SPECIAL RELATIVITY

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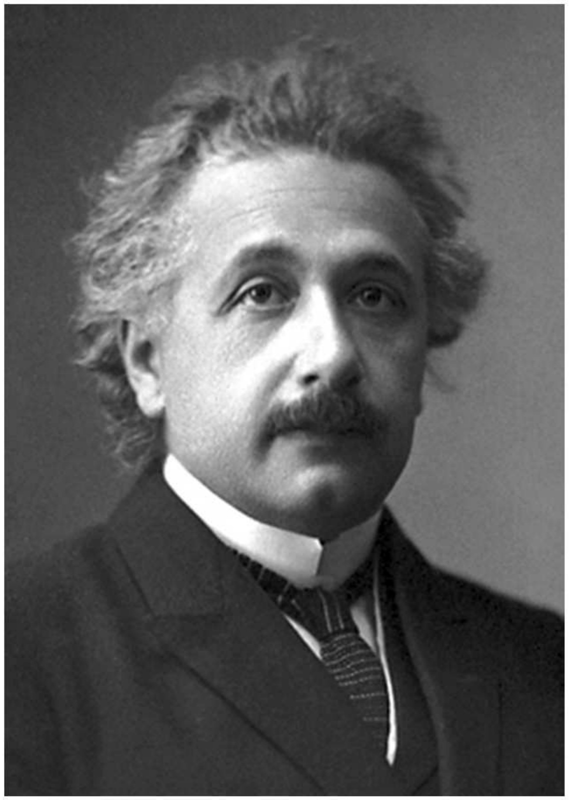
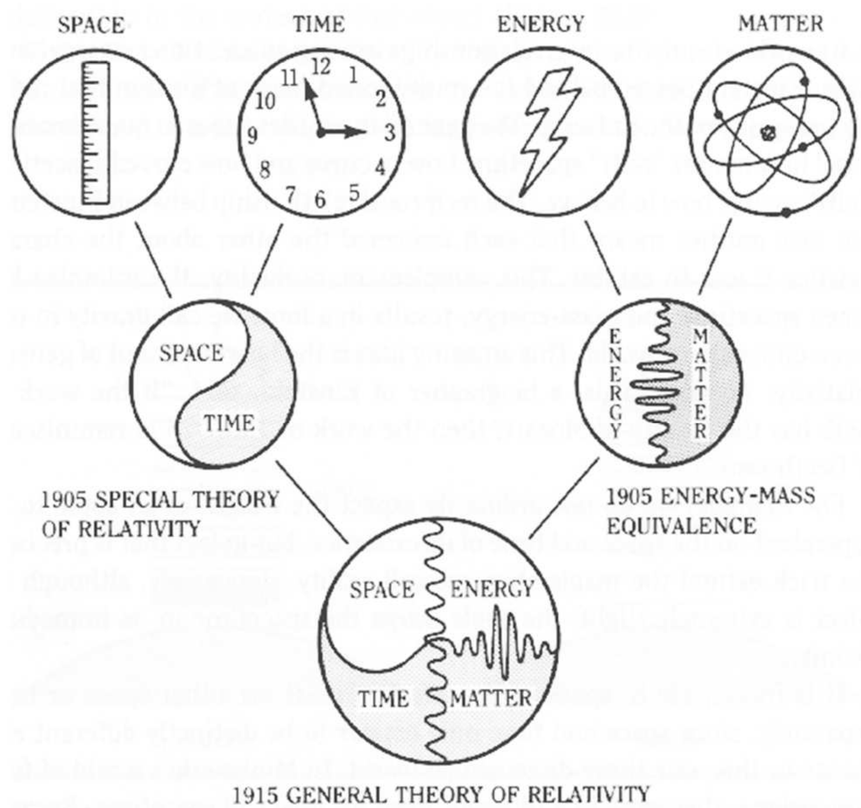
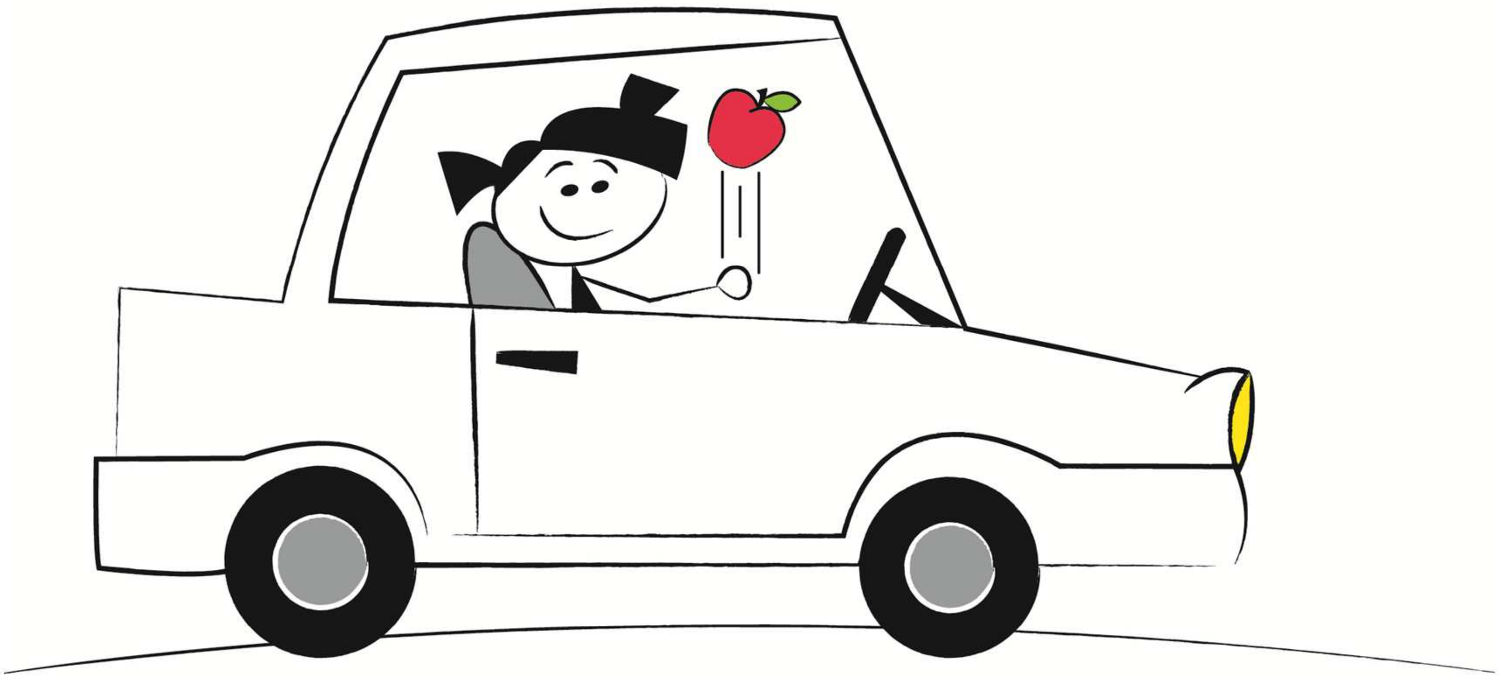


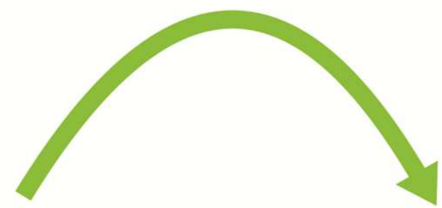

SPACE
TIME
ENERGY
MATTER
SPACE
TIME
ENERGY
MATTER
1905 SPECIAL THEORY OF RELATIVITY
1905 ENERGY-MASS EQUIVALENCE
SPACE
ENERGY
TIME
MATTER
1915 GENERAL THEORY OF RELATIVITY

Can you tell you are moving?



Alice at Rest

Alice Driving



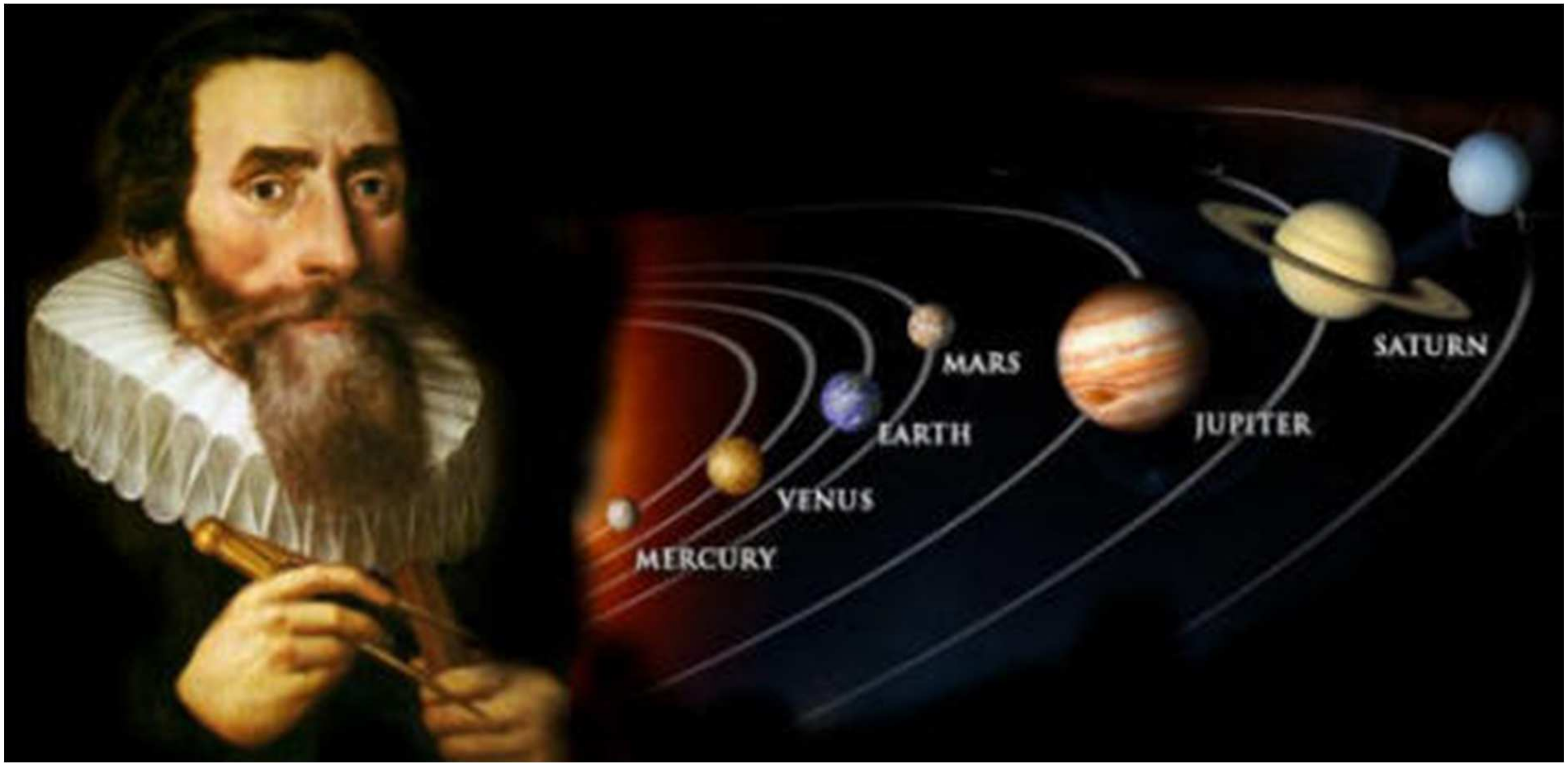
Bob Watching Alice Drive

Galilean Relativity

No Absolute Motion!

Speeding with respect to what?

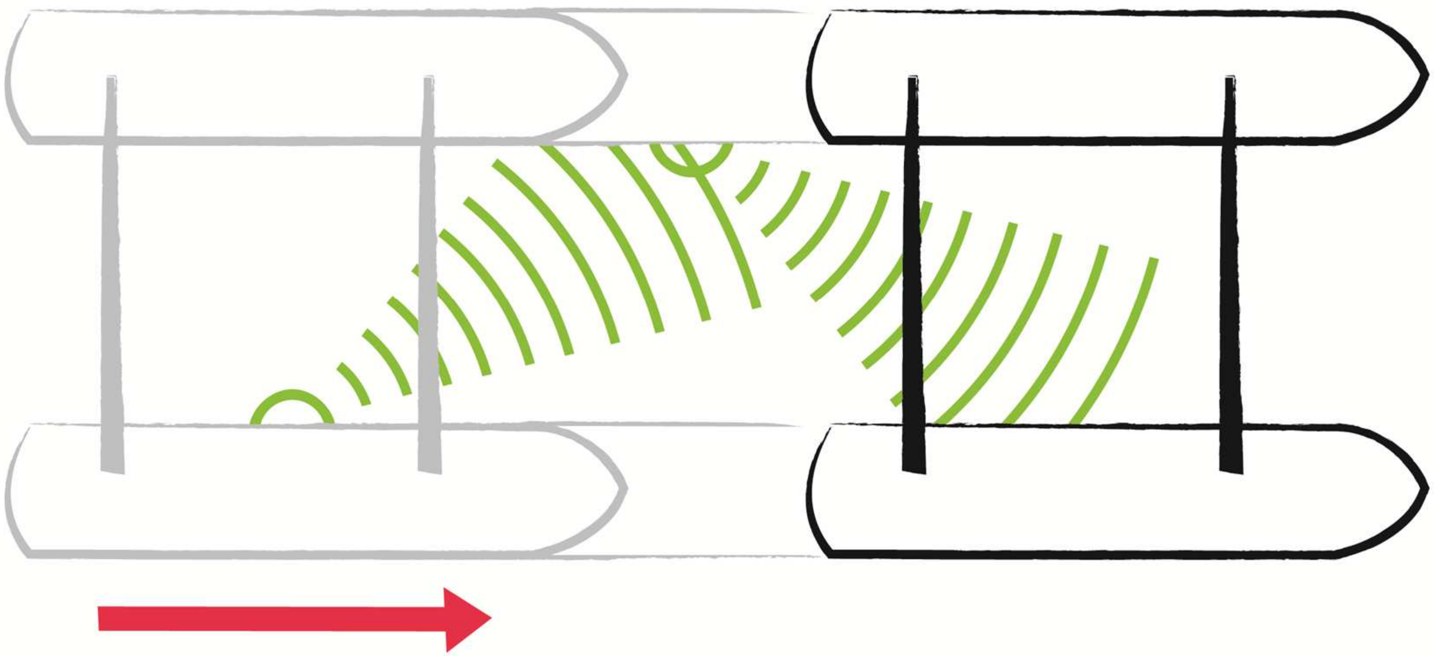
Kepler's Heliocentric Model

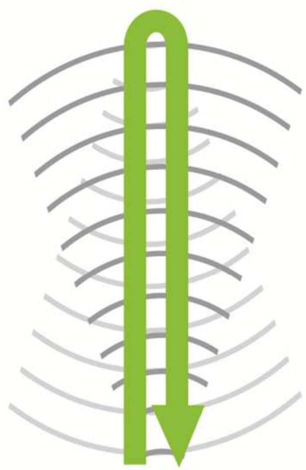


Our Everyday Experience is Stationary!

- Orbital speed around sun: 30 km/s
- Solar system around Milky Way center: 220 km/s
- Motion of Local Group wrt CMB: 600 km/s

Can you tell you are moving?





