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## ОЦЕНИВАНИЕ ПАРАМЕТРОВ РОБОТА-МАНИПУЛЯТОРА KUKA YOUNOT<sup>1)</sup>

Предложен новый алгоритм оценивания параметров механических параметров динамической модели робота.

### 1. Введение

Здесь надо что-то написать, но я пока не придумал, что.

### 2. Тест

$$W_s(1, 1) = 0$$

$$W_s(1, 2) = 0$$

$$W_s(1, 3) = 0$$

$$W_s(1, 4) = 0$$

$$W_s(1, 5) = 0$$

$$W_s(2, 1) = 0$$

$$W_s(2, 2) = 0$$

$$W_s(2, 3) = 0$$

$$W_s(2, 4) = 0$$

$$W_s(2, 5) = 0$$

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<sup>1)</sup>

$$W_s(3, 1) = 0$$

$$W_s(3, 2) = 0$$

$$W_s(3, 3) = 0$$

$$W_s(3, 4) = 0$$

$$W_s(3, 5) = 0$$

$$W_s(4, 1) = 0$$

$$W_s(4, 2) = 0$$

$$W_s(4, 3) = 0$$

$$W_s(4, 4) = 0$$

$$W_s(4, 5) = 0$$

$$W_s(5, 1) = 0$$

$$W_s(5, 2) = 0$$

$$W_s(5, 3) = 0$$

$$W_s(5, 4) = 0$$

$$W_s(5, 5) = 0$$

$$W_s(6, 1) = 0$$

$$W_s(6, 2) = -\cos(\alpha 2) * \sin(\alpha 1) * \sin(q 2)$$

$$W_s(6, 3) = 0$$

$$W_s(6, 4) = 0$$

$$W_s(6, 5) = 0$$

$$W_s(7, 1) = 0$$

$$W_s(7, 2) = 1.0 * \sin(\alpha 1) * \sin(\alpha 2) * \sin(q 2)$$

$$W_s(7, 3) = 0$$

$$W_s(7, 4) = 0$$

$$W_s(7, 5) = 0$$

$$W_s(8, 1) = 0$$

$$W_s(8, 2) = a2 * \cos(q2) * \sin(\alpha1)$$

$$W_s(8, 3) = 0$$

$$W_s(8, 4) = 0$$

$$W_s(8, 5) = 0$$

$$W_s(9, 1) = 0$$

$$W_s(9, 2) = 0$$

$$W_s(9, 3) = \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \sin(\alpha1) * \sin(q2) * \sin(q3)$$

$$W_s(9, 4) = 0$$

$$W_s(9, 5) = 0$$

$$W_s(10, 1) = 0$$

$$W_s(10, 2) = 1.0 * \sin(\alpha1) * \sin(\alpha2) * \sin(\alpha3) * \sin(q2) - 1.0 * \cos(\alpha3) * \cos(q2) * \sin(\alpha1) * \sin(q3) - \cos(\alpha2) * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)$$

$$W_s(10, 3) = -\cos(\alpha3) * \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)$$

$$W_s(10, 4) = 0$$

$$W_s(10, 5) = 0$$

$$W_s(11, 1) = 0$$

$$W_s(11, 2) = 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(\alpha 2) * \sin(q 2) + \cos(q 2) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 3) + 1.0 * \cos(\alpha 2) * \cos(q 3) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 2)$$

$$W_s(11, 3) = 1.0 * \sin(\alpha 3) * \sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 2)$$

$$W_s(11, 4) = 0$$

$$W_s(11, 5) = 0$$

$$W_s(12, 1) = 0$$

$$W_s(12, 2) = a 2 * \cos(q 2) * \sin(\alpha 1) + a 3 * \cos(q 2) * \cos(q 3) * \sin(\alpha 1) + 1.0 * d 3 * \sin(\alpha 1) * \sin(\alpha 2) * \sin(q 2) - a 3 * \cos(\alpha 2) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)$$

$$W_s(12, 3) = a 3 * \cos(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) - a 3 * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)$$

$$W_s(12, 4) = 0$$

$$W_s(12, 5) = 0$$

$$W_s(13, 1) = 0$$

$$W_s(13, 2) = 0$$

$$\begin{aligned} W_s(13, 3) = & d 4 * (1.0 * \sin(\alpha 3) * \sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \\ & \cos(q 3) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 2)) - \sin(q 4) * (\cos(\alpha 3) * \sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \\ & \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + 1.0 * \cos(\alpha 3) * \cos(q 3) * \sin(\alpha 1) * \sin(q 2)) + \cos(q 4) * (\cos(q 3) * \\ & (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) - \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) - a 4 * \sin(q 4) * \\ & (\cos(\alpha 3) * \sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + 1.0 * \cos(\alpha 3) * \cos(q 3) * \\ & \sin(\alpha 1) * \sin(q 2)) + a 4 * \cos(q 4) * (\cos(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) - \\ & \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) + a 3 * \cos(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) - \\ & a 3 * \sin(\alpha 1) * \sin(q 2) * \sin(q 3) \end{aligned}$$

