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Author(s): David P. Baron and John A. Ferejohn

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## BARGAINING IN LEGISLATURES

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DAVID P. BARON  
JOHN A. FEREJOHN  
*Stanford University*

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**B**argaining in legislatures is conducted according to formal rules specifying who may make proposals and how they will be decided. Legislative outcomes depend on those rules and on the structure of the legislature. Although the social choice literature provides theories about voting equilibria, it does not endogenize the formation of the agenda on which the voting is based and rarely takes into account the institutional structure found in legislatures. In our theory members of the legislature act noncooperatively in choosing strategies to serve their own districts, explicitly taking into account the strategies members adopt in response to the sequential nature of proposal making and voting. The model permits the characterization of a legislative equilibrium reflecting the structure of the legislature and also allows consideration of the choice of elements of that structure in a context in which the standard, institution-free model of social choice theory yields no equilibrium.

**L**asswell (1968) suggested that the fundamental question of politics is how the gains from a bargaining situation are to be distributed among members who have differing and sometimes conflicting preferences. Bargaining in legislatures reflects the structure of the legislature and the formal rules that govern agenda formation and voting. Shepsle (1979) models this structure and characterizes an equilibrium based on a concept of stability drawn from the field of social choice. We take a strategic approach to bargaining and study a stylized model of distributive of expenditure policy in a unicameral, majority rule legislature not favoring any member of the legislature or any particular outcome. For this distributive problem, Arrow's (1951) impossibility theorem implies that in the framework of social choice theory there is no voting equilibrium. The absence of an equilibrium is a consequence of the opportunity to pit simultaneously and cost-

lessly any alternative against every other alternative. In contrast to this institutionless setting, the theory presented here reflects the sequential nature of proposal making, amending, and voting, and models it as a noncooperative, multisession game. Noncooperative equilibrium can be shown to exist, and the legislative outcomes reflect the institutional structure of both the agenda formation process and the voting mechanism. Indeed, if the legislature is not too small, any distribution of the benefits can be supported as an equilibrium; that is, where social choice theory finds no equilibrium, every distribution is an equilibrium when the sequential structure of the legislative process is taken into account. These equilibria, however, are supported by complex punishment strategies that are unlikely to be self-enforcing in a large legislature with turnover. We thus focus on stationary equilibria, which have a natural focal point property.

One issue to be investigated is whether

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the legislative equilibrium reflects the majoritarian nature of the voting rule, as predicted, for example, by Riker's (1962) theory of minimal winning coalitions or whether benefits are distributed universally, as predicted by Weingast (1979). As might be expected, the answer depends on the amendment rule employed. Under a closed rule a majoritarian outcome results, and within the majority the author of the winning proposal receives between half and two-thirds of the benefits. Under an open rule, the outcome can be universalistic if the legislature is small and the members find delay in reaching an outcome to be very costly. In large legislatures, however, majoritarian outcomes result under an open rule, but the distribution among the winning majority is more equal than under a closed rule. An open rule can also result in delay in reaching a resolution, whereas the legislature completes its task in the first session under a closed rule. With respect to the choice of an amendment rule by the legislature, if delay is costly to the members of the legislature, the members will unanimously prefer that the legislature operate under a closed rule.

Since ours is a multilateral bargaining model, it is useful to contrast our results with those obtained in bilateral models. Rubinstein (1982) and Binmore (1986) find a unique equilibrium in bilateral bargaining models in which the first proposal is accepted when information is complete, players are impatient, and the rules governing who may make and accept offers are exogenous.<sup>1</sup> Legislative choice differs significantly from bilateral exchange in several respects. First, bilateral exchange requires unanimous consent for an outcome, and this requirement gives each party veto power that is reflected in the equilibrium outcomes. In a majority rule legislature, no member possesses veto power. Second, in bilateral bargaining, if agents are identical and make alternating offers, equilibrium distributions approach

equality as impatience diminishes, as Rubinstein (1982) showed.<sup>2</sup> In the legislative model considered here, majority rule equilibria do not generally tend to equal distributions, and equilibria in which some members receive nothing may occur even as impatience goes to zero. Third, in bilateral and multilateral bargaining with complete information and a unanimity rule, the equilibria are such that the first offer is always accepted. In a majority rule legislature in which amendments can be made, the first offer is rejected with positive probability in a stationary equilibrium when impatience is small. Furthermore, the probability is positive that the legislature will not reach an outcome in a finite number of sessions.

Endogenous agenda formation in legislatures has recently received renewed attention. Miller (1986) analyzes a specialized model with two competing agenda setters, but his model does not capture important features of real legislatures. Ferejohn, Fiorina, and McKelvey (1987) consider a model in which agendas are formed by members of a voting body proposing alternatives freely and voting only when an equilibrium agenda has been formed. They show that with certain special restrictions on the set of alternatives, common institutional rules about proposal making yield agenda equilibria and unique voting outcomes, but generalizations within this framework seem difficult to achieve. Banks and Gasmi (1987) consider a model of endogenous agenda formation in a legislature with three members who have Euclidean preferences and may make one or two amendments to the proposal made by the member initially recognized. Their model is sequential, as is the model considered here; but their legislature has only one session and thus differs from the sequential bargaining framework employed here. Harrington (1986) studies an  $n$ -member legislature in which a member is chosen at random to make a proposal

that must be accepted or rejected by an  $\alpha$ -majority. The legislature meets for a finite number of sessions and members are not impatient. The structure of Harrington's model is similar to that considered here in the "Illustration" below, although his focus is on the effects on the distribution of risk aversion and of the required majority. Finally, Epplé and Riordan (1987) consider a model in which a sequence of distributions is to be chosen. They employ a simplified institutional structure that resembles that studied in the third and fourth sections here, and their interest is in demonstrating that all allocations can be supported as subgame-perfect equilibria.

The model of the legislature is introduced along with an example that illustrates the equilibrium concept and the influence of legislative structure. We then show that with sufficiently little impatience on the part of members, any outcome can be supported as a subgame perfect equilibrium. Stationary equilibria are discussed, and a stationary equilibrium is characterized for a legislature governed by a closed rule that prohibits amendments, then for a legislature governed by an open rule that permits amendments. We then consider the *ex ante* choice of an amendment rule and offer conclusions.

### The Model of a Legislature

#### Legislative Structure

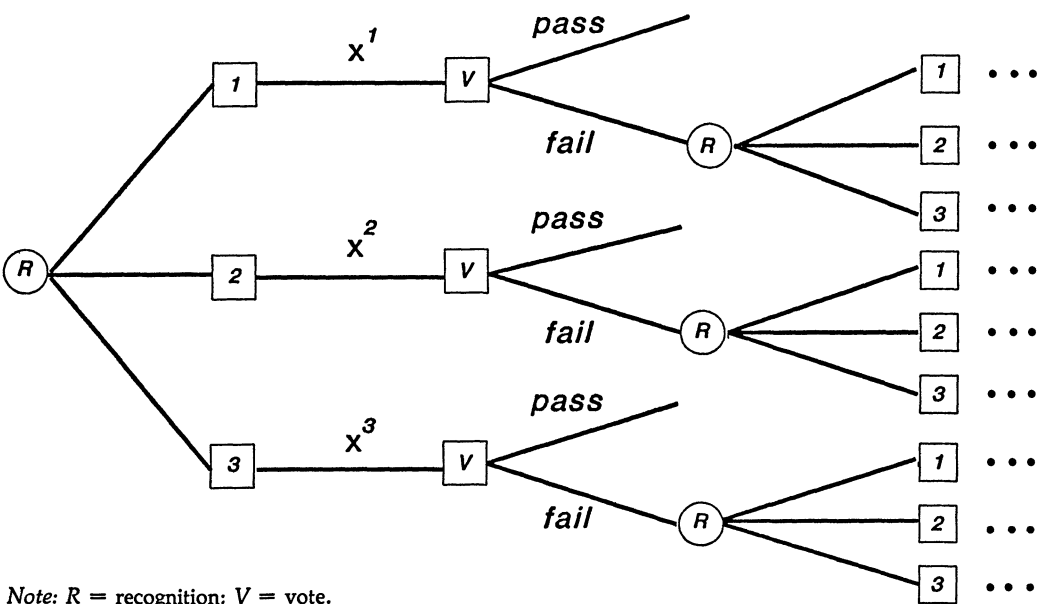
The model is intended to reflect in a stylized manner the endogenous nature of proposal making and voting, and to allow a comparison between amendment rules used to govern when an amendment can be made and when a proposal is brought to a vote. The legislature consists of (1)  $n$  members, each of whom represents a legislative district, (2) a recognition rule that determines which member may make a proposal, (3) an amendment rule, and

(4) a voting rule. The "members" could be either individuals or unified blocks of legislators who have the same preferences.

The task before the legislature is to choose a nonnegative distribution of one unit of benefits among the districts according to majority rule, with no side payments outside the legislature permitted. The task of the legislature in the model considered here thus corresponds to situations in which benefits can be distributed among districts in any manner at the discretion of the legislature. Each member is assumed to have risk-neutral preferences that depend only on the benefits distributed to the member's district, so the legislative task is one in which members have conflicting preferences and in which there is no majority rule equilibrium in the standard social choice framework. Preferences and the legislative rules are assumed to be common knowledge, and all actions are observable; so the model involves perfect information.

The legislature is governed by a recognition rule that identifies a member who is recognized to make a proposal or to bring the legislature to a vote. Since recognition is valuable in equilibrium, every member will attempt to be recognized; thus the legislature must have a rule for deciding who shall be recognized. In order not to bias the results in favor of any member or in favor of a particular amendment rule, the recognition rule is chosen to be neutral, which requires that it select a member at random to make a proposal.<sup>3</sup> Thus at the beginning of a legislative session, member  $i$  has a probability  $p_i$  of being recognized, and if recognized, make a proposal that specifies how the benefits are to be distributed. This proposal is then the motion on the floor. A proposal is a distribution  $x^i = (x_1^i, \dots, x_n^i)$ , such that  $\sum_{j=1}^n x_j^i \leq 1$ , so the set  $X$  of feasible proposals is an  $n$ -dimensional simplex. The status quo corresponds to no allocation of the benefits.

Figure 1. Legislative Process: Closed Rule



Note: R = recognition; V = vote.

The resolution of the motion on the floor depends on the amendment rule employed by the legislature. Under a *closed rule* the motion is voted on immediately (against the status quo); and if it is approved, the legislative adjourns. If the motion fails, the status quo prevails with the benefits remaining unallocated. The legislature has no means of committing not to consider distributing the benefits in the next session. Thus, when the legislature moves to the next session, the process repeats, with a member recognized to make another proposal, and so on. The legislative process under a closed rule is illustrated in Figure 1.

