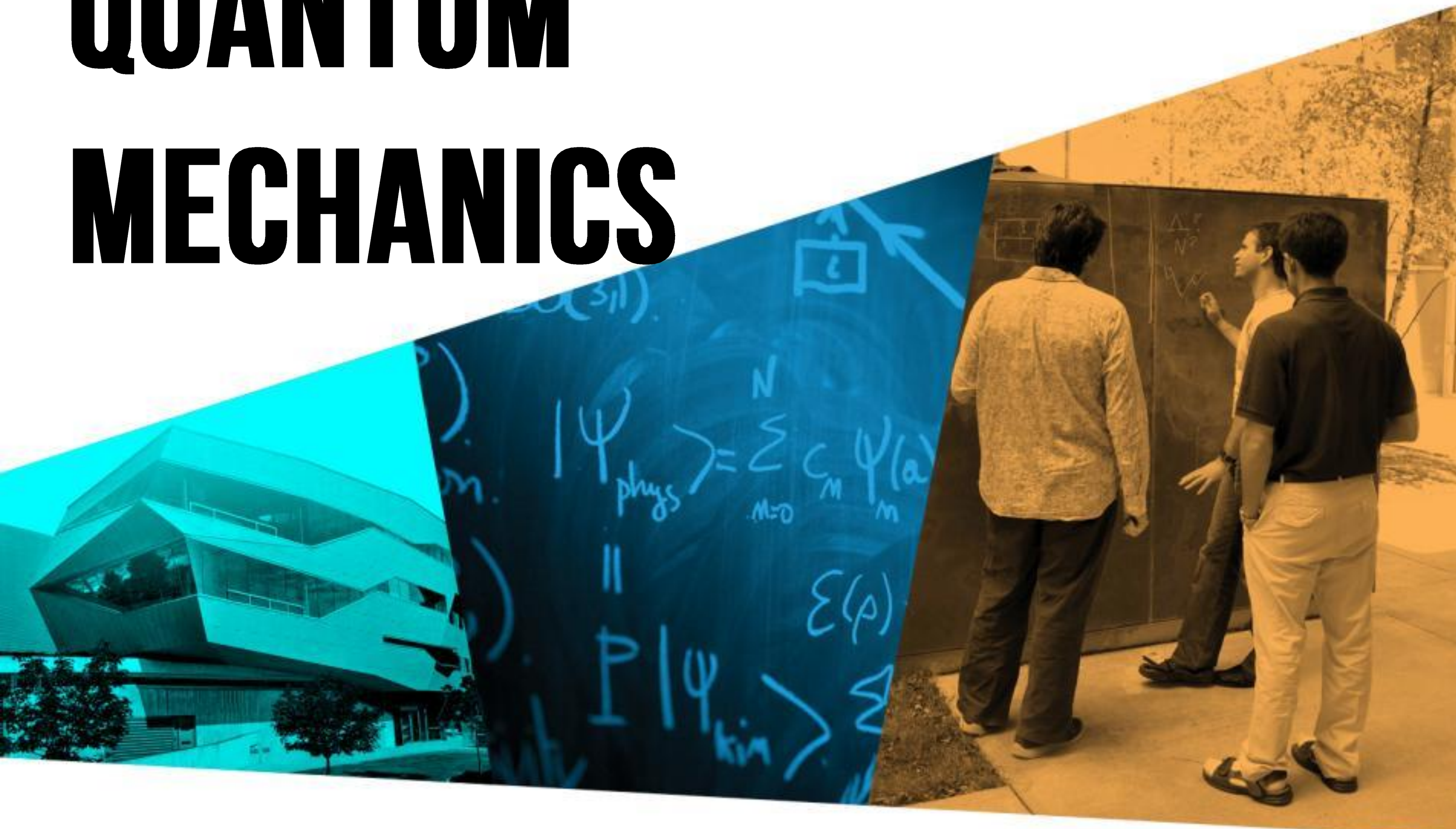
QUANTUM MECHANICS

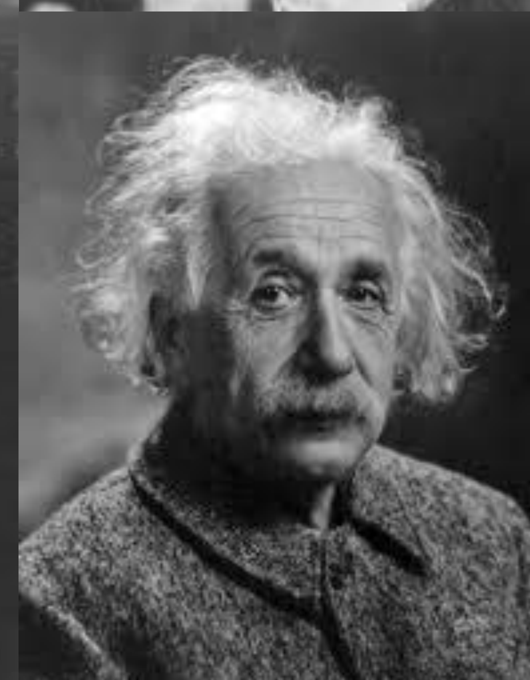


PERIMETER INSTITUTE FOR THEORETICAL PHYSICS



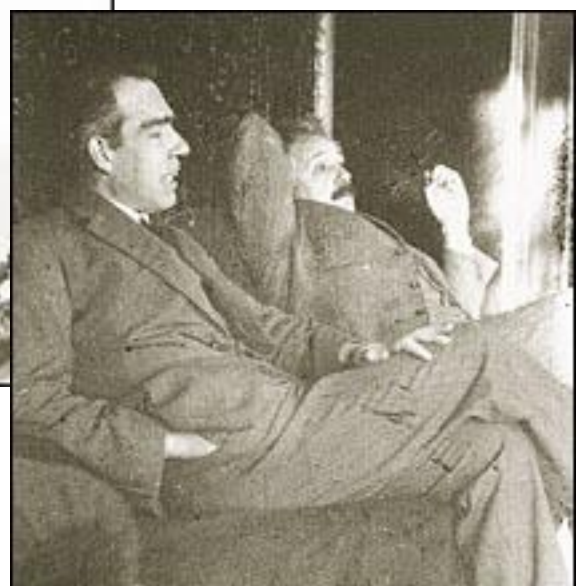


Scanned at the American
Institute of Physics

Since my talks with Bohr often continued till long after midnight and did not