*25040175*

**STATUS:** PENDING 20061107
**REQUEST DATE:** 20061107
**BORROWER:** GKJ
**RENEWAL REQ:**
**LENDERS:** *GAS, GUA, GAT, YBM, WVU

**NEED BEFORE:** 20061207
**RECEIVE DATE:**
**NEW DUE DATE:**

**OCLC #:** 1800135
**SOURCE:** ILLiad
**DUE DATE:**
**SPCL MES:**

**CALL NUMBER:**
**TITLE:** Journal of quality technology.
**ISSN:** 0022-4065
**IMPRINT:** [Milwaukee] American Society for Quality Control.
**ARTICLE:** Victor E. Kane: Process Capability Indices
**VOL:** 18
**DATE:** 1986 **PAGES:** 41-52
**VERIFIED:** OCLC
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# Process Capability Indices

VICTOR E. KANE

*Ford Motor Company, Transmission and Chassis Division, P.O. Box 2097 (7) Livonia, MI 48150*

The capability indices $C_p$, CPU, CPL, $k$ and $C_{pk}$ are presented and related to process parameters. These indices are shown to form a complementary system of measures of process performance, and can be used with bilateral and unilateral tolerances, with or without target values. A number of Japanese industries currently use the five indices and the U.S. automotive industry has started using these measures in a number of areas. Various applications of the indices are discussed along with statistical sampling considerations.

## Introduction

THE quantification of process location and variation is central to understanding the quality of units produced from a manufacturing process. Consider the situation where the process mean, $\mu$, and standard deviation, $\sigma$, are unknown and estimated by $\bar{x}$ and $s$, respectively. From a practical viewpoint, $\bar{x}$ and $s$ are not unitless and sometimes are not convenient summary statistics when hundreds of characteristics in a plant or supply base are considered. In many situations, capability indices can be used to relate the process parameters $\mu$ and $\sigma$ to engineering specifications that may include unilateral or bilateral tolerances with or without a target dimension (nominal value). The resulting indices are unitless and provide a common, easily understood language for quantifying the performance of a process.

The automotive industry is currently involved in an extensive effort to implement statistical process control (SPC) in their plants and supply base. Capability indices derived from SPC have received increased usage not only in process assessments, but in evaluation of purchasing decisions. Of particular interest have been the $C_p$ and $C_{pk}$ indices used in Japan (Sullivan [1984, 1985]) and recently in the U.S. automotive industry (e.g., Ford Motor Company [1984]). These indices relate the natural tolerance ($6\sigma$) used in the U.S. quality control literature (e.g., Juran and Gryna [1980 p. 299]) to engineering specifications. The purpose of this paper is to examine uses of capability indices, along with their sampling properties and estimation procedures. Throughout the discussion it is assumed that the process output is approximately normally distributed and in a state of statistical control. Also, a "hat" ($\hat{\ }$) will denote an estimated quantity.

## Process Potential

### $C_p$ Index

Manufacturing processes undergo several stages of development prior to actual production. The stages of "prove out" testing at the machine supplier, initial testing at the manufacturing facility, and pre-production testing all seek to determine whether machinery can produce on an on-going basis production units that meet the required engineering specifications. A typical baseline is the assessment of whether the natural tolerance ($6\sigma$) of a process is within the specification limits. An alternate formulation used in Japan (Sullivan [1984]) is to evaluate the capability index $C_p$. The above quantities are merely different ways to relate the allowable process spread (part tolerance) to the actual process spread (natural tolerance) as shown in Figure 1 where $USL$ = upper specification limit, $LSL$ = lower specification limit, and $NT$ = natural tolerance. The process potential index is

$$C_p = \frac{\text{allowable process spread}}{\text{actual process spread}} = \frac{USL - LSL}{6\sigma} = \frac{USL - LSL}{NT}. \tag{1}$$

A $C_p$ of 1.0 indicates that a process is judged to be "capable" as indicated in Figure 2. It is generally necessary to estimate the process standard deviation in (1) so an estimate $\hat{C}_p$ of the process capability is obtained. Due to sampling variation and machine testing

Dr. Kane is Manager of the Applied Systems Analysis Department within the Transmission and Chassis Division of Ford Motor Company. He is a CQE and a Member of ASQC.

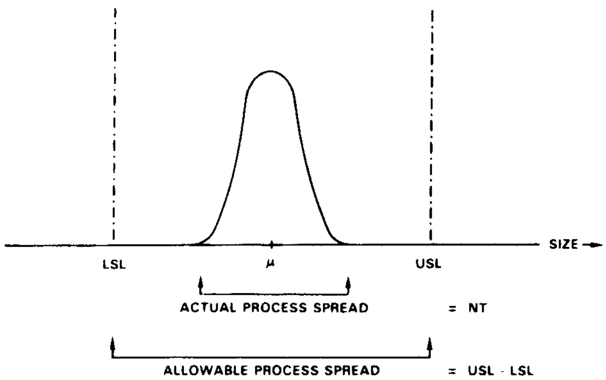



FIGURE 1. Relationship of $C_p$ Parameters.

limitations discussed below, $\hat{C}_p = 1.0$ is generally not used as a minimally acceptable value.

A capable process with an underlying stable normal distribution will in theory result in 0.27% of parts beyond the specification limits. The benchmark of 1.0 was chosen to relate $C_p$ to the standard six sigma spread used on control charts. If a process is exactly capable then

$$UCL = \frac{USL}{\sqrt{n}} \qquad LCL = \frac{LSL}{\sqrt{n}} \tag{2}$$

where

$UCL$ = upper control limit

$LCL$ = lower control limit

$n$ = subgroup sample size on control chart.

The relation in (2) indicates that sample size adjusted specification limits are equal to control limits for a process where $C_p = 1.0$.

A primary use of $C_p$ is to make various types of comparisons. Figure 3 shows how different $C_p$ values relate to the spread of a process relative to the specification width. A minimum value of $C_p = 1.33$ is generally used for an ongoing process (e.g., Juran, Gryna and Bingham [1979, pp. 9-22]). This ensures a very low reject rate (0.007%) and is thus an effective strategy for prevention of nonconforming items. A value of $C_p = 1.33$ is also often used to qualify machinery since long-term statistical control of a process is generally not established during qualification trials. Using 1.33 gives some assurance that a $C_p = 1.0$ will be possible when the additional sources of variation are experienced in production processing. It should be noted that it is seemingly more natural to use the traditional indicator

$$\% \text{ of specification used} = (100)\frac{1}{C_p}.$$

The motivation for using $C_p$ rather than the % of specification (or "capability ratio" from Charbonneau and Webster [1978]) stems from the natural relationship between process potential, quantified by $C_p$, and process performance, quantified by $C_{pk}$, which is discussed in later sections. Also, there are natural extensions to unilateral tolerances and target dimensions using the $C_p$ index.

