# Chapter 4
# Gyroscope technology 1

## 4.1 Introduction

Gyroscopes are used in various applications to sense either the angle turned through by a vehicle or structure (displacement gyroscopes) or, more commonly, its angular rate of turn about some defined axis (rate gyroscopes). The sensors are used in a variety of roles such as:

- stabilisation,
- autopilot feedback,
- flight path sensor or platform stabilisation,
- navigation.

It is possible with modern gyroscopes for a single sensor to fulfil each of the above tasks, but often two or more separate clusters of sensors are used.

The most basic and the original form of gyroscopes makes use of the inertial properties of a wheel or rotor spinning at high speed. Many people are familiar with the child's toy which has a heavy metal rotor supported by a pair of gimbals [1]. When the rotor is spun at high speed, the rotor axis continues to point in the same direction despite the gimbals being rotated. This is a crude example of a mechanical, or conventional, displacement gyroscope.

Examples of mechanical spinning wheel gyroscopes used in strapdown applications are the single-axis rate-integrating gyroscope and twin axis 'tuned' or flex gyroscopes. An alternative class designation for gyroscopes that cannot be categorised in this way, is not surprisingly called unconventional sensors, some of which are solid-state devices. The very broad and expanding class of unconventional sensors includes devices such as:

- Rate transducers which include mercury sphere and magneto-hydrodynamic sensors;
- Vibratory gyroscopes;
- Nuclear magnetic resonance (NMR) gyroscopes;

- Electrostatic gyroscopes (ESGs);
- Optical rate sensors which include ring laser gyroscopes (RLGs) and fibre optic gyroscopes (FOGs);
- Micro-machined electromechanical system (MEMS) gyroscopes.

Whilst many of the sensors in this class are strictly angular rate sensors and not gyroscopes in the sense that they do not rely on the dynamical properties of rotating bodies, it has become accepted that all such devices be referred to as gyroscopes since they all provide measurements of body rotation.

In this chapter, some conventional sensors are described followed by sections which outline the principles of operation and performance of some of the other gyroscope technologies noted above. Finally, a brief mention is made of other forms of instrument or novel techniques that may be used to sense rotational motion. Optical and MEMS gyroscope technologies are discussed separately in Chapters 5 and 7, respectively.

Throughout this and the later chapters on gyroscope technology, emphasis is placed on those sensors which are used, or have the potential to be used, in strapdown inertial systems. It is for this reason that both optical and MEMS sensors are described in some detail. Advances in interferometric fibre optic gyroscope (IFOG) technology are leading to the wider application of these devices, whilst MEMS sensors are seen very much as the technology of the future with wide application in strapdown systems. Details of fabrication of various types of gyroscopes can be found in References 2 and 3.

## 4.2 Conventional sensors

### *4.2.1 Introduction*

Conventional gyroscopes make use of the inertial properties of a wheel or rotor spinning at high speed [2, 3]. A spinning wheel tends to maintain the direction of its spin axis in space by virtue of its angular momentum vector, the product of its inertia and spin speed, and so defines a reference direction. The development of the mechanical gyroscope owes much to the excellent work of Professor C.S. Draper and his co-workers, at the Massachusetts Institute of Technology. The performance which may be achieved using gyroscopes of this type varies from the precision devices with error rates of less than 0.001°/h, to less accurate sensors with error rates of tens of degrees per hour. Many devices of this type have been developed for strapdown applications, being able to measure angular rates up to about 500°/s. Some designs are very rugged, having characteristics which allow them to operate in harsh environments such as guided weapons.

### *4.2.2 Fundamental principles*

There are several phenomena on which the operation of the conventional spinning mass gyroscope depends, namely gyroscopic inertia, angular momentum and precession. In the case of two-degrees-of-freedom gyroscopes, there are also the phenomena of nutation, gimbal lock and tumbling. These are considered in turn in the following sections.

### *4.2.2.1 Gyroscopic inertia*

Gyroscopic inertia is fundamental to the operation of all spinning mass gyroscopes, as it defines a direction in space that remains fixed in the inertial reference frame, that is, fixed in relation to a system of coordinates which do not accelerate with respect to the 'fixed stars'. The establishment of a fixed direction enables rotation to be detected, by making reference to this fixed direction. The rotation of an inertial element generates an angular momentum vector which is coincident with the axis of spin of the rotor or 'wheel'. It is the direction of this vector which remains fixed in space, given perfection in the construction of the gyroscope.

A practical reference instrument may be designed by having the rotor supported in a set of frames or gimbals which are free to rotate with respect to one another as shown in Figure 4.1. This is an external gimbal type gyroscope. The orientation of the case of the instrument with respect to the direction of the spin axis may be measured with angle pick-off devices mounted on the gimbals.

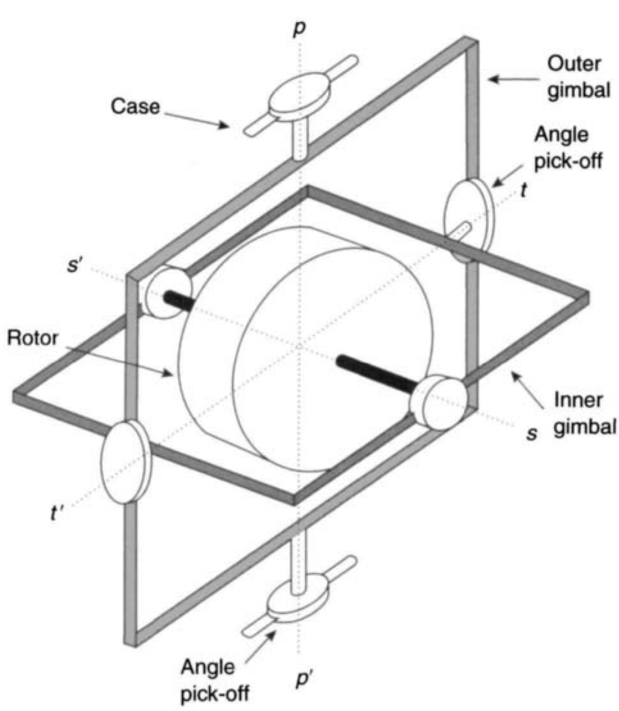


*Figure 4.1 Schematic diagram of a two-axis gyroscope*

#### 4.2.2.2 Angular momentum

The angular momentum ($H$) of a rotating body is the product of its moment of inertia ($I$) and its angular velocity ($\omega_s$) referred to the same axis of rotational motion, that is,

$$H = I\omega_s \tag{4.1}$$

where $I$ is the sum of the products of the mass elements that make up the rotor and the square of their distances from the given axis.

Angular momentum is defined by the distribution of mass on a rotor as well as by its angular velocity. For many applications, the angular momentum is chosen to be very high, so that the undesired torques that can act on a rotor and cause errors are virtually insignificant. This of course, given good design and fabrication techniques, results in a gyroscope with little movement of the direction of the spin axis. Any undesired movement of the direction of the spin axis is usually referred to as 'drift'. Clearly, one technique for producing a very high angular momentum is to have the majority of the mass of the rotor at its edge owing to the dependence of the moment of inertia on the square of the distance of its mass element from the centre of rotation.

Careful consideration must be given to the value of the angular momentum selected for a gyroscope to be used in a given application. The choice of a very high angular momentum should result in negligible drift, but there could be some considerable penalties. The gyroscope would almost certainly be relatively large and heavy, and it may take many seconds, if not minutes, for the rotor to reach its operating speed. Further, when used in a strapdown mode, the associated control system may not be capable of recording, or 'capturing', angular rates beyond a few tens of degrees per second. Hence, many compromises have to be made when selecting a gyroscope for a given application.

#### 4.2.2.3 Precession

Because the motion of a spinning mass occurs in a way that does not coincide with 'common sense' expectations, it has acquired a confusing aura of mystery. Some simple explanation may help.

First, think of the gyroscope rotor mounted in bearings in a gimbal, as shown in Figure 4.2. The gimbal axis system has one axis through the bearing axes, $ss'$, and two mutually orthogonal axes through the centre of mass of the rotor , $tt'$ and $pp'$.

*Spin* is the rotation of the gyroscope rotor relative to the gimbal.