produce a satisfactory conclusion, ...both of us became utterly exhausted and rather tense.

--Heisenberg, recollection








Double Slit with Classical Particles (POE)

PREDICT

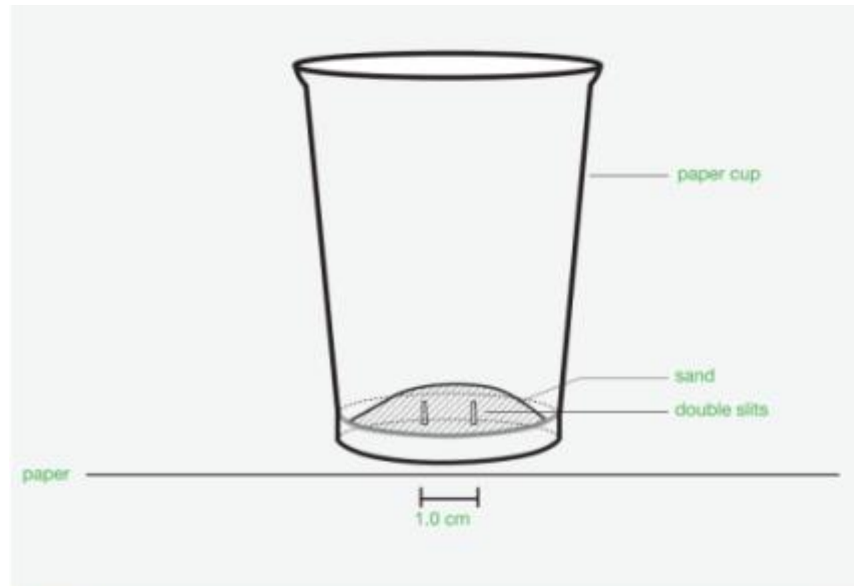


Figure 1 Be sure to keep the cup still and on the tabletop when pouring the sand through the slits.

