Special Relativity Solutions 3:

SIMULTANEITY, VELOCITY ADDITION & DOPPLER SHIFT

[easy] 1. $\lambda_{red} = 615\,nm \quad \lambda_{green} = 530\,nm$

$$\frac{530}{615} = \sqrt{\frac{1-{}^{v}/_{c}}{1+{}^{v}/_{c}}}$$

$$v = 0.15c$$

[medium] 2. Light travels at speed c, so the observed wavelength λ is related to the observed period T by $cT=\lambda$. The rest frame wavelength λ_o is related to the rest frame period τ by $c\tau=\lambda_o$. So

$$(1+z) \equiv \frac{T}{\tau} = \frac{\lambda}{\lambda_o} = \frac{950.0}{372.7}$$

$$z = 1.55$$

Assuming the velocity is radial,

$$(1+z) = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

$$\frac{v}{c} = \frac{(1+z)^2-1}{(1+z)^2+1} = 0.73$$

The galaxy is receding from us at 0.73c.

[medium] 3. a) The space station is equidistant to the two planets. According to a stationary observer both Alice and Bob will be moving towards the planets at speed v. Thus, they will arrive at the same time and the race will be a tie.

b) In Alice's rest frame planet Alpha is rushing towards her at speed v and planet Beta is rushing away from her at speed v. In order to find the speed at which Bob is moving away from her we need to use the relativistic velocity addition equation:

$$v_{Bob} = \frac{v+v}{1+\frac{v^2}{c^2}} = \frac{2v}{1+\frac{v^2}{c^2}}$$

Thus, Alice will see Bob approaching the planet Beta at speed

$$v_{BP} = \frac{2v}{1 + \frac{v^2}{c^2}} - v$$

This is clearly less than v. Thus, according to Alice, Alice wins the race.

c) In Bob's rest frame planet Beta is rushing towards him at speed v and planet Alpha is rushing away from him at speed v. In order to find the speed at which Alice is moving away from him we need to use the relativistic velocity addition equation:

$$v_{Alice} = \frac{v+v}{1+\frac{v^2}{c^2}} = \frac{2v}{1+\frac{v^2}{c^2}}$$

Thus, Bob will see Alice approaching the planet Alpha at speed

$$v_{AP} = \frac{2v}{1 + \frac{v^2}{c^2}} - v$$

This is clearly less than v. Thus, according to Bob, Bob wins the race.

[hard] 4a Step 1:

Draw the ray of light (7 units from the time axis, and 7 units up from the space axis)

[Note: this green arrow, c, is also the speed of light as measured by Bob (remember we are now in Einstein's universe)]

Step 2:

Draw the spaceship's worldline (3.5 across and 7 up, or even better: from Alice's perspective the spaceship moves half the distance the ray of light moves in a given time unit, e.g., in time t=4 units light must have moved horizontally 4 units, and the spaceship must have only moved horizontally 2 units).

[Note: the two equal horizontal black arrows at time = 4 units equally split the spaceship arrow and the speed of light arrow.]

Step 3:

To draw Bob's worldline, remember that if something moves at half the speed of light, then at any given time that object will have travelled half the distance the light pulse covered in that frame.

Distance is measured along lines parallel to the space' arrow (which has the same angle from space axis as the spaceship has from the time axis).

So picking an easy location where we can split the measured distance (along line parallel to space') such as time axis= 4 units where the two red arrows have equal length from the spaceship arrow to the speed of light arrow.

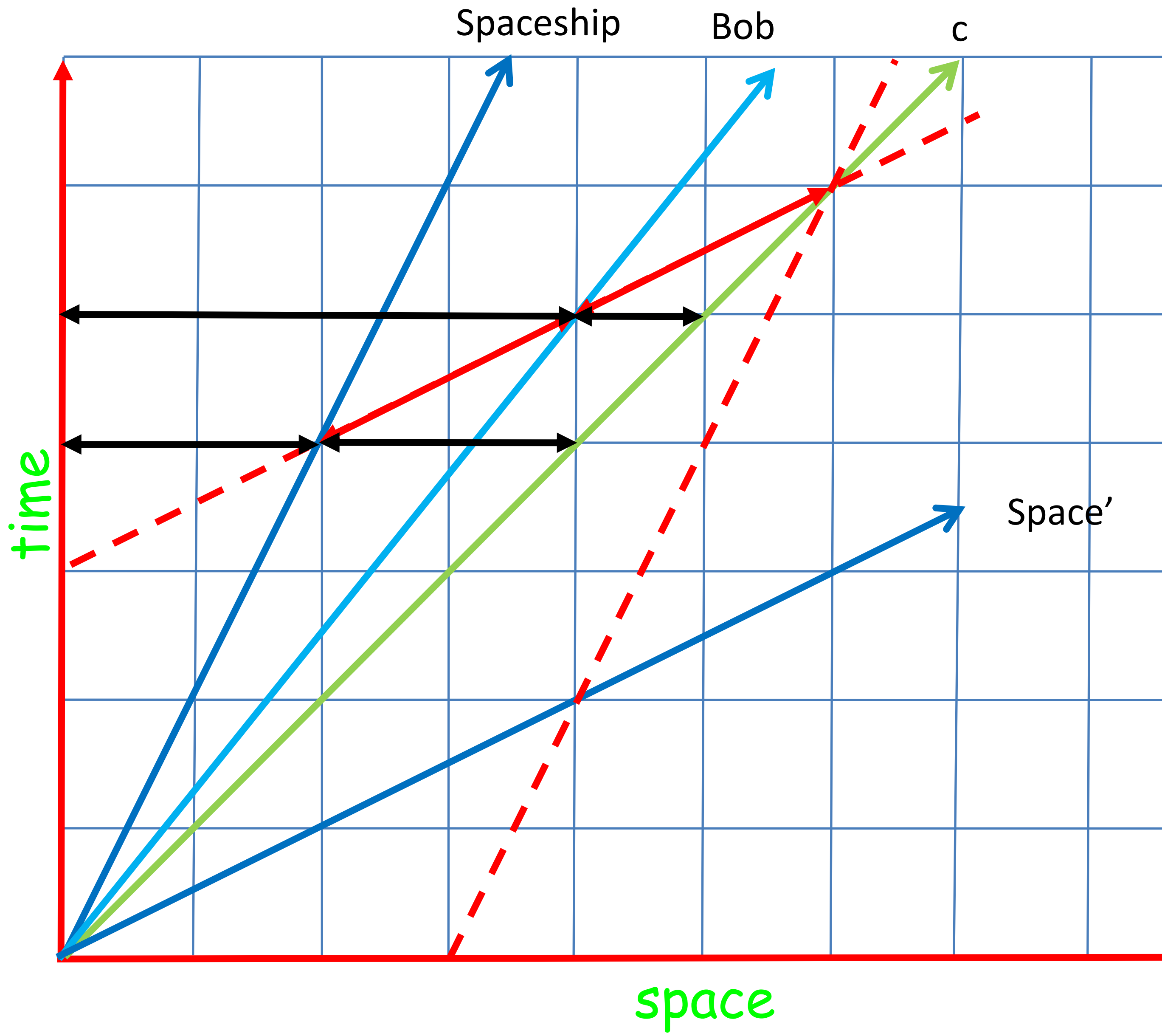
We can now draw Bob's worldline from the origin through the point where the two equal red arrows touch.

Step 4:

To determine what speed this measures in Alice's frame, we again pick a suitable location on the diagram and measure horizontally the ratio of boxes from the time axis to the speed of light and form the time axis to Bob's worldline.

Where the two black arrows at time = 5 units meet is the location I chose.

The distance to **c** is 5 units, while the distance to Bob is 4 units. Therefore Alice measures Bob's speed as 4/5 c


Spaceship
Bob
c
time
Space’
space

4b We can check this quantitatively with Einstein's velocity addition formula which states:

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

In our example: $u' = \frac{1}{2}c$, $v = \frac{1}{2}c$

Therefore

$$u = \frac{\frac{1}{2}c + \frac{1}{2}c}{1 + \frac{\left(\frac{1}{2}c\right)\left(\frac{1}{2}c\right)}{c^2}} = \frac{c}{1 + \frac{\frac{1}{4}c^2}{c^2}} = \frac{c}{\frac{5}{4}} = \frac{4}{5}c$$

The same as we obtained by reading Bob's velocity in Alice's axis.