function of (A3) and the "self-truncating" function above. These were plotted from relationships derived in reference [7]. Note the substantial error reduction achieved by the self-truncating form of $s(t)$.

W.M. WATERS

B.R. JARRETT

Radar Division
Naval Research Laboratory
Washington, D.C. 20375

REFERENCES

[1] Merrimac Industries (1980)
Signal processing components and subsystems.
1980 Merrimac Industries Catalogue. Merrimac Industries,
PCM-3 Series Phase Comparator, p. 13, 1980.

[2] Waters, W.M., and Jarrett, B.R. (1980)
Bandpass signal sampling and coherent processing.
Report 8520, Naval Research Lab., Washington, D.C., Nov. 1980.

[3] Linden, D.A. (1959)
A discussion of sampling theorems.
Proceedings of the IRE, July 1959.

[4] Kohlenberg, A. (1953)
Exact interpolation of bandlimited functions.
Journal of Applied Physics, 1953, *24*, 1432-1436.

[5] ITT Handbook (1977)
Reference Data for Radio Engineers.
New York: Howard Sams and Co., 1977.

[6] Panter, P.F. (1965)
Modulation, Noise, and Spectral Analysis.
New York: McGraw-Hill, 1965.

[7] Helms, H.D., and Thomas, J.B. (1962)
Truncation error sampling theorem expansions.
Proceedings of the IRE, 1962, *50*, 179-184.

Quadrature Sampling With High Dynamic Range

Many radio and sonar systems require signal outputs in complex low-pass form. To achieve this, it is possible to use uniform sampling of the bandpass signal, together with computation of the quadrature component by way of a Hilbert transform. The bandpass to low-pass translation is accomplished by undersampling. A hardware implementation is described which achieves 70 dB spurious-free dynamic range and a bandwidth of 30 kHz.

I. INTRODUCTION

There are a number of radio and sonar applications in which it is convenient or necessary that the sensor output signals be translated to a complex low-pass (in-phase and quadrature) representation. Examples of applications which require the processing of signals in this form are sampled-aperture arrays [1], adaptive beamforming arrays [2], and the holographic radio camera [3]. The processing of acoustic signals from hydrophone arrays may also use this approach [4].

The processing of signals in quadrature low-pass form is founded on the so-called analytic signal representation [5], whereby signals are in general treated as complex exponentials rather than as purely real functions. The low-pass case is a specialization of the more general topic of sampling of bandpass signals, which has received considerable attention [6-10].

Quadrature low-pass signals may be obtained in a number of ways. A conventional method, shown in Fig. 1, involves a pair of analog multipliers or mixers, in which the received signal, at some convenient center frequency, is multiplied by a reference signal of frequency equal to the center frequency. The output is then low-pass filtered. A 90° phase shift is imposed in one of the channels in either the signal or reference path, so that the resulting outputs from the multipliers are in-phase quadrature. A second method, second order sampling, avoids some of the limitations of the analog multiplier method by sampling the signal directly in sample pairs, the samples comprising each pair being spaced in time by $^1/_4$ period of the midband frequency [7, 9]. (More generally, the pair spacing may be $N + (^1/_4)$ periods, where N is an integer.) In some cases, it may be necessary to compensate for various errors which arise in hardware implementations of quadrature detectors [11].

In this correspondence, we describe hardware which provides a simple solution to the problem of bandpass sampling. It achieves high dynamic range and requires sampling at or above the Nyquist rate as determined by the signal bandwidth, not by the highest frequency component of the bandpass signal. By properly choosing the sampling frequency in relation to the center frequency and bandwidth, the frequency-domain periodic-repetition property of the discrete Fourier transform is exploited in such a way that the low-pass complex signal is obtained with little computational effort. The bandpass signal is sampled at uniform intervals, and the quadrature component is computed via a digital Hilbert transform [12, 13].

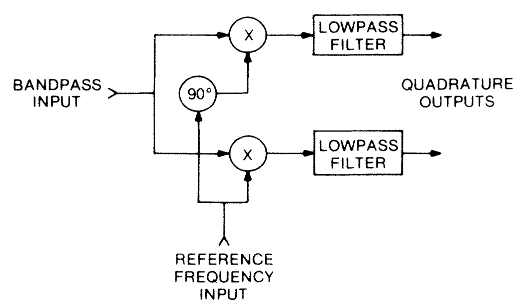


Fig. 1. Conventional quadrature detection technique, using analog multipliers (X).

