

## Brief description of research contributions.

My research is in the field of dynamical systems. More specifically, my PhD research (completed under the supervision of Yulij Ilyashenko) was on discrete dynamical systems, mainly on Milnor attractors in the context of skew products having one-dimensional fiber and hyperbolic base. During my postdoc (under the supervision of Anatoly Neishtadt), I worked on topics concerning the averaging method for small perturbations of Hamiltonian systems exhibiting separatrix crossing.

**Milnor attractors and skew products.** The major part of my PhD research was related to Milnor's definition of attractor. For a continuous map  $f : X \mapsto X$ , Milnor defined an attractor of  $f$  to be a closed subset of  $X$  that attracts almost any point with respect to Lebesgue measure, and contains no smaller subset that does so. This definition looks natural, but is quite hard to work with. One of the difficulties is that a Milnor attractor is not preserved under conjugacy by a homeomorphism. In fact, even a transitive  $C^1$ -smooth Anosov diffeomorphism on the 2-torus can have a nontrivial Milnor attractor (a result proved by C. Bonatti, S. Minkov, I. Shilin and myself), but this diffeomorphism is topologically conjugate to a linear torus diffeomorphism with Milnor attractor the whole torus. In "Milnor Attractors of Skew Products with the Fiber a Circle", I studied the properties of Milnor attractors for partially hyperbolic skew products where the fiber is a circle, and proved that generically in this setting, a Milnor attractor is Lyapunov stable and has either full or zero Lebesgue measure. This result also holds when the fiber is a segment that is mapped inside its interior by fiberwise maps. This result is in contrast with two robust examples of boundary-preserving skew products where the fiber is a segment, namely, the intermingled basins example by Ittai Kan with Lyapunov unstable Milnor attractor, and the thick attractor example by Yulij Ilyashenko, where the Milnor attractor has positive, but not full, Lebesgue measure.

In another project, Victor Kleptsyn, Yury Kudryashov and I studied generic semigroup actions on the circle. We showed that such actions are either minimal or have an absorbing domain.

**Separatrix crossing.** Averaging method is a classical powerful tool in theory of small perturbations of integrable systems. Two main obstructions to the use of averaging method are resonances and separatrix crossings. The latter was the focus of a project between myself and Anatoly Neishtadt.

For small perturbations of one-frequency systems leading to a separatrix crossing, we estimated the accuracy of the order two averaging method. This allows one to track the evolution of the phase (i.e., the angle from the pair of action-angle variables of the unperturbed system) after separatrix crossing, which is impossible using the standard (order one) averaging method. One strength of our results is that while evolution of the phase was studied in several earlier works when the perturbed system is Hamiltonian, in our work we consider arbitrary perturbations that often arise in applications, e.g., systems with any kind of friction.

In the work "*Averaging and passage through resonances in two-frequency systems near separatrices*" we study perturbations of two-frequency systems near separatrices. In this setting there are *both* resonances and separatrix crossings. We estimate the measure of initial data such that the corresponding solution of the perturbed system is approximately described by the averaging method and the accuracy of this approximation. Again we allow for arbitrary perturbations, but our results are also new even for Hamiltonian perturbations. This work is my main mathematical achievement. It extends Neishtadt's Theorem (1975) on two-frequency systems far from separatrices to the general case with separatrix crossings and provides rigorous justification for the use of averaging method in this setting, which is frequently encountered in applications. This was a challenging problem with many difficulties to overcome.

### Future plans.

- An ongoing project with Anatoly Neishtadt is to study phase change after resonance crossing in two-frequency systems.
- A natural continuation of our research on non-Hamiltonian perturbations of one-frequency systems is to obtain a formula for the jump of slow variables caused by separatrix crossing (such formulas exist for Hamiltonian perturbations). Together with our result on the evolution of the phase, such a formula allows one to study consecutive separatrix crossings for non-Hamiltonian case. Note that for Hamiltonian perturbations multiple separatrix crossings lead to a remarkable phenomenon called *stability islands*.
- It would be interesting (and important for many applications, e.g., time-periodic perturbations of chaotic systems) to obtain an analogue of our alternative for semigroup actions on the circle in the setting of partially hyperbolic skew products where the fiber is a circle.