Double Slit with Classical Particles (POE)

Using your white boards, please sketch and provide **three (3) describing words** of what you think will happen.

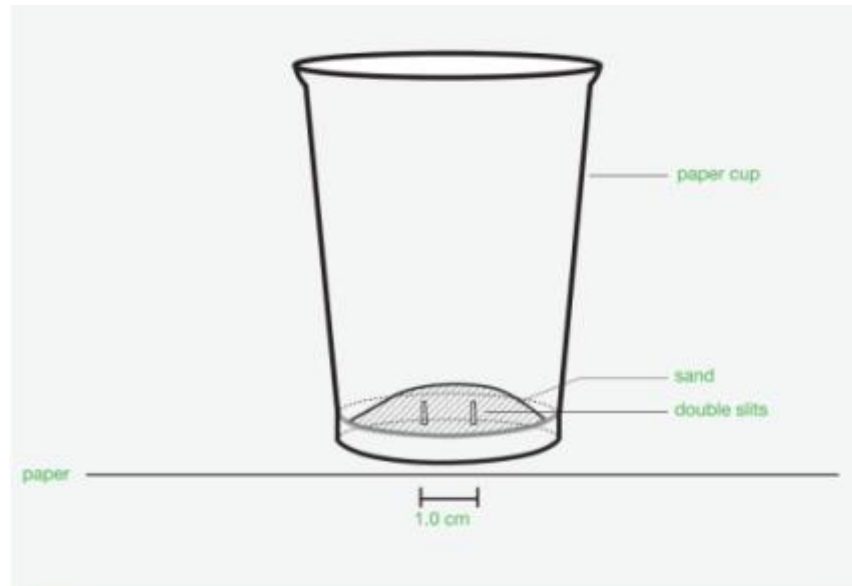


Figure 1 Be sure to keep the cup still and on the tabletop when pouring the sand through the slits.

Double Slit with Classical Particles (POE)

Classical particles…
collide
localized

Particle Model of Nature

What is a particle?

- Localized object
- Only in one place at a time
- Can bounce off other particles

Double Slit with Classical Waves (POE)

Predict what happens when waves travel through the two slits. Write down **three (3) describing words** about the properties of waves.

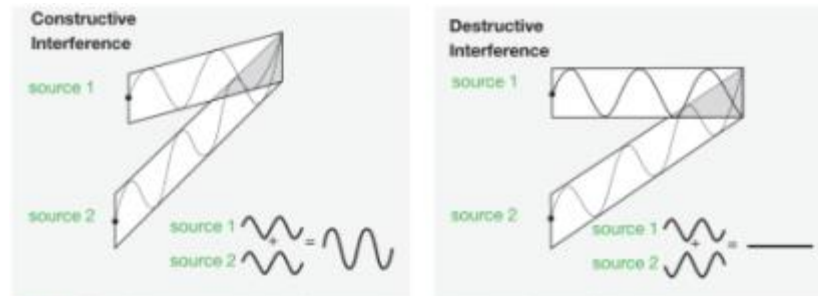


Figure 2 Recall the constructive interference and destructive interference of classical waves.