$$\begin{aligned}
W_s(13, 4) = & \cos(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \cos(q2) * \sin(\alpha1) * \sin(\alpha2)) + \\
& \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \sin(\alpha1) * \\
& \sin(q2) * \sin(q3)) - \sin(q4) * (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \\
& \cos(q3) * \sin(\alpha1) * \sin(q2)) - a4 * \sin(q4) * (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \\
& \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(q2)) + a4 * \cos(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \\
& \cos(q2) * \sin(\alpha1) * \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \\
& \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(q2) * \sin(q3))
\end{aligned}$$

$$W_s(13, 5) = 0$$

$$W_s(14, 1) = 0$$

$$\begin{aligned}
W_s(14, 2) = & \sin(\alpha4) * (1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(\alpha2) * \sin(q2) + \cos(q2) * \sin(\alpha1) * \\
& \sin(\alpha3) * \sin(q3) + 1.0 * \cos(\alpha2) * \cos(q3) * \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) + d4 * (1.0 * \cos(\alpha3) * \\
& \sin(\alpha1) * \sin(\alpha2) * \sin(q2) + \cos(q2) * \sin(\alpha1) * \sin(\alpha3) * \sin(q3) + 1.0 * \cos(\alpha2) * \cos(q3) * \\
& \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) - 1.0 * \cos(\alpha4) * \sin(q4) * (\cos(q2) * \cos(q3) * \sin(\alpha1) - \cos(\alpha2) * \\
& \sin(\alpha1) * \sin(q2) * \sin(q3)) - \cos(\alpha4) * \cos(q4) * (1.0 * \cos(\alpha3) * \cos(q2) * \sin(\alpha1) * \sin(q3) - \\
& 1.0 * \sin(\alpha1) * \sin(\alpha2) * \sin(\alpha3) * \sin(q2) + \cos(\alpha2) * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)) + \\
& a2 * \cos(q2) * \sin(\alpha1) + a4 * \cos(q4) * (\cos(q2) * \cos(q3) * \sin(\alpha1) - \cos(\alpha2) * \sin(\alpha1) * \sin(q2) * \\
& \sin(q3)) - a4 * \sin(q4) * (1.0 * \cos(\alpha3) * \cos(q2) * \sin(\alpha1) * \sin(q3) - 1.0 * \sin(\alpha1) * \sin(\alpha2) * \\
& \sin(\alpha3) * \sin(q2) + \cos(\alpha2) * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)) + a3 * \cos(q2) * \cos(q3) * \\
& \sin(\alpha1) + 1.0 * d3 * \sin(\alpha1) * \sin(\alpha2) * \sin(q2) - a3 * \cos(\alpha2) * \sin(\alpha1) * \sin(q2) * \sin(q3)
\end{aligned}$$

$$\begin{aligned}
W_s(14, 3) = & d4 * (1.0 * \sin(\alpha3) * \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \\
& \cos(q3) * \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) + \sin(\alpha4) * (1.0 * \sin(\alpha3) * \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \\
& \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) - a4 * \sin(q4) * (\cos(\alpha3) * \\
& \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + 1.0 * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) *
\end{aligned}$$

$$\begin{aligned} & \sin(q2)) + a4 * \cos(q4) * (\cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \sin(\alpha1) * \\ & \sin(q2) * \sin(q3)) - \cos(\alpha4) * \cos(q4) * (\cos(\alpha3) * \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \\ & \cos(q2) * \sin(\alpha1)) + 1.0 * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)) + a3 * \cos(q3) * (\cos(\alpha1) * \\ & \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - 1.0 * \cos(\alpha4) * \sin(q4) * (\cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \\ & \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \sin(\alpha1) * \sin(q2) * \sin(q3)) - a3 * \sin(\alpha1) * \sin(q2) * \sin(q3) \end{aligned}$$

$$\begin{aligned} W_s(14, 4) = & a4 * \cos(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \cos(q2) * \sin(\alpha1) * \sin(\alpha2)) + \\ & \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \sin(\alpha1) * \\ & \sin(q2) * \sin(q3)) - a4 * \sin(q4) * (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \\ & \cos(q3) * \sin(\alpha1) * \sin(q2)) - 1.0 * \cos(\alpha4) * \cos(q4) * (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \\ & \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(q2)) - \cos(\alpha4) * \sin(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \\ & \cos(\alpha2) - 1.0 * \cos(q2) * \sin(\alpha1) * \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \\ & \cos(q2) * \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(q2) * \sin(q3)) \end{aligned}$$