Machine potential studies often do not consider whether a machine is in statistical control. This is clearly not advisable since many simple problems could be identified if standard control charts were used, even with short production runs of $N$ parts (i.e., $N$ being between 30 and 100). Consideration of the production order on control charts is a natural problem-identification tool. However, it is not possible to determine if a machine will be in a state of statistical control over the long-term with a short-term sampling plan. The question arises whether a short-term capability study is of any value in predicting long-term production performance. If it is assumed that an unstable process will have a larger standard deviation than the same process which is stable, then a short-term process potential study will provide indication of problem areas. Typically, a process is evaluated using the estimated capability

$$\hat{C}_p = \frac{USL - LSL}{6s} \tag{3}$$

where $s$ is the sample standard deviation computed from a sample of size $N$. Unfortunately, short-term studies have other problems, such as specially prepared raw material, new tools, highly trained operators, and so on that can make the qualification process

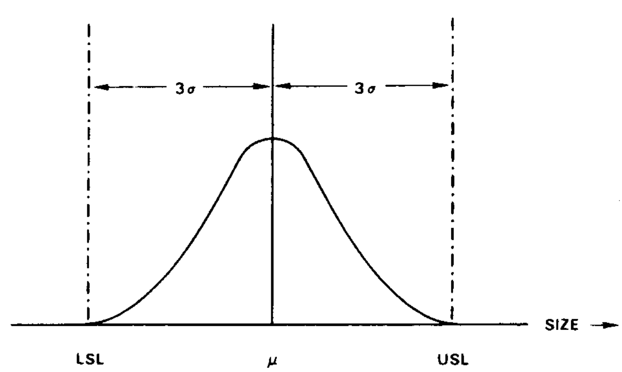



FIGURE 2. Distribution of Individual Parts for a Capable Process ($C_p = 1.0$).

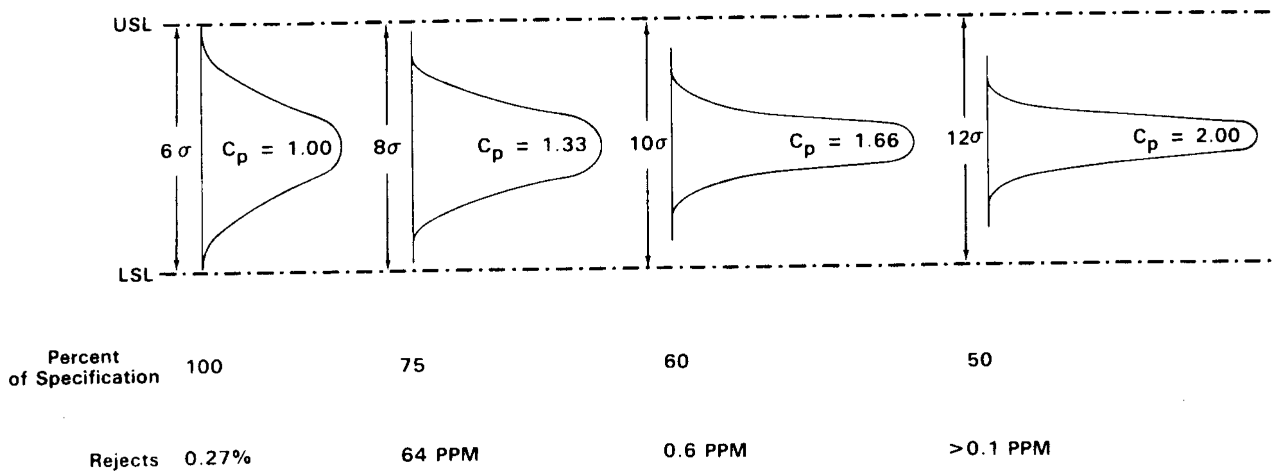



FIGURE 3. $C_p$ Indices for Varying Widths of the Process Distribution.

different from typical production operations. Generally, the special factors in short-term qualification studies reduce variability that is likely in production processing. Thus, these studies mainly serve to identify large problem areas. The $C_p$ index computed from short-term studies generally identifies the lower bound of variability (process potential) that can be expected in the initial production setting. However, it is not uncommon to be able to improve the initial production $C_p$ index with later process refinements.

**Sampling Considerations**

The sampling variation of $\hat{C}_p$ in (3) can be easily studied since it is possible to use the chi-square distribution of the sample variance ($s^2$). Burr (1976, pp. 336-341) derives the operating characteristic ($OC$) curve for testing the standard deviation using the fact that $(N-1)s^2/\sigma^2$ has a chi-square distribution with $(N-1)$ degrees of freedom. In an analogous manner, it is possible to compute the $OC$ curve for testing process capability. It should first be noted that the test ($\sigma_0 > 0$)

$$H_0: \sigma \geq \sigma_0$$

$$H_1: \sigma < \sigma_0$$

is equivalent to a test for process capability ($c_0 > 0$)

$$\begin{aligned} &H_0: C_p \leq c_0 \text{ (process is not capable)} \\ &H_1: C_p > c_0 \text{ (process is capable).} \end{aligned} \tag{4}$$

Using a critical region of $\hat{C}_p > c$, the power function of the test (4) is obtained directly from the chi-square distribution of the sample variance as

$$\begin{aligned} \pi(C_p) &= \Pr[\hat{C}_p > c | C_p] \\ &= \Pr[\chi^2_{(N-1)} < (N-1)C_p^2 c^{-2} | C_p] \end{aligned} \tag{5}$$

where $\chi^2_{(N-1)}$ is a chi-square random variable with $(N-1)$ degrees of freedom. It is convenient to use the power function for $C_p$ rather than the corresponding power function for $\sigma$, since it is not necessary to adjust for differences in process tolerances and a "standardized" process capability can be studied directly.

Using (5) the $OC$ curve $OC(C_p) = 1 - \pi(C_p)$ can be computed and used to compare testing schemes. The $OC$ curve enables evaluation of the unfortunate practice of sometimes neglecting sampling variation of $\hat{\sigma}$ when assessing process capability. Suppose we wish to evaluate whether a process is capable at the $c_0 = 1.33$ level (i.e., $H_0: C_p \leq 1.33$) using $N = 30$ samples and a sample rejection limit (critical value) of $c = 1.33$. The $OC$ curve (a) in Figure 4 shows that $OC(1.33) = 0.40$, which implies that there is a 40% chance of incorrectly judging the process not capable (accepting $H_0$). The true process capability must be $C_p = 1.6$ before

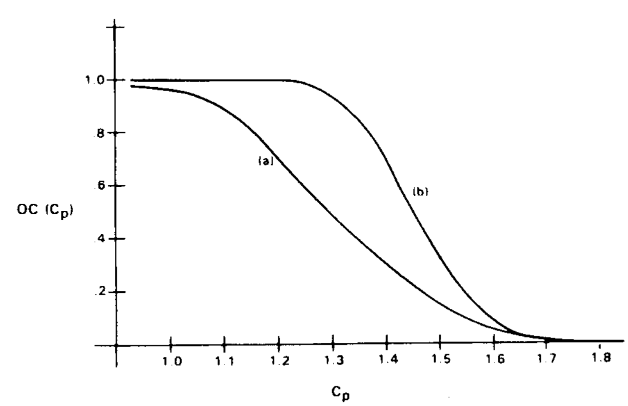



FIGURE 4. Operating Characteristic Curve for Sampling Plan that Rejects Process Capability if $\hat{C}_p < c$ Where (a) $N = 30$; $c = 1.33$, and (b) $N = 70$; $c = 1.46$.

there is only a 5% chance of judging a process not capable using a critical value of $c = 1.33$.