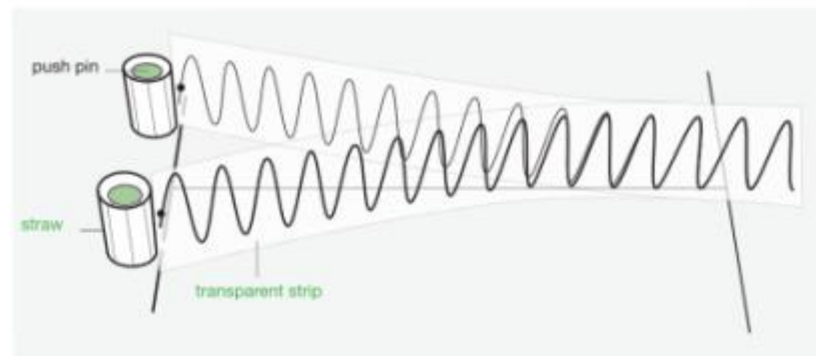
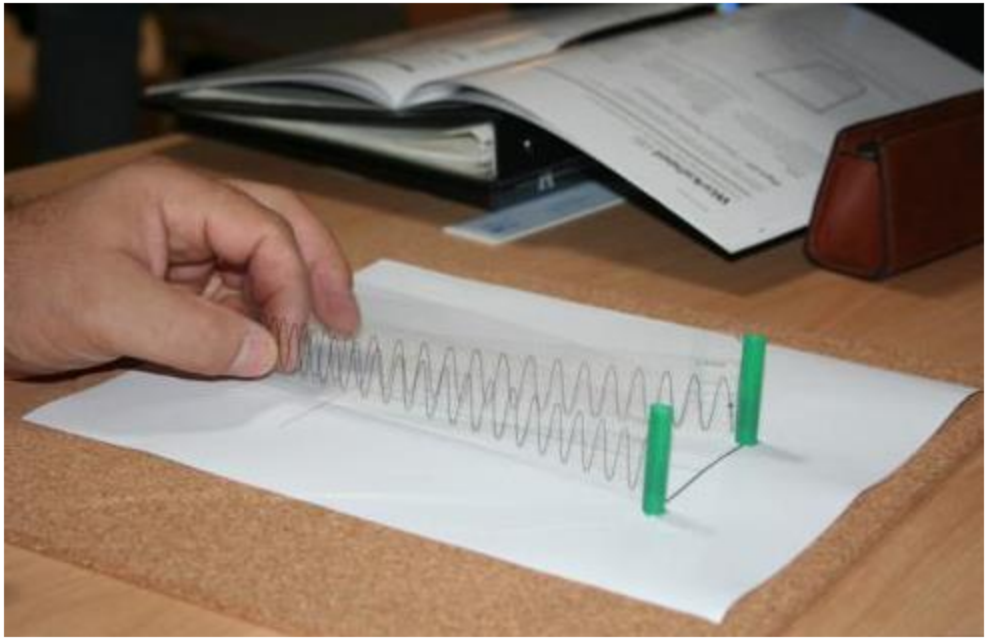


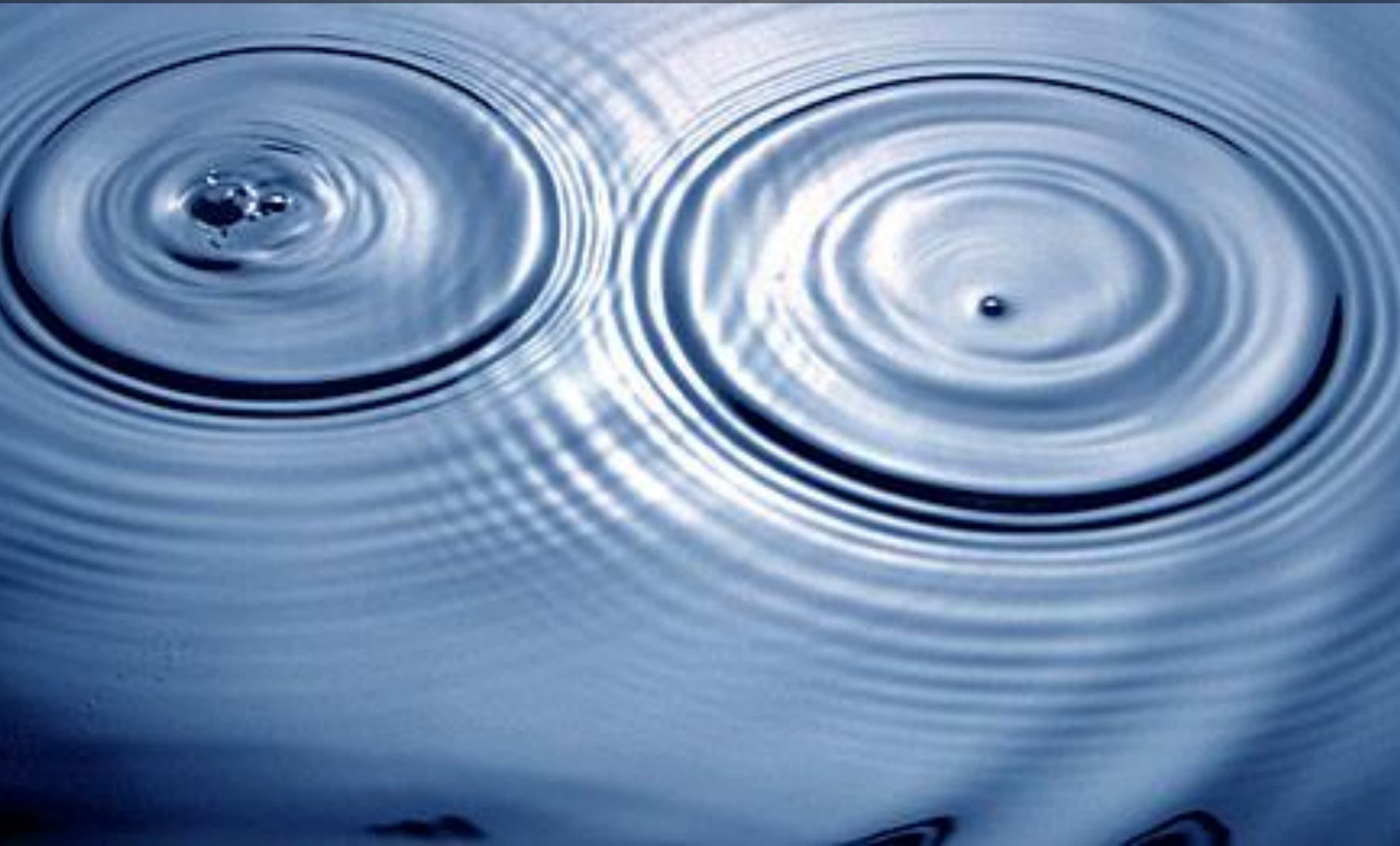
Figure 3 Use waves drawn on transparencies to observe interference.

