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$$A_{01} = \begin{pmatrix} \cos(q_1) & 0 & \sin(q_1) & 33 \cos(q_1) \\ \sin(q_1) & 0 & -\cos(q_1) & 33 \sin(q_1) \\ 0 & 1 & 0 & 147 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

$$A_{12} = \begin{pmatrix} \cos(q_2) & -\sin(q_2) & 0 & 155 \cos(q_2) \\ \sin(q_2) & \cos(q_2) & 0 & 155 \sin(q_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

$$A_{23} = \begin{pmatrix} \cos(q_3) & -\sin(q_3) & 0 & 135 \cos(q_3) \\ \sin(q_3) & \cos(q_3) & 0 & 135 \sin(q_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (3)$$

$$A_{34} = \begin{pmatrix} \cos(q_4) & 0 & \sin(q_4) & 0 \\ \sin(q_4) & 0 & -\cos(q_4) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

$$A_{45} = \begin{pmatrix} \cos(q_5) & -\sin(q_5) & 0 & 0 \\ \sin(q_5) & \cos(q_5) & 0 & 0 \\ 0 & 0 & 1 & \frac{435}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

$$A_{01} = \begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (6)$$

$$A_{02} = \begin{pmatrix} \cos(q_1) \cos(q_2) & -\cos(q_1) \sin(q_2) & \sin(q_1) & 33 \cos(q_1) \\ \cos(q_2) \sin(q_1) & -\sin(q_1) \sin(q_2) & -\cos(q_1) & 33 \sin(q_1) \\ \sin(q_2) & \cos(q_2) & 0 & 147 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (7)$$

$$A_{03} = \begin{pmatrix} \cos(q_2 + q_3) \cos(q_1) & -\sin(q_2 + q_3) \cos(q_1) & \sin(q_1) & \cos(q_1) (155 \cos(q_2) + 33) \\ \cos(q_2 + q_3) \sin(q_1) & -\sin(q_2 + q_3) \sin(q_1) & -\cos(q_1) & \sin(q_1) (155 \cos(q_2) + 33) \\ \sin(q_2 + q_3) & \cos(q_2 + q_3) & 0 & 155 \sin(q_2) + 147 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (8)$$

$$A_{04} = \begin{pmatrix} \cos(q_2 + q_3 + q_4) \cos(q_1) & -\sin(q_2 + q_3 + q_4) \cos(q_1) & \sin(q_1) & \cos(q_1) (135 \cos(q_2 + q_3) + 155 \cos(q_2) + 33) \\ \cos(q_2 + q_3 + q_4) \sin(q_1) & -\sin(q_2 + q_3 + q_4) \sin(q_1) & -\cos(q_1) & \sin(q_1) (135 \cos(q_2 + q_3) + 155 \cos(q_2) + 33) \\ \sin(q_2 + q_3 + q_4) & \cos(q_2 + q_3 + q_4) & 0 & 135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (9)$$

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$$A_{05} = \begin{pmatrix} \sin(q_1) \sin(q_5) + \cos(q_2 + q_3 + q_4) \cos(q_1) \cos(q_5) & \cos(q_5) \sin(q_1) - \cos(q_2 + q_3 + q_4) \cos(q_1) \sin(q_5) & \sin(q_2 + q_3 + q_4) \cos(q_1) & \cos(q_1) (135 \cos(q_2 + q_3) + 155 \cos(q_2) + 33) \\ \cos(q_2 + q_3 + q_4) \cos(q_5) \sin(q_1) - \cos(q_1) \sin(q_5) & -\cos(q_1) \cos(q_5) - \cos(q_2 + q_3 + q_4) \sin(q_1) \sin(q_5) & \sin(q_2 + q_3 + q_4) \sin(q_1) & \sin(q_1) (135 \cos(q_2 + q_3) + 155 \cos(q_2) + 33) \\ \sin(q_2 + q_3 + q_4) \cos(q_5) & -\sin(q_2 + q_3 + q_4) \sin(q_5) & -\cos(q_2 + q_3 + q_4) & 135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (10)$$

$$w_1 = (0 \quad 0 \quad 1 \quad 0) \quad (11)$$

$$w_2 = (\sin(q_2) \quad \cos(q_2) \quad 0 \quad 147) \quad (12)$$

$$w_3 = (\sin(q_2 + q_3) \quad \cos(q_2 + q_3) \quad 0 \quad 155 \sin(q_2) + 147) \quad (13)$$

$$w_4 = (\sin(q_2 + q_3 + q_4) \quad \cos(q_2 + q_3 + q_4) \quad 0 \quad 135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) \quad (14)$$

