# Building Java Programs
# Chapter 13

Searching and Sorting

# Sequential search

- **sequential search**: Locates a target value in an array/list by examining each element from start to finish.
  - How many elements will it need to examine?
  - Example: Searching the array below for the value **42**:

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | -4 | 2 | 7 | 10 | 15 | 20 | 22 | 25 | 30 | 36 | 42 | 50 | 56 | 68 | 85 | 92 | 103 |

i

  - Notice that the array is sorted.  Could we take advantage of this?

# Binary search (13.1)

- **binary search**: Locates a target value in a *sorted* array/list by successively eliminating half of the array from consideration.
  - How many elements will it need to examine?
  - Example: Searching the array below for the value **42**:

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | -4 | 2 | 7 | 10 | 15 | 20 | 22 | 25 | 30 | 36 | 42 | 50 | 56 | 68 | 85 | 92 | 103 |

min

mid

max

# The `Arrays` class

- Class `Arrays` in `java.util` has many useful array methods:

| Method name | Description |
|---|---|
| `binarySearch(`**array**`, `**value**`)` | returns the index of the given value in a *sorted* array (or < 0 if not found) |
| `binarySearch(`**array**`,` **minIndex**`, `**maxIndex**`, `**value**`)` | returns index of given value in a *sorted* array between indexes *min* / *max* - 1 (< 0 if not found) |
| `copyOf(`**array**`, `**length**`)` | returns a new resized copy of an array |
| `equals(`**array1**`, `**array2**`)` | returns `true` if the two arrays contain same elements in the same order |
| `fill(`**array**`, `**value**`)` | sets every element to the given value |
| `sort(`**array**`)` | arranges the elements into sorted order |
| `toString(`**array**`)` | returns a string representing the array, such as `"[10, 30, -25, 17]"` |

- Syntax: `Arrays.`**methodName**`(`**parameters**`)`

# Arrays.binarySearch

```
// searches an entire sorted array for a given value
// returns its index if found;  a negative number if not found
// Precondition: array is sorted
Arrays.binarySearch(array, value)

// searches given portion of a sorted array for a given value
// examines minIndex (inclusive) through maxIndex (exclusive)
// returns its index if found;  a negative number if not found
// Precondition: array is sorted
Arrays.binarySearch(array, minIndex, maxIndex, value)
```

- The `binarySearch` method in the `Arrays` class searches an array very efficiently if the array is sorted.
  - You can search the entire array, or just a range of indexes (useful for "unfilled" arrays)
  - If the array is not sorted, you may need to sort it first

# Using `binarySearch`

```
// index    0  1  2  3   4   5   6   7   8   9  10  11  12  13  14  15
int[] a = {-4, 2, 7, 9, 15, 19, 25, 28, 30, 36, 42, 50, 56, 68, 85, 92};

int index1 = Arrays.binarySearch(a, 0, 16, 42);   // index1 is 10
int index2 = Arrays.binarySearch(a, 0, 16, 21);   // index2 is -7
```

- `binarySearch` returns the index where the value is found
- if the value is *not* found, `binarySearch` returns:

    `-(insertionPoint + 1)`

    - where `insertionPoint` is the index where the element *would* have been, if it had been in the array in sorted order.
    - To insert the value into the array, negate `insertionPoint` + 1

```
int indexToInsert21 = -(index2 + 1);  // 6
```

# Binary search code

```
// Returns the index of an occurrence of target in a,
// or a negative number if the target is not found.
// Precondition: elements of a are in sorted order
public static int binarySearch(int[] a, int target) {
    int min = 0;
    int max = a.length - 1;

    while (min <= max) {
        int mid = (min + max) / 2;
        if (a[mid] < target) {
            min = mid + 1;
        } else if (a[mid] > target) {
            max = mid - 1;
        } else {
            return mid;    // target found
        }
    }

    return -(min + 1);    // target not found
}
```

# Recursive binary search (13.3)

- Write a recursive `binarySearch` method.
  - If the target value is not found, return its negative insertion point.