*Precession* is the rotation of the gimbal, relative to inertial space. In the case of a freely spinning body, such as the Earth (or the rotor of an electrostatic gyroscope, see Section 4.7), there is not a material frame with spin bearings. In this case, the precession must be considered to be that of the axis system which an imaginary gimbal would have – one axis through the north and south poles, and two mutually orthogonal in the plane of the Equator.

Consider now the disc shown in Figure 4.2 spinning about the axis $ss'$. If the disc is acted upon by a couple, that is, a torque, the torque being about the axis $tt'$,

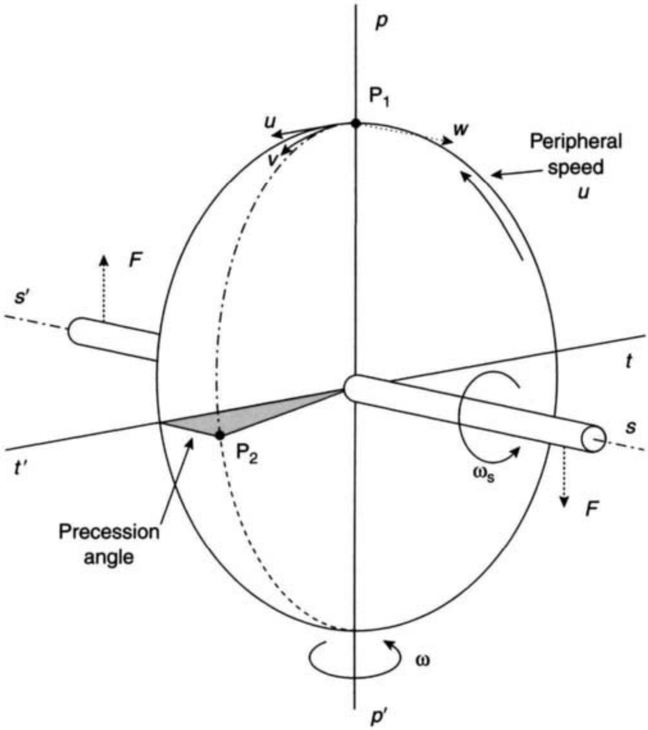


*Figure 4.2 Simple explanation of precession*

the spin axis of the disc will be forced to turn about the axis $pp'$. This turning is the precession. Note that the precession axis, $pp'$, is orthogonal to the torque axis, $tt'$. It is the unexpectedness of this result which causes confusion. However, if Newton's laws are applied carefully, the result can be explained, both qualitatively and quantitatively.

The disc is spinning about the axis $ss'$ in an anti-clockwise direction looking from $s$ to $s'$. Suppose that the disc is rigid with all the mass in the rim and that the rim has a peripheral speed $u$. Consider an element of mass at the highest point, $P_1$. Apply an impulsive couple $FF$, as shown in the figure in an anti-clockwise direction looking from $t$ to $t'$.

The instantaneous velocity of the mass is changed by adding the velocity $w$ in the same sense as the couple $FF$. The resultant velocity, $v$, is now in a different direction. It is noted that the other elements of the rim change their velocities in proportion to their distance from the axis $tt'$. After the disc has spun through 90°, the element of mass arrives at the point $P_2$, which is not in the expected line $tt'$, but in a plane which has precessed about the axis $pp'$.

This simple picture indicates how the spinning disc reacts to the impulsive couple, and shows the axis and sense of the precession. Of course, the process is normally continuous, not impulsive. The dynamics can be analysed using co-ordinate geometry and applying Newton's laws. The result agrees with eqn. (4.5) which is arrived at using vectors.

The particles making up a spinning body undergo:

1. accelerations caused by accelerations of the centre of mass of the body;
2. centripetal accelerations caused by the spinning of the body;
3. Coriolis accelerations as a result of the precession of the body.

The Coriolis accelerations are simply the additional accelerations experienced by a mass moving relative to an axis system when that axis system is itself rotating in inertial space. The precession torque is simply the torque necessary to produce the sum of the particle masses times their Coriolis accelerations.

Mathematical description of precession

Consider a heavy spinning disc, as shown in cross section in Figure 4.3, with angular momentum $H$ defined by the vector OA, that is, $H\mathbf{a}$, where $\mathbf{a}$ is a unit vector.

From Newton's first law, applied to angular motion, the angular momentum vector $\mathbf{H}$ remains constant unless the disc is acted upon the torque. Let us suppose that a torque $T$ is applied to the disc which causes it to precess at a rate $\omega(= \omega\mathbf{c}$, where $\mathbf{c}$ is also a unit vector) about an axis which will lie in the plane of the disc and may be taken to be normal to the plane of the paper. Over a short period of time $\delta t$ the disc will have precessed through an angle $\omega\delta t$ about $\mathbf{c}$, and the angular momentum vector will have changed to OB, that is, to $(H + \delta H)\mathbf{b}$, where $\mathbf{b} = \mathbf{a} + \omega\delta t\ (\mathbf{c} \times \mathbf{a})$.

The change in angular momentum over this time is represented by the vector AB and may be expressed as:

$$\begin{aligned}\delta H &= (H + \delta H)\mathbf{b} - H\mathbf{a}\\ &= H(\mathbf{b} - \mathbf{a}) + \delta H\mathbf{b}\end{aligned}$$

that is,

$$\delta H = H\omega\delta t(\mathbf{c} \times \mathbf{a}) + \delta H\mathbf{b} \tag{4.2}$$

Thus, in the limit, as $\delta t \to 0$, the rate of change of angular momentum is given by

$$\frac{\mathrm{d}H}{\mathrm{d}t} = H\omega(\mathbf{c} \times \mathbf{a}) + \frac{\mathrm{d}H}{\mathrm{d}t}\mathbf{b}$$

that is,

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \omega \times \mathbf{H} + \frac{\mathrm{d}H}{\mathrm{d}t}\mathbf{b} \tag{4.3}$$

From Newton's second law, the rate of change of angular momentum is equal to the torque $\mathbf{T}$ applied to the body, hence

$$\mathbf{T} = \omega \times \mathbf{H} + \frac{\mathrm{d}H}{\mathrm{d}t}\mathbf{b} \tag{4.4}$$

Thus, the component of the torque which is along the spin axis $\mathbf{b}$ gives rise to an acceleration in the spin rate. In a practical gyroscope, this is normally negligible and countered by the effect of the spin motor. The component normal to the spin axis

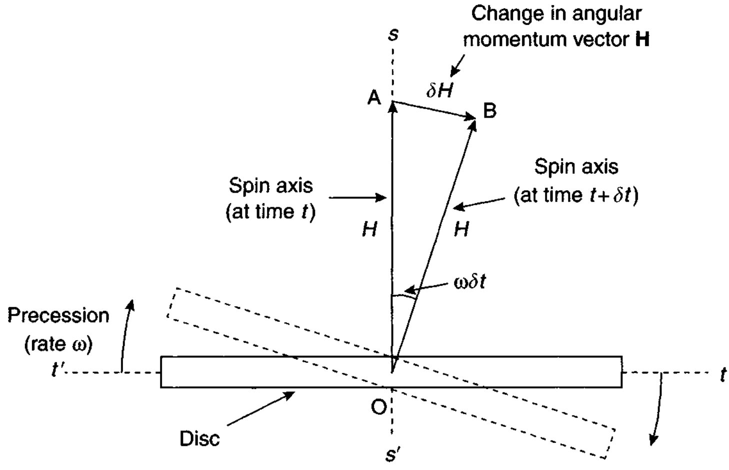


*Figure 4.3 Illustration of precession*

gives rise to a precession ω which is normal to both the torque and the spin axes, and from inspection of the figure, the direction of the precession is such as to try to align the spin axis with the torque axis.

Neglecting the component along the spin axis, in vector terms we may write

$$\mathbf{T} = \omega \times \mathbf{H} \tag{4.5}$$

and in magnitude terms

$$T = \omega H \tag{4.6}$$

This is sometimes known as the law of gyroscopes.

The application of the precession principle

The principle of precession can be exploited to provide a very accurate measure of angular rotation or rotation rate. Since a spinning wheel, or rotor, will only precess if a torque is applied to it, a rotor suspended in an instrument case by gimbals will maintain its spin axis in a constant direction in space. Changes in the angles of the gimbals will then reflect any changes in orientation of the case with reference to the spin axis direction.

