# Building Java Programs
# Chapter 12

Recursion

# Recursion

- **recursion**: The definition of an operation in terms of itself.
  - Solving a problem using recursion depends on solving smaller occurrences of the same problem.

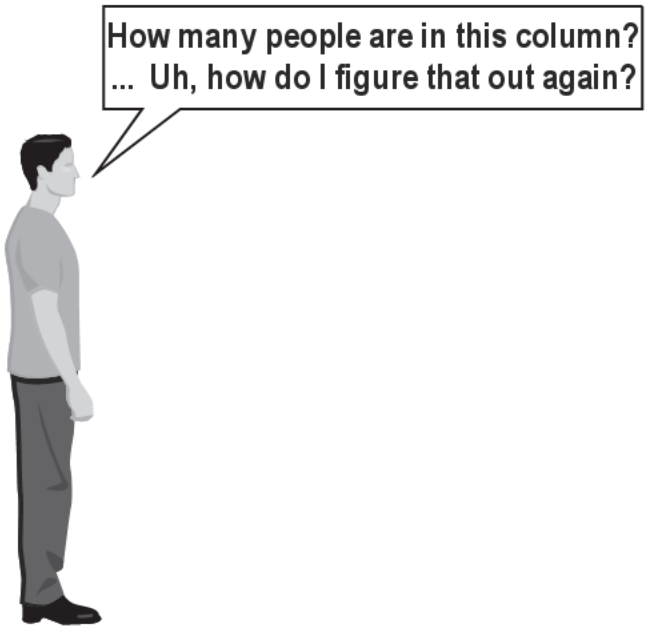
- **recursive programming**: Writing methods that call themselves to solve problems recursively.
  - An equally powerful substitute for *iteration* (loops)
  - Particularly well-suited to solving certain types of problems

# Why learn recursion?

- "cultural experience" - A different way of thinking of problems
- Can solve some kinds of problems better than iteration
- Leads to elegant, simplistic, short code (when used well)
- Many programming languages ("functional" languages such as Scheme, ML, and Haskell) use recursion exclusively (no loops)

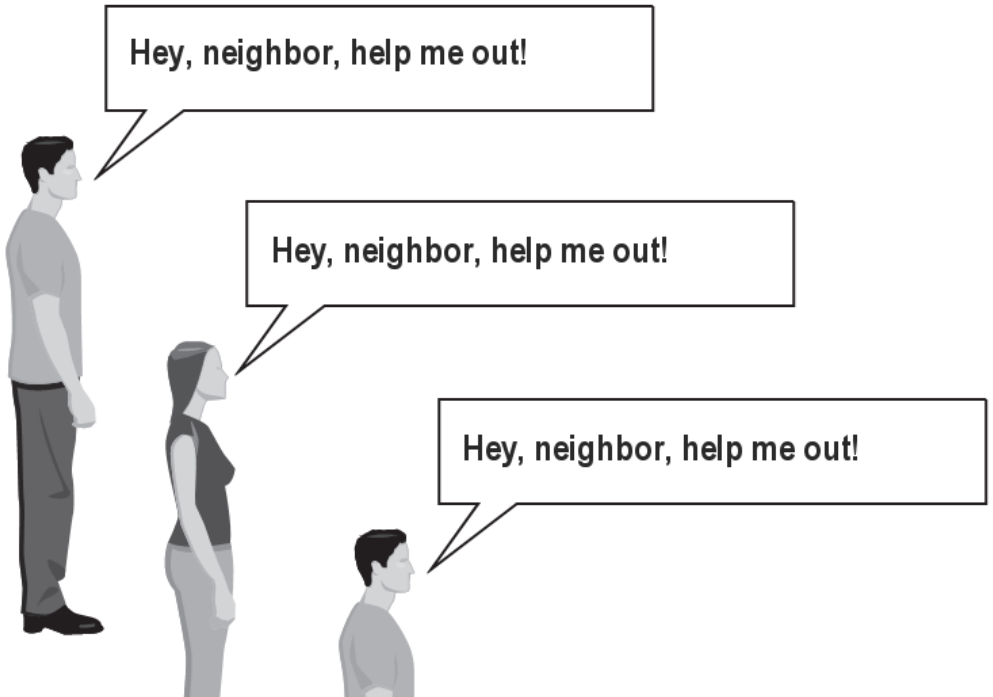
# Exercise

- (To a student in the front row)
  How many students total are directly behind you in your "column" of the classroom?
  - You have poor vision, so you can see only the people right next to you. So you can't just look back and count.
  - But you are allowed to ask questions of the person next to you.
  - How can we solve this problem? (*recursively* )



