Special Relativity Exercises 5:
MINKOWSKIAN GEOMETRY II

The point of these exercises is to convince yourself of the ***reciprocal*** nature of both *time dilation* and *length contraction*. For example, if I see your moving clock running slowly compared to mine, you will see mine running equally slowly (not faster). You will be constructing a spacetime diagram which is to be drawn very carefully and to scale: use provided grid paper and a ruler, and make the diagram as large as possible, but ensure that whole diagram still fits on the page. These exercises are continued from **Minkowskian Geometry I**.

[easy] 1. How much time must elapse on **Bob's** clock for the light signal he sent to **Bob-Prime** to return to him? Find and label this time on **Bob's** worldline using the known time scale along this line.

[easy] 2. At what time on **Bob-Prime's** clock should the light signal have been received and reflected back? Label this time on **Bob-Prime's** worldline.

[easy] 3. Find and label the event on **Bob's** worldline that is simultaneous with **Bob-Prime** receiving the light signal from **Bob**. Draw a line connecting these two simultaneous events. What should the slope of this line be? Does your graph confirm this?

[easy] 4. Knowing the reading on **Bob-Prime's** clock when he received the light signal from **Bob**, count backwards along **Bob-Prime's** worldline (using the known time scale on **Bob's** and **Bob-Prime's** worldliness) to determine the reading on **Bob-Prime's** clock when he passes **Alice-Prime**. If you have drawn your diagram correctly and neatly you should find that this time is **zero**. In other words, the event of **Bob** sending the light signal to **Bob-Prime** and the event of **Bob-Prime** passing **Alice-Prime** are simultaneous in the **Bob**/**Bob-Prime** frame.

[medium] 5. Consider the segment of **Bob's** worldline as he moves from **Alice** to **Alice-Prime**. How much time elapses for **Bob** (he is using one clock)? How much time elapses for **Alice** and **Alice-Prime** (using two clocks)? Whose time is "*running slow*", and by what factor?

[medium] 6. Now consider the segment of **Alice-Prime's** worldline as she moves from **Bob-Prime** to **Bob**. How much time elapses for **Alice-Prime** (one clock)? How much time elapses for **Bob** and **Bob-Prime** (two clocks)? Whose time is "*running slow*", and by what factor?

The two factors you found (in questions "**5**" and "**6**") should be the same – this is the relativity of time dilation. Each observer sees the other's time "*running slow*" by the *same* factor.

[hard] 7. In Part **4)** we saw the event of **Bob** sending the light signal to **Bob-Prime** and the event of **Bob-Prime** passing **Alice-Prime** are simultaneous in the **Bob**/**Bob-Prime** frame. Draw a line connecting these two events: This line can be considered a platform with **Bob** and **Bob-Prime** standing at each end (platform is tilted in time when viewed in the **Alice**/**Alice-Prime** frame) whose proper length was calculated in **question 10** in ***Minkowskian Geometry I*)**. Label the spacetime event at which this line intersects **Alice's** worldline as *****. Notice that, according to **Bob** and **Bob-Prime**, this is the same time that **Alice-Prime** is at **Bob-Prime's** location and **Alice** is at *****, which is at a point along the platform somewhere between **Bob** and **Bob-Prime**. Therefore **Bob** and **Bob-Prime** consider the distance between **Alice** and **Alice-Prime** to be the distance between ***** and **Bob-Prime** along their platform at $\mathbf{t_B=t_{B'}=0}$. Determine this distance *graphically* in two different ways.

i) Graphically measure the *ratio* of the "***** to **Bob-Prime**" distance to the "**Bob** to **Bob-Prime**" distance, and then multiply by the known proper length of the platform found in question **10** in ***Minkowskian Geometry I*.**

ii) You already know the proper distance between **Alice** and **Alice-Prime** from question **2** in ***Minkowskian Geometry I***. Measure the elapsed time $c\Delta t_A$ along **Alice's** worldline between ***** and when **Alice** passes **Bob**, and use Minkowskian geometry ("Pythagoras with a minus sign") to calculate the length of the diagonal.

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[easy] 8. Notice that the distance determined in the previous part is less than the proper distance between **Alice** and **Alice-Prime**, i.e. **Bob** and **Bob-Prime** see the distance between **Alice** and **Alice-Prime** as length contracted. By what factor is this distance length contracted? Does this agree with by how much it *should* be length contracted (using the length contraction formula)? By what factor is the distance between **Bob** and **Bob-Prime** length contracted as seen by **Alice** and **Alice-Prime** (see part **10** in ***Minkowskian Geometry I***)? This is the relativity of length contraction: Each observer sees the other length contracted by the *same* factor.