Selection of a meaningful critical value for a capability test requires specification of an acceptable quality level ($AQL$) and rejectable quality level ($RQL$) for the $C_p$ value (see Burr [1976, p. 336–341]). The $AQL$ is a sufficiently high process capability such that we would like to accept processes with capabilities above the $AQL$. The $RQL$ is a sufficiently low process capability such that we would like to reject processes with capabilities below the $RQL$. Thus,

$$AQL = C_p(\text{high}) > C_p(\text{low}) = RQL.$$

Using (5) the $\alpha$ and $\beta$ risks are

$$\begin{aligned}\alpha &= \pi(C_p(\text{low})) \\ &= \Pr[\chi^2_{(N-1)} < (N-1)C_p^2(\text{low})c^{-2} | C_p(\text{low})] \\ 1-\beta &= \pi(C_p(\text{high})) \\ &= \Pr[\chi^2_{(N-1)} < (N-1)C_p^2(\text{high})c^{-2} | C_p(\text{high})].\end{aligned}$$

Solving for $C_p(\text{high})/C_p(\text{low})$ and $c$ gives

$$\frac{C_p(\text{high})}{C_p(\text{low})} = \frac{\chi^2_{(N-1)}(1-\beta)}{\chi^2_{(N-1)}(\alpha)}$$

$$c = C_p(\text{low}) \frac{(N-1)}{\chi^2_{(N-1)}(\alpha)}$$

where $\chi^2_{(N-1)}(\gamma)$ is the $\gamma(100)$-th percentile of the chi-square distribution with $(N-1)$ degrees of freedom. Table 1 gives values of $C_p(\text{high})/C_p(\text{low})$ and $c/C_p(\text{low})$ for varying sample size and $\alpha = \beta$ equal to 0.10 and 0.05. The critical value $c$ and sample size $N$ can be determined for any specified $C_p(\text{high})$ and $C_p(\text{low})$. For example, if $\alpha = \beta = 0.05$, $C_p(\text{high}) = 1.6$, and $C_p(\text{low}) = 1.2$; then $C_p(\text{high})/C_p(\text{low}) = 1.33$, and from Table 1: $N = 70$ and $c = (1.14)C_p(\text{low}) = 1.37$. Thus, using a critical value of 1.37 and a sample size of 70, there is a 5% risk of judging a process with $C_p$ above 1.6 not capable (accepting $H_0$) and a 5% risk of judging a process with $C_p$ below 1.2 as capable (rejecting $H_0$).

TABLE 1. Sample Size and Critical Value Determination for Testing $C_p$

| Sample Size | $\alpha = \beta = .10$ $C_p(\text{high})/C_p(\text{low})$ | $\alpha = \beta = .10$ $c/C_p(\text{low})$ | $\alpha = \beta = .05$ $C_p(\text{high})/C_p(\text{low})$ | $\alpha = \beta = .05$ $c/C_p(\text{low})$ |
|---|---|---|---|---|
| 10 | 1.88 | 1.27 | 2.26 | 1.37 |
| 20 | 1.53 | 1.20 | 1.73 | 1.26 |
| 30 | 1.41 | 1.16 | 1.55 | 1.21 |
| 40 | 1.34 | 1.14 | 1.46 | 1.18 |
| 50 | 1.30 | 1.13 | 1.40 | 1.16 |
| 60 | 1.27 | 1.11 | 1.36 | 1.15 |
| 70 | 1.25 | 1.10 | 1.33 | 1.14 |
| 80 | 1.23 | 1.10 | 1.30 | 1.13 |
| 90 | 1.21 | 1.10 | 1.28 | 1.12 |
| 100 | 1.20 | 1.09 | 1.26 | 1.11 |

In order to guarantee that any process with a $C_p$ below 1.33 (where $USL - LSL = 8\sigma$) has a high probability of being judged not capable ($\alpha = 0.1$), and a process with a $C_p$ above 1.66 (where $USL - LSL = 10\sigma$) has a high probability of being judged capable ($\beta = 0.1$), then $C_p(\text{high})/C_p(\text{low}) = 1.25$, and from Table 1: $N = 70$ and $c/C_p(\text{low}) = 1.10$, which gives $c = 1.46$. The $OC$ curve for this sampling plan is curve (b) in Figure 4.

The above example indicates that some past automotive industry machine qualification practices using $N = 30$ were not generally adequate. Also, current practices using a minimum $c = 1.33$ may account for nonsampling problems that make qualification runs different from production runs, but a minimum of 1.33 may not adequately account for sampling variability. Larger $N$ and/or higher minimum $C_p$ values may be necessary. Charbonneau and Webster (1978, p. 112) suggest a capability index of 1.5 for new equipment, but the sample size is not specified.

### Tolerance Interval Approach

An alternate approach for assessing the performance of a process is to construct an interval with length $2Ks$ such that $100p$ percent of the process output is covered by the interval with probability $\gamma$. This is a standard tolerance interval approach to process capability. Tolerance intervals, together with other types of statistical intervals, are discussed by Hahn (1970). It could be argued that the $\hat{C}_p$ index in (4) should be generalized to

$$\hat{C}_t = \frac{USL - LSL}{2Ks} \tag{6}$$

where $\hat{C}_t = \hat{C}_p$ if $K = 3$. If we choose $N = 70$, $p = .99$ and $\gamma = .95$, then from Table 2 the tolerance interval has length $6.042s$, which is essentially the same as the standard approach. However, other intervals would be obtained for different $N$, $p$, and $\gamma$. For example, with $N = 30$ and the same $p$ and $\gamma$ as before; $K = 3.35$ so the denominator of (6) becomes $6.7s$. Thus, the increased uncertainty for $N = 30$ is reflected by a larger multiplier of $s$ in the denominator of (6).