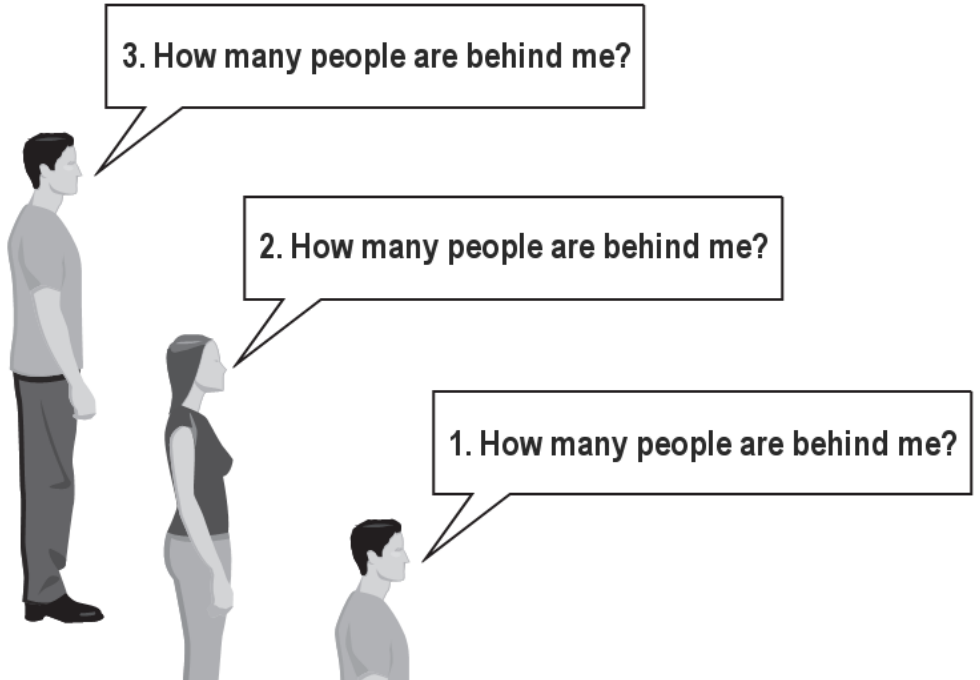
# The idea

- Recursion is all about breaking a big problem into smaller occurrences of that same problem.
  - Each person can solve a small part of the problem.
    - What is a small version of the problem that would be easy to answer?
    - What information from a neighbor might help me?



# Recursive algorithm

- Number of people behind me:
  - If there is someone behind me,
    ask him/her how many people are behind him/her.
    - When they respond with a value **N**, then I will answer **N + 1**.
  - If there is nobody behind me, I will answer **0**.



# Recursion and cases

- Every recursive algorithm involves at least 2 cases:
  - **base case**: A simple occurrence that can be answered directly.
  - **recursive case**: A more complex occurrence of the problem that cannot be directly answered, but can instead be described in terms of smaller occurrences of the same problem.
  - Some recursive algorithms have more than one base or recursive case, but all have at least one of each.
  - A crucial part of recursive programming is identifying these cases.

# Recursion in Java

- Consider the following method to print a line of * characters:

```
// Prints a line containing the given number of stars.
// Precondition: n >= 0
public static void printStars(int n) {
    for (int i = 0; i < n; i++) {
        System.out.print("*");
    }
    System.out.println();   // end the line of output
}
```

- Write a recursive version of this method (that calls itself).
  - Solve the problem without using any loops.
  - Hint: Your solution should print just one star at a time.

# A basic case

- What are the cases to consider?
  - What is a very easy number of stars to print without a loop?

```
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else {
        ...
    }
}
```

# Handling more cases

- Handling additional cases, with no loops (in a bad way):

```
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else if (n == 2) {
        System.out.print("*");
        System.out.println("*");
    } else if (n == 3) {
        System.out.print("*");
        System.out.print("*");
        System.out.println("*");
    } else if (n == 4) {
        System.out.print("*");
        System.out.print("*");
        System.out.print("*");
        System.out.println("*");
    } else ...
}
```

# Handling more cases 2

- Taking advantage of the repeated pattern (somewhat better):

```
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else if (n == 2) {
        System.out.print("*");
        printStars(1);     // prints "*"
    } else if (n == 3) {
        System.out.print("*");
        printStars(2);     // prints "**"
    } else if (n == 4) {
        System.out.print("*");
        printStars(3);     // prints "***"
    } else ...
}
```

# Using recursion properly

- Condensing the recursive cases into a single case:

```
public static void printStars(int n) {
    if (n == 1) {
        // base case; just print one star
        System.out.println("*");
    } else {
        // recursive case; print one more star
        System.out.print("*");
        printStars(n - 1);
    }
}
```

# "Recursion Zen"

- The real, even simpler, base case is an `n` of 0, not 1:

```
public static void printStars(int n) {
    if (n == 0) {
        // base case; just end the line of output
        System.out.println();
    } else {
        // recursive case; print one more star
        System.out.print("*");
        printStars(n - 1);
    }
}
```

  - **Recursion Zen**: The art of properly identifying the best set of cases for a recursive algorithm and expressing them elegantly.

# Recursive tracing

- Consider the following recursive method:

```
public static int mystery(int n) {
    if (n < 10) {
        return n;
    } else {
        int a = n / 10;
        int b = n % 10;
        return mystery(a + b);
    }
}
```

  - What is the result of the following call?

```
mystery(648)
```

# A recursive trace

mystery(648):

- `int a = 648 / 10;` `// 64`
- `int b = 648 % 10;` `//  8`
- `return mystery(a + b);` `// mystery(72)`

mystery(72):

- `int a = 72 / 10;` `// 7`
- `int b = 72 % 10;` `// 2`
- `return mystery(a + b);` `// mystery(9)`

mystery(9):

- `return 9;`

# Recursive tracing 2

- Consider the following recursive method:

```
public static int mystery(int n) {
    if (n < 10) {
        return (10 * n) + n;
    } else {
        int a = mystery(n / 10);
        int b = mystery(n % 10);
        return (100 * a) + b;
    }
}
```

  - What is the result of the following call?

```
mystery(348)
```

# A recursive trace 2

`mystery(348)`

- `int a = mystery(34);`

```
• int a = mystery(3);
    return (10 * 3) + 3;    // 33
• int b = mystery(4);
    return (10 * 4) + 4;    // 44
• return (100 * 33) + 44;   // 3344
```

- `int b = mystery(8);`

```
return (10 * 8) + 8;        // 88
```

- `return (100 * 3344) + 88;   // 334488`

– What is this method really doing?

# Exercise

- Write a recursive method `pow` accepts an integer base and exponent and returns the base raised to that exponent.
  - Example: `pow(3, 4)` returns 81
  - Solve the problem recursively and without using loops.

# pow solution

```
// Returns base ^ exponent.
// Precondition: exponent >= 0
public static int pow(int base, int exponent) {
    if (exponent == 0) {
        // base case; any number to 0th power is 1
        return 1;
    } else {
        // recursive case:  x^y = x * x^(y-1)
        return base * pow(base, exponent - 1);
    }
}
```

# Exercise

- Write a recursive method `printBinary` that accepts an integer and prints that number's representation in binary (base 2).
  - Example: `printBinary(7)` prints 111
  - Example: `printBinary(12)` prints 1100
  - Example: `printBinary(42)` prints 101010

| place | 10 | 1 |
|---|---|---|
| value | **4** | **2** |

| 32 | 16 | 8 | 4 | 2 | 1 |
|---|---|---|---|---|---|
| **1** | **0** | **1** | **0** | **1** | **0** |

  - Write the method recursively and without using any loops.

