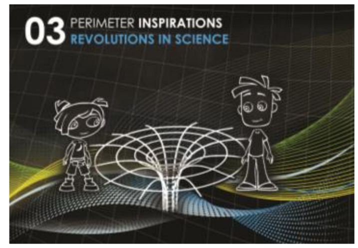
Revolutions in Science

***Supplementary Material*: Additional explanations for "accelerating up but not moving up"**



1: Compare Bob's path to the straight path of a dropped apple.

To create the straight path of an apple dropped by Bob, lay a strip of tape on the ball that starts at the bottom of the left ladder, moving initially "due East" in the same direction as Bob's path. If the strip of tape is straight, i.e., if it lies flat on the ball (no crinkling), then it will eventually diverge from Bob's path. Compared to this *straight* path, Bob's path is a parabola going up (i.e., the vertical distance from the straight path to Bob's path increases quadratically with time). Bob's path is clearly *curving up* (i.e., accelerating up) when compared to this straight path. Bob, and the ground he is standing on, really *are* accelerating up relative to something "at rest" (Einstein would consider the falling apple to be "at rest" since it experiences no force and so no acceleration). You can also imagine that there is a deep well next to Bob. You fall into the well. As you are free-falling, you feel nothing: it feels exactly as if you are "at rest", floating weightlessly in deep space. You look up and see Bob accelerating up and away from you. You would consider yourself to be at rest, and you would see Bob, and the ground he is standing on, accelerating up! If Bob were standing on a trampoline, you would explain this as the bending of the trampoline fabric exerting an elastic force on Bob, causing him to continually accelerate up.

2: What is the force *causing* the ground to accelerate up?

It is a fact that the pressure of water in the oceans, or rocks inside the Earth, increases with depth. If you consider any block of rock inside the Earth, the pressure in the rock *below* the block (which exerts an *upward* force on the block) is *greater* than the pressure above the rock (which exerts a downward force on the block). So there is *net upward force on the block*. It is this net upward force that is causing the block to continually accelerate up. But it doesn't *go* up because of the "magic" of warped spacetime! If you could magically remove all of the rock around the block, the block would begin to free-fall, i.e., it would begin to move along a *straight line path* through curved spacetime (like Alice's path). This is its natural, unforced motion. If we now replace the rocks around the block again, they exert a net upward force on it, pushing it *off* of this natural, unforced, straight line path; the net force *bends* the block's path through spacetime, causing it to *curve upwards* (like Bob's path). But what *causes* the increase in pressure with depth in the first place? The warping of spacetime! In a warped spacetime, what each block in the Earth would *like* to do is fall toward the centre of the Earth. This would be their natural, unforced, straight line path through warped spacetime. This natural tendency of each block to move toward the centre compresses the blocks below, and *causes* the increasing pressure with depth.

3: Compare Bob's path with the straight path of an apple tossed upwards.

This is similar to #1 above, but instead of showing that Bob is accelerating up, it shows what is really happening when an object is tossed straight up and comes back down: there are no forces acting on the object, and it is moving on a *straight* path though *curved* spacetime! To create the straight path of an apple that Bob tosses upwards, lay a strip of tape on the ball that starts at the bottom of the left ladder and ends at the bottom of the right ladder. With a bit of trial and error, you can make this strip of tape *straight*, i.e., lie flat on the ball (no crinkling). Notice that, relative to Bob's path, this straight strip of tape first goes up, then comes back down. (The curvature of the ball is not great, so the tape will only rise a little bit above Bob's path before coming back down again, but nevertheless it illustrates the point. The effect is more noticeable if you make Bob's path an arc of a circle of latitude closer to the North Pole.) This is the natural, unforced path through spacetime of an apple that is tossed straight up and comes back down. While it is in the air there are no forces acting on the apple (gravity is not a force), and so no acceleration—the tossed apple follows a *straight* path in *curved* spacetime.

4: Comparison with an Accelerating Rocket.

