods to characterize design parameter variability or uncertainty, methods to measure the resulting performance variability, and a design optimization formulation for incorporating this variability and the design quality goals. In the next section, such an implementation is presented.

3 Robust optimization implementation: optimizing for six sigma quality

Designing for six sigma quality includes quality assessment and quality improvement. Quality assessment is performed for a given product design. To facilitate quality assessment, uncertainty and variability in input design parameters are characterized through probability distributions and their statistical properties. Quality improvement is facilitated by modifying a design through an optimization procedure, for example, assessing the performance and quality of each new design, and including the quality measures in the design formulation. Thus a quality assessment procedure is implemented within a design/quality improvement procedure. A procedure for six sigma quality assessment and improvement – a robust optimization formulation that has been implemented in a commercial design environment (Koch et al. 2000) – is presented in this section. An overview of this six sigma-based robust optimization formulation for quality improvement is provided first, followed by a description of the specific quality assessment methods implemented for measuring performance variation within this formulation.

3.1 Quality improvement: six sigma-based robust optimization formulation

Combining the probabilistic elements of reliability, robust design, and six sigma, variability is incorporated into a robust optimization formulation through the definition of uncertain random variables, formulation of reliable input constraints (random design variable bounds) and output constraints (sigma level quality constraints or reliability constraints), and objective robustness (minimize variation in addition to mean performance on target). The formulation for implementing six sigma-based robust optimization is given as follows: find the set of design variables $\mathbf{X}$ that:

$$\begin{aligned} &\text{Minimizes:} && F(\mu_y(\mathbf{X}), \sigma_y(\mathbf{X})) \\ &\text{subject to:} && g_i(\mu_y(\mathbf{X}), \sigma_y(\mathbf{X})) \le 0 \\ & && \mathbf{X}_L + n\sigma_{\mathbf{X}} \le \mu_{\mathbf{X}} \le \mathbf{X}_U - n\sigma_{\mathbf{X}}. \end{aligned} \tag{3}$$

Here $\mathbf{X}$ includes input parameters that may be design variables, random variables, or both. Both input and output constraints are formulated to include mean performance and a desired "sigma level", or number of standard deviations within specification limits, as follows:

$$\mu_y - n\sigma_y \ge \text{Lower specification limit} \tag{4}$$

$$\mu_y + n\sigma_y \le \text{Upper specification limit.} \tag{5}$$

The robust design objective for this formulation, including "mean on target" and "minimize variation" robust design goals, is generally formulated as follows:

$$F = \sum_{i=1}^{l} \left[\frac{w_{1_i}}{s_{1_i}} \left(\mu_{Y_i} - M_i\right)^2 + \frac{w_{2_i}}{s_{2_i}} \sigma_{Y_i}^2 \right], \tag{6}$$

where w_{1_i} and w_{2_i} are the weights and s_{1_i} and s_{2_i} are the scale factors for the "mean on target" and "minimize variation" objective components, respectively, for performance response i, M_i is the target for performance response i, and l is the number of performance responses included in the objective. For the case where the mean performance is to be minimized or maximized rather than directed toward a target, the objective formulation of Eq. 6 can be modified as shown in Eq. 7, where the "+" sign is used before the first term when the response mean is to be minimized and the "–" sign is to be used when the response mean is to be maximized.[1]

$$F = \sum_{i=1}^{l} \left[(+/-) \frac{w_{1_i}}{s_{1_i}} \mu_{Y_i} + \frac{w_{2_i}}{s_{2_i}} \sigma_{Y_i}^2 \right]. \tag{7}$$

[1] NOTE: Limitations are known to exist with weighted-sum type objective functions when used to generate Pareto sets (Athan and Papalambros 1996), and the identification of alternate formulations is a topic of current research.

This formulation also facilitates the optimization of design tolerances. By defining the variance or standard deviation of uncertain design parameters not as fixed but as design variables themselves, tolerance design optimization is implemented by seeking standard deviation settings for these parameters that produce acceptable performance variation (objective components, constraints). Note that when implementing tolerance design, a cost model is essential to reflect the cost of reducing tolerances so as to evaluate the performance vs. cost tradeoff (to prevent the optimizer from reducing tolerances to zero in attempt to reduce performance variation).

The key to implementing this six sigma-based robust optimization formulation is the ability to estimate performance variability statistics to allow the reformulation of constraints and objectives as defined in Eqs. 3–7. Several methods employed in this work for estimating performance variation are discussed in the next section.

3.2 Quality assessment: sampling methods for measuring variation

Estimation of performance variability requires the definition of uncertain design parameters and their expected variability that leads to this performance variability. Uncertainty in design parameters can be characterized in many ways: through distributions and associated statistical properties (as random variables are often characterized) (Hahn and Shapiro 1994), by a range of expected variation (low and high values, as noise factors in Taguchi's robust design are commonly defined) (Thoft-Christensen and Baker 1982), or by a delta or percent variation around a nominal, baseline value.

Given the definition of uncertain parameters and their variability, the resulting variability of performance parameters can be measured. Many techniques exist for estimating performance variability; the three employed within the six sigma robust optimization procedure are discussed here:

– Monte Carlo simulation,
– Design of experiments,
– Sensitivity-based variability estimation.