# Case analysis

- Recursion is about solving a small piece of a large problem.
  - What is 69743 in binary?
    - Do we know *anything* about its representation in binary?
  - Case analysis:
    - What is/are easy numbers to print in binary?
    - Can we express a larger number in terms of a smaller number(s)?
  - Suppose we are examining some arbitrary integer `N`.
    - if `N`'s binary representation is **10010101011**
    - `(N / 2)`'s binary representation is **1001010101**
    - `(N % 2)`'s binary representation is **1**

# printBinary solution

```
// Prints the given integer's binary representation.
// Precondition: n >= 0
public static void printBinary(int n) {
    if (n < 2) {
        // base case; same as base 10
        System.out.println(n);
    } else {
        // recursive case; break number apart
        printBinary(n / 2);
        printBinary(n % 2);
    }
}
```

– Can we eliminate the precondition and deal with negatives?

# Exercise

- Write a recursive method `isPalindrome` accepts a `String` and returns `true` if it reads the same forwards as backwards.
  - `isPalindrome("madam")` → `true`
  - `isPalindrome("racecar")` → `true`
  - `isPalindrome("step on no pets")` → `true`
  - `isPalindrome("able was I ere I saw elba")` → `true`
  - `isPalindrome("Java")` → `false`
  - `isPalindrome("rotater")` → `false`
  - `isPalindrome("byebye")` → `false`
  - `isPalindrome("notion")` → `false`

# Exercise solution

```
// Returns true if the given string reads the same
// forwards as backwards.
// Trivially true for empty or 1-letter strings.
public static boolean isPalindrome(String s) {
    if (s.length() < 2) {
        return true;   // base case
    } else {
        char first = s.charAt(0);
        char last  = s.charAt(s.length() - 1);
        if (first != last) {
            return false;
        }              // recursive case
        String middle = s.substring(1, s.length() - 1);
        return isPalindrome(middle);
    }
}
```

# Exercise solution 2

```
// Returns true if the given string reads the same
// forwards as backwards.
// Trivially true for empty or 1-letter strings.
public static boolean isPalindrome(String s) {
    if (s.length() < 2) {
        return true;   // base case
    } else {
        return s.charAt(0) == s.charAt(s.length() - 1)
            && isPalindrome(s.substring(1, s.length() - 1));
    }
}
```

# Exercise

- Write a recursive method `reverseLines` that accepts a file `Scanner` and prints the lines of the file in reverse order.
  - Example input file:

```
Roses are red,
Violets are blue.
All my base
Are belong to you.
```

Expected console output:

```
Are belong to you.
All my base
Violets are blue.
Roses are red,
```

  - What are the cases to consider?
    - How can we solve a small part of the problem at a time?
    - What is a file that is very easy to reverse?

# Reversal pseudocode

- Reversing the lines of a file:
  - Read a line L from the file.
  - Print the rest of the lines in reverse order.
  - Print the line L.

- If only we had a way to reverse the rest of the lines of the file....

# Reversal solution

```
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {
        // recursive case
        String line = input.nextLine();
        reverseLines(input);
        System.out.println(line);
    }
}
```

- Where is the base case?

# Tracing our algorithm

- **call stack**: The method invocations running at any one time.

```
reverseLines(new Scanner("poem.txt"));
```

```
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {
        String line = input.nextLine();  // "Roses are red,"
        reverseLines(input);
        System.out.println(line);
    }
}
```

```
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {
        String line = input.nextLine();  // "Violets are blue."
        reverseLines(input);
        System.out.println(line);
    }
}
```

```
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {
        String line = input.nextLine();  // "All my base"
        reverseLines(input);
        System.out.println(line);
    }
}
```

# Tracing our algorithm

```
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {
        String line = input.nextLine();  // "Are belong to you."
        reverseLines(input);
        System.out.println(line);
    }
}
```

```
public static void reverseLines(Scanner input) {
    if (input.hasNextLine()) {   // false
        ...
    }
}
```

```
Roses are red,
Violets are blue.
All my base
Are belong to you.
```

```
Are belong to you.
All my base
Violets are blue.
Roses are red,
```

# Exercise

- Write a method `crawl` accepts a `File` parameter and prints information about that file.
  - If the `File` object represents a normal file, just print its name.
  - If the `File` object represents a directory, print its name and information about every file/directory inside it, indented.

```
csc2010
    handouts
        syllabus.doc
        lecture_schedule.xls
    homework
        1-sortedintlist
            ArrayIntList.java
            SortedIntList.java
            index.html
            style.css
```

  - **recursive data**: A directory can contain other directories.

# `File` objects

- A `File` object (from the `java.io` package) represents a file or directory on the disk.

| Constructor/method | Description |
|---|---|
| `File(`**String**`)` | creates `File` object representing file with given name |
| `canRead()` | returns whether file is able to be read |
| `delete()` | removes file from disk |
| `exists()` | whether this file exists on disk |
| `getName()` | returns file's name |
| `isDirectory()` | returns whether this object represents a directory |
| `length()` | returns number of bytes in file |
| `listFiles()` | returns a `File[]` representing files in this directory |
| `renameTo(`**File**`)` | changes name of file |

# Public/private pairs

- We cannot vary the indentation without an extra parameter:

```
public static void crawl(File f, String indent) {
```

- Often the parameters we need for our recursion do not match those the client will want to pass.