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | -4 | 2 | 7 | 10 | 15 | 20 | 22 | 25 | 30 | 36 | 42 | 50 | 56 | 68 | 85 | 92 | 103 |

```
int index  = binarySearch(data, 42);   // 10
int index2 = binarySearch(data, 66);   // -14
```

# Exercise solution

```java
// Returns the index of an occurrence of the given value in
// the given array, or a negative number if not found.
// Precondition: elements of a are in sorted order
public static int binarySearch(int[] a, int target) {
    return binarySearch(a, target, 0, a.length - 1);
}

// Recursive helper to implement search behavior.
private static int binarySearch(int[] a, int target,
                                int min, int max) {
    if (min > max) {
        return -1;          // target not found
    } else {
        int mid = (min + max) / 2;
        if (a[mid] < target) {            // too small; go right
            return binarySearch(a, target, mid + 1, max);
        } else if (a[mid] > target) {  // too large; go left
            return binarySearch(a, target, min, mid - 1);
        } else {
            return mid;   // target found; a[mid] == target
        }
    }
}
```

# Binary search and objects

- Can we `binarySearch` an array of Strings?
  - Operators like `<` and `>` do not work with `String` objects.
  - But we do think of strings as having an alphabetical ordering.

- **natural ordering**: Rules governing the relative placement of all values of a given type.

- **comparison function**: Code that, when given two values *A* and *B* of a given type, decides their relative ordering:
  - A < B,     A == B,     A > B

# The `compareTo` method (10.2)

- The standard way for a Java class to define a comparison function for its objects is to define a `compareTo` method.
  - Example: in the `String` class, there is a method:

    `public int compareTo(String other)`

- A call of **A**`.compareTo(`**B**`)` will return:

| | | |
|---|---|---|
| a value < | 0 | if **A** comes "before" **B** in the ordering, |
| a value > | 0 | if **A** comes "after" **B** in the ordering, |
| or | 0 | if **A** and **B** are considered "equal" in the ordering. |

# Runtime Efficiency (13.2)

- **efficiency**: A measure of the use of computing resources by code.
  - can be relative to speed (time), memory (space), etc.
  - most commonly refers to run time

- Assume the following:
  - Any single Java statement takes the same amount of time to run.
  - A method call's runtime is measured by the total of the statements inside the method's body.
  - A loop's runtime, if the loop repeats N times, is N times the runtime of the statements in its body.

# Efficiency examples

```
statement1;
statement2;
statement3;
```

3

```
for (int i = 1; i <= N; i++) {
    statement4;
}
```

N

```
for (int i = 1; i <= N; i++) {
    statement5;
    statement6;
    statement7;
}
```

3N

4N + 3

# Efficiency examples 2

```
for (int i = 1; i <= N; i++) {
    for (int j = 1; j <= N; j++) {
        statement1;
    }
}

for (int i = 1; i <= N; i++) {
    statement2;
    statement3;
    statement4;
    statement5;
}
```

$N^2$

$4N$

$N^2 + 4N$

- How many statements will execute if N = 10? If N = 1000?

# Algorithm growth rates (13.2)

- We measure runtime in proportion to the input data size, N.
  - **growth rate**: Change in runtime as N changes.

- Say an algorithm runs $\mathbf{0.4N^3 + 25N^2 + 8N + 17}$ statements.
  - Consider the runtime when N is *extremely large* .
  - We ignore constants like 25 because they are tiny next to N.
  - The highest-order term ($N^3$) dominates the overall runtime.
  - We say that this algorithm runs "on the order of" $N^3$.
  - or $\mathbf{O(N^3)}$ for short ("**Big-Oh** of N cubed")

# Complexity classes

- **complexity class**: A category of algorithm efficiency based on the algorithm's relationship to the input size N.

| Class | Big-Oh | If you double N, ... | Example |
|---|---|---|---|
| constant | $O(1)$ | unchanged | 10ms |
| logarithmic | $O(\log_2 N)$ | increases slightly | 175ms |
| linear | $O(N)$ | doubles | 3.2 sec |
| log-linear | $O(N \log_2 N)$ | slightly more than doubles | 6 sec |
| quadratic | $O(N^2)$ | quadruples | 1 min 42 sec |
| cubic | $O(N^3)$ | multiplies by 8 | 55 min |
| ... | ... | ... | ... |
| exponential | $O(2^N)$ | multiplies drastically | $5 * 10^{61}$ years |