Monte Carlo simulation

Monte Carlo simulation (MCS) techniques are implemented by randomly simulating a design or process, given the stochastic properties of one or more random variables, with a focus on characterizing the statistical nature (mean, variance, range, distribution type, etc.) of the responses (outputs) of interest (Hammersley and Handscomb 1964). Monte Carlo methods have long been recognized as the most exact method for all calculations that require knowledge of the probability distribution of responses of uncertain systems to uncertain inputs. To implement a MCS, a defined number of system simulations to be analyzed are generated by sampling values of random variables (uncertain inputs) following the probabilistic distributions and associated properties defined for each.

Several sampling techniques exist for simulating a population; the two techniques implemented in this work are *simple random sampling* (Hahn and Shapiro 1994), the traditional approach to Monte Carlo, and *descriptive sampling* (Saliby 1990; Ziha 1995), a more efficient variance reduction technique. These two techniques are compared in Fig. 4. With simple random sampling, the traditional MCS approach shown in Fig. 4a, sample points are generated randomly from each distribution, as its name implies. Sufficient sample points (often thousands or tens of thousands) must be taken to ensure the probability distributions are fully sampled.

Variance reduction sampling techniques have been developed to reduce the sample size (number of simulations) without sacrificing the quality of the statistical description of the behavior of the system. Descriptive sampling is one such sampling technique. In this technique, the probability distribution of each random variable is divided into subsets of *equal probability* and the analysis is performed with each subset of each random variable only once (each subset of one random variable is combined with only one subset of every other random variable). This sampling technique is similar to *Latin hypercube* experimental design techniques and is best described through illustration as in Fig. 4b for two random

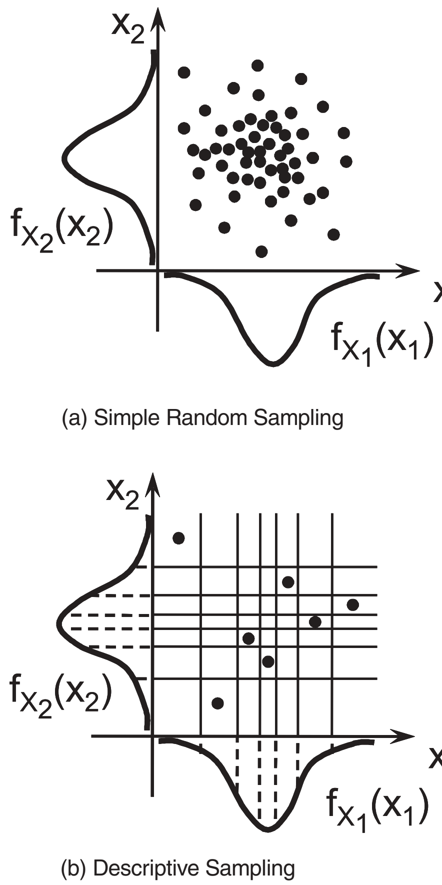


Fig. 4 Monte Carlo sampling comparison

variables. Each row and column in the discretized two-variable space is sampled only once, in random combinations and random order. Only seven points/subsets are shown in Fig. 4b for clarity in illustration; obviously more points are necessary for acceptable estimation, but often an order of magnitude fewer sample points are necessary when compared to simple random sampling.

MCS is also the traditional method of reliability analysis. The probability of failure is estimated simply as the ratio of the number of points violating constraints (points on failure side of failure function, or limit state function, $g(\mathbf{X}) > 0$) to the total number of sample points: $P_f = N_{failed}/N_{total}$. The reliability is then $1 - P_f$, and the design sigma level can be calculated directly from this reliability. Other information gathered through MCS includes response statistical information (including standard deviation/variance for assessing robustness), visual response scatter, response PDF/CDF (probability density function, cumulative distribution function) information, and a measure of the relative effects of each random variable on a response (identifying key random variables that drive response variability). A sample of such results is presented in Fig. 5.

Design of experiments (DOE)

A second approach to estimating performance variability due to uncertain design parameters is through more structured sampling, using a designed experiment. In DOE, a design matrix is constructed, in a systematic fashion, that specifies the values for the design parameters (uncertain parameters in this context) for each sampled point or experiment. A number of experimental design techniques exist for efficiently sampling values of design parameters; for details on experimental design techniques, see Mongomery (1996).

With DOE, potential values for uncertain design parameters are not defined through probability distributions but rather are defined by a range (low and high values), a nominal baseline plus/minus some delta or percent, or through specified values or levels. In this case, each run in the designed experiment is a combination of the defined levels of each parameter. Consequently, this approach represents a higher level of uncertainty in uncertain design parameters, with expected ranges rather than specific distributions. Given data from a designed experiment, capturing ranges of uncertain parameters, robustness or quality can be assessed by using response statistics to determine sigma level, percent variation within specification limits, probability of meeting specification limits, or defects per million parts based on defined specification limits.