  In these cases, we instead write a pair of methods:

  1) a public, non-recursive one with the parameters the client wants
  2) a private, recursive one with the parameters we really need

# Exercise solution 2

```
// Prints information about this file,
// and (if it is a directory) any files inside it.
public static void crawl(File f) {
    crawl(f, "");   // call private recursive helper
}

// Recursive helper to implement crawl/indent behavior.
private static void crawl(File f, String indent) {
    System.out.println(indent + f.getName());
    if (f.isDirectory()) {
        // recursive case; print contained files/dirs
        for (File subFile : f.listFiles()) {
            crawl(subFile, indent + "    ");
        }
    }
}
```

# Recursive Backtracking

# Exercise: Permutations

- Write a method `permute` that accepts a string as a parameter and outputs all possible rearrangements of the letters in that string. The arrangements may be output in any order.
  - Example: `permute("MARTY")` outputs the following sequence of lines:

| | | | | | |
|---|---|---|---|---|---|
| MARTY | MYRAT | ATYMR | RTMAY | TARMY | YMTAR |
| MARYT | MYRTA | ATYRM | RTMYA | TARYM | YMTRA |
| MATRY | MYTAR | AYMRT | RTAMY | TAYMR | YAMRT |
| MATYR | MYTRA | AYMTR | RTAYM | TAYRM | YAMTR |
| MAYRT | AMRTY | AYRMT | RTYMA | TRMAY | YARMT |
| MAYTR | AMRYT | AYRTM | RTYAM | TRMYA | YARTM |
| MRATY | AMTRY | AYTMR | RYMAT | TRAMY | YATMR |
| MRAYT | AMTYR | AYTRM | RYMTA | TRAYM | YATRM |
| MRTAY | AMYRT | RMATY | RYAMT | TRYMA | YRMAT |
| MRTYA | AMYTR | RMAYT | RYATM | TRYAM | YRMTA |
| MRYAT | ARMTY | RMTAY | RYTMA | TYMAR | YRAMT |
| MRYTA | ARMYT | RMTYA | RYTAM | TYMRA | YRATM |
| MTARY | ARTMY | RMYAT | TMARY | TYAMR | YRTMA |
| MTAYR | ARTYM | RMYTA | TMAYR | TYARM | YRTAM |
| MTRAY | ARYMT | RAMTY | TMRAY | TYRMA | YTMAR |
| MTRYA | ARYTM | RAMYT | TMRYA | TYRAM | YTMRA |
| MTYAR | ATMRY | RATMY | TMYAR | YMART | YTAMR |
| MTYRA | ATMYR | RATYM | TMYRA | YMATR | YTARM |
| MYART | ATRMY | RAYMT | TAMRY | YMRAT | YTRMA |
| MYATR | ATRYM | RAYTM | TAMYR | YMRTA | YTRAM |

# Examining the problem

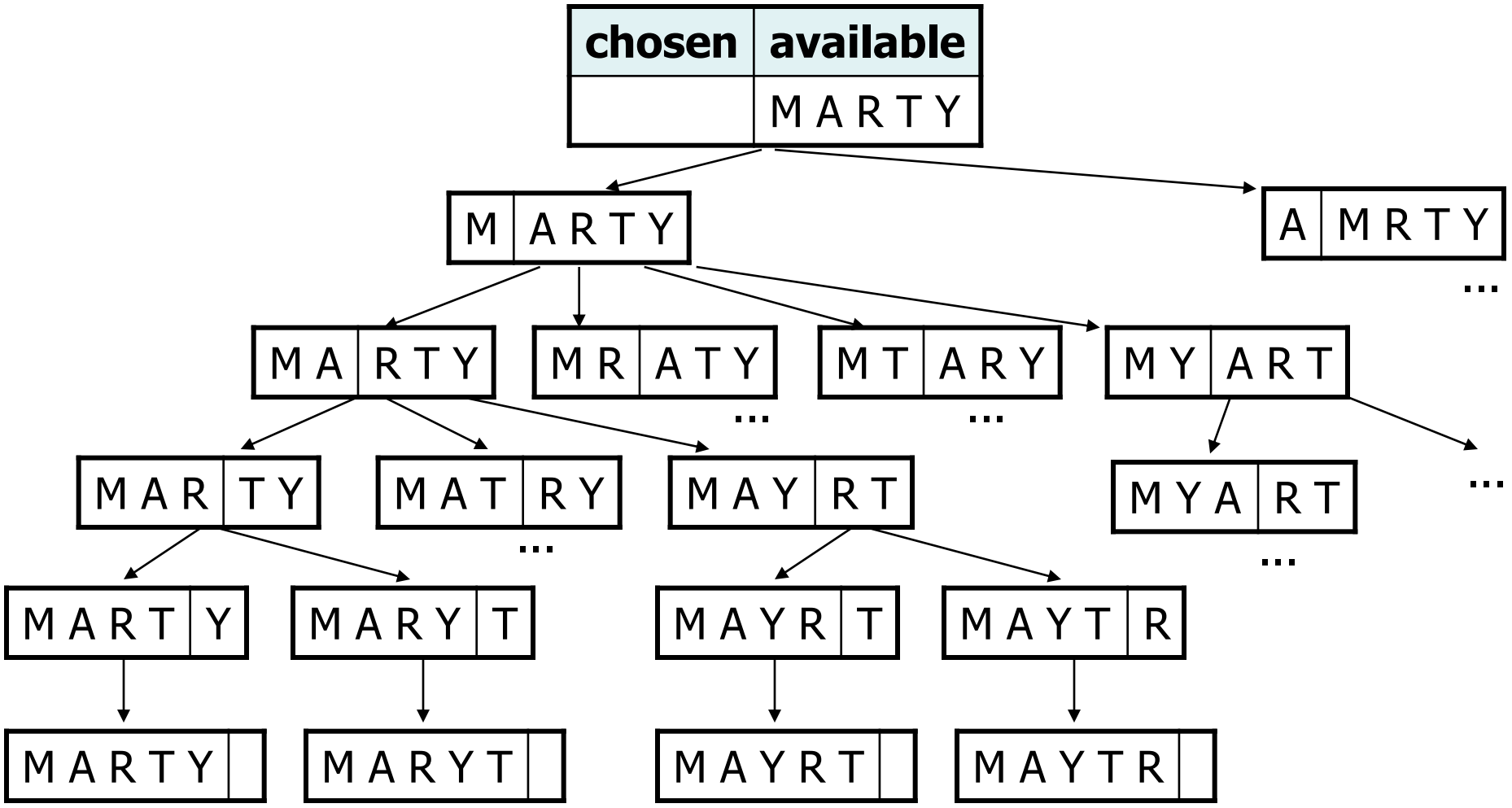
- Think of each permutation as a set of choices or **decisions**:
  - Which character do I want to place first?
  - Which character do I want to place second?
  - ...
  - **solution space**: set of all possible sets of decisions to explore