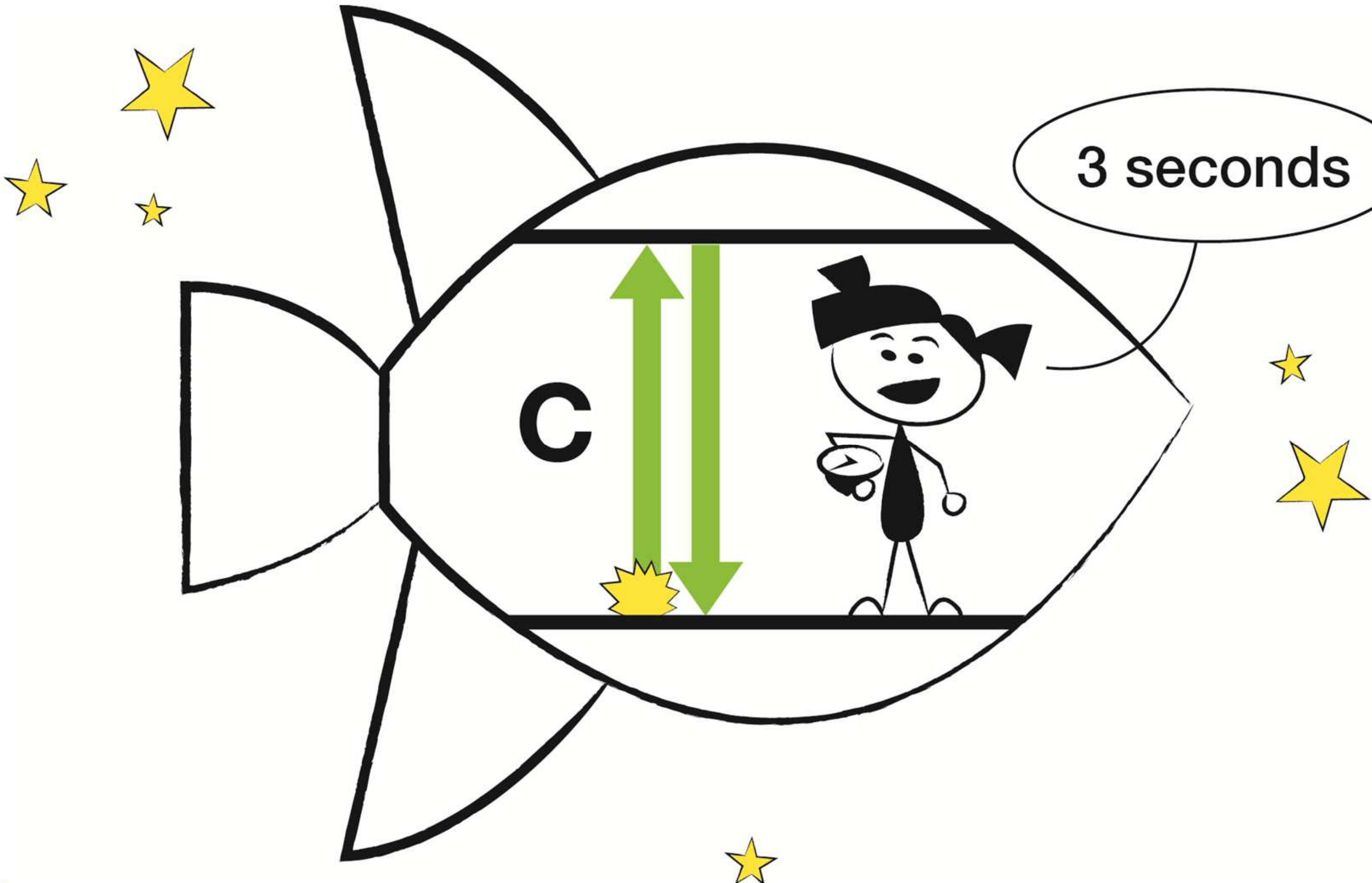
Alice at Rest

Alice Moving

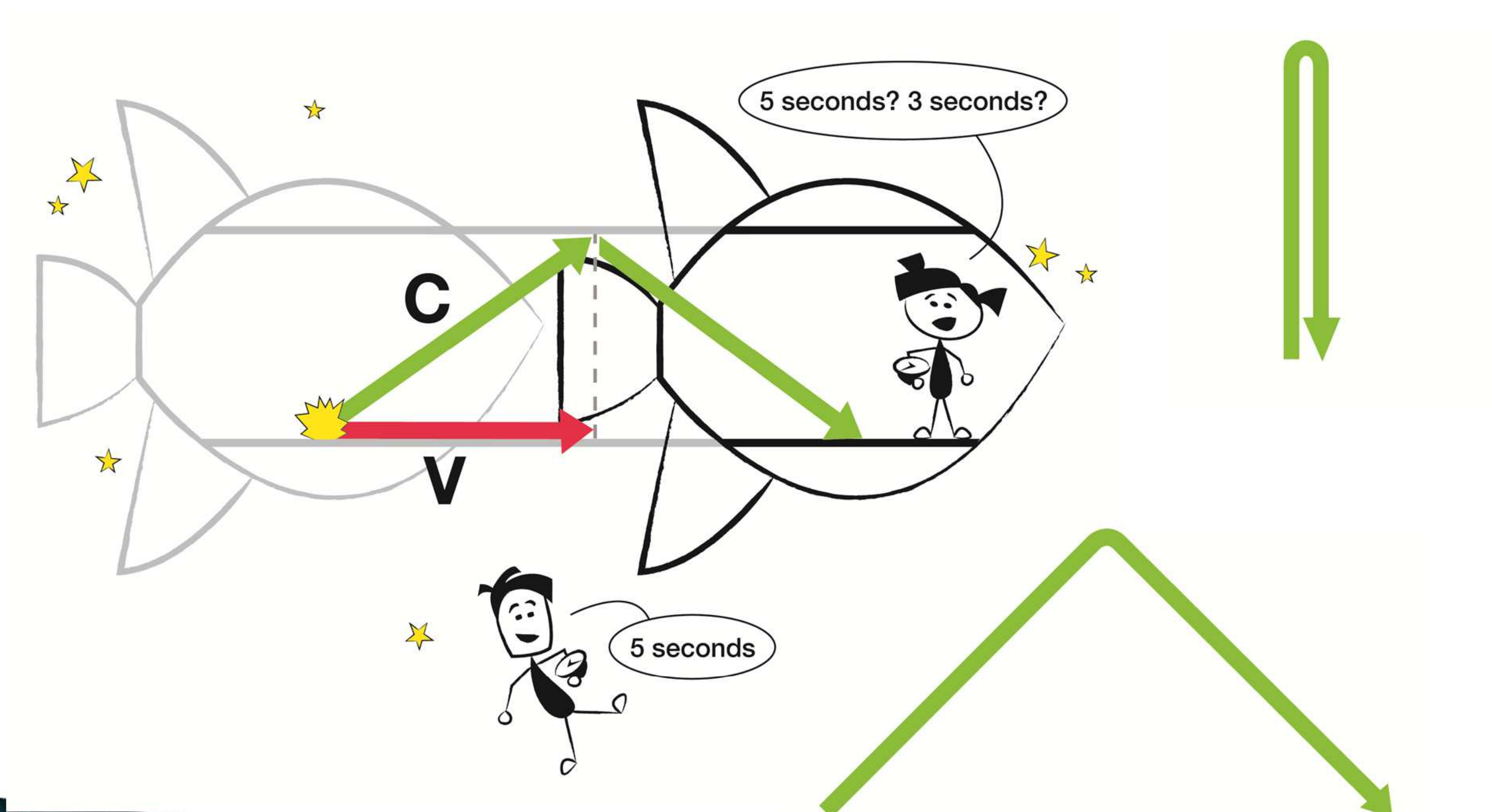
Something to consider…

The speed of a wave NEVER depends on the speed of the source.

Can you tell you are moving?



Will Bob and Alice agree?



If Alice measures the same time as Bob:

- She would get a different answer then when she is at rest!

- She would be able to tell she is moving so,

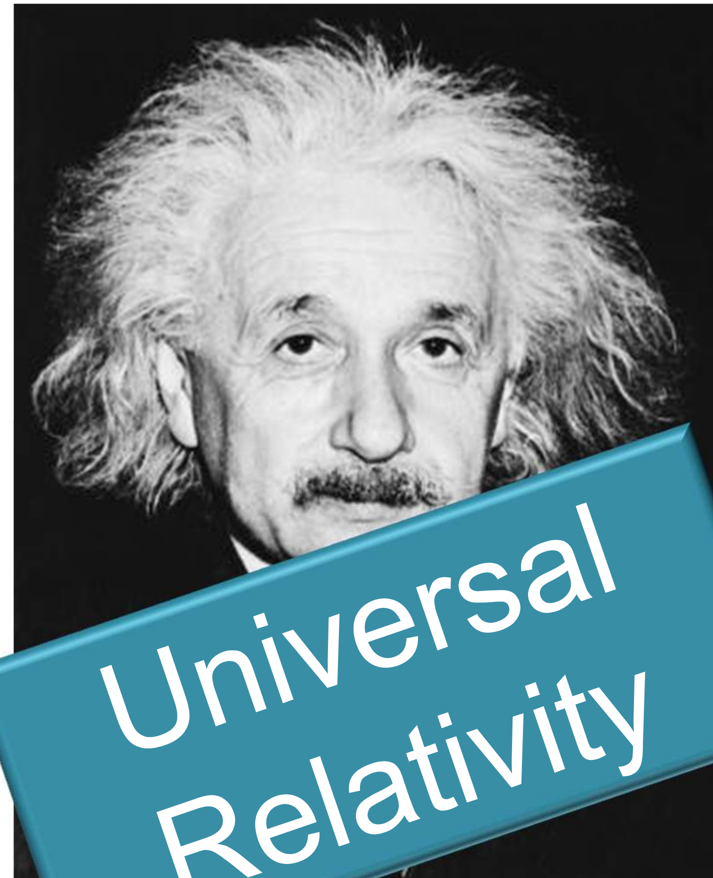
- ~~Universal Relativity~~

If Alice measures the same time as when at rest:

- She would not know she is moving
- But her time is different from Bob
- Time is not absolute!!

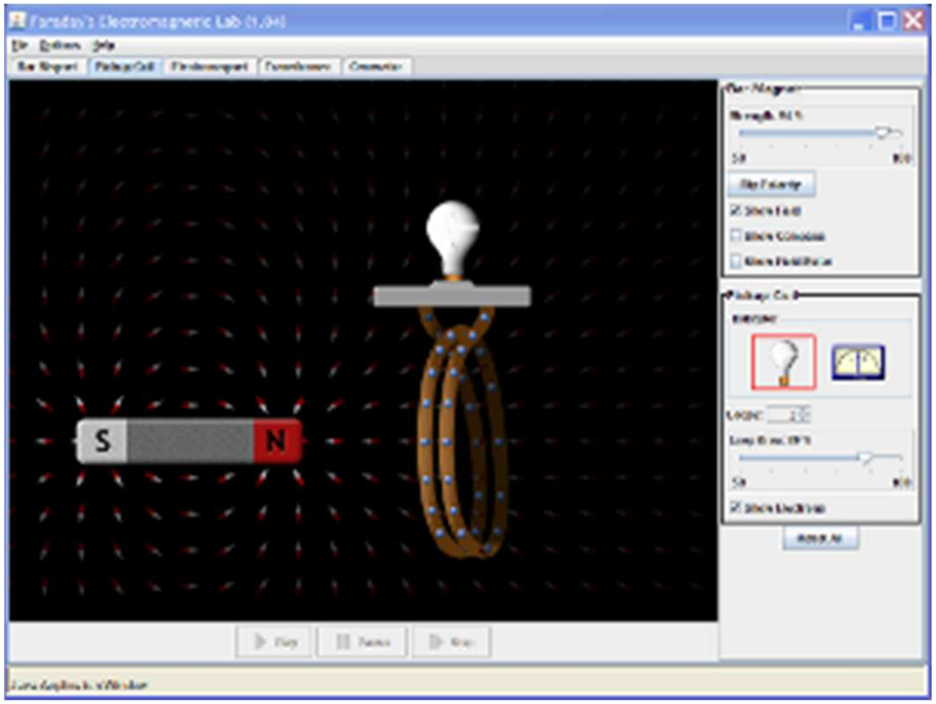
Newton vs Einstein

Absolute Space & Time



Universal Relativity

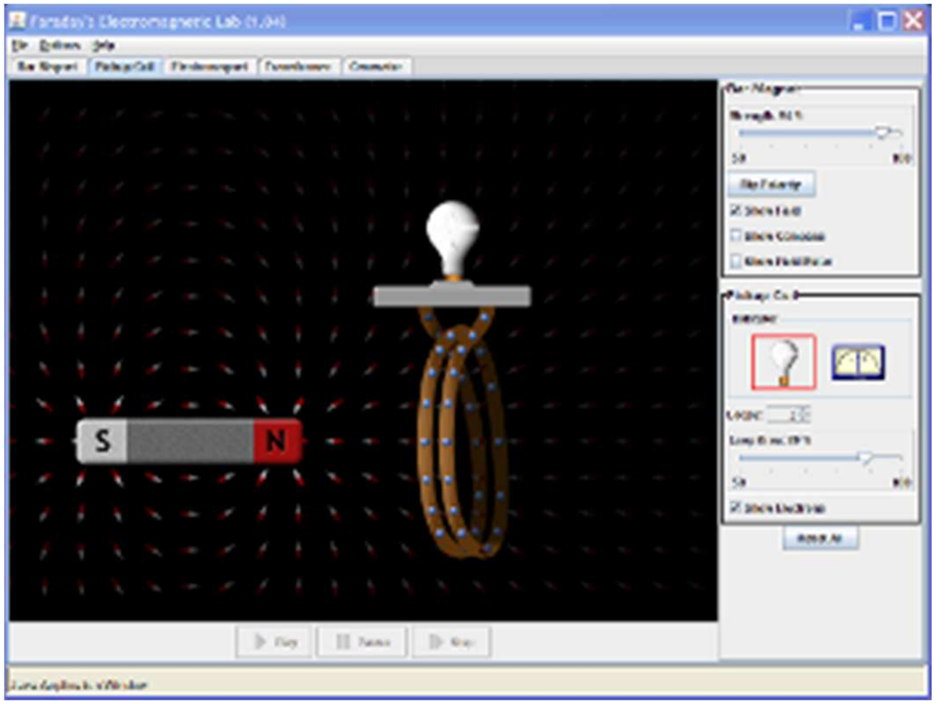
Einstein's Problem



A moving magnet creates an electric current in the coil.

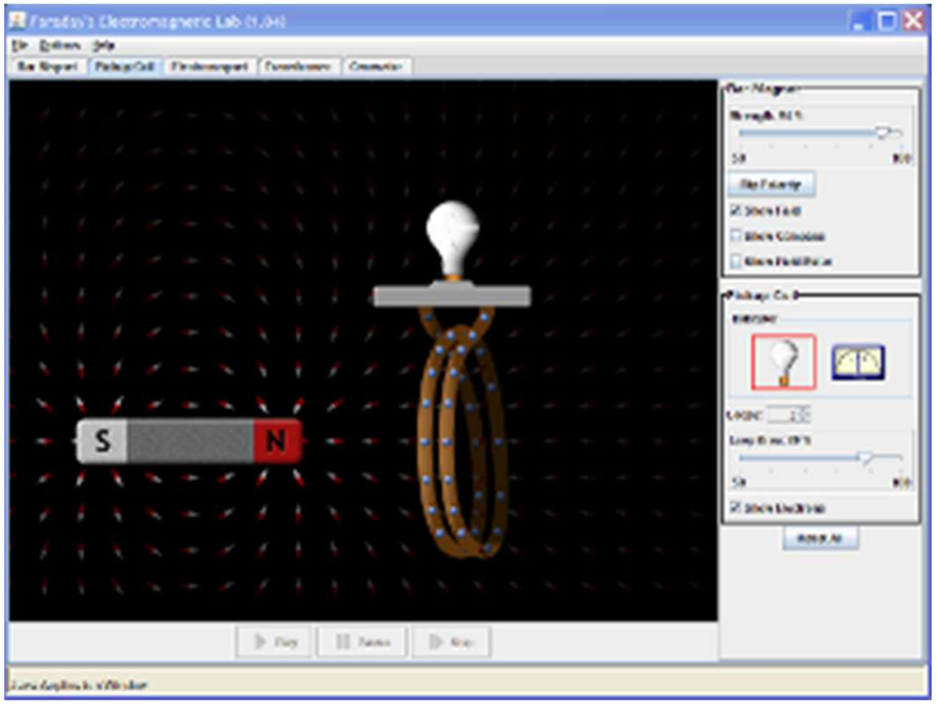
HOW?

