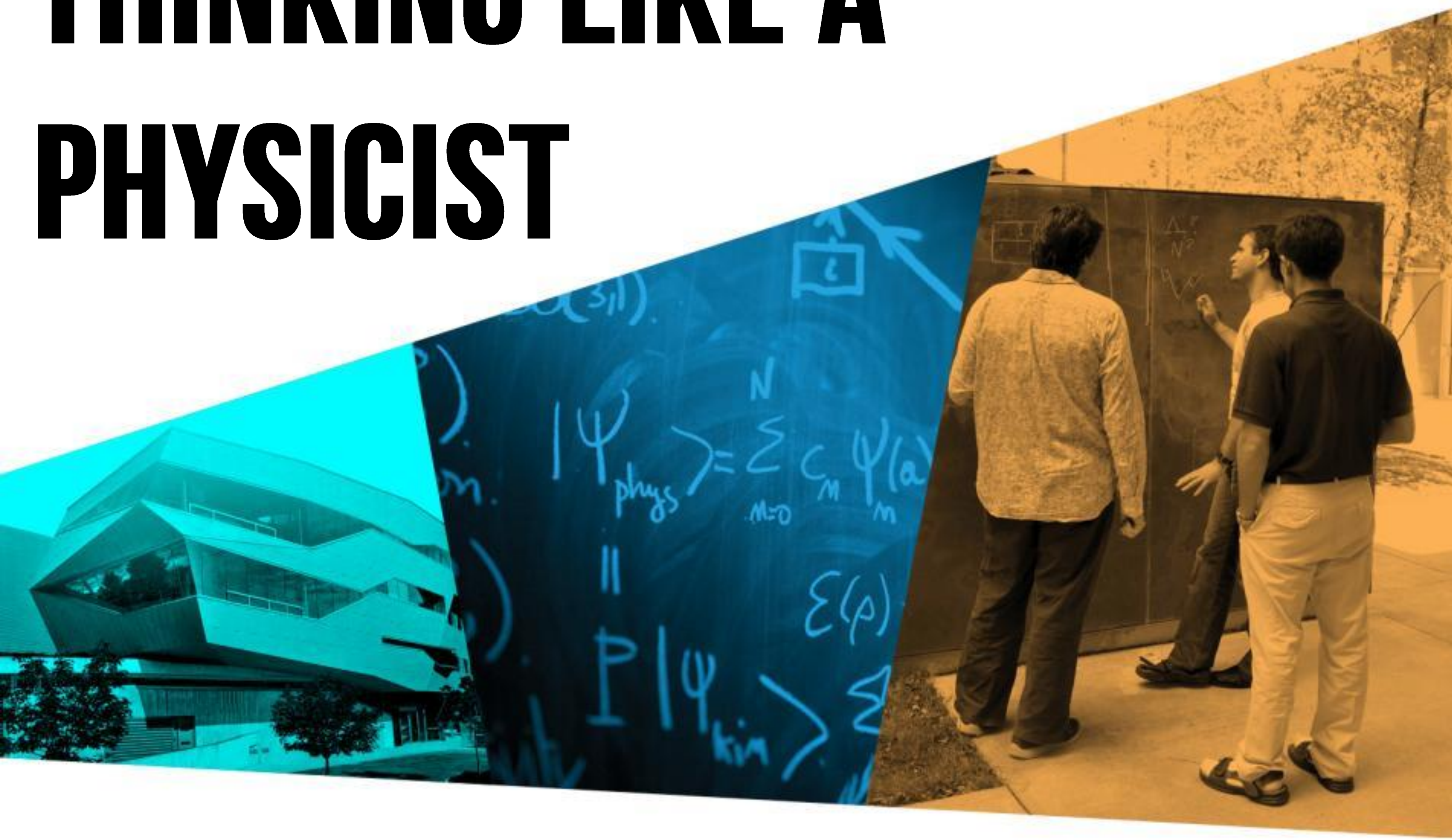
THINKING LIKE A PHYSICIST



PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Why is the Zero Point Energy ½ hf?

Recall the classical HO $$E = \frac{1}{2}mp^2(t) + \frac{k}{2}x^2(t)$$

p and x oscillate as sine and cosine functions of t. We see that E=0 requires that both p=0 and x=0

From QM, E=0 is impossible

- ***Argue based on HUP:*** If x=0 then Δx = 0. This means Δ p = ∞ (not 0!).
- ***Argue based on de Broglie:*** If E=0 then p=0 and this means $\lambda = \infty$. This is not possible for a bound particle!

Probabilities

- We know that QM deals in probabilities so let's set up many oscillators each at E_{min} and take a snapshot of their positions and take the average.

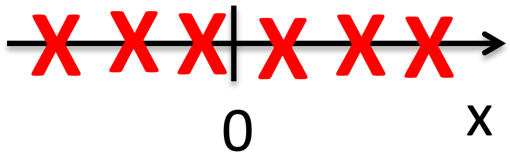
$$\langle x^2 \rangle$$

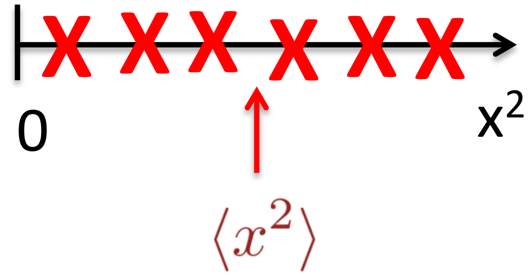
- Do the same for the momentum

$$\langle p^2 \rangle$$

- Together this gives:

$$\langle E \rangle = \frac{1}{2m}\langle p^2 \rangle + \frac{k}{2}\langle x^2 \rangle$$





The average of the square of the positions will not be zero!

- The larger $\langle x^2 \rangle$ the wider the scattering (like uncertainty!).
- Mathematically for HUP: $\sqrt{\langle x^2 \rangle} = \Delta x$
- Like a "standard deviation"
- Rename: $\Delta x \longrightarrow A_q$ (Quantum Amplitude)
- The quantum amplitude is not a "classical" amplitude. It describes the size of the quantum fuzziness.

- Follow the same line of reasoning for the momentum:

$$\sqrt{\langle p^2 \rangle} = \Delta p \geq \frac{h}{4\pi \Delta x} = \frac{h}{4\pi A_q}$$

- Let's go back to the energy equation:

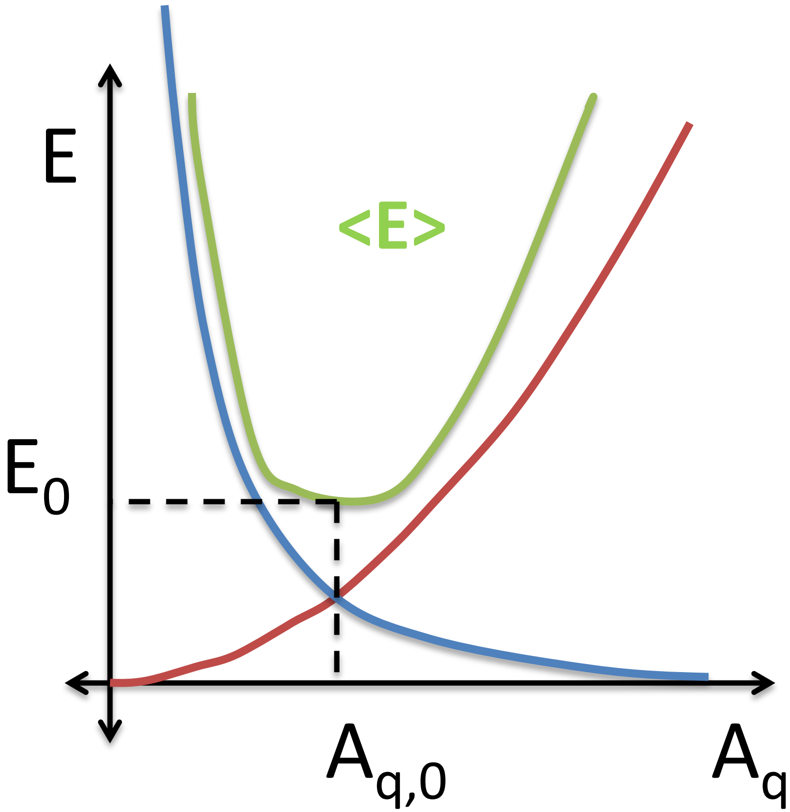
$$\langle E \rangle = \frac{1}{2m}\langle p^2 \rangle + \frac{k}{2}\langle x^2 \rangle \geq \left(\frac{h^2}{32\pi^2 m} \right) \frac{1}{A_q^2} + \left(\frac{k}{2} \right) A_q^2$$

QUANTUM TERM CLASSICAL TERM

Classical Term $\propto A_q^2$ If it was just this term then E = 0 when A_q =0

Quantum Term $\propto \frac{1}{A_q^2}$ Prevents A_q =0 because E blows up when A_q =0

$$\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A_q^2} + \left(\frac{k}{2}\right)A_q^2$$



Classical Term $\propto A_q^2 \quad PE_q$

The total energy stored in the oscillator decreases as the quantum amplitude gets smaller.

Quantum Term $\propto \dfrac{1}{A_q^2} \quad KE_q$

The total energy stored in the oscillator *increases* as the quantum amplitude gets smaller (claustrophobia!).

What is the minimum energy?

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A_q^2} + \left(\frac{k}{2}\right)A_q^2$$

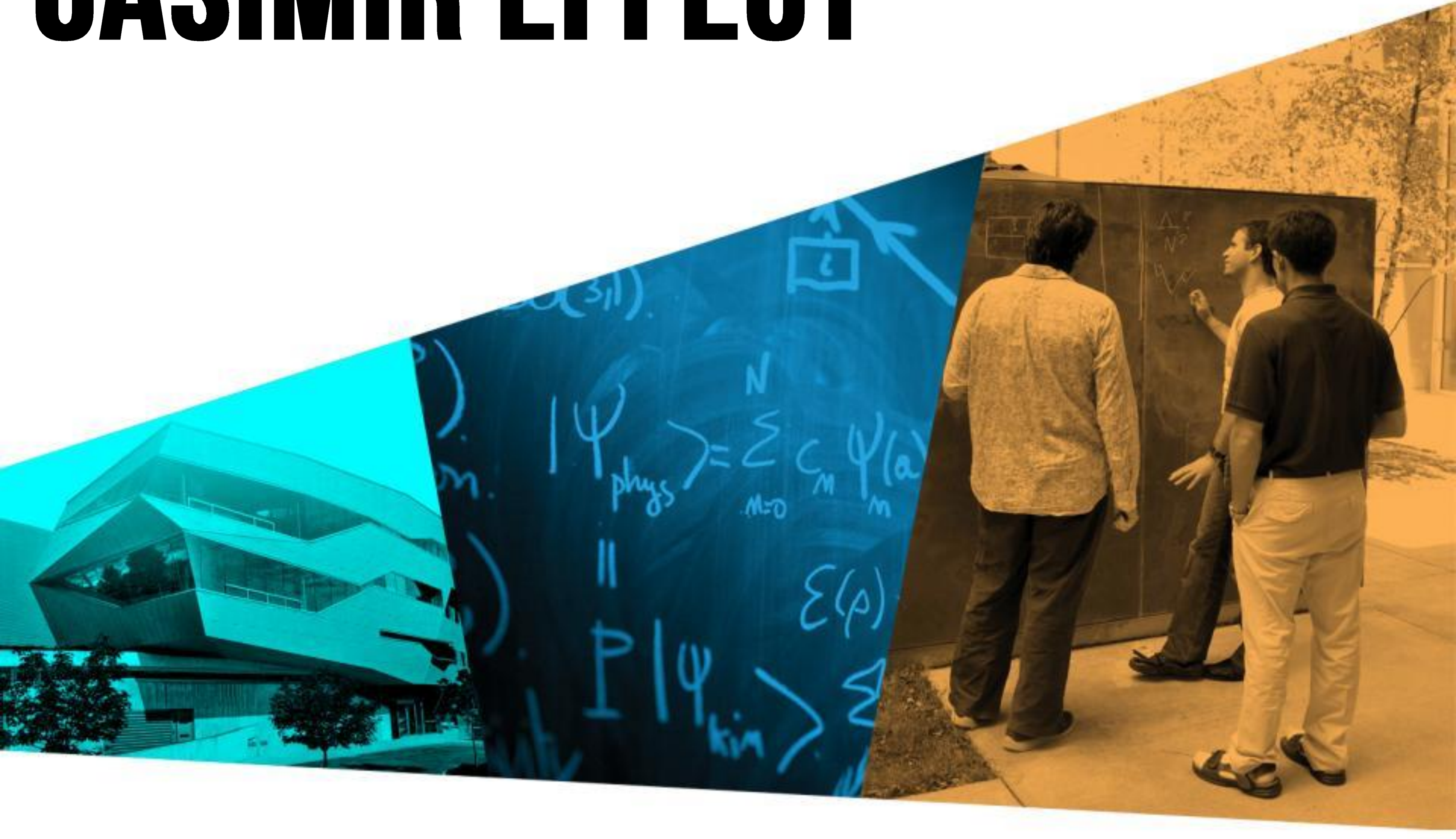
If you know calculus differentiate <E> as a function of A_q and set derivative to zero.

$$A_{q,0}^2 = \frac{h}{4\pi\sqrt{mk}}$$

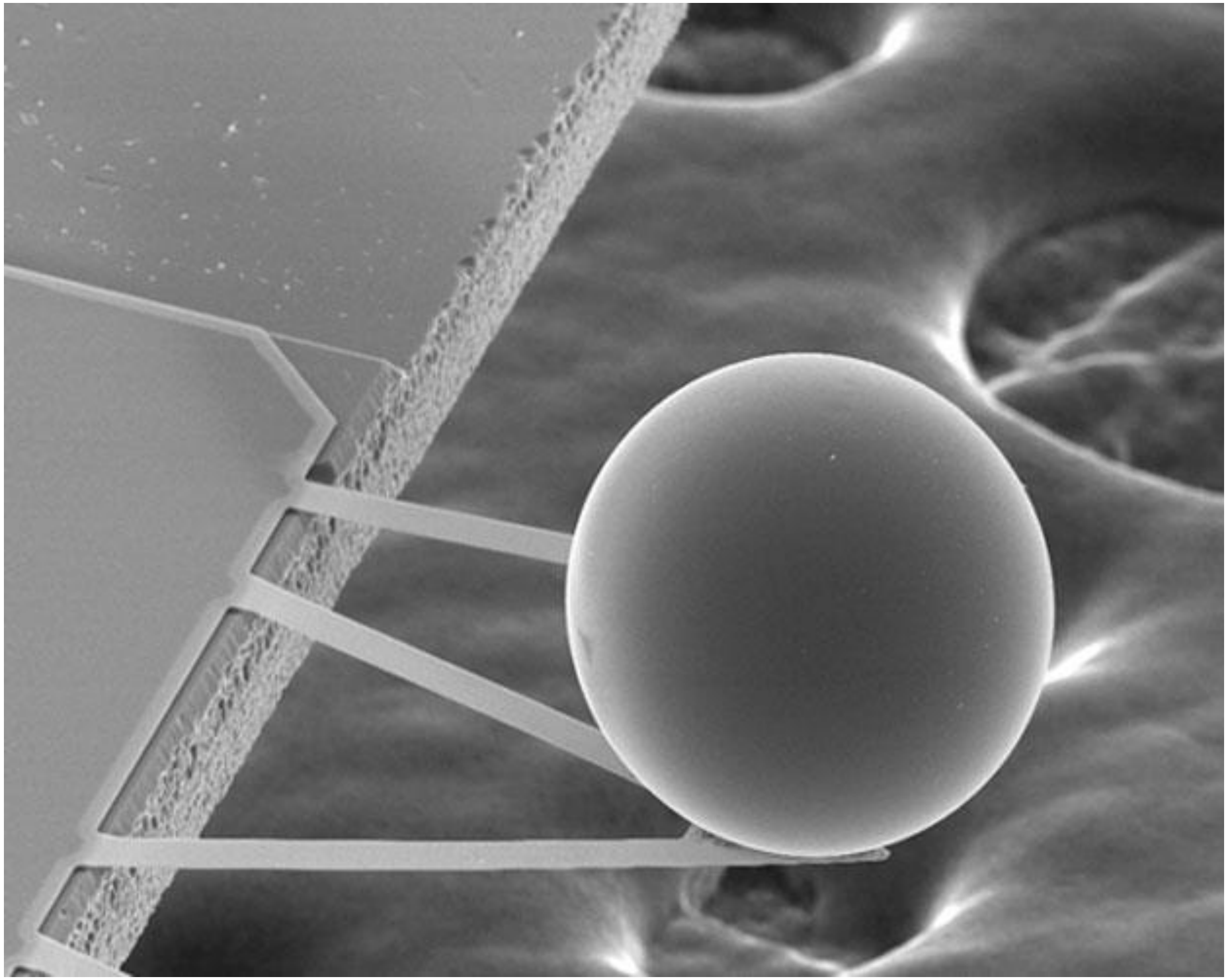
If you don't know calculus, sub the result into <E> and solve for E_0

$$E_0 = \frac{h}{4\pi}\sqrt{\frac{k}{m}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \longrightarrow E_0 = \frac{1}{2}hf$$