# Binary search (13.1, 13.3)

- **binary search** successively eliminates half of the elements.
  - *Algorithm:* Examine the middle element of the array.
    - If it is too big, eliminate the right half of the array and repeat.
    - If it is too small, eliminate the left half of the array and repeat.
    - Else it is the value we're searching for, so stop.
  - Which indexes does the algorithm examine to find value **22**?
  - What is the runtime complexity class of binary search?

| *index* | *0* | *1* | *2* | *3* | *4* | *5* | *6* | *7* | *8* | *9* | *10* | *11* | *12* | *13* | *14* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *value* | -4 | -1 | 0 | 2 | 3 | 5 | 6 | 8 | 11 | 14 | 22 | 29 | 31 | 37 | 56 |

# Binary search runtime

- For an array of size N, it eliminates ½ until 1 element remains.

  N, N/2, N/4, N/8, ..., 4, 2, 1

  – How many divisions does it take?

- Think of it from the other direction:
  – How many times do I have to multiply by 2 to reach N?

  1, 2, 4, 8, ..., N/4, N/2, N

  – Call this number of multiplications "x".

  $2^x = N$

  $\mathbf{x = \log_2 N}$

- Binary search is in the **logarithmic** complexity class.

# Range algorithm

What complexity class is this algorithm? Can it be improved?

```
// returns the range of values in the given array;
// the difference between elements furthest apart
// example: range({17, 29, 11, 4, 20, 8}) is 25
public static int range(int[] numbers) {
    int maxDiff = 0;     // look at each pair of values
    for (int i = 0; i < numbers.length; i++) {
        for (int j = 0; j < numbers.length; j++) {
            int diff = Math.abs(numbers[j] - numbers[i]);
            if (diff > maxDiff) {
                maxDiff = diff;
            }
        }
    }
    return diff;
}
```

# Range algorithm 2

The algorithm is **O(N²)**.  A slightly better version:

```
// returns the range of values in the given array;
// the difference between elements furthest apart
// example: range({17, 29, 11, 4, 20, 8}) is 25
public static int range(int[] numbers) {
    int maxDiff = 0;     // look at each pair of values
    for (int i = 0; i < numbers.length; i++) {
        for (int j = i + 1; j < numbers.length; j++) {
            int diff = Math.abs(numbers[j] - numbers[i]);
            if (diff > maxDiff) {
                maxDiff = diff;
            }
        }
    }
    return diff;
}
```

# Range algorithm 3

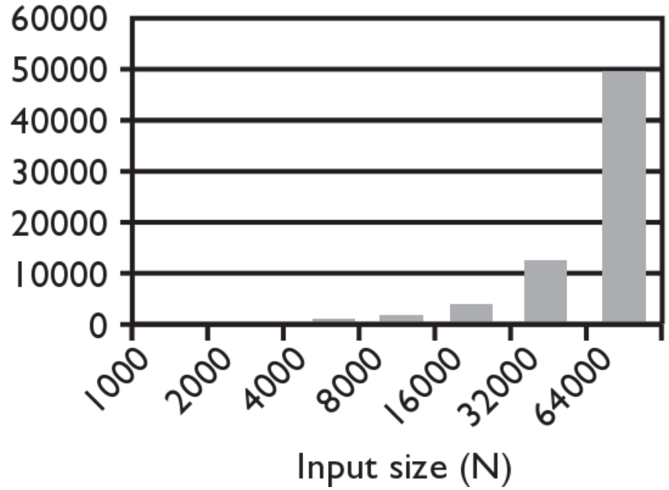
This final version is O(N). It runs MUCH faster:

```
// returns the range of values in the given array;
// example: range({17, 29, 11, 4, 20, 8}) is 25
public static int range(int[] numbers) {
    int max = numbers[0];        // find max/min values
    int min = max;
    for (int i = 1; i < numbers.length; i++) {
        if (numbers[i] < min) {
            min = numbers[i];
        }
        if (numbers[i] > max) {
            max = numbers[i];
        }
    }
    return max - min;
}
```