- We want to generate all possible sequences of decisions.

```
for (each possible first letter):
    for (each possible second letter):
        for (each possible third letter):
            ...
                print!
```

  - This is called a **depth-first search**

# Decision trees



# Backtracking

- **backtracking**: A general algorithm for finding solution(s) to a computational problem by trying partial solutions and then abandoning them ("backtracking") if they are not suitable.
  - a "brute force" algorithmic technique  (tries all paths; not clever)
  - often (but not always) implemented recursively

  Applications:
  - producing all permutations of a set of values
  - parsing languages
  - games: anagrams, crosswords, word jumbles, 8 queens
  - combinatorics and logic programming

# Backtracking algorithms

*A general pseudo-code algorithm for backtracking problems:*

explore(**choices**):

- if there are no more **choices** to make: stop.
- else:
  - Make a single choice **C** from the set of choices.
    - Remove **C** from the set of **choices**.
  - explore the remaining **choices**.
  - Un-make choice **C**.
    - Backtrack!

# Backtracking strategies

- When solving a backtracking problem, ask these questions:
  - What are the "choices" in this problem?
    - What is the "base case"?  (How do I know when I'm out of choices?)
  - How do I "make" a choice?
    - Do I need to create additional variables to remember my choices?
    - Do I need to modify the values of existing variables?
  - How do I explore the rest of the choices?
    - Do I need to remove the made choice from the list of choices?
  - Once I'm done exploring the rest, what should I do?
  - How do I "un-make" a choice?

# Permutations revisited

- Write a method `permute` that accepts a string as a parameter and outputs all possible rearrangements of the letters in that string. The arrangements may be output in any order.
  - Example: `permute("MARTY")` outputs the following sequence of lines:

```
MARTY  MYRAT  ATYMR  RTMAY  TARMY  YMTAR
MARYT  MYRTA  ATYRM  RTMYA  TARYM  YMTRA
MATRY  MYTAR  AYMRT  RTAMY  TAYMR  YAMRT
MATYR  MYTRA  AYMTR  RTAYM  TAYRM  YAMTR
MAYRT  AMRTY  AYRMT  RTYMA  TRMAY  YARMT
MAYTR  AMRYT  AYRTM  RTYAM  TRMYA  YARTM
MRATY  AMTRY  AYTMR  RYMAT  TRAMY  YATMR
MRAYT  AMTYR  AYTRM  RYMTA  TRAYM  YATRM
MRTAY  AMYRT  RMATY  RYAMT  TRYMA  YRMAT
MRTYA  AMYTR  RMAYT  RYATM  TRYAM  YRMTA
MRYAT  ARMTY  RMTAY  RYTMA  TYMAR  YRAMT
MRYTA  ARMYT  RMTYA  RYTAM  TYMRA  YRATM
MTARY  ARTMY  RMYAT  TMARY  TYAMR  YRTMA
MTAYR  ARTYM  RMYTA  TMAYR  TYARM  YRTAM
MTRAY  ARYMT  RAMTY  TMRAY  TYRMA  YTMAR
MTRYA  ARYTM  RAMYT  TMRYA  TYRAM  YTMRA
MTYAR  ATMRY  RATMY  TMYAR  YMART  YTAMR
MTYRA  ATMYR  RATYM  TMYRA  YMATR  YTARM
MYART  ATRMY  RAYMT  TAMRY  YMRAT  YTRMA
MYATR  ATRYM  RAYTM  TAMYR  YMRTA  YTRAM
```

# Exercise solution

```
// Outputs all permutations of the given string.
public static void permute(String s) {
    permute(s, "");
}

private static void permute(String s, String chosen) {
    if (s.length() == 0) {
        // base case: no choices left to be made
        System.out.println(chosen);
    } else {
        // recursive case: choose each possible next letter
        for (int i = 0; i < s.length(); i++) {
            char c = s.charAt(i);                    // choose
            s = s.substring(0, i) + s.substring(i + 1);
            chosen += c;

            permute(s, chosen);                      // explore

            s = s.substring(0, i) + c + s.substring(i + 1);
            chosen = chosen.substring(0, chosen.length() - 1);
        }                                            // un-choose
    }
}
```

# Exercise solution 2

```
// Outputs all permutations of the given string.
public static void permute(String s) {
    permute(s, "");
}

private static void permute(String s, String chosen) {
    if (s.length() == 0) {
        // base case: no choices left to be made
        System.out.println(chosen);
    } else {
        // recursive case: choose each possible next letter
        for (int i = 0; i < s.length(); i++) {
            String ch = s.substring(i, i + 1);   // choose
            String rest = s.substring(0, i) +    // remove
                          s.substring(i + 1);

            permute(rest, chosen + ch);          // explore
        }
    }        // (don't need to "un-choose" because
}            //  we used temp variables)
```

# Exercise: Combinations

- Write a method `combinations` that accepts a string *s* and an integer *k* as parameters and outputs all possible *k*-letter words that can be formed from unique letters in that string. The arrangements may be output in any order.
  - Example:
    `combinations("GOOGLE", 3)`
    outputs the sequence of lines at right.

```
EGL  LEG
EGO  LEO
ELG  LGE
ELO  LGO
EOG  LOE
EOL  LOG
GEL  OEG
GEO  OEL
GLE  OGE
GLO  OGL
GOE  OLE
GOL  OLG
```

  - To simplify the problem, you may assume that the string *s* contains at least *k* unique characters.