$$W_s(14, 5) = 0$$

$$W_s(15, 1) = 0$$

$$\begin{aligned} W_s(15, 2) = & \cos(\alpha4) * (1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(\alpha2) * \sin(q2) + \cos(q2) * \sin(\alpha1) * \\ & \sin(\alpha3) * \sin(q3) + 1.0 * \cos(\alpha2) * \cos(q3) * \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) + d4 * (1.0 * \cos(\alpha3) * \\ & \sin(\alpha1) * \sin(\alpha2) * \sin(q2) + \cos(q2) * \sin(\alpha1) * \sin(\alpha3) * \sin(q3) + 1.0 * \cos(\alpha2) * \cos(q3) * \\ & \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) + \sin(\alpha4) * \sin(q4) * (\cos(q2) * \cos(q3) * \sin(\alpha1) - \cos(\alpha2) * \sin(\alpha1) * \\ & \sin(q2) * \sin(q3)) + 1.0 * \cos(q4) * \sin(\alpha4) * (1.0 * \cos(\alpha3) * \cos(q2) * \sin(\alpha1) * \sin(q3) - 1.0 * \\ & \sin(\alpha1) * \sin(\alpha2) * \sin(\alpha3) * \sin(q2) + \cos(\alpha2) * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)) + a2 * \\ & \cos(q2) * \sin(\alpha1) + a4 * \cos(q4) * (\cos(q2) * \cos(q3) * \sin(\alpha1) - \cos(\alpha2) * \sin(\alpha1) * \sin(q2) * \\ & \sin(q3)) - a4 * \sin(q4) * (1.0 * \cos(\alpha3) * \cos(q2) * \sin(\alpha1) * \sin(q3) - 1.0 * \sin(\alpha1) * \sin(\alpha2) * \\ & \sin(\alpha3) * \sin(q2) + \cos(\alpha2) * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)) + a3 * \cos(q2) * \cos(q3) * \end{aligned}$$

$$\sin(\alpha 1) + 1.0 * d3 * \sin(\alpha 1) * \sin(\alpha 2) * \sin(q2) - a3 * \cos(\alpha 2) * \sin(\alpha 1) * \sin(q2) * \sin(q3)$$

$$\begin{aligned} W_s(15, 3) = & d4 * (1.0 * \sin(\alpha 3) * \sin(q3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q2) * \sin(\alpha 1)) + \\ & \cos(q3) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q2)) + \cos(\alpha 4) * (1.0 * \sin(\alpha 3) * \sin(q3) * (\cos(\alpha 1) * \sin(\alpha 2) + \\ & \cos(\alpha 2) * \cos(q2) * \sin(\alpha 1)) + \cos(q3) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q2)) - a4 * \sin(q4) * (\cos(\alpha 3) * \\ & \sin(q3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q2) * \sin(\alpha 1)) + 1.0 * \cos(\alpha 3) * \cos(q3) * \sin(\alpha 1) * \\ & \sin(q2)) + a4 * \cos(q4) * (\cos(q3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q2) * \sin(\alpha 1)) - \sin(\alpha 1) * \\ & \sin(q2) * \sin(q3)) + 1.0 * \cos(q4) * \sin(\alpha 4) * (\cos(\alpha 3) * \sin(q3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \\ & \cos(q2) * \sin(\alpha 1)) + 1.0 * \cos(\alpha 3) * \cos(q3) * \sin(\alpha 1) * \sin(q2)) + a3 * \cos(q3) * (\cos(\alpha 1) * \\ & \sin(\alpha 2) + \cos(\alpha 2) * \cos(q2) * \sin(\alpha 1)) + \sin(\alpha 4) * \sin(q4) * (\cos(q3) * (\cos(\alpha 1) * \sin(\alpha 2) + \\ & \cos(\alpha 2) * \cos(q2) * \sin(\alpha 1)) - \sin(\alpha 1) * \sin(q2) * \sin(q3)) - a3 * \sin(\alpha 1) * \sin(q2) * \sin(q3) \end{aligned}$$

$$\begin{aligned} W_s(15, 4) = & 1.0 * \sin(\alpha 4) * \sin(q4) * (\sin(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - 1.0 * \cos(q2) * \sin(\alpha 1) * \\ & \sin(\alpha 2)) + \cos(\alpha 3) * \cos(q3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q2) * \sin(\alpha 1)) - 1.0 * \cos(\alpha 3) * \\ & \sin(\alpha 1) * \sin(q2) * \sin(q3)) - a4 * \sin(q4) * (\sin(q3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q2) * \\ & \sin(\alpha 1)) + \cos(q3) * \sin(\alpha 1) * \sin(q2)) + a4 * \cos(q4) * (\sin(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - 1.0 * \\ & \cos(q2) * \sin(\alpha 1) * \sin(\alpha 2)) + \cos(\alpha 3) * \cos(q3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q2) * \\ & \sin(\alpha 1)) - 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(q2) * \sin(q3)) + \cos(q4) * \sin(\alpha 4) * (\sin(q3) * (\cos(\alpha 1) * \\ & \sin(\alpha 2) + \cos(\alpha 2) * \cos(q2) * \sin(\alpha 1)) + \cos(q3) * \sin(\alpha 1) * \sin(q2)) \end{aligned}$$