$$w_5 = (\sin(q_2 + q_3 + q_4) \cos(q_5) \quad -\sin(q_2 + q_3 + q_4) \sin(q_5) \quad -\cos(q_2 + q_3 + q_4) \quad 135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) \quad (15)$$

$$p_1 = g m_1 \text{rz}_1 \quad (16)$$

$$p_2 = g m_2 (\text{ry}_2 \cos(q_2) + \text{rx}_2 \sin(q_2) + 147) \quad (17)$$

$$p_3 = g m_3 (155 \sin(q_2) + \text{ry}_3 \cos(q_2 + q_3) + \text{rx}_3 \sin(q_2 + q_3) + 147) \quad (18)$$

$$p_4 = g m_4 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + \text{ry}_4 \cos(q_2 + q_3 + q_4) + \text{rx}_4 \sin(q_2 + q_3 + q_4) + 147) \quad (19)$$

$$p_5 = g m_5 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) - g m_5 \text{rz}_5 \cos(q_2 + q_3 + q_4) + g m_5 \text{rx}_5 \sin(q_2 + q_3 + q_4) \cos(q_5) - g m_5 \text{ry}_5 \sin(q_2 + q_3 + q_4) \sin(q_5) \quad (20)$$

$$\begin{aligned} P_1 &= 147 g m_2 + g m_1 \text{rz}_1 + g m_3 (155 \sin(q_2) + 147) + g m_4 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + g m_5 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + \\ &g m_3 \text{ry}_3 \cos(q_2 + q_3) + g m_3 \text{rx}_3 \sin(q_2 + q_3) + g m_2 \text{ry}_2 \cos(q_2) + g m_2 \text{rx}_2 \sin(q_2) + g m_4 \text{ry}_4 \cos(q_2 + q_3 + q_4) - g m_5 \text{rz}_5 \cos(q_2 + q_3 + q_4) + \\ &g m_4 \text{rx}_4 \sin(q_2 + q_3 + q_4) + g m_5 \text{rx}_5 \sin(q_2 + q_3 + q_4) \cos(q_5) - g m_5 \text{ry}_5 \sin(q_2 + q_3 + q_4) \sin(q_5) \\ P_2 &= 147 g m_2 + g m_3 (155 \sin(q_2) + 147) + g m_4 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + g m_5 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + \\ &g m_3 \text{ry}_3 \cos(q_2 + q_3) + g m_3 \text{rx}_3 \sin(q_2 + q_3) + g m_2 \text{ry}_2 \cos(q_2) + g m_2 \text{rx}_2 \sin(q_2) + g m_4 \text{ry}_4 \cos(q_2 + q_3 + q_4) - g m_5 \text{rz}_5 \cos(q_2 + q_3 + q_4) + \\ &g m_4 \text{rx}_4 \sin(q_2 + q_3 + q_4) + g m_5 \text{rx}_5 \sin(q_2 + q_3 + q_4) \cos(q_5) - g m_5 \text{ry}_5 \sin(q_2 + q_3 + q_4) \sin(q_5) \\ P_3 &= g m_3 (155 \sin(q_2) + 147) + g m_4 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + g m_5 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + g m_3 \text{ry}_3 \cos(q_2 + q_3) + \\ &g m_3 \text{rx}_3 \sin(q_2 + q_3) + g m_4 \text{ry}_4 \cos(q_2 + q_3 + q_4) - g m_5 \text{rz}_5 \cos(q_2 + q_3 + q_4) + g m_4 \text{rx}_4 \sin(q_2 + q_3 + q_4) + g m_5 \text{rx}_5 \sin(q_2 + q_3 + q_4) \cos(q_5) - \\ &g m_5 \text{ry}_5 \sin(q_2 + q_3 + q_4) \sin(q_5) \\ P_4 &= g m_4 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + g m_5 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) + g m_4 \text{ry}_4 \cos(q_2 + q_3 + q_4) - g m_5 \text{rz}_5 \cos(q_2 + q_3 + q_4) + \\ &g m_4 \text{rx}_4 \sin(q_2 + q_3 + q_4) + g m_5 \text{rx}_5 \sin(q_2 + q_3 + q_4) \cos(q_5) - g m_5 \text{ry}_5 \sin(q_2 + q_3 + q_4) \sin(q_5) \\ P_5 &= g m_5 (135 \sin(q_2 + q_3) + 155 \sin(q_2) + 147) - g m_5 \text{rz}_5 \cos(q_2 + q_3 + q_4) + g m_5 \text{rx}_5 \sin(q_2 + q_3 + q_4) \cos(q_5) - g m_5 \text{ry}_5 \sin(q_2 + q_3 + q_4) \sin(q_5) \end{aligned}$$

