Quantum Mechanics Exercises 2:

STANDING WAVES REAL

In this excise you will work through the mathematics of how a standing wave is really a superposition of two traveling waves, traveling in opposite directions, and vice versa. This concept is crucial to appreciating the central mystery of our quantum universe - superposition. This is relevant, for example, to our discussion in the lectures of a particle in a box that is in a weird state of moving both to the left and to the right at the same time!

[easy] 1. Consider a standing wave of wavelength λ and period **T**. Convince yourself that mathematically it can be described by the function: $\psi(x,t)=\cos\left(\frac{2\pi t}{T}\right)\sin\left(\frac{2\pi x}{\lambda}\right)$. Do so by sketching this function between x = 0 and x = λ at the following successive instants of time: t = 0; T/4; T/2; 3T/4; T. Observe that the standing wave oscillates up and down, returning to its original state after one full period, **T**.

[medium] 2. Use the trigonometric identity $\cos(a)\sin(b)=\frac{1}{2}[\sin(a+b)-\sin(a-b)]$ to re-express $\psi(x,t)$ as a sum of a left-moving and a right-moving wave, $\psi_L(x,t)$ and $\psi_R(x,t)$. (The right-moving wave has a minus sign out front; also, ignore the factors of 1/2.) Convince yourself that $\psi_R(x,t)$ (the one with "a - b") is a right-moving wave by sketching it between x = 0 and x = λ at the following successive instants of time: t = 0; T/4; T/2; 3T/4; T. Observe that a given crest of the wave covers a distance λ in a time T. What is the velocity of the wave? Repeat for the left-moving wave, $\psi_L(x,t)$ (the one with "a + b").

Type equation here.

[hard]

3a. The wavelength, λ, and period, **T**, can be expressed in terms of physical properties of the corresponding particle they refer to by using the de Broglie relation ($p = \frac{h}{\lambda}$) and Einstein's formula ($E = hf$, where f = 1/T is the frequency of the wave). Thus, re-express $\psi_R(x,t)$ and $\psi_L(x,t)$ in terms of **p** and **E** instead of λ and **T**. Factor out the 2π and Planck's constant, **h**.

Physicists usually further simplify these expressions by defining the **reduced Planck's constant**, $\hbar = \frac{h}{2\pi}$ (read as "h bar"). Notice that one wave has **+p** and the other has **–p**. The point of this exercise is that **both** waves must be present, simultaneously, for a wave confined to a box, forming a standing wave. Thus, a particle in a box has momentum **+p and –p** simultaneously! This is natural for a wave, but decidedly strange for a particle!