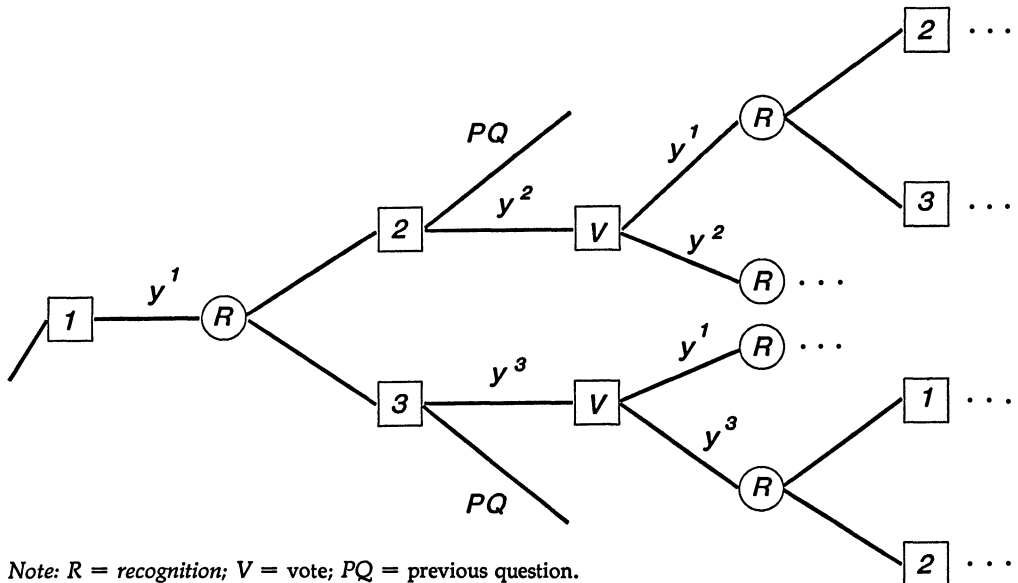
Under an *open rule* amendments can be offered to the motion on the floor, where an amendment is in the nature of a substitute and hence is an element of  $X$ ; that is, in the same session another member  $j$  ( $j \neq i$ ) may be recognized with probability  $p_j / \sum_{k \neq i} p_k$  and may either offer an amendment to the proposal or move the

previous question. The term *previous question* is used here to indicate that the amending process is concluded and a vote is required on the motion on the floor. This differs somewhat from congressional usage and perhaps corresponds most closely to the House's rising from the Committee of the Whole to vote on amendments and bills.<sup>5</sup> If the previous question is approved, the legislature votes on whether to accept the proposed distribution of the benefits. Because a member would never move the previous question unless he or she anticipated that the motion on the floor would be adopted, the vote on the previous question need not be incorporated in the model. Thus, moving the previous question is equivalent to bringing the motion on the floor to an immediate vote. If that motion is approved, the legislature adjourns.

If the member recognized does not move the previous question, the member offers an amendment, which is itself a

# Bargaining in Legislatures

Figure 2. Legislative Process: Simple Open Rule



Note: R = recognition; V = vote; PQ = previous question.

proposed distribution of the benefits and may be viewed as a substitute for the motion on the floor. To facilitate the analysis, a simple open rule will be considered allowing no more than one amendment to be on the floor at any time;<sup>6</sup> that is, once an amendment is offered, it must be disposed of by bringing it to an immediate vote against the proposal on the floor.<sup>7</sup> Given that amendments are in the nature of a substitute, such a restriction seems natural. If the amendment fails, the proposal remains on the floor, and the next session commences. If the amendment obtains a majority, the amendment becomes the proposal on the floor, and the legislature continues to the next session. At this point, a member is recognized who may offer another amendment or may move the previous question on the proposal on the floor. The process continues with members proposing amendments or moving the previous question until the proposal or one of the amendments has

defeated the motion, if any, on the floor and has been passed upon movement of the previous question. The structure of the legislative process under an open rule is illustrated in Figure 2.

In contrast to models in which the agenda is exogenously imposed, members in the legislature considered here must form expectations as to which future proposals will be made and how members will vote on them. If a member fails to vote for the proposal on the floor, that member runs the risk that in the next session a proposal could be passed allocating no benefits to his or her district. This provides an incentive for the member to vote for the proposal on the floor if it provides an allocation to the member's district at least as great as can be expected from future legislative sessions. Because the legislature may not distribute the benefits in the first session, the time preference of members may play a role in legislative behavior. Members are thus assumed to have a common discount factor  $\delta \leq 1$ ,



which may reflect the political imperative resulting from reelection concerns to distribute the benefits sooner rather than later. The parameter may also represent the probability that the member will be returned to office in the next election; when  $\delta$  is interpreted in this manner, the member is assumed to have preferences only for the benefits he or she delivers to the district. The preferences of member  $j$  are thus represented by the utility function  $u^j(x^k, t) = \delta^t x_t^k$ , where  $t$  is the session in which the legislature adopts the distribution  $x^k$ .

The process of proposal generation and voting yields an extensive form game with an infinite game tree. A *history*  $h_t$  of the game up to time  $t$  is a specification of who had a move at each time, the move selected by each member every time a move was allowed, and the vote when a vote was required. A pure strategy  $s_t^i$  at time  $\tau$  is a prescription of what motion to make when a member is recognized and of how to vote whenever a vote is required; that is, if  $H_\tau$  denotes the set of histories, a pure strategy is  $s_t^i : H_\tau \rightarrow X$  if  $\tau$  is the beginning of a session and  $i$  is recognized and is  $s_t^i : H_\tau \rightarrow \{\text{yes}, \text{no}\}$  if  $\tau$  is a time to vote, where  $\{\text{yes}, \text{no}\}$  are the two voting alternatives. A *strategy*  $s^i$  of member  $i$  is a sequence of functions  $s_t^i$  mapping  $H_\tau$  into that member's available actions at time  $\tau$ . A randomized strategy  $\sigma_\tau^i$  at time  $\tau$  is a probability distribution over the strategies  $s_t^i$  available at time  $\tau$ . An important feature of this model is that whenever a member is to take an action, he or she knows which history has occurred; so the game is one of perfect information.

Members are not able to make binding commitments either to vote in a particular manner or to offer a particular proposal. Thus, an equilibrium strategy must be self-enforcing in the sense that the member would wish to execute it at each point in the game at which an action can be taken. Therefore, the equilibrium is required to be subgame-perfect. An equilib-

rium configuration of strategies is subgame-perfect if the restriction of those strategies to any subgame constitutes a Nash equilibrium in that subgame. For any particular subgame-perfect equilibrium, the *value*  $v(t, g)$  of subgame  $g$  after  $t$  sessions is defined as the vector of values  $v_i(t, g)$ , with  $i = 1, \dots, n$ , to the members that results from the play of that subgame-perfect equilibrium strategy configuration. The *continuation value*  $\delta v_i(t, g)$  is the value if the legislature moves to subgame  $g$ . The ex ante value at the beginning of the game is denoted by  $v_i$ ,  $i = 1, \dots, n$ .

Whenever a vote is required, voting takes place sequentially and in public; that is, there is a fixed order in which each member must announce how he or she will vote on the question before the body and every other member may observe each vote as it is cast, so members always know the entire history of play whenever they must take an action. Because of the "coarseness" of voting rules—the fact that if a proposal wins by more than one vote no member can change the outcome by changing this vote—we restrict attention to a refinement of the subgame-perfect equilibria, in which weakly dominated strategies are eliminated. Because votes and proposals occur sequentially and openly, the weakly undominated subgame-perfect equilibria correspond to dominant-solvable solutions.<sup>8</sup> (For simplicity's sake, we shall no longer explicitly refer to the elimination of weakly dominated strategies.)

#### An Illustration: A Closed Rule and a Finite Number of Sessions

To illustrate the basic structure of the model, consider the simple case of an  $n$ -member, two-session legislature governed by majority rule, with equal probabilities of recognition and in which a closed rule prohibits amendments once a proposal has been made. In this setting, if

by the end of the second session no bill has been passed, the legislature dissolves, and each member receives zero. This might correspond to the case of benefits that expire after a fixed time.

A history in this case is simply a description of who was recognized, what proposal was made, and how each member voted for each session. A strategy for a member specifies the bill to offer when recognized and the set of bills to vote for when required to vote. A member will be assumed to vote for a bill if indifferent between the distribution in the bill and the continuation value of the subgame beginning in the next session.<sup>9</sup> Similarly, if a member knows that his or her vote will not be decisive (because, e.g., a majority will vote for or against the bill), the member is assumed to vote for the bill if and only if the member's distribution in the bill is at least as great as the member's continuation value.

Unlike the more general case considered later, the extensive form of this game is finite, which permits backwards induction to be used to derive the equilibrium. In such a legislature, there is a subgame-perfect equilibrium in which each member, if recognized, makes the same "canonical" majoritarian proposal to distribute the benefits to a minimal winning majority. The equilibrium strategies and distributions are thus unique up to the specification of who receives the benefits.

**PROPOSITION 1.** *A strategy configuration is a subgame-perfect equilibrium for a two-session,  $n$ -member (with  $n$  odd) legislature with a closed rule and equal probabilities of recognition if and only if it has the following form:*

- a. If recognized in the first session, a member makes a proposal to distribute  $\delta/n$  to any  $(n - 1)/2$  other members and to keep  $1 - \delta(n - 1)/2n$  for his or her district. If recognized in the second session, a member proposes to keep all the benefits.*

- b. Each member votes for any first-session proposal in which the member receives at least  $\delta/n$  and votes for any second-session proposal.*

*The first proposal is thus accepted, and the legislature adjourns in the first session.*

*Proof.* To establish necessity, note that since the game ends after the second session, the continuation value  $v_i(2, g) = 0$  for all (null) subgames  $g$  and for all members. The member recognized at the beginning of session 2 can thus successfully propose to take all the benefits, since each member will vote for it because the member's allocation is at least as great as the member's continuation value. The continuation value  $v_i(1, g) = 1/n$  for any second-session subgame  $g$  is the expectation of member  $i$ 's distribution contingent on who is recognized. Thus, in the first session, a member will vote for a proposal only if the member receives at least  $\delta/n$ , the value of defeating a proposal and continuing to the second session. Any member recognized at the beginning of the first session will thus offer  $\delta/n$  to a minimal number  $(n - 1)/2$  of other members needed to obtain a majority vote. Recognized thus, the member keeps  $1 - \delta(n - 1)/2n$ . The  $(n - 1)/2$  other members and the proposer vote for the proposal, and the other members vote against it.

Sufficiency follows from direct verification that strategies satisfying conditions a and b constitute a subgame-perfect equilibrium, using the fact that before anyone is recognized, the continuation value  $v_i(1, g) = 1/n$  for any subgame  $g$ . QED.