$$W_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (21)$$

$$W_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{22}$$

$$W_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{23}$$

$$W_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{24}$$

$$W_5 = \begin{pmatrix} 0 \\ \cos(q_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{25}$$

$$W_6 = \begin{pmatrix} 0 \\ -\sin(q_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{26}$$

$$W_7 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{27}$$

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$$W_8 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{28}$$

$$W_9 = \begin{pmatrix} 0 \\ \cos(q_2 + q_3) \\ \cos(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \tag{29}$$

$$W_{10} = \begin{pmatrix} 0 \\ -\sin(q_2 + q_3) \\ -\sin(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \tag{30}$$

$$W_{11} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{31}$$

$$W_{12} = \begin{pmatrix} 0 \\ 155 \cos(q_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{32}$$

$$W_{13} = \begin{pmatrix} 0 \\ \cos(q_2 + q_3 + q_4) \\ \cos(q_2 + q_3 + q_4) \\ \cos(q_2 + q_3 + q_4) \\ 0 \end{pmatrix} \tag{33}$$

$$W_{14} = \begin{pmatrix} 0 \\ -\sin(q_2 + q_3 + q_4) \\ -\sin(q_2 + q_3 + q_4) \\ -\sin(q_2 + q_3 + q_4) \\ 0 \end{pmatrix} \quad (34)$$

$$W_{15} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (35)$$

$$W_{16} = \begin{pmatrix} 0 \\ 135 \cos(q_2 + q_3) + 155 \cos(q_2) \\ 135 \cos(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \quad (36)$$

$$W_{17} = \begin{pmatrix} 0 \\ \cos(q_2 + q_3 + q_4) \cos(q_5) \\ \cos(q_2 + q_3 + q_4) \cos(q_5) \\ \cos(q_2 + q_3 + q_4) \cos(q_5) \\ -\sin(q_2 + q_3 + q_4) \sin(q_5) \end{pmatrix} \quad (37)$$

$$W_{18} = \begin{pmatrix} 0 \\ -\cos(q_2 + q_3 + q_4) \sin(q_5) \\ -\cos(q_2 + q_3 + q_4) \sin(q_5) \\ -\cos(q_2 + q_3 + q_4) \sin(q_5) \\ -\sin(q_2 + q_3 + q_4) \cos(q_5) \end{pmatrix} \quad (38)$$

$$W_{19} = \begin{pmatrix} 0 \\ \sin(q_2 + q_3 + q_4) \\ \sin(q_2 + q_3 + q_4) \\ \sin(q_2 + q_3 + q_4) \\ 0 \end{pmatrix} \quad (39)$$

$$W_{20} = \begin{pmatrix} 0 \\ 135 \cos(q_2 + q_3) + 155 \cos(q_2) \\ 135 \cos(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \quad (40)$$

$$W_{1_R} = \begin{pmatrix} \cos(q_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (41)$$

$$W_{2_R} = \begin{pmatrix} -\sin(q_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (42)$$

$$W_{3_R} = \begin{pmatrix} \cos(q_2 + q_3) \\ \cos(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \quad (43)$$

$$W_{4_R} = \begin{pmatrix} -\sin(q_2 + q_3) \\ -\sin(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \quad (44)$$

$$W_{5_R} = \begin{pmatrix} 155 \cos(q_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (45)$$

$$W_{6_R} = \begin{pmatrix} \cos(q_2 + q_3 + q_4) \\ \cos(q_2 + q_3 + q_4) \\ \cos(q_2 + q_3 + q_4) \\ 0 \end{pmatrix} \quad (46)$$

$$W_{7_R} = \begin{pmatrix} -\sin(q_2 + q_3 + q_4) \\ -\sin(q_2 + q_3 + q_4) \\ -\sin(q_2 + q_3 + q_4) \\ 0 \end{pmatrix} \quad (47)$$

$$W_{8_R} = \begin{pmatrix} 135 \cos(q_2 + q_3) + 155 \cos(q_2) \\ 135 \cos(q_2 + q_3) \\ 0 \\ 0 \end{pmatrix} \quad (48)$$

$$W_{9_R} = \begin{pmatrix} \cos(q_2 + q_3 + q_4) \cos(q_5) \\ \cos(q_2 + q_3 + q_4) \cos(q_5) \\ \cos(q_2 + q_3 + q_4) \cos(q_5) \\ -\sin(q_2 + q_3 + q_4) \sin(q_5) \end{pmatrix} \quad (49)$$

$$\infty \quad W_{10_R} = \begin{pmatrix} -\cos(q_2 + q_3 + q_4) \sin(q_5) \\ -\cos(q_2 + q_3 + q_4) \sin(q_5) \\ -\cos(q_2 + q_3 + q_4) \sin(q_5) \\ -\sin(q_2 + q_3 + q_4) \cos(q_5) \end{pmatrix} \quad (50)$$