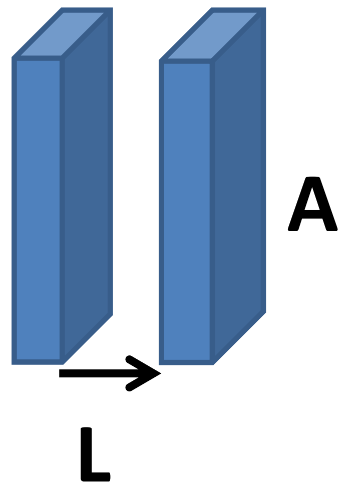
CASIMIR EFFECT



PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS



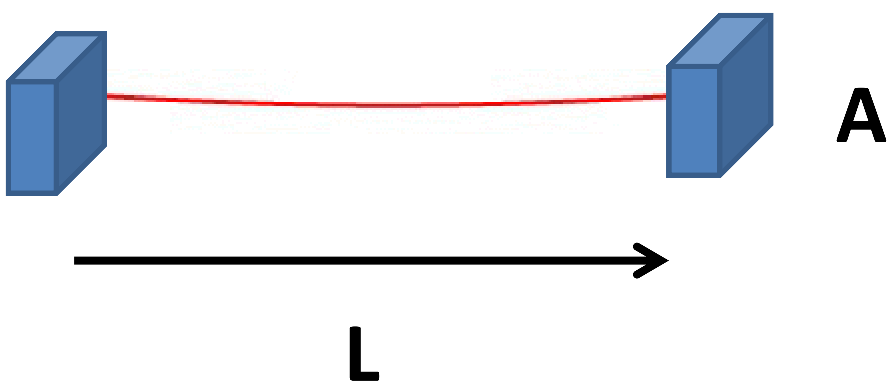
Plane Parallel Mirrors



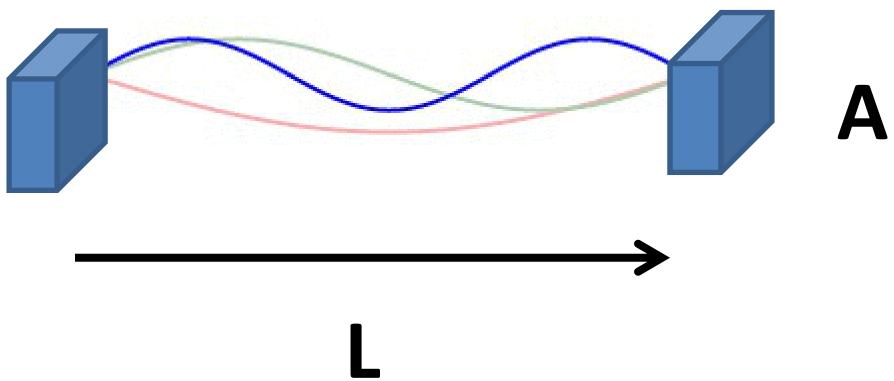
In vacuum conditions these mirror plates will move together!

Fundamental Mode

$$\lambda_0 = 2L$$



Simple Harmonic Oscillators



Fundamental Mode

$$f_0 = \frac{c}{\lambda_0} = \frac{c}{2L}$$

First Harmonic Mode

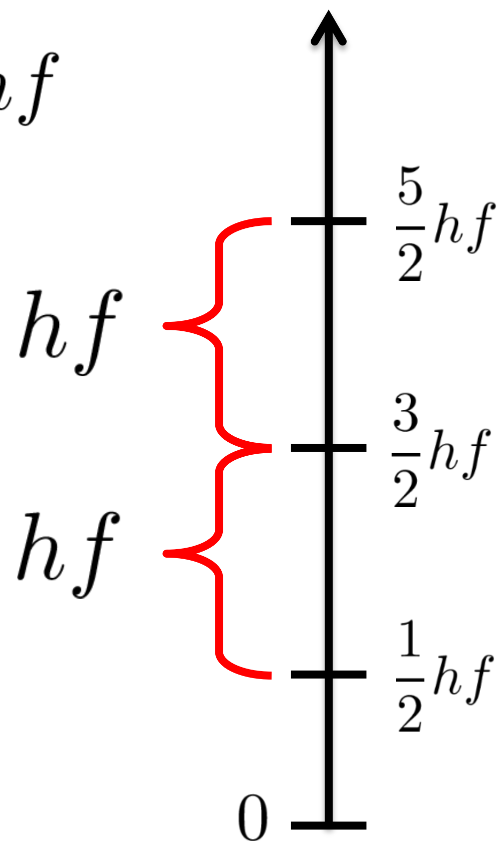
$$f_1 = \frac{c}{\lambda_1} = \frac{c}{L} = 2f_0$$

Second Harmonic Mode

$$f_2 = \frac{c}{\lambda_2} = \frac{3c}{2L} = 3f_0$$

Energy Values are Discrete

$$E_n = (n + \frac{1}{2})hf$$



Example $E_n = (n + \frac{1}{2})hf$

Three photons of frequency f_0 , **five** photons of frequency f_1 and no photons of higher frequencies.

$$E_{total} = (3 + \frac{1}{2})hf_0 + (5 + \frac{1}{2})hf_1 + (0 + \frac{1}{2})hf_2 + (0 + \frac{1}{2})hf_3 + ...$$

Infinite Series

Remove all Photons

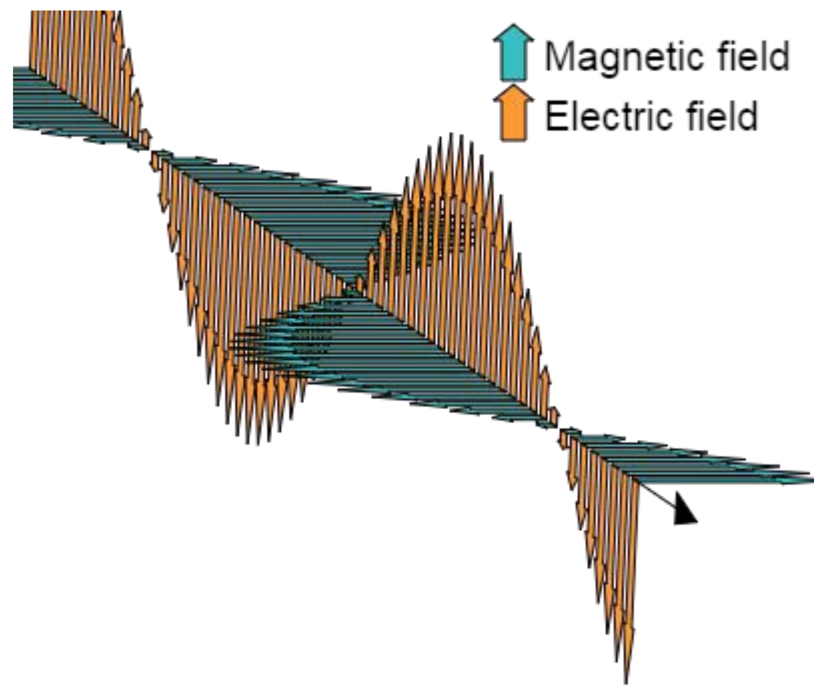
We still have energy!!!

$$E_{vac} = (0 + \frac{1}{2})hf_0 + (0 + \frac{1}{2})hf_1 + (0 + \frac{1}{2})hf_2 + (0 + \frac{1}{2})hf_3 + ...$$

Each mode of frequency f contributes a so-called *zero point energy*.

Electromagnetic Field

Even when there are no photons present, the electromagnetic field is still very much alive, continuously fluctuating in a purely quantum mechanical way.



Vacuum Energy

The energy associated with each mode is finite!

$$E_{vac} = (0 + \frac{1}{2})hf_0 + (0 + \frac{1}{2})hf_1 + (0 + \frac{1}{2})hf_2 + (0 + \frac{1}{2})hf_3 + ...$$

But the total energy between the two mirrors *appears* to be infinite!

Does x = 1+2+4+8+16+… = -1?

- We were able to represent this series as:

$$1 + x^2 + x^3 + \ldots$$

- However, it is only valid for -1<x<1

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$

$$x = \frac{1}{2}$$

- LHS $= \frac{1}{1-1/2} = 2$
- RHS $= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = 2$

$$x = 2$$

- LHS $= \frac{1}{1-2} = -1$
- RHS $= 1 + 2 + 4 + 8 + 16 + \ldots = \infty \neq \frac{1}{1-2}$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$

all $x \neq 1$

only $-1 < x < 1$

$$1 + 2 + 4 + 8 + ... = -1$$

$$1 + 2 + 3 + 4 + ... = -\frac{1}{12}$$

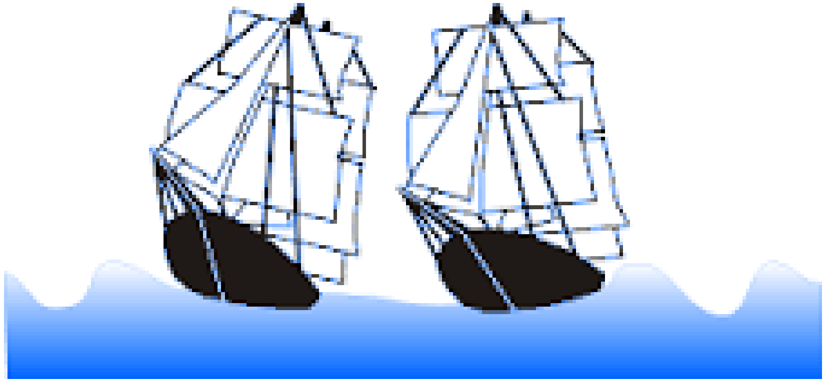
Energy in the Vacuum

$$E_{vac} = \frac{1}{2} h f_0 (1 + 2 + 3 + 4 + 5 + ...) = -\frac{1}{24} h f_0$$

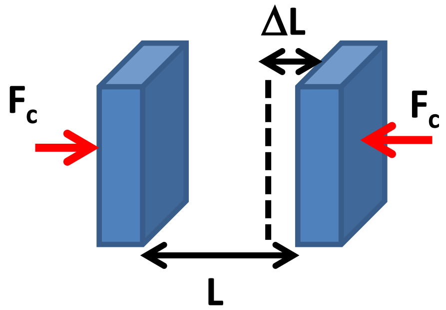
$$f_0 = \frac{c}{2L} \qquad E_{vac} = -\frac{hc}{48L}$$

A *finite, negative* value that depends on L.

Casimir Force



Casimir Force



$$E'_{vac} = \frac{-hc}{48(L - \Delta L)}$$

$$W = \text{force} \times \text{distance} = F_c \Delta L$$

$$W = E_{vac} - E'_{vac} = -\frac{hc}{48L} + \frac{hc}{48(L - \Delta L)}$$

Binomial Series

$$\frac{1}{L - \Delta L} = \frac{1}{L}(1 - \frac{\Delta L}{L})^{-1} \approx \frac{1}{L}\left(1 + \frac{\Delta L}{L}\right)$$

$$W \approx -\frac{hc}{48L} + \frac{hc}{48L}(1 + \frac{\Delta L}{L}) = \frac{hc}{48L^2}\Delta L$$

$$W = \text{force} \ \times \ \text{distance} = F_c \Delta L$$

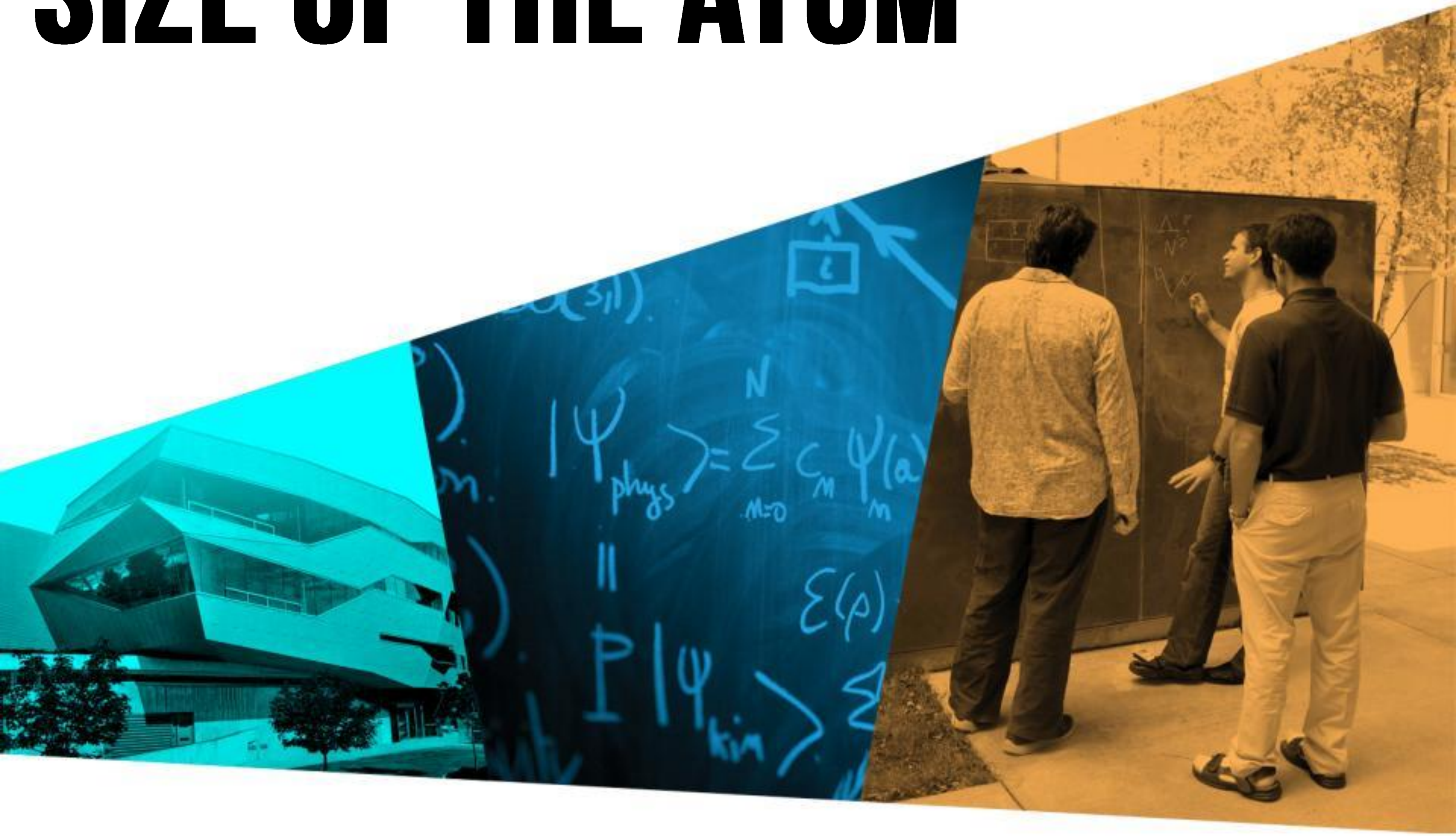
Casimir Force

$$F_c = \frac{hc}{48L^2}$$

But we neglected 3D volume and polarization!

$$F_c = \frac{\pi hc}{480L^4} A$$

SIZE OF THE ATOM



PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Predicting the Size of Atoms $[L]$