# Runtime of first 2 versions

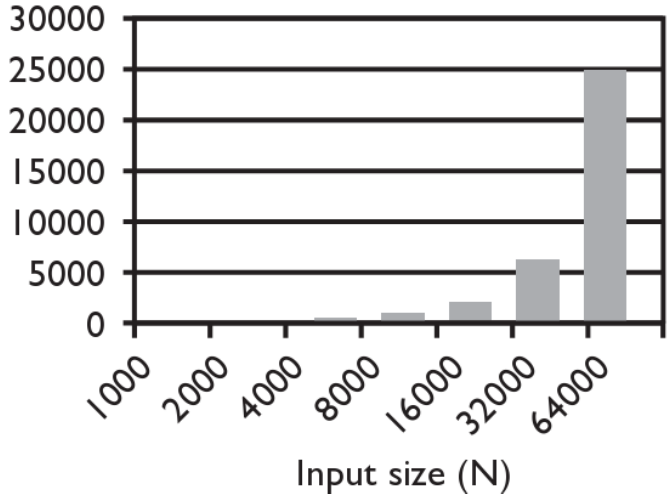
- Version 1:

| N | Runtime (ms) |
|---|---|
| 1000 | 15 |
| 2000 | 47 |
| 4000 | 203 |
| 8000 | 781 |
| 16000 | 3110 |
| 32000 | 12563 |
| 64000 | 49937 |




- Version 2:

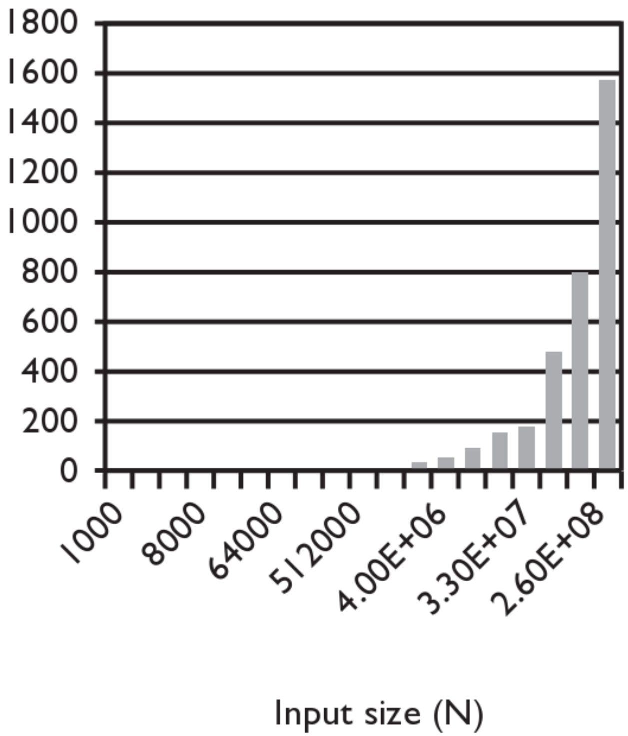
| N | Runtime (ms) |
|---|---|
| 1000 | 16 |
| 2000 | 16 |
| 4000 | 110 |
| 8000 | 406 |
| 16000 | 1578 |
| 32000 | 6265 |
| 64000 | 25031 |



# Runtime of 3rd version

- Version 3:

| N | Runtime (ms) |
|---|---|
| 1000 | 0 |
| 2000 | 0 |
| 4000 | 0 |
| 8000 | 0 |
| 16000 | 0 |
| 32000 | 0 |
| 64000 | 0 |
| 128000 | 0 |
| 256000 | 0 |
| 512000 | 0 |
| 1e6 | 0 |
| 2e6 | 16 |
| 4e6 | 31 |
| 8e6 | 47 |
| 1.67e7 | 94 |
| 3.3e7 | 188 |
| 6.5e7 | 453 |
| 1.3e8 | 797 |
| 2.6e8 | 1578 |



# Sorting

- **sorting**: Rearranging the values in an array or collection into a specific order (usually into their "natural ordering").
  - one of the fundamental problems in computer science
  - can be solved in many ways:
    - there are many sorting algorithms
    - some are faster/slower than others
    - some use more/less memory than others
    - some work better with specific kinds of data
    - some can utilize multiple computers / processors, ...