## Process Performance

It is apparent that the $C_p$ index measures potential process performance since only the process spread is related to the specification limits; the location of the

TABLE 2. Tolerance Interval Factors

| Sample Size | $\gamma = 0.95$ | | | $\gamma = 0.99$ | | |
|---|---|---|---|---|---|---|
| | $p = 0.95$ | 0.99 | 0.999 | 0.95 | 0.99 | 0.999 |
| 10 | 3.379 | 4.433 | 5.649 | 4.265 | 5.594 | 7.129 |
| 20 | 2.752 | 3.615 | 4.614 | 3.168 | 4.161 | 5.312 |
| 30 | 2.549 | 3.350 | 4.278 | 2.841 | 3.733 | 4.768 |
| 40 | 2.445 | 3.213 | 4.104 | 2.677 | 3.518 | 4.493 |
| 50 | 2.379 | 3.126 | 3.993 | 2.576 | 3.385 | 4.323 |
| 60 | 2.333 | 3.066 | 3.916 | 2.506 | 3.293 | 4.206 |
| 70 | 2.299 | 3.021 | 3.859 | 2.454 | 3.225 | 4.120 |
| 80 | 2.272 | 2.986 | 3.814 | 2.414 | 3.173 | 4.053 |
| 90 | 2.251 | 2.958 | 3.778 | 2.382 | 3.130 | 3.999 |
| 100 | 2.233 | 2.934 | 3.748 | 2.355 | 3.096 | 3.954 |

process mean is not considered. It would be possible to have any percentage of parts outside the specification limits with a high $C_p$ by merely locating the process mean sufficiently close to a specification limit. Thus, only potential performance of a process is quantified by $C_p$ and can only be attained for the process mean equal to the midpoint of the specification limits.

The $C_{pk}$ index is related to the $C_p$ index, but utilizes the process mean and can be considered a measure of process performance. There are two equivalent forms of the $C_{pk}$ index. The first formulation is derived by considering the upper and lower specification limits separately. The second formulation uses the deviation of the process mean from the midpoint of the specification limits. Both derivations will be given since a different insight is gained from each approach.

**Upper and Lower Capability Index**

Consider the unilateral tolerance situation where only a single specification limit is given. Figure 5 shows the process distribution related to an upper specification limit. Relating the actual and allowable spreads in the same manner as (1) gives a new *CPU* index

$$CPU = \frac{\text{allowable upper spread}}{\text{actual upper spread}} = \frac{USL - \mu}{\frac{NT}{2}} = \frac{USL - \mu}{3\sigma}. \tag{7}$$

The *CPU* index was developed in Japan and is utilized by a number of Japanese companies. In a similar manner to (7) the index for the lower specification limit is

$$CPL = \frac{\mu - LSL}{3\sigma}.$$

Estimates of *CPU* and *CPL* are denoted by $\widehat{CPU}$ and $\widehat{CPL}$ and are obtained by replacing $\mu$ and $\sigma$ by $\bar{x}$ and $s$, respectively. The *CPU* ($C_p$ upper) and *CPL* ($C_p$ lower) indices are related to $C_p$ by

$$C_p = (CPU + CPL)/2$$

and are used to measure process capability in the single specification limit situation. Of course, in the single limit case a $CPU = 1.0$ implies half as many nonconforming items (0.136%) as when $C_p = 1.0$ in the two limit case.

**$C_{pk}$ Index**

Since *CPU* and *CPL* utilize process location, a natural index for the bilateral specification case is $C_{pk}$ where

$$C_{pk} = \text{Min}\{CPL, CPU\}. \tag{8}$$

Thus, the $C_{pk}$ index relates the scaled distance between the process mean and the closest specification limit. A slightly different approach is to reduce the value of $C_p$ for a process by a factor $(1 - k)$, a scaled distance that the process is off-center. Denote the midpoint of the specification range by

$$m = (USL + LSL)/2.$$

The distance between the process mean, $\mu$, and the optimum, the midpoint $m$, is $\mu - m$ where we are assuming $m \le \mu \le USL$ as shown in Figure 6. The scaled distance is

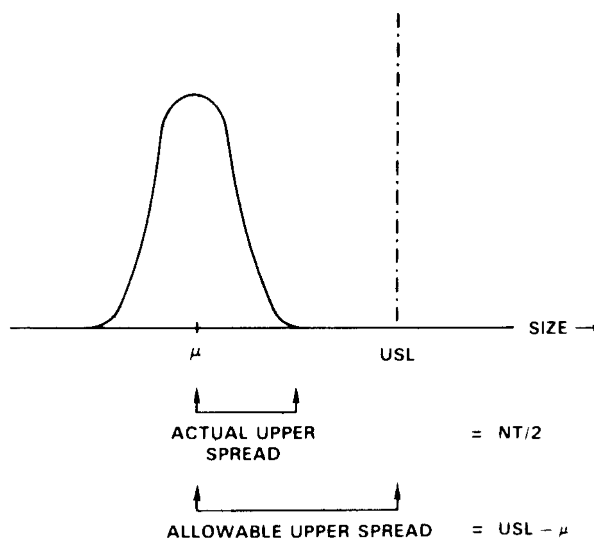



FIGURE 5. Relationship of CPU Parameters.

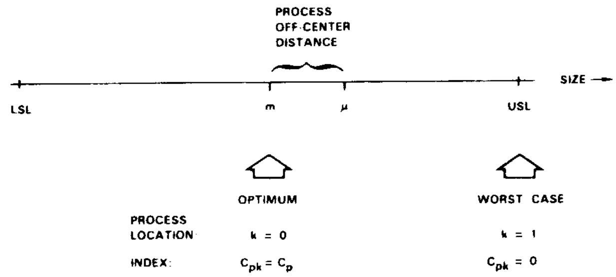



FIGURE 6. Relationship of $C_{pk}$ Parameters.

$$k = \frac{|m - \mu|}{\frac{USL - LSL}{2}}. \qquad (9)$$

The absolute value sign was added for the case when $LSL \leq \mu \leq m$. An estimate of $k$ is denoted by $\hat{k}$ and is obtained by replacing $\mu$ by $\bar{x}$ in (9). The $k$ factor is used in Japan as an index describing the amount the process mean is off-center. The $C_p$ index adjusted by the $k$ factor in (9) is

$$C_{pk} = C_p(1 - k) \qquad (10)$$

which is algebraically equivalent to (8). Since $0 \leq k \leq 1$, it follows that $C_{pk} \leq C_p$ where it is assumed $LSL \leq \mu \leq USL$.

Consider an artifical example where $USL = 20$, $LSL = 8$, $s = 2$ and $\bar{x} = 16$. First, note that $\hat{C}_p = 1.0$ so the process is potentially capable if it is located at the midpoint $m = 14$. However, $CPL = 1.33$, $CPU = 0.67$ and $\hat{k} = 0.33$ which results in $\hat{C}_{pk} = 0.67$. An acceptable process will require reducing $\sigma$ and/or centering $\mu$.

