Quantum Mechanics Solutions 5:

STANDING WAVES COMPLEX

[easy] 1. Make sure this step makes sense to you!

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[medium] 2. The superposition of right- and left-moving complex traveling waves is simply (putting the factors of 1/2 back in):

$$\begin{aligned}
\psi(x,t) &= \psi_L(x,t) + \psi_R(x,t) \\
&= \frac{1}{2} e^{\frac{i}{\hbar}(Et+px)} - \frac{1}{2} e^{\frac{i}{\hbar}(Et-px)} \\
&= \frac{1}{2} e^{\frac{i}{\hbar}Et} \left(e^{\frac{i}{\hbar}px} - e^{-\frac{i}{\hbar}px} \right) \\
&= \frac{1}{2} e^{\frac{i}{\hbar}Et} \left(\cos\left(\frac{1}{\hbar}px\right) + i\sin\left(\frac{1}{\hbar}px\right) - \cos\left(-\frac{1}{\hbar}px\right) - i\sin\left(-\frac{1}{\hbar}px\right) \right) \\
&= \frac{1}{2} e^{\frac{i}{\hbar}Et} \left(\cos\left(\frac{1}{\hbar}px\right) + i\sin\left(\frac{1}{\hbar}px\right) - \cos\left(\frac{1}{\hbar}px\right) + i\sin\left(\frac{1}{\hbar}px\right) \right) \\
&= \frac{1}{2} e^{\frac{i}{\hbar}Et} \left(i2\sin\left(\frac{1}{\hbar}px\right) \right) \\
&= ie^{\frac{i}{\hbar}Et} \left(\sin\left(\frac{1}{\hbar}px\right) \right)
\end{aligned}$$

[hard] 3. Observe that $|i| = 1$ and that $e^{i\theta} = 1$, i.e., the magnitude, or length of both of these complex numbers is just one. Thus $|\psi(x,t)|^2 = \sin^2\left(\frac{1}{\hbar}px\right) = \sin^2\left(\frac{2\pi x}{\lambda}\right)$ See sketch: