Special Relativity Exercises 4:
MINKOWSKIAN GEOMETRY I

The point of these exercises is to convince yourself of the ***reciprocal*** nature of both *time dilation* and *length contraction*. For example, if I see your moving clock running slowly compared to mine, you will see mine running equally slowly (not faster). You will be constructing a spacetime diagram which is to be drawn very carefully and to scale: use provided grid paper and a ruler, and make the diagram as large as possible, but ensure that whole diagram still fits on the page. These exercises will continue with **Minkowskian Geometry II**.

[easy] 1. On the attached grid sheet I have drawn **Alice's** worldline slightly to the left of centre on the page, and labeled her time axis $\mathbf{ct_A}$. I have labeled the event $\mathbf{ct_A=0}$ on her worldline. Note that the spacing between grid lines (both horizontal and vertical) is **1m,** and light moves on lines with slope **±1**.

[easy] 2. **Alice** sees **Alice-Prime** located some distance to the right of her, and at rest with respect to her. **Alice** conducts a radar ranging experiment by sending a light signal at $\mathbf{ct_A=0}$ to **Alice-Prime,** and finds that the light signal is reflected by **Alice-Prime** and returns back at time $\mathbf{ct_A=24m}$. What is the distance between **Alice** and **Alice-Prime**? Draw the appropriate light signals and the **Alice-Prime** worldline, and include relevant distance and time labels on **Alice's** worldline ($\mathbf{ct_A}$) to illustrate the experiment. Label **Alice-Prime's** worldline as $\mathbf{ct_{A'}}$.

[easy] 3. Assuming the two clocks (**Alice** and **Alice-Prime**) are synchronized, label the time event on the **Alice-Prime** worldline, $\mathbf{ct_{A'}}$, where **Alice-Prime** receives the light signal from **Alice**.

[easy] 4. A short time later at $\mathbf{ct_A=25m}$, **Alice** notices **Bob-Prime** pass by her moving to the right with some constant velocity. Label the time of this event on **Alice's** worldline. **Alice-Prime** notices **Bob-Prime** pass by her at $\mathbf{ct_{A'}=40m}$; label this event on **Alice-Prime's** worldline. What is **Bob-Prime's** velocity, $\boldsymbol{v}$, relative to **Alice** and **Alice-Prime**?

[easy] 5. Simultaneous with **Bob-Prime** passing **Alice-Prime**, **Bob** passes **Alice** (simultaneous for **Alice** and **Alice-Prime**). Label the time of this event on **Alice's** worldline. If **Bob** is moving with the same velocity as **Bob-Prime** (i.e. **Bob** and **Bob-Prime** are at rest relative to each other), what is the time on **Alice-Prime's** clock when **Bob** passes **Alice-Prime**? Label this time on **Alice-Prime's** worldline. Draw **Bob** and **Bob-Prime's** worldliness, labeling them $\mathbf{ct_B}$ and $\mathbf{ct_{B'}}$ respectively.

