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$$A_{01} = \begin{pmatrix} \cos(q_1) & 0 & \sin(q_1) & 33 \cos(q_1) \\ \sin(q_1) & 0 & -\cos(q_1) & 33 \sin(q_1) \\ 0 & 1 & 0 & 147 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

$$A_{12} = \begin{pmatrix} \cos(q_2) & -\sin(q_2) & 0 & 155 \cos(q_2) \\ \sin(q_2) & \cos(q_2) & 0 & 155 \sin(q_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

$$A_{23} = \begin{pmatrix} \cos(q_3) & -\sin(q_3) & 0 & 135 \cos(q_3) \\ \sin(q_3) & \cos(q_3) & 0 & 135 \sin(q_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (3)$$

$$A_{34} = \begin{pmatrix} \cos(q_4) & 0 & \sin(q_4) & 0 \\ \sin(q_4) & 0 & -\cos(q_4) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

$$A_{45} = \begin{pmatrix} \cos(q_5) & -\sin(q_5) & 0 & 0 \\ \sin(q_5) & \cos(q_5) & 0 & 0 \\ 0 & 0 & 1 & \frac{435}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

$$A_{01} = \begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (6)$$

$$A_{02} = \begin{pmatrix} \cos(q_1) \cos(q_2) & -\cos(q_1) \sin(q_2) & \sin(q_1) & 33 \cos(q_1) \\ \cos(q_2) \sin(q_1) & -\sin(q_1) \sin(q_2) & -\cos(q_1) & 33 \sin(q_1) \\ \sin(q_2) & \cos(q_2) & 0 & 147 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (7)$$

$$A_{03} = \begin{pmatrix} \cos(q_2 + q_3) \cos(q_1) & -\sin(q_2 + q_3) \cos(q_1) & \sin(q_1) & \cos(q_1) (155 \cos(q_2) + 33) \\ \cos(q_2 + q_3) \sin(q_1) & -\sin(q_2 + q_3) \sin(q_1) & -\cos(q_1) & \sin(q_1) (155 \cos(q_2) + 33) \\ \sin(q_2 + q_3) & \cos(q_2 + q_3) & 0 & 155 \sin(q_2) + 147 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (8)$$

$$A_{04} = \begin{pmatrix} \cos(q_2 + q_3 + q_4) \cos(q_1) & -\sin(q_2 + q_3 + q_4) \cos(q_1) & \sin(q_1) & \cos(q_1) (135 \cos(q_2 + q_3) + 155 \cos(q_2) + 33) \\ \cos(q_2 + q_3 + q_4) \sin(q_1) & -\sin(q_2 + q_3 + q_4) \sin(q_1) & -\cos(q_1) & \sin(q_1) (135 \cos(q_2 + q_3) + 155 \cos(q_2) + 33) \\ \sin(q_2 + q_3 + q_4) & \cos(q_2 + q_3 + q_4) & 0 & 135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (9)$$

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$$A_{05} = \begin{pmatrix} \sin(q_1) \sin(q_5) + \cos(q_2 + q_3 + q_4) \cos(q_1) \cos(q_5) & \cos(q_5) \sin(q_1) - \cos(q_2 + q_3 + q_4) \cos(q_1) \sin(q_5) & \sin(q_2 + q_3 + q_4) \cos(q_1) & \cos(q_1) (135 \cos(q_2 + q_3) + 155 \cos(q_2) + 33) \\ \cos(q_2 + q_3 + q_4) \cos(q_5) \sin(q_1) - \cos(q_1) \sin(q_5) & -\cos(q_1) \cos(q_5) - \cos(q_2 + q_3 + q_4) \sin(q_1) \sin(q_5) & \sin(q_2 + q_3 + q_4) \sin(q_1) & \sin(q_1) (135 \cos(q_2 + q_3) + 155 \cos(q_2) + 33) \\ \sin(q_2 + q_3 + q_4) \cos(q_5) & -\sin(q_2 + q_3 + q_4) \sin(q_5) & -\cos(q_2 + q_3 + q_4) & 135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (10)$$

$$w_1 = (0 \quad 0 \quad 1 \quad 0) \quad (11)$$

$$w_2 = (\sin(q_2) \quad \cos(q_2) \quad 0 \quad 147) \quad (12)$$

$$w_3 = (\sin(q_2 + q_3) \quad \cos(q_2 + q_3) \quad 0 \quad 155 \sin(q_2) + 147) \quad (13)$$