  - *comparison-based sorting* : determining order by comparing pairs of elements:
    - `<`, `>`, `compareTo`, ...

# Sorting methods in Java

- The `Arrays` and `Collections` classes in `java.util` have a static method `sort` that sorts the elements of an array/list

```
String[] words = {"foo", "bar", "baz", "ball"};
Arrays.sort(words);
System.out.println(Arrays.toString(words));
// [ball, bar, baz, foo]

List<String> words2 = new ArrayList<String>();
for (String word : words) {
    words2.add(word);
}
Collections.sort(words2);
System.out.println(words2);
// [ball, bar, baz, foo]
```

# `Collections` class

| Method name | Description |
|---|---|
| `binarySearch(`**list**`, `**value**`)` | returns the index of the given value in a sorted list (< 0 if not found) |
| `copy(`**listTo**`, `**listFrom**`)` | copies **listFrom**'s elements to **listTo** |
| `emptyList(), emptyMap(), emptySet()` | returns a read-only collection of the given type that has no elements |
| `fill(`**list**`, `**value**`)` | sets every element in the list to have the given value |
| `max(`**collection**`), min(`**collection**`)` | returns largest/smallest element |
| `replaceAll(`**list**`, `**old**`, `**new**`)` | replaces an element value with another |
| `reverse(`**list**`)` | reverses the order of a list's elements |
| `shuffle(`**list**`)` | arranges elements into a random order |
| `sort(`**list**`)` | arranges elements into ascending order |

# Sorting algorithms

- **bogo sort**: shuffle and pray
- **bubble sort**: swap adjacent pairs that are out of order
- **selection sort**: look for the smallest element, move to front
- **insertion sort**: build an increasingly large sorted front portion
- **merge sort**: recursively divide the array in half and sort it
- **heap sort**: place the values into a sorted tree structure
- **quick sort**: recursively partition array based on a middle value

other specialized sorting algorithms:

- **bucket sort**: cluster elements into smaller groups, sort them
- **radix sort**: sort integers by last digit, then 2nd to last, then ...
- ...

# Bogo sort

- **bogo sort**: Orders a list of values by repetitively shuffling them and checking if they are sorted.
  - name comes from the word "bogus"

  The algorithm:
  - Scan the list, seeing if it is sorted.  If so, stop.
  - Else, shuffle the values in the list and repeat.

- This sorting algorithm (obviously) has terrible performance!

# Bogo sort code

```
// Places the elements of a into sorted order.
public static void bogoSort(int[] a) {
    while (!isSorted(a)) {
        shuffle(a);
    }
}

// Returns true if a's elements are in sorted order.
public static boolean isSorted(int[] a) {
    for (int i = 0; i < a.length - 1; i++) {
        if (a[i] > a[i + 1]) {
            return false;
        }
    }
    return true;
}
```

# Bogo sort code, cont'd.

```
// Shuffles an array of ints by randomly swapping each
// element with an element ahead of it in the array.
public static void shuffle(int[] a) {
    for (int i = 0; i < a.length - 1; i++) {
        // pick a random index in [i+1, a.length-1]
        int range = a.length - 1 - (i + 1) + 1;
        int j = (int) (Math.random() * range + (i + 1));
        swap(a, i, j);
    }
}

// Swaps a[i] with a[j].
public static void swap(int[] a, int i, int j) {
    if (i != j) {
        int temp = a[i];
        a[i] = a[j];
        a[j] = temp;
    }
}
```

# Selection sort

- **selection sort**: Orders a list of values by repeatedly putting the smallest or largest unplaced value into its final position.

  The algorithm:
  - Look through the list to find the smallest value.
  - Swap it so that it is at index 0.

  - Look through the list to find the second-smallest value.
  - Swap it so that it is at index 1.

    ...

  - Repeat until all values are in their proper places.