- What will the size depend on:
  - Electric charge $[C]$
  - Electron Mass $[M]$
  - Electrostatic Force constant $[\frac{ML^3}{C^2T^2}]$
  - Planck's constant $[\frac{ML^2}{T^2}]$

$$\frac{\hbar^2}{km_e e^2}$$

Predicting the Size of Atoms

$$F = ma$$

$$\frac{ke^2}{r^2} = \frac{mv^2}{r}$$

$$\frac{ke^2}{r^2} = \frac{m}{r}\frac{p^2}{m^2} = \frac{p^2}{rm}$$

$$\frac{ke^2}{r^2} = \frac{h^2}{\lambda^2}\frac{1}{rm}$$

Predicting the Size of Atoms

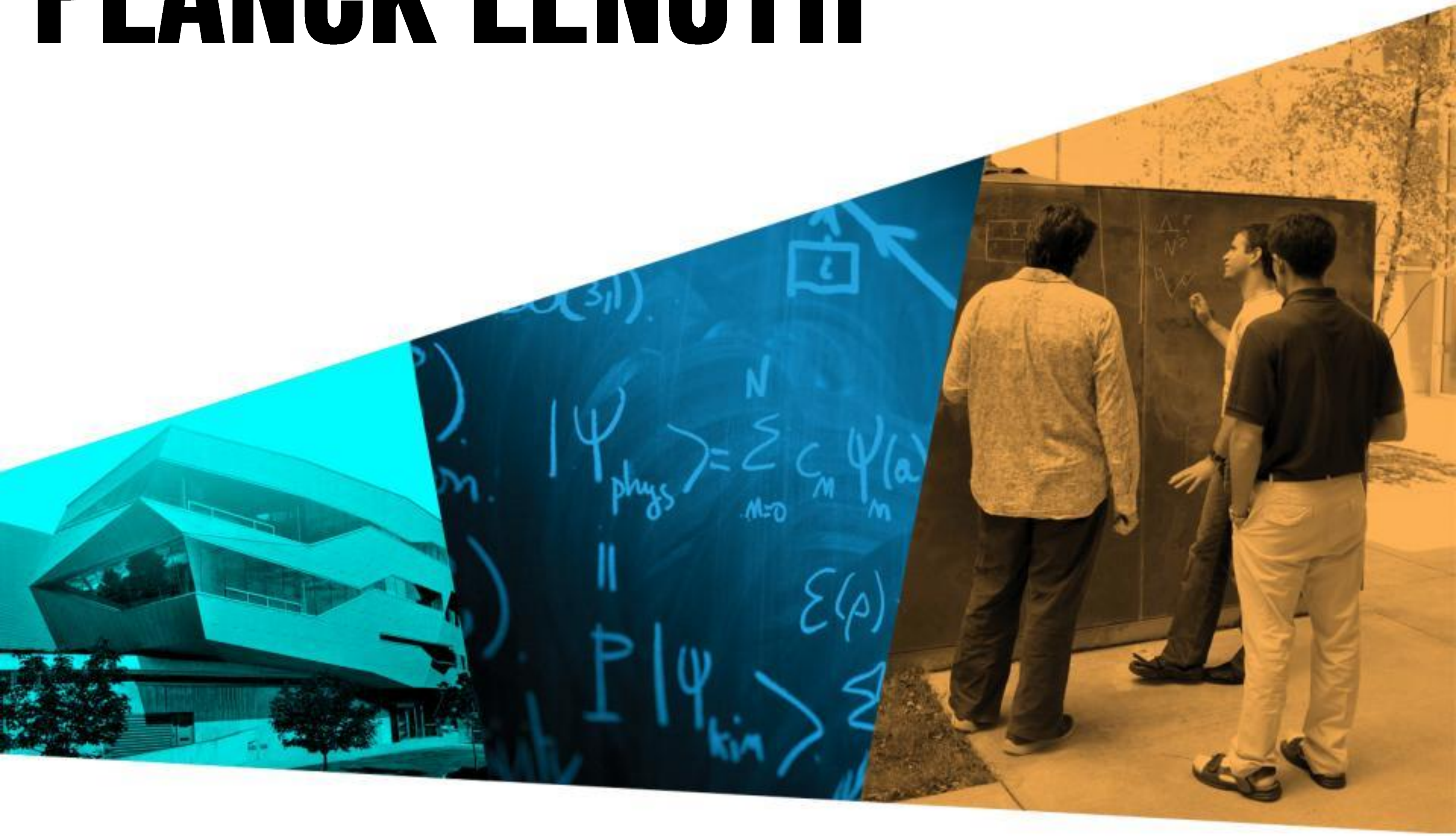
$$\frac{ke^2}{r^2} = \frac{h^2}{\lambda^2}\frac{1}{rm}$$

Ground State -> Smallest possible orbit $2\pi r = \frac{\lambda}{2}$

$$\frac{ke^2}{r^2} = \frac{h^2}{4\pi r^2}\frac{1}{rm}$$

$$r = \frac{\hbar^2}{4ke^2m_e}$$

PLANCK LENGTH



PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Planck Length

- Is the Universe *smooth* or *grainy*?

Dimensional Analysis $[L]$

- What will the size depend on:
  - Planck's constant $[\frac{ML^2}{T^2}]$
  - Speed of Light $[\frac{L}{T}]$
  - Newton's Constant $[\frac{L^3}{MT^2}]$

$$\sqrt{\frac{G\hbar}{c^3}}$$

GUP

- Quantum $E = \frac{hc}{\lambda}$
- Special Relativity $M_{eff} = \frac{h}{c\lambda}$
- General Relativity $a = \frac{Gh}{c\lambda r^2}$

GUP

- Random Gravitational Displacement

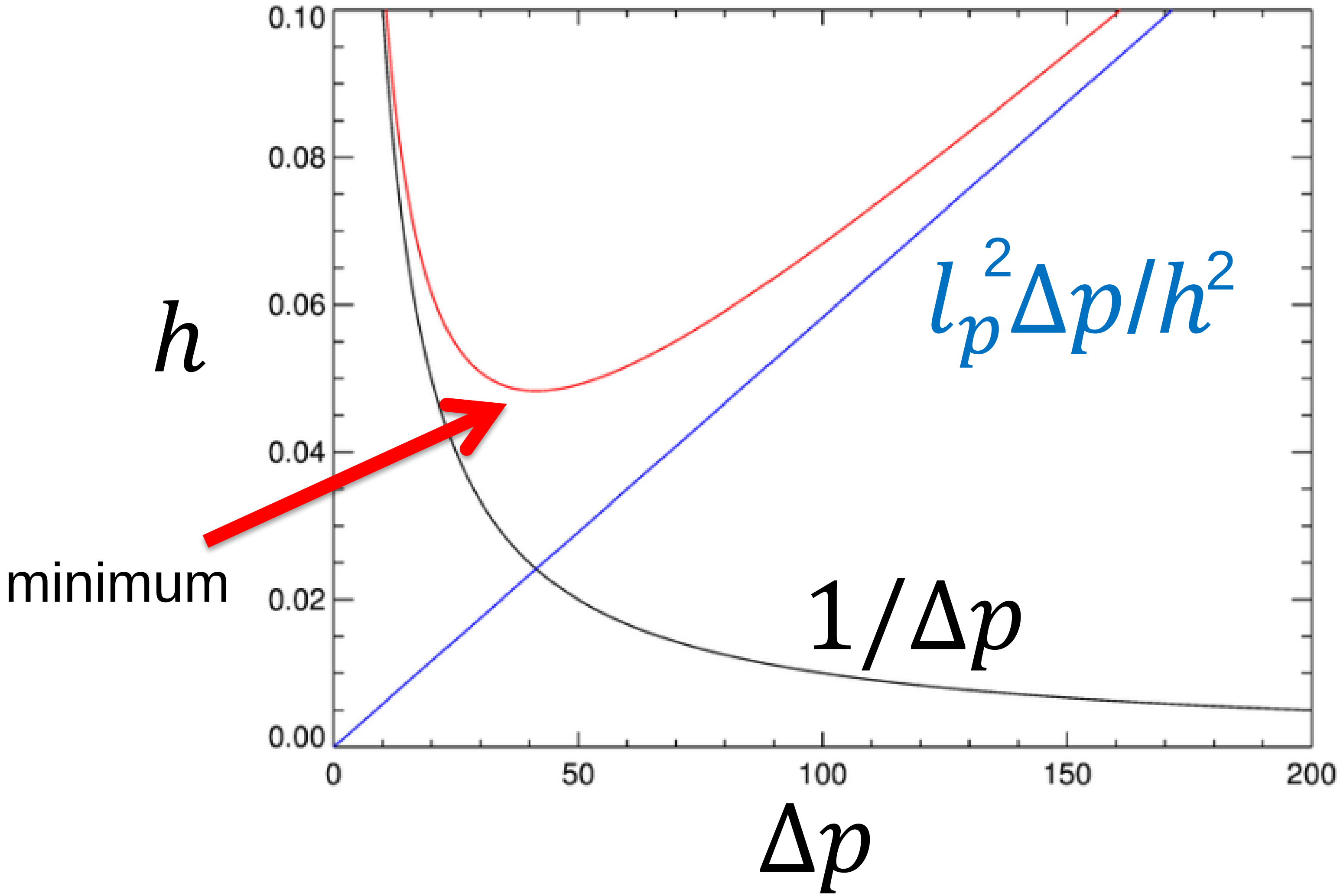
$$\Delta x_{grav} = \frac{Gh}{c\lambda r^2} t_{eff}^2$$

- Simplified $$\Delta x_{grav} = \frac{Gh}{\lambda c^3}$$

- HUP $$\Delta x_{grav} = \frac{l_p^2 \Delta p}{h}$$

GUP

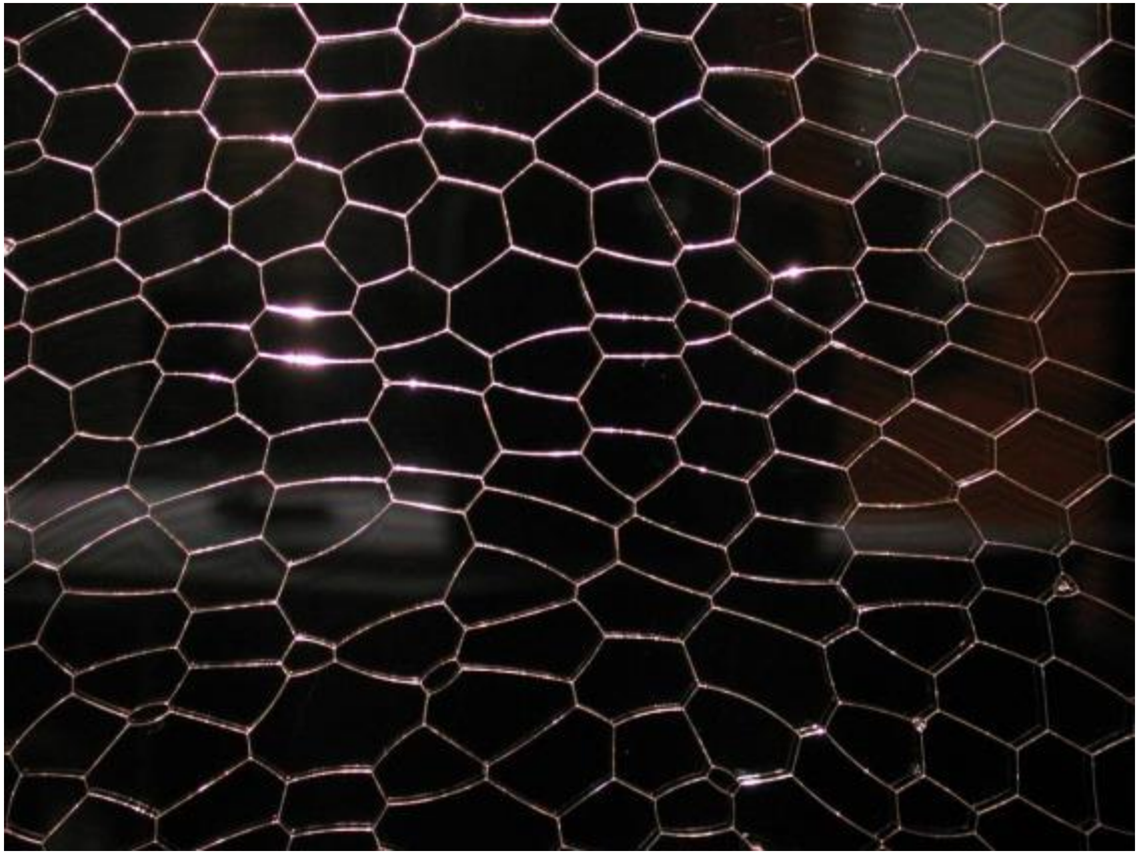
$$\Delta x \approx \frac{h}{\Delta p} + \frac{l_p^2 \Delta p}{h}$$



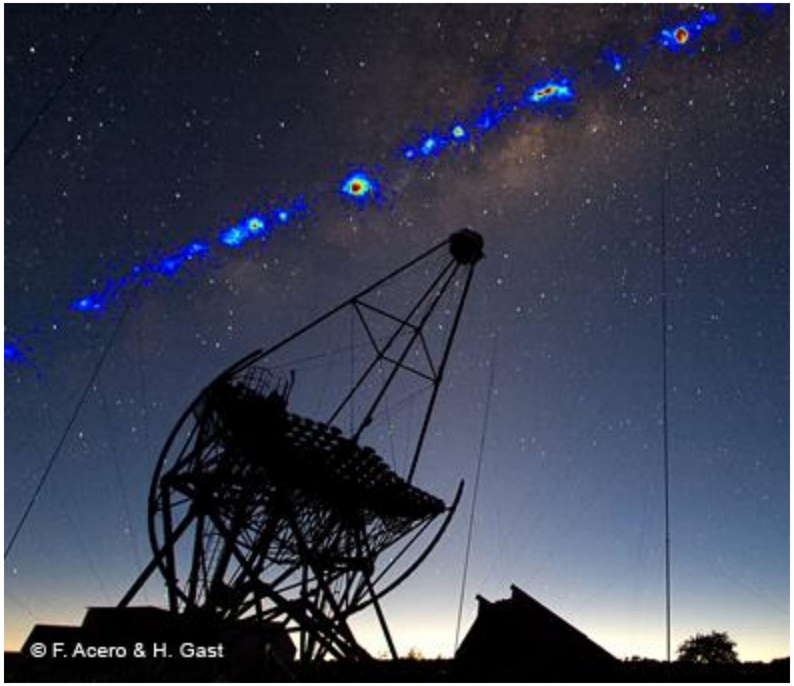
0.10
0.08
0.06
0.04
0.02
0.00
0
50
100
150
200
h
Δp
$l_p^2 \Delta p / h^2$
$1/\Delta p$
minimum

Smooth or Lumpy?

Spacetime Foam



HESS and Fermi



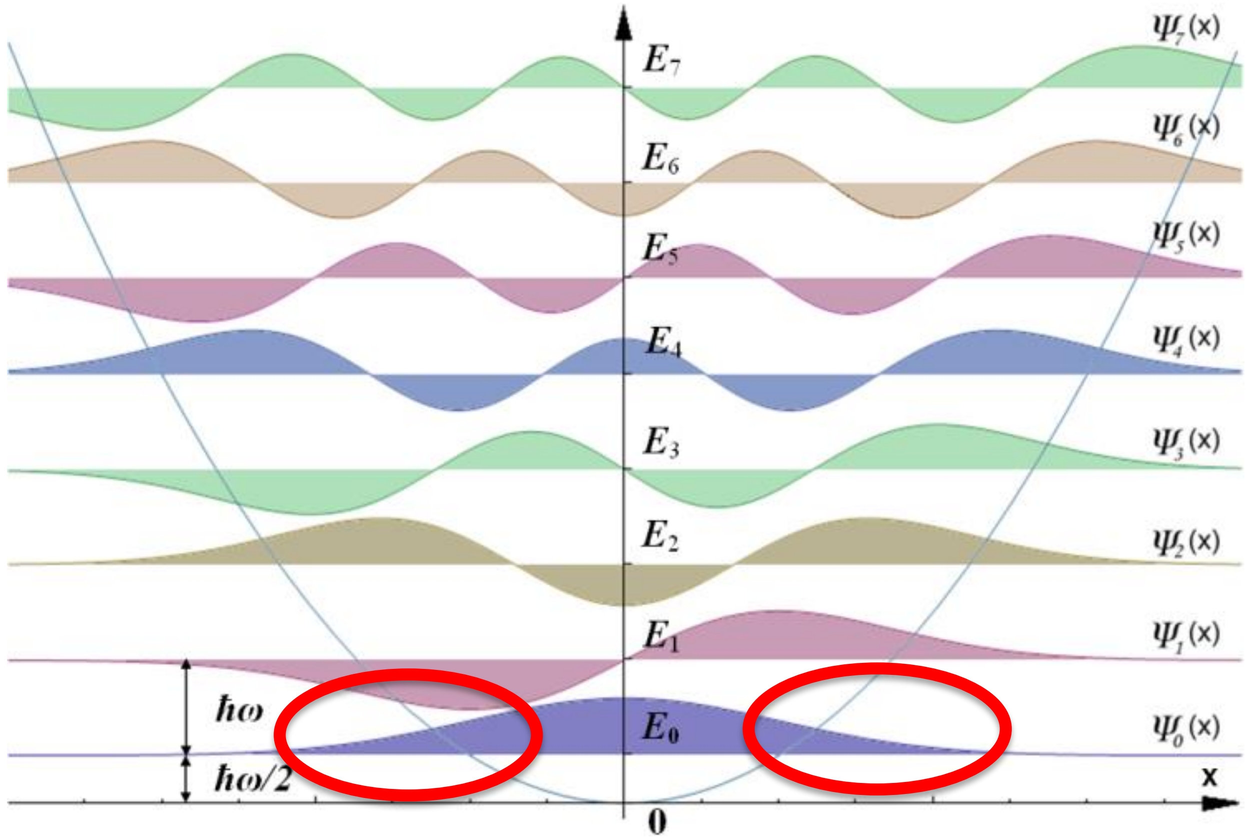
A limit on the variation of the speed of light arising from quantum gravity effect

