Pontificia Universidad Católica de Chile
School of Engineering
Department of Electrical Engineering
IEE3463 – Advanced Optimization for Electric Power Systems
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First Semester 2018

Homework 2
Convex Optimization, DC-OPF, and Power System Simulation

Instructions

- The homework is due on Monday April 9th at 11:59 PM.
- You must turn in all files developed in one ZIP file through the SIDING system, including a solution report in pdf format which can be written in English or Spanish. There is no need to turn in a printed copy of your report.
- Your report has to be concise and self-contained. The files you turn in are just a back-up.
- Your work is individual, i.e. one report per person, but you can collaborate with anyone.

Problem 1 (40 points)

Once again, you are the system operator at Coconut Island. It is now year 2050 and the island has become a metropolis with a complex power system. Under this new context, renewable generation has gained a large share in the system thanks to a series of environmental policies. However, due to the irresponsible behavior of several nations over many decades, global warming left a mess in the world and now natural disasters have become more and more frequent in the island and the coordination of the system must now take into account the possibility of multiple *contingencies*.

Fortunately, thanks to your engineering team's work you currently have a simulation tool to study Coconut Island's power system under different scenarios of renewable generation. However, such tool still needs to be tested and validated. The tool is available through several scripts written in Python. The main script is called *Experiment1.py*, which employs a script called *Models.py* and some other scripts for data processing. *Models.py* has functions for generating stochastic trajectories for renewable power availability in the system, and for solving a deterministic *Direct-Current Optimal Power Flow Problem* (DC-OPF). The details of the functions implemented in *Models.py* are listed below:

- **Renewable generation**: The resulting available renewable power $\widetilde{p}^r_{it}$ for the renewable plant i at time t is the result of a deterministic trend μ_{it} plus a gaussian random noise ε_{it}:

  $$\widetilde{p}^r_{it} = \mu_{it} + \varepsilon_{it}, \quad \varepsilon_{it} \sim \mathcal{N}(0, \sigma_{it})$$

  with all ε_{it} independent from each other. *Models.py* has a function for generating trajectories for renewable power from this stochastic model.

- **DC-OPF**: *Models.py* has a function for solving the following DC-OPF problem:

$$\min_{\mathbf{p}^g,\,\theta} \quad \sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}} C^g_{it}\, p^g_{it} \tag{1a}$$

$$\text{s.t.} \quad \underline{p}^g_{it} \le p^g_{it} \le \overline{p}^g_{it} \quad \forall\, i\in\mathcal{G},\, t\in\mathcal{T} \tag{1b}$$

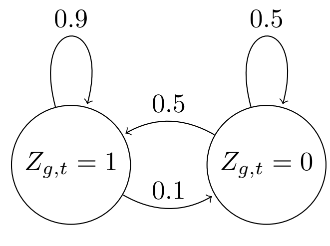
$$-RD_{it} \le p^g_{it} - p^g_{i,t-1} \le RU_{it} \quad \forall\, i\in\mathcal{G},\, t\in\mathcal{T} \tag{1c}$$

$$\sum_{k\in\mathcal{G}(i)} p^g_{kt} - p^d_{it} = \sum_{j\in\delta(i)} B_{ij}\,(\theta_{it}-\theta_{jt}) \quad \forall i\in\mathcal{B}, t\in\mathcal{T} \tag{1d}$$

$$B_{ij}\,(\theta_{it}-\theta_{jt}) \le f^{max}_{ij} \quad \forall\, (i,j)\in\mathcal{L},\, t\in\mathcal{T}. \tag{1e}$$