Einstein’s Problem



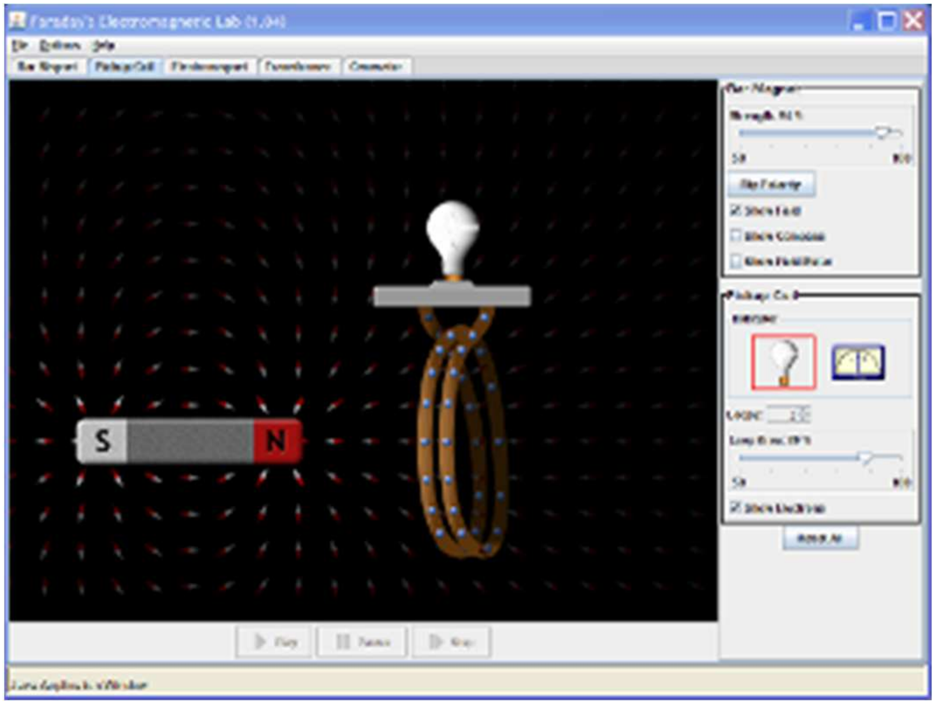
Moving magnet creates an electric field that produces a current in the coil.

Einstein's Problem



What if we move the coil instead of the magnet??

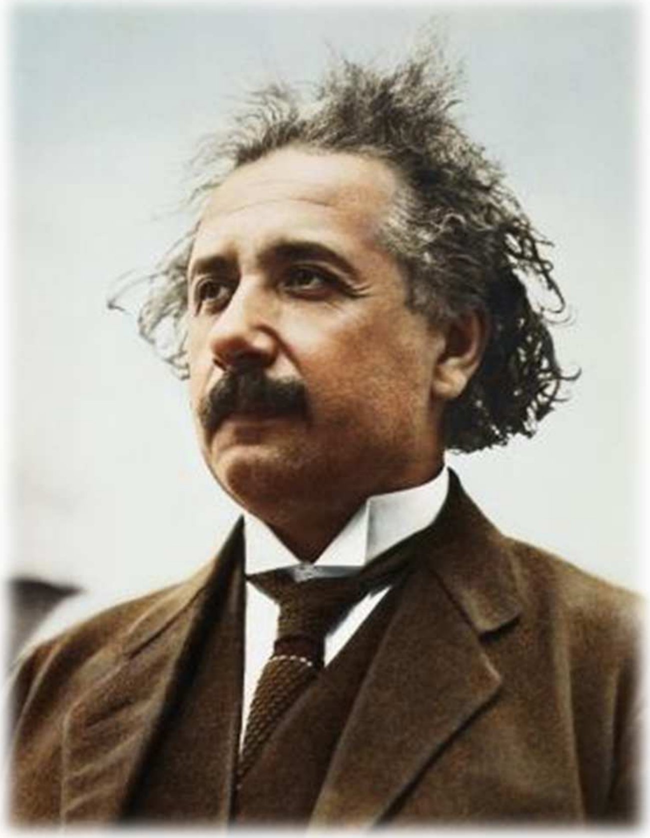
Einstein’s Problem



Magnetic field exerts a force on electrons to produce the current in the coil.

Einstein's Problem

Why should a change in frame of reference result in different explanations for induced current?



Principle of Relativity

- *Given any two inertial observers in uniform relative motion, both are* ***equally entitled*** *to consider themselves* ***"at rest"****.*

Principle of Consistency of Speed of Light

- *For an **observer "at rest" the speed of light is c**, independent of the motion of the source of light.*

- *There is at least one inertial reference frame in which the velocity of light is c.*

Nothing revolutionary yet!

- **P1:** *Given any two inertial observers in uniform relative motion, both are* ***equally entitled*** *to consider themselves* ***"at rest".***
- **P2**: *For an* ***observer*** *"at rest"* ***the speed of light is c****, independent of the motion of the source of light.*

Revolutionary!

- P1 + P2 imply:

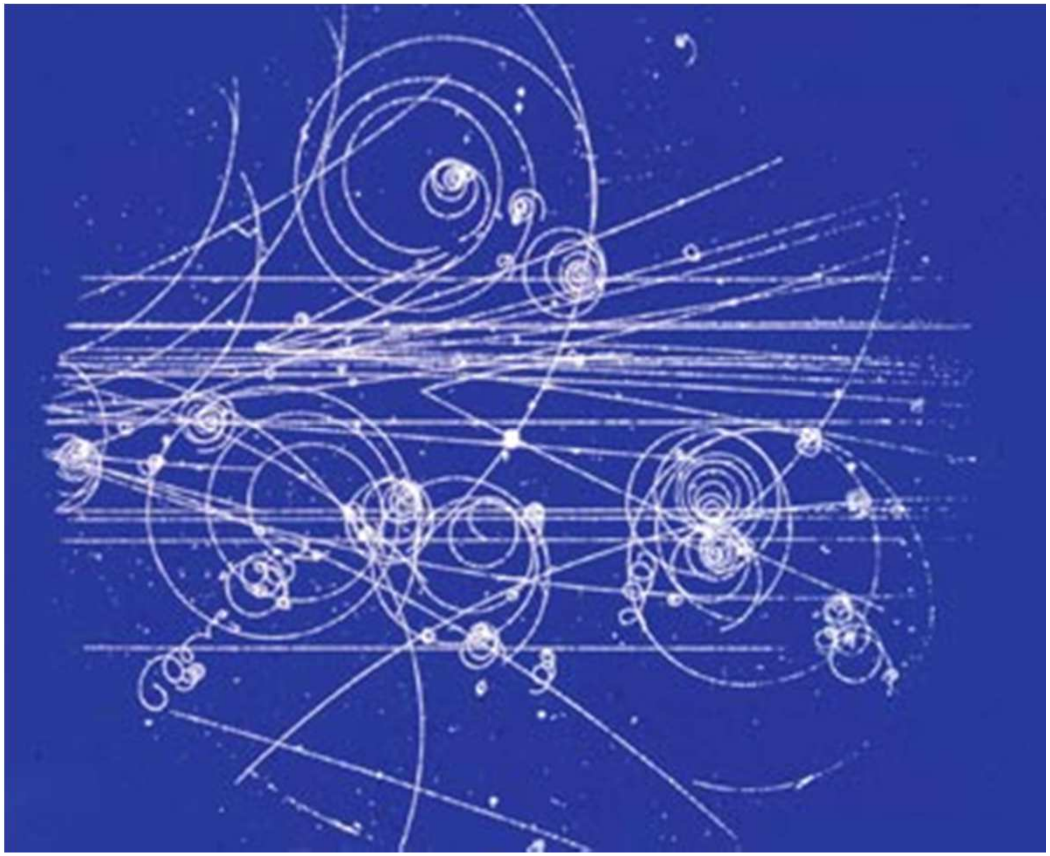
- *For a <u>source of light "at rest"</u> the speed of light is **c, independent of the motion of the observer!***

Principle of Relativity: The laws of physics are the same in all inertial frames of reference

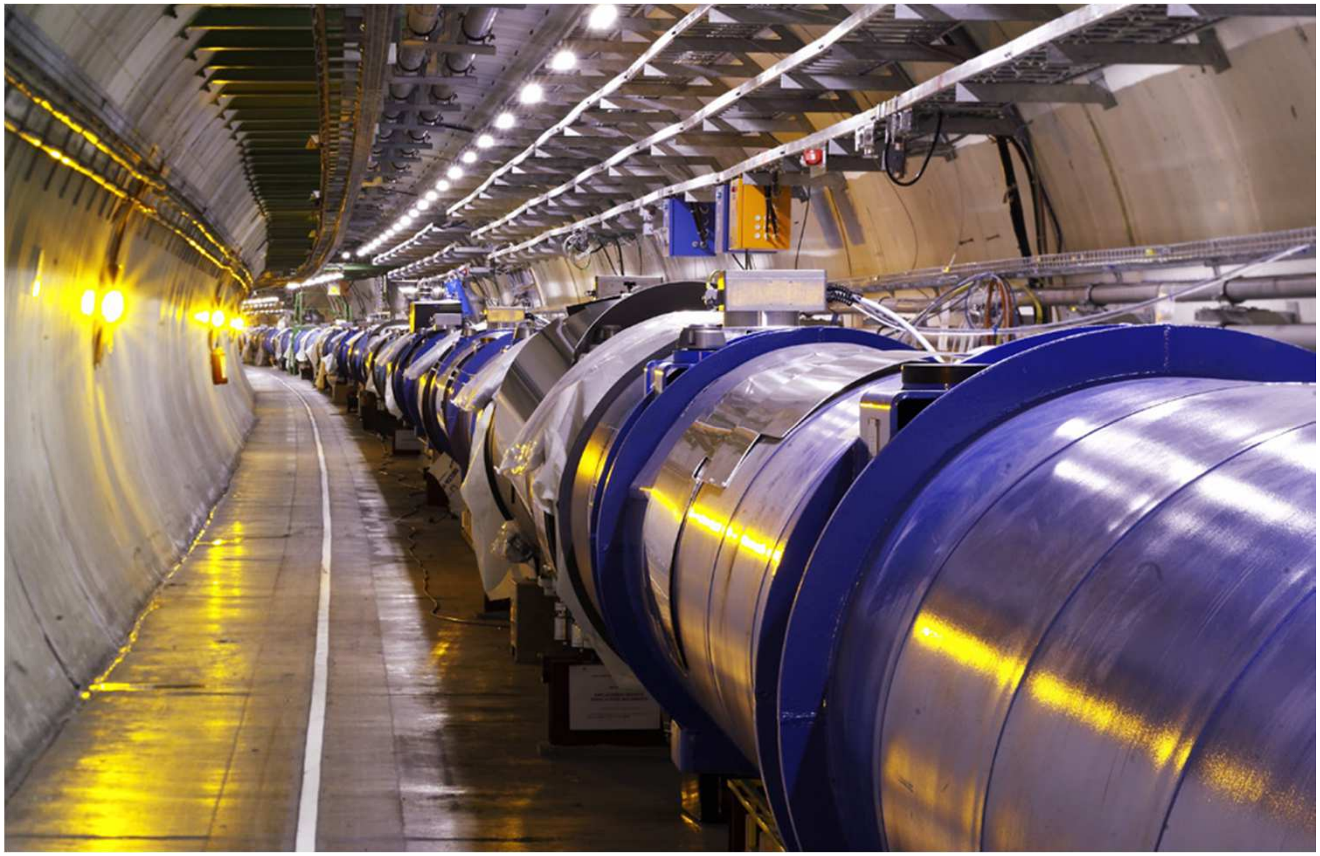
- *All experiments run the same in all inertial frames of reference.*

- *No experiment can reveal the absolute motion of the observer.*

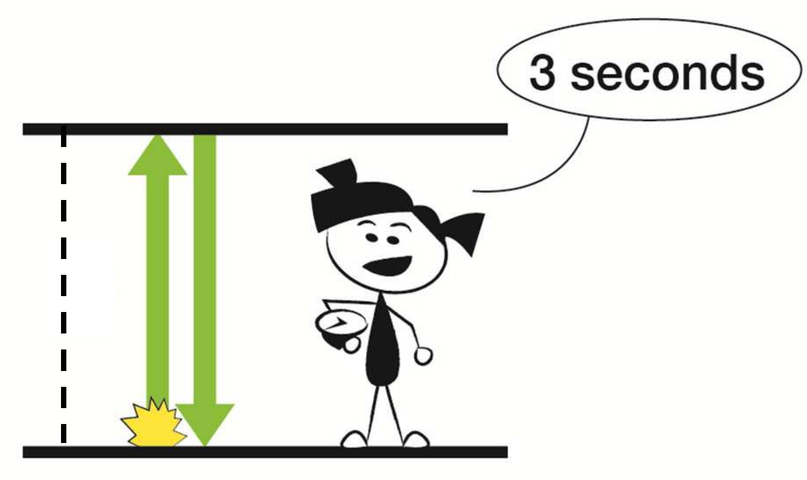
Experimental Evidence



LHC



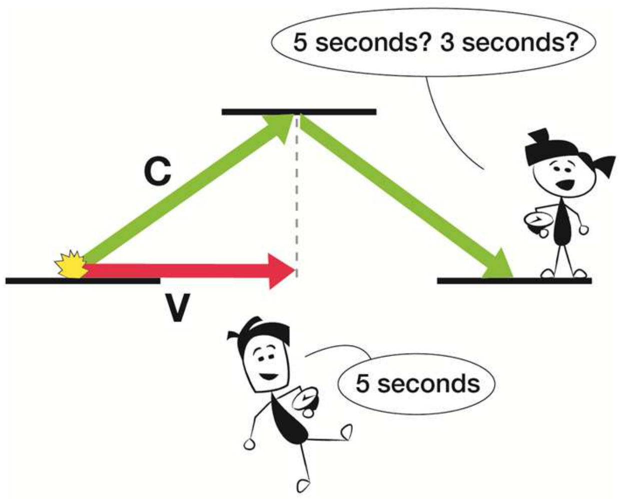
Time Dilation



$$t_A = \frac{d}{c}$$

d is the length of path UP and DOWN.