Manuscript received April 1, 1982.

This work was supported by the Program for Industry/Laboratory Projects of the Government of Canada. The design and implementation of this work were undertaken at Miller Communications Systems Ltd., 300 Legget Dr., Kanata, Ont., Canada, K2K 1Y5.

0018-9251/82/1100-0736 $00.75 © 1982 Canadian Crown Copyrighter.

The procedure of directly sampling the bandpass signal and computing the quadrature component by way of a Hilbert transform has several advantages. The quadrature component can be computed with arbitrary accuracy (limited only by word size and processor speed) so that the level of image spurious responses can be controlled to any desired value. The effect of dc offset is removed from the center of the bilateral baseband spectrum, where it occurs with quadrature mixers, to a band edge where it is easily filtered out. If a variable signal bandwidth is desired, this can be provided by sampling at a high rate commensurate with the widest bandwith and then digitally filtering the signal and decimating the sample rate as appropriate. Thus only one analog bandpass filter per sensor channel is needed, an important consideration if phase- and gain-matched receiving channels are required. The phase match of analog antialiasing filters normally degrades as the band edges are approached. Thus digital filtering can take advantage of the better phase match available in the central part of the analog filter passband. By their nature, digital filters eliminate many of the problems of analog methods such as temperature sensitivity and drifts in component values.

II. HILBERT TRANSFORM RELATIONS

A discrete-time signal may be represented by a complex time series of the form:

$$s(n) = s_r(n) + js_i(n) \tag{1}$$

where n is a time index and $s_r(n)$ and $s_i(n)$ are real sequences. Given that the spectrum $S(\omega)$ of the complex signal $s(n)$ is zero for $-\pi \leq \omega < 0$, where ω is a normalized frequency, $\omega = 2\pi(f/f_s)$, and f_s is the sampling frequency, it can be shown [12] that the real and imaginary parts $s_r(n)$ and $s_i(n)$ are related by a Hilbert transform. Thus, given $s_r(n)$, the other part $s_i(n)$ may be computed.

If $s_i(n)$ is regarded as being obtained by passing $s_r(n)$ through a linear, time-invariant system, then the required system has frequency response:

$$H(\omega) = \begin{cases} -j, & 0 \leq \omega < \pi \\ +j, & -\pi \leq \omega < 0 . \end{cases} \tag{2}$$

Thus, the system is a broadband 90° phase shifter. The corresponding impulse response may be approximated by a finite-duration impulse response (FIR) filter, following the procedures in [13]. The computation of the imaginary part of the sequence $s_i(n)$ then involves a convolution of the real part of the sequence $s_r(n)$ with the impulse response of the Hilbert transformer.

Fig. 2(A) illustrates a complex, continuous-time bandpass spectrum, which is nonzero only over the frequency interval $\{f_L, f_U\}$. Suppose that the time-domain signal is complex-sampled at discrete intervals, such that the sampling waveform spectrum is that shown in Fig. 2(B). The spectrum of the sampled signal is the convolution of (A) and (B), as shown in Fig. 2(C). Given that the sampling frequency has been chosen in relation to the bandpass spectrum as illustrated, then there is a Hilbert transform relation between the real and imaginary parts of the time domain signal, since the negative half of the spectrum, $-\pi \leq \omega < 0$, is zero. Thus there is no need to complex-sample the bandpass signal; the imaginary part of the sequence can be computed from the real part. Once this is done, the sample rate may be decimated by 2, yielding