# Selection sort example

- Initial array:

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | 22 | 18 | 12 | -4 | 27 | 30 | 36 | 50 | 7 | 68 | 91 | 56 | 2 | 85 | 42 | 98 | 25 |

- After 1st, 2nd, and 3rd passes:

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | **-4** | 18 | 12 | **22** | 27 | 30 | 36 | 50 | 7 | 68 | 91 | 56 | 2 | 85 | 42 | 98 | 25 |

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | -4 | **2** | 12 | 22 | 27 | 30 | 36 | 50 | 7 | 68 | 91 | 56 | **18** | 85 | 42 | 98 | 25 |

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | -4 | 2 | **7** | 22 | 27 | 30 | 36 | 50 | **12** | 68 | 91 | 56 | 18 | 85 | 42 | 98 | 25 |

# Selection sort code

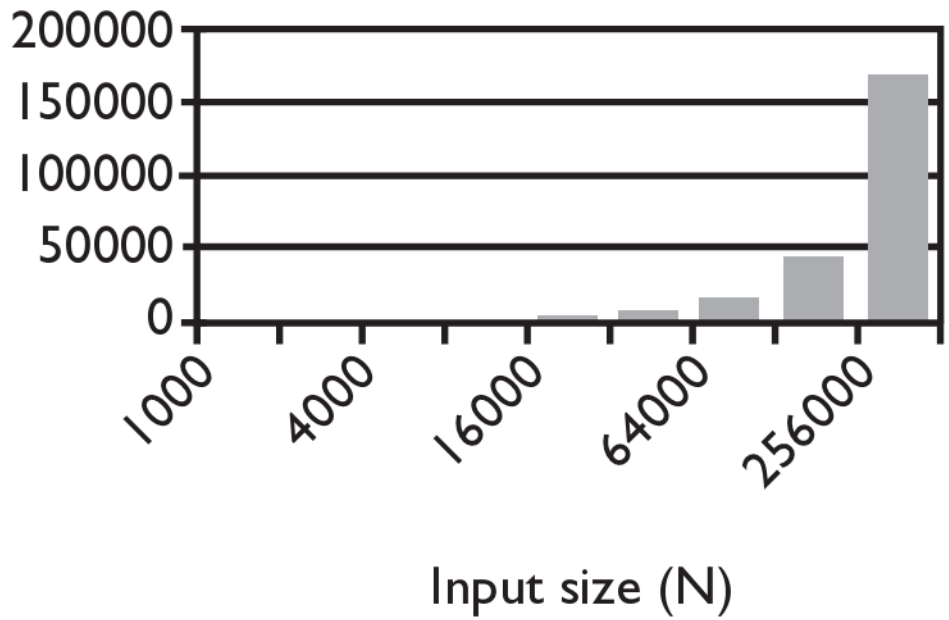
```
// Rearranges the elements of a into sorted order using
// the selection sort algorithm.
public static void selectionSort(int[] a) {
    for (int i = 0; i < a.length - 1; i++) {
        // find index of smallest remaining value
        int min = i;
        for (int j = i + 1; j < a.length; j++) {
            if (a[j] < a[min]) {
                min = j;
            }
        }

        // swap smallest value its proper place, a[i]
        swap(a, i, min);
    }
}
```

# Selection sort runtime (Fig. 13.6)

- What is the complexity class (Big-Oh) of selection sort?

| N | Runtime (ms) |
|---|---|
| 1000 | 0 |
| 2000 | 16 |
| 4000 | 47 |
| 8000 | 234 |
| 16000 | 657 |
| 32000 | 2562 |
| 64000 | 10265 |
| 128000 | 41141 |
| 256000 | 164985 |



# Similar algorithms

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | 22 | 18 | 12 | -4 | 27 | 30 | 36 | 50 | 7 | 68 | 91 | 56 | 2 | 85 | 42 | 98 | 25 |

- **bubble sort**: Make repeated passes, swapping adjacent values
  - slower than selection sort (has to do more swaps)

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | 18 | 12 | -4 | 22 | 27 | 30 | 36 | 7 | 50 | 68 | 56 | 2 | 85 | 42 | 91 | 25 | 98 |