The computational cost of implementing DOE depends on the particular experiment design chosen and the levels of the uncertain parameters but is generally significantly less than that of MCS. However, the estimates may not be as accurate. This method is recommended when distributions are not known and can also be used when uncertain parameters follow a uniform distribution.

Sensitivity-based variability estimation

A third approach to estimating performance variability is a sensitivity-based approach, based on Taylor's series expansions. In this approach, rather than sampling across known distributions or ranges for uncertain design parameters, gradients for performance parameters are taken with respect to the uncertain design parameters, hence the "sensitivity-based" nature of the approach. Generally, with this approach the Taylor's expansion is either first order, neglecting higher-order terms, or second order. Obviously there is a tradeoff between expense (number of gradient calculations) and accuracy when choosing to include or neglect the higher-order terms in the expansion. The first-order and second-order formulations are given as follows.

First-order Taylor's expansion. Neglecting higher-order terms, the Taylor's series expansion for a performance response, Y, is:

$$Y(x) = y + \frac{\mathrm{d}Y}{\mathrm{d}x} \triangle x. \tag{8}$$

The mean of this performance response is then calculated by setting the uncertain design parameters to their mean value, μ_x:

$$\mu_y = Y(\mu_x) \tag{9}$$

and the standard deviation of $Y(x)$ is given by:

$$\sigma_Y = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial Y}{\partial x_i}^2 (\sigma_{x_i})^2 \right)}, \tag{10}$$

where σ_{x_i} is the standard deviation of the i-th parameter and n is the number of uncertain parameters. For more details on this approach for variability estimation for robust design purposes, see Phadke (1989) and Chen et al. (1996).

Since first-order derivatives of responses with respect to random variables are needed in Eq. 10 and the mean value point is needed in Eq. 9, the first-order Taylor's expansion estimates require $n+1$ analyses for evaluation.

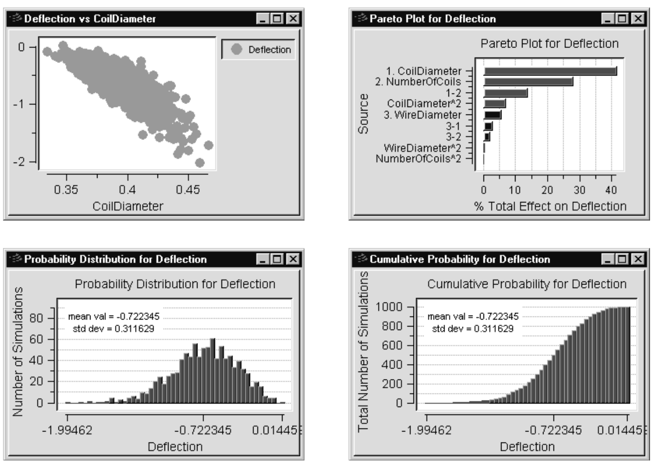


Fig. 5 Monte Carlo measures of performance variability

Consequently, this approach is significantly more efficient than MCS and often more efficient than DOE while including distribution properties. However, the approach loses accuracy when responses are not close to linear in the region being sampled. This method is therefore recommended when responses are known to be linear or close to linear and also when computational cost is high and rough estimates are acceptable.

Second-order Taylor's expansion. Adding the second-order terms, the Taylor's series expansion for a performance response, Y, is:

$$Y(x) = y + \frac{\mathrm{d}Y}{\mathrm{d}x}\triangle x + \frac{1}{2}\triangle x^T \frac{\mathrm{d}^2 Y}{\mathrm{d}x^2}\triangle x. \quad (11)$$

The mean of this performance response is then obtained by taking the expectation of both sides of the expansion

$$\mu_Y = Y(\mu_x) + \frac{1}{2}\sum_{i=1}^{n} \frac{\mathrm{d}^2 Y}{\mathrm{d}x_i^2}, \sigma_{x_i}^2 \quad (12)$$

and the standard deviation of $Y(X)$ is given by:

$$\sigma_Y = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial Y}{\partial x_i}\right)^2 (\sigma_{x_i})^2 + \frac{1}{2}\sum_{i}^{n}\sum_{j}^{n}\left(\frac{\partial^2 Y}{\partial x_i \partial x_j}\right)^2 (\sigma_{x_i})^2(\sigma_{x_j})^2}, \quad (13)$$

where σ_{X_i} is the standard deviation of the i-th parameter and σ_{X_j} is the standard deviation of the j-th parameter. For more details on this approach for variability estimation and its use for robust design, see Hsieh and Oh (1992).

Since second-order derivatives of responses, including crossed terms, with respect to random variables are needed in Eq. 13, and the mean value point is needed in Eq. 12, the second-order Taylor's expansion estimates require $(n+1)(n+2)/2$ analyses for evaluation. Consequently, this approach is significantly more computationally expensive than the first-order Taylor's expansion and usually will be more expensive than DOE. For low numbers of uncertain parameters, this approach can still be more efficient than MCS but becomes less efficient with increasing n. (MCS is not dependent on the number of parameters; for 100 parameters, second-order Taylor's expansion would require 5151 sample points, which may be more than required by MCS for the same level of accuracy in the estimates.) For efficiency in this implementation, the cross terms of Eq. 13 are ignored, and only pure second-order terms are included (diagonal terms). The number of necessary evaluations is then reduced to $2n+1$. The second-order Taylor's expansion approach is recommended when curvature exists and the number of uncertain parameters is not excessive.