Time Dilation

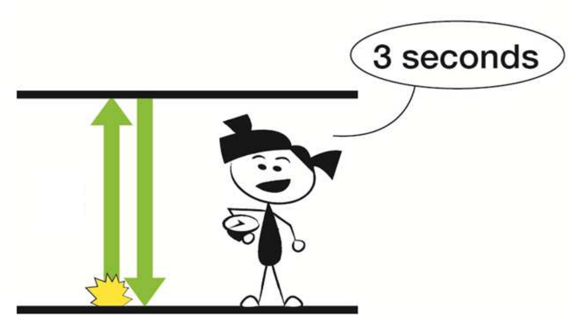


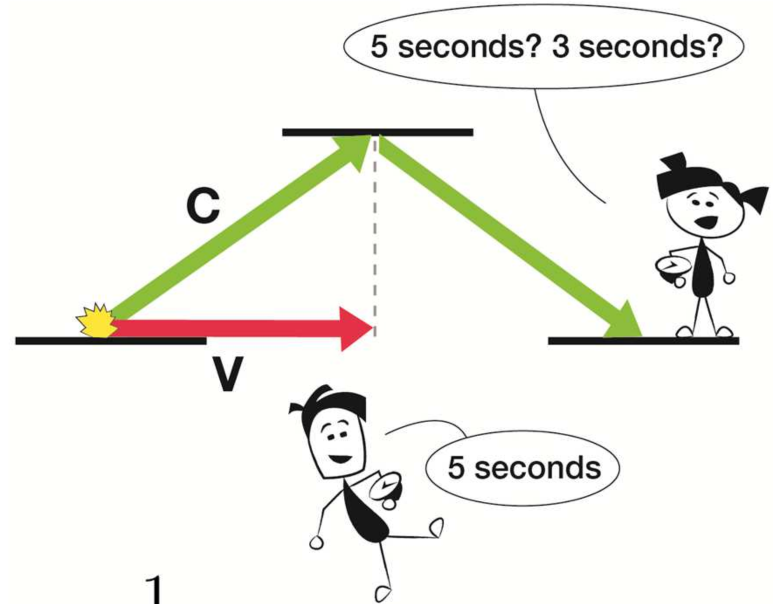
Vertical speed measured by Bob via Pythagoras.

$$v_{B,vertical} = \sqrt{c^2 - v^2}$$

$$t_B = \frac{d}{c} = \frac{d}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

Time Dilation





$$\frac{t_B}{t_A} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

A moving clock run slow!

Length Contraction

- Alice and Bob disagree on how far she travels.
- Bob will see Alice travel the distance

$$L_{Bob} = vt_{Bob}$$

- Alice sees Bob traveling in the opposite direction at speed v.
- Alice sees Bob travel

$$L_{Alice} = vt_{Alice}$$

Length Contraction

- Since $t_{Alice} < t_{Bob}$

- We have $L_{Alice} < L_{Bob}$

- From Alice's perspective, Bob's space and everything in it is contracted, so from her point of view she has not traveled as far.

Something to consider…

Time Dilation and Length Contraction are ***logical consequences*** of Universal Relativity.

Time Dilation & Length Contraction

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Greater than or equal to 1.

$$\Delta t = \gamma \Delta t_o$$

Time between ticks of a moving clock is longer, so moving clocks run slow.

$$L = \frac{L_o}{\gamma}$$

Moving meter sticks are shorter.

Alice measures 1m/s

- Bob measures:

$$L = \frac{L_o}{\gamma} = \frac{1 \text{ m}}{\gamma}$$

Shorter distance

$$\Delta t = \gamma \Delta t_o = \gamma 1 \text{ s}$$

Longer time

$$v = \frac{1 \text{ m/s}^2}{\gamma^2}$$

Lower speed

Speed Limit

- As the speed of Alice's rocket approaches c, γ becomes *larger and larger* so her changes in speed will appear to Bob to be *less and less*.

$$v = \frac{1\ \mathrm{m/s}^2}{\gamma^2}$$

Speed Barrier

$v = c$

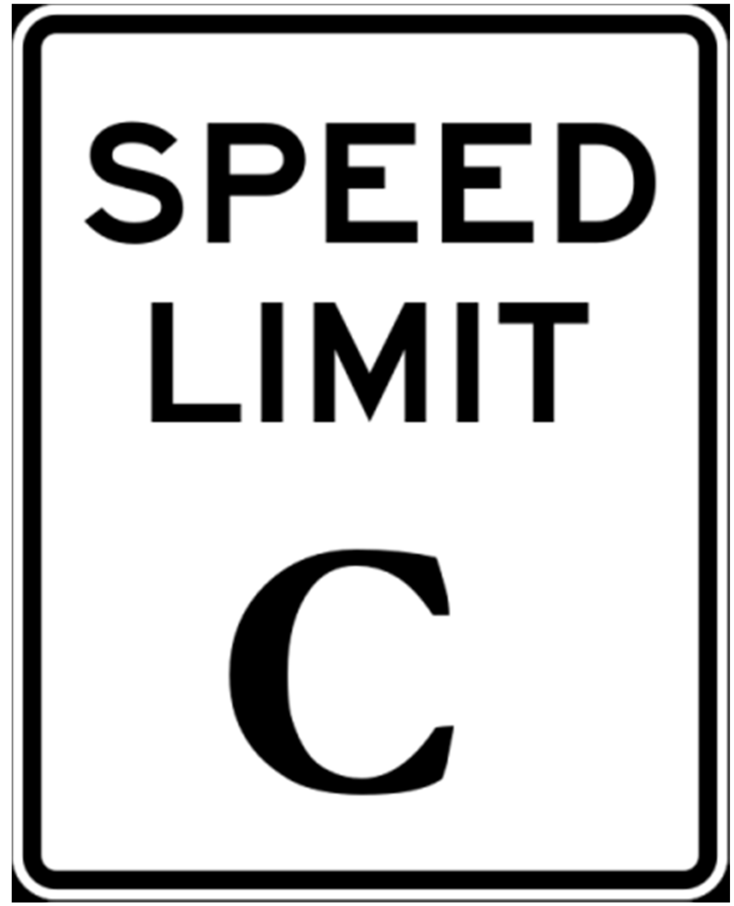
$v < c$

$v > c$

We cannot accelerate something through the speed of light!

Something to consider…

The speed limit is a ***logical consequence*** of time dilation and length contraction.



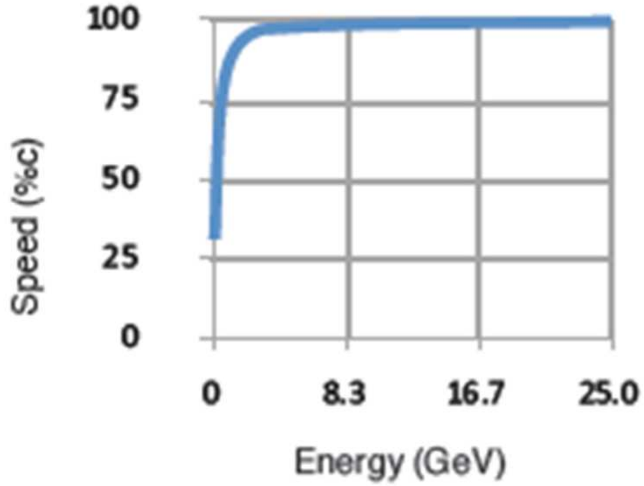
Applying a Force

- If Bob applies the same force F as Alice for the same amount of time:
  - Alice will claim he applied the force for *less* time
  - He needs to apply the force for more time according to his clock.

Time dilation makes his efforts *less* effective than he thinks!

Effective Inertia

- As her speed increases it gets harder and harder to accelerate her.
- It is as though her mass increases
- In fact, the real cause is time dilation
- We call this effective inertia



The added energy increases the particle's momentum, not speed.

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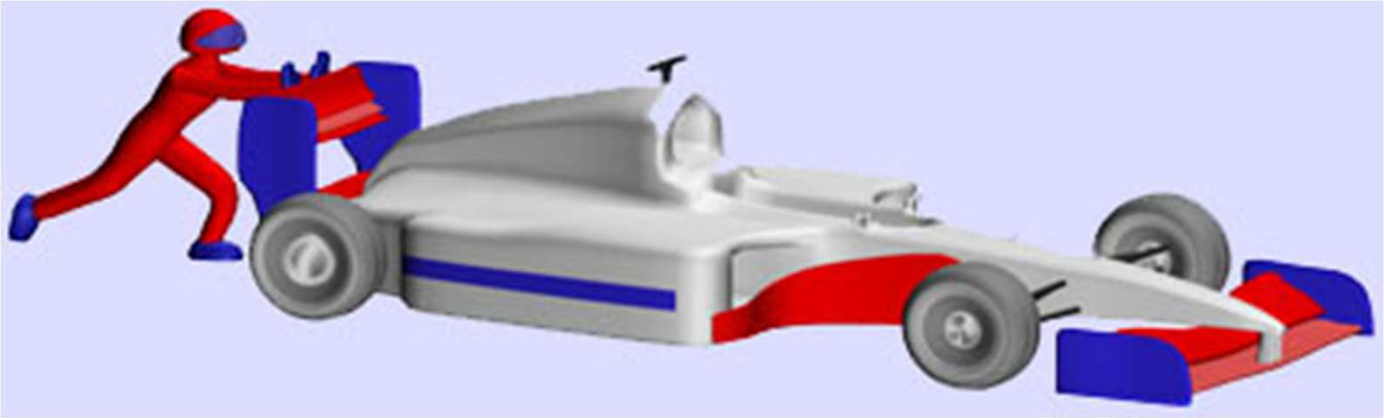
Something to consider…

Mass NEVER increases. Time dilation and length contraction just make the applied force less effective.



Universal Relativity → Length Contraction & Time Dilation → Speed of Light Limit

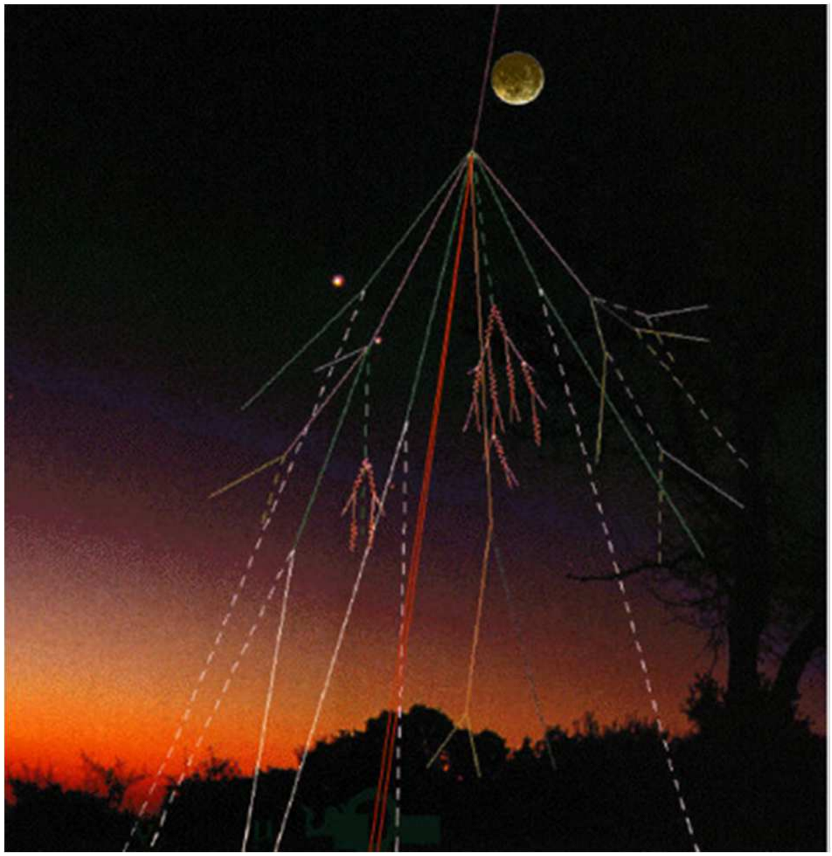
In summary, the core ideas in Relativity…

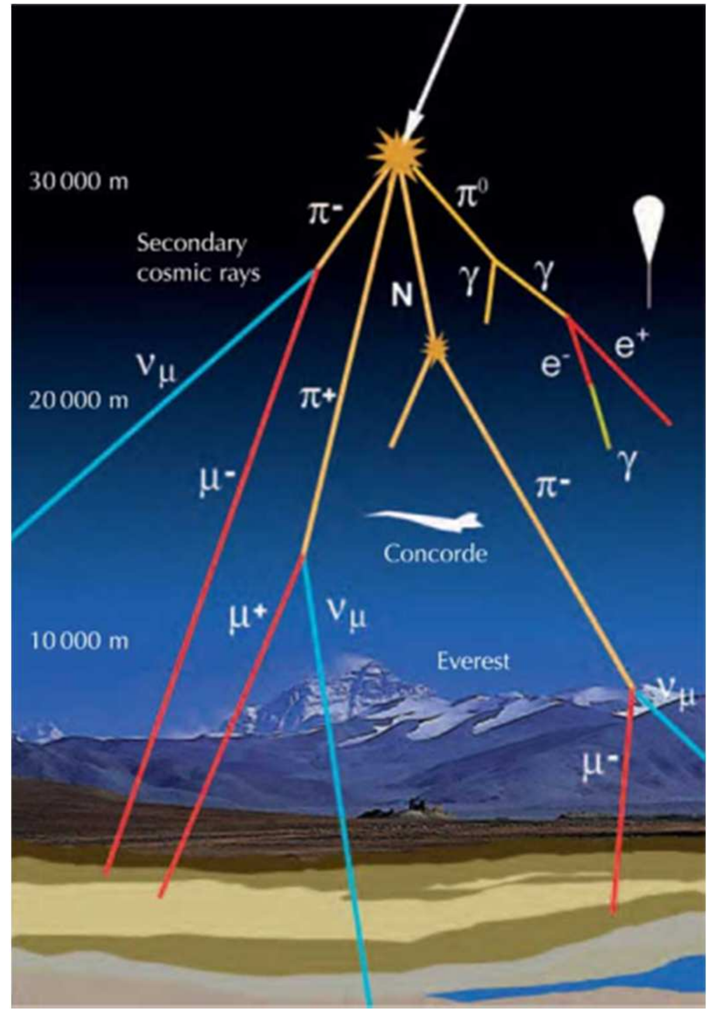
You cannot tell that you are moving!!

Time and space are relative.

Nothing mysterious here, just simple logical consequences…

Cosmic Rays




30 000 m
20 000 m
10 000 m
Secondary cosmic rays
π^-
π^0
N
γ
γ
e^-
e^+
ν_μ
π^+
γ
μ^-
π^-
Concorde
μ^+
ν_μ
Everest
ν_μ
μ^-

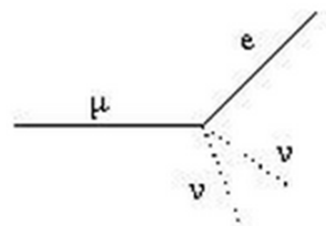
Cosmic Rays

Cosmic Rays

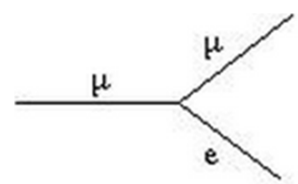
- Hydrogen nuclei
- Helium nuclei (alpha particles)
- Electrons (beta rays)
- Produced by solar flares and SNe
- Collide with atmosphere gas
  - Produce pions, muons and neutrinos
- 1 particle passes through your hand per second

Cloud Chamber – What to Look For

- Straight skinny track
  - Muon
- Kinks to the right
  - Muon decay
  - μ -> e- ν ν

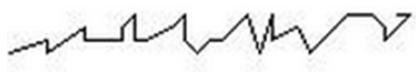


- Three tracks meet at pt.
  - Incoming ray hits e- and knocks it out and muon is deflected



Cloud Chamber – What to Look For

- Zizag
  - Low energy cosmic ray bounces off atoms



Muon Lifetime = 2 x10^{-6} sec

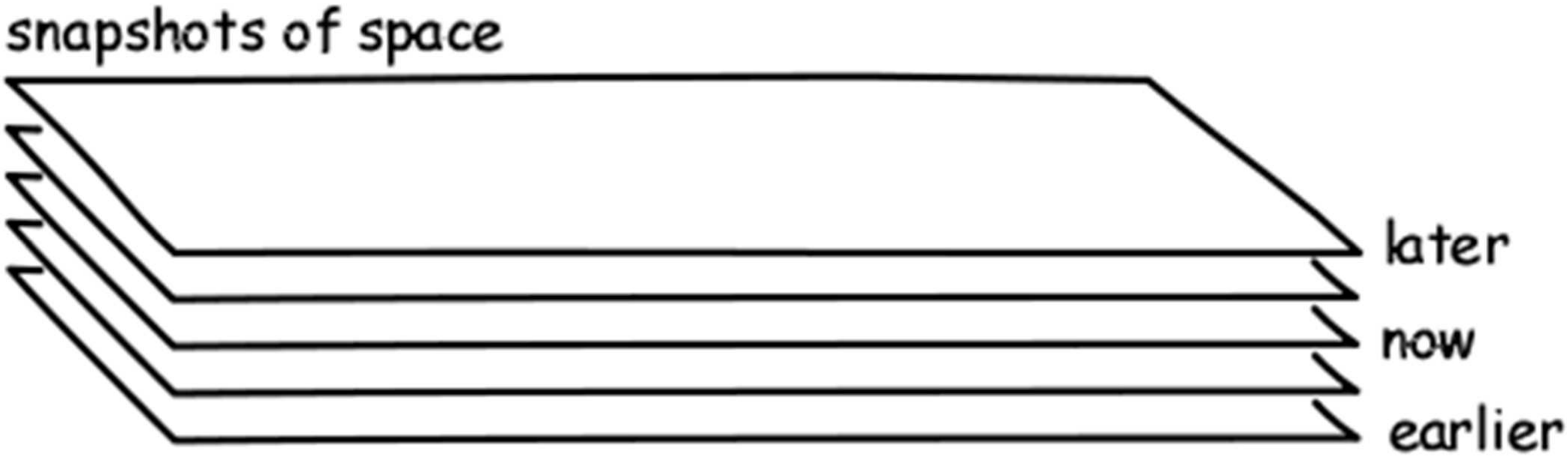
- If muons have speeds close to c, how far should they travel before decaying?
- Should any muons reach the ground?

Space versus Spacetime

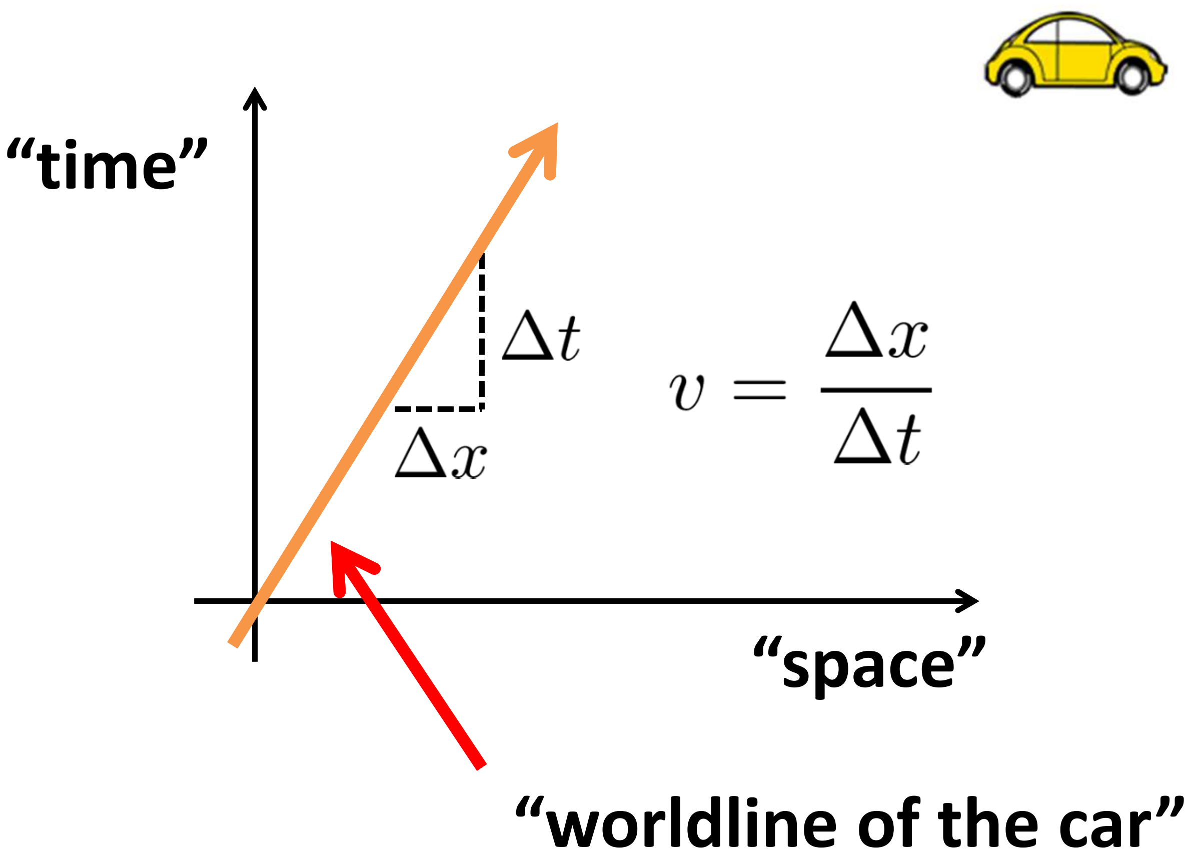
We are used to thinking of objects moving through space.

Any object is also moving through ***time*** even if it is at rest!

Spacetime

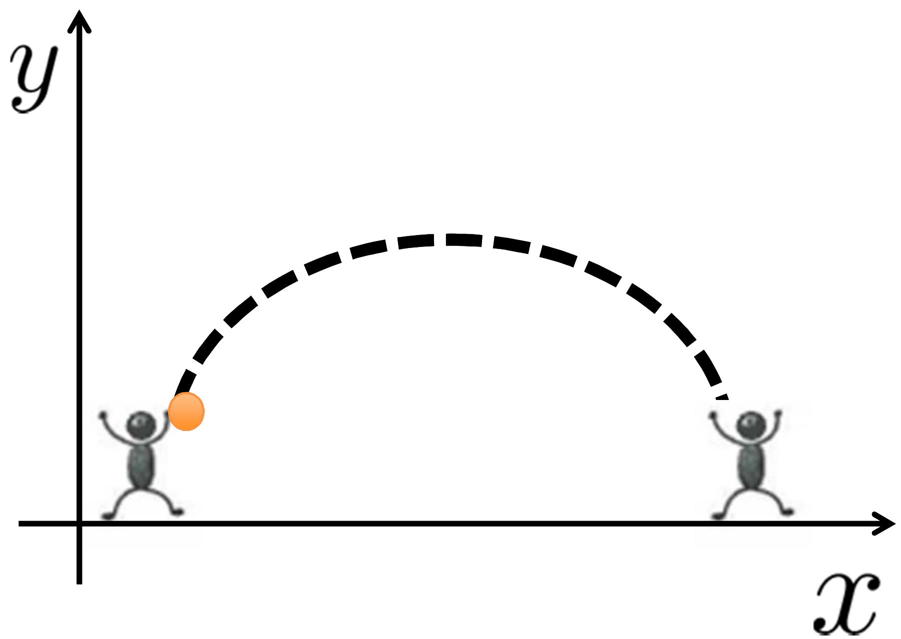


Spacetime Diagram

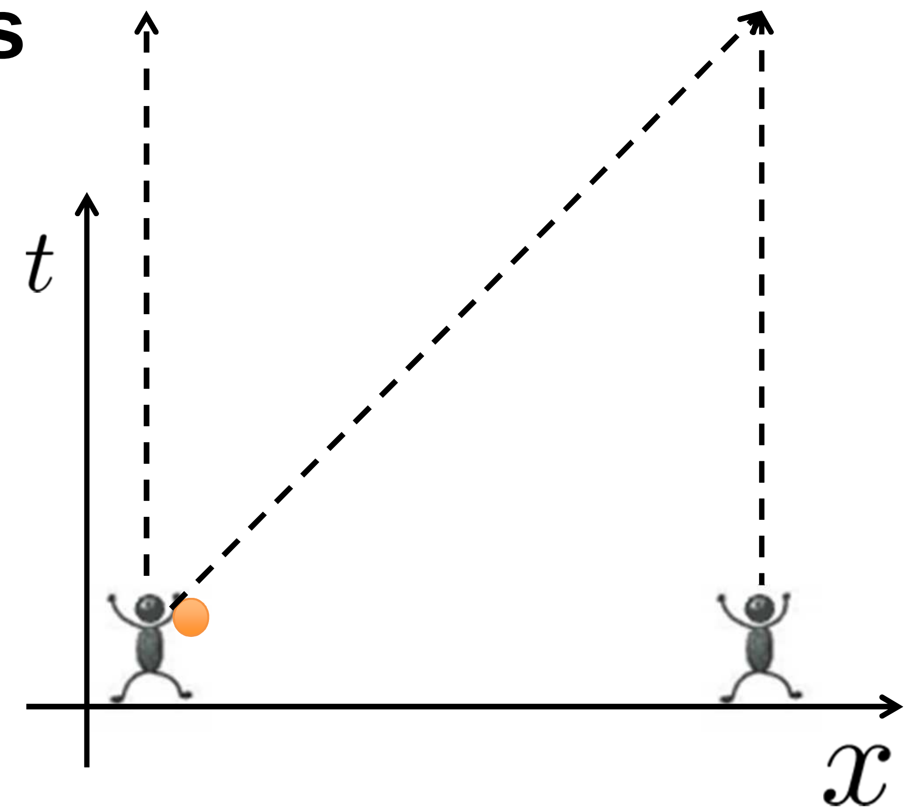


$$v = \frac{\Delta x}{\Delta t}$$

Alice & Bob Ball Toss



Space Diagram



Spacetime Diagram

Spacetime Diagrams

- The Players
  - Alice
  - Bob
  - Cleopatra
  - Doug
  - Einstein

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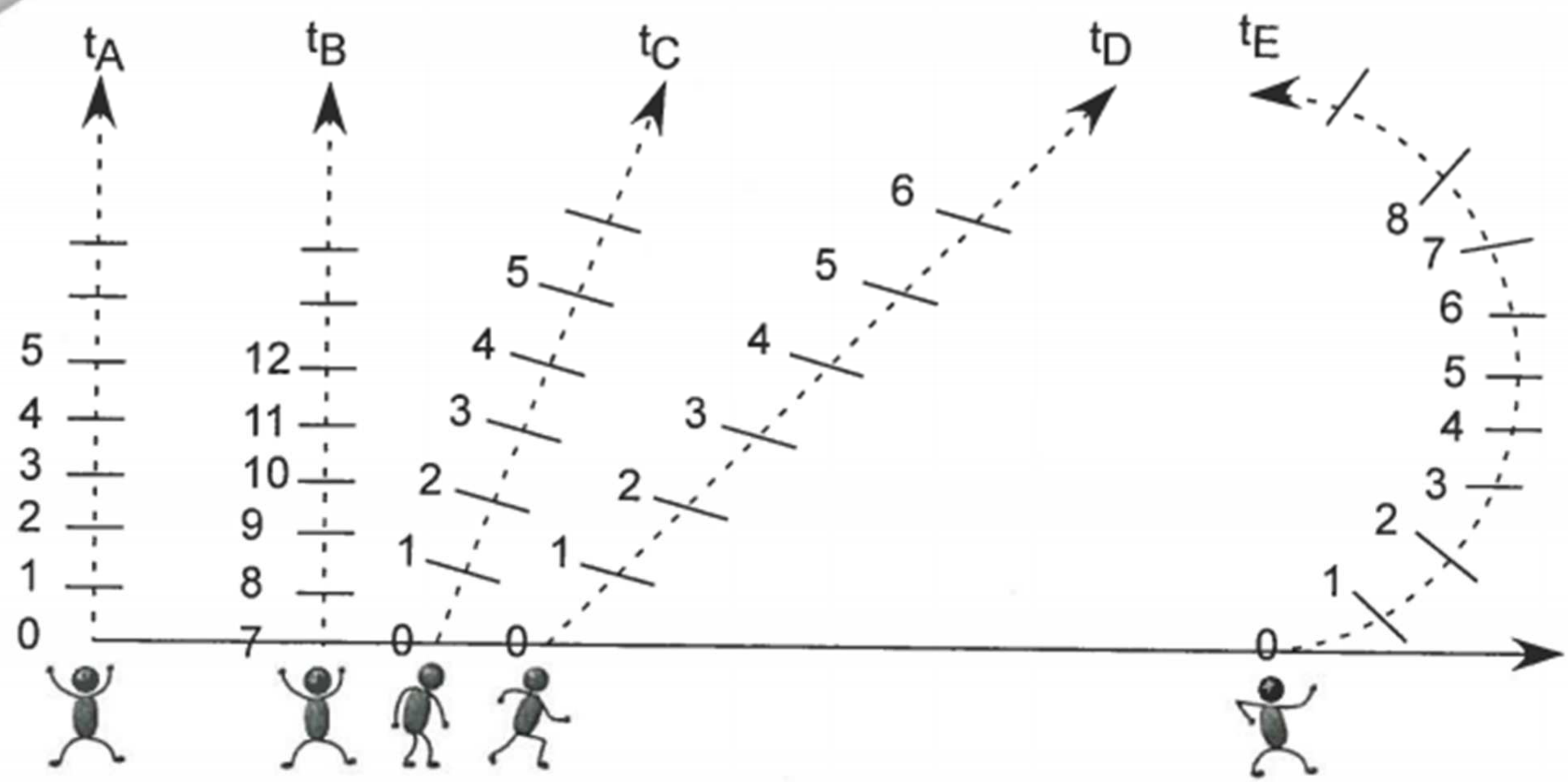
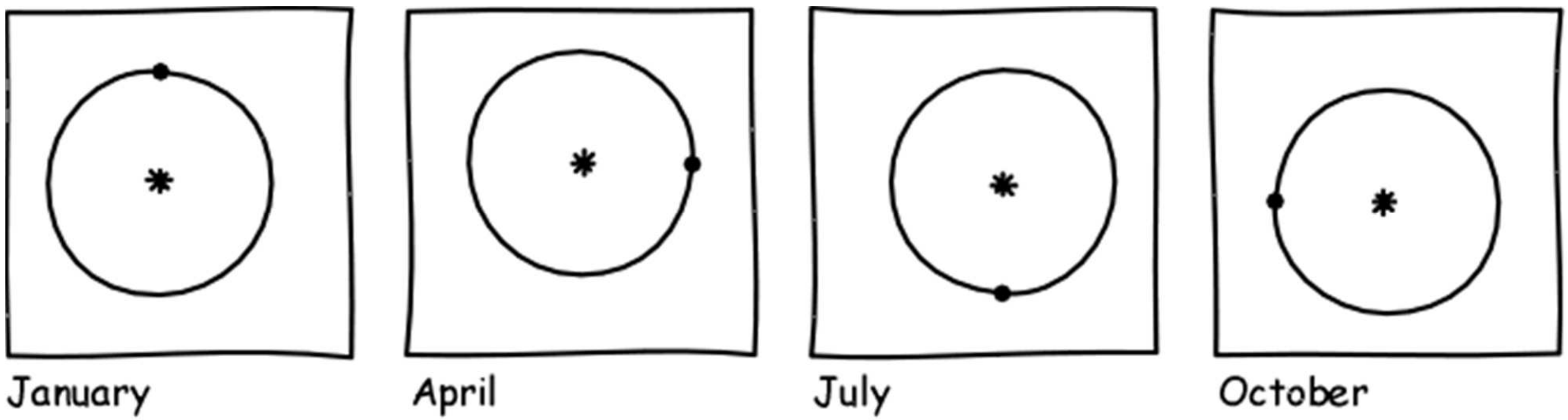
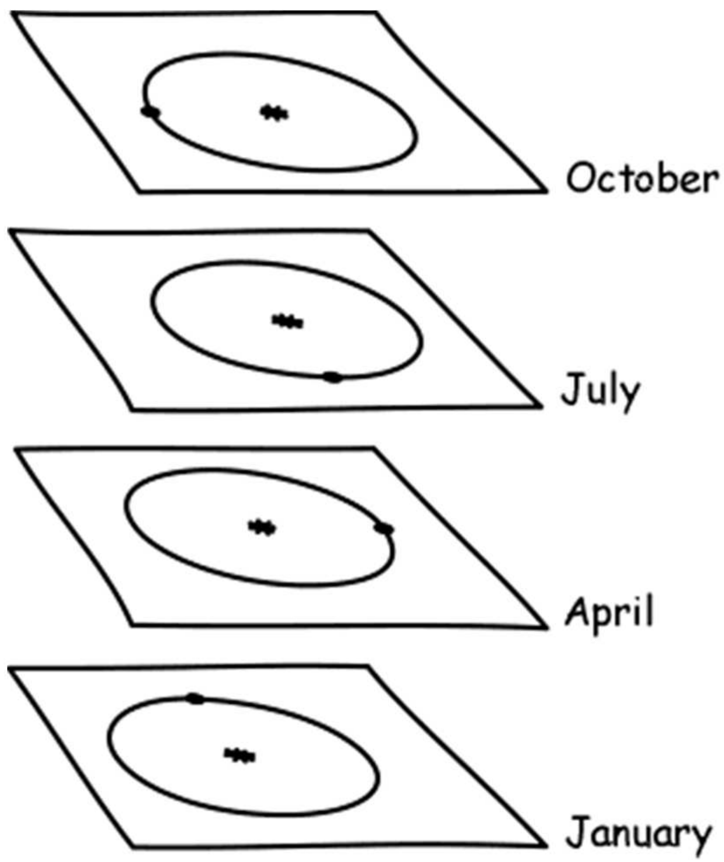


