Quantum Mechanics Exercises 5:

STANDING WAVES COMPLEX

This is a follow up to the Quantum Exercise: Standing Waves Real. As discussed in the lectures, a free or bound particle is described quantum mechanically by a ***complex*** wave, which solves the problem of the particle "blinking in and out of existence."

[easy] 1. In the Quantum Exercise: Standing Waves Real you found that a particle with momentum **+p** and energy $E = \frac{p^2}{2m}$ (right-moving) could be described by the **real** traveling wave: $\psi_R(x,t) = -\sin\left(\frac{1}{\hbar}(Et - px)\right)$, and similarly for a left-moving particle we have $\psi_L(x,t) = \sin\left(\frac{1}{\hbar}(Et + px)\right)$. We can construct ***complex*** traveling waves by simply replacing the sine function with a ***complex exponential***:

$$\psi_R(x,t) = -e^{\frac{i}{\hbar}(Et-px)}$$

$$\psi_L(x,t) = e^{\frac{i}{\hbar}(Et+px)}$$

where $\boldsymbol{i}$ is the unit imaginary number: $i^2 = -1$

Make sure you understand this.

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[medium] 2. If a particle is confined to a box, both the right- and left-moving complex traveling waves will necessarily be present due to reflections off the ends of the box. Thus, consider a superposition of the right- and left-moving complex traveling waves: $\psi(x,t) = \psi_R(x,t) + \psi_L(x,t)$. Factor out the $e^{\frac{i}{\hbar}Et}$ then use Euler's Formula $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ to express the remainder in terms of a sine function.

[hard] 3. As discussed in class, the squared magnitude of a complex wavefunction is to be interpreted as a function describing the probability of finding the particle at various positions (as a function of time). Refer back to exercise 2 and calculate $\left|\psi\left(x,t\right)\right|^2$. Observe that it is *independent of time*, i.e., the probability pattern is ***static*** (physicists call this a **"*stationary state*"**). Using the de Broglie relation $p=\frac{h}{\lambda}$, re-express $\left|\psi\left(x,t\right)\right|^2$ in terms of λ instead of **p**, and sketch it for x = 0 to x = λ. The particle no longer "blinks in and out of existence"!