Any of the sampling methods summarized in this section can be used to estimate performance variability. These estimates can then be used within the robust optimization formulation to improve design quality. One technology that has made this sampling for quality assessment and improvement more feasible for real engineering design problems is parallel processing (Koch et al. 2000). The sample points defined by MCS or DOE are independent points, and the complete set can be defined prior to execution and thus executed in parallel. For the first- or second-order Taylor's expansion sensitivity methods, if finite differencing is used for calculating the partial derivatives of the performance responses with respect to each random variable, these gradient points are also known prior to execution and can be executed in parallel. Thus any number of machines and/or processors can be used to expedite the sampling procedure.

This six sigma-based robust optimization formulation is demonstrated in the next section for an automotive design for a crashworthiness example. Results obtained are compared to deterministic optimization results and to results obtained using a reliability-based optimization approach, focusing only on probability of constraint satisfaction.

4 Application: automotive crashworthiness

One quality engineering design application currently of high visibility in the automotive industry is vehicle structural design for crashworthiness. These design problems are not only particularly complex in terms of understanding the problem and defining design requirements and design alternatives, but they also involve a very high degree of uncertainty and variability: *velocity of impact, mass of vehicle, angle of impact,* and *mass/stiffness of barrier* are just a few of many uncertain parameters. A balance must be struck in designing for crashworthiness between designing the vehicle structure to absorb/manage the crash energy (through structure deformation) and maintaining passenger compartment integrity, all in the face of uncertainty.

With the expense of physical crash testing and with recent increases in computing power and improvement in numerical modeling techniques, much of crash analysis is done today using computer simulation models. However, these simulations bring a different kind of expense. Vehicle finite element models for crash simulation can be upwards of 150 000 elements. Processing can range from 20 h to over 150 h for a single analysis. Even with parallel processing, the expense of crash simulations prohibits extensive optimization, let alone probabilistic optimization. In addition, solver instability often prevents successful application of sequential (optimization) methods since runs will often fail. It is essential to create approximate models from a number of allowable actual simulations.

In this paper, one specific crash scenario is investigated – *side impact* – using the six sigma robust optimization formulation presented in the previous section. A typical vehicle side impact model is shown in Fig. 6. Including a finite element dummy model, the total number

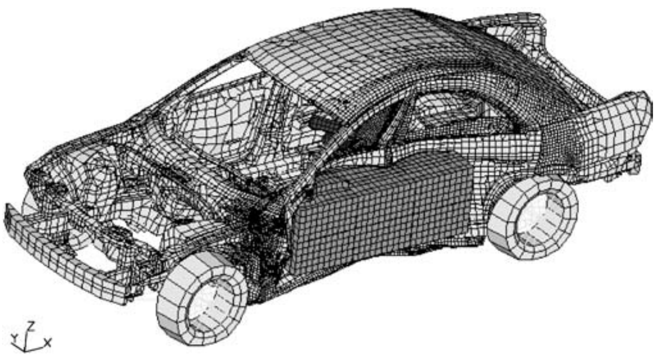


Fig. 6 Side impact model

of elements in this model is about 90 000 and the total number of nodes is around 96 000. The initial lateral velocity of the side deformable barrier is 31 MPH. The CPU time for a RADIOSS simulation of the model is about 20 h on an SGI Origin 2000.

For side impact protection, the vehicle design should meet the requirements for the National Highway Traffic Safety Administration (NHTSA) side impact procedure specified in Federal Motor Vehicle Safety Standards (FMVSS) or European Enhanced Vehicle-Safety Committee (EEVC) side impact procedure. In our study, the EEVC side impact test configuration was used. The dummy performance is the main concern in side impact, which includes head injury criterion (HIC), abdomen load, pubic symphysis force (pelvic load), V*C (viscous criterion), and rib deflections (upper, middle, and lower). These dummy responses must at least meet EEVC requirements. A finite element dummy model was employed for prediction. Other concerns in side impact design are the velocity of the B-pillar at middle point and the velocity of the front door at B-pillar.

For side impact, increase of gage design variables tends to produce better dummy performance. However, it also increases vehicle weight, which is undesirable. Therefore, a balance must be sought between weight reduction and safety concerns. The objective is to reduce the weight with imposed constraints on the dummy safety. The dummy safety performance is usually measured by the EEVC side impact safety rating score. In the EEVC side impact safety rating system, the safety rating score is derived from four measurements of the dummy: HIC, abdomen load, rib deflection or V*C, and pubic symphysis force. The highest score for each of these measurements is 4 points; a total of 16 points may be obtained for vehicle side impact. Table 2 lists all the design targets for the benchmark model along with the EEVC regulation and the top safety rating criteria for vehicle side impact.