Abdo et al. 2009, Nature

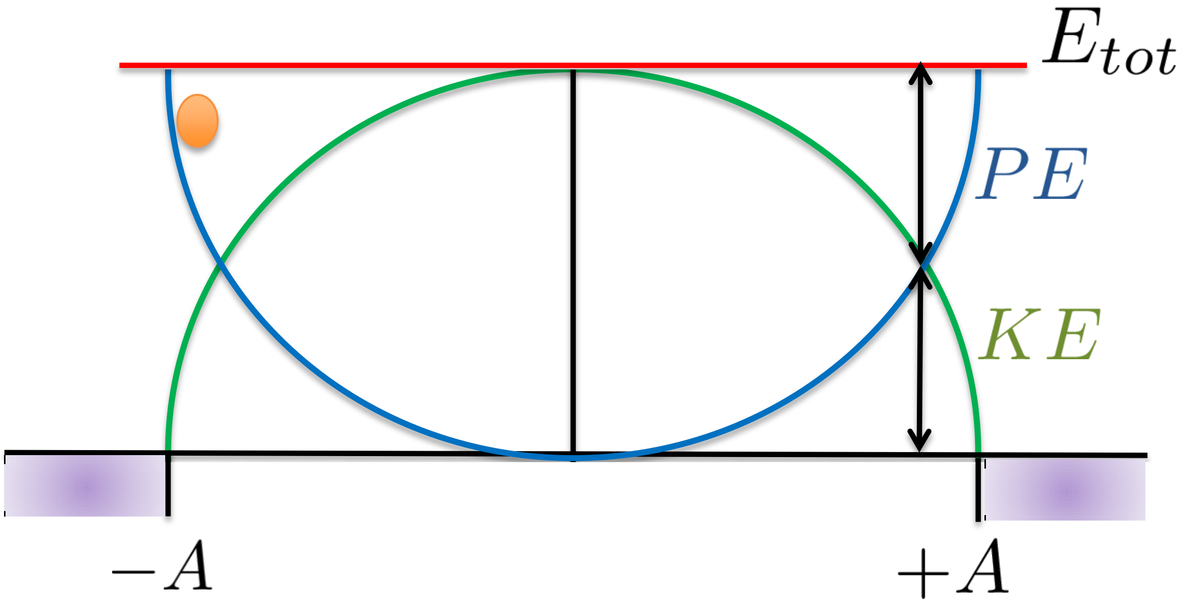
Abstract

A cornerstone of Einstein's special relativity is Lorentz invariance—the postulate that all observers measure exactly the same speed of light in vacuum, independent of photon-energy. While special relativity assumes that there is no fundamental length-scale associated with such invariance, there is a fundamental scale (the Planck scale, $l_{Planck} \sim 1.62\times10^{-33}$cm or $E_{Planck} = M_{Planck}c^2 \sim 1.22\times10^{19}$GeV), at which quantum effects are expected to strongly affect the nature of space-time. There is great interest in the (not yet validated) idea that Lorentz invariance might break near the Planck scale. A key test of such violation of Lorentz invariance is a possible variation of photon speed with energy. Even a tiny variation in photon speed, when accumulated over cosmological light-travel times, may be revealed by observing sharp features in γ-ray burst (GRB) light-curves. Here we report the detection of emission up to ~31GeV from the distant and short GRB090510. We find no evidence for the violation of Lorentz invariance, and place a lower limit of $1.2E_{Planck}$ on the scale of a linear energy dependence (or an inverse wavelength dependence), subject to reasonable assumptions about the emission (equivalently we have an upper limit of $l_{Planck}/1.2$ on the length scale of the effect). Our results disfavour quantum-gravity theories in which the quantum nature of space-time on a very small scale linearly alters the speed of light.

How can a mass move into a classically forbidden region?

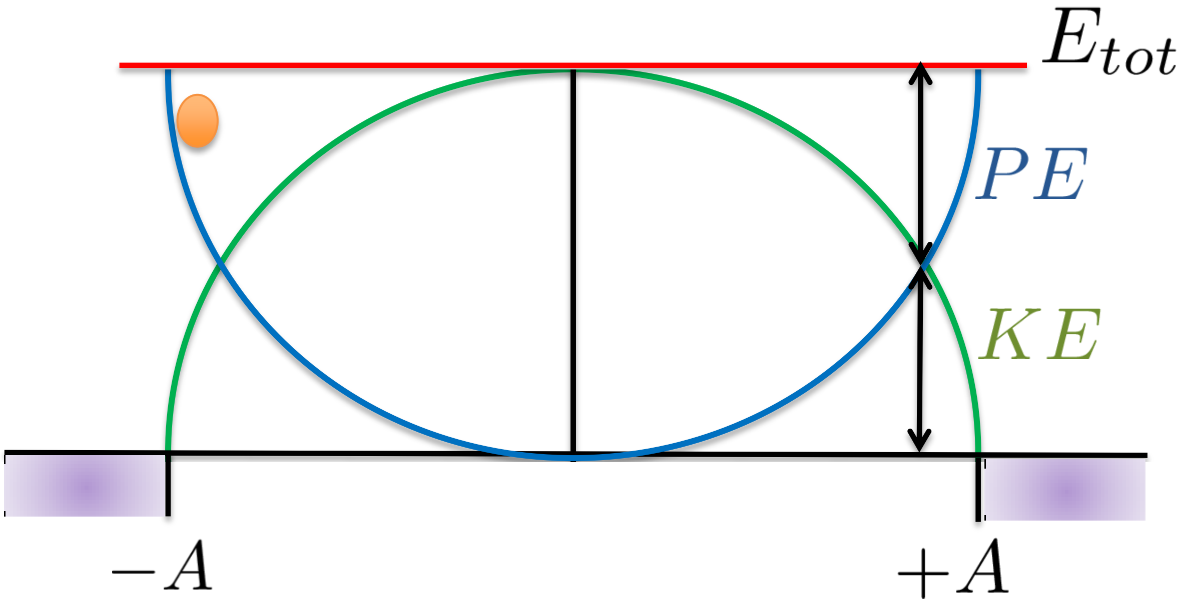


$$E_{total} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

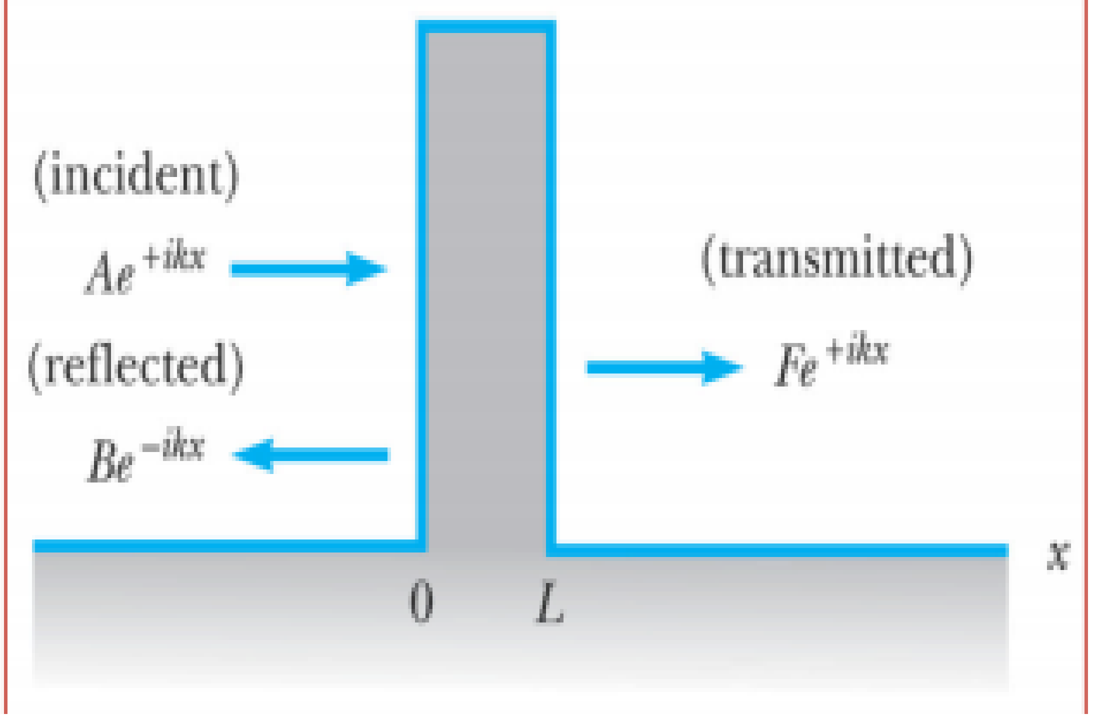


- Classically the turning points are $\pm A$
- As x approaches A the KE goes to zero. This why the mass stop!

$$E_{total} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$



- How could the mass go past $\pm A$?
- This would mean: $PE > E_{tot}$
- This would require KE to be negative (i.e. $p^2 < 0$)
- p is an imaginary number!



$$(x < 0) \qquad \psi = Ae^{ikx} + Be^{-ikx}$$

$$(0 < x < L) \qquad \psi = Ce^{\kappa x} + De^{-\kappa x}$$

$$(x > L) \qquad \psi = Fe^{ikx}$$

$$y(x) = y_{edge} e^{-\frac{x}{h}}$$

$$h = \frac{\hbar}{\sqrt{2m(U - E)}}$$

- Sun shines via nuclear fusion
  - 4 protons stick together to form He
  - These protons need energy to overcome the Coulomb Barrier

Temperature: 15 000 000 000 degrees

Sun: 10 000 000 000 degrees

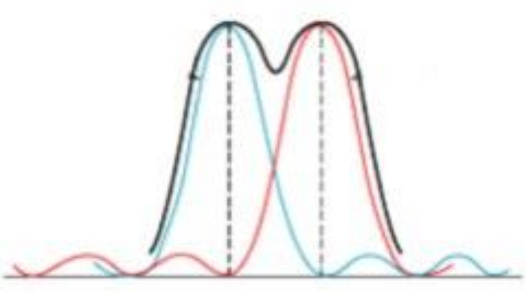
Solution: Tunneling!

There is a small probability of particles passing through impenetrable barriers!

This is what powers the sun!

Electron microscopy vs. Light microscopy

• The resolution of a microscope corresponds to the shortest distance between two points that can still be distinguished by the observer as separate entities.

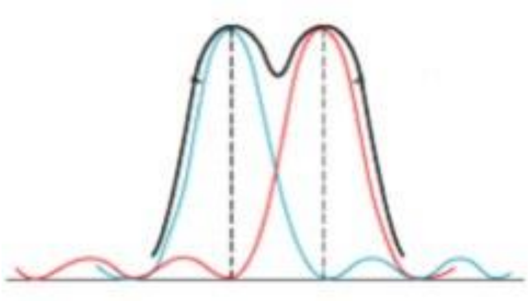


Electron microscopy vs. Light microscopy

• Because of HUP, the best resolution that can be achieved using radiation or particles with wavelength λ is ~λ/2.

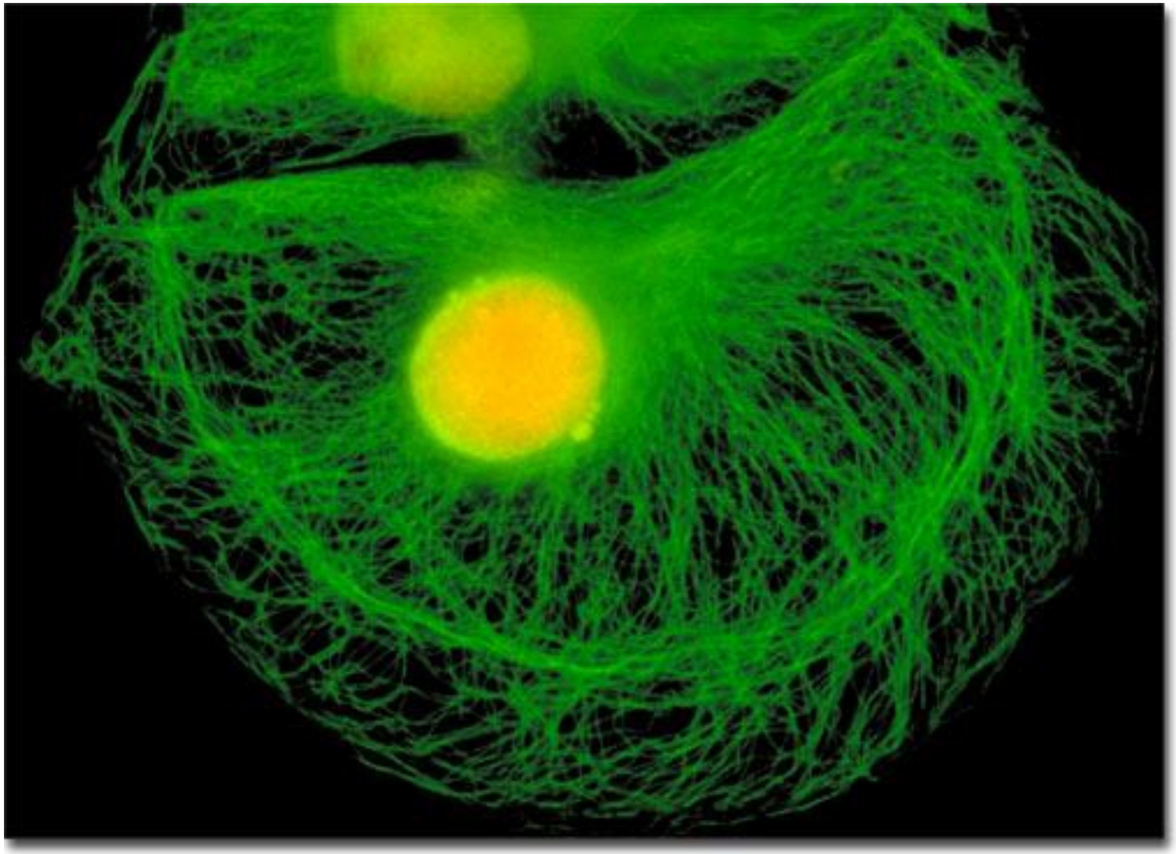
• Visible light microscope the best resolution is ~200 nm

• Electron microscopes on the order of ~0.2 nm



Light Microscopy

- This means that light microscopy is useful to visualize eukaryotic cells (size: 10 – 100 μm).