1 W

$$\begin{array}{c}
 \mathbf{g} \\
 W =
 \end{array}
 \begin{bmatrix}
 -0.488 & -0.873 & 0.200 & 0.980 & -75.682 & 0.899 & -0.437 & -48.656 & 0.440 & -0.784 \\
 0.000 & 0.000 & 0.200 & 0.980 & 0.000 & 0.899 & -0.437 & 27.026 & 0.440 & -0.784 \\
 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.899 & -0.437 & 0.000 & 0.440 & -0.784 \\
 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & -0.381 & -0.214 \\
 0.767 & -0.641 & 0.501 & 0.866 & 118.919 & 0.248 & -0.969 & 186.514 & -0.109 & 0.223 \\
 0.000 & 0.000 & 0.501 & 0.866 & 0.000 & 0.248 & -0.969 & 67.595 & -0.109 & 0.223 \\
 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.248 & -0.969 & 0.000 & -0.109 & 0.223 \\
 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.870 & 0.425 \\
 0.974 & -0.227 & -0.883 & -0.469 & 150.938 & 0.876 & 0.483 & 31.738 & 0.483 & -0.730 \\
 0.000 & 0.000 & -0.883 & -0.469 & 0.000 & 0.876 & 0.483 & -119.200 & 0.483 & -0.730 \\
 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.876 & 0.483 & 0.000 & 0.483 & -0.730 \\
 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.403 & 0.266 \\
 0.156 & -0.988 & 0.195 & -0.981 & 24.216 & -0.099 & -0.995 & 50.524 & 0.084 & 0.053 \\
 0.000 & 0.000 & 0.195 & -0.981 & 0.000 & -0.099 & -0.995 & 26.307 & 0.084 & 0.053 \\
 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & -0.099 & -0.995 & 0.000 & 0.084 & 0.053 \\
 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & -0.529 & 0.843 \\
 -0.812 & -0.584 & -0.776 & 0.631 & -125.800 & 0.981 & -0.196 & -230.570 & -0.073 & 0.978 \\
 0.000 & 0.000 & -0.776 & 0.631 & 0.000 & 0.981 & -0.196 & -104.770 & -0.073 & 0.978 \\
 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.981 & -0.196 & 0.000 & -0.073 & 0.978 \\
 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.196 & 0.015
 \end{bmatrix}
 \tag{51}$$

$$W^T W = \begin{bmatrix} 2.458 & 0.032 & 0.087 & -0.936 & 381.058 & -0.207 & -0.055 & 392.789 & 0.244 & -0.943 \\ 0.032 & 2.542 & -0.034 & -0.703 & 5.019 & -1.618 & 1.991 & 0.411 & -0.464 & 0.085 \\ 0.087 & -0.034 & 3.421 & 0.727 & 13.469 & -2.498 & -2.081 & 475.350 & -0.640 & -0.298 \\ -0.936 & -0.703 & 0.727 & 6.579 & -145.156 & 2.800 & -1.283 & -47.010 & -0.036 & 0.665 \\ 381.058 & 5.019 & 13.469 & -145.156 & 59063.928 & -32.132 & -8.598 & 60882.300 & 37.848 & -146.113 \\ -0.207 & -1.618 & -2.498 & 2.800 & -32.132 & 7.825 & -0.915 & -369.380 & 2.135 & -1.009 \\ -0.055 & 1.991 & -2.081 & -1.283 & -8.598 & -0.915 & 7.175 & -289.527 & 0.232 & -1.411 \\ 392.789 & 0.411 & 475.350 & -47.010 & 60882.300 & -369.380 & -289.527 & 125054.597 & -48.495 & -186.348 \\ 0.244 & -0.464 & -0.640 & -0.036 & 37.848 & 2.135 & 0.232 & -48.495 & 2.737 & -2.252 \\ -0.943 & 0.085 & -0.298 & 0.665 & -146.113 & -1.009 & -1.411 & -186.348 & -2.252 & 7.480 \end{bmatrix} \quad (52)$$
[illegible]