# Initial attempt

```
public static void combinations(String s, int length) {
    combinations(s, "", length);
}

private static void combinations(String s, String chosen, int length) {
    if (length == 0) {
        System.out.println(chosen);    // base case: no choices left
    } else {
        for (int i = 0; i < s.length(); i++) {
            String ch = s.substring(i, i + 1);
            if (!chosen.contains(ch)) {
                String rest = s.substring(0, i) + s.substring(i + 1);
                combinations(rest, chosen + ch, length - 1);
            }
        }
    }
}
```

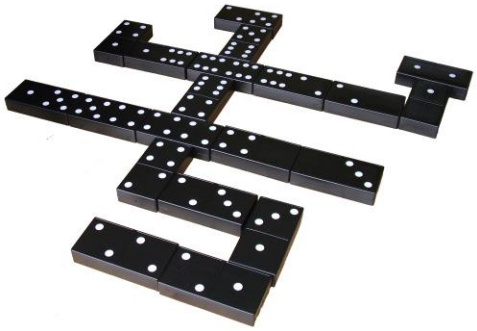
- Problem: Prints same string multiple times.

# Exercise solution

```
public static void combinations(String s, int length) {
    Set<String> all = new TreeSet<String>();
    combinations(s, "", all, length);
    for (String comb : all) {
        System.out.println(comb);
    }
}

private static void combinations(String s, String chosen,
                                 Set<String> all, int length) {
    if (length == 0) {
        all.add(chosen);          // base case: no choices left
    } else {
        for (int i = 0; i < s.length(); i++) {
            String ch = s.substring(i, i + 1);
            if (!chosen.contains(ch)) {
                String rest = s.substring(0, i) + s.substring(i + 1);
                combinations(rest, chosen + ch, all, length - 1);
            }
        }
    }
}
```

# Exercise: Dominoes



- The game of dominoes is played with small black tiles, each having 2 numbers of dots from 0-6. Players line up tiles to match dots.

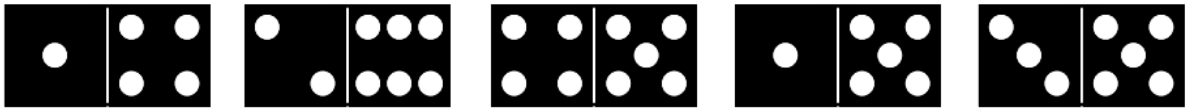
- Given a class `Domino` with the following methods:

```
public int first()              // first dots value
public int second()             // second dots value
public String toString()        // e.g. "(3|5)"
```

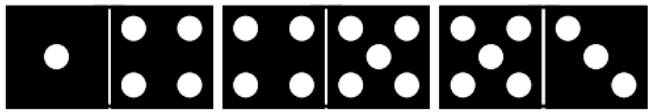
- Write a method `hasChain` that takes a `List` of dominoes and a starting/ending dot value, and returns whether the dominoes can be made into a chain that starts/ends with those values.
  - If the chain's start/end are the same, the answer is always `true`.

# Domino chains

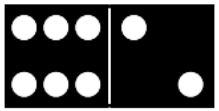
- Suppose we have the following dominoes:



- We can link them into a chain from 1 to 3 as follows:
  - Notice that the 3|5 domino had to be flipped.



- We can "link" one domino into a "chain" from 6 to 2 as follows:



# Exercise client code

```
import java.util.*;   // for ArrayList

public class SolveDominoes {
    public static void main(String[] args) {
        // [(1|4), (2|6), (4|5), (1|5), (3|5)]
        List<Domino> dominoes = new ArrayList<Domino>();
        dominoes.add(new Domino(1, 4));
        dominoes.add(new Domino(2, 6));
        dominoes.add(new Domino(4, 5));
        dominoes.add(new Domino(1, 5));
        dominoes.add(new Domino(3, 5));
        System.out.println(hasChain(dominoes, 5, 5));   // true
        System.out.println(hasChain(dominoes, 1, 5));   // true
        System.out.println(hasChain(dominoes, 1, 3));   // true
        System.out.println(hasChain(dominoes, 1, 6));   // false
        System.out.println(hasChain(dominoes, 1, 2));   // false
    }

    public static boolean hasChain(List<Domino> dominoes,
                                   int start, int end) {
        ...
    }
}
```

# Exercise solution

```
public static boolean hasChain(List<Domino> dominoes,
                               int start, int end) {
    if (start == end) {
        return true;                                  // base case
    } else {
        for (int i = 0; i < dominoes.size(); i++) {
            Domino d = dominoes.remove(i);    // choose
            if (d.first() == start) {         // explore
                if (hasChain(dominoes, d.second(), end)) {
                    return true;
                }
            } else if (d.second() == start) {
                if (hasChain(dominoes, d.first(), end)) {
                    return true;
                }
            }
            dominoes.add(i, d);               // un-choose
        }
        return false;
    }
}
```

# Exercise: Print chain

- Write a variation of your `hasChain` method that also prints the chain of dominoes that it finds, if any.

```
hasChain(dominoes, 1, 3);
```

```
[(1|4), (4|5), (5|3)]
```