22 → 50 → 91 → 98 →

- **insertion sort**: Shift each element into a sorted sub-array
  - faster than selection sort (examines fewer values)

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| value | -4 | 12 | 18 | 22 | 27 | 30 | 36 | 50 | 7 | 68 | 91 | 56 | 2 | 85 | 42 | 98 | 25 |

sorted sub-array (indexes 0-7)

← 7

# Merge sort

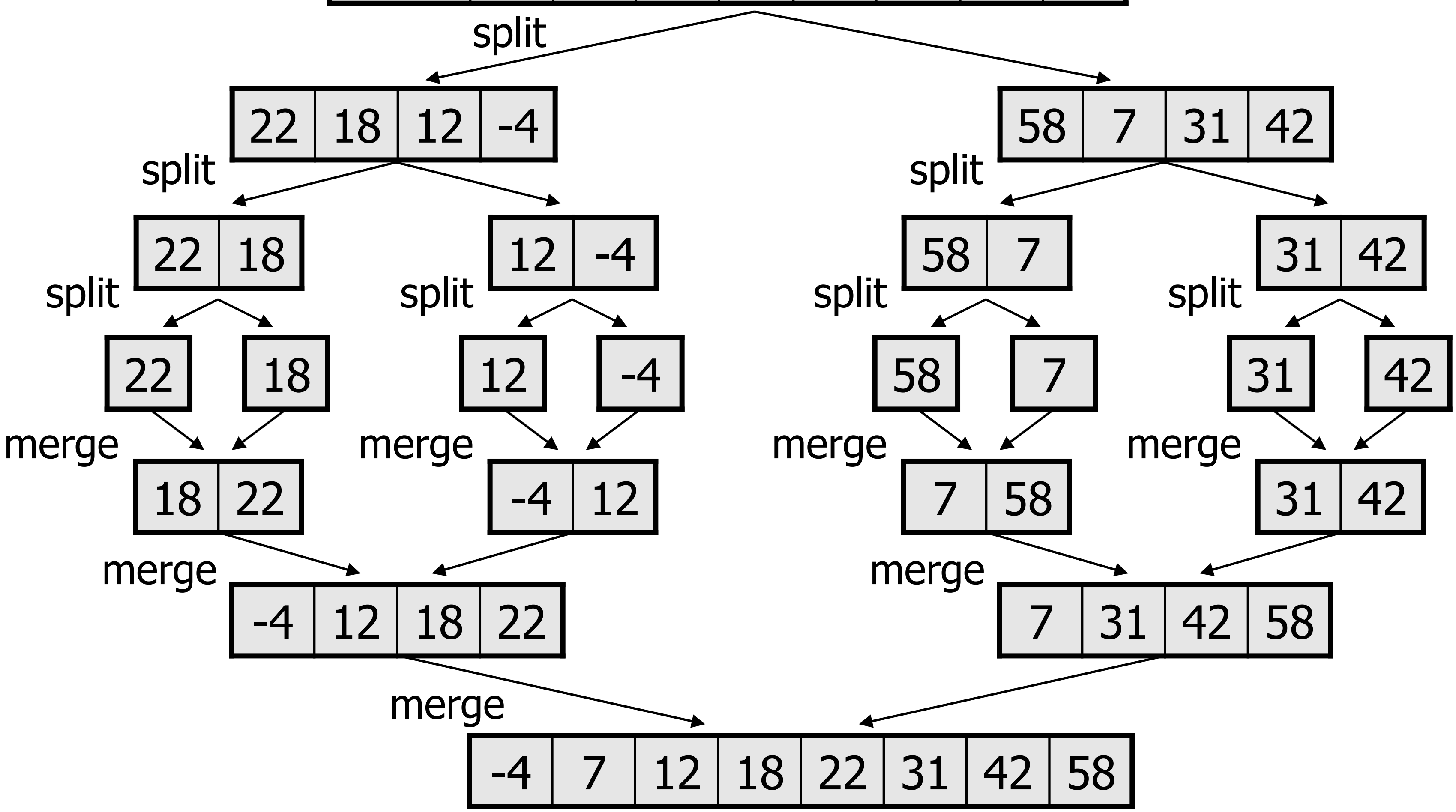
- **merge sort**: Repeatedly divides the data in half, sorts each half, and combines the sorted halves into a sorted whole.