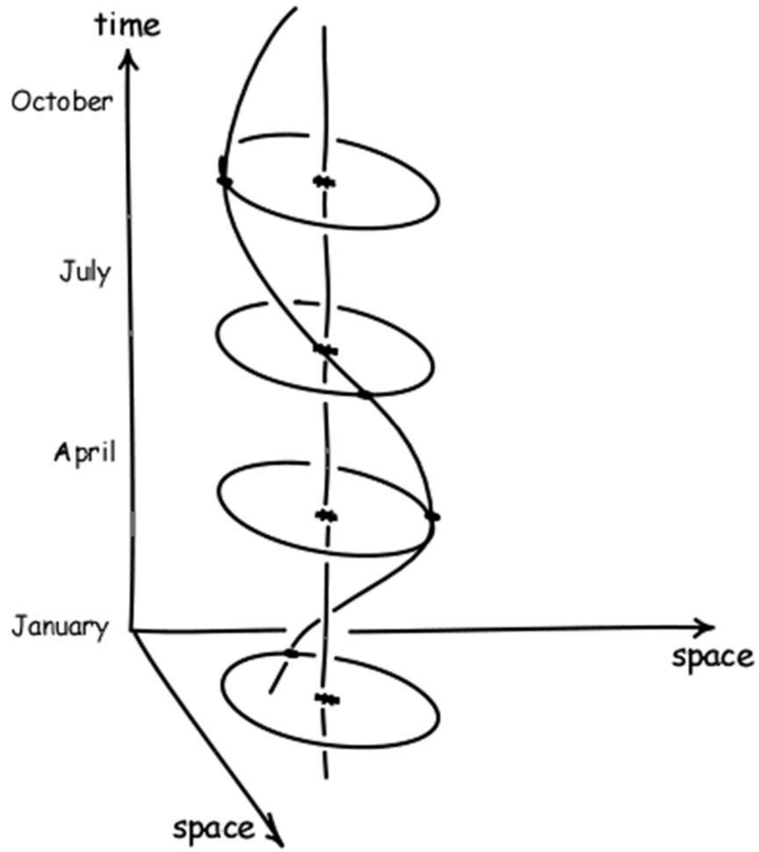
Earth's Orbit



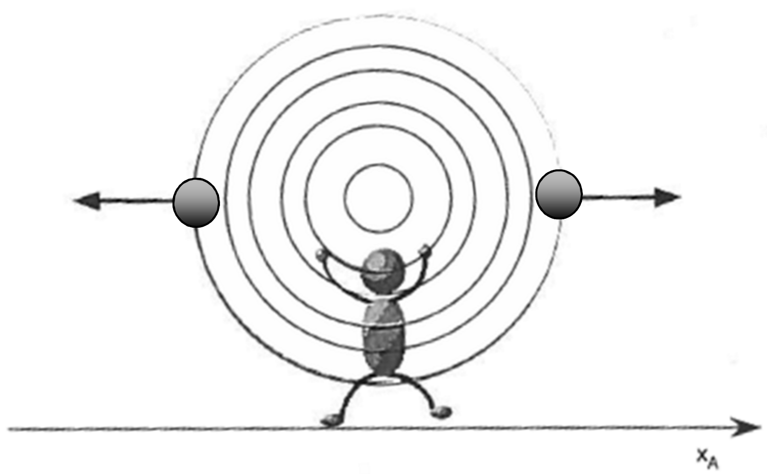
Earth's Orbit



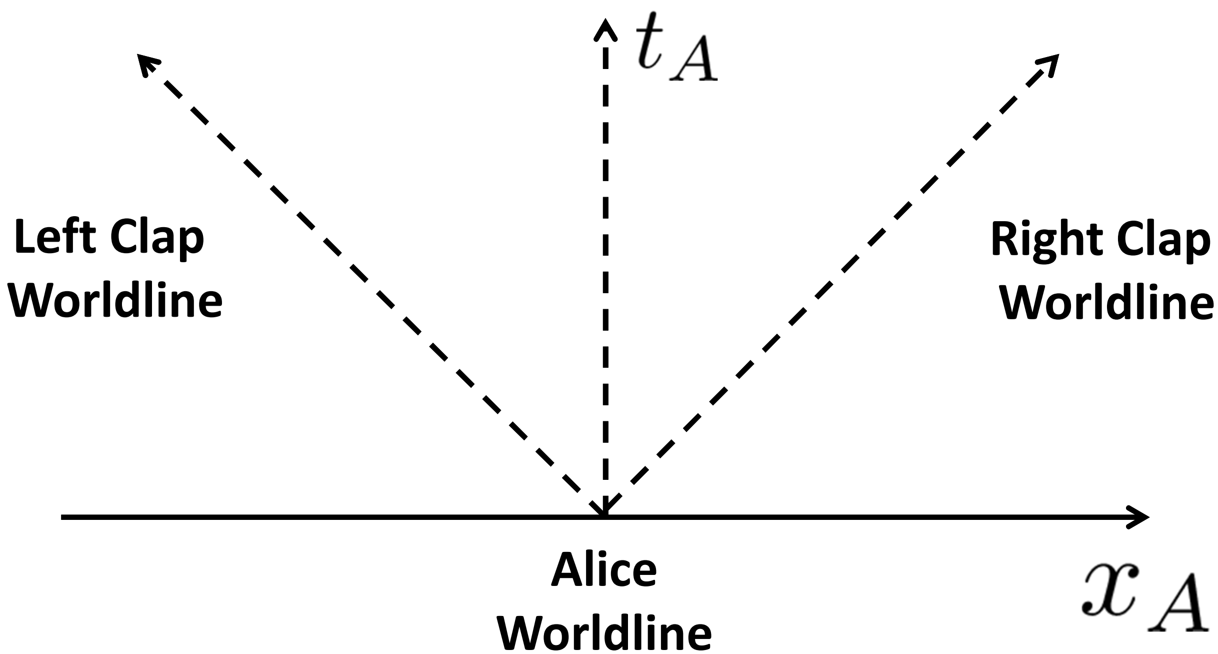
Earth’s Orbit



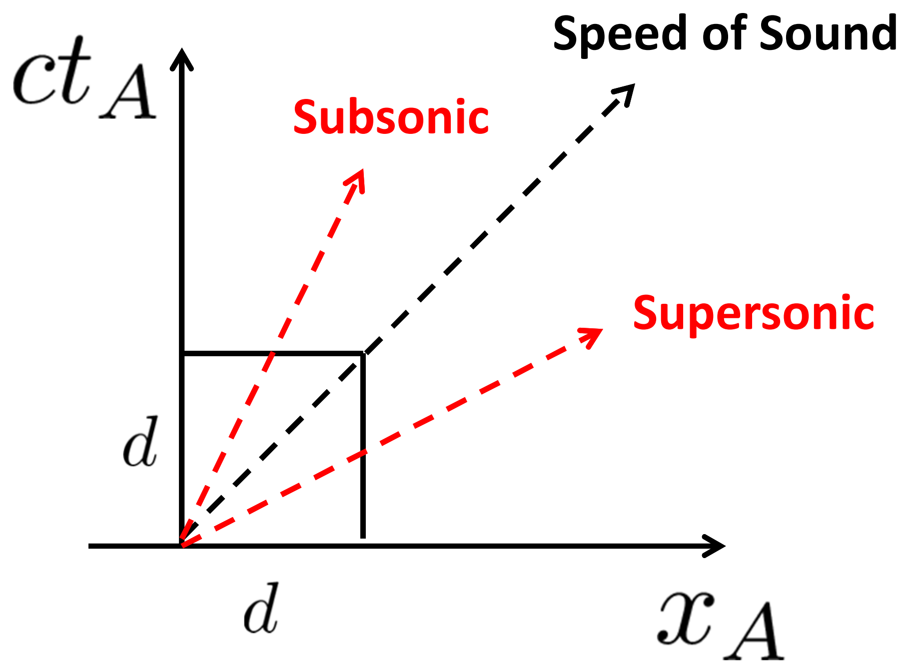
Space Diagram Clap!



Spacetime Diagram

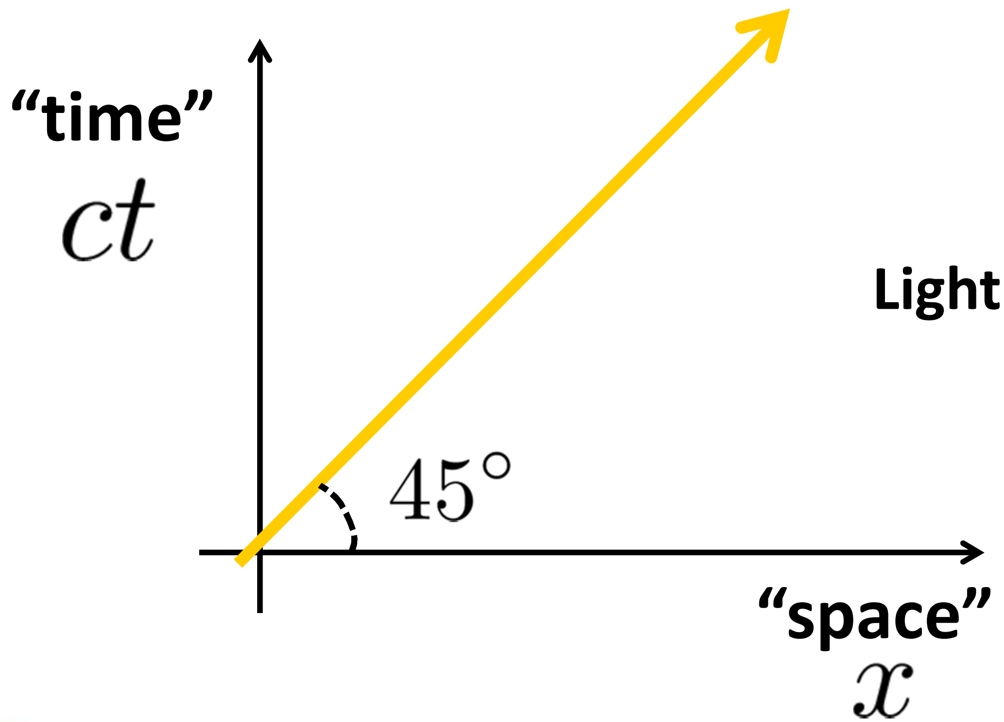


Spacetime Diagram



Space and time are on the same footing!

Spacetime Diagram



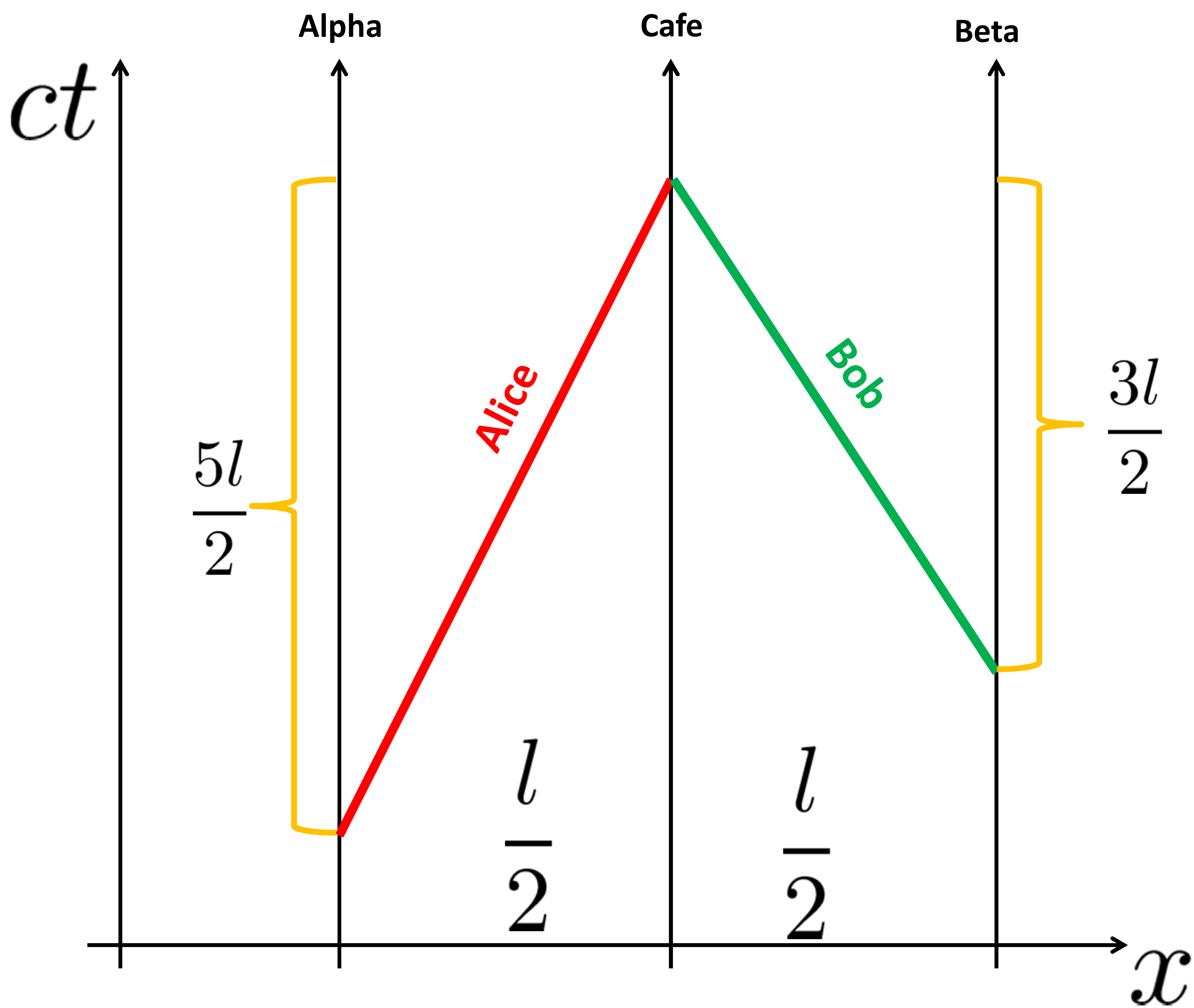
Light rays are drawn at 45°

Alice & Bob meet at a Café

Alice and Bob live on planets Alpha and Beta separated by 6 x10^{11} m. Halfway between their home planets is an interplanetary café where they decide to meet at noon.

