Quantum Mechanics Exercises 4:
HARMONIC OSCILATOR

[easy] 1. The classical simple harmonic oscillator, such as a spring governed by Hooke's Law F = –kx, results in an oscillation of the form $x(t) = A\cos(\omega t)$, where **A** is the amplitude of oscillation and $\boldsymbol{\omega} = \frac{\boldsymbol{\theta}}{\boldsymbol{t}}$ is the frequency. Sketch this cosine function as **x** versus ***t***, and draw a circular representation of simple harmonic motion in terms of angle $\boldsymbol{\theta}$. Knowing how an oscillator stops and reverses its direction of motion when it reaches its point of greatest displacement, and that it attains its highest speed when at its equilibrium position, sketch and deduce the general form of its momentum p(t) on your **x** versus **t** sketch.

[hard] 2. Next, consider the law of conservation of energy: $E = E_{\text{kinetic}} + E_{\textit{potential}} = \text{const}$. For a harmonic oscillator, this takes the form $E = \frac{p^2}{2m} + \frac{1}{2}kx^2$. More generally, the first term represents the energy of a free particle (unrestrained, ergo only kinetic), while the second one can be substituted with any function, which will then represent a physical constraint on the freedom of the particle; it represents the *potential energy* of the system. In this exercise, we're using the *harmonic potential energy*, $V(x) = \frac{1}{2}kx^2$. As another example, for a particle trapped in a box we would have $V(x) = 0$ for 0 < x < L, and $V(x) = \infty$ elsewhere.

Using the above system with harmonic potential energy, apply the constancy of E and the fact that $p(t) = mv = m\frac{dx}{dt}$ ($\frac{dx}{dt}$, a derivative, is the ratio of the change in position, **x** to the change in time **t**) to solve for the constant coefficient from p(t), then solve for the frequency $\boldsymbol{\omega}$ in terms of **k** and **m**.

Hint: $\frac{d}{dt}(\cos(\omega t)) = -\omega\sin(\omega t)$