The equilibrium in proposition 1 follows from five features of the model: (1) subgame perfection, (2) a finite legislature, (3) random recognition, (4) a closed rule, and (5) majority rule. Subgame perfection and finiteness are used to identify the equilibrium strategies in the second session, and random recognition and sub-



game perfection determine for each member the continuation values of the subgames. The closed rule prohibits others from making counterproposals or amendments to the proposal on the floor. Majority rule implies that in the first session the member recognized need only obtain the votes of  $(n - 1)/2$  other members, and those votes can be obtained by offering each a distribution equal to the member's continuation value.

Several important features are evident in this equilibrium. First, the distribution of the benefits reflects the majoritarian distribution of power in the legislature in that only a minimal majority of members receives a positive allocation of benefits. Second, the member recognized in the first session receives the largest allocation. The power of the member recognized first results both from agenda power and from the closed rule that prohibits amendments. Agenda power results from recognition which gives the proposer an opportunity to choose strategically a proposal for which a majority of members will vote. The closed rule prohibits other members from challenging the motion on the floor with an amendment, so those who in the motion on the floor will receive a distribution less than their continuation value can respond only by voting against the motion and, if they constitute a majority, continuing to the next session.<sup>10</sup> To indicate the extent of this power, note that the distribution received by the proposer can be as large as  $[1 - (\delta/3)]$  in a three-member legislature, and the limit as  $n$  becomes arbitrarily large is  $[1 - (\delta/2)]$ . Thus, the benefits received by the member recognized are in the interval  $[1 - (\delta/2), 1 - (\delta/3)]$ , so the member recognized always receives at least half the benefits.

Third, the initial offer is accepted and the legislature adjourns after only one bill has been proposed and voted on. This results from the incentives created by majority rule rather than from impatience

as in the bargaining models of Rubinstein (1982) and Binmore (1986). Majority rule—or more generally any rule that does not require unanimity among all the members—and a closed rule allow the proposer to form a majority to distribute the benefits among themselves to the exclusion of the other members. The member recognized need distribute only the continuation value to  $(n - 1)/2$  other members, and the opportunity to keep the rest leads the member recognized to make a proposal that will be accepted. The fact that the legislature adjourns after the first session is thus not due to impatience but rather to majority rule and to the inability of other members to amend the proposal of the member recognized. As will be demonstrated, if amendments are governed by an open rule, the benefits are distributed more evenly and the legislature may not complete its task in the first session.

Although the equilibrium in proposition 1 distributes the benefits among a minimal majority of the legislature, this minimal majority is not a coalition in the sense the term is used in cooperative game theory or in the social choice literature. The members of the majority in the legislature considered here act noncooperatively, and find it in their own individual interests to act as specified in the equilibrium. Even though the equilibrium has the property that the benefits are distributed among a minimal majority of members, this conclusion differs in two ways from Riker's (1962) theory of minimal winning coalitions. First, Riker's theory is based on the theory of cooperative games, and the theory developed here is noncooperative. Second, the noncooperative theory provides predictions about the distribution of the benefits among the members of the majority.

The result in proposition 1 is directly generalizable in a number of directions. First, the result can be extended to any finite number of sessions. Second, in a

two-session legislature if the members have different probabilities  $p_i$  of being recognized, each has a continuation value  $v_i(1, g) = p_i$  for any second-session subgame. Then, if member  $k$  is recognized in the first stage, he or she can offer  $\delta p_i$  to the  $i$ th member and that member will vote for the proposal. Member  $k$  will thus choose the  $(n - 1)/2$  members with the lowest  $p_i$ . Note that depending on the probabilities the member with the lowest probability of recognition may have the highest ex ante value of the game, and the member with the highest probability of recognition may have the lowest ex ante value of the game.<sup>11</sup> For example, if  $n = 3$ , and  $p_2 = 1/3$ ,  $p_1 = 1/3 + \epsilon$ , and  $p_3 = 1/3 - \epsilon$ ,  $\epsilon > 0$ , the ex ante values  $v_i$  of the game have limits  $v_1 = 2/9$ ,  $v_2 = 1/3$ ,  $v_3 = 4/9$  as  $\epsilon \rightarrow 0$ . The member with the lowest probability of recognition thus can do better than the other members because he or she is a less costly member of any majority.

There are a continuum of subgame-perfect, Nash equilibria satisfying the conditions of proposition 1. For example, in the first session member  $i$  may make a proposal that distributes  $\delta/n$  to any of the  $(n - 1)/2$  other members. One equilibrium thus has the member recognized randomizing among the other members resulting in an ex ante value  $v_i = 1/n$  for each member. This equilibrium has the property that the continuation values for all subgames are the same, or stationary, and equal to  $1/n$ . This stationary equilibrium is also symmetric in both the continuation values and the strategies. As demonstrated later, in an infinite-session legislative game with  $\delta$  sufficiently high, any distribution of the benefits can be supported as a subgame-perfect, Nash equilibrium. Later, stationary equilibria are argued to possess a focal point property.

## Equilibria in an Infinite-Session Legislature with a Closed Rule

In the finite-session legislative game just considered, backward induction could be used to characterize the equilibria. When the number of sessions is unlimited, however, any distribution of the benefits may be supported as a subgame-perfect equilibrium if there is a sufficient number of members and if they are not too impatient. In contrast to the uniqueness result for two-member bargaining, multimember bargaining under majority rule allows virtually all distributions of the benefits to be supported as equilibria.<sup>12</sup>

**PROPOSITION 2.** *For an  $n$ -member, majority-rule legislature with an infinite number of sessions and a closed rule, if  $1 > \delta > (n + 2)/[2(n - 1)]$  and  $n \geq 5$ , any distribution  $x$  of the benefits may be supported as a subgame-perfect equilibrium. To support an arbitrary distribution  $x \in X$ , employ the following strategy configuration: (1) whenever a member is recognized, he or she is to propose  $x$ , everyone is to vote for  $x$ ; (2) if a majority rejects  $x$ , the next member recognized is to propose  $x$ ; (3) if a member  $j$  is recognized and deviates by proposing  $y \neq x$ ,*

- a. a majority  $M(y)$  is to reject  $y$*
- b. the member  $k$  recognized next is to propose  $z(y)$  such that for the deviator  $z_j(y) = 0$ , and everyone in  $M(y)$  is to vote for  $z(y)$  over  $y$ ;*

*(4) if in strategy 3b member  $k$  is recognized and proposes  $s \neq z(y)$ , repeat strategy 3 with  $s$  replacing  $y$  and  $k$  replacing  $j$ .*

The proof is presented in the Appendix. The idea of the proof is simple: the required strategy configurations are such that any member who deviates from the prescribed distribution or from the prescribed punishment is certain to be pun-

ished, since subgame strategies constitute perfect equilibria. Members expect the punishment to be enforced because they expect that anyone who fails to punish a deviator will in turn be punished, and so on. When punishments of this sort are available, any outcome can be supported as a subgame-perfect equilibrium for  $n \geq 5$  and a sufficiently high  $\delta$ . Consequently, any vector of ex ante values of the game to the members may be supportable by a subgame-perfect equilibrium.

It should be clear that the degree of impatience plays very little role in the proof. As long as members do not discount the future too much, any distribution may be supported. For very small legislatures, this restriction is consequential, but as  $n$  increases, the required  $\delta$  declines rapidly to  $1/2$ . If there is no impatience, however, a member may find a deviation attractive, since under the recognition rule there is a positive probability that that member will be the one recognized in each session. If the member fails to punish him- or herself and  $\delta < 1$ , the value from such deviations is zero in an infinite session legislature, so deviation does not pay. If  $\delta = 1$ , however, that value does not go to zero. Under an alternative recognition rule in which no member may be recognized in consecutive sessions, punishment of a deviation is certain, and the result of proposition 2 holds for  $\delta = 1$  and  $n \geq 5$ .

In contrast to the theory of social choice, which finds there is no equilibrium for this distribution problem, the theory presented here predicts that all distributions may be achieved as noncooperative equilibria in a model that captures in a simple manner the sequential nature of proposal making and voting.

### Stationary Equilibria

In view of proposition 2, a theory of agenda formation that yields sharp pre-

dictions must be based on some refinement of the set of equilibria. The refinement considered here locates a "focal" equilibrium as discussed by Schelling (1960). We argue that the equilibria based on the punishment strategies of the sort displayed in proposition 2 are not "natural" focal points, since each member believes that if there is a deviation all the other members will carry out their designated punishment strategies at each of the infinitely many times they might be recognized. But if the game were to be played, a potential deviator would know that at some point some member would be required to make a punishment proposal in a circumstance in which that member is actually indifferent to doing so. Such a member may renege without cost, so a potential deviator may come to suspect that some such member in some session would renege with positive probability. Believing this, he or she would assign probability less than one to the event that all punishments would be carried out and would, moreover, know that everyone else would share this skepticism. In this sense, the punishment strategies that support any distribution  $x$  are not robust: if members are indifferent as to whether or not they punish deviators, the required strategies are only weakly credible.

Indeed, if  $\delta$  is interpreted as a reelection probability, nonstationary strategies of the sort required for proposition 2 hardly seem plausible.<sup>13</sup> Unless new members are assumed to be able to know the whole history of play when they enter and to find it in their interests to act on that history, strategies conditioned on that history may be impossible to implement. Furthermore, with new members entering each period, sitting members might be expected to have less reason to believe that punishment strategies would be implemented.

From a different perspective, note that in equilibrium the punishment strategies are never actually used, since the first proposal is  $x$  and it is approved. If there is a

cost to devising and being prepared to employ complex strategies of the type needed to support the equilibrium in proposition 2, the set of equilibria may be much smaller. In the context of a repeated game, Abreu and Rubinstein (1988) have provided such a theory.

The fragile nature of the equilibria in proposition 2 thus suggests that infinitely nested punishment strategies might not actually be played in legislatures because members would be unwilling to rest their fates on such weak incentives. Instead, members might find it natural to restrict their behavior in a way that calls for the same strategy in structurally equivalent subgames.<sup>14</sup> Two subgames are *structurally equivalent* if (1) the extant agenda at the initial nodes of the subgames are identical, (2) the sets of members who may be recognized at the next recognition node are the same, and (3) the strategy sets of members are identical. With a closed rule, two subgames commencing with the null agenda (i.e., no motion on the floor) are thus structurally equivalent, so all subgames commencing after the defeat of the proposal on the floor are structurally equivalent. A subgame commencing with a bill  $x$  on the agenda for a vote and a subgame commencing with a bill  $y$  on the agenda for a vote are not structurally equivalent if  $x \neq y$  even though the strategy sets of the members are the same.

An equilibrium is said to be *stationary* if the continuation values for each structurally equivalent subgame are the same. A stationary equilibrium necessarily has strategies that are stationary; that is, they dictate that a member take the same action in structurally equivalent subgames. Thus, if a member is recognized when there are no motions pending at each of two different sessions, he or she makes the same proposal in both sessions. Stationary strategies are necessarily history-independent and thus are chosen in response only to the incentives presented

by the play in future sessions.<sup>15</sup> Clearly, the strategies in proposition 2 fail to satisfy this restriction, since what a member is required to propose depends on the history of play leading to the subgame.