## Target Values

The effort to provide quality products passes through four phases. The first phase is to simply "make a part to print" where inspection is commonly used in go, no-go screening to hopefully ensure that all parts are within the engineering print specification limits. The second phase is the use of control charts to establish the stability of processes. The third phase is to minimize variability to enhance the uniformity of products. The ideas of defect prevention (i.e., preventing nonconforming parts from occurring) and continuous improvement are part of this phase. The fourth phase is to use enhanced process capability to target the means of processes to optimize product function, or more traditionally, to minimize cost (e.g., assembly time). The last phase is no less significant than its predecessors. The product design engineers are forced to address the question of "what dimensional value functions best" rather than "what dimensional values allow the product to function." Also, the manufacturing process engineers must now attempt to obtain the required mean (i.e., target value), as well as maintain reduced variability to be within process specifications.

The most common target value used today is the midpoint of the specification limits. This value is often selected as a process target to minimize cost since the chance of a part beyond the specification limit is minimized. Most off-center target values used today involve targeting mating part dimensions for ease of assembly or targeting upstream operations to minimize problems with downstream processing. Jessup (1983) discusses use of the Taguchi loss function that assigns a monetary value to departures from a selected target value. Figure 7 illustrates how target values ($T$), process capability, and cost are related.

There are a variety of ways to extend the previous capability indices to the target value case. The following development utilizes the analogy of actual and allowable process spread shown in Figure 1. The process potential index relates the actual process spread to the allowable spread. From Figure 8, for $T$ closest to $USL$

$$\begin{aligned} C_p &= \frac{\text{allowable process spread}}{\text{actual process spread}} \\ &= \frac{2(USL - T)}{NT} \\ &= \frac{USL - T}{3\sigma}. \end{aligned}$$

In general,

$$C_p = \text{Min}\left\{\frac{T - LSL}{3\sigma}, \frac{USL - T}{3\sigma}\right\}. \qquad (11)$$

The $C_p$ index in (11) is a generalization of $C_p$ in (1) obtained by forming the ratio of the allowable and actual process spread. The indices are equal when $T = m$.

A similar analogy can be used for the $k$ index. From Figure 9, for $T$ closest to $USL$

$$\begin{aligned} k &= \frac{\text{actual deviation}}{\text{½ allowable spread}} \\ &= \frac{\mu - T}{USL - T}. \end{aligned}$$

In general,

$$k = \frac{|T - \mu|}{\text{Min}\{T - LSL, USL - T\}} \qquad (12)$$

where we assume $\mu$ is between the specification limits and $0 \leq k \leq 1$. The $k$ index in (12) is a generalization

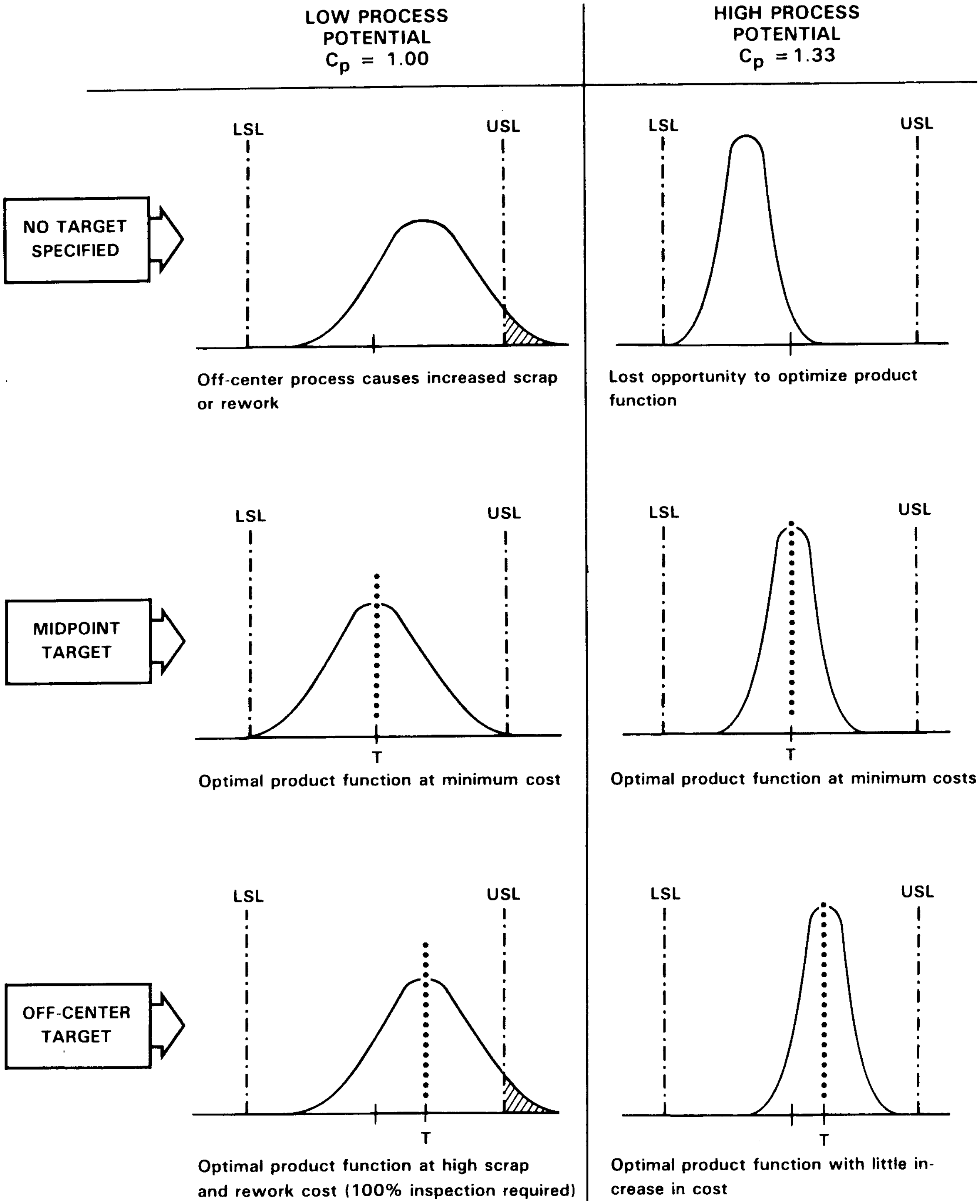



FIGURE 7. Process Capability Related to Use of Target Values.