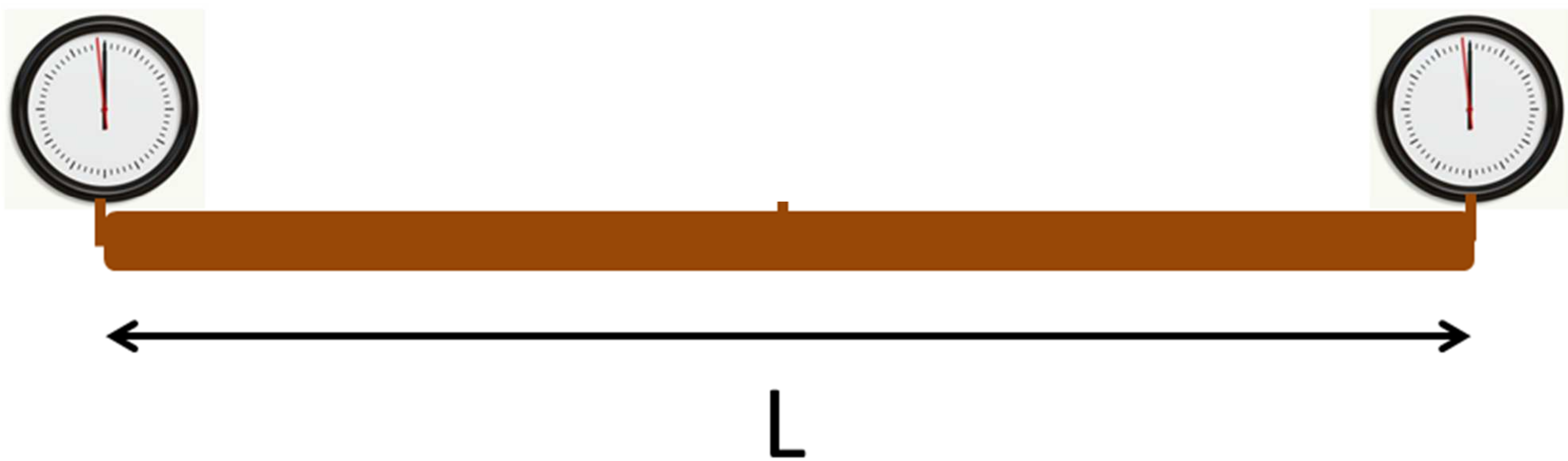
Alice has a standard spaceship, which travels at c/5. Bob has a sporty model, which travels at c/3.

Plot the worldlines in the frame of the planets.

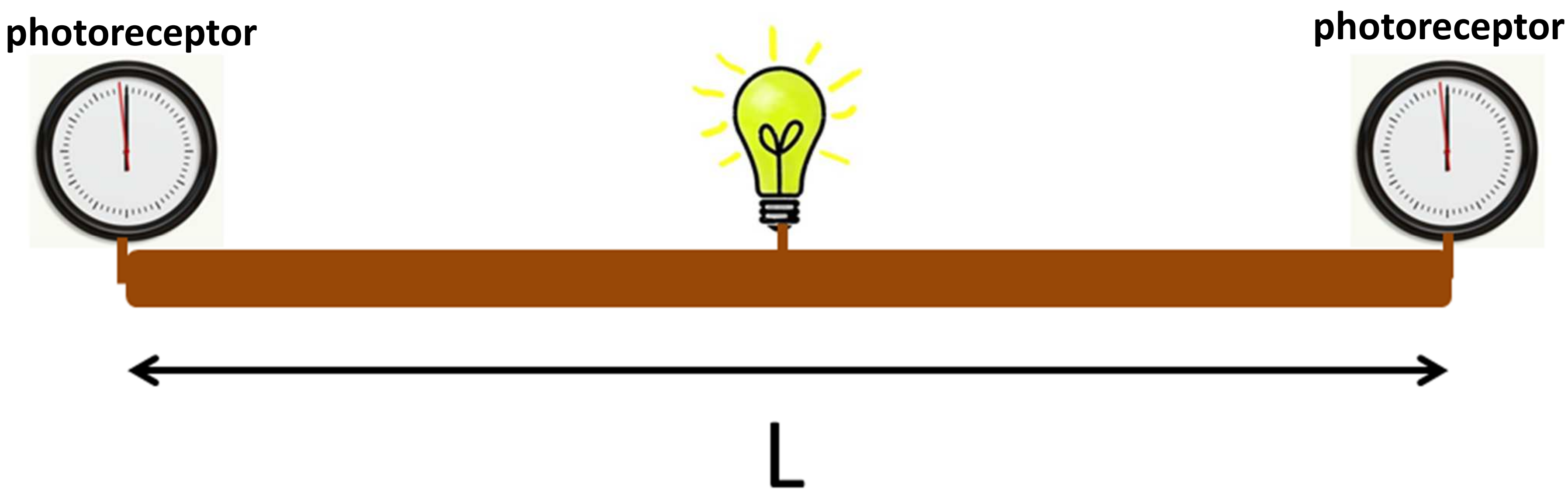


Alpha
Cafe
Beta
ct
Alice
Bob
$\frac{5l}{2}$
$\frac{3l}{2}$
$\frac{l}{2}$
$\frac{l}{2}$
x

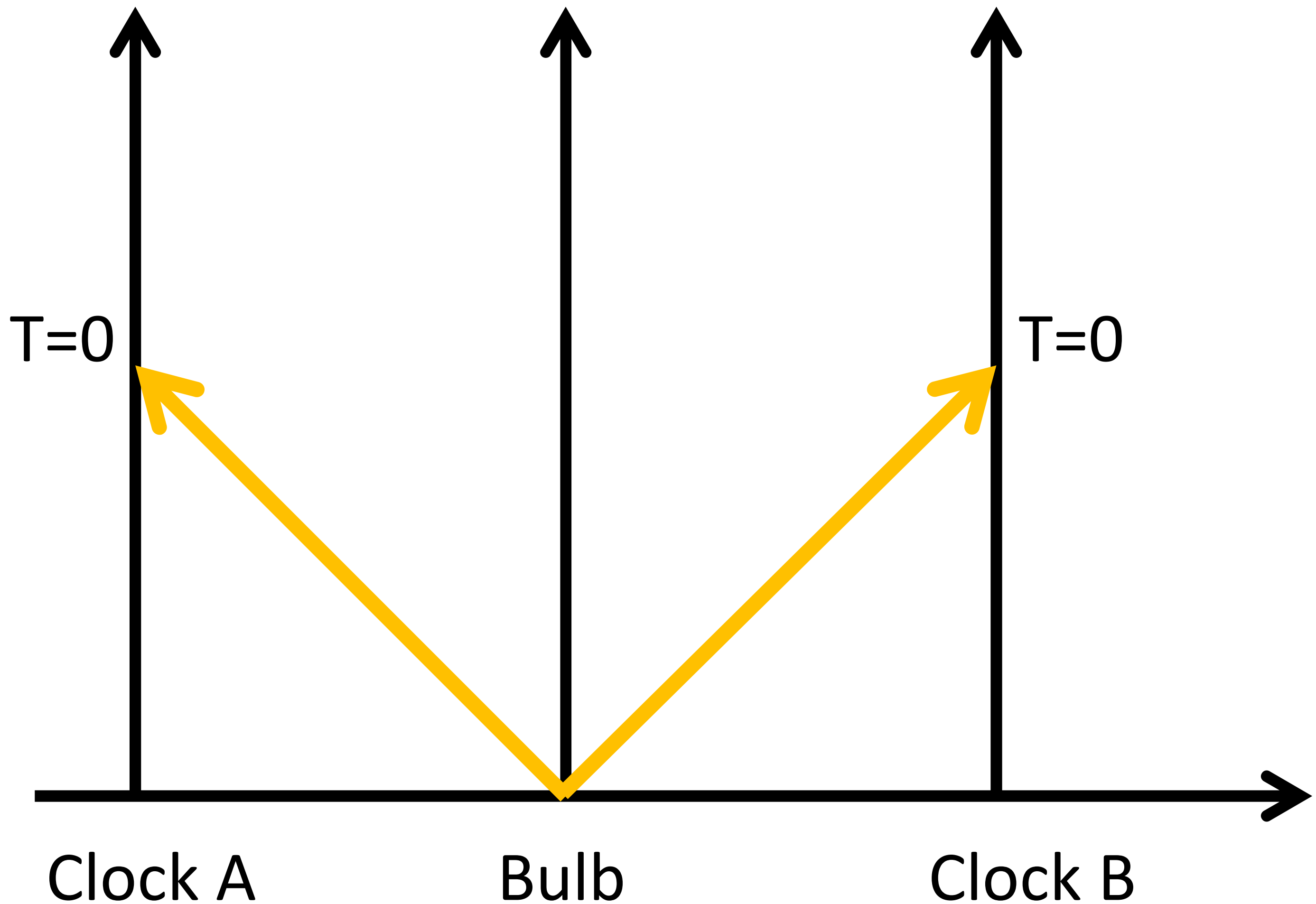
How Can we Synchronize these Clocks?