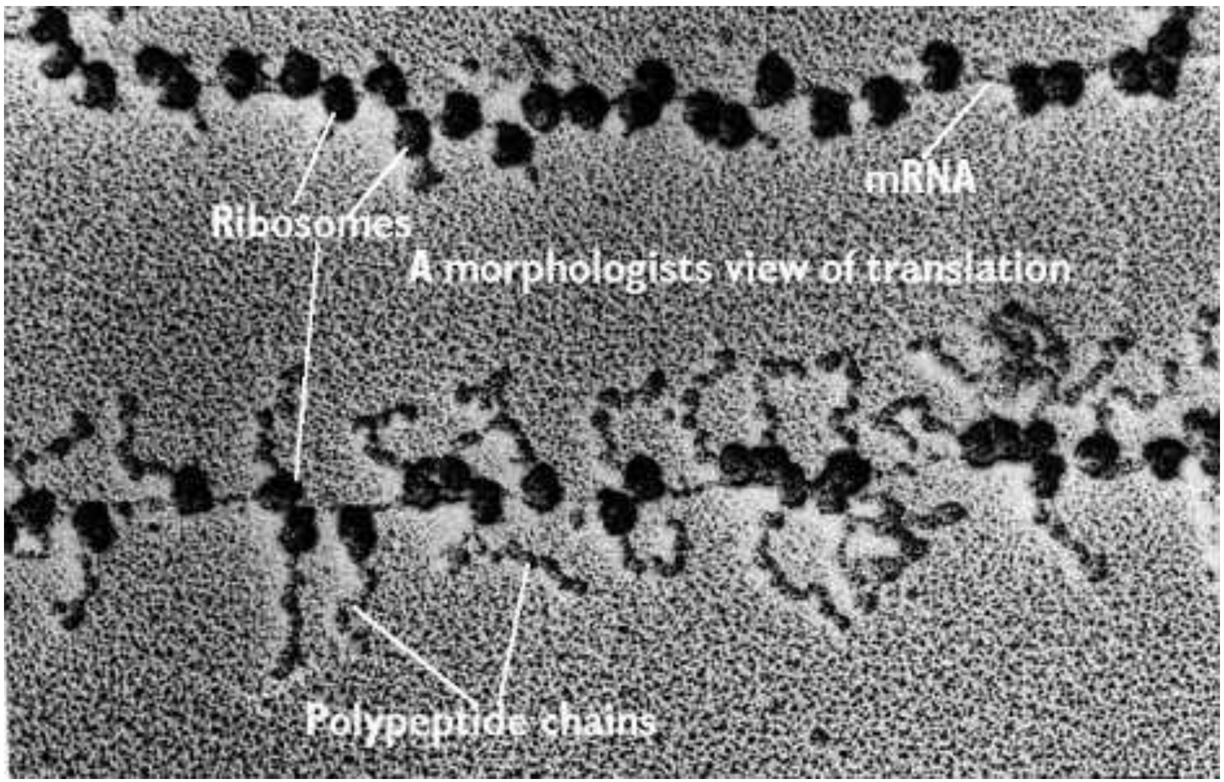


The size of a cell nucleus is ~10 μm.

http://www.olympusmicro.com/galleries/fluorescence/images/p2k2microtubulescellnuclarge.jpg

Electron microscopy

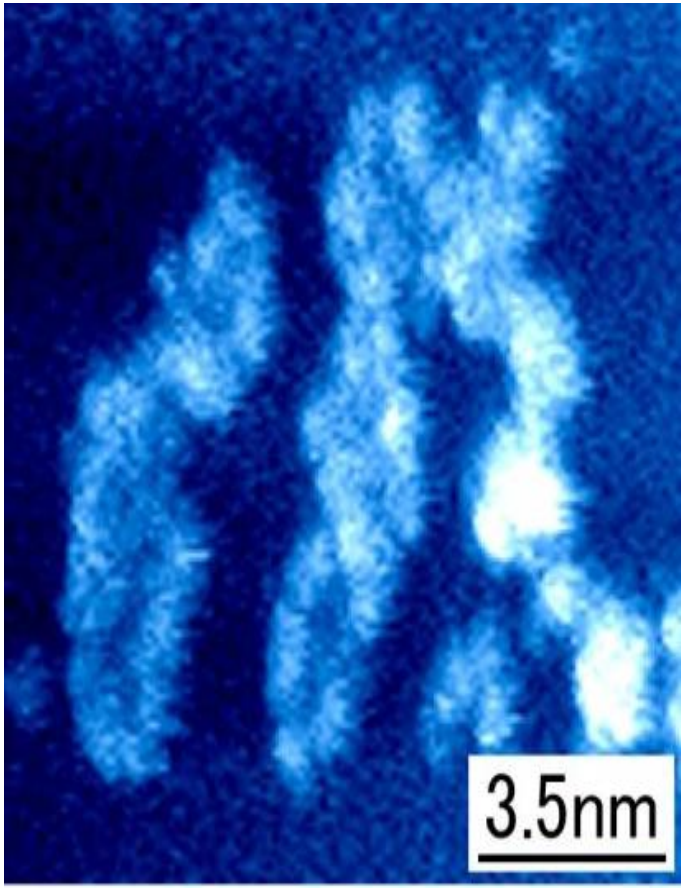
• But smaller cells (e.g. bacteria, size ~1-2 μm), or the inside structure of cells can only be seen well using electron microscopy.



http://nanobiologynotes.blogspot.com/

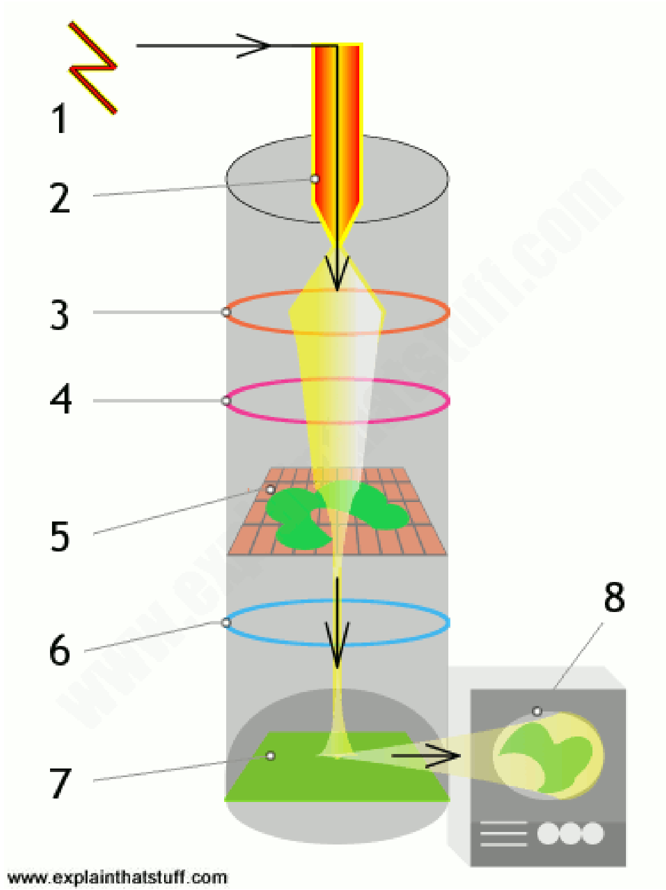
The size of a ribosome is ~11 nm.

• Direct imaging of single DNA molecules

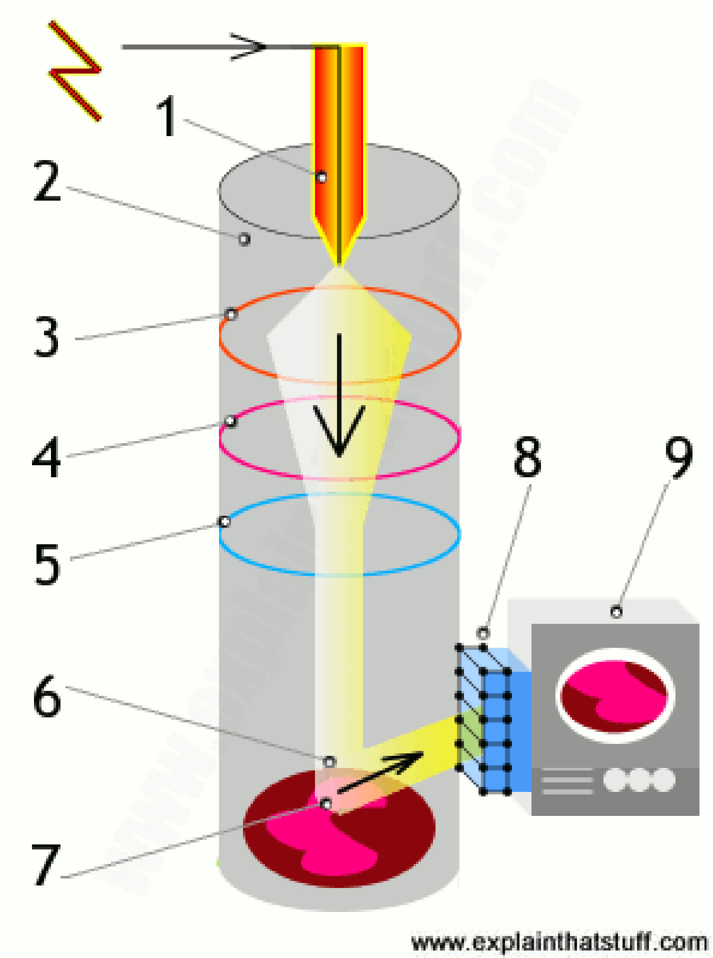


Electron Microscopy

TEM: Transmission Electron Microscope

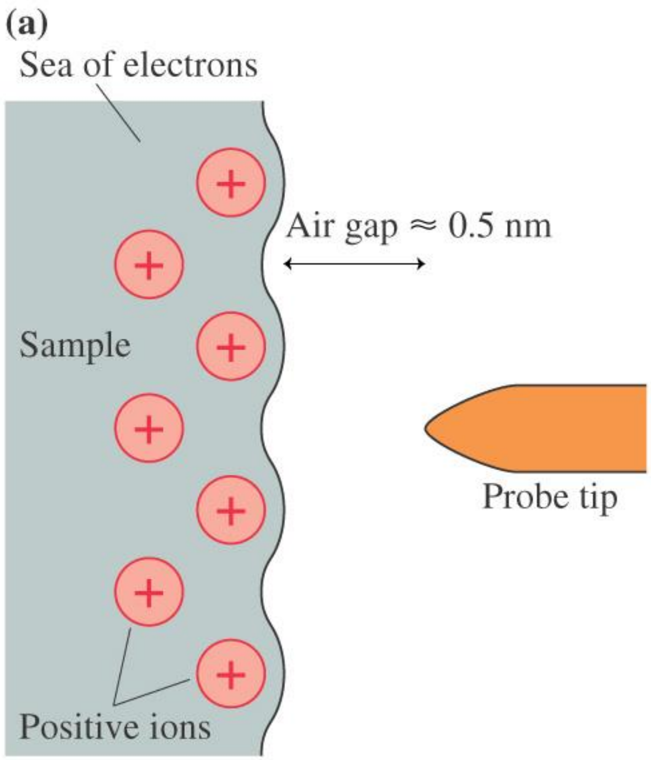


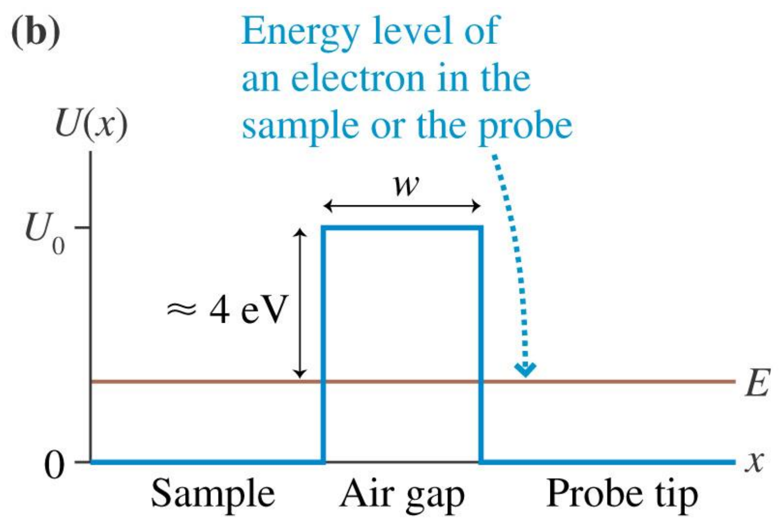
SEM: Scanning Electron Microscope



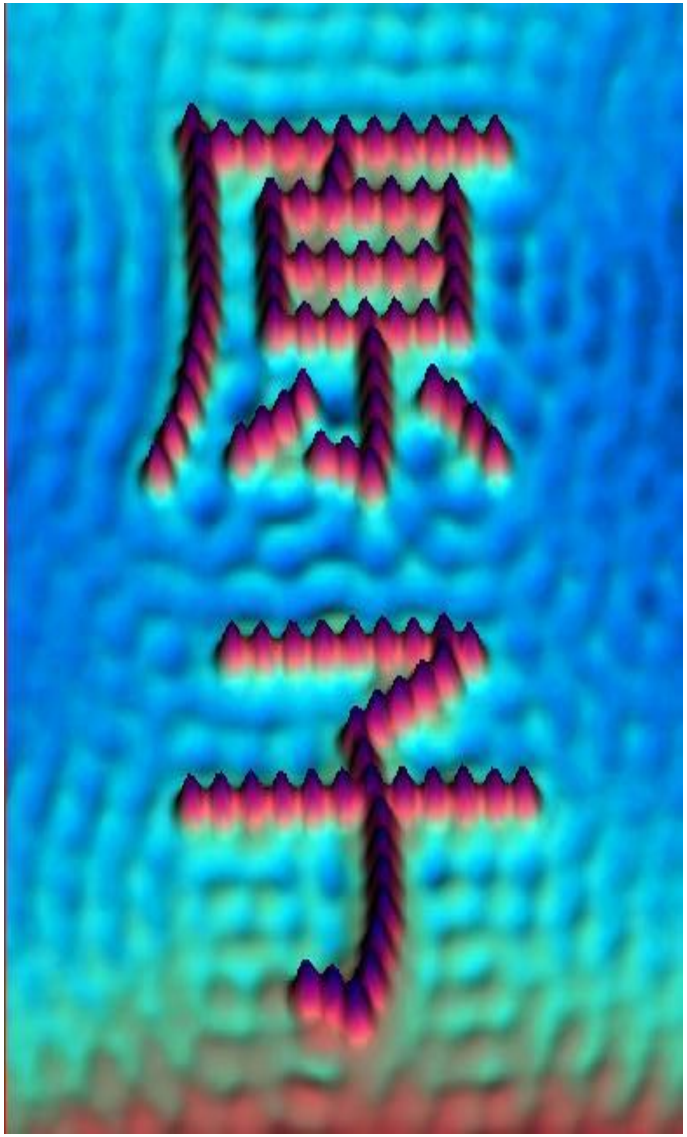
Scanning Tunneling Microscopy

• STM (developed in 1981) makes use of the tunneling effect to achieve 0.01 nm resolution or better.

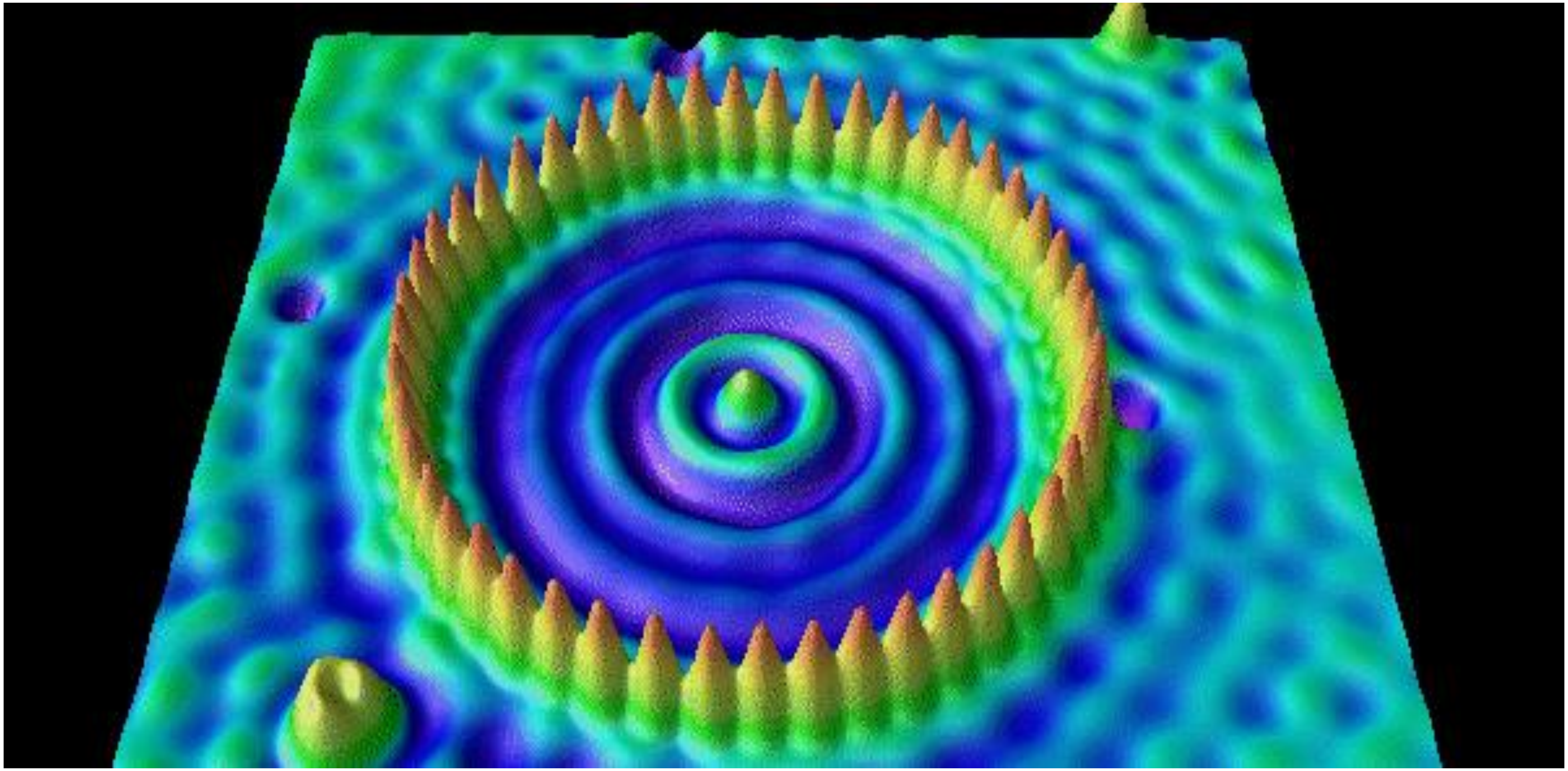




- Imaging of single iron atoms at the surface of copper.