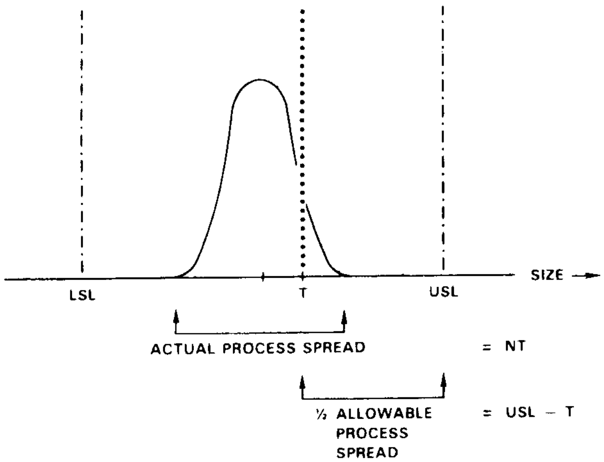



FIGURE 8. Relationship of $C_p$ Parameters Using Target Values.

of $k$ in (9), equivalence holds when $T = m$. Thus, the $C_{pk}$ index in (10) can be used by replacing $C_p$ and $k$ by (11) and (12), respectively. It is easy to show that

$$C_{pk} = \text{Min}\{CPL, CPU\}$$

where

$$CPL = \frac{T - LSL}{3\sigma}\left\{1 - \frac{|T-\mu|}{T - LSL}\right\}$$

$$CPU = \frac{USL - T}{3\sigma}\left\{1 - \frac{|T-\mu|}{USL - T}\right\}.$$

Note that $CPL = 0$ if $|T - \mu| > T - LSL$, and $CPU = 0$ if $|T - \mu| > USL - T$. These indices can be used for a one-sided specification limit with a target value. The above equations are analogous to (8) for $C_{pk}$ and correspond to the $CPL$ and $CPU$ defined previously when $T = \mu$.

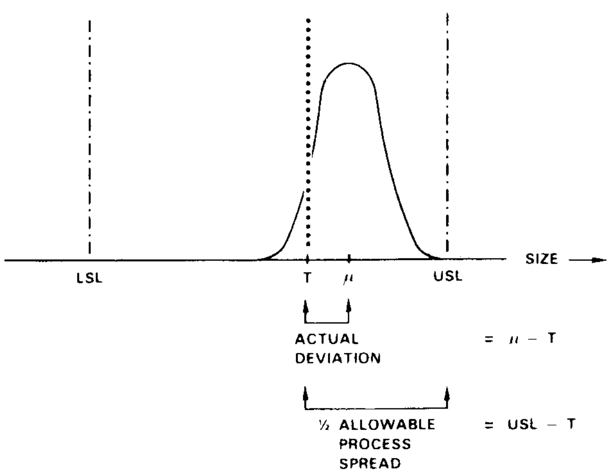
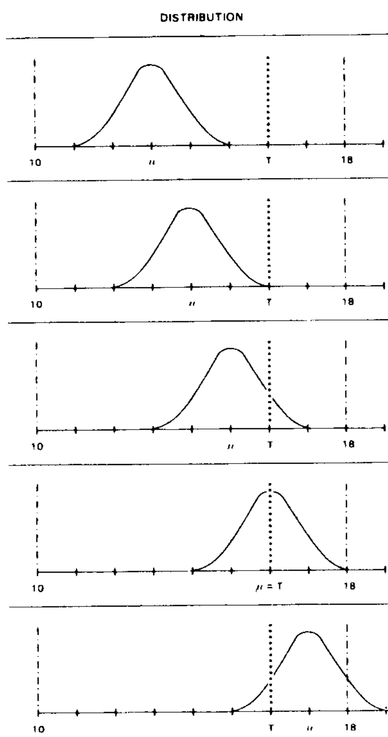



FIGURE 9. Relationship of $k$ Parameters Using Target Values.




| $\mu$ | CPL | CPU | k | $C_{pk}$ |
|---|---|---|---|---|
| 13 | 1.5 | 0.0 | 1.0 | 0.0 |
| 14 | 2.0 | 0.0 | 1.0 | 0.0 |
| 15 | 2.5 | 0.5 | 0.5 | 0.5 |
| 16 | 3.0 | 1.0 | 0.0 | 1.0 |
| 17 | 2.5 | 0.5 | 0.5 | 0.5 |

FIGURE 10. Capability Indices Using a Target Value of $T = 16$.

A simple example illustrates the use of the indices. Suppose that $USL = 18$, $LSL = 10$, $T = 16$ and $\sigma = 0.67$. Note that from (11) $C_p = 1.0$. Figure 10 shows the capability indices for selected mean values. Notice that because $C_{pk}$ adjusts for departures from $T$, it is possible to have a 0 capability and not have any parts out of the specification limits. Also, deviations on the "long side" (i.e., maximum distance between $T$ and a specification limit) of $T$ result in the same reduction in $C_{pk}$ as deviations on the "short side" (i.e., minimum distance between $T$ and a specification limit).

## Drawbacks

Experience to date has shown that there are potential problems in using $C_p$ and $C_{pk}$ on a routine basis. These drawbacks generally stem from users having an incomplete understanding of statistical principles rather than from problems with the indices. Some of the drawbacks are described below:

*(1) Statistical Control*—There is a tendency to want to know the capability of a process before statistical control has been established. Capability refers to a quantification of common cause variation and what can be expected from a process in the future. The presence of special causes of variation make predic-

tion impossible and the meaning of a capability index is unclear.

*(2) Sampling Plan*—Clearly, values of the average range $\bar{R}$, often used to estimate $\sigma$, depend on the sampling plan. Thus, it could be argued that the value of a capability index could be made to change easily by merely changing the sampling plan. The situation is seemingly complicated further when a remark is considered that W. Edwards Deming made to an engineer who was noting his achievement at attaining statistical control. Deming stated that he could make any process appear in statistical control—merely space out the samples within a subgroup over time. This statement is recognition of the fact that the number of sources of variation increases as the time interval between samples within a subgroup increases. Spreading out the samples within a subgroup over time will increase $\bar{R}$, widening the control limits, and thus makes achieving statistical control more likely. However, increasing $\bar{R}$ will increase the estimate of $\sigma$ and thus decrease process capability. Conversely, it is possible to increase process capability by using consecutive piece sampling (i.e., obtaining a small $\bar{R}$) to obtain a minimum estimate of $\sigma$ and maximum capability. However, in this case statistical control is the most difficult to attain since the distance between control limits will be narrower. Thus, considering only statistical control or process capability separately is not a valid assessment of process performance—both criteria must be evaluated jointly.

*(3) Computation*—It is sometimes difficult to compute $C_p$ and $C_{pk}$ on the plant floor where all individuals are not familiar with mathematical formulas. This issue can be addressed in a variety of ways, but often involves training and/or some type of automation.

*(4) Nonnormality*—A variety of processes result in a nonnormal distribution for a characteristic. It is probably reasonable to expect that the capability indices are somewhat sensitive to departures from normality. Data transformations may be useful to attain approximate normality. Also, it is possible to estimate the percentage of parts outside the specification limits either directly or with a fitted distribution. This percentage can be related to an equivalent capability index for a process having a normal distribution.