Double Slit with Classical Waves (POE)

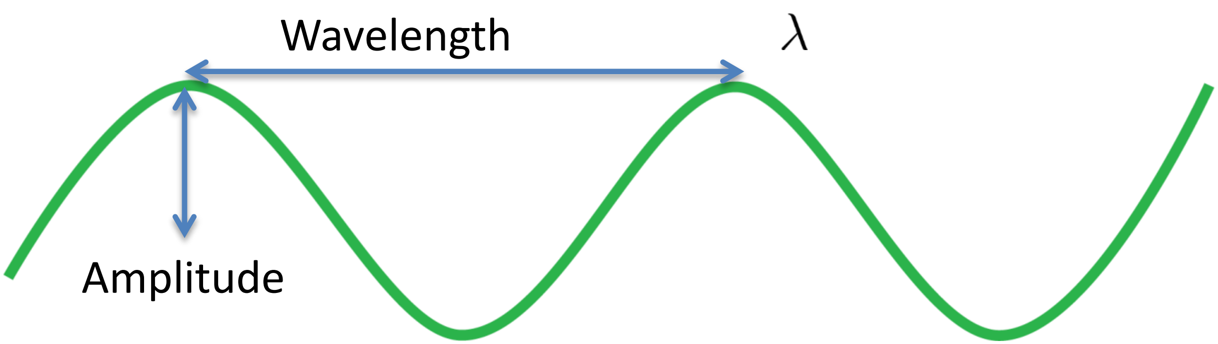


Classical waves…
- interfere
- non-localized

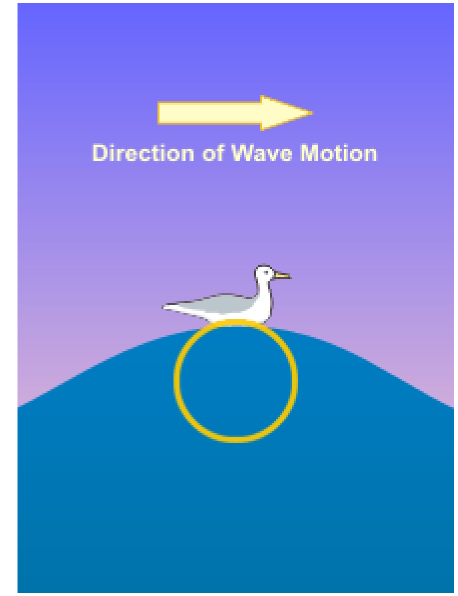
WAVES



Waves 101