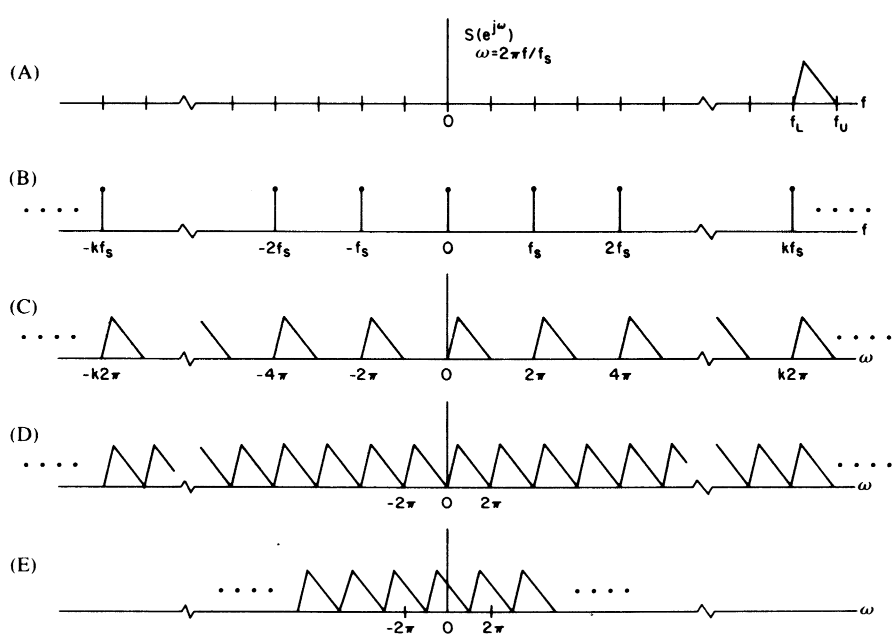


Fig. 2. (A) Representation of the frequency spectrum of a complex continuous-time bandpass signal. The spectrum is nonzero only between frequencies f_L and f_U. (B) Spectrum of a sampling signal which has repetition frequency f_s. (C) Spectrum of the signal in (A), after sampling by the function whose spectrum is shown in (B). (D) The sampled-signal spectrum after sample rate decimation by factor of 2. (E) The sampled-signal spectrum of (D) after a frequency shift by π. The low-pass repetition of the spectrum, $-\pi \leq \omega < \pi$, is the complex baseband representation of the bandpass signal in (A).

Fig. 2(D). The final desired result, Fig. 2(E), is obtained by shifting the spectrum by π, or in the time domain, by multiplying the time domain samples by $e^{-j\pi n} = (-1)^n$, $n = 0,1,2, \ldots$. The final result in Fig. 2(E) is a complex low-pass representation of the bandpass signal originally occupying the spectral range $\{f_L, f_U\}$ as shown in Fig. 2(A).

In order that the nonzero portion of the bandpass spectrum repeat into the region $0 \leq \omega < \pi$, the sampling rate f_s must be chosen such that the lower bandedge f_L is at

$$f_L = kf_s \tag{3}$$

and the upper bandedge f_U is at

$$f_U = (k + 1/2)f_s \tag{4}$$

where k is an integer.

Another set of solutions exists, if the nonzero portion of the bandpass spectrum is repeated into the region $-\pi \leq \omega < 0$. In this case the lower and upper bandedges are given by

$$f_L = (k - 1/2)f_s \tag{5}$$

$$f_U = kf_s \, . \tag{6}$$

The normal Nyquist criterion requires that

$$f_s \geq 2(f_U - f_L) \, . \tag{7}$$

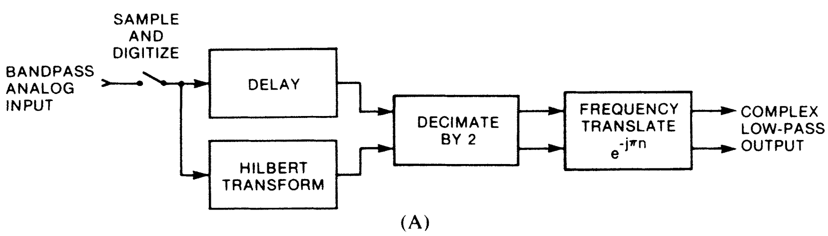
Here it is understood that f_U and f_L are the actual bandedges including guardbands, and the equality in (7) applies. For $k>0$, the bandpass signal is undersampled with respect to f_U, but not with respect to $f_U - f_L$, and no information is lost.