### A Closed Rule and Stationary Equilibria

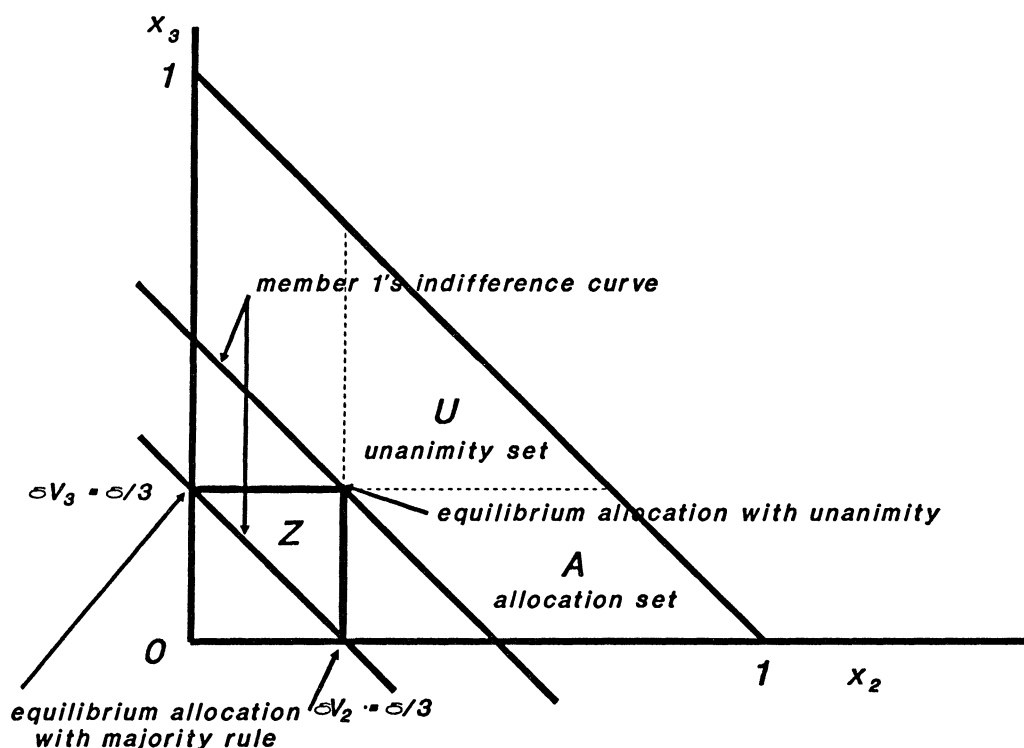
#### A Legislature with an Infinite Number of Sessions

The results of the "Illustration" indicate that with a finite number of sessions (1) the equilibrium distributions of benefits are majoritarian, (2) the member recognized has power in the sense that his or her allocation of benefits is strictly greater than that of any other member, and (3) the legislature completes its task in the first session. We now investigate an equilibrium with the stationarity property to determine whether the qualitative properties of the equilibria in the finite session case also hold when the number of sessions that the legislature can meet is not arbitrarily limited.

Initially, we treat only the symmetric case in which member  $i$  has a probability  $p_i = 1/n$  of being recognized, then the case of unequal probabilities. The following proposition indicates that the qualitative properties of the finite session case are preserved in a stationary equilibrium.

**PROPOSITION 3.** *For all  $\delta \in [0, 1]$  a configuration of pure strategies is a stationary subgame-perfect equilibrium in an infinite session, majority rule,  $n$ -member (with  $n$  odd) legislature governed by a closed rule if and only if it has the following form: (1) a member recognized proposes to receive  $1 - \delta(n - 1)/2n$  and offers  $\delta/n$  to  $(n - 1)/2$  other members selected at random; (2) each member votes for any proposal in which at least  $\delta/n$  is received.<sup>16</sup> The first proposal receives a majority vote,*

Figure 3. Closed Rule and Stationary Strategies



so the legislature completes its task in the first session. The ex ante values of the game are  $v_i = 1/n$ ,  $i = 1, \dots, n$ .

The proof is presented in the Appendix. The role of majority rule in proposition 3 can be identified from Figure 3 for the case of  $n = 3$ . The recognition rule and the requirement that strategies satisfy subgame perfection yield stationary values  $(v_1, v_2, v_3)$  for the legislative game. The member recognized is a (temporary) monopoly agenda setter, and chooses distributions  $x_2^1$  and  $x_3^1$  for the other two members subject to the restriction that  $(x_2^1, x_3^1)$  be in the feasible allocation set

$$A \equiv \{(x_2^1, x_3^1) \mid x_2^1 + x_3^1 \leq 1, x_2^1 \geq 0, x_3^1 \geq 0\}.$$

The objective of member 1 is to maximize allocation,  $x_1^1 = 1 - x_2^1 - x_3^1$ , so indifference curves in the  $x_2^1$ - $x_3^1$  plane are linear and parallel to the upper boundary of the allocation set. Member 1 thus maximizes  $1 - x_2^1 - x_3^1$  subject to  $(x_2^1, x_3^1) \in A$  and subject to the requirement that his or her proposal obtains the vote of at least one other member.

A member  $j$ ,  $j = 2, 3$ , will vote for member 1's bill if and only if  $x_j^1 \geq \delta v_j$ . A continuation set  $Z$ , defined as those allocations that will not command a majority, is thus

$$Z \equiv \{(x_2^1, x_3^1) \mid 0 \leq x_j^1 < \delta v_j, j = 2, 3\},$$

which is the interior of the square in Figure 3. Graphically, member 1 chooses an allocation  $(x_2^1, x_3^1)$  on the lowest indif-



ference curve such that  $(x_2^1, x_3^1)$  is in the majority outcome set  $M \equiv A - Z$ . Since  $v_j = \delta/3$ ,  $j = 1, 2, 3$ , two points— $(0, \delta/3)$  and  $(\delta/3, 0)$ —in the majority outcome set lie on the lowest indifference curve. The member recognized thus offers the continuation value to one other member and nothing to the other.

To identify the role of majority rule, suppose that the legislature operated under a unanimity rule so that member 1 must secure the votes of both of the other members. This would require that  $x_2^1 \geq \delta v_2$  and  $x_3^1 \geq \delta v_3$ . The unanimity set  $U$  may thus be defined as

$$U \equiv \{(x_2^1, x_3^1) \mid x_j^1 \geq \delta v_j, j = 2, 3, \\ x_2^1 + x_3^1 \leq 1\} \cap A,$$

which is illustrated in Figure 3. Member 1 thus maximizes  $1 - x_2^1 - x_3^1$  subject to  $(x_2^1, x_3^1) \in U$ . The solution is clearly  $x_j^1 = \delta v_j$ ,  $j = 2, 3$ . Majority rule thus allows the member recognized to capture the share of the member excluded from the majority.

Since a majority can exclude a minority and distribute the benefits among themselves, the recognized member has an incentive to exclude as large a minority as possible. A minimal winning majority will thus be formed with the benefits distributed among the members of that majority. To determine how the benefits will be distributed, consider the case in which  $\delta = 1$ . Majority rule allows the one-third share of the excluded member to be divided between the member recognized and the other member of the majority, and agenda power allows the member recognized to capture that entire share. With discounting, that member also captures a premium of  $(1 - \delta)/3$  due to the impatience of the other member.

For an  $n$ -member legislature the distribution of the benefits is  $\delta/n$  for  $(n - 1)/2$  members, zero for  $(n - 1)/2$  other members, and  $[1 - \delta(n - 1)/2n]$  for the member recognized. The latter is a

decreasing function of  $n$  and is bounded above by two-thirds and bounded below by one-half. The difference between the benefits received by the proposer and the benefits  $\delta/n$  received by the other members who vote for the proposal is an increasing function of  $n$ , so the distribution becomes more unequal as the size of the legislature increases. These properties are the same as those for the two-session game studied in the "Illustration." The stationary equilibrium characterized in proposition 3 is not the limit of all the equilibria characterized in proposition 1 of the finite session legislative game. It, however, is the limit of the finite session equilibrium in which at the beginning of each session the member recognized randomizes among the other members to determine whose vote will be attracted.

While proposition 3 is established in the special case of simple majority rule, the extension is straightforward to the case of  $k$ -member majority rule in which  $k$  members are required to adopt a proposal. For an  $n$ -member legislature governed by  $k$ -majority rule, random recognition, and a closed rule, the stationary equilibrium strategies are for the member recognized to offer  $\delta/n$  to  $k - 1$  members and to keep  $1 - \delta(k - 1)/n$ . Each member votes for any proposal offering at least  $\delta/n$ . Note that as  $k$  goes to  $n$ , the benefits retained by the initial proposer go to  $1 - \delta(n - 1)/n$  which, for  $\delta = 1$ , equals  $1/n$ . This represents the unanimity distribution under stationary strategies.<sup>17</sup> As  $k$  goes to one, the member recognized receives all the benefits. This may be thought of as corresponding to a property right granted to the member recognized.

## An Application: Government Formation in Parliamentary Systems

One application of this framework is to the government formation problem in multiparty parliamentary systems when no party has a majority of seats. The

benefits can then be interpreted as the ministries that are to be allocated among the parties in the majority. For simplicity of interpretation, assume that there are three parties and that the probability  $p_i$  of being asked to form a government is closely related to the proportion of seats held in the legislature after an election.<sup>18</sup> The bargaining over the allocation of ministries in a coalition government, then, corresponds closely to the model of a legislature operating under a closed rule: the party asked to form a government makes a proposal for allocating ministries (the benefits) among the parties, knowing that if its proposal does not receive a majority, another party (perhaps itself) will be asked (using the same recognition probabilities) to form a government.<sup>19</sup> The game thus has an infinite number of sessions, and the values  $v_i$  represent the expected proportion of ministries each party will have in the government. Attention will be restricted to stationary equilibria, and an argument analogous to that used to establish proposition 3 yields the stationary equilibria and provides predictions of which governments will form as well as how the ministries will be distributed. With asymmetric probabilities of recognition, however, asymmetric randomized strategies may result in equilibrium.