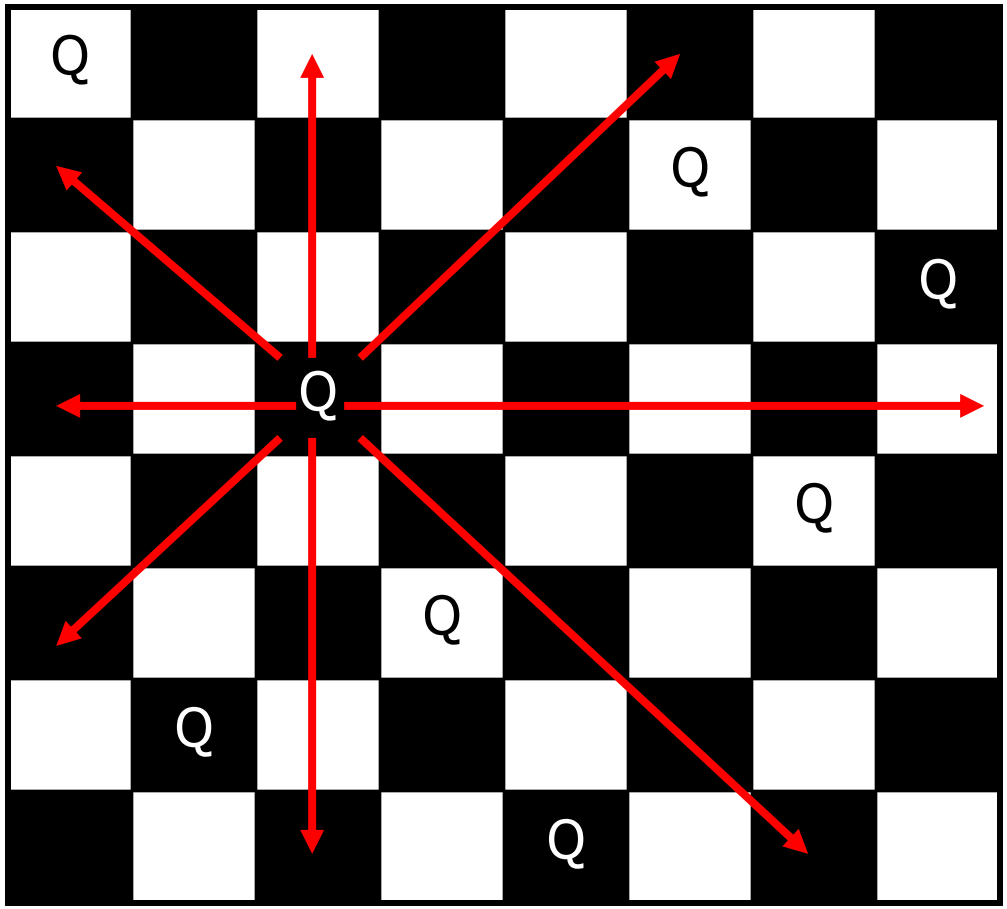
# Exercise solution

```
public static boolean hasChain(List<Domino> dominoes, int start, int end) {
    Stack<Domino> chosen = new Stack<Domino>();
    return hasChain(dominoes, chosen, start, end);
}

private static boolean hasChain(List<Domino> dominoes,
                                Stack<Domino> chosen, int start, int end) {
    if (start == end) {
        System.out.println(chosen);
        return true;                                  // base case
    } else {
        for (int i = 0; i < dominoes.size(); i++) {
            Domino d = dominoes.remove(i);            // choose
            if (d.first() == start) {                 // explore
                chosen.push(d);
                if (hasChain(dominoes, chosen, d.second(), end)) {
                    return true;
                }
                chosen.pop();
            } else if (d.second() == start) {
                d.flip();
                chosen.push(d);
                if (hasChain(dominoes, chosen, d.second(), end)) {
                    return true;
                }
                chosen.pop();
            }
            dominoes.add(i, d);                       // un-choose
        }
        return false;
    }
}
```

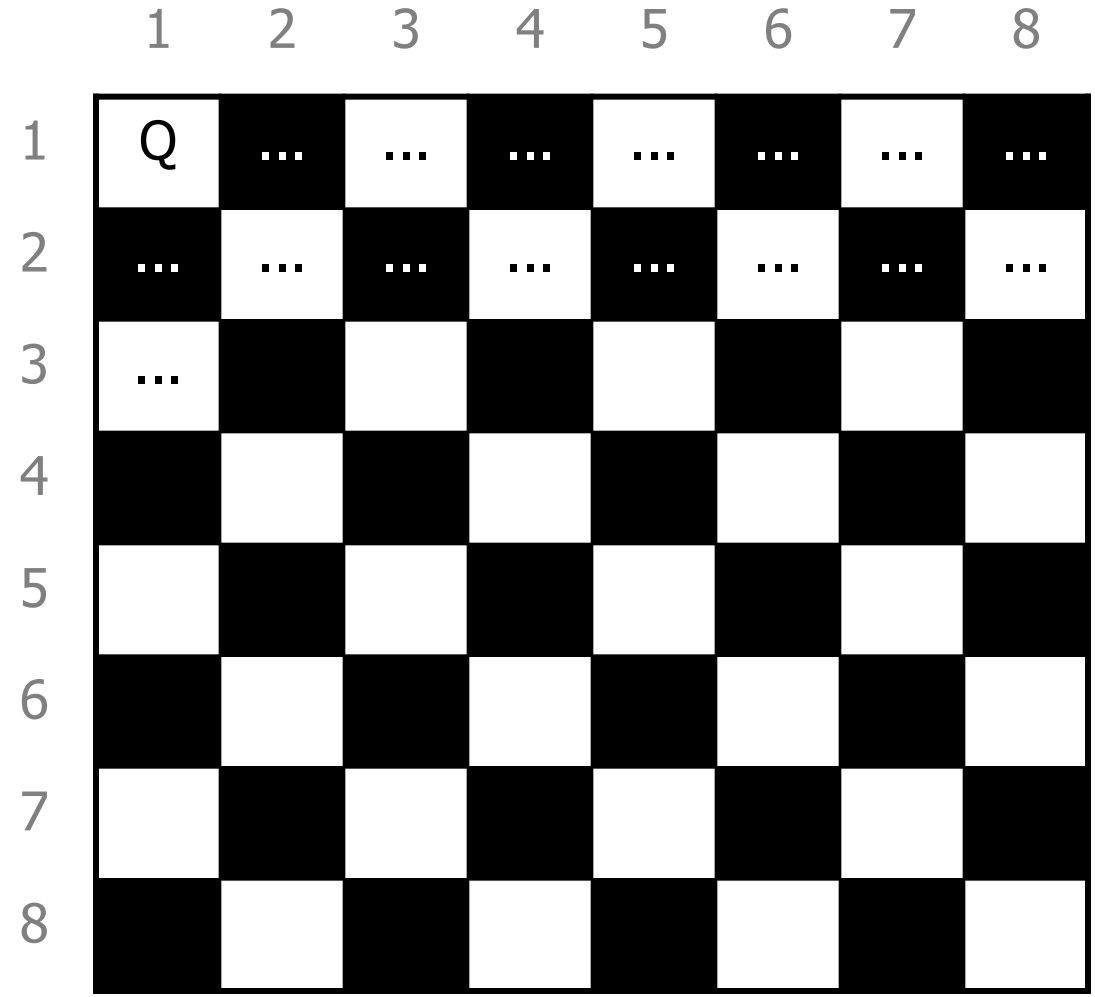
# The "8 Queens" problem

- Consider the problem of trying to place 8 queens on a chess board such that no queen can attack another queen.
  - What are the "choices"?
  - How do we "make" or "un-make" a choice?
  - How do we know when to stop?