$$w_4 = (\sin(q_2 + q_3 + q_4) \quad \cos(q_2 + q_3 + q_4) \quad 0 \quad 135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) \quad (14)$$

$$w_5 = (\sin(q_2 + q_3 + q_4) \cos(q_5) \quad -\sin(q_2 + q_3 + q_4) \sin(q_5) \quad -\cos(q_2 + q_3 + q_4) \quad 135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) \quad (15)$$

$$p_1 = g m_1 \text{rz}_1 \quad (16)$$

$$p_2 = g m_2 (\text{ry}_2 \cos(q_2) + \text{rx}_2 \sin(q_2) + 147) \quad (17)$$

$$p_3 = g m_3 (155 \sin(q_2) + \text{ry}_3 \cos(q_2 + q_3) + \text{rx}_3 \sin(q_2 + q_3) + 147) \quad (18)$$

$$p_4 = g m_4 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + \text{ry}_4 \cos(q_2 + q_3 + q_4) + \text{rx}_4 \sin(q_2 + q_3 + q_4) + 147) \quad (19)$$

$$p_5 = g m_5 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) - g m_5 \text{rz}_5 \cos(q_2 + q_3 + q_4) + g m_5 \text{rx}_5 \sin(q_2 + q_3 + q_4) \cos(q_5) - g m_5 \text{ry}_5 \sin(q_2 + q_3 + q_4) \sin(q_5) \quad (20)$$

$$\begin{aligned} P_1 &= 147 g m_2 + g m_1 \text{rz}_1 + g m_3 (155 \sin(q_2) + 147) + g m_4 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + g m_5 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + \\ &g m_3 \text{ry}_3 \cos(q_2 + q_3) + g m_3 \text{rx}_3 \sin(q_2 + q_3) + g m_2 \text{ry}_2 \cos(q_2) + g m_2 \text{rx}_2 \sin(q_2) + g m_4 \text{ry}_4 \cos(q_2 + q_3 + q_4) - g m_5 \text{rz}_5 \cos(q_2 + q_3 + q_4) + \\ &g m_4 \text{rx}_4 \sin(q_2 + q_3 + q_4) + g m_5 \text{rx}_5 \sin(q_2 + q_3 + q_4) \cos(q_5) - g m_5 \text{ry}_5 \sin(q_2 + q_3 + q_4) \sin(q_5) \\ P_2 &= 147 g m_2 + g m_3 (155 \sin(q_2) + 147) + g m_4 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + g m_5 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + \\ &g m_3 \text{ry}_3 \cos(q_2 + q_3) + g m_3 \text{rx}_3 \sin(q_2 + q_3) + g m_2 \text{ry}_2 \cos(q_2) + g m_2 \text{rx}_2 \sin(q_2) + g m_4 \text{ry}_4 \cos(q_2 + q_3 + q_4) - g m_5 \text{rz}_5 \cos(q_2 + q_3 + q_4) + \\ &g m_4 \text{rx}_4 \sin(q_2 + q_3 + q_4) + g m_5 \text{rx}_5 \sin(q_2 + q_3 + q_4) \cos(q_5) - g m_5 \text{ry}_5 \sin(q_2 + q_3 + q_4) \sin(q_5) \\ P_3 &= g m_3 (155 \sin(q_2) + 147) + g m_4 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + g m_5 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + g m_3 \text{ry}_3 \cos(q_2 + q_3) + \\ &g m_3 \text{rx}_3 \sin(q_2 + q_3) + g m_4 \text{ry}_4 \cos(q_2 + q_3 + q_4) - g m_5 \text{rz}_5 \cos(q_2 + q_3 + q_4) + g m_4 \text{rx}_4 \sin(q_2 + q_3 + q_4) + g m_5 \text{rx}_5 \sin(q_2 + q_3 + q_4) \cos(q_5) - \\ &g m_5 \text{ry}_5 \sin(q_2 + q_3 + q_4) \sin(q_5) \\ P_4 &= g m_4 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + g m_5 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + g m_4 \text{ry}_4 \cos(q_2 + q_3 + q_4) - g m_5 \text{rz}_5 \cos(q_2 + q_3 + q_4) + \\ &g m_4 \text{rx}_4 \sin(q_2 + q_3 + q_4) + g m_5 \text{rx}_5 \sin(q_2 + q_3 + q_4) \cos(q_5) - g m_5 \text{ry}_5 \sin(q_2 + q_3 + q_4) \sin(q_5) \\ P_5 &= g m_5 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) - g m_5 \text{rz}_5 \cos(q_2 + q_3 + q_4) + g m_5 \text{rx}_5 \sin(q_2 + q_3 + q_4) \cos(q_5) - g m_5 \text{ry}_5 \sin(q_2 + q_3 + q_4) \sin(q_5) \end{aligned}$$