Alternatively, if controlled torques are applied to the rotor to keep its spin axis aligned with a direction defined by the case of the instrument, then the measurement of these torques will provide measurements of the angular velocity of the instrument, and hence of the angular velocity of any body to which the instrument is attached.

Note that when a torque is applied to the rotor, which responds by precessing, then there is an equal but opposite reaction torque from the rotor to the application mechanism. However, if precession is prevented, as when the supporting gimbal hits a stop, then the reaction torque disappears and the rotor and gimbal act as a non-gyroscopic body about this axis.

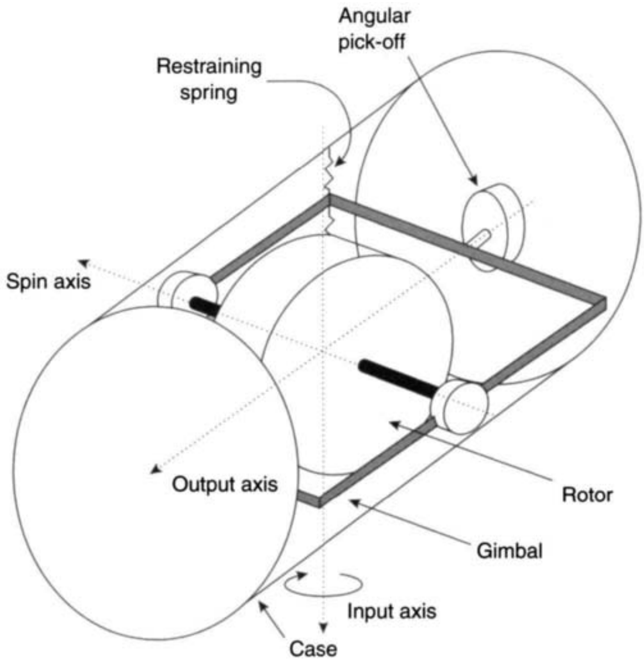


*Figure 4.4 A single-axis gyroscope*

In the single-axis rate gyroscope, shown schematically in Figure 4.4, the gyroscope's rotor is supported by a single gimbal whose axis is normal to the spin axis. The gimbal is restrained about its axis by a spring attached to the case, and there is an angular pick-off which measures the displacement of the gimbal about its axis from a 'null', or zero, position.

In the case where the instrument is rotating about the input axis and the rotor is not precessing at the same rate, the difference in rates will result in elastic compression of the gimbal pivots. This gives a torque on the gimbal (and a reaction torque on the case) which is applied to the rotor about the input axis. This torque causes the rotor to precess initially about the output axis, about which it is free to turn. The resulting displacement about the output axis causes a torque to be produced about this axis by the restraining spring.

The spring torque on the gimbal and the rotor about the output axis results in the rotor precessing about the input axis until, in the steady state, it is precessing at the same rate as the case is turning, with the deflection of the restraining spring providing just that amount of torque needed to keep the case and rotor in alignment.

Assuming that the restraining spring is linear, the deflection of the gimbal is proportional to the torque required to keep the rotor precessing with the case, and so to the turn rate of the case.

In practice, it is quite difficult to measure angular displacements accurately without resorting to sophisticated and consequently expensive equipment. However, it is quite easy to measure accurately a fixed or defined position, particularly a zero deflection or 'null' position. Hence, if the spin axis of a rotor is made to precess back to the 'null' position by the application of a suitable torque, there is potential for very accurate angular measurement, provided that the torque required to null the deflection can be generated and measured.

This is achieved in practice by replacing the restraining spring with an electromagnetic torque generator that produces a torque to cause precession of the rotor in a direction opposite to that caused by rotation of the case about its input axis. The current required can be measured very accurately by simple techniques, and when the system is 'balanced' this current is directly proportional to the applied angular rate. This technique is commonly called nulling, and is fundamental to the use of strapdown techniques. Hence, application of the reverse of the precession principle enables very accurate measurements to be made of the angular displacement or rate of turn of the case of the rate gyroscope.

A conventional single-axis gyroscope can be considered to have three orthogonal axes: its rotor or spin axis, an input precession axis and an output or torque axis. That is, torque is always applied about the output axis to cause precession about the input axis to keep it in alignment with the case. In the case of the so-called two- or dual-axis gyroscopes, these sensors have a spin axis with two orthogonal input axes. In this case, angular motion of the case of the sensor with respect to the rotor is sensed by pick-off angle sensors on the gimbals, as indicated in Figure 4.1, and each gimbal is provided with a torque mechanism.

Even the most accurate of gyroscopes will appear to drift, or have their spin axis precess. This is because the angular momentum vector is fixed with respect to space axes, not the co-ordinate system defined by the Earth. Hence, for some orientations on Earth, it is necessary to apply corrective torques to precess the gyroscope if it is to be used as an Earth reference.

#### *4.2.2.4 Nutation*

This is a natural phenomenon that occurs with so-called two-degrees-of-freedom gyroscopes, such as those in which the rotor is supported by a gimbal structure. Nutation is simply a wobbling of the spin axis of the rotor. It is a self-sustaining oscillation which physically represents a continuous transfer of energy from one degree of freedom to the other and back again. In contrast to precession, this motion does not need any external torques to sustain it. This motion has a natural frequency $\omega_R$, commonly known as the nutation frequency, given by:

$$\omega_R = \frac{H}{\sqrt{I_{ig} I_{og}}} \tag{4.7}$$

where $H$ is the angular momentum of the rotor; $I_{ig}$ the moment of inertia of the rotor and inner gimbal about the inner axis and $I_{og}$ is the moment of inertia of the rotor, inner gimbal and outer gimbal about the outer axis.

In a frictionless system, nutation would be self-perpetuating. However, friction in the gimbal bearings or deliberately applied viscous drag damps out this undesirable motion. Energy dissipation varies in proportion to the nutation frequency. Therefore, in order to minimise the occurrence of nutation, it is required to increase $\omega_R$. It is usual for the rotor to have as large an angular momentum as possible, combined with gimbals having low moments of inertia. This is achieved through the use of light but stiff materials such as beryllium (alloy) in the construction of gimbals.

#### *4.2.2.5 Gimbal lock*

Gimbal lock is an effect which prevents a two-degrees-of-freedom gyroscope having 360° of freedom about both its inner and outer gimbal axes. Gimbal lock occurs when the spin axis of the rotor coincides with the outer gimbal axis owing to a 90° rotation about the inner axis. At this point, the gyroscope loses one degree of freedom. Application of motion about an axis perpendicular to the plane containing the outer gimbal causes the outer gimbal to spin. Once this spinning motion has begun, the spin axis of the rotor and the axis of the outer gimbal remain permanently coaxial. The only method of separating them is to stop the rotation of the inertial element to allow the two axes to be reset. This undesirable effect is prevented by using mechanical stops to limit the motion of the inner gimbal. These stops usually permit up to ±85° of motion by the inner gimbal. Gimbal lock can also occur in stable platforms with three gimbals.

#### *4.2.2.6 Tumbling*

Tumbling is a consequence of using mechanical stops to prevent gimbal lock. This phenomenon occurs when the inner gimbal hits one of the mechanical stops. This causes the outer gimbal to turn through 180° about its own axis. This motion of the outer gimbal is known as tumbling. Once tumbling occurs, the reference is lost.

### *4.2.3 Components of a mechanical gyroscope*

The basic components of mechanical gyroscopes are as follows:

1. *The instrument case*: The case in which the other elements are housed and which provides the structure by which the instrument is mounted in a vehicle.
2. *The rotor or inertial element*: This is essentially a flywheel rotated at high angular velocity. The rotor usually has the majority of its mass at the outer edge, as the moment of inertia is the sum of the products of the individual masses ($m_i$) and the square of their distance ($r_i$) from the axis of rotation, $\sum m_i r_i^2$. This enables a high angular momentum ($H$) to be achieved for a given angular velocity, since $H$ is the product of moment of inertia ($I$) and angular velocity ($\omega_S$) as described in the previous section. This approach also allows a high angular momentum to be achieved with the lowest overall mass. A low rotor mass is desirable to minimise vibration and shock effects.
3. *Gimbals*: These are support frames on which the rotor or another gimbal is mounted to isolate the rotor from rotational motion, by allowing freedom of angular movement of these frames about the rotor. In the case of a two-gimbal

sensor, the axes of rotation of the two gimbals and the rotor are arranged to be mutually orthogonal as shown in Figure 4.1.