*(5) Tool Wear*—In a tool wear situation, $\bar{R}$ from a consecutive piece sampling plan is of primary interest. Technically, $\bar{R}$ can be used to estimate $C_p$, but the capability is generally quite high. The performance of the process depends on the tool change frequency. There is not a convenient extension of $C_{pk}$ to this situation.

TABLE 3. Summary of Capability Indices

| Index | Estimation Equation | Usage |
|---|---|---|
| $C_p$ | $\dfrac{USL - LSL}{6s}$ | process potential for two-sided specification limits |
| $CPU$ | $\dfrac{USL - \bar{x}}{3s}$ | process performance relative to upper specification limit |
| $CPL$ | $\dfrac{\bar{x} - LSL}{3s}$ | process performance relative to lower specification limit |
| $k$ | $\dfrac{2\lvert m - \bar{x}\rvert}{USL - LSL}$ | deviation of process mean from midpoint ($m$) of specification limits |
| $C_{pk}$ | $\mathrm{Min}\{CPL, CPU\} = C_p(1-k)$ | process performance for two-sided specification limits |

## Applications

The intent of any index is to conveniently summarize information in a more usable form. The statistics $\bar{x}$ and $s$ summarize much of the information on an assumed normal process and can be related in a variety of ways to $USL$ and $LSL$. The five indices $C_p$, $CPU$, $CPL$, $k$ and $C_{pk}$ given in Table 3 provide a unitless "language" that can be understood from the plant floor to the manager's desk. Some of the application areas are described below, but any particular application must address the drawbacks discussed earlier.

*(1) Prevention of Nonconforming Product*—For various types of machine and process qualification trials, it is sometimes reasonable to establish a benchmark capability. A typical benchmark is $C_{pk} = 1.33$ which will make non-conforming units unlikely (see Hoffer [1985]) in many situations. Note that since the capability is generally estimated, a higher critical value (c) should be used, as was discussed earlier.

*(2) Continuous Improvement*—Within a department or plant it is often useful to monitor continuous improvement, which can be accomplished by observing the changing distribution of process capabilities. For example, if there were 10% of processes with capabilities between 1 and 1.33 in a month and some of these improved to between 1.33 and 1.67 the next month, improvement has occurred. This distributional shift can easily be monitored.

*(3) Communication*—Use of $C_p$ and $C_{pk}$ establishes a common language that is dimensionless and assesses both the potential and actual performance of production processes. Engineering and manufacturing can communicate and note processes with high capabilities. Design changes may enable cost savings. Manu-

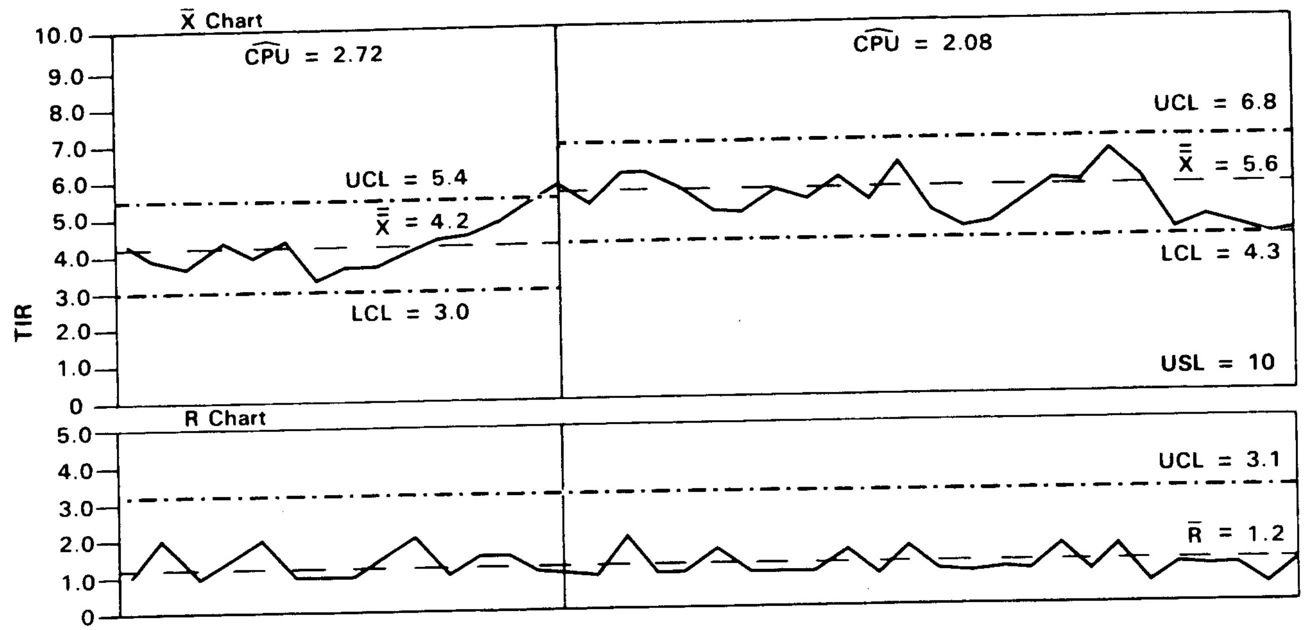



FIGURE 11. Oil Seal Groove Concentricity Showing Changing Capability *CPU* With Changing Location.

facturers can communicate with suppliers (e.g., Sullivan [1984]) in a straightforward manner.

*(4) Prioritization*—A simple computer printout of the processes with unacceptable $C_p$ or $C_{pk}$ values is an aid in establishing the priority for process improvements.

*(5) Location or Variability*—For each characteristic, it is meaningful to compare $C_p$ and $C_{pk}$. If $C_{pk}$ is too low, then $C_p$ must be examined to determine whether the variability is unacceptably high. If $C_p$ is close to $C_{pk}$, then process location is not a problem. The indices *CPU*, *CPL* and $k$ provide an assessment of how close the process mean is to a specification limit and how far the process mean is from the target mean.

*(6) Audits*—Various types of audits are used to assess the performance of quality systems. Comparison of in-process capabilities with capabilities determined from audits can help establish problem areas.

## Examples

The first example involves the concentricity of an engine oil seal groove which was monitored by the control chart in Figure 11. Concentricity is a measure of the cross-sectional coaxial relationship of two cylindrical features such as an oil seal groove and a base cylinder in the interior of the groove. Typically, concentricity is measured as a positive deviation using a dial indicator gauge which produces a total indicator reading (TIR). The control chart in Figure 11 appears in statistical control with a mean of $\bar{\bar{X}} = 4.2$ and average range of $\bar{R} = 1.2$ with $n = 3$. The resulting calculations give $CPU = 2.72$ which demonstrates good process capability. The chart shows that there was a shift in the process mean level around the seventh point to $\bar{\bar{X}} = 5.6$. This new mean results in a reduced capability with $CPU = 2.08$. However, the new process was more economical than the old process and still maintained adequate capability. This example shows that it is sometimes possible to target the process mean to achieve better product function and/or lower cost when a process has sufficiently high capability.