$$W_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (21)$$

$$W_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{22}$$

$$W_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{23}$$

$$W_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{24}$$

$$W_5 = \begin{pmatrix} 0 \\ \cos(q_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{25}$$

$$W_6 = \begin{pmatrix} 0 \\ -\sin(q_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{26}$$

$$W_7 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{27}$$

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$$W_8 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{28}$$

$$W_9 = \begin{pmatrix} 0 \\ \cos(q_2 + q_3) \\ \cos(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \tag{29}$$

$$W_{10} = \begin{pmatrix} 0 \\ -\sin(q_2 + q_3) \\ -\sin(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \tag{30}$$

$$W_{11} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{31}$$

$$W_{12} = \begin{pmatrix} 0 \\ 155 \cos(q_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{32}$$

$$W_{13} = \begin{pmatrix} 0 \\ \cos(q_2 + q_3 + q_4) \\ \cos(q_2 + q_3 + q_4) \\ \cos(q_2 + q_3 + q_4) \\ 0 \end{pmatrix} \tag{33}$$

$$W_{14} = \begin{pmatrix} 0 \\ -\sin(q_2 + q_3 + q_4) \\ -\sin(q_2 + q_3 + q_4) \\ -\sin(q_2 + q_3 + q_4) \\ 0 \end{pmatrix} \quad (34)$$

$$W_{15} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (35)$$

$$W_{16} = \begin{pmatrix} 0 \\ 135 \cos(q_2 + q_3) + 155 \cos(q_2) \\ 135 \cos(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \quad (36)$$

$$W_{17} = \begin{pmatrix} 0 \\ \cos(q_2 + q_3 + q_4) \cos(q_5) \\ \cos(q_2 + q_3 + q_4) \cos(q_5) \\ \cos(q_2 + q_3 + q_4) \cos(q_5) \\ -\sin(q_2 + q_3 + q_4) \sin(q_5) \end{pmatrix} \quad (37)$$

$$W_{18} = \begin{pmatrix} 0 \\ -\cos(q_2 + q_3 + q_4) \sin(q_5) \\ -\cos(q_2 + q_3 + q_4) \sin(q_5) \\ -\cos(q_2 + q_3 + q_4) \sin(q_5) \\ -\sin(q_2 + q_3 + q_4) \cos(q_5) \end{pmatrix} \quad (38)$$

$$W_{19} = \begin{pmatrix} 0 \\ \sin(q_2 + q_3 + q_4) \\ \sin(q_2 + q_3 + q_4) \\ \sin(q_2 + q_3 + q_4) \\ 0 \end{pmatrix} \quad (39)$$

$$W_{20} = \begin{pmatrix} 0 \\ 135 \cos(q_2 + q_3) + 155 \cos(q_2) \\ 135 \cos(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \quad (40)$$

$$W_{1_R} = \begin{pmatrix} -\sin(q_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (41)$$

$$W_{2_R} = \begin{pmatrix} \cos(q_2 + q_3) \\ \cos(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \quad (42)$$

$$W_{3_R} = \begin{pmatrix} -\sin(q_2 + q_3) \\ -\sin(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \quad (43)$$

$$W_{4_R} = \begin{pmatrix} 155 \cos(q_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (44)$$

$$W_{5_R} = \begin{pmatrix} \cos(q_2 + q_3 + q_4) \\ \cos(q_2 + q_3 + q_4) \\ \cos(q_2 + q_3 + q_4) \\ 0 \end{pmatrix} \quad (45)$$

$$W_{6_R} = \begin{pmatrix} -\sin(q_2 + q_3 + q_4) \\ -\sin(q_2 + q_3 + q_4) \\ -\sin(q_2 + q_3 + q_4) \\ 0 \end{pmatrix} \quad (46)$$

$$W_{7_R} = \begin{pmatrix} 135 \cos(q_2 + q_3) + 155 \cos(q_2) \\ 135 \cos(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \quad (47)$$