  The algorithm:
  - Divide the list into two roughly equal halves.
  - Sort the left half.
  - Sort the right half.
  - Merge the two sorted halves into one sorted list.

  - Often implemented recursively.
  - An example of a "divide and conquer" algorithm.
    - Invented by John von Neumann in 1945

# Merge sort example

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| value | 22 | 18 | 12 | -4 | 58 | 7 | 31 | 42 |



# Merging sorted halves

| Subarrays (left) 0 1 2 3 | i1 | Subarrays (right) 0 1 2 3 | i2 | Next include | Merged array 0 1 2 3 4 5 6 7 | i |
|---|---|---|---|---|---|---|
| 14 32 67 76 | 0 | 23 41 58 85 | 0 | 14 from left | 14 | 0 |
| 14 32 67 76 | 1 | 23 41 58 85 | 0 | 23 from right | 14 23 | 1 |
| 14 32 67 76 | 1 | 23 41 58 85 | 1 | 32 from left | 14 23 32 | 2 |
| 14 32 67 76 | 2 | 23 41 58 85 | 1 | 41 from right | 14 23 32 41 | 3 |
| 14 32 67 76 | 2 | 23 41 58 85 | 2 | 58 from right | 14 23 32 41 58 | 4 |
| 14 32 67 76 | 2 | 23 41 58 85 | 3 | 67 from left | 14 23 32 41 58 67 | 5 |
| 14 32 67 76 | 3 | 23 41 58 85 | 3 | 76 from left | 14 23 32 41 58 67 76 | 6 |
| 14 32 67 76 | | 23 41 58 85 | 3 | 85 from right | 14 23 32 41 58 67 76 85 | 7 |

# Merge halves code

```
// Merges the left/right elements into a sorted result.
// Precondition: left/right are sorted
public static void merge(int[] result, int[] left,
                                       int[] right) {
    int i1 = 0;   // index into left array
    int i2 = 0;   // index into right array

    for (int i = 0; i < result.length; i++) {
        if (i2 >= right.length ||
           (i1 < left.length && left[i1] <= right[i2])) {
            result[i] = left[i1];    // take from left
            i1++;
        } else {
            result[i] = right[i2];   // take from right
            i2++;
        }
    }
}
```

# Merge sort code

```
// Rearranges the elements of a into sorted order using
// the merge sort algorithm.
public static void mergeSort(int[] a) {
    // split array into two halves
    int[] left  = Arrays.copyOfRange(a, 0, a.length/2);
    int[] right = Arrays.copyOfRange(a, a.length/2, a.length);

    // sort the two halves
    ...

    // merge the sorted halves into a sorted whole
    merge(a, left, right);
}
```

# Merge sort code 2

```
// Rearranges the elements of a into sorted order using
// the merge sort algorithm (recursive).
public static void mergeSort(int[] a) {
    if (a.length >= 2) {
        // split array into two halves
        int[] left  = Arrays.copyOfRange(a, 0, a.length/2);
        int[] right = Arrays.copyOfRange(a, a.length/2, a.length);

        // sort the two halves
        mergeSort(left);
        mergeSort(right);

        // merge the sorted halves into a sorted whole
        merge(a, left, right);
    }
}
```

# Merge sort runtime

- What is the complexity class (Big-Oh) of merge sort?

| N | Runtime (ms) |
|---|---|
| 1000 | 0 |
| 2000 | 0 |
| 4000 | 0 |
| 8000 | 0 |
| 16000 | 0 |
| 32000 | 15 |
| 64000 | 16 |
| 128000 | 47 |
| 256000 | 125 |
| 512000 | 250 |
| 1e6 | 532 |
| 2e6 | 1078 |
| 4e6 | 2265 |
| 8e6 | 4781 |
| 1.6e7 | 9828 |
| 3.3e7 | 20422 |
| 6.5e7 | 42406 |
| 1.3e8 | 88344 |