The optimization problem of side impact can be described as:

Minimize
  Weight
Subject to
  Abdomen load ≤ 1.0 KN
  Rib deflection, D ≤ 32 mm
  V*C ≤ 0.32 m/s
  Pubic symphysis force, FPS ≤ 4.0 KN
  Velocity of B-pillar at middle point
    $V_{\text{B-pillar}} \leq 9.9$ mm/ms
  Velocity of front door at B-pillar
    $V_{\text{Door}} \leq 15.70$ mm/ms

Note that rib deflection and V*C are measured at three chest locations – upper, middle, and low – for a total of six constraints.

There are 11 design parameters used for side impact optimization, as shown in Table 3. The first 9 design parameters in Table 3 are design variables: 7 thickness parameters and 2 materials of critical parts. All the thickness design variables are continuous and allowed to be varied over the range of $0.5 \times t_0$ to $1.5 \times t_0$, where t_0 is the baseline value. The material design variables are discrete, which can only be either mild steel (MS) or high-strength steel (HSS). All thickness and material design variables are also random variables, normally distributed with standard deviation of 3% of the mean for the thickness variables and 0.6% for the material prop-

Table 2 Regulations and requirements for side impact

Performance		EEVC regulation	Top safety rating criteria	Baseline value	Design constraints
Abdomen load (KN)		2.5	≤ 1.0	0.663	≤ 1.0
Rib deflection, D (mm)	upper	42	≤ 22	28.5	≤ 32
	middle	42	≤ 22	29.0	≤ 32
	low	42	≤ 22	34.0	≤ 32
V*C (m/s)	upper	1.0	≤ 0.32	0.22	≤ 0.32
	middle	1.0	≤ 0.32	0.21	≤ 0.32
	low	1.0	≤ 0.32	0.31	≤ 0.32
Pubic symphysis force, FPS (KN)		6	≤ 3.0	4.067	≤ 4.0
HIC		1000	≤ 650	229.4	≤ 650

Table 3 Design parameters

Parameter	Type	Side Constraints
1. Thickness of B-pillar inner	Random design variable	$0.5x_1^0 \leq x_1 \leq 1.5x_1^0$
2. Thickness of B-pillar reinforcement	Random design variable	$0.5x_2^0 \leq x_2 \leq 1.5x_2^0$
3. Thickness of floor side inner	Random design variable	$0.5x_3^0 \leq x_3 \leq 1.5x_3^0$
4. Thickness of cross members	Random design variable	$0.5x_4^0 \leq x_4 \leq 1.5x_4^0$
5. Thickness of door beam	Random design variable	$0.5x_5^0 \leq x_5 \leq 1.5x_5^0$
6. Thickness of door belt line Reinforcement	Random design variable	$0.5x_6^0 \leq x_6 \leq 1.5x_6^0$
7. Thickness of roof rail	Random design variable	$0.5x_7^0 \leq x_7 \leq 1.5x_7^0$
8. Material of B-pillar inner	Random design variable	x_8 : MS or HSS
9. Material of floor side inner	Random design variable	x_9 : MS or HSS
10. Barrier height	Pure random variable	$30 \leq x_{10} \leq 30$
11. Barrier hitting position	Pure random variable	$30 \leq x_{11} \leq 30$

MS: Mild Steel
HSS: high-strength steel

Table 4 Side impact results: optimization, reliability-based optimization

		Optimization		Reliability optimization	
	Baseline	Solution (SQP)	Reliability (MCS)	Solution	Reliability (MCS)
Weight (Kg)	29.17	23.59		24.99	
Abdomen load (KN)	0.7325	0.5635	100%	0.4243	100%
D_{up} (mm)	28.85	28.41	100%	28.59	100%
D_{mid}(mm)	29.64	27.67	99.60%	27.11	99.90%
D_{low} (mm)	34.97	32.00	49.65%	31.14	90.45%
V^*C_{up}	0.2345	0.2299	100%	0.2330	100%
V^*C_{mid}	0.2065	0.2029	100%	0.2115	100%
V^*C_{low}	0.3181	0.2924	100%	0.2895	100%
FPS(KN)	4.049	4.000	39.25%	3.88	90.70%
$V_{B\text{-}Pillar}$ (mm/ms)	9.636	9.342	100%	9.254	100%
V_{Door} (mm/ms)	14.88	15.68	67.45%	15.47	99.45%
# function evals	1	30	2000	1174	2000

erties. The final two parameters in Table 3, the barrier height and hitting position, are pure random variables that are continuous and vary from −30 mm to 30 mm according to the physical test. These pure random variables are used only in the reliability and robustness assessment (fixed during deterministic optimization at given baseline values).

Before running deterministic optimization with this side impact formulation, a sample of 40 crash simulations were executed, in parallel, based on a uniform Latin hypercube design experiment. A stepwise regression approach was used to fit polynomial response surface models to the obtained data, and following verification of these models they were used to obtain the results presented in this paper. For the deterministic optimization, a sequential quadratic programming (SQP) algorithm is employed (appropriate for quadratic response surface models) and a solution is obtained after 30 evaluations of the response surface models. The side impact deterministic optimization results are compared to the baseline design in Table 4. As seen in this table, the optimization solution results in a weight reduction of 5.58 kg but also in three active constraints: the lower rib deflection constraint, pubic symphysis force, and door velocity.