4 *Pick-off*: This device is used to detect relative motion between the rotor and the gimbals or, in some cases, between the rotor and the instrument case. The pick-off produces an electrical signal, indicating the direction and amplitude of the motion from a reference position. There are three basic forms of pick-off technology commonly used with mechanical gyroscopes operating in torque re-balance mode:
   - moving coil – using a small receiver coil and an a.c. excitation coil, so that any relative movement between the two modifies the flux sensed by the receiver coil;
   - variable reluctance – the excitation and receiver coils are fixed to the case of the gyroscope, with a soft iron assembly attached to the moving component so that it is in the flux return path between the excitation and receiver coils. Motion of the soft iron components causes a change in its orientation in the excitation field, thereby modifying the return flux to the receiver coil;
   - capacitive – there is a stationary plate close to the rotor, or moving component, whilst the rotor acts as the other plate of the capacitor. Movement of the rotor about its input axis, or axes for a two axis sensor, causes a change in separation between the two plates of the capacitor and hence there is a change in capacitance.

   Open-loop gyroscopes, such as simple rate sensors, often use a potentiometer to sense angular displacement of a gimbal. Generally, this form of sensor is not used for navigation purposes.

5 *Torque motor or electromagnetic torquer*: When a gyroscope is used in a closed loop or torque re-balance mode, it is necessary to generate a torque on the rotor in order to return the rotor to the 'null', or zero, position. This is achieved using a torque generator, which usually takes one of two common forms:
   - Permanent magnet – this type relies on the interaction between the field generated by a permanent magnet and that of an electromagnetic coil. Particularly with single-axis sensors, a coil 'cup' is fixed to the moving element and the permanent magnet attached to the case. This has several advantages such as reducing the sensitivity to external magnetic fields and allowing the magnet to be outside of the flotation fluid. However, it does require a pair of flexible leads to the coil which can generate error torques. In general, as a result of other constraints, dynamically tuned gyroscopes have the opposite configuration with the permanent magnet fixed to the moving element.
   - Electromagnet – a soft iron component is attached to the sensing element and a coil is fixed to the case. When a current is applied to the coil, a magnetic field is produced that interacts with the soft iron producing a torque on the sensing element.

6 *Re-balance loop*: This is the term given to the electronic circuitry that receives and uses the signals from the pick-off assembly. It interprets these signals in terms of the current required in the torquer coils to return the inertial element to its 'null'

position. The re-balance loop electronics can either be analogue or digital. In the case of the analogue re-balance loop, a continuously variable current is passed through the coil to return the inertial element to its 'null' position. When there is no displacement, then there is no current flow. A digital re-balance system generates precision current pulses of particular duration to force the inertial element back to its 'null' position. With some implementations, pulses of equal amplitude but opposite sign are passed into the torque generator even when there is no displacement of the rotor. Imbalance in the number of pulses applied in each direction gives rise to a net torque.

7 *Spin motor*: This is the motor used to rotate the inertial element and give it the angular momentum that is vital for the operation of the mechanical gyroscope. Usually the spin motor is either a hysteresis motor or an inductive device. Some gyroscopes that have a short run-time use a blast of air or a small explosive charge to spin the inertial element, and for cheap and crude applications, a d.c. electric motor may be used.

8 *Float*: Rate-integrating gyroscopes, as discussed below in Section 4.2.5, have their rotor and spin motor sealed in a can that is immersed in a fluid to reduce the load on the gimbal bearings. This can, with its encapsulated components, is known as the float. Careful choice of the flotation fluid can reduce the load of the rotor assembly on the gimbal to zero. In such a design, bellows are used to compensate for changes in the volume of the fluid when the temperature inside the case of the gyroscope changes. The centre of buoyancy is arranged to be close to the centre of gravity of the float and along the output axis in order to minimise acceleration sensitive errors. There is further consideration of the effect of this on sensor performance in Section 4.2.4.

9 *Flotation fluid*: This is the fluid in the gyroscope that gives buoyancy to the float in a floated rate-integrating gyroscope. It also provides damping of the motion of the float which gives rise to the integration function for the single-axis rate-integrating gyroscope.

10 *Bearings*: For the spin axis of the rotor, most gyroscopes use ball bearings in a race with a retainer, and are chosen to have low noise characteristics. This form of bearing needs a suitable lubricant with the following characteristics:
- it should not separate into solid and liquid components;
- it should have small or negligible change in viscosity over the temperature range of the sensor;
- it should not leak out from the bearing;
- it should retain its physical and chemical properties for the required shelf-life of the gyroscope.

Lubricants can severely limit the environmental performance and shelf-life potential of a sensor. An alternative form of bearing that overcomes the well-known problems of 'ball bearings' is the gas bearing. This form of bearing can be either self-acting or externally pumped. In the former case, the bearing draws a gas, usually air, into a series of grooves which, owing to the viscosity of the fluid, supports the structure in the other part of the bearing. In the latter case, the fluid is pumped into the grooves to support the structure. The drawbacks with

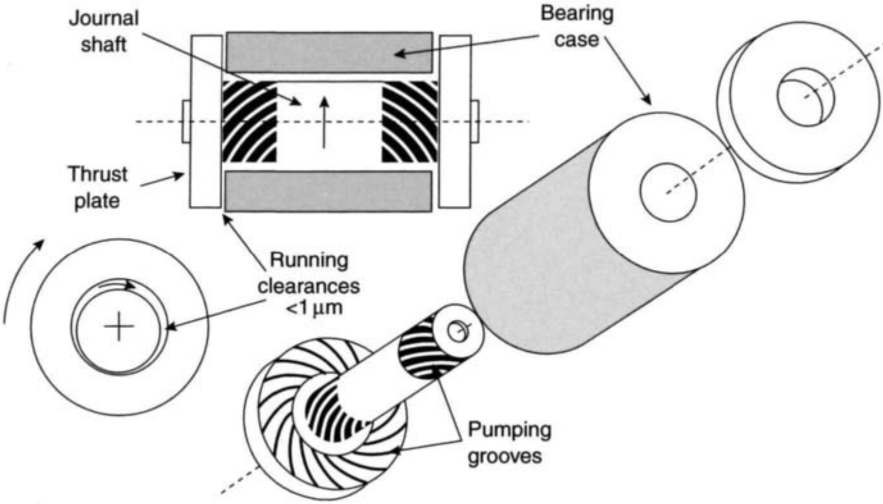


*Figure 4.5 Self-pumping gas bearing*

gas bearings are the need for very tight tolerances in manufacture, of the order of a micrometre or better, and the use of very hard materials such as boron carbide as the two bearing surfaces touch and rub during both starting and stopping of the rotor. However, they are very low noise bearings and can last a very long time, particularly if the bearing runs continuously. A schematic diagram of a self-pumping gas bearing is shown in Figure 4.5.

### *4.2.4 Sensor errors*

All gyroscopic sensors are subject to errors which limit the accuracy to which the angle of rotation or applied turn rate can be measured. Spurious and undesired torques, caused by design limitations and constructional deficiencies, act on the rotors of all mechanical gyroscopes. These imperfections give rise to precession of the rotor, which manifests itself as a 'drift' in the reference direction defined by the spin axis of the rotor. In a free gyroscope, that is, one which measures angular displacements from a given direction, it is customary to describe the performance in terms of an angular drift rate. For a restrained gyroscope, that is, one operating in a nulling or re-balance loop mode to provide a measure of angular rate, any unwanted torques act to produce a 'bias' on the measurement of angular rate.

The terms 'drift' and 'bias' are commonly used interchangeably. In this book, we reserve the term 'drift' for the motion of the spin axis in a free gyroscope, whereas 'bias' is used with 'nulled' sensors. In practice, the way in which the errors are quoted often depends on the accuracy band of the sensor rather than whether the gyroscope is used with its spin axis fixed in space or restrained in some way.

The major sources of error which arise in mechanical gyroscopes are itemised overleaf. Further details relating to specific types of gyroscope will be given later in the chapter where the physical effects which give rise to each type of error are discussed in more detail.