$$W_s(15, 5) = 0$$

$$W_s(16, 1) = 0$$

$$W_s(16, 2) = 0$$

$$W_s(16, 3) = 0$$

$$W_s(16, 4) = 0$$

$$W_s(16, 5) = 0$$

$$W_s(17, 1) = 0$$

$$\begin{aligned} W_s(17, 2) = & \cos(q5) * (\cos(q4) * (\cos(q2) * \cos(q3) * \sin(\alpha1) - \cos(\alpha2) * \sin(\alpha1) * \sin(q2) * \\ & \sin(q3)) - \sin(q4) * (1.0 * \cos(\alpha3) * \cos(q2) * \sin(\alpha1) * \sin(q3) - 1.0 * \sin(\alpha1) * \sin(\alpha2) * \sin(\alpha3) * \\ & \sin(q2) + \cos(\alpha2) * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2))) - \sin(q5) * (1.0 * \cos(\alpha4) * \sin(q4) * \\ & (\cos(q2) * \cos(q3) * \sin(\alpha1) - \cos(\alpha2) * \sin(\alpha1) * \sin(q2) * \sin(q3)) - \sin(\alpha4) * (1.0 * \cos(\alpha3) * \\ & \sin(\alpha1) * \sin(\alpha2) * \sin(q2) + \cos(q2) * \sin(\alpha1) * \sin(\alpha3) * \sin(q3) + 1.0 * \cos(\alpha2) * \cos(q3) * \\ & \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) + \cos(\alpha4) * \cos(q4) * (1.0 * \cos(\alpha3) * \cos(q2) * \sin(\alpha1) * \sin(q3) - \\ & 1.0 * \sin(\alpha1) * \sin(\alpha2) * \sin(\alpha3) * \sin(q2) + \cos(\alpha2) * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2))) \end{aligned}$$

$$\begin{aligned} W_s(17, 3) = & -\cos(q5) * (\sin(q4) * (\cos(\alpha3) * \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \\ & \cos(q2) * \sin(\alpha1)) + 1.0 * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)) - \cos(q4) * (\cos(q3) * (\cos(\alpha1) * \\ & \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \sin(\alpha1) * \sin(q2) * \sin(q3))) - \sin(q5) * (\cos(\alpha4) * \\ & \cos(q4) * (\cos(\alpha3) * \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + 1.0 * \cos(\alpha3) * \\ & \cos(q3) * \sin(\alpha1) * \sin(q2)) - \sin(\alpha4) * (1.0 * \sin(\alpha3) * \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \\ & \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) + 1.0 * \cos(\alpha4) * \sin(q4) * (\cos(q3) * \\ & (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \sin(\alpha1) * \sin(q2) * \sin(q3))) \end{aligned}$$

$$\begin{aligned} W_s(17, 4) = & \cos(q5) * (\cos(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \cos(q2) * \sin(\alpha1) * \\ & \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \\ & \sin(\alpha1) * \sin(q2) * \sin(q3)) - \sin(q4) * (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \\ & \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(q2))) - \sin(q5) * (\cos(\alpha4) * \sin(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \\ & \cos(\alpha2) - 1.0 * \cos(q2) * \sin(\alpha1) * \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \\ & \cos(q2) * \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(q2) * \sin(q3)) + 1.0 * \cos(\alpha4) * \cos(q4) * \\ & (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(q2))) \end{aligned}$$

$$W_s(17, 5) = \cos(q5) * (\sin(\alpha4) * (\cos(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \cos(q2) * \sin(\alpha1) * \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(q2) * \sin(q3)) + 1.0 * \cos(\alpha4) * \cos(q4) * (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(q2)))$$



$$\begin{aligned}
& \sin(\alpha 2)) - 1.0 * \cos(q 3) * \sin(\alpha 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \sin(\alpha 1) * \\
& \sin(\alpha 3) * \sin(q 2) * \sin(q 3)) + \cos(\alpha 4) * \cos(q 4) * (\sin(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - 1.0 * \cos(q 2) * \\
& \sin(\alpha 1) * \sin(\alpha 2)) + \cos(\alpha 3) * \cos(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) - \\
& 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) - 1.0 * \cos(\alpha 4) * \sin(q 4) * (\sin(q 3) * (\cos(\alpha 1) * \\
& \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(q 2))) - \sin(q 5) * (\sin(q 4) * \\
& (\sin(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - 1.0 * \cos(q 2) * \sin(\alpha 1) * \sin(\alpha 2)) + \cos(\alpha 3) * \cos(q 3) * (\cos(\alpha 1) * \\
& \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) - 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) + \cos(q 4) * \\
& (\sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(q 2)))
\end{aligned}$$

$$W_s(18, 1) = 0$$

$$\begin{aligned}
W_s(18, 2) = & \sin(\alpha 5) * (\cos(\alpha 4) * (1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(\alpha 2) * \sin(q 2) + \cos(q 2) * \\
& \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 3) + 1.0 * \cos(\alpha 2) * \cos(q 3) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 2)) + \sin(\alpha 4) * \\
& \sin(q 4) * (\cos(q 2) * \cos(q 3) * \sin(\alpha 1) - \cos(\alpha 2) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) + 1.0 * \cos(q 4) * \\
& \sin(\alpha 4) * (1.0 * \cos(\alpha 3) * \cos(q 2) * \sin(\alpha 1) * \sin(q 3) - 1.0 * \sin(\alpha 1) * \sin(\alpha 2) * \sin(\alpha 3) * \\
& \sin(q 2) + \cos(\alpha 2) * \cos(\alpha 3) * \cos(q 3) * \sin(\alpha 1) * \sin(q 2))) - 1.0 * \cos(\alpha 5) * \sin(q 5) * (\cos(q 4) * \\
& (\cos(q 2) * \cos(q 3) * \sin(\alpha 1) - \cos(\alpha 2) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) - \sin(q 4) * (1.0 * \cos(\alpha 3) * \\
& \cos(q 2) * \sin(\alpha 1) * \sin(q 3) - 1.0 * \sin(\alpha 1) * \sin(\alpha 2) * \sin(\alpha 3) * \sin(q 2) + \cos(\alpha 2) * \cos(\alpha 3) * \\
& \cos(q 3) * \sin(\alpha 1) * \sin(q 2))) - \cos(\alpha 5) * \cos(q 5) * (1.0 * \cos(\alpha 4) * \sin(q 4) * (\cos(q 2) * \cos(q 3) * \\
& \sin(\alpha 1) - \cos(\alpha 2) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) - \sin(\alpha 4) * (1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(\alpha 2) * \\
& \sin(q 2) + \cos(q 2) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 3) + 1.0 * \cos(\alpha 2) * \cos(q 3) * \sin(\alpha 1) * \sin(\alpha 3) * \\
& \sin(q 2)) + \cos(\alpha 4) * \cos(q 4) * (1.0 * \cos(\alpha 3) * \cos(q 2) * \sin(\alpha 1) * \sin(q 3) - 1.0 * \sin(\alpha 1) * \\
& \sin(\alpha 2) * \sin(\alpha 3) * \sin(q 2) + \cos(\alpha 2) * \cos(\alpha 3) * \cos(q 3) * \sin(\alpha 1) * \sin(q 2)))
\end{aligned}$$

