Three-Lens System Solution

Ali Onsi

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Problem

An object with its bottom placed at Origin (0,0) and the top of the object is (5,0) cm, and a lens is placed at the coordinates (0,30) cm, and the two other lenses are separated from this lens with only a y-shift from this lens by 35 cm, and 40 cm, respectively. the focal point of the first lens and second lenses coincide at (0,50) cm, and the third lens is at (0,80) cm; if the first and the third lenses are convergent, and the second lens is divergent find the coordinates of the final image, and total magnification and the coordinates of the top and the bottom of the final image, and determine its orientation.

Problem Statement

An object with its bottom at $(0,0)$ and top at $(5,0)$ (height $h = 5$ cm along x-axis). Three lenses are placed along the y-axis:

- **L1:** at $(0,30)$ cm, focal point at $(0,50)$ cm $\Rightarrow f_1 = +20$ cm (converging)
- **L2:** at $(0,65)$ cm, focal point at $(0,50)$ cm $\Rightarrow f_2 = -15$ cm (diverging)
- **L3:** at $(0,70)$ cm, focal point at $(0,80)$ cm $\Rightarrow f_3 = +10$ cm (converging)

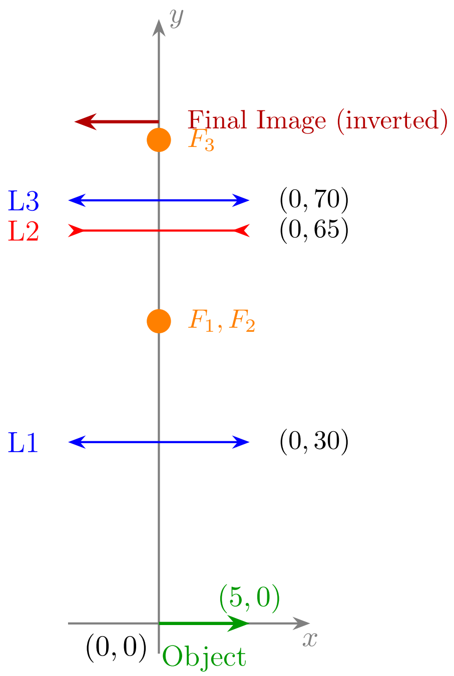

Figure 1: Lens system configuration (not to scale)

Solution

Step 1: Lens 1

Object distance: $s_1 = 30$ cm

$$\frac{1}{s_1'} = \frac{1}{f_1} - \frac{1}{s_1} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60} \implies s_1' = +60 \text{ cm} \tag{1}$$

Image 1: at $y = 30 + 60 = 90$ cm (the image is formd beyond L2 and L3 which means this will act as a virtual object for L2)

Magnification: $m_1 = -\frac{60}{30} = -2$

Step 2: Lens 2

Image from L1 at $y = 90$ is on the "wrong side" of L2 at $y = 65$.

Virtual object: $s_2 = 65 - 90 = -25$ cm

$$\frac{1}{s_2'} = \frac{1}{-15} - \frac{1}{-25} = -\frac{1}{15} + \frac{1}{25} = -\frac{2}{75} \implies s_2' = -37.5 \text{ cm} \tag{2}$$

Image 2: at $y = 65 - 37.5 = 27.5$ cm (Which is positioned before L1, this is image serves as a real object for L3, this image is located at y=27.5 this means it is 2.5 cm before L1)

Magnification: $m_2 = -\frac{-37.5}{-25} = -1.5$

Step 3: Lens 3

Object distance: $s_3 = 70 - 27.5 = 42.5$ cm (Another way to think of this is that the distance between the lens L1 and L3 is 40 cm, and the the distance between this image and L1 is 2.5 cm, so the distance between this image and L3 is 40 + 2.5 = 42.5 cm)

$$\frac{1}{s_3'} = \frac{1}{10} - \frac{1}{42.5} = \frac{32.5}{425} \implies s_3' = 13.08 \text{ cm} \tag{3}$$

Final image: at $y = 70 + 13.08 = 83.08$ cm

Magnification: $m_3 = -\frac{13.08}{42.5} = -0.308$

Final Answers

Total Magnification

$$M = m_1 \times m_2 \times m_3 = (-2)(-1.5)(-0.308) = \boxed{-0.923} \tag{4}$$

Final Image Coordinates

Object: bottom at $(0, 0)$, top at $(5, 0)$

The transverse magnification affects the x-coordinate:

$$x_{\text{image}} = M \times x_{\text{object}} \tag{5}$$

	Object	**Final Image**
Bottom (on axis)	$(0, 0)$	$\boxed{(0,\ 83.08)}$
Top (off axis)	$(5, 0)$	$\boxed{(-4.615,\ 83.08)}$

Image Properties

- **Image height:** $|M| \times 5 = 0.923 \times 5 = 4.615$ cm
- **Orientation: INVERTED** (upside down) since the total magnification is negative
  - Object top at $x = +5$ cm
  - Image top at $x = -4.615$ cm (flipped to negative x)
- **Image type:** Real (forms on right side of L3)
- **Size:** Reduced (smaller than object) since the magnitude of the total magnification is less than 1

Summary Table

Lens	**Position**	**f** (cm)	**s** (cm)	**s′** (cm)	**m**
L1	$(0, 30)$	$+20$	$+30$	$+60$	-2
L2	$(0, 65)$	-15	-25	-37.5	-1.5
L3	$(0, 70)$	$+10$	$+42.5$	$+13.08$	-0.308
				Total Magnification:	$\mathbf{-0.923}$