Let  $r_i$  denote that the probability that party  $i$  makes an offer of  $\delta v_{i+1}$  to party  $i + 1 \pmod{3}$ , where  $\delta v_{i+1}$  is the proportion of ministries allocated to party  $i + 1$ . Party 1, for example, makes a proposal  $x^{1,2} = (1 - \delta v_2, \delta v_2, 0)$  with probability  $r_1$  and a proposal  $x^{1,3} = (1 - \delta v_3, 0, \delta v_3)$  with probability  $1 - r_1$ . In general, necessary conditions for a stationary equilibrium are that the values  $v_i$  and the randomization probabilities  $r_i \in [0, 1]$  satisfy

$$v_1 = p_1[1 - r_1\delta v_2 - (1 - r_1)\delta v_3] + p_2(1 - r_2)\delta v_1 + p_3r_3\delta v_1 \quad (1)$$

$$v_2 = p_2[1 - r_2\delta v_3 - (1 - r_2)\delta v_1] + p_3(1 - r_3)\delta v_2 + p_1r_1\delta v_2 \quad (2)$$

$$v_3 = p_3[1 - r_3\delta v_1 - (1 - r_3)\delta v_2] + p_1(1 - r_1)\delta v_3 + p_2r_2\delta v_3. \quad (3)$$

In equilibrium, a party  $i$  will adopt a strictly mixed strategy  $r_i \in (0, 1)$  only if the continuation values of the other two parties are equal; and if two parties use strictly mixed strategies, all three continuation values will be equal. In the case in which two parties randomize, the values are  $v_i = 1/3$ ,  $i = 1, 2, 3$ . Then, the rank of the matrix of coefficients of  $(r_1, r_2, r_3)$  in equations 1-3 is two, so there may be a continuum of equilibria. As an example, with  $\delta = .8$ ,  $p_1 = .45$ ,  $p_2 = .35$ ,  $p_3 = .2$ , the equilibrium randomizations parameterized on  $r_3$  are  $r_1 = .19 + .44r_3$  and  $r_2 = .96 + .57r_3$ , for  $r_3 \in [0, .0625]$ .

To see why the continuation values can be equal even though the recognition probabilities are unequal, note that since the smallest party 3 is least likely to be recognized to form a government, its continuation value (out of equilibrium) would be expected to be smaller than the continuation values of the other two parties. The two largest parties would thus prefer to form a government with the smallest party. The smallest party would recognize this preference and, to join a government, would require a higher allocation of ministries than would be suggested by the likelihood it would be asked to form a government. If the smallest party were to demand more than one-third, however, at least one of the other parties would prefer to form a government with a party other than the smallest. In equilibrium the values thus must be equalized.

For the case in which  $r_3 = 0$ , party 1 with probability .19 offers to form a government with party 2 and with probability .81 offers to form a government with party 3. Party 2 with probability .96

offers to form a government with party 3 and with probability .04 offers to form a government with party 1. The probabilities  $q_i$  that party  $i$  will be in the government are then  $q_1 = .46$ ,  $q_2 = .64$ ,  $q_3 = .90$ . The smallest party is thus most likely to be in the government. In this example, however, the probability is .10 that the government will be formed by the two largest parties.

### Amendments: A Simple Open Rule

With an open rule, proposals on the floor may be subject to amendment. This provides an opportunity for a member not allocated his or her continuation value in the proposal on the floor to make a substitute proposal. A substitute can always be chosen that will defeat the proposal on the floor, so the excluded member has an opportunity to assume the position of the member who made the prior proposal. The power of the member recognized first is thus diminished. That member, however, will take into account the possibility of an amendment in making an initial proposal. The issues to be investigated are thus the effect of the possibility of amendments on the size of the majority and on the distribution among the members of the majority and whether the possibility of an amendment may cause the legislature not to conclude its task in the first session. The characterization of the equilibrium for the case of an open rule also provides the basis for the analysis of the *ex ante* choice of an amendment rule, as considered later.

In the legislative process illustrated in Figure 2, each member has a  $1/n$  probability of gaining initial recognition to make a proposal. Then each of the other members has a  $1/(n - 1)$  probability of being recognized and may either make an amendment or move the previous question. If the previous question is moved, the pro-

posal on the floor is voted on; and if it is approved, the benefits are distributed. If an amendment is offered, it must be voted against the proposal before either another amendment or moving the previous question is in order.<sup>20</sup> The winner of the vote then becomes the motion on the floor, and the next session begins. Each member other than the author of the proposal on the floor then has a probability of  $1/(n - 1)$  of recognition for the purpose of offering an amendment or moving the previous question.<sup>21</sup> The process continues in this manner until the previous question is moved on a proposal and the proposal passes. In this setting, discounting is assumed to occur whenever an amendment is moved and voted on.

Under a simple open rule no member will ever be in a position to make a proposal or an amendment that cannot be pitted directly against another proposal. Thus, whoever proposes a bill or an amendment must take account of the fact that other members may subsequently be recognized. Another difference between an open rule and a closed rule is that a vote takes place only after a member has been recognized. The member initially recognized thus faces a trade-off. If the member wants to be certain that the proposal will be accepted, it must be sufficiently attractive that whoever is recognized next will move the previous question and then vote for the proposal. The equilibrium in this case would exhibit universalism. Making a proposal attractive to all members is expensive, however, so the member recognized may prefer to offer a proposal that is attractive to a subset including only  $m$  of the  $n - 1$  other members. Then, if one of the  $m$  is recognized next and has been allocated at least as much as his or her continuation value corresponding to offering an amendment  $y$ , the member will move the previous question and, if  $m + 1 \geq (n + 1)/2$ , a majority will approve it. That continuation value will be denoted  $\delta V_i^m(y)$ , which

is the discounted value of the game for  $j$  when  $j$  makes a successful amendment  $y$  and the legislature continues to the next session. If one of the  $n - 1 - m$  other members is recognized, the member will offer an amendment chosen so as to defeat the motion on the floor, and the legislature will move to the next session. The member recognized thus can determine through the choice of  $m$  the likelihood that the proposal will be accepted. Intuition suggests that the lower the discount factor  $\delta$ , the greater the incentive to offer a proposal that will be accepted with high probability, since continuing to the next session is then more costly. The higher the discount factor, the less costly continuation and the smaller the majority  $m$  to whom benefits are allocated. The majority  $m$  characterized in the following proposition is a (weakly) decreasing function of  $\delta$ .

The equilibrium for a legislature with a simple open rule is characterized in the following proposition. To simplify the analysis, a member who is indifferent between two proposals will be assumed to vote for the one proposed last.<sup>22</sup>

**PROPOSITION 4.** *In an  $n$ -member, majority-rule legislature governed by a simple open rule and equal probabilities of recognition, a stationary equilibrium is a strategy configuration in which (1) the member recognized first makes a proposal  $\hat{y}$  that allocates  $[(1 - \hat{y}^a)/m(\delta, n)]$  to  $m(\delta, n)$  other members, where  $(n - 1)/2 \leq m(\delta, n) \leq n - 1$ , and allocates  $\hat{y}^a$  to him- or herself; (2) if one of those  $m(\delta, n)$  members is recognized next, this member moves the previous question, the proposal is approved by the proposer of  $\hat{y}$  plus the  $m(\delta, n)$  members, and the legislature adjourns; and (3) if one of the  $n - 1 - m(\delta, n)$  other members is recognized next, that member offers an amendment that allocates  $\hat{y}^a$  to him- or herself and  $[(1 - \hat{y}^a)/m(\delta, n)]$  to  $m(\delta, n)$  other*

*members excluding the member that made the previous proposal. Those members include all those offered zero in the proposal on the floor plus  $n - 1 - m(\delta, n)$  others chosen randomly from those offered  $[(1 - \hat{y}^a)/m(\delta, n)]$  in that proposal. This amendment defeats the prior motion and becomes the motion on the floor. Then repeat strategies 2 and 3. The number  $m(\delta, n)$ , defined in equation 4 is a weakly decreasing function of  $\delta$  and a weakly increasing function of  $n$ . The function  $m(\delta, n)$  is given by*

$$m(\delta, n) \in \arg_m \max V^m(\hat{y}), \quad (4)$$

where

$$\begin{aligned} V_I^m(\hat{y}) = & \left( \frac{m}{n-1} \right) / \{1 + \delta \left( \frac{m^2}{n-1} \right) \\ & - \delta^2 \left(1 - \frac{m}{n-1}\right) \left[ \left( \frac{1}{n-1} \right) \right. \\ & \left. + \left(1 - \frac{m+1}{n-1}\right) \gamma(\delta, m, n) \right]\}; \\ \gamma(\delta, m, n) = & \frac{\delta}{B} \left[ \left( \frac{m}{n-1} \right) \right. \\ & \left. + \delta \left(1 - \frac{m}{n-1}\right) \left( \frac{n-m-2}{m} \right) \right. \\ & \left. \left( \frac{1}{n-1} \right) \right]; \end{aligned}$$

and

$$\begin{aligned} B = & 1 - \left( \frac{2m-n+2}{m} \right) \left(1 - \frac{m}{n-1}\right) \\ \delta - & \left( \frac{n-m-2}{m} \right) \left(1 - \frac{m+1}{n-1}\right) \\ & \left(1 - \frac{m}{n-1}\right) \delta^2. \end{aligned}$$

The benefits  $\hat{y}^a$  retained by the author of the winning proposal are  $\hat{y}^a = 1 - m(\delta, n)\delta V_I^m(\hat{y}^1)$ .

# Bargaining in Legislatures

Table 1. Open Rule, Stationary Equilibria

Discount Factor ( $\delta$ )	Size ( $n$ )	Majority ( $m[\delta, n]$ )	Share of Proposer $\hat{y}^a$	Values of Games			Probability First Proposal Approved
				$V_1^m(\hat{y}_1^a)$	$V_k^m(\hat{y}_1^a)$	$V_j^m(\hat{y}_1^a)$	
1	3	1	.60	.40	.40	.20	.5
1	5	2	.44	.28	.23	.12	.5
1	51	25	.06	.04	.03	.01	.5
1	101	50	.03	.02	.01	.01	.5
.8	3	1	.68	.40	.27	.16	.5
.8	5	2	.52	.30	.17	.09	.5
.8	51	25	.08	.05	.02	.01	.5
.8	101	50	.04	.02	.01	.00	.5
.6	3	2	.45	.45	.27	<sup>a</sup>	1.0
.6	5	2	.42	.33	.13	.07	.5
.6	51	25	.11	.06	.02	.01	.5
.6	101	50	.06	.03	.01	.00	.5
.4	3	2	.56	.56	.22	<sup>a</sup>	1.0
.4	5	3	.52	.40	.13	.04	.75
.4	51	25	.17	.08	.02	.00	.5
.4	101	50	.09	.05	.01	.00	.5
.2	3	2	.71	.71	.14	<sup>a</sup>	1.0
.2	5	4	.56	.56	.11	<sup>a</sup>	1.0
.2	51	25	.29	.14	.01	.00	.5
.2	101	50	.17	.08	.01	.00	.5

<sup>a</sup>No member receives zero in a proposal.

The proof is presented in the Appendix. The number  $m(\delta, n)$  of members who receive benefits in the equilibrium proposal is a (weakly) decreasing function of  $\delta$ , so the higher the discount factor, the smaller the majority. This results because the higher  $\delta$ , the smaller the loss from continuing to the next session. Table 1 presents  $m(\delta, n)$  for legislatures of varying sizes. As observed above, the legislature will complete its task in the first session with probability one only if the equilibrium proposal is universalistic. As indicated in Table 1, this will result only if  $\delta$  is low. Similarly, the majority may be larger than the minimum, although as the size  $n$  of the legislature increases the closer  $m(\delta, n)$  is to a minimal majority. If the model is interpreted as representing  $n$  blocks of legislators, each of which will share benefits equally among their members, a legislature with three blocks, for example, may yield universalistic out-

comes if impatience is sufficiently great.