$$\begin{aligned}
W_s(18, 3) = & \sin(\alpha 5) * (\cos(\alpha 4) * (1.0 * \sin(\alpha 3) * \sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \\
& \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 2)) + 1.0 * \cos(q 4) * \sin(\alpha 4) * (\cos(\alpha 3) *
\end{aligned}$$

$$\begin{aligned}
& \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + 1.0 * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \\
& \sin(q2)) + \sin(\alpha4) * \sin(q4) * (\cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \\
& \sin(\alpha1) * \sin(q2) * \sin(q3))) + 1.0 * \cos(\alpha5) * \sin(q5) * (\sin(q4) * (\cos(\alpha3) * \sin(q3) * (\cos(\alpha1) * \\
& \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + 1.0 * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)) - \cos(q4) * \\
& (\cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \sin(\alpha1) * \sin(q2) * \sin(q3))) - \\
& \cos(\alpha5) * \cos(q5) * (\cos(\alpha4) * \cos(q4) * (\cos(\alpha3) * \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \\
& \sin(\alpha1)) + 1.0 * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)) - \sin(\alpha4) * (1.0 * \sin(\alpha3) * \sin(q3) * (\cos(\alpha1) * \\
& \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) + 1.0 * \cos(\alpha4) * \\
& \sin(q4) * (\cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \sin(\alpha1) * \sin(q2) * \sin(q3)))
\end{aligned}$$

$$\begin{aligned}
W_s(18, 4) = & \sin(\alpha5) * (1.0 * \sin(\alpha4) * \sin(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \cos(q2) * \\
& \sin(\alpha1) * \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \\
& 1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(q2) * \sin(q3)) + \cos(q4) * \sin(\alpha4) * (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \\
& \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(q2))) - \cos(\alpha5) * \cos(q5) * (\cos(\alpha4) * \\
& \sin(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \cos(q2) * \sin(\alpha1) * \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * \\
& (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(q2) * \sin(q3)) + \\
& 1.0 * \cos(\alpha4) * \cos(q4) * (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \\
& \sin(\alpha1) * \sin(q2))) - 1.0 * \cos(\alpha5) * \sin(q5) * (\cos(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \\
& \cos(q2) * \sin(\alpha1) * \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \\
& \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(q2) * \sin(q3)) - \sin(q4) * (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \\
& \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(q2)))
\end{aligned}$$

$$\begin{aligned}
W_s(18, 5) = & -\cos(\alpha5) * \sin(q5) * (\sin(\alpha4) * (\cos(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \cos(q2) * \\
& \sin(\alpha1) * \sin(\alpha2)) - 1.0 * \cos(q3) * \sin(\alpha3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \\
& \sin(\alpha1) * \sin(\alpha3) * \sin(q2) * \sin(q3)) + \cos(\alpha4) * \cos(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 *
\end{aligned}$$

$$\begin{aligned}
& \cos(q2) * \sin(\alpha1) * \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \\
& \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(q2) * \sin(q3)) - 1.0 * \cos(\alpha4) * \sin(q4) * (\sin(q3) * (\cos(\alpha1) * \\
& \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(q2))) - 1.0 * \cos(\alpha5) * \cos(q5) * \\
& (\sin(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \cos(q2) * \sin(\alpha1) * \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * \\
& (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(q2) * \sin(q3)) + \\
& \cos(q4) * (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(q2)))
\end{aligned}$$