When a reliability analysis is performed at this design solution using Monte Carlo descriptive sampling with 2000 points, the reliability values shown in the fourth column of Table 4 are obtained. As expected with the three active constraints, the reliability values are less than desirable: 49.65% for the lower rib deflection constraint, 39.25% for pubic symphysis force, and 67.45% for door velocity. To improve the reliability associated with these active constraints, probabilistic optimization is implemented. Both reliability-based optimization and the six sigma-based robust optimization formulations are exercised for this problem.

4.1 Side impact reliability-based optimization

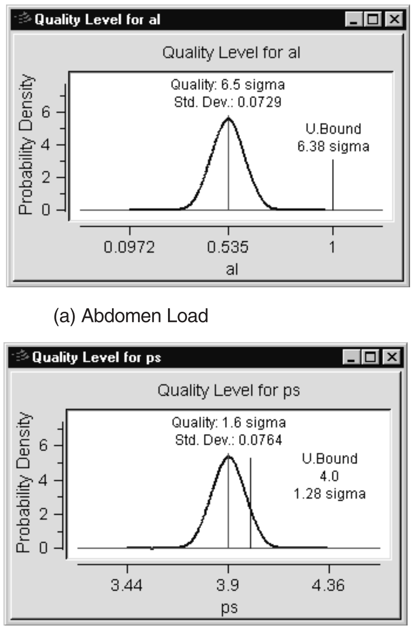
For the reliability-based optimization implementation, the optimization formulation is modified by converting the constraints to probabilistic constraints as follows:

Minimize Weight

Subject to
$\Pr[\text{Abdomen load} > 1.0] \leq 10\%$
$\Pr[\text{Rib Deflection} > 32] \leq 10\%$
$\Pr[\text{V*C} > 0.32] \leq 10\%$
$\Pr[\text{FPS} > 4.0] \leq 10\%$
$\Pr[\text{V}_{\text{B-pillar}} > 9.9] \leq 10\%$
$\Pr[\text{V}_{\text{Door}} > 15.70] \leq 10\%$

With the acceptable probability of failure initially set to 10% for this problem, the desired reliability is 90% for all constraints. The 11 random variables defined previously are incorporated, and reliability-based optimization is performed, starting with the optimized design. For this reliability-based optimization, a modified version of the single-loop algorithm presented in Chen et al. (1997) is employed. With this algorithm, reliability analysis is not actually performed at each optimization step, as with double-loop approaches (optimization algorithm calling reliability analysis algorithm), but rather the constraints are adjusted for the desired reliability level and optimization is performed with the adjusted constraints. Reliability analysis is performed at the final design to verify the achieved reliability levels. The algorithm presented in Chen et al. (1997) is modified in this work to allow updating of the constraint adjustment during the optimization at each step. With this approach, the reliability-based optimization is implemented within the optimization algorithm itself, as opposed to the decoupled nature of double-loop approaches. This strategy has been implemented using a sequential linear programming (SLP) algorithm within the design environment (Koch et al. 2002) employed in this work, and thus the algorithm is different than the SQP algorithm employed for deterministic optimization.

The reliability-based optimization results are also presented in Table 4, columns five and six. This reliability-based optimization requires 1174 evaluations of the response surface models. The reliability at the final design point was verified by again running 2000 Monte Carlo descriptive sampling points. The 90% reliability goal is achieved or exceeded for all constraints. Two of the three previously active constraints drive the reliability as seen in Table 4, the lower rib deflection constraint and pubic symphysis force; the door velocity constraint is no longer active even for the specified 90% reliability. The tradeoff in achieving the 90% reliability goal is the objective value;



(a) Abdomen Load

(b) Public Symphysis Force, FPS

Fig. 7 Quality level at reliability optimization solution

the weight increases slightly but still represents a 4.18 kg weight savings compared to the baseline design.

When we review the reliability-based optimization results for the side impact problem in the context of "six sigma", converting the reliabilities to a sigma level, or quality level, the range is from a high value of over 8σ (beyond the limits of double precision in calculating the probability values) for abdomen load to a low value of 1.7σ for pubic symphysis force, as shown in Fig. 7. When performing a robustness analysis of the objective, weight, it is discovered that the standard deviation of weight is a small percent of the mean weight value – 1.5% – and thus objective robustness is not a primary concern for this problem.

4.2 Side impact six sigma robust optimization

In an attempt to increase the quality level and observe the resulting tradeoff, the reliability optimization formulation is converted to a "six sigma" robust optimization formulation. This robust optimization formulation is given as follows:

Minimize $\mu[\text{Weight}] + \sigma[\text{Weight}]$

Subject to

$$\mu[\text{Abdomen load}] + n\sigma[\text{Abdomen load}] \leq 1.0\,\text{KN}$$
$$\mu[\text{Rib deflection}] + n\sigma[\text{ Rib deflection}] \leq 32\,\text{mm}$$
$$\mu[\text{V*C}] + n\sigma[\text{V*C}] \leq 0.32\,\text{m/s}$$
$$\mu[\text{FPS}] + n\sigma[\text{FPS}] \leq 4.0\,\text{KN}$$
$$\mu[\text{V}_{\text{B-pillar}}] + n\sigma[\text{V}_{\text{B-pillar}}] \leq 9.9\,\text{mm/ms}$$
$$\mu[\text{V}_{\text{Door}}] + n\sigma[\text{V}_{\text{Door}}] \leq 15.70\,\text{mm/ms}.$$