*Fixed bias*: This refers to the sensor output which is present even in the absence of an applied input rotation. It may be a consequence of a variety of effects, including residual torques from flexible leads within the sensor, spurious magnetic fields and temperature gradients, which produce such biases. The size of the bias is independent of any motion to which the gyroscope may be subjected and is sometimes referred to as the acceleration (or $g$)-independent bias. It is usually expressed in units of degrees per hour (°/h), or for the less accurate sensors in degrees per second (°/s).

*Acceleration-dependent bias (g-dependent bias)*: Biases which are proportional to the magnitude of the applied acceleration. Such errors arise in spinning mass gyroscopes as a result of mass unbalance in the rotor suspension, that is, non-coincidence of the rotor centre of gravity and the centre of the suspension mechanism. The relationship between these components of bias and the applied acceleration can be expressed by means of coefficients having units of °/h/$g$. In general, such terms relate accelerations in each of the principal axes of the gyroscope, that is, accelerations which act both along and orthogonal to the sensitive axis of the sensor, to errors in the measurement of turn rate. In the presence of a steady acceleration, a fixed bias in the measured rate occurs.

*Anisoelastic bias ($g^2$-dependent bias)*: Biases which are proportional to the product of acceleration along orthogonal pairs of axes. Such biases arise in spinning mass gyroscopes because the gyroscope rotor suspension structure, particularly the bearings, has finite compliances which are unequal in different directions. The anisoelastic coefficients have units of °/h/$g^2$.

*Anisoinertia errors*: Such errors arise in spinning mass gyroscopes and introduce biases owing to inequalities in gyroscope moments of inertia about different axes. Anisoinertia is frequency sensitive if the rotor is driven by a hysterisis motor. This is a consequence of the elastic coupling between the magnetic ring on the rotor and the rotating magnetic field. The resulting biases are proportional to the product of angular rates applied about pairs of orthogonal axes. The anisoinertia coefficients may be expressed in units of °/h/(rad/s)$^2$.

*Scale-factor errors*: Errors in the ratio relating the change in the output signal to a change in the input rate which is to be measured. Scale-factor error is commonly expressed as a ratio of output error to input rate, in parts per million (ppm), or as a percentage figure for the lower performance class of sensor. Additional errors arise as a result of scale-factor non-linearity and scale-factor asymmetry. Scale-factor non-linearity refers to the systematic deviations from the least-squares straight line or non-linear function fitted to the measurements, which relates the output signal to the applied angular rate. The latter term includes differences in the magnitude of the output signal for equal rotations of the sensor in opposite directions. In spinning mass gyroscopes, scale-factor non-linearity relates to thermal changes that result in changes of the magnetic flux.

*Cross-coupling errors*: Erroneous gyroscope outputs resulting from gyroscope sensitivity to turn rates about axes normal to the input axis. Such errors arise through non-orthogonality of the sensor axes and may also be expressed as parts per million or a percentage of the applied angular rate.

*Angular acceleration sensitivity*: This error is also known as the gyroscopic inertial error. All mechanical gyroscopes are sensitive to angular acceleration owing to the inertia of the rotor. Such errors become important in wide bandwidth applications. This error increases with increasing frequency of input motion. Hence, it is necessary to compensate for this error if accuracy is to be preserved. A detailed analysis is given by Edwards in Reference 4 for both the rate-integrating gyroscope and the dynamically tuned gyroscope, which are described in Sections 4.2.5 and 4.2.6.

It is important to realise that each of the errors described will, in general, include some or all of the following components:

- fixed or repeatable terms;
- temperature induced variations;
- switch-on to switch-on variations;
- in-run variations.

For instance, the measurement of angular rate provided by a gyroscope will include:

(i) a bias component which is predictable and is present each time the sensor is switched on and can therefore be corrected;
(ii) a temperature-dependent bias component which can be corrected with suitable calibration;
(iii) a random bias which varies from gyroscope switch-on to switch-on but is constant for any one run;
(iv) an in-run random bias which varies throughout a run; the precise form of this error varies from one type of sensor to another.

The fixed components of error, and to a large extent the temperature induced variations, can be corrected to leave residual errors attributable to switch-on to switch-on variation and in-run effects, that is, the random effects caused by instabilities within the gyroscope. Assuming that the systematic errors are compensated, it is mainly the switch-on to switch-on and in-run variations which influence the performance of the inertial system in which the sensors are installed. Compensation techniques are discussed further in Chapter 8.

A number of different types of mechanical gyroscope of interest in strapdown applications are now described.

### 4.2.5 Rate-integrating gyroscope

#### 4.2.5.1 Introductory remarks

The design of this type of mechanical gyroscope was conceived in the late 1950s for use on stabilised platforms, the early examples appearing at the start of the 1960s.

This basic concept is capable of achieving a wide spectrum of performance from a very small gyroscope that fits into a cylinder of diameter 25 mm (1 in.) and length 50 mm (2 in.). Typically, the drift performance of the miniature versions of this type of sensor is in the 1–10°/h class, although substantially better than 0.01°/h can be achieved with the larger 'top of the range' sensors. The smaller sensors are able to measure turn rates typically of the order of 400°/s or better. This type of sensor has found many different applications as a result of this wide spectrum of performance, including navigation systems in aircraft, ships and guided weapons.

#### *4.2.5.2 Detailed description of sensor*

A rate-integrating gyroscope has one input axis and so it is known as a single-axis gyroscope. Besides the case, it has three main component parts, as illustrated in Figure 4.6:

- the float, which contains the rotor and its motor. It is supported in precision bearings to allow rotation about an axis perpendicular to the spin axis of the rotor;
- the angle pick-off which senses rotation of the float assembly;
- the torque motor, which is used to apply precise torques to return the float to its 'null' position.

These components are sealed into a case and the small gap between the float and the case is filled with a highly viscous liquid. This liquid provides some support for the float in its bearings, thus reducing undesired torques, and, in some very

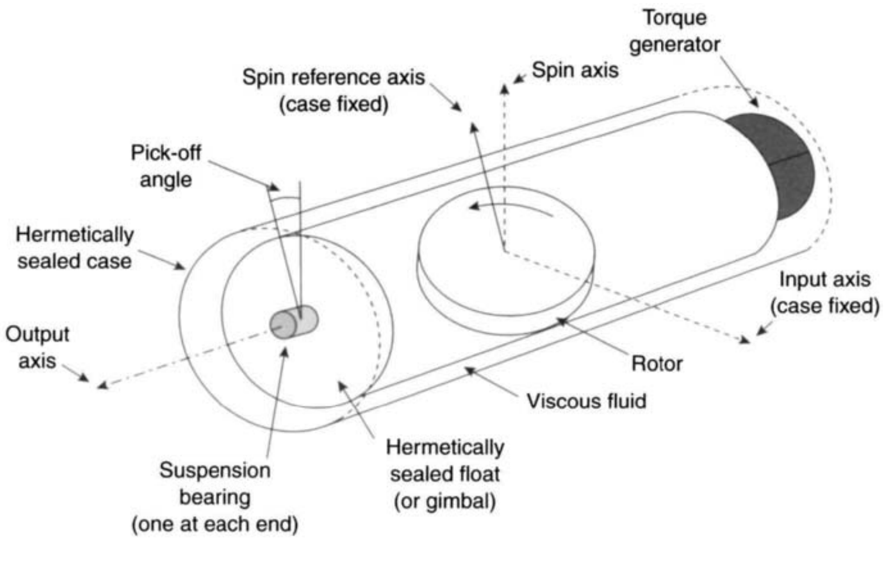


*Figure 4.6 Single degree-of-freedom rate-integrating gyroscope*

particular instances, it provides total buoyancy. The flotation fluid also provides viscous damping between the float and the case. Electrical signals and power are transmitted between the case and the float via delicate flexible (flex) leads.

When an angular rate is applied about the input axis, the float develops a precessional rate about the output axis shown in the figure. As a result of the damping fluid which supports the float, the output axis rate gives rise to a viscous torque about the output axis. This torque causes the float to precess about the input axis at the input rate and so follow the case rotation. The output axis rate therefore becomes proportional to the input rate. The gyroscope operates in this manner, as a precision rate-integrating gyroscope. In other words, the output which is sensed by the pick-off, is proportional to the integral of the input axis rate, that is, to the change in input angle.