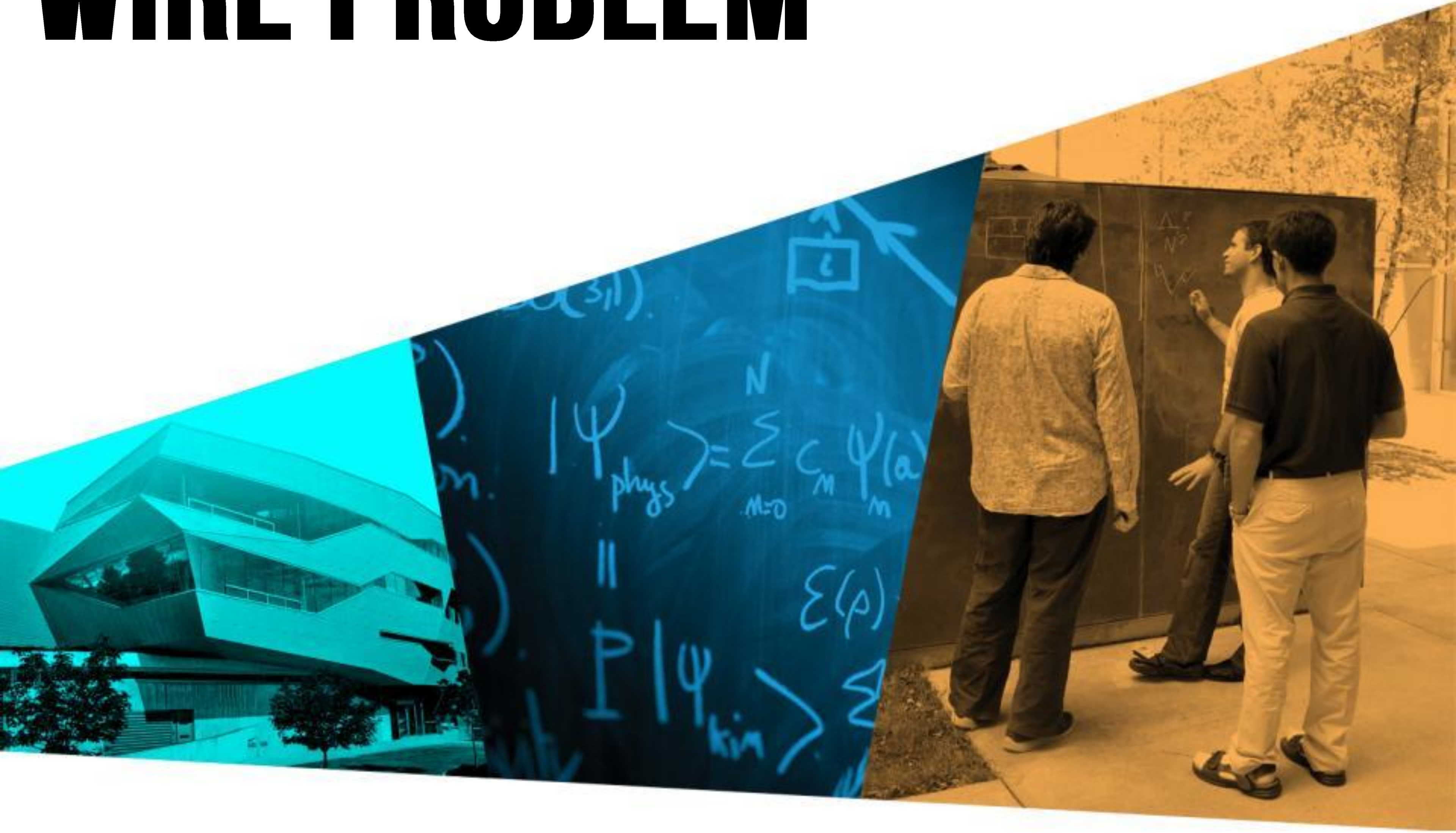
Quantum “Corral”



[redacted]

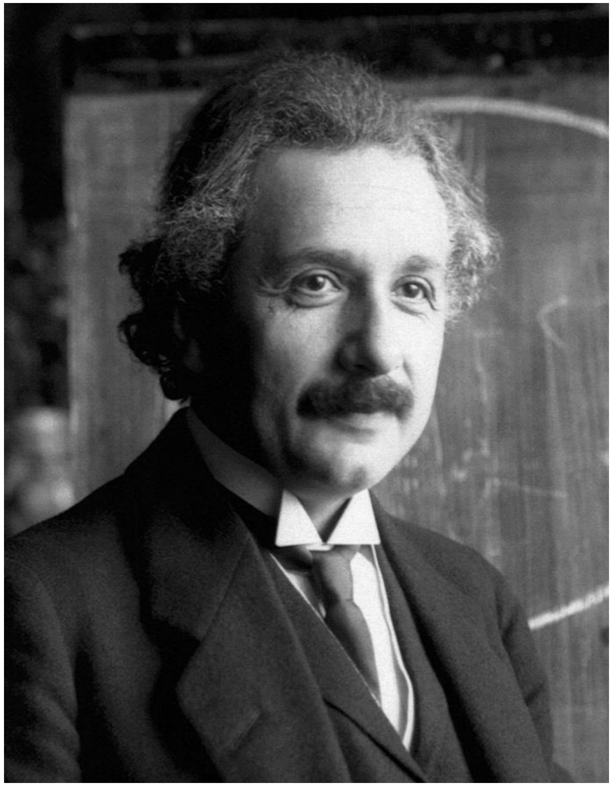
[redacted]

WIRE PROBLEM



PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Einstein’s Motivation

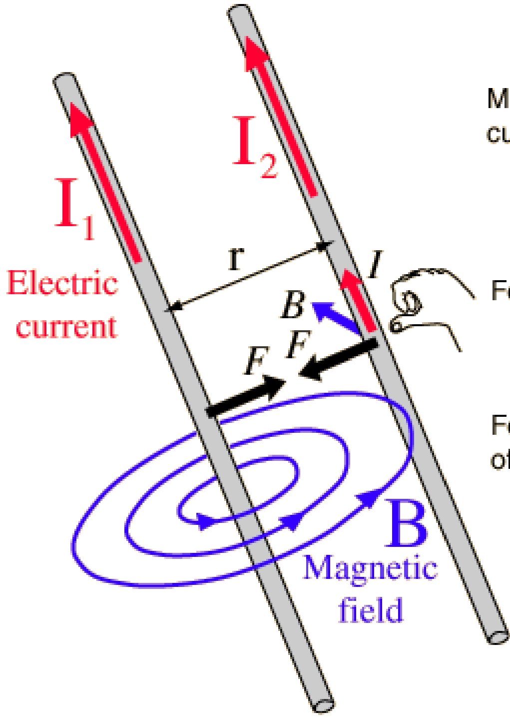


*“… **the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good.** We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that **light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies.** The introduction of a "luminiferous ether" will prove to be superfluous in as much as the view here to be developed will not require an "absolutely stationary space" provided with special properties, nor assign a velocity-vector to a point of the empty space in which electromagnetic processes take place.*

The theory […] is based—like all electrodynamics—on the kinematics of the rigid body, since the assertions of any such theory have to do with the relationships between rigid bodies (systems of co-ordinates), clocks, and electromagnetic processes. Insufficient consideration of this circumstance lies at the root of the difficulties which the electrodynamics of moving bodies at present encounters.”

Electrostatic

Magnetic Force



Magnetic field at wire 2 from current in wire 1:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

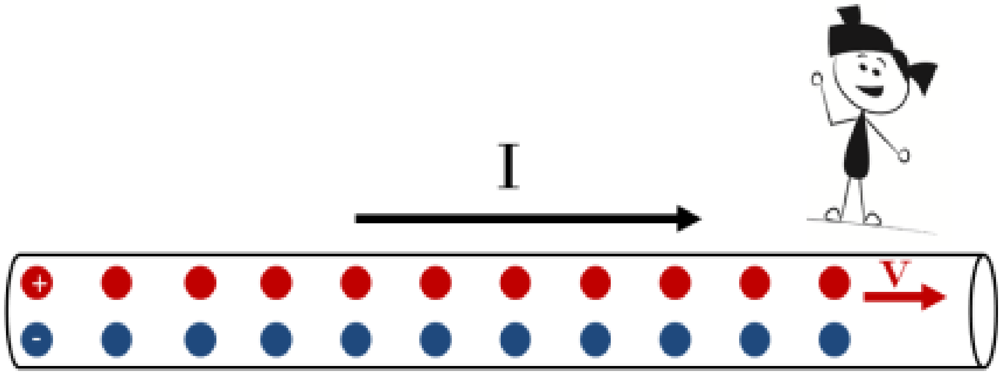
Force on a length ΔL of wire 2:

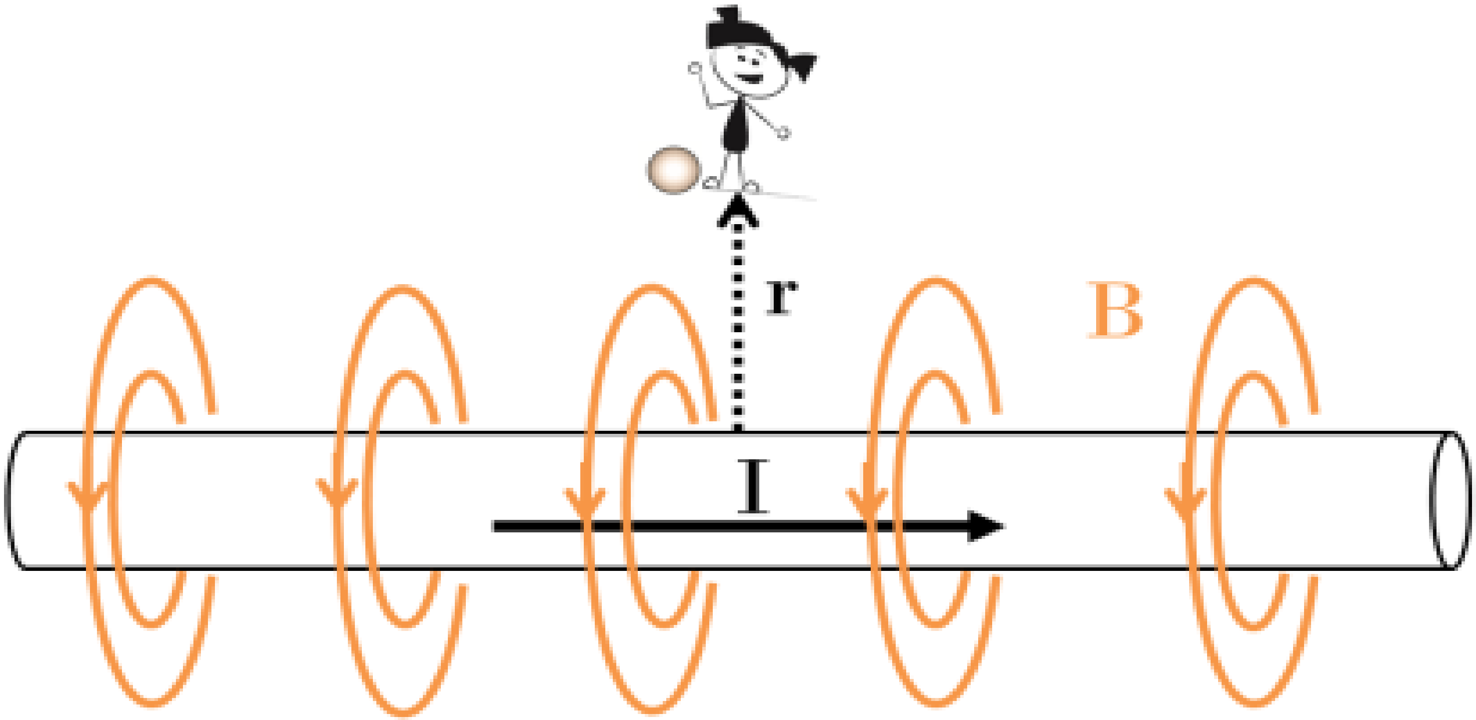
$$F = I_2 \Delta L B$$

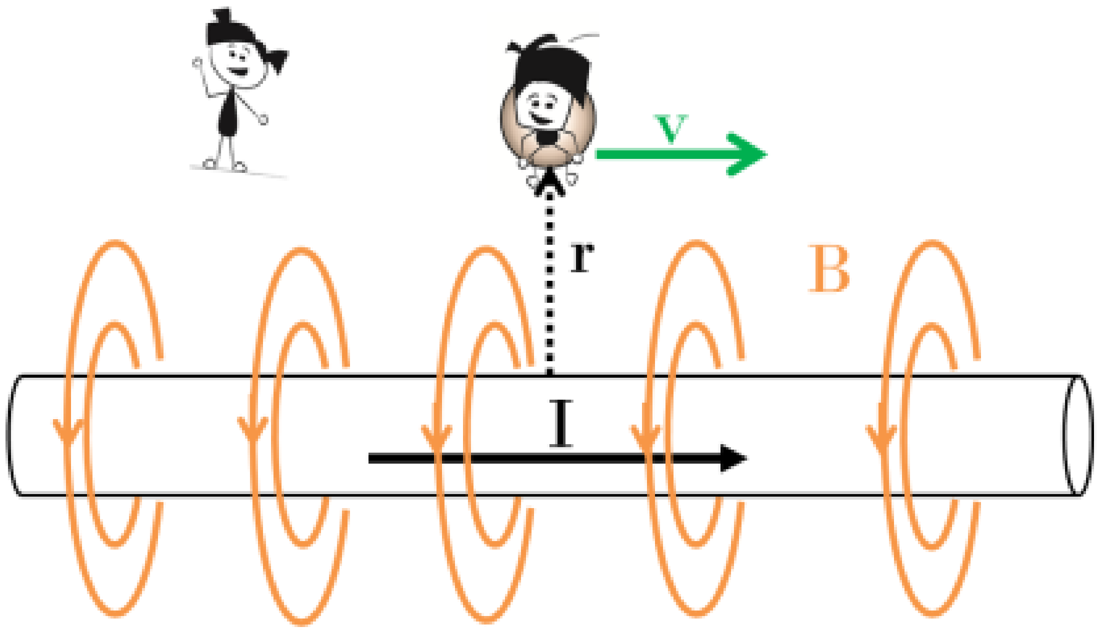
Force per unit length in terms of the currents:

$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

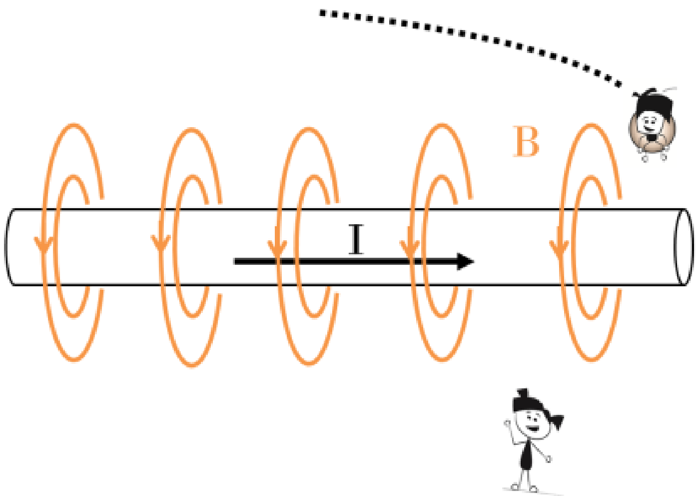
Einstein's Motivation




r
B
I


v
r
B
I

Alice’s Frame

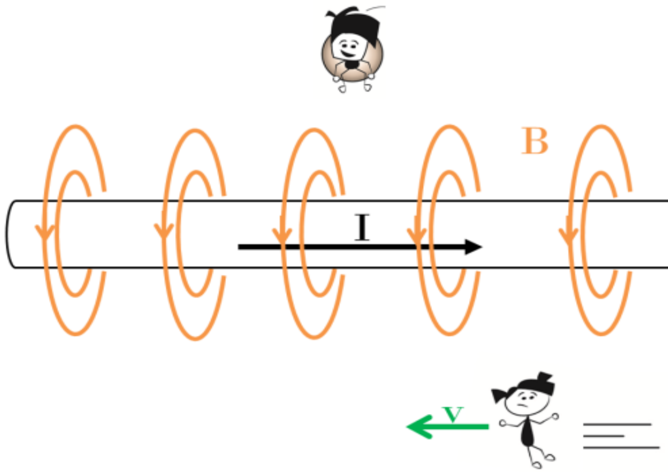


$$F = qvB$$

$$F = -qv\frac{\mu_o I}{2\pi r}$$

Because Bob and his charge is moving he will experience a force and be **deflected**.