As indicated in Table 1, even when the majority is of minimal size under a simple open rule, the distribution among the members is more equal than under a closed rule. A member who votes for the winning proposal receives  $[(1 - \hat{y}^a)/m(\delta, n)]$ , and the member who makes the winning proposal receives  $1 - m(\delta, n)\{(1 - \hat{y}^a)/[m(\delta, n)]\} = \hat{y}^a$ . For a legislature with 101 members and  $\delta = 1$ ,  $\hat{y}^a = .032$  and  $[(1 - \hat{y}^a)/m(\delta, n)] = .019$ . Under a closed rule, the member making the winning proposal receives .5, and the members voting for the proposal receive .01. An open rule thus greatly reduces the power of the member recognized compared to a closed rule even though in a large legislature without great impatience the benefits are distributed among a minimal majority under both rules.

Recognition is still valuable, however, since the value for the member recognized



is at least as great as the value to any other members. To illustrate the value of recognition, consider the case of a three-member legislature. As indicated below,  $m(\delta, 3) = 1$  if  $\delta \in [\sqrt{3} - 1, 1]$  and  $m(\delta, 3) = 2$  if  $\delta \in [0, \sqrt{3} - 1]$ . For  $\delta \in [\sqrt{3} - 1, 1]$ , let  $V_1^1(\hat{y}^1)$  and  $V_3^1(\hat{y}^1)$  denote, respectively, the values of the game to the member offered  $1 - \hat{y}^a$  and to the member offered zero in the proposal  $\hat{y}^1 = \hat{y}$  of member 1, where the superscript is  $m(\delta, 3)$ . The values are  $V_1^1(\hat{y}^1) = 2/(4 + 2\delta - \delta^2)$ ,  $V_2^1(\hat{y}^1) = 2\delta/[(2 - \delta)(4 + 2\delta - \delta^2)]$ , and  $V_3^1(\hat{y}^1) = \delta/(4 + 2\delta - \delta^2)$ . It is straightforward to show that  $V_1^1(\hat{y}^1) + V_2^1(\hat{y}^1) + V_3^1(\hat{y}^1) < 1$  for  $\delta \in (\sqrt{3} - 1, 1)$ . For  $\delta = 1$ , the continuation values are  $V_1^1(\hat{y}^1) = 2/5$ ,  $V_2^1(\hat{y}^1) = 2/5$ , and  $V_3^1(\hat{y}^1) = 1/5$ . For the case of a closed rule, the values of the game conditional on being recognized are, for  $\delta = 1$ , two-thirds, one-third, and zero respectively. Consequently, compared to the equilibrium with a closed rule, the value of recognition is lower and the values of the game are closer together with an open rule. The possibility of amendments thus substantially reduces the value of recognition.

As  $\delta$  increases,  $V_1^1(\hat{y}^1)$  decreases, so (as under a closed rule) whoever is recognized first benefits from greater impatience. For  $\delta < \sqrt{3} - 1$  the member recognized first prefers to make a universalistic [ $m(\delta, 3) = 2$ ] proposal, and the value for member 1 is  $V_1^2(\hat{y}^1) = \hat{y}^a = 1/(1 + 2\delta)$ . The values for the other two members equal  $(1 - \hat{y}^a)/2 = \delta/(1 + 2\delta)$ . The sum of the values equals one, since the previous question is moved by whoever is recognized next. The value  $V_1^2(\hat{y}^1)$  of the member recognized first is a decreasing function of  $\delta$  and is less than the corresponding value under a closed rule for  $\delta > 0$ .

In summary, the principal features of the stationary equilibrium for a simple open rule are the following: First, if impatience is sufficiently low, the benefits are distributed among a minimal majority; but if impatience is high, the benefits may

be shared among a supermajority. Second, the initial proposal is not necessarily accepted in the first session when impatience is low. Instead, with some probability an amendment is offered and the legislature continues to the next session. When impatience is low, the probability thus is positive that the legislature will not reach an agreement in any finite number of sessions; thus, the sum of the continuation values is less than one. Third, as indicated in Table 1, the value to the member initially recognized is greater, the greater the impatience of the members. Fourth, the benefits are distributed more evenly among the winning majority under an open rule than under a closed rule. Fifth, even though amendments may be offered in equilibrium, the initial proposer is still advantaged. Recognition is valuable; but, as indicated in Table 1 and proposition 3, recognition is not as valuable as under a closed rule.

### The Choice of an Amendment Rule

Legislatures often are delegated the duty of choosing their own method of organization; and in the model considered here, that choice corresponds to the selection of an amendment rule. In this model, that selection is made at the ex ante stage before any member has gained recognition to make a proposal; so all members are in a symmetric position and thus have identical preferences over amendment rules.<sup>23</sup> The members, however, weakly prefer a closed rule because it assures that delay will not result. Whenever delay is costly (but not so costly that a universalistic proposal is made under an open rule), all members strictly prefer a closed rule to an open rule; that is, for all  $\delta < 1$  such that  $m(\delta, n) < n - 1$ , the sum of the values of the game is strictly less under an open rule than under a closed rule. Because each member has the same chance

of being recognized, each member prefers a closed rule.<sup>24</sup>

Of course, the distribution problem studied here is not representative of the bulk of the proposals that legislatures consider, so the extent to which these results generalize is not clear. The basic implication of this analysis, however, conforms with political intuition. If delay in the passage of legislation is costly but is not so costly that immediate passage is a necessity, strategic considerations can come into play. The incentive to use the opportunity to make a proposal to benefit one's own district is strong, and under an open rule the pursuit of that opportunity can result in costly delay. In the context of the distribution problem and legislative process considered here, that cost can be avoided by adopting a closed rule.

At least in the Congress, this prediction is not supported by the evidence; so it is useful to explore in more detail the structure of the model considered here and the additional considerations needed to provide a complete theory of the choice of an amendment rule. First, the legislature modeled here does not have committees with jurisdictions, so it is natural to represent each member as having the same probability of recognition. If a committee had the right to make the first proposal under a closed rule, it would, if modeled as a unitary actor, have an *ex ante* value of the game equal to  $1 - [(n - 1)/2](\delta/n)$ , and the rest of the members would have an expected value equal to only  $(\delta/2n)$ . In such a case, the legislature would prefer an open rule to a closed rule as a means of diminishing the power of the committee. We investigate this subject in more detail in Baron and Ferejohn 1989.

A legislature might adopt an open rule for a variety of other reasons. An open rule results in more equal *ex post* distributions than does a closed rule, so it might be chosen to further equity considerations. Similarly, risk aversion on the part of members of the legislature could

result in a preference for the less unequal *ex post* distribution that results with an open rule.<sup>25</sup> Furthermore, an open rule seems to comport better with democratic theory in the sense that it allows greater opportunity to members than does a closed rule, which immediately closes the amendment process when a proposal is made. Whereas these considerations may explain the relatively infrequent use of a closed rule, the theory presented here indicates that the cost of an open rule is the possibility of delay.

In practice, the House of Representatives employs a variety of amendment rules to govern floor consideration of legislative proposals. Krehbiel (1989) and Bach and Smith (1988) found that a closed rule is not used frequently in the House, although much legislation is considered under modified rules. These rules do restrict the amendments that can be made on the floor and thus serve to reduce the likelihood of delay. Similarly, the Senate operates without amendment rules, but it has increasingly resorted to unanimous consent agreements to limit time or amendment activity.

## Conclusions

We provide a theory of a majority rule legislature in a context in which the standard institution-free model of social choice theory yields no equilibrium. The structural aspect of the legislature that yields equilibria is the sequential process by which proposals are made and voted on. The member recognized to make a proposal is shown to have agenda power that results from the chance that a member may be excluded by a majority in a future session. Impatience adds to that agenda power. With a closed rule, the equilibrium outcomes are majoritarian, the first proposal is passed, and benefits are distributed to a minimal majority. Since each member acts noncooperatively, the ma-

jority is not a coalition in the sense the term is used in cooperative game theory. Compared to a closed rule, the opportunity to make an amendment under an open rule dramatically reduces the agenda power of the member recognized first and results in an outcome that more evenly distributes the benefits among the winning majority. Unless impatience is great, however, the distribution is majoritarian, and the proposal made by the first member recognized is not necessarily accepted. If, however, the number of members is small and there is substantial impatience (as there would be if members faced a significant electoral risk), the equilibrium is "universalistic" in the sense that every member receives benefits. Thus, if a legislature has a few internally homogeneous blocks that can act in a concerted manner, universalistic outcomes might be expected if impatience is substantial. The member recognized to make a proposal nevertheless retains some agenda power and receives more benefits than any other member.

The amendment rule used by a legislature is endogenous, and the theory presented here predicts that unless there is no impatience, the legislature prefers a closed rule to a simple open rule. This results in part because the internal structure of the legislature is not modeled; and if that structure (e.g., a committee system) gave agenda power to a particular individual, the legislature might be expected to adopt an open rule that to some extent lessened that power. Predictions about the implications of the internal organization of legislatures from this perspective await further research.

These predictions of the noncooperative bargaining theory of legislatures contrast with those that arise in cooperative models that focus on coalition formation. Cooperative models of politics abstract from the process by which alternatives arise and assume that coalitions will freely form to defeat alternatives when a major-

ity of members prefers another available alternative. Cooperative theories generate weak predictions about legislative outcomes because they ignore the sequential aspects of legislative consideration of alternatives and ignore the implications of structure for undertaking coordinated activity (or coalition formation) in a legislative setting. One prediction from this approach is Riker's minimal winning coalition theorem. The noncooperative theory presented here demonstrates that Riker's conclusion holds under a closed rule and also under a simple open rule if impatience is sufficiently small or the legislature is large. Using this bargaining model, Baron (1989) considers the possibility of coordinated proposal making (but not voting) among members and finds that benefits are allocated to more than a minimal number of districts but not to all.

By taking seriously the sequential structure of legislative action, definite predictions about the outcomes of legislative processes are obtained. In addition to the paradigmatic case of the U.S. Congress, for which our theory provides a strong prediction as to how a single distributional issue will be resolved, the theory has implications for the problem of cabinet formation in multiparty democracies. It is also clear that once the preferences of the members are specified, the bargaining theory approach yields predictions about the relations between the president and Congress in the appointments process (in which the president monopolizes the proposal power) and in the legislative process (in which Congress monopolizes the proposal power).<sup>26</sup> Indeed, in any setting in which the proposal-making process can be clearly specified, the bargaining approach will generate testable predictions.