If an additional torque is applied electrically via the torque motor, the pick-off angle rate becomes proportional to the difference between the input rate and the precessional rate induced by the torque motor. Hence, the pick-off angle becomes proportional to the integral of the difference between the input and torque motor rates. For strapdown operation, the pick-off angle is 'nulled' by feeding back the pick-off output to the torque motor. In this situation, the time integral of the difference between the input and torque motor rates becomes zero. It follows that the current applied to the torque motor to maintain the 'null' position is proportional to the applied input rate. This gyroscope is used as a closed loop sensor as this leads to a better definition of the input axis and more accurate measurement of rotation.

Figure 4.7 shows the components of a rate-integrating gyroscope in more detail.

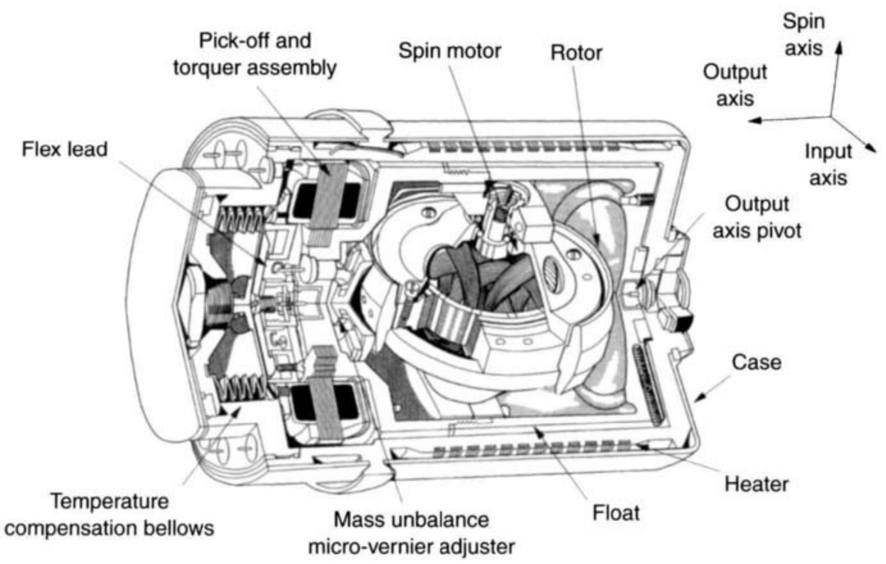


*Figure 4.7 Rate-integrating gyroscope*

#### 4.2.5.3 Sources of error

The major error processes that influence the performance of this type of gyroscope are shown outlined below:

*g-insensitive bias*, resulting from a variety of causes which include residual flex lead torques, thermal gradients across the sensor which result in fluid flow around the float assembly and pivot stiction.

*g-sensitive bias* caused by:

- mass unbalance of the float relative to the pivots of the gimbal along the spin motor axis – principally the result of rotor movement along the spin axis caused by spin motor bearing compliance.
- mass unbalance of the float along the input axis.

*anisoelastic bias*, which results from unequal compliance of the gyroscope's float assembly along the input and spin axes.

*scale-factor error*, caused by imperfections and temperature fluctuations in the pick-off and nulling components, which may be expressed as the sum of a 'fixed' error and a set of non-linear components.

*cross-coupling*, which arises through imperfections in the construction of the sensor.

*zero-mean random bias*, caused by instabilities in the gyroscope which have short correlation times, variations in pivot friction and random movements of the rotor along the spin axis, for instance.

This sensor is intended to measure angular rates, but unfortunately it is also sensitive to linear and angular accelerations and vibrations and these can give rise to errors in measurements. Careful shielding is required to eliminate errors resulting from stray magnetic fields interacting with the torque generator. Changes in temperature alter the characteristics of the magnetic materials within the sensor. Without at least approximate compensation, these changes in temperature give rise to scale factor errors. Generally, heating effects in conjunction with magnetic imperfections give rise to first, second and third order scale-factor errors. The significant error sources are usually systematic and can be readily corrected.

The angular rate measurement ($\tilde{\omega}_x$) provided by a rate-integrating gyroscope may be expressed in terms of the true input rate and the error terms as follows:

$$\tilde{\omega}_x = (1 + S_x)\omega_x + M_y\omega_y + M_z\omega_z + B_{\mathrm{f}x} + B_{gx}a_x + B_{gz}a_z + B_{\mathrm{a}xz}a_xa_z + n_x \tag{4.8}$$

where $\omega_x$ is the turn rate of the gyroscope about its input axis; $\omega_y$ and $\omega_z$ are the turn rates of the gyroscope about its output and spin axes, respectively; $a_x$ and $a_z$ are the accelerations of the gyroscope along its input and spin axes, respectively. $B_{\mathrm{f}x}$ is the $g$-insensitive bias, $B_{gx}$, $B_{gz}$ are the $g$-sensitive bias coefficients, $B_{\mathrm{a}xz}$ is the anisoelastic bias coefficient, $n_x$ is the zero-mean random bias, $M_y$, $M_z$ are the cross-coupling coefficients and $S_x$ is the scale-factor error which may be expressed as a polynomial in $\omega_x$ to represent scale factor non-linearities.

#### 4.2.5.4 *Typical performance characteristics*

Typical $1\sigma$ values for the major error sources are:

| | |
|---|---|
| $g$-Independent bias | 0.05–10°/h |
| $g$-Dependent/mass unbalance bias | 1–10°/h/$g$ |
| Anisoelastic bias | 1–2°/h/$g^2$ |
| Scale-factor error (uncompensated temperature effects) | up to 400 ppm/°C |
| Scale-factor non-linearities (at high rotation rates) | 0.01–0.1% |
| Bandwidth | up to 60 Hz |
| Maximum input rate | up to 400°/s |

In certain applications, other systematic error effects may become important, but generally, those given above are dominant.

### 4.2.6 *Dynamically tuned gyroscope*

#### 4.2.6.1 *Introductory remarks*

This sensor is sometimes also called the tuned rotor gyroscope, or dry tuned gyroscope. It has two input axes which are mutually orthogonal and which lie in a plane which is perpendicular to the spin axis of the gyroscope. Work to demonstrate this form of technology was underway at the Royal Aircraft Establishment, Farnborough (now the DSTI and QinetiQ) by Philpot and Mitchell [5] during the late 1940s. Although demonstration of the tuning phenomenon took place in the early 1950s, it is only since the 1970s that this type of gyroscope has been fully developed. The original concept was developed for stabilised platform applications, but has been applied to strapdown systems since the mid- to late 1970s in many types of vehicle.

Generally, the performance of these gyroscopes is very similar to that achieved by the rate-integrating gyroscope. Miniature instruments of this type developed for strapdown applications are typically about 30 mm in diameter and 50 mm in length. Sub-miniature devices have also been produced, with some slight degradation in performance, which are about 20 mm by 25 mm. These gyroscopes have found many applications similar to the floated rate-integrating gyroscope.

#### 4.2.6.2 *Detailed description of sensor*

The sensor consists of three major sub-assemblies as indicated in Figure 4.8:

(i) the body block, which consists of the spin motor and angle pick-off arrangement;
(ii) the rotor assembly, which also includes the torque generator magnets and the Hooke's joint suspension;
(iii) the case and torque generator coil assembly.