Bob’s Frame



Only moving charges experience a force so Bob should **not be deflected**!

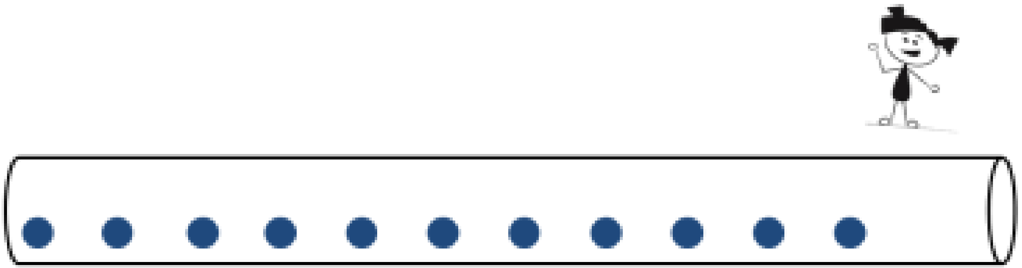
Paradox?

- It would seem that the laws of physics depend on your reference frame.

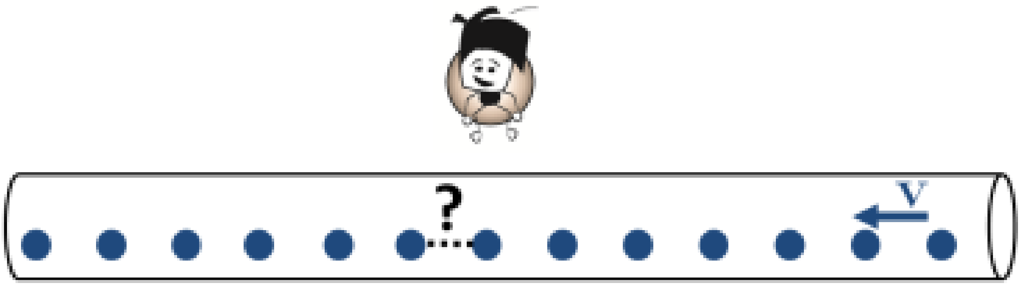
- That would be very weird!

- Einstein postulated that the laws of physics should be the same regardless of your frame of reference!

Negative Charges

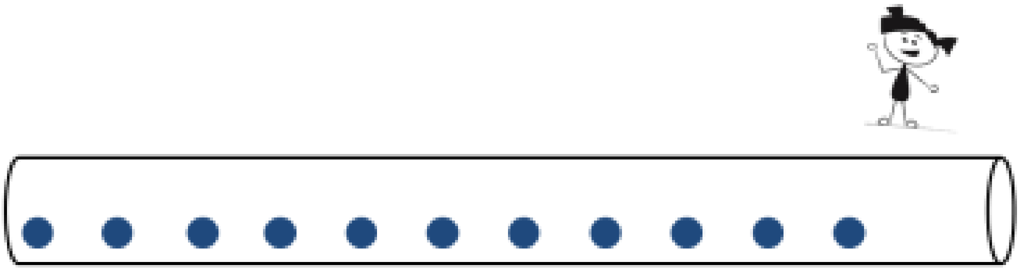


For Alice, the negative charges are at rest!

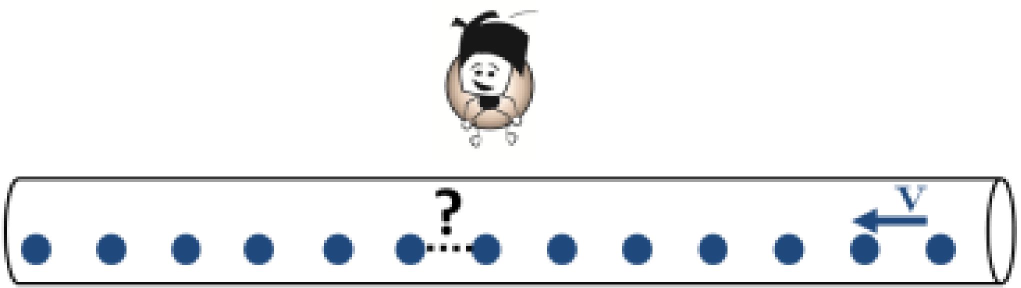


For Bob, the negative charges move to the left with speed v.

Negative Charges



$$N_-$$

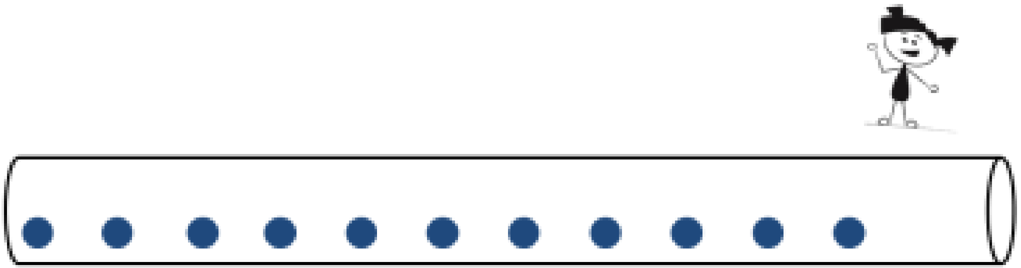


$$N'_- = \gamma N_-$$

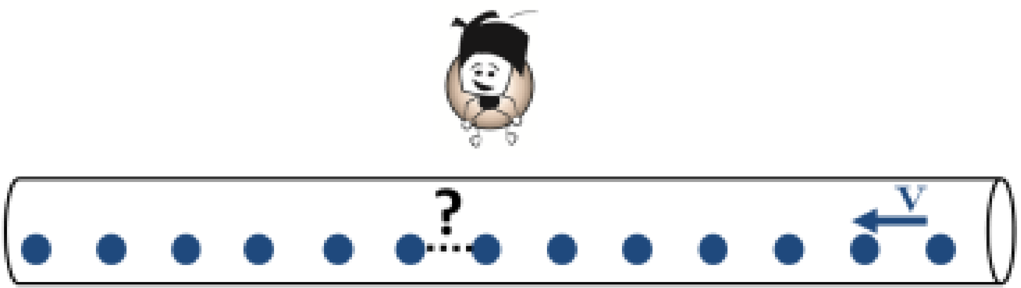
Binomial Expansion

$$(1+x)^n \approx 1 + nx$$

Negative Charges

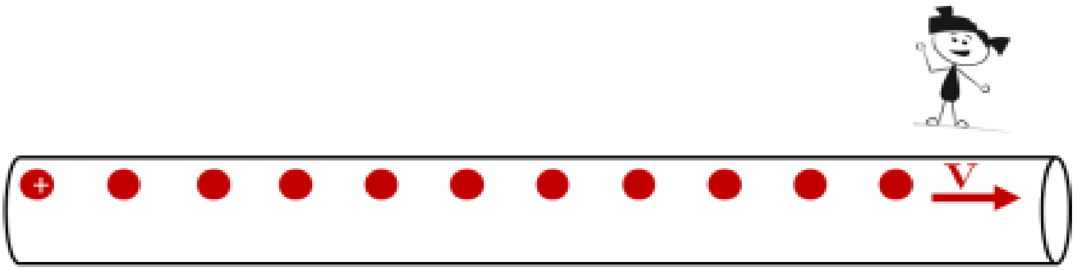


$$N_-$$

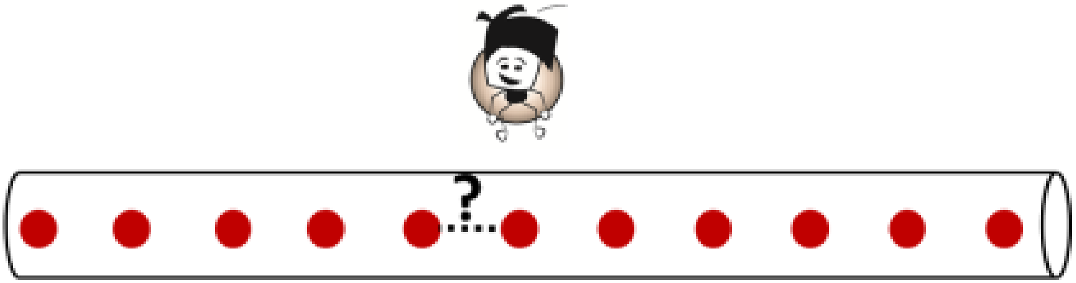


$$N'_- = N(1 + \frac{v^2}{2c^2})$$

Positive Charges

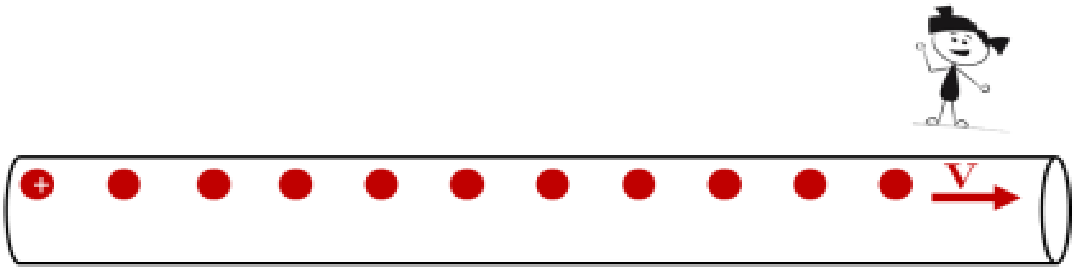


For Alice, the positive charges are moving to the right.

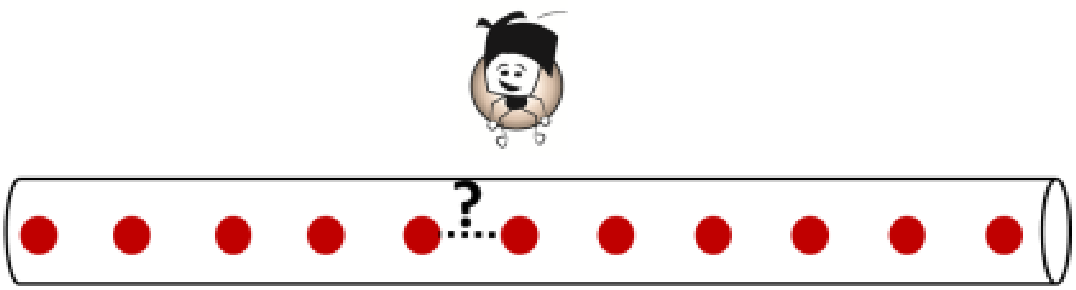


For Bob, the positive charges are at rest.

Positive Charges

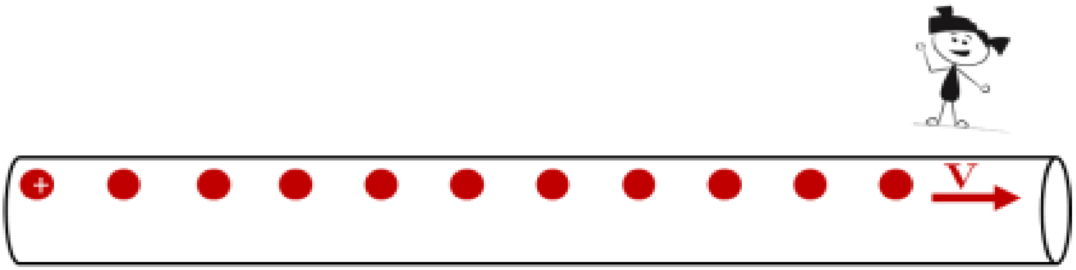


$$N_+$$

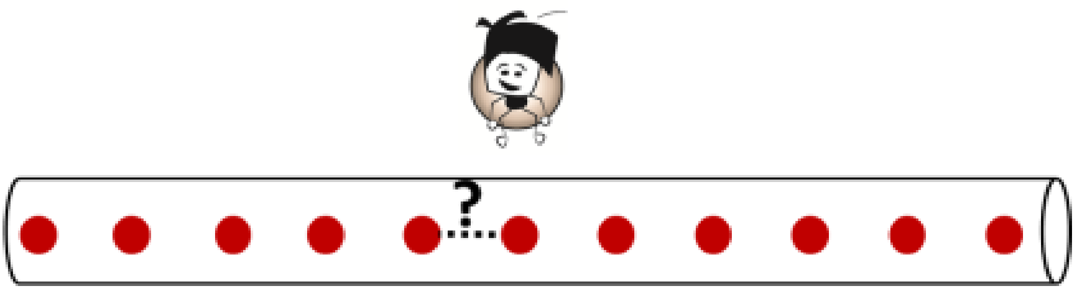


$$N'_+ = \frac{N}{\gamma}$$

Positive Charges



$$N_+$$



$$N'_+ = N\left(1 - \frac{v^2}{2c^2}\right)$$

Bob's Charge Line Density

$$\lambda' = e(\frac{N}{\gamma} - N\gamma)$$

To Bob, the wire is negative. To Alice, it is neutral.

Perpendicular to Motion

- Bob measures the same distance r, because distances perpendicular to motion are not affected by length contraction.

Electric Field

$$E' = \frac{\lambda'}{2\pi\epsilon_o r}$$

$$E' = \frac{-vI}{2\pi\epsilon_o r c^2}$$

$$E' = -v\frac{\mu_o I}{2\pi r}$$

Force measured by Bob

$$F' = qE' = -qv\frac{\mu_o I}{2\pi r}$$

This is the same force measured by Alice!
Bob also will experience a deflection!

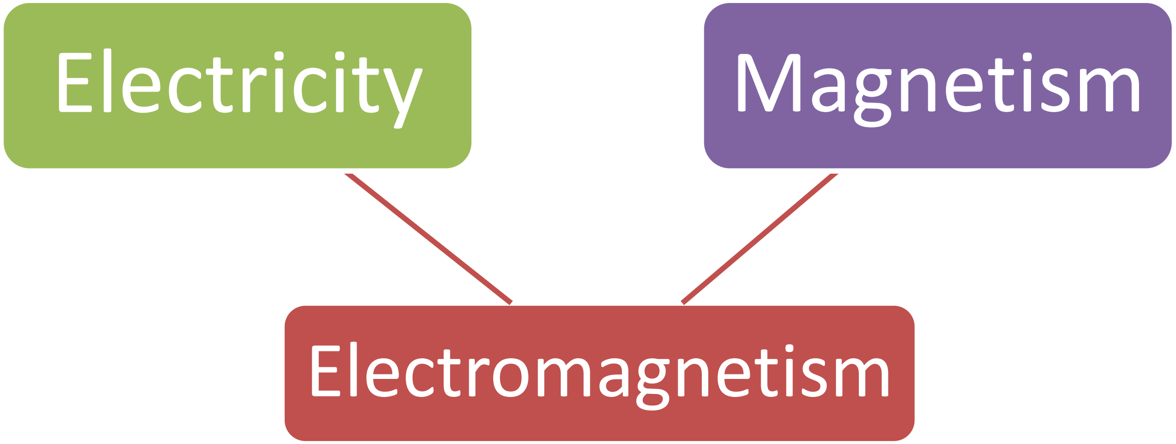
Alice

- Rest wrt Wire
- Measures a neutral wire
- Bob is deflected due to magnetic field of wire

Bob

- Bob moves to the right
- Measures a negative wire
- Bob is deflected due to the electric field

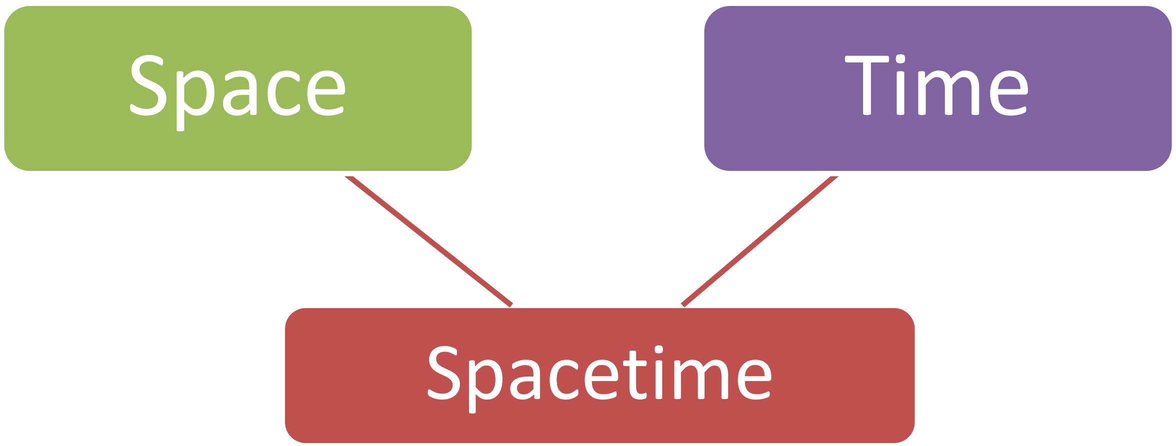
Einstein



Einstein

- Electric and magnetic fields are interchangeable
- The relative contribution of one to the other depends on your reference frame
- In this example, Alice measured everything as magnetic and Bob measured everything as electric.