Finally, it seems appropriate to speculate about repetition. Our model is one in which a fixed quantity of benefits is to be distributed once and for all. In actual

legislatures, many issues are decided; and the members may wish to make use of voting and proposal strategies that link their behavior on certain issues to those on others. Our model is useful in considering this richer setting in several senses. First, it represents a useful way to think about the resolution of the "final" issue, if there is one. In this sense, there is a finite set of issues and backwards induction can be used to characterize equilibria for the multi-issue legislature. Second, when there are multiple issues to be resolved, a construction of the sort given in proposition 2 is available; so a great variety of legislative outcomes may be supported by nonstationary strategies.<sup>27</sup> Third, if multiple issues are considered simultaneously, different members (e.g., committees) would be recognized to make proposals in their policy jurisdictions. Then, if preferences were as represented here, more universalistic outcomes might be expected. Since universalistic outcomes also result under an open rule if impatience is sufficiently great and the number of members or blocks is small, stationary equilibria may still have some importance as a focal point in a repeated setting.

## Appendix

*Proof of Proposition 2.*<sup>28</sup> The proof requires that the punishment in strategy 3 make any deviation  $y \neq x$  yield a continuation value of zero for the deviator  $j$ . This requires specification of  $z(y)$  and the majority  $M$ . Given  $y \neq x$ , choose some majority  $M$  of members that does not contain  $j$ , and let  $Y_M(y) \equiv \sum_{i \in M} y_i$  denote the total distribution to that majority. Let  $M^*(y)$  denote a majority with the smallest cost or

$$M^*(y) \in \arg \min_{M \in M_j} Y_M(y),$$

where  $M_j$  is the set of all majorities not containing  $j$ . The number of members of

$M^*(y)$  is  $(n + 1)/2$  and let  $m^*$  be the number of members of  $M^*(y)$  such that  $y_i = 0$ ,  $i \in M^*(y)$ . Then define the distribution  $z(y)$ , for  $i \in M^*(y)$ , by

$$z_i(y) = \begin{cases} \frac{y_i}{Y_{M^*(y)}(y)} - \eta\epsilon & \text{if } y_i > 0 \\ \epsilon & \text{if } y_i = 0, \end{cases}$$

where  $\epsilon > 0$  and small,  $\eta = m^*/[(n + 1)/2 - m^*]$  if  $m^* < (n + 1)/2$ , and  $\eta = 0$  if  $m^* = (n + 1)/2$ . For  $i \notin M^*(y)$ , let  $z_i(y) = 0$ . By construction, the proposal  $z(y)$  is such that  $z_i(y) > y_i$ ,  $\forall i \in M^*(y)$ . All members of the majority  $M^*(y)$ , knowing that each subsequent proposal will be  $z(y)$  and that it will be approved, will thus reject  $y_i$  if  $\delta$  is such that

$$\delta z_i(y) > y_i \quad \forall i \in M^*(y). \quad (\text{A-1})$$

The majority knows this because it anticipates that any deviation from the proposal  $z(y)$  will be punished for the same reason that the proposer of  $y$  is being punished.

Next, the values of  $\delta$  and  $n$  such that the inequality in relationship A-1 is satisfied must be determined. For  $y_i = 0$ , relationship A-1 is satisfied for all  $\delta > 0$ ; and for  $y_i > 0$ , the inequality implies that

$$\begin{aligned} \delta > \frac{y_i}{z_i(y)} &= \frac{y_i}{[y_i/Y_{M^*(y)}(y)] - \eta\epsilon} \\ &= \frac{Y_{M^*(y)}(y)}{1 - \eta\epsilon Y_{M^*(y)}(y)/y_i} \quad \forall i \in M^*(y). \end{aligned}$$

Since  $x$  is arbitrary, the least upper bound on  $Y_{M^*(y)}(y)$  must be determined. It is straightforward to verify that if  $\epsilon$  is sufficiently small, the least upper bound  $Y^*$  on  $Y_{M^*(y)}(y)$  is

$$\begin{aligned} Y^* &\equiv \max_y Y_{M^*(y)}(y) \\ &= \begin{cases} (n + 1)/[2(n - 1)] & \text{if } n \text{ is odd,} \\ (n + 2)/[2(n - 1)] & \text{if } n \text{ is even.} \end{cases} \end{aligned}$$

The upper bound  $Y^*$  is determined by the deviation  $y$  that is most costly to defeat, that is,  $y = (1/(n-1), \dots, 1/(n-1), 0, 1/(n-1), \dots, 1/(n-1))$ , where the zero is in the position for the deviator  $j$ . For  $n$  odd, a majority thus requires  $1/(n-1)$  for  $(n+1)/2$  members. Thus, an arbitrary  $x$  can be supported in equilibrium if  $\delta > Y^* = (n+1)/[2(n-1)]$ . For  $n$  odd, such a  $\delta$  exists if  $n \geq 5$ ; and for  $n$  even, such a  $\delta$  exists if  $n \geq 6$ . Any deviator  $j$  will thus find his or her proposal  $y$  defeated by anticipating that  $z(y)$  will be passed, in which case the deviator receives zero.

The remaining case to consider is that in which the deviator is recognized in consecutive sessions. Strategy 3 thus calls for the deviator to punish him- or herself, but suppose that  $j$  deviates from that as well. Then if in the next session a member  $k \neq j$  is recognized,  $k$  will punish  $j$ . The only event in which  $j$  can avoid punishment is if he or she is recognized in every session; but no majority will vote for the deviation, so  $\lim_{\tau \rightarrow \infty} \delta^\tau y_j = 0$  for all  $\delta < 1$ .

Any deviation thus yields zero to the deviator. Thus, no member has an incentive to deviate, and  $x$  is supported as a subgame-perfect equilibrium. QED

*Proof of Proposition 3.* Sufficiency follows from direct verification that the configuration of offers is an equilibrium in every subgame. To establish necessity, first note that all subgames including the entire game are structurally identical. Let  $v_i$  denote the stationary continuation value for member  $i$ ,  $i = 1, \dots, n$ , for each subgame and let the proposal made by member  $i$  when recognized be denoted by  $x^i = (x_1^i, \dots, x_n^i)$ . Any member recognized will prefer to make a proposal that will be accepted, since making a proposal that will be rejected is equivalent to making no proposal. Subgame perfection implies that member 1 need be no more generous than to offer the discounted continuation value to a minimum number of members and zero to the others. Conse-

quently, if a member,  $j$ , is recognized, that member will offer  $x^j$  such that  $x_i^j \geq \delta v_i$  to  $(n-1)/2$  other members needed to constitute a majority. Any member receiving an offer  $x_i^j \geq \delta v_i$  will vote for it. Consider then the case in which member  $i$  chooses at random a bill that attracts the vote of  $(n-1)/2$  members; that is, the probability that a member will receive a positive share of the benefits is one-half. The continuation values  $v_i$  are, using symmetry,

$$v_i = \frac{1}{n} [1 - \frac{n-1}{2} \delta v_i] + (\frac{n-1}{n})(1/2)\delta v_i, \quad i = 1, \dots, n,$$

where the first term corresponds to the event that  $i$  is recognized and the second term corresponds to the event that one of the other members is recognized. This simplifies to  $v_i = 1/n$ ,  $i = 1, \dots, n$ . Then  $x_i^j = 1 - \delta(n-1)/2n$ ,  $i = 1, \dots, n$ ; and  $x_i^j = \delta/n$ ,  $i = 1, \dots, n$ ,  $i \neq j$ . QED

*Proof of Proposition 4.* A constructive proof will be presented. Because of the symmetry of the legislature, the member (denoted member  $j$ ) recognized first will make a proposal  $y^j$  that offers  $(1 - y^a)/m$  to  $m$  other members selected at random—where  $(n-1)/2 \leq m \leq n-1$ —and offers  $y^a$  to him- or herself. It can be shown that recognition is valuable in this game, which is to say that  $y^a \geq (1 - y^a)/m$  so that member  $j$  receives at least as much as any other member. Let  $V_j^m(y^j)$  denote the value of the game to member  $j$  when proposal  $y^j$  is on the floor and let  $\hat{y}^j$  denote the proposal that maximizes  $V_j^m(y^j)$ , so  $\hat{y}^a$  denotes the allocation to the proposer. A member  $i$  to whom  $(1 - \hat{y}^a)/m$  is offered will move the previous question rather than make an amendment if and only if

$$\frac{1 - \hat{y}^a}{m} \geq \delta V_i^{m*}(\hat{y}^i), \quad (\text{A-2})$$



where  $V_i^m(y^i)$  is the continuation value to  $i$  if  $i$  proposes an amendment  $y^i$  that defeats  $\hat{y}^i$ . Note that if  $y^i$  defeats  $\hat{y}^i$  and  $y^i$  becomes the proposal on the floor, then  $V_i^m(y^i) = V_i^m(\hat{y}^i)$ . Then if  $\hat{y}^i$  maximizes  $V_j^m(y)$ , then  $\hat{y}^i$ , defined by  $\hat{y}_i^i = \hat{y}^a$  and  $(1 - \hat{y}^a)/m$  offered to  $m$  other members selected at random, maximizes  $V_i^m(y^i)$ . Clearly the  $m$  members to be offered  $(1 - \hat{y}^a)/m$  can be chosen such that  $\hat{y}^i$  defeats  $\hat{y}^j$ .<sup>29</sup> Since  $\hat{y}^a \geq (1 - \hat{y}^a)/m$ , member  $j$  will be offered zero unless  $m = n - 1$ , in which case every member receives a positive amount of the benefits. Consequently, any member  $i$  who offers an amendment rather than moving the previous question will have a continuation value  $V_i^m(\hat{y}^i) = V_i^m(\hat{y}^j)$ . Consequently, relationship A-2 is

$$\frac{1 - \hat{y}^a}{m} \geq \delta V_j^m(\hat{y}^j). \quad (\text{A-3})$$

In the following, the member recognized first is denoted 1, a member who receives  $(1 - \hat{y}^a)/m$  in 1's proposal  $\hat{y}^1$  is denoted  $k$ , and  $j$  and  $l$  denote members who receive zero in  $\hat{y}^1$ . The value  $V_1^m(\hat{y}^1)$  is given by

$$V_1^m(\hat{y}^1) = \frac{m}{n-1} \hat{y}^a + (1 - \frac{m}{n-1}) \delta V_1^m(\hat{y}^j),$$

where  $m/(n - 1)$  is the probability that one of those offered  $(1 - \hat{y}^a)/m$  is recognized and moves the previous question and  $[1 - m/(n - 1)]$  is the probability that one, denoted  $j$ , of the members offered zero by member 1 is recognized. Member  $j$  will propose  $\hat{y}^j$ , and since  $\hat{y}^a \geq (1 - \hat{y}^a)/m$ , the component  $\hat{y}_1^j = 0$  for all  $j$ . The above argument indicates that  $\hat{y}^j$  is a permutation of  $\hat{y}^1$ , so  $V_1^m(\hat{y}^j) = V_j^m(\hat{y}^1)$  and thus