Waves Transport Energy

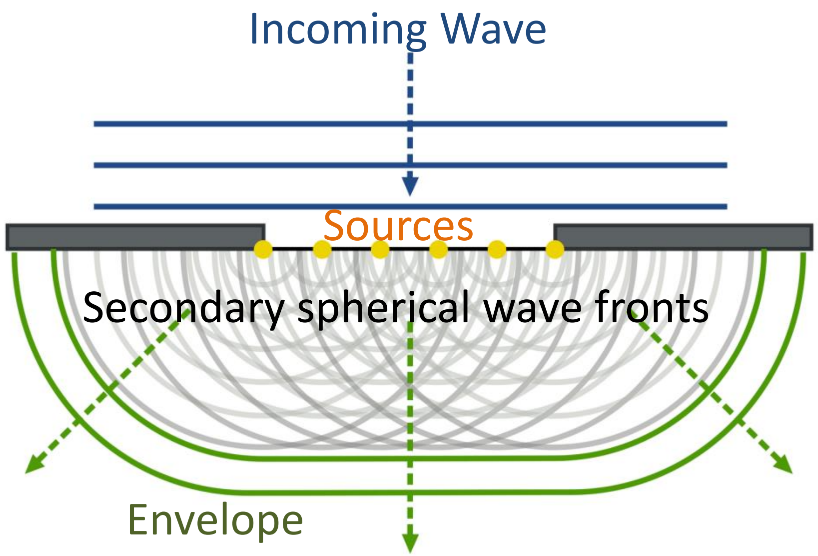


$$v = f \times \lambda$$

The speed of the wave is determined by the medium.

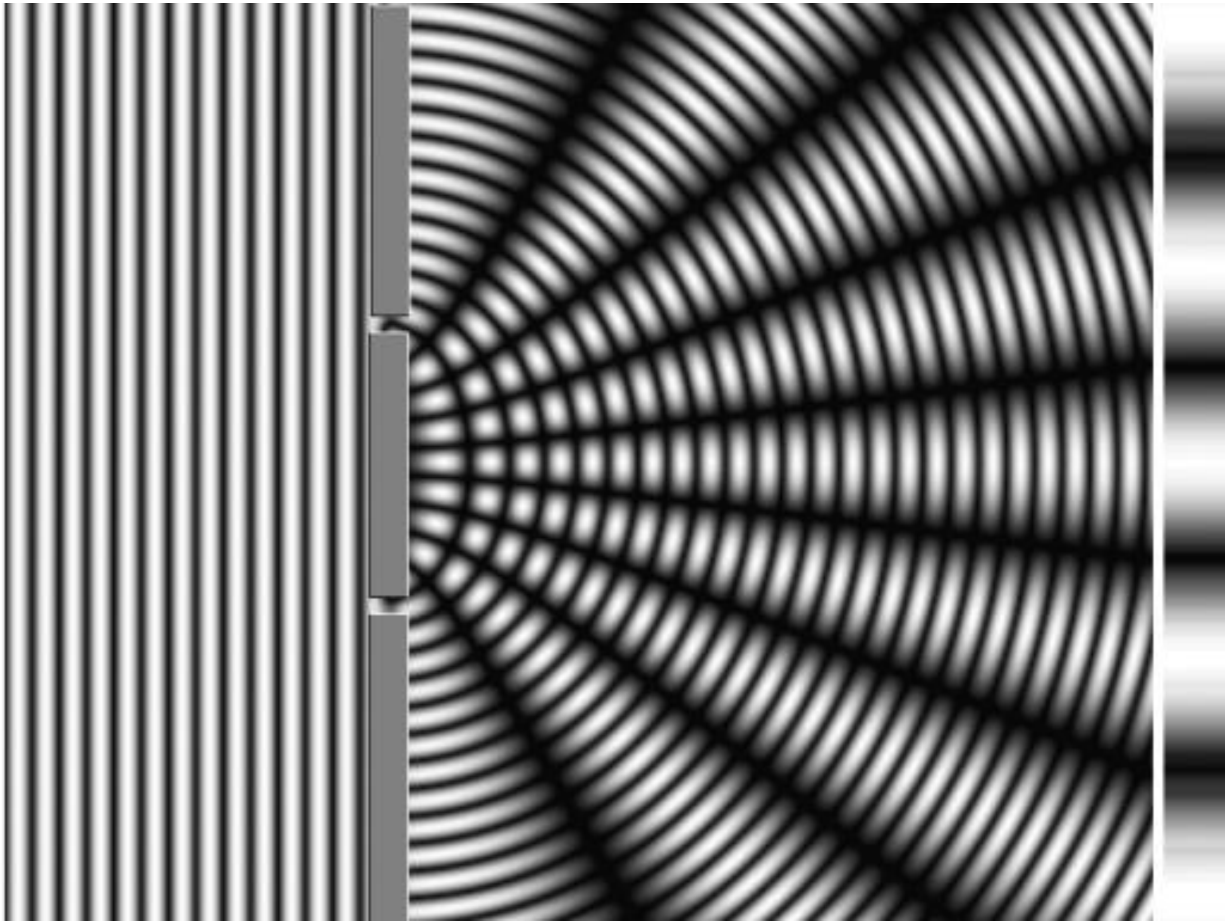
Waves Spread Out (Diffract)