$$W_s(19, 1) = 0$$

$$\begin{aligned}
W_s(19, 2) = & \cos(\alpha5) * (\cos(\alpha4) * (1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(\alpha2) * \sin(q2) + \cos(q2) * \\
& \sin(\alpha1) * \sin(\alpha3) * \sin(q3) + 1.0 * \cos(\alpha2) * \cos(q3) * \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) + \sin(\alpha4) * \\
& \sin(q4) * (\cos(q2) * \cos(q3) * \sin(\alpha1) - \cos(\alpha2) * \sin(\alpha1) * \sin(q2) * \sin(q3)) + 1.0 * \cos(q4) * \\
& \sin(\alpha4) * (1.0 * \cos(\alpha3) * \cos(q2) * \sin(\alpha1) * \sin(q3) - 1.0 * \sin(\alpha1) * \sin(\alpha2) * \sin(\alpha3) * \\
& \sin(q2) + \cos(\alpha2) * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2))) + 1.0 * \cos(q5) * \sin(\alpha5) * (1.0 * \\
& \cos(\alpha4) * \sin(q4) * (\cos(q2) * \cos(q3) * \sin(\alpha1) - \cos(\alpha2) * \sin(\alpha1) * \sin(q2) * \sin(q3)) - \\
& \sin(\alpha4) * (1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(\alpha2) * \sin(q2) + \cos(q2) * \sin(\alpha1) * \sin(\alpha3) * \sin(q3) + \\
& 1.0 * \cos(\alpha2) * \cos(q3) * \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) + \cos(\alpha4) * \cos(q4) * (1.0 * \cos(\alpha3) * \\
& \cos(q2) * \sin(\alpha1) * \sin(q3) - 1.0 * \sin(\alpha1) * \sin(\alpha2) * \sin(\alpha3) * \sin(q2) + \cos(\alpha2) * \cos(\alpha3) * \\
& \cos(q3) * \sin(\alpha1) * \sin(q2))) + \sin(\alpha5) * \sin(q5) * (\cos(q4) * (\cos(q2) * \cos(q3) * \sin(\alpha1) - \\
& \cos(\alpha2) * \sin(\alpha1) * \sin(q2) * \sin(q3)) - \sin(q4) * (1.0 * \cos(\alpha3) * \cos(q2) * \sin(\alpha1) * \sin(q3) - \\
& 1.0 * \sin(\alpha1) * \sin(\alpha2) * \sin(\alpha3) * \sin(q2) + \cos(\alpha2) * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)))
\end{aligned}$$

$$\begin{aligned}
W_s(19, 3) = & \cos(\alpha5) * (\cos(\alpha4) * (1.0 * \sin(\alpha3) * \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \\
& \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) + 1.0 * \cos(q4) * \sin(\alpha4) * (\cos(\alpha3) * \\
& \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + 1.0 * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \\
& \sin(q2)) + \sin(\alpha4) * \sin(q4) * (\cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) -
\end{aligned}$$

$$\begin{aligned} & \sin(\alpha 1) * \sin(q 2) * \sin(q 3))) - \sin(\alpha 5) * \sin(q 5) * (\sin(q 4) * (\cos(\alpha 3) * \sin(q 3) * (\cos(\alpha 1) * \\ & \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + 1.0 * \cos(\alpha 3) * \cos(q 3) * \sin(\alpha 1) * \sin(q 2)) - \cos(q 4) * \\ & (\cos(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) - \sin(\alpha 1) * \sin(q 2) * \sin(q 3))) + 1.0 * \\ & \cos(q 5) * \sin(\alpha 5) * (\cos(\alpha 4) * \cos(q 4) * (\cos(\alpha 3) * \sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \\ & \sin(\alpha 1)) + 1.0 * \cos(\alpha 3) * \cos(q 3) * \sin(\alpha 1) * \sin(q 2)) - \sin(\alpha 4) * (1.0 * \sin(\alpha 3) * \sin(q 3) * (\cos(\alpha 1) * \\ & \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 2)) + 1.0 * \cos(\alpha 4) * \\ & \sin(q 4) * (\cos(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) - \sin(\alpha 1) * \sin(q 2) * \sin(q 3))) \end{aligned}$$

$$\begin{aligned} W_s(19, 4) = & \cos(\alpha 5) * (1.0 * \sin(\alpha 4) * \sin(q 4) * (\sin(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - 1.0 * \\ & \cos(q 2) * \sin(\alpha 1) * \sin(\alpha 2)) + \cos(\alpha 3) * \cos(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \\ & \sin(\alpha 1)) - 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) + \cos(q 4) * \sin(\alpha 4) * (\sin(q 3) * (\cos(\alpha 1) * \\ & \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(q 2))) + 1.0 * \cos(q 5) * \sin(\alpha 5) * \\ & (\cos(\alpha 4) * \sin(q 4) * (\sin(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - 1.0 * \cos(q 2) * \sin(\alpha 1) * \sin(\alpha 2)) + \cos(\alpha 3) * \\ & \cos(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) - 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(q 2) * \\ & \sin(q 3)) + 1.0 * \cos(\alpha 4) * \cos(q 4) * (\sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \\ & \cos(q 3) * \sin(\alpha 1) * \sin(q 2))) + \sin(\alpha 5) * \sin(q 5) * (\cos(q 4) * (\sin(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - \\ & 1.0 * \cos(q 2) * \sin(\alpha 1) * \sin(\alpha 2)) + \cos(\alpha 3) * \cos(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \\ & \sin(\alpha 1)) - 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) - \sin(q 4) * (\sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \\ & \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(q 2))) \end{aligned}$$

