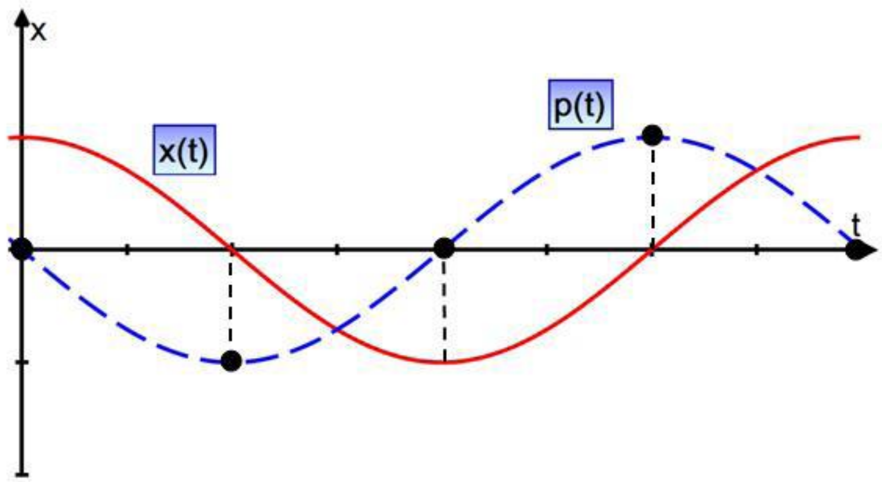
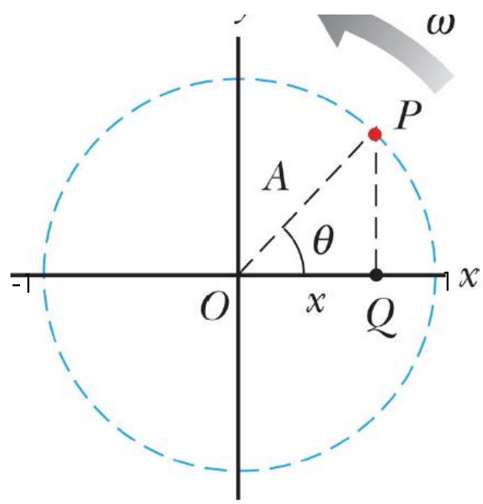
Quantum Mechanics Solutions 4:

HARMONIC OSCILATOR

[easy]

1. We are given that $x(t) = A\cos(\omega t)$, for some positive amplitude **A**. We can plot this, which is a simple cosine curve, and consider what the points along it mean for the physical oscillator it describes. Clearly, x = 0 is the oscillator's equilibrium position, to which it always tends and at which it moves the most rapidly. At these points, therefore, its speed would be largest. As speed is directly proportional to the oscillator's momentum, it would have its lowest (most negative) and highest values of momentum at these points in time, depending on the direction of motion. In contrast, at $x = \pm 1$, the oscillator is at its furthest extension and is forced to stop and reverse direction by the restoring force that governs it. So, at these points in time, the oscillator's momentum is 0.





Type equation here.

Plotting these points, we can convince ourselves that the function of time which describes the momentum has the general form of an inverted sine wave, i.e.

$p(t) = -B\sin(\omega t)$ for some $B > 0$ (B is the magnitude of peak momentum).

[hard] 2. Having functions for both **x** and **p**, we can now plug them into the energy equation:

$$E = const = \frac{p^2}{2m} + \frac{1}{2}kx^2 = \frac{B^2}{2m}\sin^2(\omega t) + \frac{1}{2}kA^2\cos^2(\omega t)$$

We already know the value of this constant, in terms of **A**, but we can easily convince ourselves of this fact by simply evaluating the function at time t = 0. More importantly, we can also evaluate it at any other time, knowing that the result will be equal to what we already know. In particular, evaluating at $t = \frac{\pi}{2\omega}$ we get:

$$E(t=0) = E\left(t = \frac{\pi}{2\omega}\right) = E$$

$$\frac{k}{2}A^2 = \quad \frac{1}{2m}B^2 \quad = E$$

Solving this gives us $B = \pm\sqrt{mk}A$B, of which we choose the positive value because we have defined B to be positive.

Now, therefore, our momentum function is: $p(t) = -\sqrt{mk}A\sin(\omega t)$

It is also true, however, that $p(t) = mv(t) = m\frac{dx}{dt} = m\left[-\omega\sin(\omega t)\right]$

Therefore, we have that $\sqrt{mk} = m\omega$, where $\omega = \sqrt{\frac{k}{m}}$