Posting Date: October 28, 2015.

Problem 1: In the lecture, we introduced the concept of code-carrier divergence by considering the effect of the dispersive ionosphere on a simplified signal impinging on a GNSS receiver

$$r(t) = C(t)\sin(2\pi f_c t),$$

where $C(t)$ is a spreading waveform with null-to-null bandwidth B_1 of approximately 2 MHz and f_c is a GNSS L-band carrier frequency (somewhere between 1.2 and 1.6 GHz). We subsequently expressed $r(t)$ as a function of the Fourier transform $\tilde{C}(f) = \mathcal{F}\{C(t)\}$ and introduced ionospheric delay effects to arrive at

$$r(t) = \int_{-\infty}^{\infty} \frac{\tilde{C}(f)}{2j}\left[e^{j2\pi(f+f_c)(t-\Delta\tau_{p,H})} - e^{j2\pi(f-f_c)(t-\Delta\tau_{p,L})}\right] df, \tag{1}$$

where

$$\Delta\tau_{p,H} = \frac{-K}{(f+f_c)^2}$$

is the ionospheric delay (excess delay) for the frequency constituent at the "high" frequency $f + f_c$ and

$$\Delta\tau_{p,L} = \frac{-K}{(f-f_c)^2}$$

is the ionospheric delay for the frequency constituent at the "low" frequency $f - f_c$. It was claimed that after some derivation, one could show that (1) reduces to

$$r(t) = C(t-\Delta\tau_g)\sin\left[2\pi f_c(t-\Delta\tau_p)\right],$$

where $\Delta\tau_g = K/f_c^2$ is the so-called group delay and $\Delta\tau_p = -K/f_c^2$ is the so-called phase delay, with $K = \frac{(40.3)\text{TEC}}{c}$. Derive this latter expression for $r(t)$ from (1). Hint: Recognize that the effective range of the integral is small compared to f_c. This allows you to approximate a term $K/(f_c^2 - f^2)$ as K/f_c^2 over the range of integration.

Problem 2: Show that the group velocity v_g and the phase index of refraction n_p are related by $v_g = cn_p$ for small group and phase velocity departures from the speed of light c.

Problem 3: In this problem, you will generate a plot for the ionospheric delay estimates obtained from code and carrier phase measurements on both GPS L_1 and L_2. Download the files `gpsL1CAch01.txt`, `gpsL2CLch02.txt`, `gpsL1CAch01.txt`, `gpsL2CLch02.txt`, which contain the log of tracked GPS signals of two different satellite vehicles (SVs) for L_1 C/A and L_2 CL channels. The log file definition is provided in `logfiledef.txt`. Plot the code- and carrier-derived ionospheric delay time histories.

Problem 4: Write a function in MATLAB for computing the ionospheric delay from a model of the ionosphere. Your function should adhere to the interface described below. Only develop calculations for the broadcast (Klobuchar) model. The function can later be augmented to accommodate other model types. You can learn about the broadcast model on pages 168–169 of the Misra and Enge text. More details can be found on pages 128–130 of the GPS interface specification `IS-GPS-200F.pdf` posted on iLearn. You will also need to write your own function for computing satellite elevation and azimuth angles. Assume the WGS84 model for the shape of the Earth.

```
function [delTauG] = getIonoDelay(ionodata,fc,rRx,rSv,tGPS,model)
% getIonoDelay : Return a model-based estimate of the ionospheric delay
%                experienced by a transionospheric GNSS signal as it
%                propagates from a GNSS SV to the antenna of a terrestrial
%                GNSS receiver.
%
% INPUTS
%
% ionodata ------- Structure containing a parameterization of the
%                  ionosphere that is valid at time tGPS.  The structure is
%                  defined differently depending on what ionospheric model
%                  is selected:
%                  broadcast --- For the broadcast (Klobuchar) model, ionodata
%                  is a structure containing the following fields:
%                     alpha0 ... alpha3 -- power series expansion coefficients
%                     for amplitude of ionospheric TEC
%                     beta0 .. beta3 -- power series expansion coefficients
%                     for period of ionospheric plasma density cycle
%                  Other models TBD ...
%
% fc ------------- Carrier frequency of the GNSS signal, in Hz.
%
% rRx ------------ A 3-by-1 vector representing the receiver antenna position
%                  at the time of receipt of the signal, expressed in meters
%                  in the ECEF reference frame.
%
% rSv ------------ A 3-by-1 vector representing the space vehicle antenna
%                  position at the time of transmission of the signal,
%                  expressed in meters in the ECEF reference frame.
%
% tGPS ----------- A structure containing the true GPS time of receipt of
%                  the signal.  The structure has the following fields:
%                  week -- unambiguous GPS week number
%                  seconds -- seconds (including fractional seconds) of the
%                  GPS week
%
% model ---------- A string identifying the model to be used in the
%                  computation of the ionospheric delay:
%                  broadcast --- The broadcast (Klobuchar) model.
%                  Other models TBD ...
%
% OUTPUTS
%
% delTauG -------- Modeled scalar excess group ionospheric delay experienced
%                  by the transionospheric GNSS signal, in seconds.
%
%+------------------------------------------------------------------------------+
% References: For the broadcast (Klobuchar) model, see IS-GPS-200F
% pp. 128-130.
%
%+==============================================================================+
```