Here, "$n\sigma$" can be any desired sigma level. Recall that the "six" in "six sigma" does not imply that only 6σ solutions are sought in a six sigma process (6σ is a quality measure, six sigma is a quality improvement philosophy). A six sigma process seeks to explore the tradeoff between performance and quality and to investigate the quality levels achievable for a given design. In many cases, 6σ quality is not achievable for a given design concept, configuration, and/or set of requirements; in many cases the cost of achieving 6σ quality is not bearable with current technology. New technology is not investigated in this side impact problem. In this investigation, 3σ and 6σ solutions are sought ($n = 3$ and $n = 6$ in the six sigma robust optimization formulation above).

The results obtained after executing this robust optimization formulation with 3σ and 6σ quality constraints are presented in Table 5. The robust optimization problems are solved employing an SQP algorithm, coupled with the second-order Taylor method of Sect. 3.2 for estimating the mean and standard deviation of all outputs. For the 3σ formulation, robust optimization is performed starting from the deterministic optimization solution given in Table 4. A total of 1236 evaluations of the response surface models are required to obtain this solution, and the desired 3σ quality level is achieved for all constraints. The three active constraints after deterministic optimization drive the 3σ solution, as shown in Table 5. The tradeoff results in a weight increase of 5.92 kg over the deterministic optimization starting point, 0.34 kg over the baseline design. The standard deviation of weight does not change (0.36 kg at both the deterministic optimization solution and the 3σ solution), and thus the objective *robustness* is not affected.

When the six sigma robust optimization formulation is changed to seek 6σ quality with respect to all constraints, the solution shown in the last two columns of Table 5 is obtained. Once again the SQP algorithm is employed with the second-order Taylor's method for statistic estimations. This robust optimization is performed starting with the 3σ solution and requires an additional 225 evaluations of the response surface models. For this problem, given its current formulation and requirements, 6σ quality is not achievable for all constraints. While the quality level is increased for every constraint, five of the ten constraints do not satisfy the 6σ level. In this case, the design variables are driven to their bounds and thus this solution represents the best quality allowable given the defined design space. The quality driver in this case is the pubic symphysis force, with the lowest quality level of 3.76σ. For comparison with Fig. 7b, the quality level for pubic symphysis force is shown graphically in Fig. 8a for the 3σ formulation results and in Fig. 8b for the 6σ formulation results. Notice in Fig. 8 (as well as in Fig. 7) that the overall quality or sigma level displayed is different than the sigma position of the upper bound. The position of the upper bound, as a number of standard deviations, or "sigmas", from the mean, is not equivalent to the sigma level or overall quality level with respect to this constraint. With only one bound present, "sigma level" is a metric that reflects the percent variation outside the bound, the probability of failure, and the equivalent $\pm n\sigma$. The total probability of failure is the consistent basis for the quality measurement; with a single upper bound at 3.58σ as in Fig. 8b, the percent of the distribution variation outside the constraint is equivalent to that with symmetric lower and upper bounds at $\pm 3.8\sigma$ around the mean, and thus the overall quality level is 3.8σ.

The tradeoff between performance and quality for this side impact crashworthiness problem is captured in Fig. 9, where the sigma level quality of each solution presented – deterministic optimum, 90% reliability solution, 3σ solution, and 6σ formulation solution – is plotted

Table 5 Side impact results: optimization, six sigma robust optimization

	Baseline	Robust optimization – 3σ Solution	Sigma level	Robust optimization – 6σ Solution	Sigma level
Weight (Kg)	29.17	29.51		35.34	
Abdomen load (KN)	0.7325	0.3326	> 8 (∼ 100%)	0.3698	> 8 (∼ 100%)
D_{up} (mm)	28.85	27.87	4.93 (99.99992%)	24.36	>8 (∼100%)
D_{mid}(mm)	29.64	25.61	4.12 (99.9963%)	20.82	6.99 (99.9999999997%)
D_{low} (mm)	34.97	29.36	3.0 (99.73%)	26.32	4.61 (99.9996%)
$V{*}C_{up}$	0.2345	0.2104	> 8 (∼ 100%)	0.1491	> 8 (∼ 100%)
$V{*}C_{mid}$	0.2065	0.2020	> 8 (∼ 100%)	0.1640	> 8 (∼ 100%)
$V{*}C_{low}$	0.3181	0.2722	4.83 (99.99986%)	0.2459	5.48 (99.999996%)
FPS(KN)	4.049	3.764	3.0 (99.73%)	3.751	3.76 (99.98%)
$V_{B\text{-}pillar}$ (mm/ms)	9.636	8.928	4.77 (99.99982%)	8.307	5.47 (99.999996%)
V_{Door} (mm/ms)	14.88	15.27	3.0 (99.73%)	15.00	4.42 (99.9990%)
# function evals	1	1236		225	