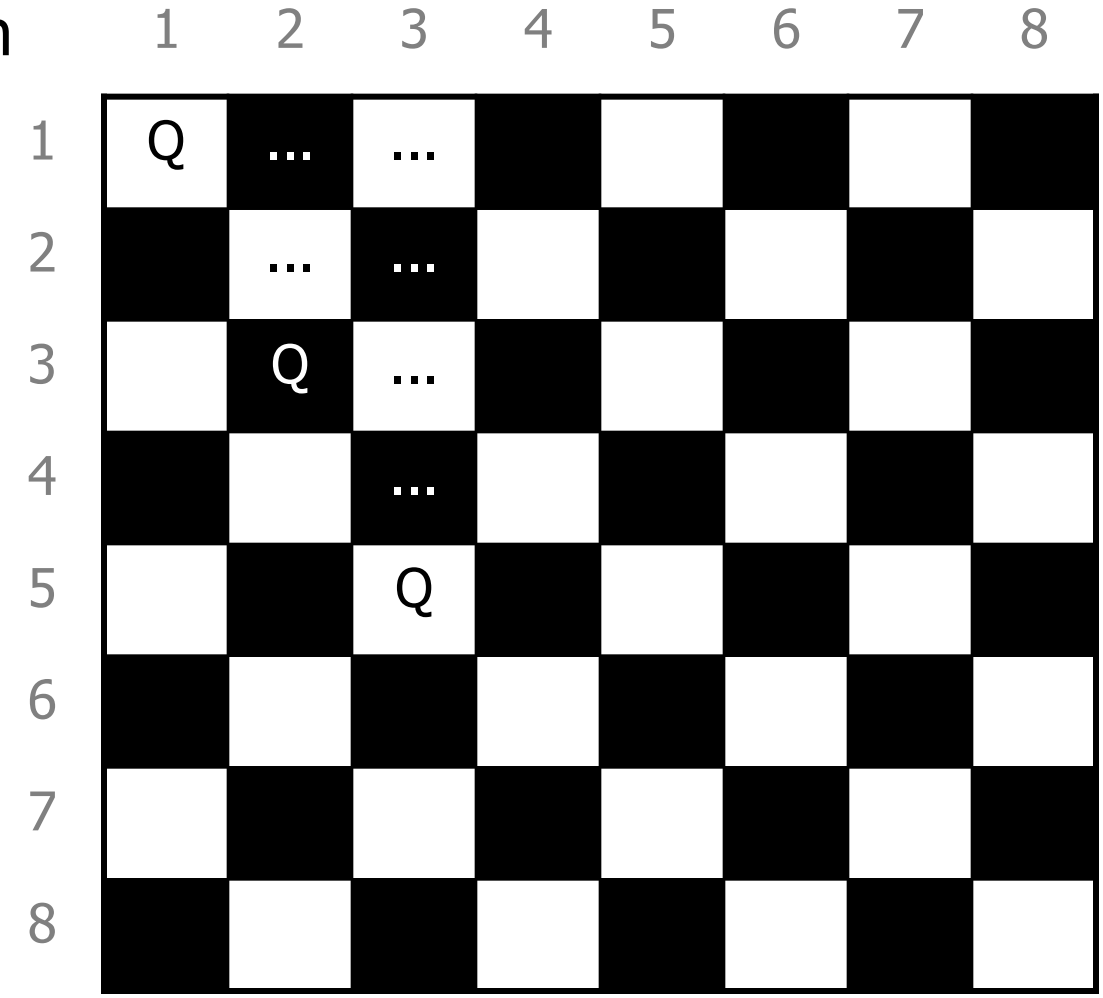
# Naive algorithm

- for (each square on board):
  - Place a queen there.
  - Try to place the rest of the queens.
  - Un-place the queen.
  - How large is the solution space for this algorithm?
    - 64 * 63 * 62 * ...



# Better algorithm idea

- Observation: In a working solution, exactly 1 queen must appear in each row and in each column.
  - Redefine a "choice" to be valid placement of a queen in a particular column.
  - How large is the solution space now?
    - 8 * 8 * 8 * ...



# Exercise

- Suppose we have a `Board` class with the following methods:

| Method/Constructor | Description |
|---|---|
| `public` **`Board`**`(int size)` | construct empty board |
| `public boolean` **`isSafe`**`(int row, int column)` | `true` if queen can be safely placed here |
| `public void` **`place`**`(int row, int column)` | place queen here |
| `public void` **`remove`**`(int row, int column)` | remove queen from here |
| `public String` **`toString`**`()` | text display of board |

- Write a method `solveQueens` that accepts a `Board` as a parameter and tries to place 8 queens on it safely.
  - Your method should stop exploring if it finds a solution.

# Exercise solution

```
// Searches for a solution to the 8 queens problem
// with this board, reporting the first result found.
public static void solveQueens(Board board) {
    if (!explore(board, 1)) {
        System.out.println("No solution found.");
    } else {
        System.out.println("One solution is as follows:");
        System.out.println(board);
    }
}

...
```

# Exercise solution, cont'd.

```
// Recursively searches for a solution to 8 queens on this
// board, starting with the given column, returning true if a
// solution is found and storing that solution in the board.
// PRE: queens have been safely placed in columns 1 to (col-1)
public static boolean explore(Board board, int col) {
    if (col > board.size()) {
        return true;   // base case: all columns are placed
    } else {
        // recursive case: place a queen in this column
        for (int row = 1; row <= board.size(); row++) {
            if (board.isSafe(row, col)) {
                board.place(row, col);              // choose
                if (explore(board, col + 1)) {      // explore
                    return true;   // solution found
                }
                b.remove(row, col);                 // un-choose
            }
        }
        return false;   // no solution found
    }
}
```