Problem 5: The following broadcast ionospheric parameters are valid for the data provided in the Problem 3.

```
alpha0: 4.6566e-009
alpha1: 1.4901e-008
alpha2: -5.9605e-008
alpha3: -5.9605e-008
 beta0: 79872
 beta1: 65536
 beta2: -65536
 beta3: -393220
```

Assume a true GPS time of receipt given by

```
week = 1490
seconds = 146238.774036515
```

and a static ECEF receiver antenna location given in meters by

```
X =  1101972.5309609
Y = -4583489.78279095
Z =  4282244.3010423
```

Use the function you wrote for Problem 4 to determine the ionospheric delay as calculated by the broadcast model for SVIDs 29 and 31, with the ECEF SV position of SVID 29 at time of transmission given in meters by

```
X = 24597807.6872883
Y = -3065999.1384585
Z =  9611346.77939927
```

and the ECEF SV position of SVID 31 at time of transmission given in meters by

```
X =   2339172.27088689
Y = -16191391.3551878
Z =  21104185.0481546
```

Problem 6: Download the Cornell Scintillation Simulation Toolkit `scintSim.zip` from iLearn. Read the instructions in `UserGuide.pdf` for generating synthetic scintillation. Experiment with the graphical user interface by typing `guiscint` at the MATLAB prompt. You'll be able to generate a time history of the complex channel response function $z(t)$ by setting the parameters S_4 and τ_0 and pressing "Simulate". The quantity T_e above the bar chart is the mean time between differentially-detected bit errors, which serves as a proxy for the mean time between cycle slips. Notice how T_e changes as you experiment with different values of S_4, τ_0, and C/N_0.

Write your own MATLAB function to calculate the S_4 index and the decorrelation time τ_0 corresponding to a given scintillation time history produced by the scintillation simulator and stored in `scintDat.mat`. Test your function to see how well your calculated S_4 and τ_0 values match the values that were used as parameters in generating the data. Your function should adhere to the following interface:

```
function [S4,tau0] = computeS4AndTau0(zkhist,tkhist)
% computeS4AndTau0 : Compute the scintillation index S4 and the decorrelation
%                    time tau0 corresponding to the input complex channel
%                    response function time history zkhist.
%
%
% INPUTS
%
% zkhist ----- Nt-by-1 vector containing the normalized complex scintillation
%              time history in the form of averages over Ts with sampling
%              interval Ts.  zkhist(kp1) is the average over tk to tkp1.
%
% tkhist ----- Nt-by-1 vector of time points corresponding to zkhist.
%
%
% OUTPUTS
%
% S4 --------- Intensity scintillation index of the scintillation time history
%              in zkhist, equal to the mean-normalized standard deviation of
%              the intensity abs(zkhist).^2.
%
% tau0 ------- The decorrelation time of the scintillation time history in
%              zkhist, in seconds.
%
%
%+------------------------------------------------------------------------------+
% References:
%
%
%+==============================================================================+
```