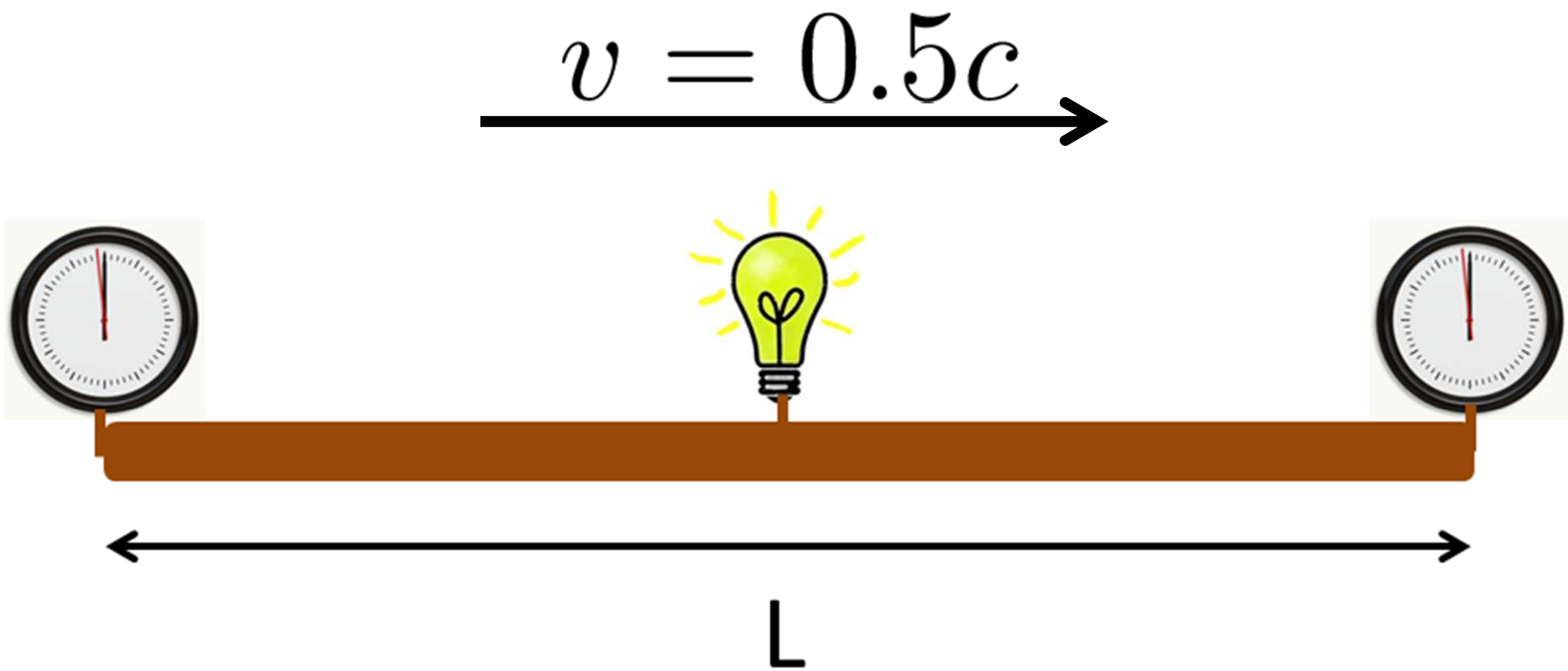


Synchronizing Clocks



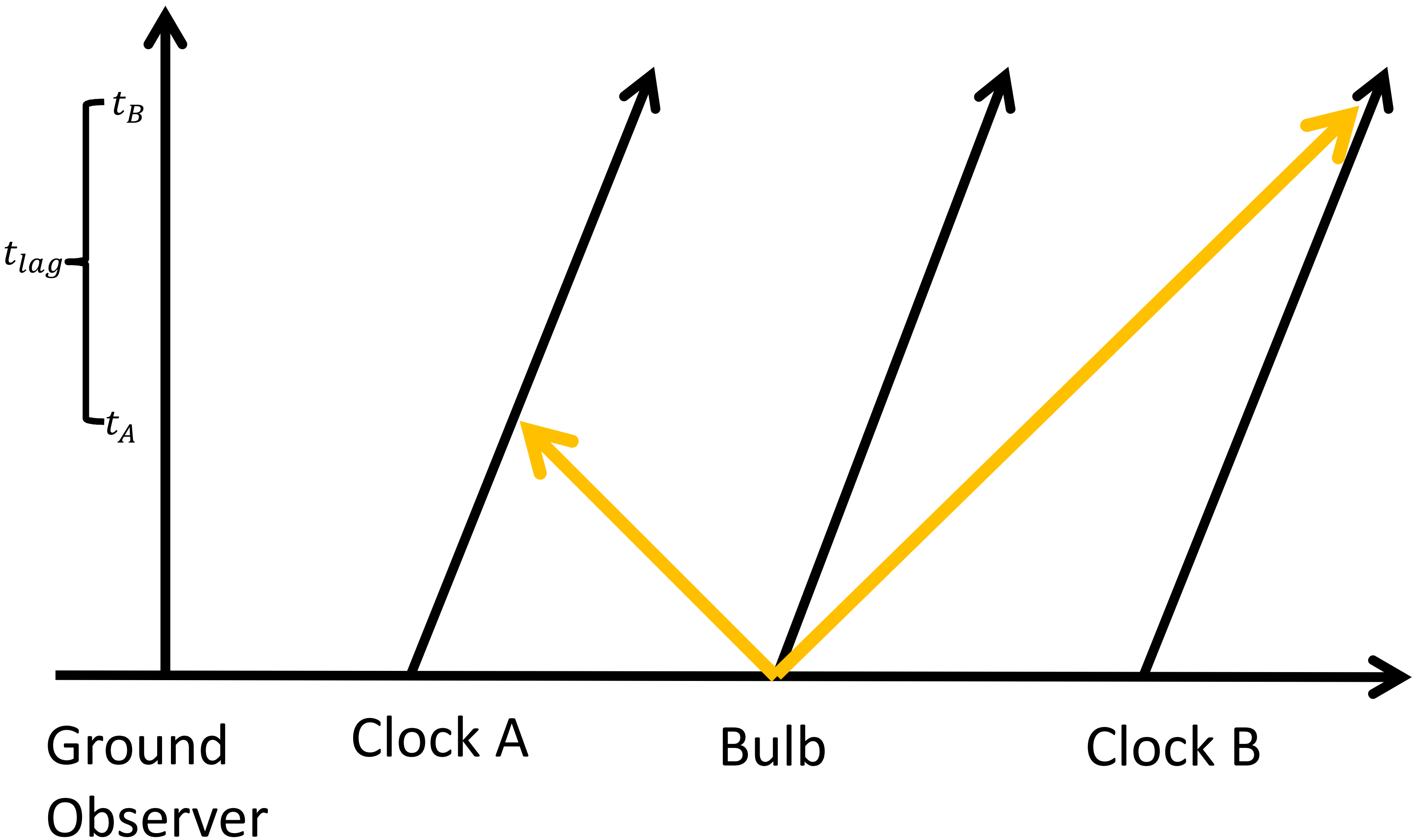
Frame on the Platform

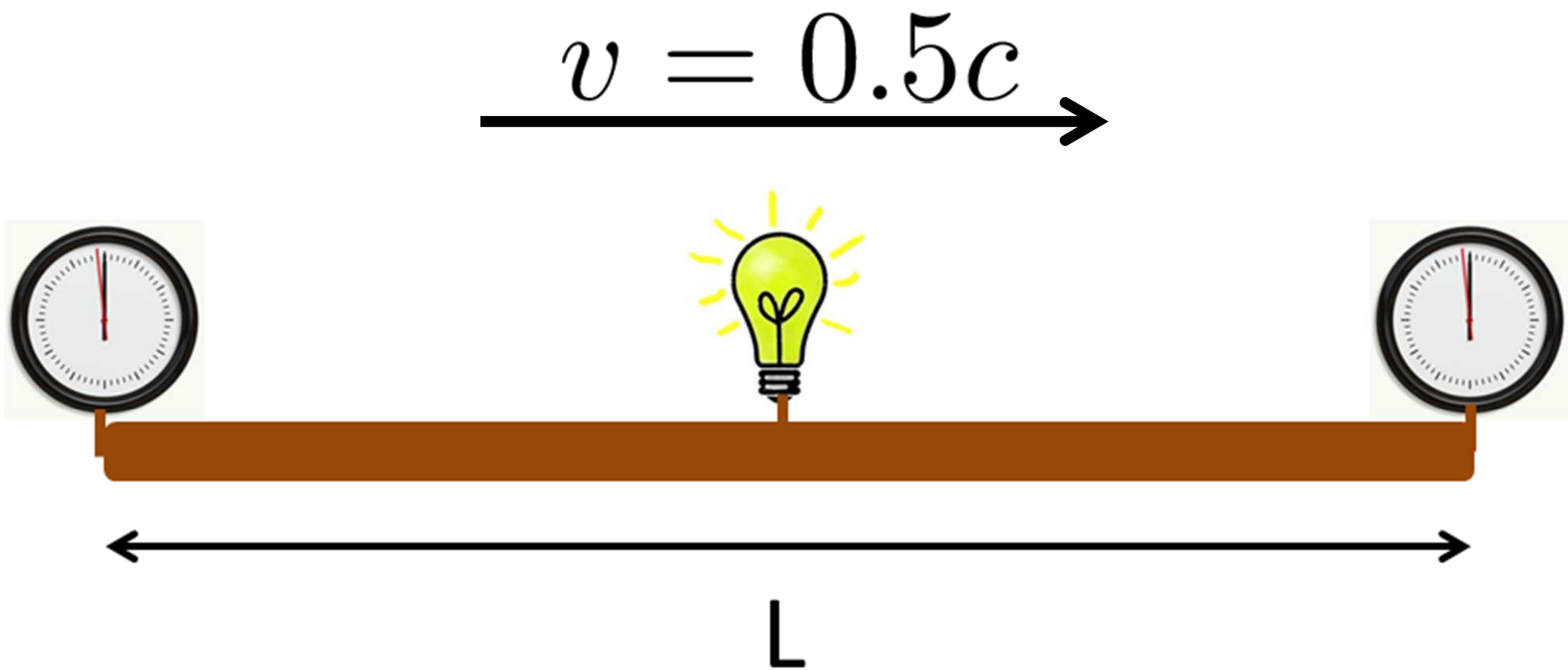




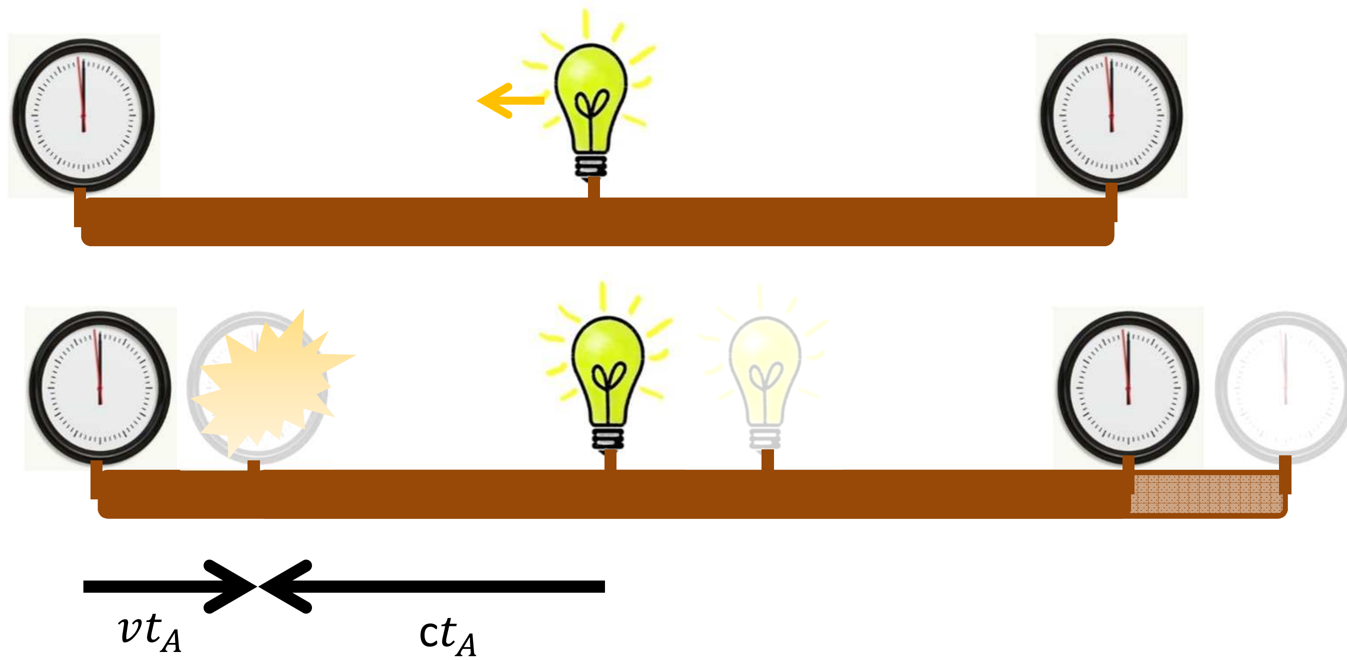
$v = 0.5c$
L

Frame on the Ground





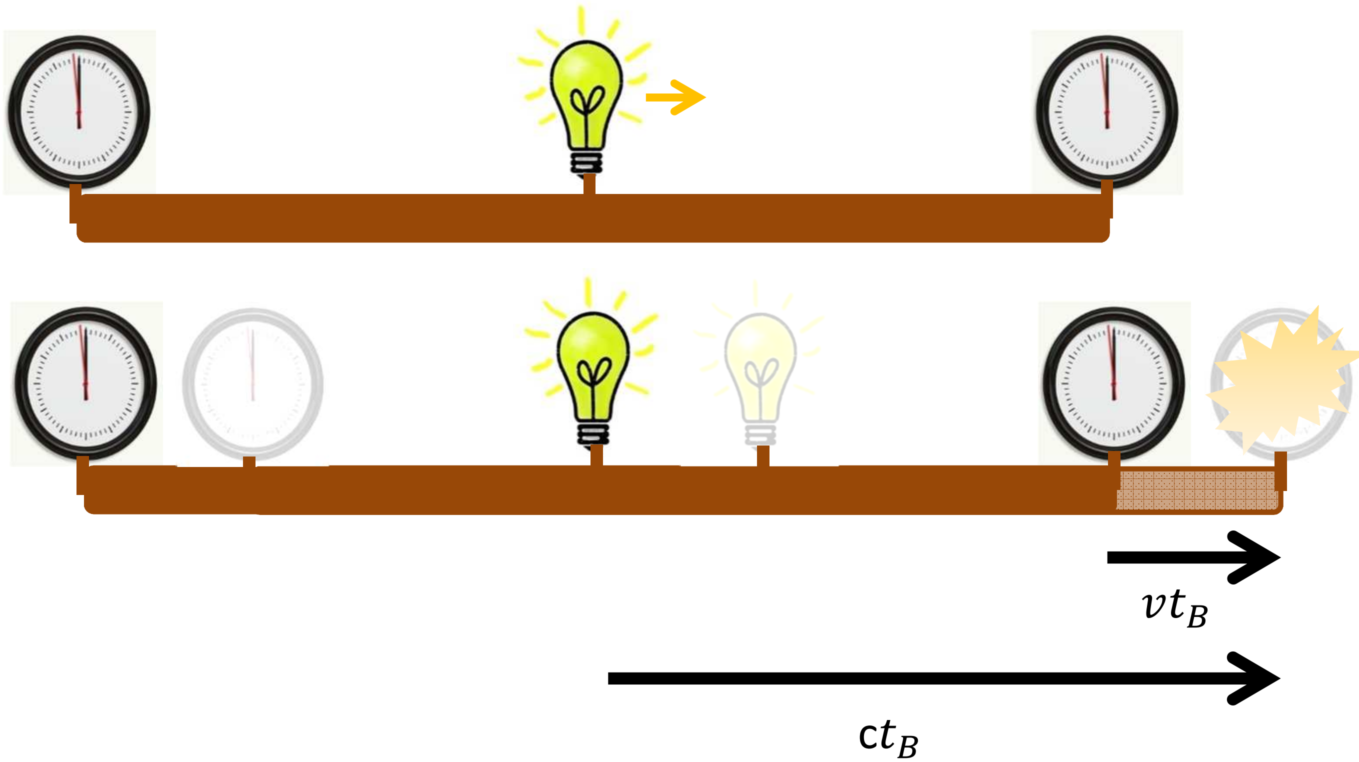
$$\frac{L}{\gamma} = L\sqrt{1-\frac{v^2}{c^2}} \qquad d = \frac{L}{2\gamma} = \frac{L}{2}\sqrt{1-\frac{v^2}{c^2}}$$



$$ct_A = d - vt_A$$

$$t_A = \frac{L}{2(v+c)}\sqrt{1-\frac{v^2}{c^2}}$$

$$ct_A + vt_A = \frac{L}{2}\sqrt{1-\frac{v^2}{c^2}}$$



$$ct_B = d + vt_B$$

$$ct_B - vt_B = \frac{L}{2}\sqrt{1 - \frac{v^2}{c^2}}$$

$$t_B = \frac{L}{2(c-v)}\sqrt{1 - \frac{v^2}{c^2}}$$

Time Lag for Ground Observer

$$t_B - t_A = \left(\frac{1}{c-v} - \frac{1}{c+v}\right)\frac{L}{2}\sqrt{1-\frac{v^2}{c^2}}$$

$$= \left(\frac{2v}{c^2 - v^2}\right)\frac{L}{2}\sqrt{1-\frac{v^2}{c^2}}$$

$$= \frac{2v}{c^2}\left(\frac{1}{1-\frac{v^2}{c^2}}\right)\frac{L}{2}\sqrt{1-\frac{v^2}{c^2}}$$

$$= \frac{Lv}{c^2}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

Ground Observer Reading Platform Clocks

- To the observer on the ground the clocks on the platform appear to tick more slowly, by a factor ɣ, so less time will elapse.

- Thus, even though the ground observer measures a time of lag of

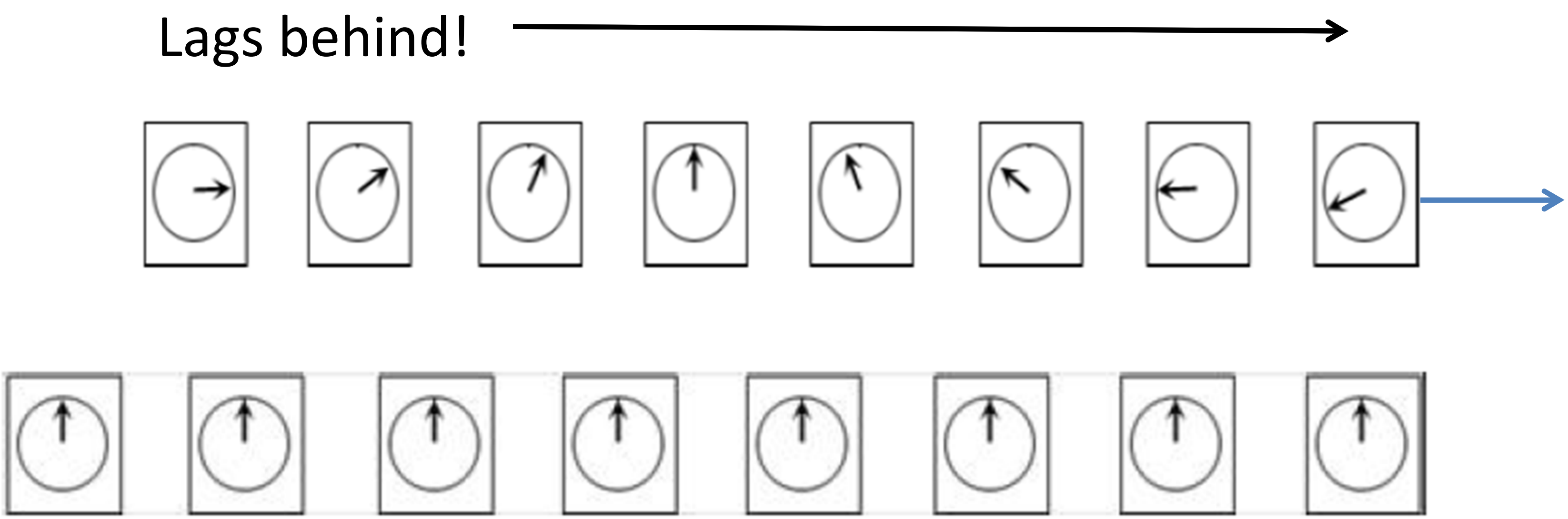
$$t_B - t_A = \frac{Lv}{c^2}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

- The observer will "see" the slow running clock at the back actually reading $\frac{Lv}{c^2}$ seconds the instant they see the front clock start.

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Lack of Synchronicity

$$\frac{Lv}{c^2}$$



There is no absolute way to know if:

- A rocket is moving uniformly or is at rest
- A meter stick is one meter long
- A process is one minute long
- Two spatially separated events are simultaneous
- Two spatially separated clocks are synchronous