$$W_{8_R} = \begin{pmatrix} \cos(q_2 + q_3 + q_4) \cos(q_5) \\ \cos(q_2 + q_3 + q_4) \cos(q_5) \\ \cos(q_2 + q_3 + q_4) \cos(q_5) \\ -\sin(q_2 + q_3 + q_4) \sin(q_5) \end{pmatrix} \quad (48)$$

$$\infty \quad W_{9_R} = \begin{pmatrix} -\cos(q_2 + q_3 + q_4) \sin(q_5) \\ -\cos(q_2 + q_3 + q_4) \sin(q_5) \\ -\cos(q_2 + q_3 + q_4) \sin(q_5) \\ -\sin(q_2 + q_3 + q_4) \cos(q_5) \end{pmatrix} \quad (49)$$

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$$W = \begin{bmatrix} -0.831 & -0.994 & 0.110 & 86.145 & 0.227 & 0.974 & -48.043 & -0.097 & -0.205 \\ 0.000 & -0.994 & 0.110 & 0.000 & 0.227 & 0.974 & -134.188 & -0.097 & -0.205 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.227 & 0.974 & 0.000 & -0.097 & -0.205 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.880 & -0.418 \\ -0.669 & -0.687 & 0.727 & -115.274 & 0.979 & -0.202 & -208.032 & -0.056 & -0.978 \\ 0.000 & -0.687 & 0.727 & 0.000 & 0.979 & -0.202 & -92.758 & -0.056 & -0.978 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.979 & -0.202 & 0.000 & -0.056 & -0.978 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & -0.201 & 0.011 \\ -0.991 & 0.423 & -0.906 & 20.408 & -0.846 & -0.533 & 77.569 & -0.628 & -0.568 \\ 0.000 & 0.423 & -0.906 & 0.000 & -0.846 & -0.533 & 57.161 & -0.628 & -0.568 \\ 0.000 & 0.000 & 0.000 & 0.000 & -0.846 & -0.533 & 0.000 & -0.628 & -0.568 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.357 & -0.395 \end{bmatrix} \quad (50)$$

$$W^T W = \begin{bmatrix} 2.121 & 0.866 & 0.321 & -14.783 & -0.005 & -0.147 & 102.118 & 0.740 & 1.387 \\ 0.866 & 3.279 & -1.983 & 2.218 & -2.514 & -2.110 & 444.854 & -0.261 & 1.271 \\ 0.321 & -1.983 & 2.721 & -92.809 & 3.006 & 0.886 & -360.555 & 1.035 & -0.437 \\ -14.783 & 2.218 & -92.809 & 21125.485 & -110.623 & 96.271 & 21424.890 & -14.771 & 83.476 \\ -0.005 & -2.514 & 3.006 & -110.623 & 5.181 & 1.423 & -449.989 & 1.363 & -1.572 \\ -0.147 & -2.110 & 0.886 & 96.271 & 1.423 & 3.819 & -188.596 & 0.752 & 0.900 \\ 102.118 & 444.854 & -360.555 & 21424.890 & -449.989 & -188.596 & 81480.247 & -50.055 & 255.016 \\ 0.740 & -0.261 & 1.035 & -14.771 & 1.363 & 0.752 & -50.055 & 2.162 & 0.781 \\ 1.387 & 1.271 & -0.437 & 83.476 & -1.572 & 0.900 & 255.016 & 0.781 & 4.292 \end{bmatrix} \quad (51)$$

$$R = \begin{bmatrix} -103.220 & -439.843 & 343.413 & -18170.737 & 429.365 & 200.375 & -77549.497 & 47.363 & -240.436 \\ 0.000 & 66.834 & -143.258 & 23729.152 & -173.636 & 66.284 & 32751.713 & -22.058 & 118.046 \\ 0.000 & 0.000 & 13.430 & -3467.557 & 16.413 & -17.639 & -3467.557 & 0.251 & -16.582 \\ 0.000 & 0.000 & 0.000 & 278.045 & 0.808 & 0.981 & 278.045 & 1.087 & 0.877 \\ 0.000 & 0.000 & 0.000 & 0.000 & -1.634 & -0.313 & 0.000 & -0.282 & 1.452 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & -1.938 & -0.000 & -0.761 & -2.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.059 & 0.148 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & -1.138 & 0.098 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.973 \end{bmatrix} \quad (52)$$