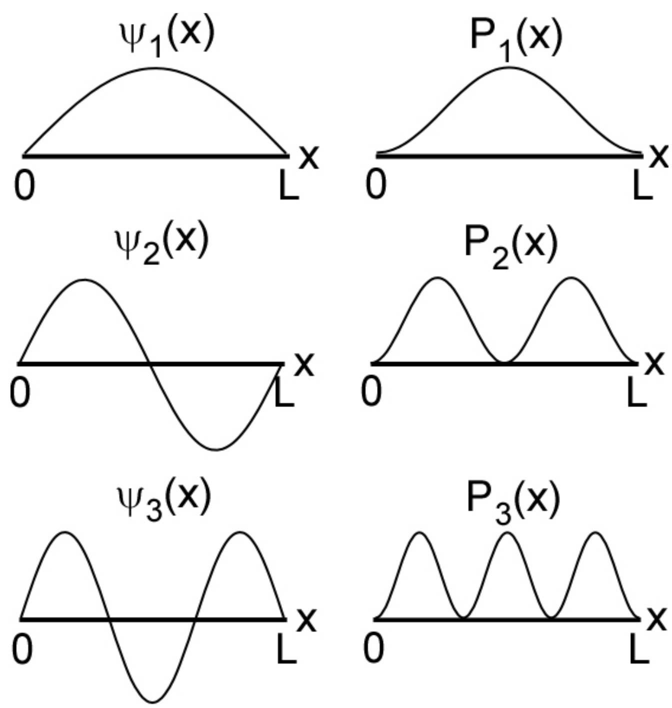
Quantum Mechanics Solutions 3:

PARTICLE IN A BOX

[easy] 1. $\sin(\theta)=0$ for $\theta = n\pi$, where n is any integer. Thus, $\psi(x=L)=\sin\left(\frac{2\pi L}{\lambda}\right)=0$ for $\frac{2\pi L}{\lambda}=n\pi$, for any integer n. Solving this equation for λ tells us that the wavelength can only take on certain discrete values, which we will denote as λ_n, where $\lambda_n=\frac{2L}{n}$, for n = 1, 2, 3, … . (Think about why we have excluded the value n = 0 and all negative integer values of n.)

[medium] 2. For these allowed values of wavelength, the corresponding allowed wavefunctions are $\psi_n(x)=\sin\left(\frac{n\pi x}{L}\right)$. For n = 1 this is $\psi_1(x)=\sin\left(\frac{\pi x}{L}\right)$, for n = 2 this is $\psi_2(x)=\sin\left(\frac{2\pi x}{L}\right)$, and so on. The corresponding probability functions are $P_n(x)=\left|\psi_n(x)\right|^2=\sin^2\left(\frac{n\pi x}{L}\right)$. See sketch:



[hard] 3. From the de Broglie relation $p = \frac{h}{\lambda}$ we have the allowed momentum values: $p_n = \frac{h}{\lambda_n} = \frac{hn}{2L}$, for n = 1, 2, 3, … . Thus, from the relation $E = \frac{p^2}{2m}$, we have the allowed energy values: $E_n = \frac{p_n^2}{2m} = \frac{h^2 n^2}{8mL^2}$. The lowest allowed energy (zero point energy) is therefore $E_{zeropoint} = E_1 = \frac{h^2}{8mL^2}$. (Note that in a Newtonian universe, Planck's constant would be zero (h = 0), and we would have $E_{zeropoint} = 0$) As the box is made smaller (L decreases), the zero point energy increases. Why would squeezing the box to make it smaller give it more energy?