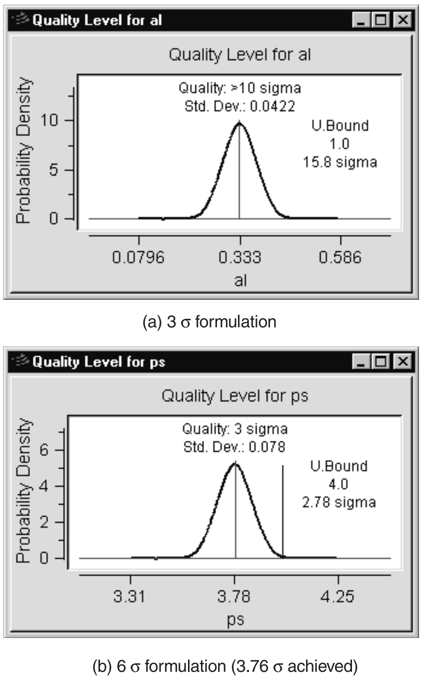


Fig. 8 Quality level, pubic symphysis force

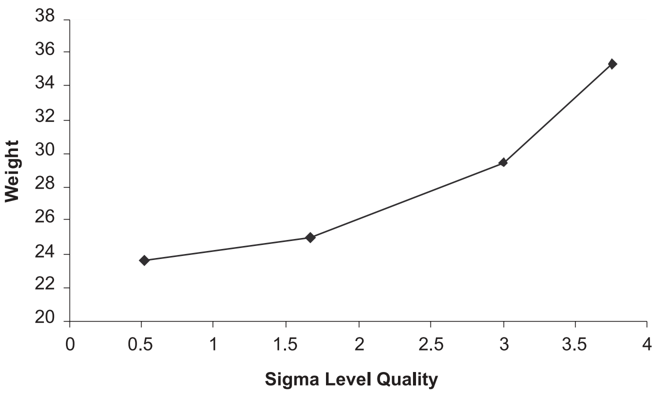


Fig. 9 Side impact performance vs. quality tradeoff

against the weight objective. As can be seen, as design quality is increased, weight not only increases but increases at an increasing rate. Design quality comes at a cost. Design engineers and managers are faced with the challenge of evaluating this tradeoff and making decisions. Implementing six sigma design strategies facilitates the generation of the data necessary to evaluate these tradeoffs and make informed decisions.

5
Discussion and closing remarks

In an engineering design context, the concepts and philosophy of six sigma can be combined with methods from structural reliability and robust design to formulate a design for six sigma strategy for measuring and improving design quality. Inherent uncertainty and known variability in engineering design problems are captured by characterizing independent design parameters through probability distributions and associated statistics, or ranges of variability. Resulting performance variation is measured and design quality assessed. Two goals for design quality have been discussed: maintaining performance variation within acceptable limits or constraints and reducing performance variation. These concepts of distribution "shift", to improve reliability, and distribution "shrink", to improve robustness, together define the overall achievable sigma level of a design.

A robust optimization formulation has been presented that incorporates these design quality concepts and is framed within the six sigma philosophy and terminology. This flexible formulation is implemented within a commercial design environment (Koch et al. 2002), providing fully automated design and quality assessment and improvement and parallel processing for improved efficiency (Koch et al. 2000). Multiple sampling methods incorporated within this environment for estimating performance variability have been summarized. The key to effective and practical implementation of this six sigma robust optimization strategy is acceptably accurate and efficient sampling and performance variability estimation. Unfortunately, there is usually a conflict between accuracy and efficiency (in most cases, accuracy increases with increasing sampling points, leading to reduced computational efficiency). Performance variation estimation is an ongoing topic of research. Currently, in most cases the complexity and computational expense of the analyses involved will dictate the allowable degree of sampling and thus the accuracy level. For many complex design problems, the computational expense of a single analysis precludes even deterministic optimization, let alone probabilistic, robust optimization. For these cases, accurate and efficient approximation models are essential. When using approximation models, as in the example presented in this paper, the number of function evaluations necessary to achieve a reliable, robust solution is not a primary concern since the models are extremely inexpensive to execute. Creation of approximation models of acceptable accuracy becomes the primary concern and is another topic of ongoing research that is key to these design strategies. Currently, approximation methods capable of more accurately modeling nonlinear responses, such as *kriging* (Simpson et al. 1998, 2001), are being investigated for complex, computationally expensive engineering design problems.

The six sigma robust optimization formulation is demonstrated in this paper for an automotive application – vehicle structural design for side impact crashworthiness. In this example, the tradeoff between "optimized" structural weight and reliability and robustness, or "sigma level", with respect to strict safety regulations is assessed. While optimization can reduce the weight and satisfy the safety constraints when variability is not considered, increasing the desired reliability or overall quality level results in an increase of weight. A 3σ solution is achieved, with a weight essentially equal to that of the baseline design. In pushing the limits of achievable quality in formulating 6σ quality constraints, only 3.76σ is achievable with the current problem formulation. However, side impact is only one crash scenario. Future investigations are necessary to combine various crash modes – front impact, rear impact, roof crush – for full vehicle robust design for crashworthiness.

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