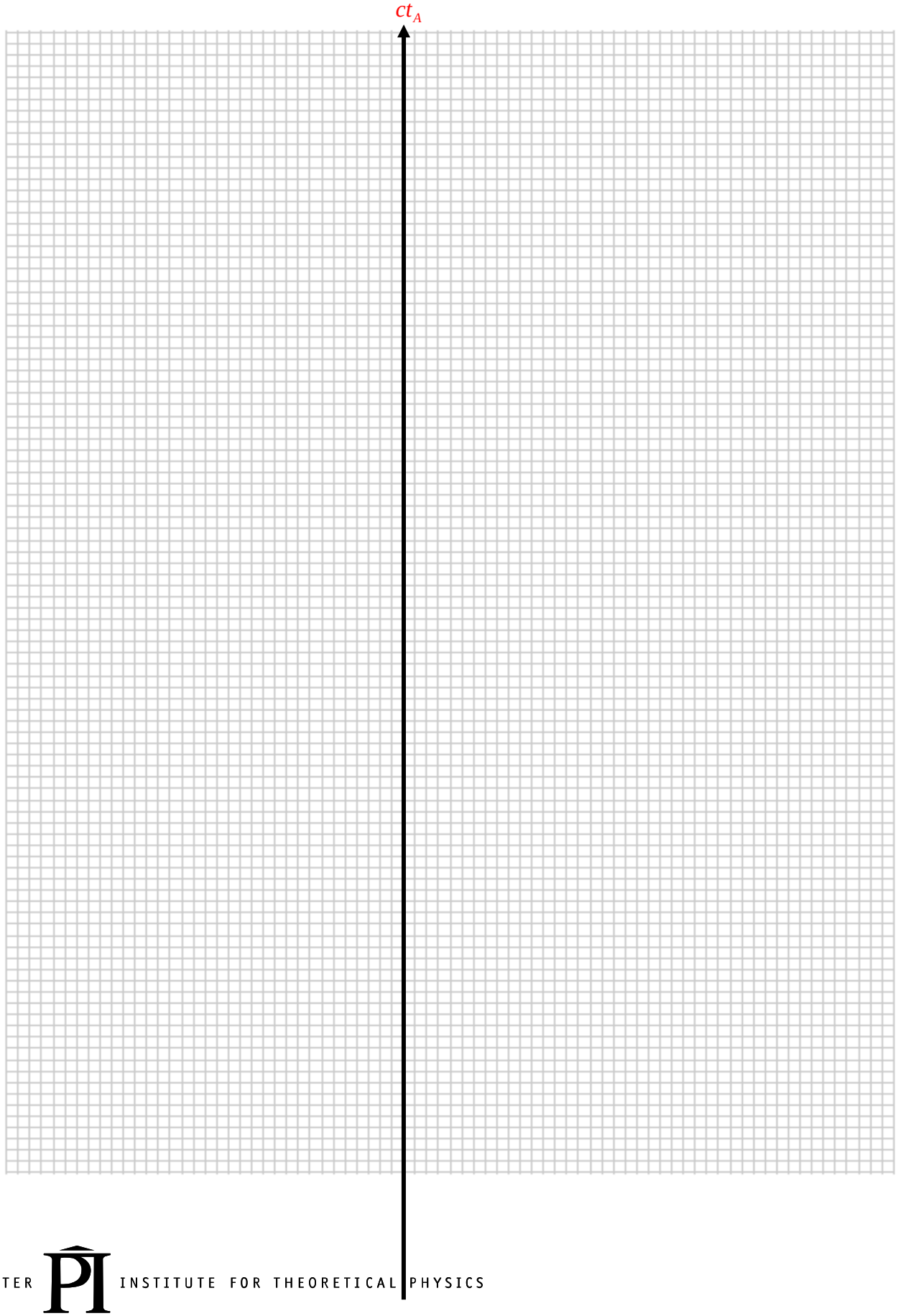
[medium] 6. Using Minkowskian geometry ("Pythagoras with a minus sign"), calculate how much time has elapsed for **Bob's** clock $\left(c\Delta t_B\right)$ between passing **Alice** and passing **Alice-Prime**? Calculate this time also using the ***time dilation formula***. Do the two calculations agree? If **Bob's** clock happens to read $\mathbf{ct_B=16m}$ as he passes by **Alice**, what will his clock reading be when he passes **Alice-Prime**? Label these two times on **Bob's** worldline. The graphical distance between these events *sets the scale* of time along **Bob's** (and **Bob-Prime's**) worldliness, as well as the scale of spatial distances in their frame.

[easy] 7. Prior to **Bob** passing **Alice**, **Bob** started a radar ranging experiment to determine the distance between himself and **Bob-Prime** and to check the synchronization of their clocks. Using the timescale on **Bob's** worldline (as determined in the previous question) find and label this event as $\mathbf{ct_B=0}$ on **Bob's** clock (just count backwards from the time he passes **Alice**).

[easy] 8. Suppose this is the time that **Bob** sent his radar ranging light signal to **Bob-Prime**, who then reflects it back to **Bob**. Draw the outgoing and reflected light signals (remember light signals **always** travel at $\mathbf{45^0}$ with respect to the diagram).

[easy] 9. What is the length-contracted distance between **Bob** and **Bob-Prime** as measured by **Alice** and **Alice-Prime** (this is what **Alice** and **Alice prime** measure as the distance on their diagram when **Bob** and **Bob-Prime** are simultaneously passing them).

[easy] 10. Knowing the length contraction factor, what must the *proper distance* between **Bob** and **Bob-Prime** be (as measured in their frame)?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