Fig. 3(A) shows the steps which are required to accomplish the above operations. In fact, if the Hilbert transform is implemented as a symmetric FIR filter with an odd number of stages [13], the alternate impulse response coefficients of the filter are zero. The in-phase component is derived from the sampled data with a delay of $(N-1)/2$ (where N is the filter length), which coincides with the zero coefficient in the center of the FIR filter. Thus input samples can be fed alternately to a delay register and the Hilbert transformer as shown in Fig. 3(B). In addition, this operation automatically provides a decimation of 2. Finally, the frequency translation of π is applied.

III. IMPLEMENTATION

The basic scheme shown in Fig. 3(B) has been implemented in special-purpose hardware, referred to herein as a baseband processor. The input signal is sampled to 12 bit precision, but 16 bits are used for computation. The Hilbert transform is implemented as a 63 point convolution (but only 32 coefficients are nonzero). The hardware includes a TRW 16 × 16 bit multiplier/accumulator. Temporary data storage is in shift registers. The filter coefficients are stored in ROM, as is the timing state information (8 bits × 584 states). Also included in the hardware is a digital bandpass filter which is applied to the sampled signal before conversion to complex form. This filter is also an FIR design, but of length 128. The filter bandwidth is selectable through storage of eight different sets of filter coefficients in ROM. The output sample rate is decimated by a factor depending on the ratio of Nyquist frequency and filter bandwidth. The digital bandpass filter and the Hilbert transform filter share the same multiplier/accumulator. The maximum rate of sampling of the input signal is 60 kHz for the hardware as implemented. Even higher rates could easily be achieved by simplifying the bandpass filter, which is the dominant computational load. For example, the filter could be made odd length and symmetric so that the alternate impulse response coefficients are zero.

The implementation of the digital part of the hardware was straightforward. More care was required with the analog portion (at the input), and with the track-and-hold



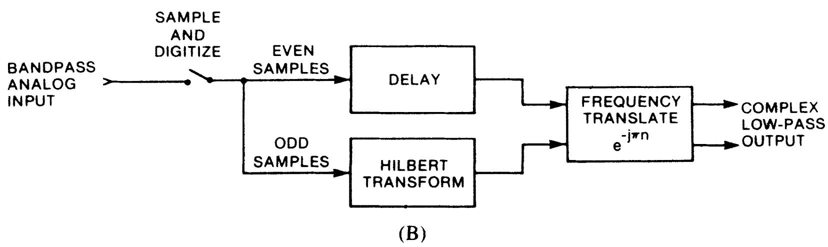


Fig. 3. (A) Sequence of operations to obtain low-pass complex samples from uniform real samples of a bandpass signal. (B) Simplification of (A) for more efficient computation.

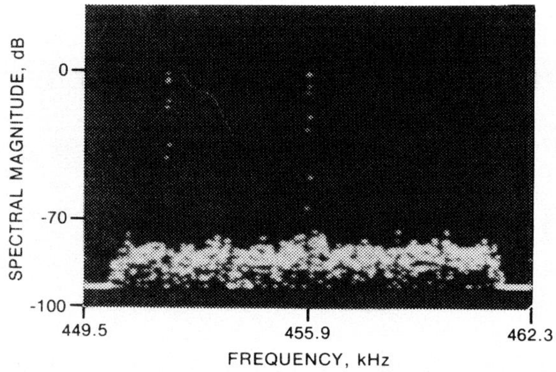


Fig. 4. Spectrum of the baseband processor output with two equal-amplitude test tones as input. There are no spurious responses exceeding -70 dB with respect to the test tone levels.

amplifier. A cascade of two high-performance operational amplifiers, separated by a bandpass filter, was required to amplify the 100 mV input signal to 5 V needed by the A/D converter, without introducing intermodulation or harmonic products above the desired -70 dB floor (harmonics would alias into the passband). The track-and-hold amplifier was an Analog Devices model HTC-0300 with a harmonic distortion specification of -75 dB typical but -62 dB maximum. Two units were tested; they yielded results of -65 dB and -75 dB. This amplifier also has a specified maximum aperture uncertainty time of 100 ps. In the present application, this is sufficient to avoid introducing phase noise through sampling time jitter.