The rotor is connected to the drive shaft by a pair of flexure hinges to an inner gimbal ring. This inner 'gimbal' is also connected to the drive shaft by a pair of

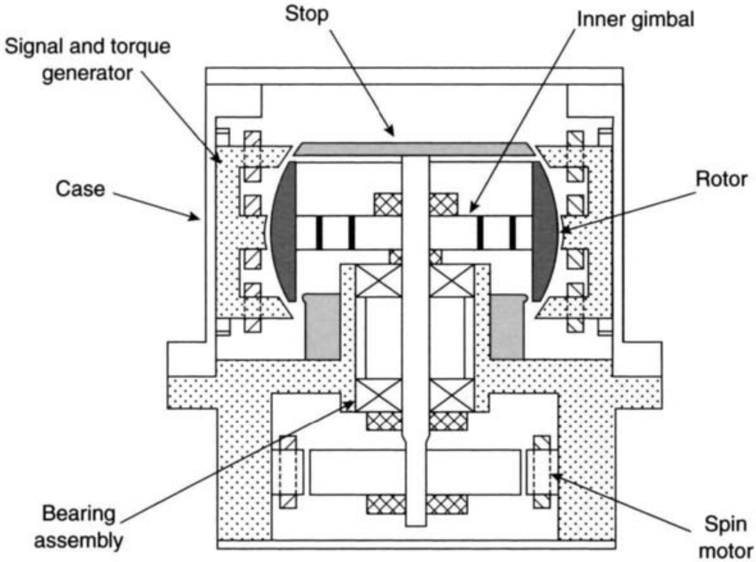


*Figure 4.8 Typical tuned rotor gyroscope configuration*

flexure hinges, the two axes of freedom being orthogonal as illustrated schematically in Figure 4.9. This is often called a Hooke's joint or a Cardan joint and allows torsional flexibility. This is an internal type of gimbal and is far more compact than the external gimbal shown in Figure 4.1. At the other end of the drive shaft is a synchronous motor.

Rotation of the gimbal causes a reaction at the rotor that is equivalent to a negative torsional spring stiffness. This effect occurs when the angular momentum of the shaft does not coincide with that of the rotor, the angular momentum of the gimbal jumping between that of the shaft and the rotor, at twice the speed of the rotor. Thus careful selection of the torsional stiffness of the gimbal components and the rotational speed of the rotor, allows the rotor suspension to have a net zero spring stiffness at a particular rotor speed, known as the tuned speed. Under these conditions, the rotor is decoupled from the motion of the rest of the sensor and hence is 'free'. In practice, this condition is usually adjusted or trimmed by the use of screws set into the inner gimbal ring that allow minor changes in the mass properties of the gimbal.

Normally, the decoupling of the rotor is not complete or perfect and residual elastic restraints restrict the useful angular range of movement of the rotor. Therefore, the sensor is usually used in a torque re-balance mode allowing only very small deflections of the rotor. Deflections of the rotor are sensed about two orthogonal axes, and are directly proportional to the motion of the gyroscope case about the respective axes in inertial space.

A figure of merit [6] is sometimes used for describing the quality of a dynamically tuned gyroscope. The figure of merit relates the inertias of the rotor to the inertias of

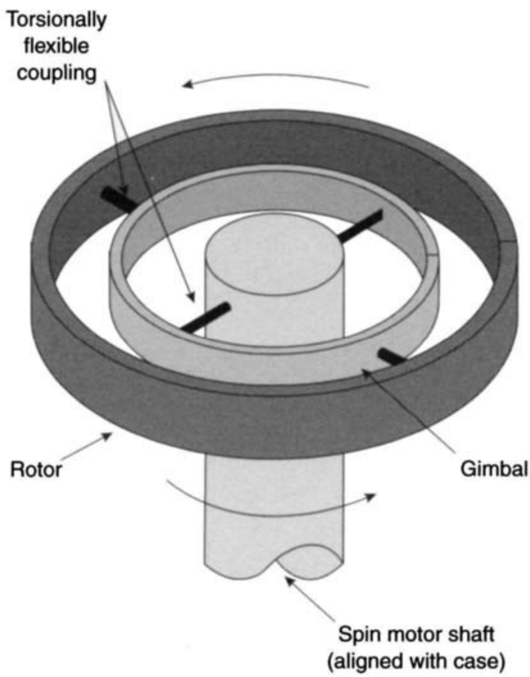


*Figure 4.9 Dynamically tuned gyroscope rotor and drive shaft assembly*

the gimbal, given as

$$\text{figure of merit} = \frac{C}{I_g + J_g - K_g}$$

where $C$ is the spin inertia of the rotor, $I_g$, $J_g$ are the gimbal transverse inertias and $K_g$ is the gimbal polar inertia.

Typical values for figure of merit for a moderate performance instrument are in the region of 50.

### *4.2.6.3 Sources of error*

It will be noticed that the errors are of similar form to those given for the rate-integrating gyroscope errors.

*g-Insensitive bias*, principally the result of stray internally generated magnetic fields which interact with the torque motor magnet mounted on the rotor plus re-balance loop biases. The effects of tuning errors and gimbal damping are often included in this error.

*g-Sensitive bias*, caused by mass unbalance of the rotor assembly and geometrical imperfections in the torsional elements. The flexures can also generate a torque when loaded axially, leading to an acceleration sensitive bias about the axis opposite to that axis along which the acceleration is acting. This is known as quadrature mass unbalance.

*Anisoelastic bias*, which results from unequal compliance of the rotor assembly in the $x$-, $y$- and $z$-directions.

*Anisoinertia bias*, results from differences in rotor inertias in the $x$-, $y$- and $z$-directions, and is frequency sensitive.

*Scale-factor errors*, mainly caused by thermally induced changes in magnets and coils used in the re-balance system.

*Zero-mean random bias*, caused, for example, by error torques resulting from changes in spin motor–shaft orientation owing to variations in the bearing pre-load.

As in the case of the rate-integrating gyroscope, this sensor is sensitive to linear and angular accelerations, vibratory motion, stray magnetic fields and temperature changes, all of which give rise to errors in measurements. This type of sensor is sensitive to vibrations at integer multiples of the spin speed, not only at the spin frequency, as in the single degree of freedom gyroscope, but also vibrations at twice this frequency. Vibration about the input axis interacts with the gimbal angular momentum, and is rectified to give a fixed bias.

The angular rate measurements provided by the sensor ($\tilde{\omega}_x$ and $\tilde{\omega}_y$) may be expressed mathematically as follows:

$$\begin{aligned}\tilde{\omega}_x &= (1+S_x)\omega_x + M_y\omega_y + M_z\omega_z + B_{fx} + B_{gx}a_x + B_{gy}a_y + B_{axz}a_xa_z + n_x\\ \tilde{\omega}_y &= (1+S_y)\omega_y + M_x\omega_x + M_z\omega_z + B_{fy} + B_{gy}a_y - B_{gx}a_x + B_{ayz}a_ya_z + n_y\end{aligned} \tag{4.9}$$

where $\omega_x$ and $\omega_y$ are the turn rates of the gyroscope about its input axes, $a_x$ and $a_y$ are the accelerations along its input axes and $a_z$ is the acceleration along its spin axis. $B_{fx}$, $B_{fy}$ are the $g$-insensitive bias coefficients, $B_{gx}$, $B_{gy}$ are the $g$-sensitive bias coefficients, $B_{axz}$, $B_{ayz}$ are the anisoelastic bias coefficients, $n_x$, $n_y$ represent the zero-mean random bias, $S_x$, $S_y$ are the scale-factor errors and $M_x$, $M_y$, $M_z$ denote the cross-coupling coefficients.

### 4.2.6.4 *Typical performance characteristics*

Typical values for the significant error sources and performance parameters are given below:

| | |
|---|---|
| $g$-Independent bias | 0.05–10°/h |
| $g$-Dependent/mass unbalance bias | 1.00–10°/h/$g$ |
| Anisoelastic bias | 0.1–0.5°/h/$g^2$ |
| Scale-factor error (uncompensated temperature effects) | up to 400 ppm/°C |
| Scale-factor non-linearities (at high rotation rates) | 0.01–0.1% |
| Bandwidth | up to 100 Hz |
| Maximum input rate | up to 1000°/s |

