KOM4510 – Fall 2025
Homework 1 – g–h Filters

Due: October 20, 2025 **Submission:** MATLAB Script or Jupyter Notebook
Collaboration: Individual work only

Assignment Description

You are to explore the behavior of the **g–h (alpha–beta)** filter in various tracking scenarios using simulated position measurements of a moving object.
Use a simple constant-velocity motion model and noisy position measurements to evaluate how the parameters g and h affect responsiveness, stability, and noise suppression.
All simulations, plots, and quantitative analyses must be included in a single, well-documented MATLAB Live Script or Jupyter Notebook.

AI Usage Policy

Students are permitted to use **AI-based tools** (e.g., ChatGPT, Copilot, or MATLAB AI Assistant) **only for conceptual clarification, syntax guidance, or debugging minor errors.** All simulation design, parameter selection, performance analysis, plots, and written explanations must represent the student's own understanding and effort.
Any output copied directly from an AI system without understanding, modification, or citation constitutes a violation of academic integrity.
If AI assistance is used responsibly:

- Clearly indicate its use with a short note (e.g., "ChatGPT was used to clarify the difference between Kalman and g–h filters").
- Ensure the submission still demonstrates independent reasoning and coding.

Purpose: AI tools are meant to support learning, not replace critical analysis. Responsible use is encouraged; overreliance is not.

Starter Algorithm – g–h Filter (Pseudo-code)

Inputs: Measurement sequence $z[1..N]$, initial position x_0, initial velocity v_0, parameters g, h, time step Δt

Outputs: Estimated positions $x_{\text{est}}[1..N]$, estimated velocities $v_{\text{est}}[1..N]$

Initialize:

$$x_{\text{est}}[1] = x_0, \quad v_{\text{est}}[1] = v_0$$

for $k = 2$ to N **do**

  Prediction step:

$$x_{\text{pred}} = x_{\text{est}}[k-1] + v_{\text{est}}[k-1] \cdot \Delta t$$

$$v_{\text{pred}} = v_{\text{est}}[k-1]$$

  Innovation:

$$r = z[k] - x_{\text{pred}}$$

  Update step:

$$x_{\text{est}}[k] = x_{\text{pred}} + g \cdot r$$

$$v_{\text{est}}[k] = v_{\text{pred}} + h \cdot \frac{r}{\Delta t}$$

end for

Homework Tasks (80 pts total)

Q1. Overlay and Quantitative Error (10 pts)
Simulate a constant-velocity motion and compare the true trajectory, noisy measurements, and filtered estimates on the same plot. Compute:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_k (x_{\text{est}}(k) - x_{\text{true}}(k))^2}, \qquad \text{MAI} = \frac{1}{N}\sum_k |z(k) - x_{\text{pred}}(k)|$$

Briefly explain how the parameters g and h influence smoothing versus responsiveness.

Q2. Bad Initial Guess Stress Test (10 pts)
Repeat the tracking experiment with a poor initial position estimate (e.g., $x_0 = 200$). Plot and discuss the convergence behavior and how g and h affect recovery from initialization errors.

Q3. Measurement Noise Sensitivity (10 pts)
Repeat your simulation for two noise levels: moderate ($\sigma_z = 10$) and severe ($\sigma_z = 100$). Tune g if necessary and report RMSE for each case. Discuss how filter stability and smoothness change with increasing measurement noise.

Q4. Acceleration in the Plant (10 pts)
Introduce acceleration into the true motion (e.g., $x(t) = 10 + 2t + 0.5at^2$, with $a = 0.5$). Evaluate the tracking error using the same constant-velocity g–h filter and then a version that includes the acceleration term in prediction. Discuss when an α–β–γ filter would be more suitable.

Q5. Effect of g under Heavy vs. Light Noise (20 pts)
Simulate a 75-step trajectory ($t = 0{:}74$) with high noise ($\sigma_z = 50$) and low noise ($\sigma_z = 10$). Run the g–h filter with $g \in \{0.01, 0.1, 0.8\}$ while keeping $h = 0.3$. Compare the RMSEs and discuss how the optimal g value depends on the signal-to-noise ratio (SNR).

Q6. Role of h under Good vs. Poor Velocity Priors (20 pts)
Generate a 100-step trajectory with true velocity $v = 0.1$. Analyze three cases:

- Correct velocity initialization ($v_0 = 0.1, h = 0.05$)
- Poor initialization ($v_0 = 3, h = 0.05$)
- Poor initialization with larger correction ($v_0 = 3, h = 0.5$)

Compare position estimates and RMSE values. Explain how h controls the velocity correction bandwidth and why too large or small h can degrade performance.

Deliverables and Grading

Item	Points
Q1	10
Q2	10
Q3	10
Q4	10
Q5	20
Q6	20
Total	**80**

Academic Integrity

All work must be performed individually. Code, plots, and explanations must be original and clearly documented. Cite any references or libraries used for theoretical background. Reports showing identical figures, parameters, or explanations will receive a grade of zero.