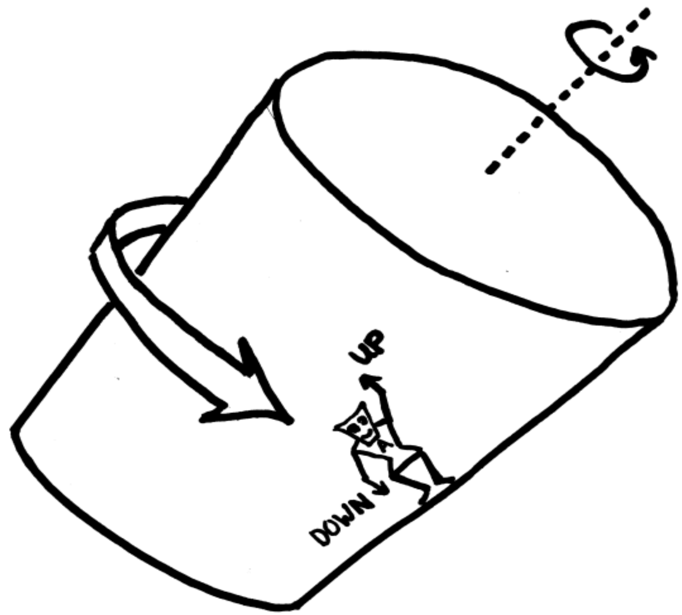
Einstein



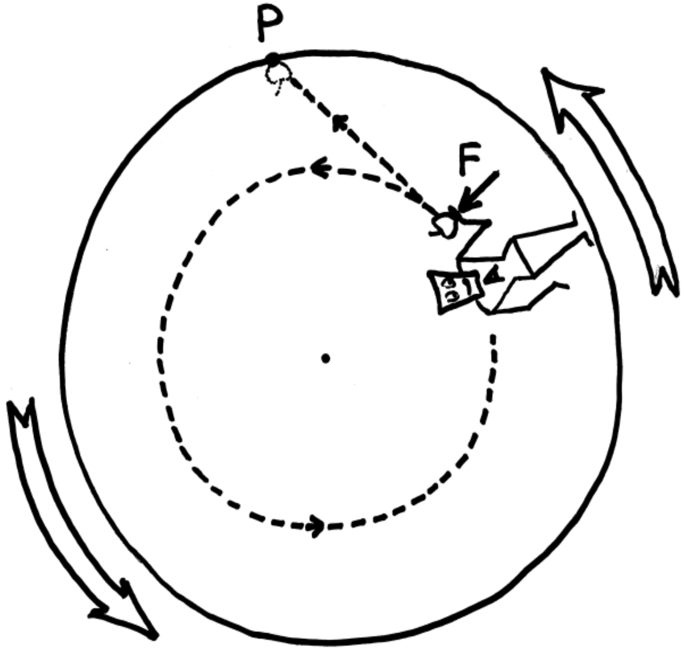
EINSTEIN'S ROTATING DISK

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

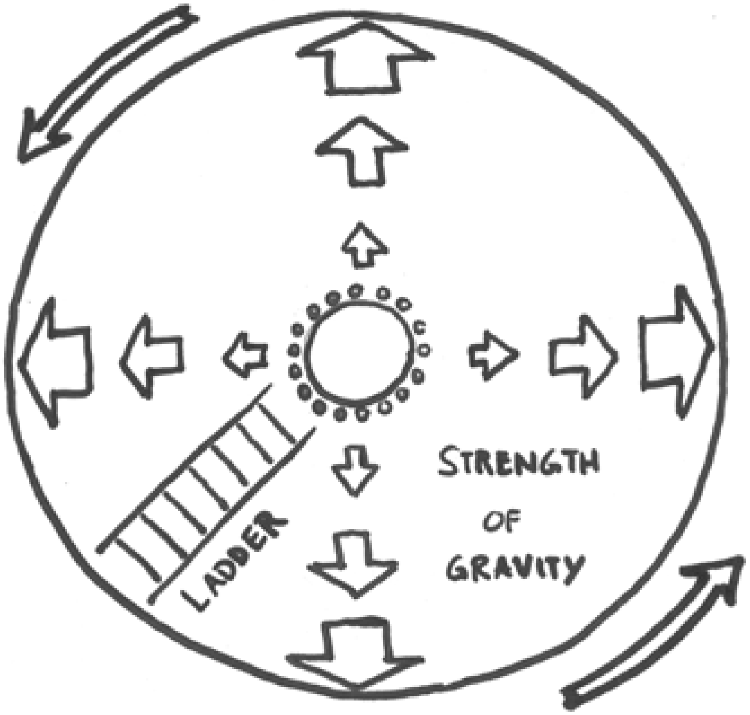
Equivalence Principle



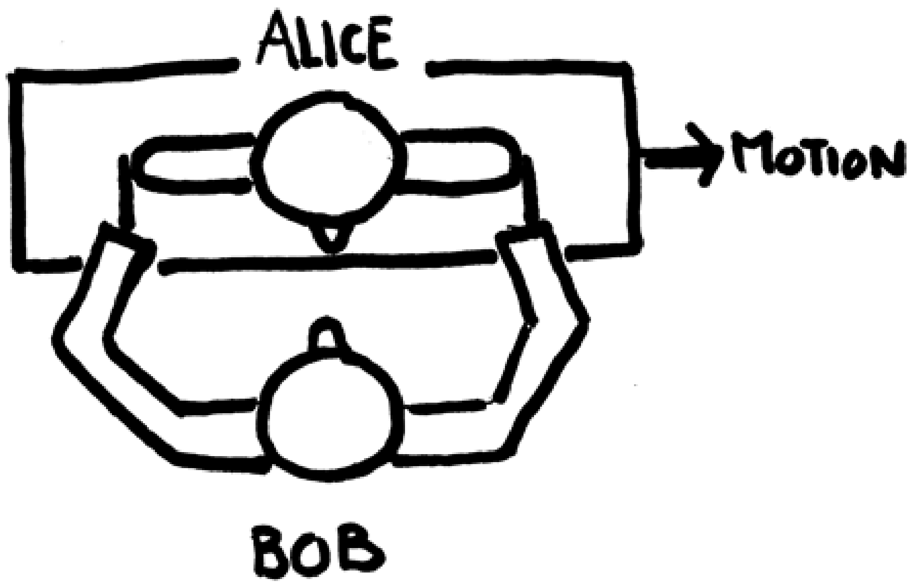
Alice follows the curved path!



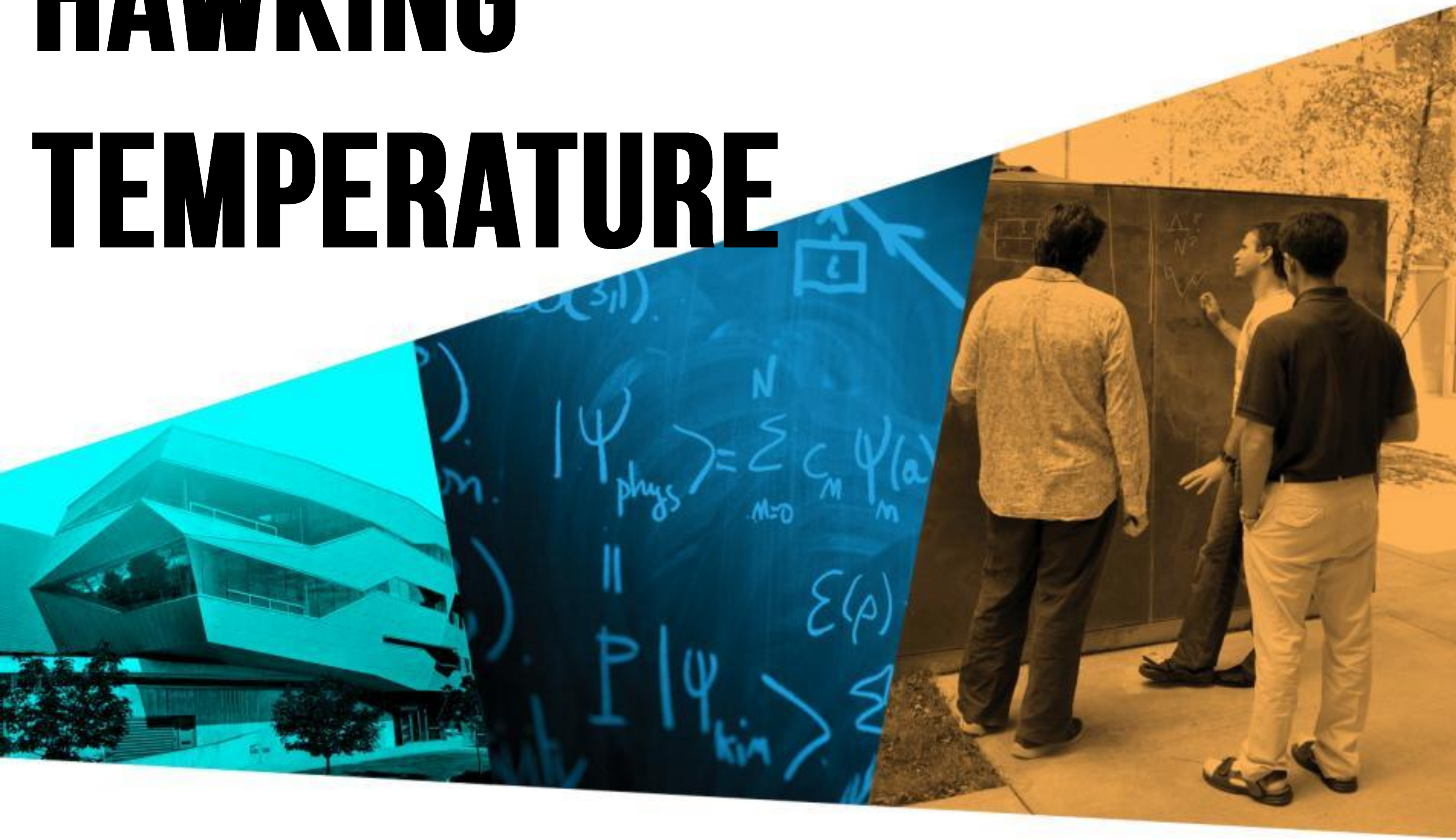
Warping of Time



Warping of Space



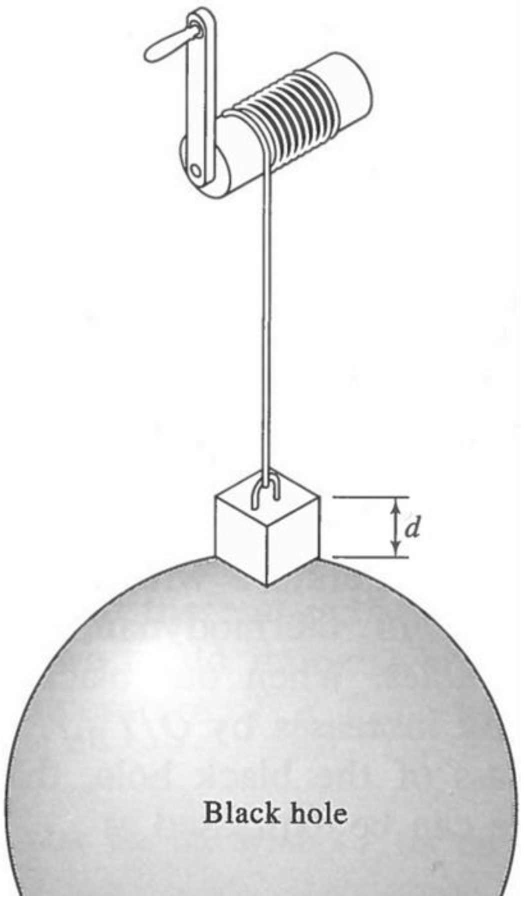
HAWKING TEMPERATURE



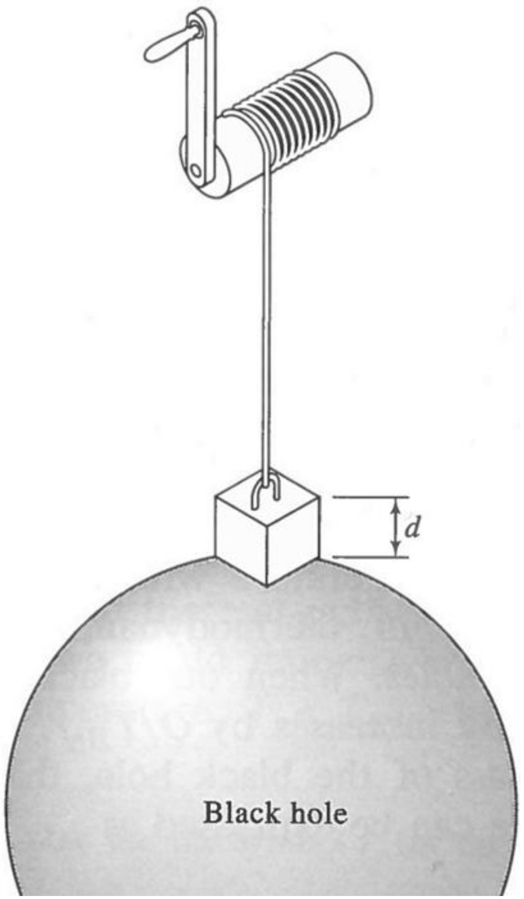
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

General Relativity

We can extract energy by lowering a box of mass m, down to the black hole!



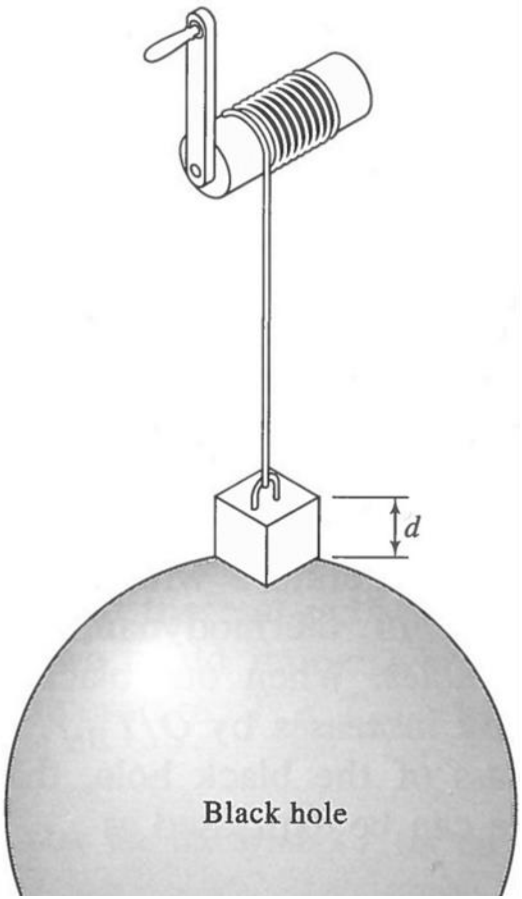
Special Relativity



Light has inertial mass. Trapping it in the box, adds to the amount of energy we can extract.

Thermodynamics

By allowing thermal energy to escape, we can create a heat engine.



Quantum Theory

But we can't lower the box the whole way down. This leads us to conclude that the black hole must have a non-zero temperature.