  In this problem, $\mathcal{B}$, $\mathcal{G}$ and $\mathcal{T}$ are the set of buses, generators, and time periods, respectively. Set $\mathcal{L}$ is the set of transmission lines in the network, including both directions, that is, if buses i and j are connected through a transmission line, then $(i,j)\in\mathcal{L}$ and $(j,i)\in\mathcal{L}$. The variable p^g_{it} stands for the active power output of generator i at time t, and θ_{it} is the voltage angle at bus i, time t. For generator i at time t, parameters C^g_{it}, $\underline{p}^g_{it}$, $\overline{p}^g_{it}$, RD_{it}, RU_{it} correspond to the unit cost, lower and upper bounds of active power output, and ramp-down and ramp-up capacities, respectively. For bus i, $\mathcal{G}(i)$ is the set of generators at this bus, $\delta(i)$ is the set of buses connected to this bus through a transmission line, that is, $\delta(i) = \{j \in \mathcal{B} : (i, j) \in \mathcal{L}\}$, and p^d_{it} is the active power demand at the bus. For buses i, j, the parameter B_{ij} stands for the (i, j)-th element of the susceptance matrix, and f^{max}_{ij} is the maximum apparent power flow allowed on transmission line $(i, j) \in \mathcal{L}$.

The current version of the simulation tool that your team developed needs to be upgraded to account for the unstable system dynamics resulting from the frequent occurrence of constant hurricanes that affect the island, which generate the contingencies mentioned above. These contingencies consist of making generators unavailable for some time. In order to account for this in the simulation tool, a series of meetings with the engineering team were held, and after multiple discussions you have decided to model contingencies exclusively for thermal generation units through a Discrete-Time Markov Chain, which is described through the following graph:



Here, $Z_{it} \in \{0, 1\}$ is a state variable defining the availability of thermal unit i at time t: if $Z_{it} = 1$ the unit is operational, whereas if $Z_{it} = 0$ the unit is faulty (unavailable). Also, the arcs in the figure indicate the probability of switching or staying in the same state for the next period. This model holds for each generator separately (the Markov Chain describing the temporal availability for one generator is independent of the one describing any other generator).

Finally, the data for Coconut Island's system is available in file Case014.xls, and it can be read using *DataInterface.py*.

1. Take a close look at the scripts *Experiment1.py* and *Models.py*. Use them to generate three trajectories for wind and solar power. For each of these trajectories, run a simulation of the dispatch process throughout the day, using four time periods in the DC-OPF problem. Notice that *Experiment1.py* automates this process moving from hour to hour over the 24 hours of the day, and solving a DC-OPF problem in each of these hours. For each of these three simulations, plot over the 24 hours the power production of each thermal generator and each renewable generator. Try to use a reasonable plot that allows to easily compare the daily dynamics between the three cases. Use a *penalty cost* of \$5000/MWh for unserved demand.

2. Upgrade the scripts to also generate random contingencies according to the Markov Chain model above. How could you add this new extension without modifying significantly the original code? In which part of the code should this extension be located? Please describe your main changes to the original codes.

3. Simulate $N = 100$ random trajectories of renewable power and thermal unit contingencies, and run the daily simulation of the dispatch process under each of these trajectories. Discuss the results obtained. In particular, calculate expected daily penalty costs of the system due to unserved demand. Also calculate the expected daily dispatch costs, and the expected demand not served. You can also present histograms for any relevant metrics.

4. Using the same previously simulated scenarios, identify the scenario that achieves the highest total daily cost and explain the main drivers that explain the difficulties encountered.

5. Assume now that some enhancements have been performed at the generators, and in the Markov Chain model the transition probabilities are modified as follows: the 0.9 is replaced by 0.99, and the 0.1 is replaced by 0.01. Simulate another $N = 100$ random trajectories of renewable power and unit contingencies, in this new context, and run the daily simulation of the dispatch process under each of these trajectories. Compare your results to the previous case.

Problem 2 (20 points)

We have discussed the structure and importance of second-order cone optimization and of semidefinite optimization. We ask here for you to do some research and find a concrete application of one of these two classes of problems, and then write a summary about it. The application you discuss can be in any area of knowledge. You do not need to understand all details, but indicate at least the key decisions involved and why second-order cone or semidefinite constraints appear. Try to also explain the relevance of the application studied. Include citations to any references you use.