Imagine you are floating inside a rocket, adrift in deep space, far from the gravitational influence of any planets or stars. When the rocket fires its engines and accelerates "up", you feel as if you are being pulled "down" into the floor by a mysterious invisible force. Held objects appear to have weight, and dropped objects appear to accelerate toward the floor. The artificial gravity you are experiencing is indistinguishable from the effects we call gravity on Earth. In his *special* theory of relativity (spacetime without gravity), Einstein showed that inside such an accelerating rocket the acceleration *warps time* in the sense that time passes at a different rate at the ceiling of the rocket compared to at the floor. In other words, the hallmark of acceleration is a warping of time, or *time dilation*. (This kind of time dilation is not the same as the usual time dilation considered in special relativity.) Now suppose that, unbeknownst to you, your accelerating rocket is placed at rest on the surface of the Earth, and the rocket engines are turned off. If the acceleration of the rocket used to be one *g*, you wouldn't be able to tell that your situation had changed. In fact, you would observe the *same time dilation* that you noticed in the accelerating rocket in deep space (no gravity). In other words, the mass of the Earth warps time in order to reproduce *exactly the same conditions* you experience in an accelerating rocket in deep space (no gravity). You really *are* accelerating up when you are at rest on the Earth! Now imagine that the rocket engines are turned on again so that the rocket is hovering 1 metre off the ground. Again, from inside the rocket you can't tell the difference. Your situation is identical to that of a rocket accelerating in deep space (no gravity). So gravity is a kind of warping of spacetime that allows a rocket to accelerate, *exactly like in deep space*, but not go anywhere! Hovering above the ground (or standing on the ground—same thing) is a bit like rowing a boat against the current, at the right speed so that you are not moving relative to the banks of the river. Like a river flowing downstream, warped spacetime causes "space *itself* to flow toward the centre of the Earth"; you must turn on your rocket engines just to keep from being swept up in this flow, i.e., to keep from free-falling. (Be careful with this analogy: with the river we are talking about velocity; with space we are talking about acceleration.)

5: Analogy with centripetal acceleration.

We are used to thinking of motion and velocity in three-dimensional space. But we are also *moving in time*—inexorably moving into the future. Einstein showed us that we must consider not only our motion through space, but also our motion through time. We must consider velocity in four-dimensional spacetime, i.e., velocity as a vector with four components: three spatial and one time. We are familiar with the three spatial components. The time component is closely related to energy. In terms of the ball and tape activity, the four-dimensional velocity vector points in the direction of (i.e., lies tangent to) the strips of tape for Alice or Bob. (Of course we can't show all four components; we show just the two non-zero components: the time and vertical space components.) The fact is that the *length* of this four-dimensional velocity vector for *any* object *is always c*, the speed of light, whether the object is "at rest" or not. Even if the object is "at rest" it is still moving through time (i.e., moving into the future); the "rate" of this motion is the speed of light. For example, the upward acceleration Bob experiences standing on the Earth does not change the *length* of his four-dimensional velocity vector (which, as just stated, is always c); but it *does* change its direction. Bob's four-dimensional velocity vector (of which we see just the two relevant components) is continually changing direction as he moves along his path through spacetime: it is rotating counterclockwise as he moves along his path. (This is particularly evident if you move Bob's path up to a higher circle of latitude nearer to the North Pole). It's clear that as you drag the base of his velocity vector along his path, you must continually rotate it counterclockwise to keep it tangent to his path. In this sense, the acceleration we experience standing on the Earth is *exactly* like centripetal acceleration, except in four dimensions. For example, in uniform circular motion, the acceleration is perpendicular to the velocity, meaning the direction of motion is continually changing, but the speed remains constant. The same is true in four-dimensional spacetime: the acceleration is perpendicular to the four-dimensional velocity, meaning the direction of motion is continually changing (rotating counterclockwise), but the speed remains constant (equal to the speed of light). Regarding Alice, notice that her velocity vector (which is tangent to her path) is *not* changing direction as she moves along her path: as you drag the base of her velocity vector along her path, you need *not* rotate it in order to keep it tangent to her path. This is because her path is *a straight line*. ("Not having to turn as one goes forward" is as good a definition of a straight line as any other.)