$$V_1^m(\hat{y}^1) = \frac{m}{n-1} \hat{y}^a$$

$$+ (1 - \frac{m}{n-1}) \delta V_j^m(\hat{y}^1). \quad (\text{A-4})$$

The value  $V_j^m(\hat{y}^1)$  is given by

$$V_j^m(\hat{y}^1) = \frac{m}{n-1} \cdot 0 + \frac{1}{n-1} \delta V_j^m(\hat{y}^j) + (1 - \frac{m+1}{n-1}) \delta V_j^m(\hat{y}^l),$$

where  $m/(n - 1)$  is the probability that a member,  $k$ , offered  $(1 - \hat{y}^a)/m$  in 1's proposal will move the previous question, in which case  $\hat{y}^1$  will be adopted;  $1/(n - 1)$  is the probability that  $j$  is recognized and will successfully offer  $\hat{y}^j$  yielding a value  $V_j^m(\hat{y}^j)$ ; and  $1 - (m + 1)/(n - 1)$  is the probability that another member  $l$  offered zero in member 1's proposal is recognized and offers an amendment  $\hat{y}^l$ . If member  $l$  who was offered zero in  $\hat{y}^1$  is recognized, he or she will make a similar proposal that defeats  $\hat{y}^1$ , and that proposal  $\hat{y}^l$  will allocate  $(1 - \hat{y}^a)/m$  to  $j$ ; so the value  $V_j^m(\hat{y}^l)$  to  $j$  equals the value  $V_k^m(\hat{y}^1)$  to  $k$ , given  $\hat{y}^1$ . Thus,

$$V_j^m(\hat{y}^1) = \frac{1}{n-1} \delta V_1^m(\hat{y}^1) + (1 - \frac{m+1}{n-1}) \delta V_k^m(\hat{y}^1). \quad (\text{A-5})$$

We can write the expression for  $V_k^m(\hat{y}^1)$  as follows:

$$V_k^m(\hat{y}^1) = \frac{m}{n-1} \cdot \frac{1 - \hat{y}^a}{m} + (1 - \frac{m}{n-1}) \delta V_k^m(\hat{y}^l). \quad (\text{A-6})$$

Using relationship A-3, the expression for  $V_k^m(\hat{y}^l)$  can be seen to take the following form:

$$V_k^m(\hat{y}^l) = \frac{2m - n + 2}{m} \cdot$$

$$\begin{aligned} & [(\frac{m}{n-1})\delta V_1^m(\hat{y}^1) \\ & + (1 - \frac{m}{n-1})\delta V_k^m(\hat{y}^1)] \\ & + \frac{n-m-2}{m} [\frac{m}{n-1} \cdot 0 \\ & + \frac{1}{n-1} \delta V_1^m(\hat{y}^1) \\ & + (1 - \frac{m+1}{n-1})\delta V_k^m(\hat{y}^1)], \quad (\text{A-7}) \end{aligned}$$

where, with probability  $(2m - n + 2)/m$ , member  $k$  will receive  $(1 - \hat{y}^a)/m = \delta V_1^m(\hat{y}^1)$  in  $\hat{y}^1$ , which in turn will be accepted with probability  $m/(n - 1)$  or rejected with probability  $1 - m/(n - 1)$ . In the latter case  $k$  is in the same position as before, when  $\hat{y}^1$  was proposed, so  $k$ 's value is  $\delta V_k^m(\hat{y}^1)$ . With probability  $(n - m - 2)/m$ ,  $k$  will receive zero in  $\hat{y}^1$ , and then one of three possibilities will occur: with probability  $m/(n - 1)$ ,  $\hat{y}^1$  will be accepted, and  $k$  will receive 0; with probability  $1/(n - 1)$ ,  $k$  will be recognized to make a proposal, in which case he or she can be assured of  $\delta V_1^m(\hat{y}^1)$ ; and with probability  $1 - (m + 1)/(n - 1)$ , someone else who received zero in  $\hat{y}^1$  will be recognized and will make a proposal in which  $k$  will receive  $(1 - \hat{y}^a)/m$ , so that  $k$  will be in the same position as it was following the proposal of  $\hat{y}^1$ , and  $k$ 's value at that point will be  $\delta V_k^m(\hat{y}^1)$ .

Substituting equation A-6 into A-1 and rearranging terms allows us to solve for  $V_k^m(\hat{y}^1)$  in terms of  $V_1^m(\hat{y}^1)$ :

$$V_k^m(\hat{y}^1) = \gamma(\delta, m, n)V_1^m(\hat{y}^1), \quad (\text{A-8})$$

where

$$\begin{aligned} \gamma(\delta, m, n) = & \frac{\delta}{B} [(\frac{m}{n-1}) \\ & + \delta(1 - \frac{m}{n-1})(\frac{n-m-2}{m}) \end{aligned}$$

$$\begin{aligned} & (\frac{1}{n-1})] \\ \text{and} \\ B = & 1 - (\frac{2m-n+2}{m})(1 - \frac{m}{n-1})\delta \\ & - (\frac{n-m-2}{m})(1 - \frac{m+1}{n-1}) \\ & (1 - \frac{m}{n-1})\delta^2. \end{aligned}$$

Then, substituting equations A-5 and A-8 and relationship A-3 into A-4 yields a closed form expression for  $V_1^m(\hat{y}^1)$  or

$$\begin{aligned} V_1^m(\hat{y}) = & (\frac{m}{n-1}) / \{1 + \delta(\frac{m^2}{n-1}) \\ & - \delta^2(1 - \frac{m}{n-1})[(\frac{1}{n-1}) \\ & + (1 - \frac{m+1}{n-1})\gamma(\delta, m, n)]\}. \quad (\text{A-9}) \end{aligned}$$

For  $m = n - 1$ , this simplifies to  $V_1^m(\hat{y}^1) = 1/[1 + \delta(n - 1)]$ . Member 1 will choose  $m = m(\delta, n)$  to maximize  $V_1^m(\hat{y}^1)$ , so  $m(\delta, n) \in \text{argmax}_m V_1^m(\hat{y}^1)$ . Then,  $\hat{y}^a$  is determined from relationship A-2 as  $\hat{y}^a = 1 - m(\delta, n)\delta V_1^m(\hat{y}^1)$ . The values  $V_k^m(\hat{y}^1)$  and  $V_j^m(\hat{y}^1)$  are obtained by substituting equation A-9 into A-8 and equations A-9 and A-8 into A-5, respectively. QED

## Notes

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1. See Sutton 1986 for a survey of these results.
2. If a unanimity rule is employed and if members are recognized sequentially, the same result is obtained by Dutta and Gevers (1981) in a finite session legislative model and by Herrero (1985) in a stationary equilibrium for an infinite number of sessions.

## Bargaining in Legislatures

3. Other recognition rules are discussed briefly later.

4. Germaneness thus may be thought of as requiring proposals and amendments to be in the set  $X$ .

5. See Oleszek 1984.

6. That is, second-degree amendments are not allowed.

7. In the terminology of Ferejohn, Fiorina, and McKelvey (1987) such amendment rules exhibit a "depth" limitation. They are properly termed open rules because there is no limit to the number of amendments that may be offered before any proposal is brought to a vote.

8. See Moulin 1979.

9. If a member votes for a bill only if the member strictly prefers it, the distributions necessary to achieve a majority lie in an open set. Then an equilibrium does not exist, since for any distribution that achieves a majority, there is another distribution that also achieves a majority and is better for the proposer. To avoid these complications and the use of an equilibrium concept such as an  $\epsilon$ -equilibrium, a member who is indifferent between a bill and continuing to the next session is assumed to vote for the bill.

10. The power of the proposer in this setting is investigated in more detail in Baron and Ferejohn 1989.

11. This observation is due to Douglas Rivers.

12. This observation has been made by Herrero (1985) who considered  $n$ -person bargaining under a unanimity rule and sequential recognition.

13. While we take the reelection probability to be exogenous for the present argument, the same caveat would apply to a more complete treatment of reelection.

14. Stationary equilibria have been studied by Herrero in the context of  $n$ -person bargaining under a unanimity rule. Binmore (1985) shows that if the strategies are required to satisfy a continuity condition on the histories, there is a unique equilibrium and the equilibrium strategies are stationary.

15. A stationary equilibrium provides a complete specification of the strategies of members on the equilibrium path. If a move off-the-equilibrium path is made, members are assumed to believe that a "mistake" has been made and that mistakes will not be made in subsequent play. Members will thus base their responses on the equilibrium path strategies.

16. Instead of randomizing among the members, a strategy configuration such that each member receives an offer of  $\delta/n$  from exactly  $(n-1)/2$  other members (e.g., the  $(n-1)/2$  members on the left) is also a stationary equilibrium.

17. Dutta and Gevers (1981) also obtain this result.

18. One feature of government formation in parliamentary systems is that failure of a vote of confidence can dissolve the government. The assumption

here is that an acceptable allocation of ministries is sufficient to maintain the government.

19. The analysis can be generalized to the case in which the probabilities of recognition are different at each recognition.

20. With the interpretation of the discount factor representing impatience rather than a reelection probability, the model can be understood as allowing a number  $a$  of amendments per session; that is, the discount factor can be expressed as  $\delta(a)$ , where  $\delta(a) = \exp^{-i/a}$  and  $i$  is the time preference or interest rate. Then,  $a = 1$  represents one amendment and vote in each session, which corresponds to the case of a closed rule or  $\delta = \delta(1)$ . If an open rule allows a number  $a > 1$  amendments and votes per session, then  $\delta(a) > \delta$ .

21. Thus, there is never consecutive recognition.

22. Otherwise, the set of amendments that defeat the proposal on the floor is open, and the equilibrium concept has to be weakened to that of an  $\epsilon$ -equilibrium.

23. That is, under a closed rule and under an open rule the ex ante values of the game are equal for each member.

24. Gilligan and Krehbiel (1987, 1989) provide another explanation for the use of a restrictive rule. They demonstrate that the parent chamber may grant a closed rule to generate incentives for a committee to develop information and expertise that the parent body can use to its benefit. The benefits from the information in their theory can offset the strategic advantage the closed rule gives to the committee.

25. See Harrington 1987 for a study of the effect of risk aversion in a bargaining context.

26. See Baron and Ferejohn 1989.

27. See Epple and Riordan 1987.

28. The construction in this proof is related to that in Herrero 1985.

29. To see this, note that member  $j$  who has  $y_j^i = 0$  will vote for  $y^i$  if  $y_j^i \geq 0$  and member  $k$  who has  $y_k^i = (1 - y^a)/m$  will vote for  $y^i$  if  $y_k^i \geq (1 - y^a)/m$ .

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David P. Baron is Professor at the Graduate School of Business and John A. Ferejohn is Senior Fellow, Hoover Institution, and Professor of Political Science, Stanford University, Stanford, CA 94305.