Quantum Mechanics Solutions 6:

LIGHT QUANTA & UNCERTAINTY

[medium] 1. The energy of a photon is given by the expression $E = hf$ and since the wavelength and frequency are related through $c = \lambda f$ the energy is $E = hc/\lambda$.

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34})(3\times10^{8})}{520\times10^{-9}} = 3.825 \times 10^{-19}\,\mathrm{J}$$

Since the power emitted as visible light is $10 \times 0.05 = 0.5$ J/s, the number of 520 nm photons emitted per second is given by $\frac{0.5}{3.825\times10^{-19}} = 1.31 \times 10^{18}$. We assume that these are emitted uniformly in all directions, so that the fraction entering the eye at a distance of 1000 m is given by [area of eye]/[area of sphere at 1000 m radius] = F. Then $F = (0.35 \times 10^{-2})^2/(1000)^2 = 1.225 \times 10^{-11}$. The number per second entering the eye is just F x [no. emitted], or 1.6×10^7 per second. Since the eye can respond to a signal of only 1000 photons per second, the eye can in principle see the signal from the light bulb very easily at a distance of 1000 m. This does assume that there is little "background" signal from other sources.

[medium] 2. The frequency range 400-700 nm corresponds to an energy range given by $E = hf$ of $E_{400} = 3.11$ eV and $E_{700} = 1.78$ eV. Visible light of all colors can cause transitions from the valence band to the conduction band and therefore be absorbed by the atom. Since silicon can absorb light it will appear opaque. Visible light cannot cause similar transitions in diamond. The light therefore cannot be absorbed, and diamond will appear transparent.

[hard] 3. According to the uncertainty principle, $\Delta y \Delta p_y \geq h$, so that if $\Delta y = 0.4 \times 10^{-3}$ m, then the uncertainty in the momentum is $\Delta p_y = 1.66 \times 10^{-30}$ kg m/s. But $\Delta p_y = \Delta(mv_y) = m\Delta v_y$ so that $\Delta v_y = 1.66 \times 10^{-30}/9.1 \times 10^{-31} = 1.82$ m/s. The kinetic energy of the electron beam $\frac{1}{2}mv^2$ is given by the voltage

KE = 15000 eV=$15000(1.6 \times 10^{-19})$ J.

Therefore, the x component of velocity of the electron is $v_x = 7.26 \times 10^7$m/s. The time for the beam to reach the screen after passing through the aperture is therefore only $\frac{s}{v_x} = \frac{0.35}{7.26\times10^7} =$

4.82×10^{-9}s. The uncertainty in position of the electrons at the screen resulting from the uncertainty principle is therefore only Δv_y [time to reach screen]=$1.82(4.82 \times 10^{-9}) = 8.8 \times 10^{-9}$m or 8.8nm. This uncertainty is totally insignificant.