$$\begin{aligned} W_s(19, 5) = & 1.0 * \sin(\alpha 5) * \sin(q 5) * (\sin(\alpha 4) * (\cos(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - 1.0 * \cos(q 2) * \\ & \sin(\alpha 1) * \sin(\alpha 2)) - 1.0 * \cos(q 3) * \sin(\alpha 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \\ & \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 2) * \sin(q 3)) + \cos(\alpha 4) * \cos(q 4) * (\sin(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - \\ & 1.0 * \cos(q 2) * \sin(\alpha 1) * \sin(\alpha 2)) + \cos(\alpha 3) * \cos(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \\ & \sin(\alpha 1)) - 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) - 1.0 * \cos(\alpha 4) * \sin(q 4) * (\sin(q 3) * (\cos(\alpha 1) * \end{aligned}$$

$$\begin{aligned} & \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(q 2))) + \cos(q 5) * \sin(\alpha 5) * \\ & (\sin(q 4) * (\sin(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - 1.0 * \cos(q 2) * \sin(\alpha 1) * \sin(\alpha 2)) + \cos(\alpha 3) * \cos(q 3) * \\ & (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) - 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) + \\ & \cos(q 4) * (\sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(q 2))) \end{aligned}$$

$$W_s(20, 1) = 0$$

$$\begin{aligned} W_s(20, 2) = & d4 * (1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(\alpha 2) * \sin(q 2) + \cos(q 2) * \sin(\alpha 1) * \sin(\alpha 3) * \\ & \sin(q 3) + 1.0 * \cos(\alpha 2) * \cos(q 3) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 2)) + d5 * (\cos(\alpha 4) * (1.0 * \cos(\alpha 3) * \\ & \sin(\alpha 1) * \sin(\alpha 2) * \sin(q 2) + \cos(q 2) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 3) + 1.0 * \cos(\alpha 2) * \cos(q 3) * \\ & \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 2)) + \sin(\alpha 4) * \sin(q 4) * (\cos(q 2) * \cos(q 3) * \sin(\alpha 1) - \cos(\alpha 2) * \sin(\alpha 1) * \\ & \sin(q 2) * \sin(q 3)) + 1.0 * \cos(q 4) * \sin(\alpha 4) * (1.0 * \cos(\alpha 3) * \cos(q 2) * \sin(\alpha 1) * \sin(q 3) - 1.0 * \\ & \sin(\alpha 1) * \sin(\alpha 2) * \sin(\alpha 3) * \sin(q 2) + \cos(\alpha 2) * \cos(\alpha 3) * \cos(q 3) * \sin(\alpha 1) * \sin(q 2))) + a2 * \\ & \cos(q 2) * \sin(\alpha 1) + a4 * \cos(q 4) * (\cos(q 2) * \cos(q 3) * \sin(\alpha 1) - \cos(\alpha 2) * \sin(\alpha 1) * \sin(q 2) * \\ & \sin(q 3)) - a5 * \sin(q 5) * (1.0 * \cos(\alpha 4) * \sin(q 4) * (\cos(q 2) * \cos(q 3) * \sin(\alpha 1) - \cos(\alpha 2) * \\ & \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) - \sin(\alpha 4) * (1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(\alpha 2) * \sin(q 2) + \cos(q 2) * \\ & \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 3) + 1.0 * \cos(\alpha 2) * \cos(q 3) * \sin(\alpha 1) * \sin(\alpha 3) * \sin(q 2)) + \cos(\alpha 4) * \\ & \cos(q 4) * (1.0 * \cos(\alpha 3) * \cos(q 2) * \sin(\alpha 1) * \sin(q 3) - 1.0 * \sin(\alpha 1) * \sin(\alpha 2) * \sin(\alpha 3) * \sin(q 2) + \\ & \cos(\alpha 2) * \cos(\alpha 3) * \cos(q 3) * \sin(\alpha 1) * \sin(q 2))) + a5 * \cos(q 5) * (\cos(q 4) * (\cos(q 2) * \cos(q 3) * \\ & \sin(\alpha 1) - \cos(\alpha 2) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) - \sin(q 4) * (1.0 * \cos(\alpha 3) * \cos(q 2) * \sin(\alpha 1) * \\ & \sin(q 3) - 1.0 * \sin(\alpha 1) * \sin(\alpha 2) * \sin(\alpha 3) * \sin(q 2) + \cos(\alpha 2) * \cos(\alpha 3) * \cos(q 3) * \sin(\alpha 1) * \\ & \sin(q 2))) - a4 * \sin(q 4) * (1.0 * \cos(\alpha 3) * \cos(q 2) * \sin(\alpha 1) * \sin(q 3) - 1.0 * \sin(\alpha 1) * \sin(\alpha 2) * \\ & \sin(\alpha 3) * \sin(q 2) + \cos(\alpha 2) * \cos(\alpha 3) * \cos(q 3) * \sin(\alpha 1) * \sin(q 2)) + a3 * \cos(q 2) * \cos(q 3) * \\ & \sin(\alpha 1) + 1.0 * d3 * \sin(\alpha 1) * \sin(\alpha 2) * \sin(q 2) - a3 * \cos(\alpha 2) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3) \end{aligned}$$

$$W_s(20, 3) = d4 * (1.0 * \sin(\alpha 3) * \sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) +$$