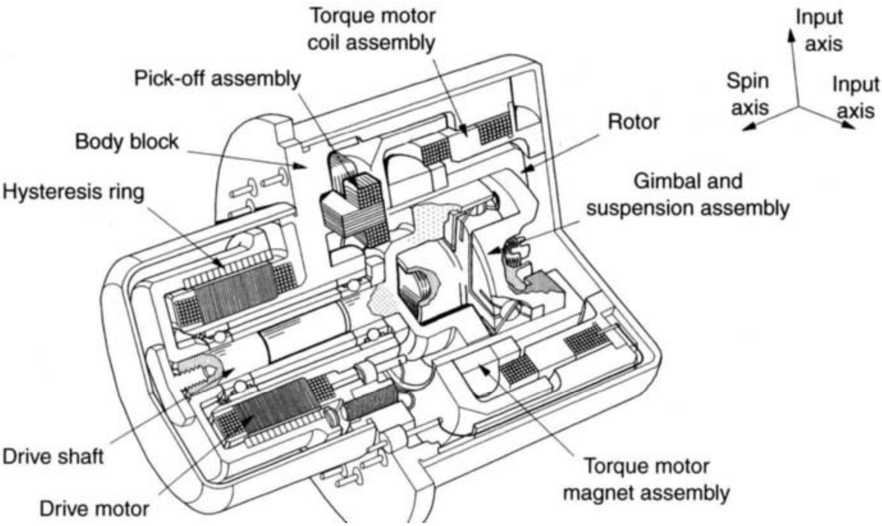


*Figure 4.10 Sectional diagram of a dynamically tuned gyroscope*

The dynamically tuned gyroscope offers a number of significant advantages for many applications, when compared with the rate-integrating gyroscope. These are usually quoted as fewer parts, a fluid free suspension, no flex lead torques, simplified spin motor bearing design and a fast warm up characteristic. Of course, it offers the ability to measure angular motion about two axes, and additionally, its construction allows the sensor either to be re-worked more easily, or to have its performance optimised before final sealing of the case. One potential drawback is its susceptibility to disturbances and oscillations at the tuned frequency and harmonics of this frequency. Its suspension is analogous to a mass on a spring. For this reason, careful design is required to ensure that mechanical resonances do not interact with the suspension and destroy it. For reliable performance in a harsh environment, careful design of the suspension and mounting is crucial. The rate-integrating gyroscope is generally more resilient in this type of environment owing to its inherently rugged design. Figure 4.10 shows a typical arrangement of the various components of a dynamically tuned gyroscope.

Miniature instruments of this type developed for strapdown applications are typically about 30 mm in diameter and 50 mm in length. Sub-miniature devices have also been produced which are about 20 mm by 25 mm in diameter (see Figure 4.11).

## 4.2.7 *Flex gyroscope*

### 4.2.7.1 *Introductory remarks*

This sensor bears a close resemblance to the dynamically tuned gyroscope and operates in a similar manner, as the rotor acts as a free inertial element. It also has two


*Figure 4.11 Photograph of a modern dynamically tuned gyroscope (published courtesy of Northrop Grumman Corporation, Litton Systems)*

sensitive input axes. Development of this inertial instrument has progressed dramatically since the mid-1970s. The form of construction allows a very small instrument to be made, typically about 20 mm in diameter and 30 mm in length. These sensors have found many applications in aerospace and industrial applications.

#### *4.2.7.2 Detailed description of sensor*

The major difference in construction between the flex gyroscope and the dynamically tuned gyroscope is that the flex device does not have a Hooke's joint type of flexure pivot arrangement but has a flexible pivot where the drive shaft is reduced in diameter, as shown in Figure 4.12.

The rotor is attached to the main shaft usually using a spider and strut arrangement. Flexible joint torques arising from this form of suspension are compensated by small permanent magnets attached to the rim of the rotor which attract a set of high permeability screws mounted on a plate attached to the shaft. This use of magnetic forces to balance the flex pivot torques has the effect of decoupling the rotor from the drive shaft, as depicted in Figure 4.12. Generally, magnetic shielding is crucial with this sensor to ensure effective decoupling of the rotor. A schematic diagram of such a sensor is given in Figure 4.13.

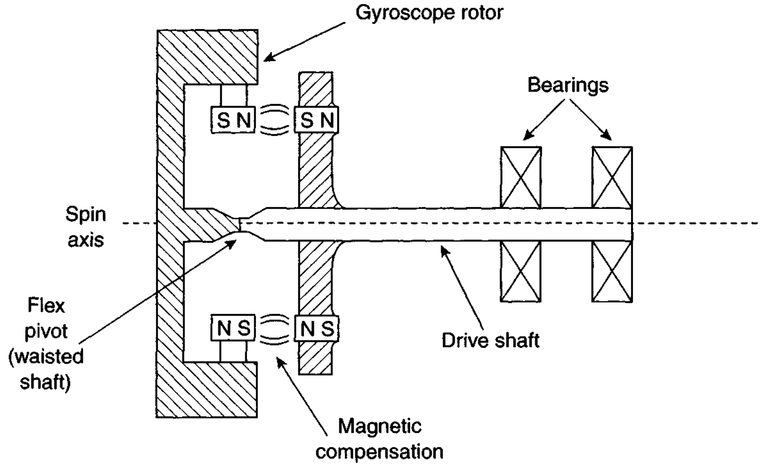


*Figure 4.12 Shaft assembly of a flex gyroscope*

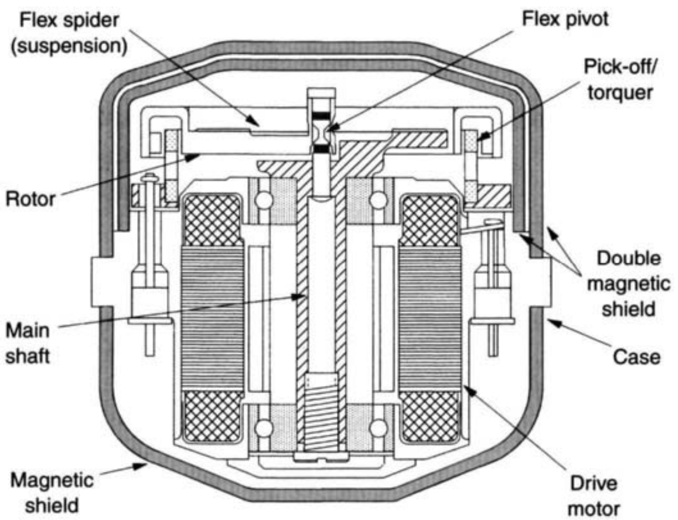


*Figure 4.13 Flex gyroscope*

### *4.2.7.3 Sources of error*

The error mechanisms associated with this sensor are very similar to the dynamically tuned gyroscope described above and will not be repeated here. The outputs may be expressed mathematically in the same form as those for the dynamically tuned gyroscope in eqn. (4.9). Use of magnetic tuning allows the option of running the rotor at different speeds to fulfil different needs or applications. Additionally, this type of suspension gives very good resilience to vibratory inputs.

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#### 4.2.7.4 Typical performance characteristics

Typical values for the significant error sources and performance parameters are given below:

| | |
|---|---|
| $g$-Independent bias | 1–50°/h |
| $g$-Dependent/mass unbalance bias | 1–10°/h/$g$ |
| Anisoelastic bias | 0.05–0.25°/h/$g^2$ |
| Scale-factor error (uncompensated temperature effects) | up to 400 ppm/°C |
| Scale-factor non-linearities (at high rotation rates) | 0.01–0.1% |
| Bandwidth | up to 100 Hz |
| Maximum input rate | >500°/s |

It can be seen that the error parameters are very similar to those quoted for the dynamically tuned gyroscope in Section 4.2.6.4. Typically, the drift performance of such a device is in the range 1–50°/h with the capability to capture rotation rates up to at least 500°/s. Additionally, the anisoelasticity is often slightly smaller, typically by a factor of 2–5.

## 4.3 Rate sensors

There is a class of mechanical sensors designed to sense angular rate using various physical phenomena which are suitable for use in some strapdown applications. Such devices resemble conventional gyroscopes in that they make use of the principles of gyroscopic inertia and precession described in Section 4.2.2. They are suitable for some lower accuracy strapdown applications, particularly those that do not require navigational data, but stabilisation. These devices tend to be rugged and to be capable of measuring rotation rates up to about 500°/s with typical drift accuracies of a few hundred degrees per hour. A number of devices of this type are discussed in the following sections.

### 4.3.1 Dual-axis rate transducer (DART)

#### 4.3.1.1 Introductory remarks

Development of this type of gyroscope started in the United States during the 1960s. It has, as its name implies, the ability to sense angular rate about two orthogonal axes. Its basic performance is certainly sub-inertial, typically having a drift in the region of 0.5°/s or less. Its size is somewhat smaller than the rate-integrating gyroscope being about 18 mm in diameter and 40 mm in length.

#### 4.3.1.2 Detailed description of sensor

The inertial element in this form of gyroscope is a sphere of heavy liquid, such as mercury, contained in a spherical cavity. This cavity is rotated at high speed about an axis along the case in order to give high angular momentum to the fluid sphere. There