IV. EXPERIMENTAL RESULTS

Fig. 4 shows a typical result from a two-tone test of the baseband processor hardware. One tone is at the center frequency of 455.9 kHz, the other at a frequency of 452.0 kHz. The sampling rate was 25 685 Hz, thus defining the passband from 449.5 to 462.3 kHz (bandpass case) or -6.4 to $+6.4$ kHz (at baseband). Both tones have amplitudes of 50 mV peak at the input, one-half the full scale input. The spectrum in Fig. 4 was computed via an FFT using samples in complex low-pass form as provided by the hardware. As shown, the spurious-free dynamic range, in the presence of two equal-amplitude tones, exceeds 70 dB.

D.W. RICE*

K.H. WU

Communications Research Center
P.O. Box 11490, Station H
Ottawa, Ont., Canada K2H 8S2

*Deceased.

REFERENCES

[1] Rice, D.W. (1973)
Phase characteristics of ionospherically-propagated radio waves.
Nature Physical Science, 1973, *244* (136), 86.

[2] Compton, R.T. (1976)
An experimental four-element adaptive array.
IEEE Transactions on Antennas and Propagation, Sept. 1976, *AP-24* (5), 697.

[3] Stone, W.R. (1976)
A holographic radio camera technique for the three-dimensional reconstruction of ionospheric inhomogeneities.
Journal of Atmospheric and Terrestrial Physics, 1976, *38*, 583.

[4] Pridham, R.G., and Mucci, R.A. (1979)
Shifted sideband beamformer.
IEEE Transactions on Acoustics, Speech, and Signal Processing, Dec. 1979, *ASSP-27*, 713.

[5] Bedrosian, E. (1962)
The analytic signal representation of modulated waveforms.
Proceedings of the IRE, Oct. 1962, *50*, 2071.

[6] Linden, D.A. (1959)
A discussion of sampling theorems.
Proceedings of the IRE, July 1959, *47*, 1219.

[7] Grace, O.D., and Pitt, S.P. (1968)
Quadrature sampling of high-frequency waveforms.
Journal of the Acoustical Society of America, 1968, *44*, 1453(L).

[8] Grace, O.D., and Pitt, S.P. (1970)
Sampling and interpolation of bandlimited signals by quadrature methods.
Journal of the Acoustical Society of America, 1970, *48* (6) (Pt. 1), 1311.

[9] Brown, J.L., Jr. (1979)
On quadrature sampling of bandpass signals.
IEEE Transactions on Aerospace and Electronic Systems, May 1979, *AES-15*, 366.

[10] Pridham, R.G., and Mucci, R.A. (1979)
Digital interpolation beamforming for low-pass and bandpass signals.
Proceedings of the IEEE, June 1979, *67*, 904.

[11] Churchill, F.E., Ogar, G.W., and Thompson, B.J. (1981)
The correction of I and Q Errors in a Coherent Processor,"
IEEE Transactions on Aerospace and Electronic Systems, Jan. 1981, *AES-17*, 131-137.

[12] Oppenheim, A.V., and Schafer, R.W. (1975)
Digital Signal Processing.
Englewood Cliffs, N.J.: Prentice-Hall, 1975, ch. 7.

[13] Rabiner, L.R., and Gold, B. (1975)
Theory and Application of Digital Signal Processing.
Englewood Cliffs, N.J.: Prentice-Hall, 1975, ch. 3.

Product Device Analysis for Radar Applications

By applying the theory of two-dimensional Fourier transforms to the response of a product device, results are developed that may be applied to tracking radar range gates, phase detectors, and angle-error detectors. These responses are general in that noises that are present can be nonstationary with arbitrary power spectrum while the radar signal, a pulse stream, can have any envelope shape.

I. INTRODUCTION

In many forms of radar, range gates, phase detectors, and angle-error detectors are routinely needed [1-3]. Of-

Manuscript received December 9, 1981; revised June 7, 1982.

Work reported herein was sponsored by the U.S. Naval Research Laboratory while the author was at University of Tennessee, Knoxville, TN 37916, under contract N00014-80-C-0847 from the Office of Naval Research.

0018-9251/83/0100-0739 $00.75 © 1983 IEEE