$$\begin{aligned}
& \cos(q3) * \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) + d5 * (\cos(\alpha4) * (1.0 * \sin(\alpha3) * \sin(q3) * (\cos(\alpha1) * \\
& \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(\alpha3) * \sin(q2)) + 1.0 * \cos(q4) * \\
& \sin(\alpha4) * (\cos(\alpha3) * \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + 1.0 * \cos(\alpha3) * \\
& \cos(q3) * \sin(\alpha1) * \sin(q2)) + \sin(\alpha4) * \sin(q4) * (\cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \\
& \cos(q2) * \sin(\alpha1)) - \sin(\alpha1) * \sin(q2) * \sin(q3))) - a4 * \sin(q4) * (\cos(\alpha3) * \sin(q3) * (\cos(\alpha1) * \\
& \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + 1.0 * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)) + a4 * \\
& \cos(q4) * (\cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \sin(\alpha1) * \sin(q2) * \\
& \sin(q3)) - a5 * \sin(q5) * (\cos(\alpha4) * \cos(q4) * (\cos(\alpha3) * \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \\
& \cos(q2) * \sin(\alpha1)) + 1.0 * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)) - \sin(\alpha4) * (1.0 * \sin(\alpha3) * \\
& \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(\alpha3) * \\
& \sin(q2)) + 1.0 * \cos(\alpha4) * \sin(q4) * (\cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \\
& \sin(\alpha1) * \sin(q2) * \sin(q3))) - a5 * \cos(q5) * (\sin(q4) * (\cos(\alpha3) * \sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \\
& \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + 1.0 * \cos(\alpha3) * \cos(q3) * \sin(\alpha1) * \sin(q2)) - \cos(q4) * (\cos(q3) * \\
& (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \sin(\alpha1) * \sin(q2) * \sin(q3))) + a3 * \cos(q3) * \\
& (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - a3 * \sin(\alpha1) * \sin(q2) * \sin(q3)
\end{aligned}$$

$$\begin{aligned}
W_s(20, 4) = & d5 * (1.0 * \sin(\alpha4) * \sin(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \cos(q2) * \\
& \sin(\alpha1) * \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - \\
& 1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(q2) * \sin(q3)) + \cos(q4) * \sin(\alpha4) * (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \\
& \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \sin(\alpha1) * \sin(q2))) - a5 * \sin(q5) * (\cos(\alpha4) * \sin(q4) * \\
& (\sin(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \cos(q2) * \sin(\alpha1) * \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \\
& \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) - 1.0 * \cos(\alpha3) * \sin(\alpha1) * \sin(q2) * \sin(q3)) + 1.0 * \\
& \cos(\alpha4) * \cos(q4) * (\sin(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) + \cos(q3) * \\
& \sin(\alpha1) * \sin(q2))) + a5 * \cos(q5) * (\cos(q4) * (\sin(\alpha3) * (\cos(\alpha1) * \cos(\alpha2) - 1.0 * \cos(q2) * \\
& \sin(\alpha1) * \sin(\alpha2)) + \cos(\alpha3) * \cos(q3) * (\cos(\alpha1) * \sin(\alpha2) + \cos(\alpha2) * \cos(q2) * \sin(\alpha1)) -
\end{aligned}$$

$$\begin{aligned}
& 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) - \sin(q 4) * (\sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \\
& \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(q 2))) - a 4 * \sin(q 4) * (\sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \\
& \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(q 2)) + a 4 * \cos(q 4) * (\sin(\alpha 3) * (\cos(\alpha 1) * \\
& \cos(\alpha 2) - 1.0 * \cos(q 2) * \sin(\alpha 1) * \sin(\alpha 2)) + \cos(\alpha 3) * \cos(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \\
& \cos(q 2) * \sin(\alpha 1)) - 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3))
\end{aligned}$$

$$\begin{aligned}
W_s(20, 5) = & a 5 * \cos(q 5) * (\sin(\alpha 4) * (\cos(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - 1.0 * \cos(q 2) * \sin(\alpha 1) * \\
& \sin(\alpha 2)) - 1.0 * \cos(q 3) * \sin(\alpha 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \sin(\alpha 1) * \\
& \sin(\alpha 3) * \sin(q 2) * \sin(q 3)) + \cos(\alpha 4) * \cos(q 4) * (\sin(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - 1.0 * \cos(q 2) * \\
& \sin(\alpha 1) * \sin(\alpha 2)) + \cos(\alpha 3) * \cos(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) - \\
& 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) - 1.0 * \cos(\alpha 4) * \sin(q 4) * (\sin(q 3) * (\cos(\alpha 1) * \\
& \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(q 2))) - a 5 * \sin(q 5) * (\sin(q 4) * \\
& (\sin(\alpha 3) * (\cos(\alpha 1) * \cos(\alpha 2) - 1.0 * \cos(q 2) * \sin(\alpha 1) * \sin(\alpha 2)) + \cos(\alpha 3) * \cos(q 3) * (\cos(\alpha 1) * \\
& \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) - 1.0 * \cos(\alpha 3) * \sin(\alpha 1) * \sin(q 2) * \sin(q 3)) + \cos(q 4) * \\
& (\sin(q 3) * (\cos(\alpha 1) * \sin(\alpha 2) + \cos(\alpha 2) * \cos(q 2) * \sin(\alpha 1)) + \cos(q 3) * \sin(\alpha 1) * \sin(q 2)))
\end{aligned}$$