The second example is a machine qualification run where the parallelism and radial length of a machined hole in a transmission differential case were measured and charted. Parallelism relates the axis of the hole to the axis of another hole on the part. Radial length is the distance between the hole center and the centerline of the part. Both measurements were performed by a coordinate measuring machine. Table 4 shows the results from three stages of development. In the first stage a machine compensator was used which resulted in poor cycle time and process capability. In the second stage the compensator was disconnected and the machine cutting tool feed rate increased. The process capability for radial length decreased. In the third stage, the compensator remained disconnected but the feed rate was reduced. The parallelism still does not demonstrate adequate capability

TABLE 4. Machine Qualification Data for a Transmission Differential Case

| Parallelism ($\times 10^4$ inches)[a] Stage | $N$ | $\bar{x}$ | $s$ | $\widehat{CPU}$ | |
|---|---|---|---|---|---|
| 1 | 268 | 8.8 | 8.3 | 0.45 | |
| 2 | 79 | 8.3 | 7.8 | 0.50 | |
| 3 | 300 | 5.5 | 4.3 | 1.12 | |
| **Radial Length ($\times 10^3$ inches)[b] Stage** | $N$ | $\bar{x}$ | $s$ | $\hat{C}_p$ | $\hat{C}_{pk}$ |
| 1 | 201 | 4.7 | 8.7 | 0.77 | 0.59 |
| 2 | 96 | 10.4 | 21.1 | 0.32 | 0.15 |
| 3 | 316 | 5.0 | 5.4 | 1.23 | 0.93 |

[a] One-sided spec with $USL = 20$.
[b] Two-sided spec with $USL = 20$, $LSL = -20$.

(i.e., $\widehat{CPU} = 1.12$), but the improvement is apparent. The radial length demonstrates the value of using $C_p$ and $C_{pk}$ together. Since $\hat{C}_p = 1.23$ and $\hat{C}_{pk} = 0.93$ it is apparent that the process variability needs to be improved slightly and the process needs to be centered to attain a 1.33 benchmark.

The third example uses selected data collected from a bore and value diameter size capability assessment for a transmission valve body. Figure 12 shows the specification limits and process capabilities of 4 valves and the associated bores. This type of representation of mating parts (i.e., the valve must fit into the bore) allows assessment of how process capabilities may impact product function. Note that for the 1-2 shift the processes are centered around a midpoint target. However, for the 1-2 TV modulator case, the lack of valve process capability is compensated for to some degree by the high bore process capability. Design actions may now be necessary to optimize product function.

## Summary

Capability indices can be used to effectively summarize process information in a succinct manner. The indices $C_p$, $CPU$, $CPL$, $k$ and $C_{pk}$ form a group of complementary measures that comprise a convenient unitless system. These measures collectively determine whether a process has sufficiently low variability to meet the process specification or whether process location is a problem. They can be used with one-sided or two-sided specification limits and can be generalized to use with target dimensions.

Perhaps the greatest value of these indices is that their use encourages efforts to prevent production of nonconforming products and they provide a method to monitor continuous improvement on a broad scale.

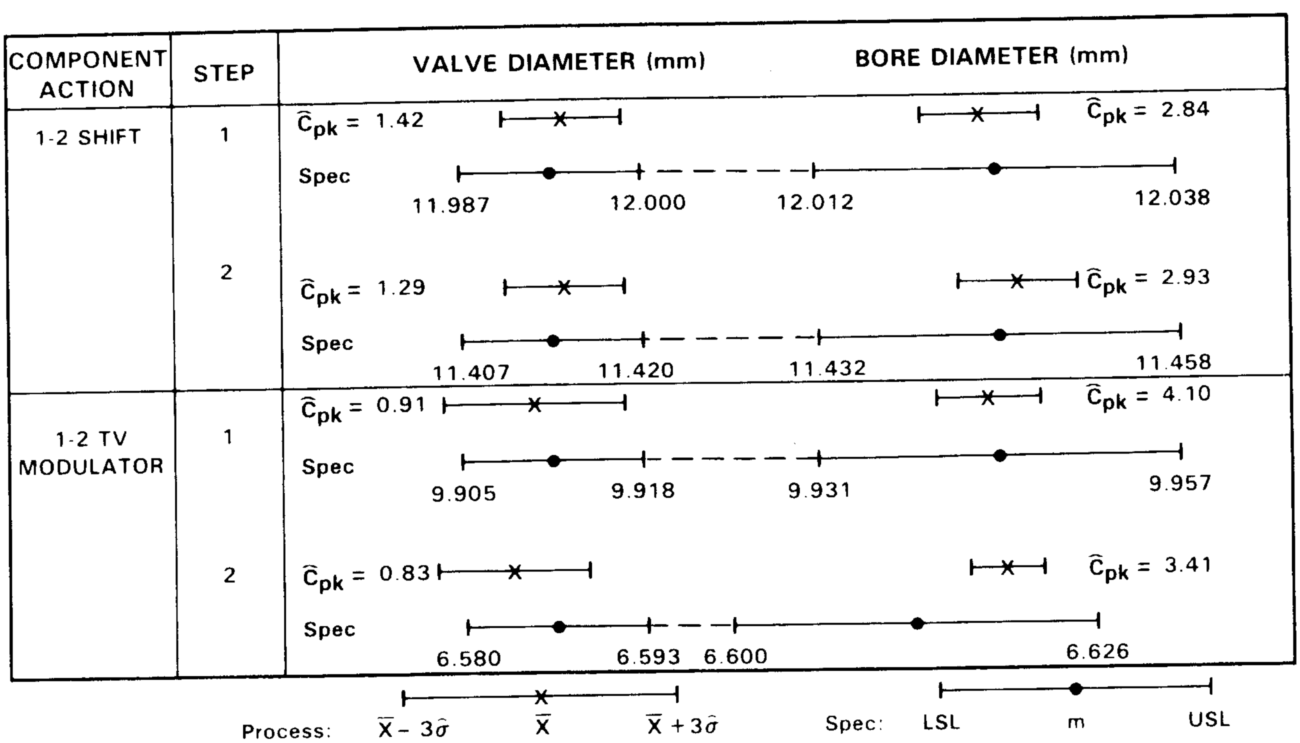



FIGURE 12. Transmission Valve and Bore Process Capabilities Related to Specification Limits.

Finally, these measures enable effective communication of process potential and performance information in a 'language' that can be easily understood.

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Key Words: *Capability, Target Value, Process Potential, Process Performance.*