Quantum Mechanics Exercises 1:

QUANTUM DOUBLE SLIT

In this exercise you will work through the details of measuring Planck's constant using the double slit experiment, as outlined in the lectures.

[easy] 1. Make a sketch of the double slit experiment discussed in the first quantum lecture, assuming that the source emits waves of wavelength λ. Show the source, the barrier with the two slits, and the observing screen. Label the distance between the slits as **w** and the distance from the barrier to the observing screen as **L**. Sketch the waves before they hit the barrier, after they pass through the two slits (showing diffraction), and the final interference pattern on the observing screen (showing several successive minima and maxima). Notice that the interference pattern on the observing screen has a maximum at the centre point, directly in line with the point midway between the slits. Explain why this is so. (Think of the length of the path a wave would take in getting from the source to the centre point on the screen via the upper slit compared with the lower slit.) Let **d** denote the distance on the observing screen from the central maximum to first minimum. Label this distance on your diagram.

[medium] 2. Our first task is to find an approximate relationship between λ, **w**, **d** and **L**. To do so, draw a straight line from the centre of the upper slit to the first minimum on the observing screen. Then draw a straight line from the centre of the lower slit to the first minimum on the observing screen. Convince yourself that the difference in the lengths of these two lines is the path difference for the wave in getting from the source to the first minimum via the upper or lower slits. For most experimental laboratory set ups, **w** is much less than **L**, which means that the lines from the two slits to the screen are nearly parallel. (So here assume they **are** parallel.) Let the angle they make with the horizontal be θ. Again, for most experimental laboratory set ups, θ will be a very small angle, i.e., much less than 1 (in radians). Using some trigonometry, determine the ***path difference*** in terms of **w** and θ.

[hard] 3a. A minimum in the interference pattern occurs when the path difference is such that a crest coming from one slit adds with a trough coming from the other slit, cancelling each other out. Knowing that the distance from one wave crest to the next is λ, what must the path difference be for waves from both slits hitting the first minimum?

b. Use simple trigonometry to express the ratio **d/L** in terms of the angle θ. Finally, using your previous two results, and the approximation $\tan(\theta) \approx \sin(\theta)$ (valid when θ is very small compared to 1), to find an approximate relationship between λ, **w**, **d** and **L**.

c. In the lecture we discussed the fact that when particles of momentum **p** pass through a double slit apparatus they will hit the observing screen at random locations according to a certain probability pattern. Moreover, we motivated the idea that this probability pattern can be calculated by assuming that, instead of particles we have waves of wavelength λ passing through the apparatus, where λ is inversely proportional to **p**. In other words, $\lambda = \frac{h}{p}$ (the de Broglie relation) for some constant of proportionality, **h**. Substituting this de Broglie relation into your result from the previous part, derive a formula for **h** in terms of **d**, **w**, **p** and **L**.

d. In a typical experiment we might have electrons with a kinetic energy of **5 keV** ("kilo electron volts"), i.e. the electrons are accelerated from rest through an electric potential difference of **5000 volts**. Convert this energy to Joules using the conversion: **1 eV = 1.6 x** $\mathbf{10^{-19}}$ **J**. Assuming the electron is non-relativistic (so that kinetic energy formula $\mathbf{E = p^2/2m}$ applies, where **m = 9.11 x** $\mathbf{10^{-31}}$ **kg** is the mass of the electron), determine the corresponding electron momentum, **p**. Also, in a typical experiment we might have: **L = 50 cm** (distance from barrier to screen) and **w =** $\mathbf{10^{-9}}$ **m** (distance between the slits; notice that this is a distance of only about 10 atoms - it's a hard experiment to do!). If the experimentally measured distance from the central maximum to the first minimum is **d = 4.34 mm**, use your result from the previous part to give an estimate for Planck's constant, **h**. (Make sure you use the proper units in your calculation!) How close is this value to the documented value of **h = 6.626 x** $\mathbf{10^{-34}}$ $\mathbf{m^2}$ **kg/s**?