Huygens' Principle

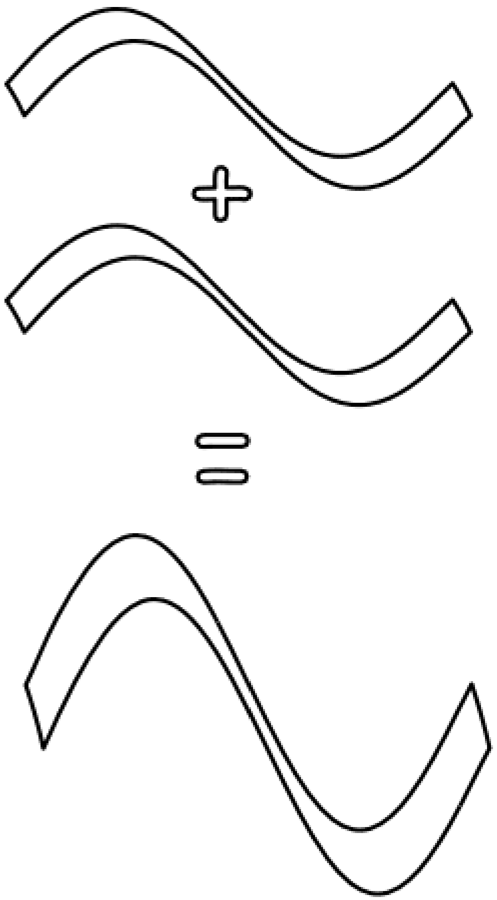


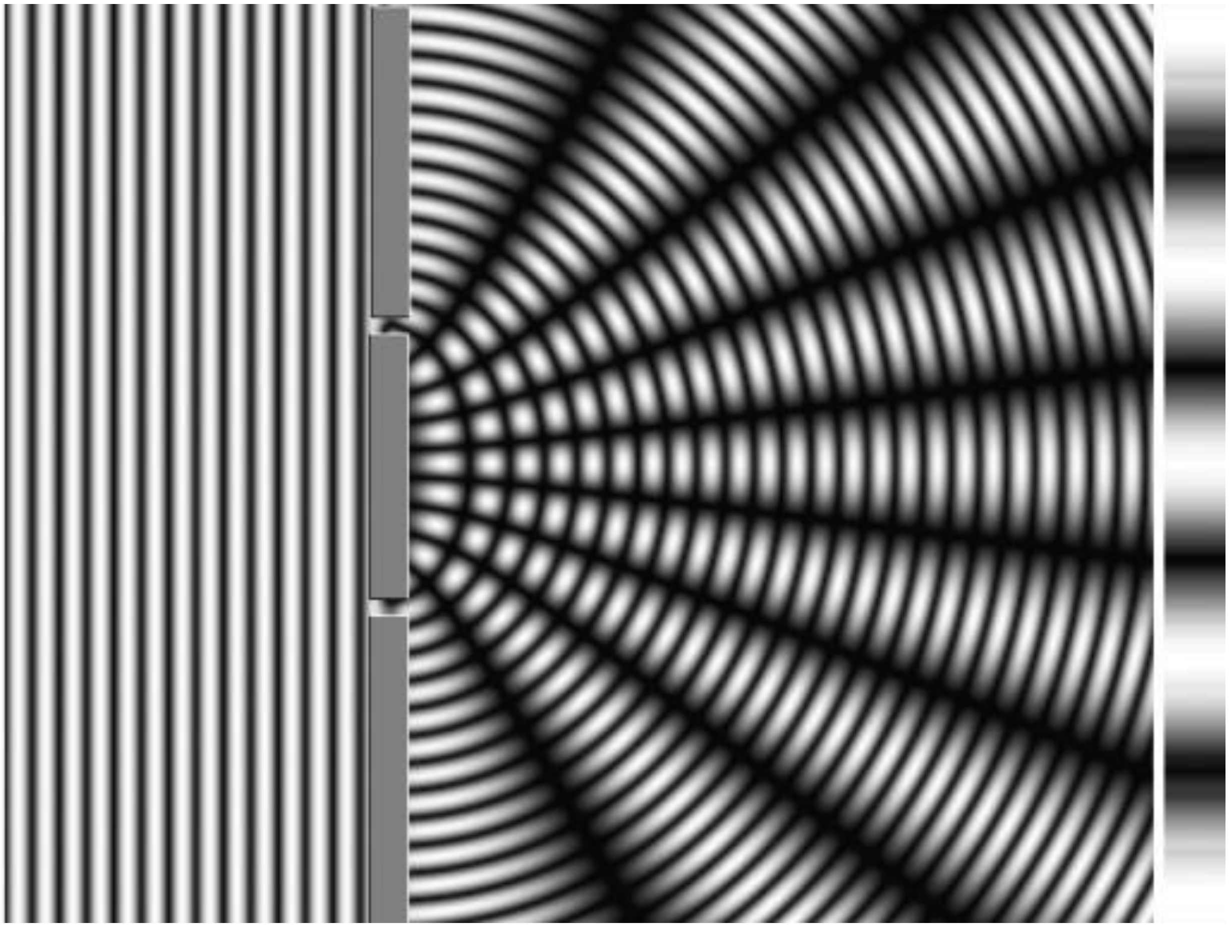
Each point disturbed by the advancing wave front can be viewed as a source of spherical waves. The new front is the envelope of these secondary spherical wave fronts.

WAVES Interfere



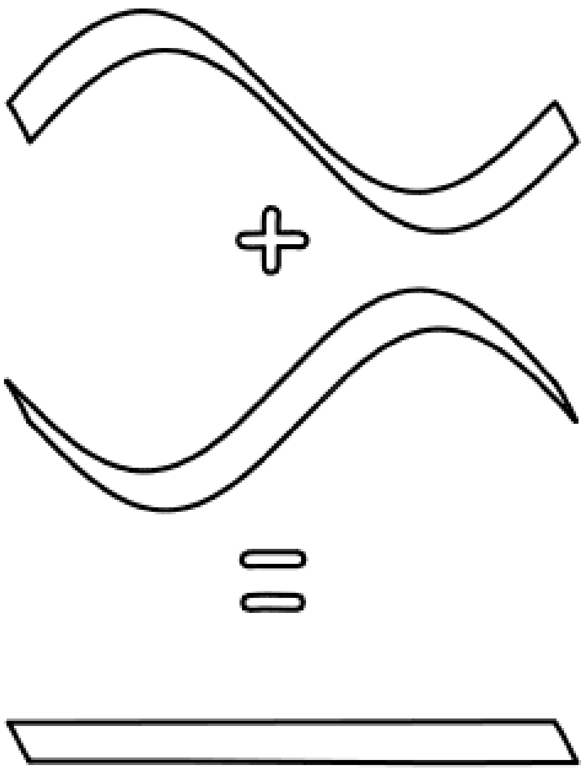
WAVES *Interfere*

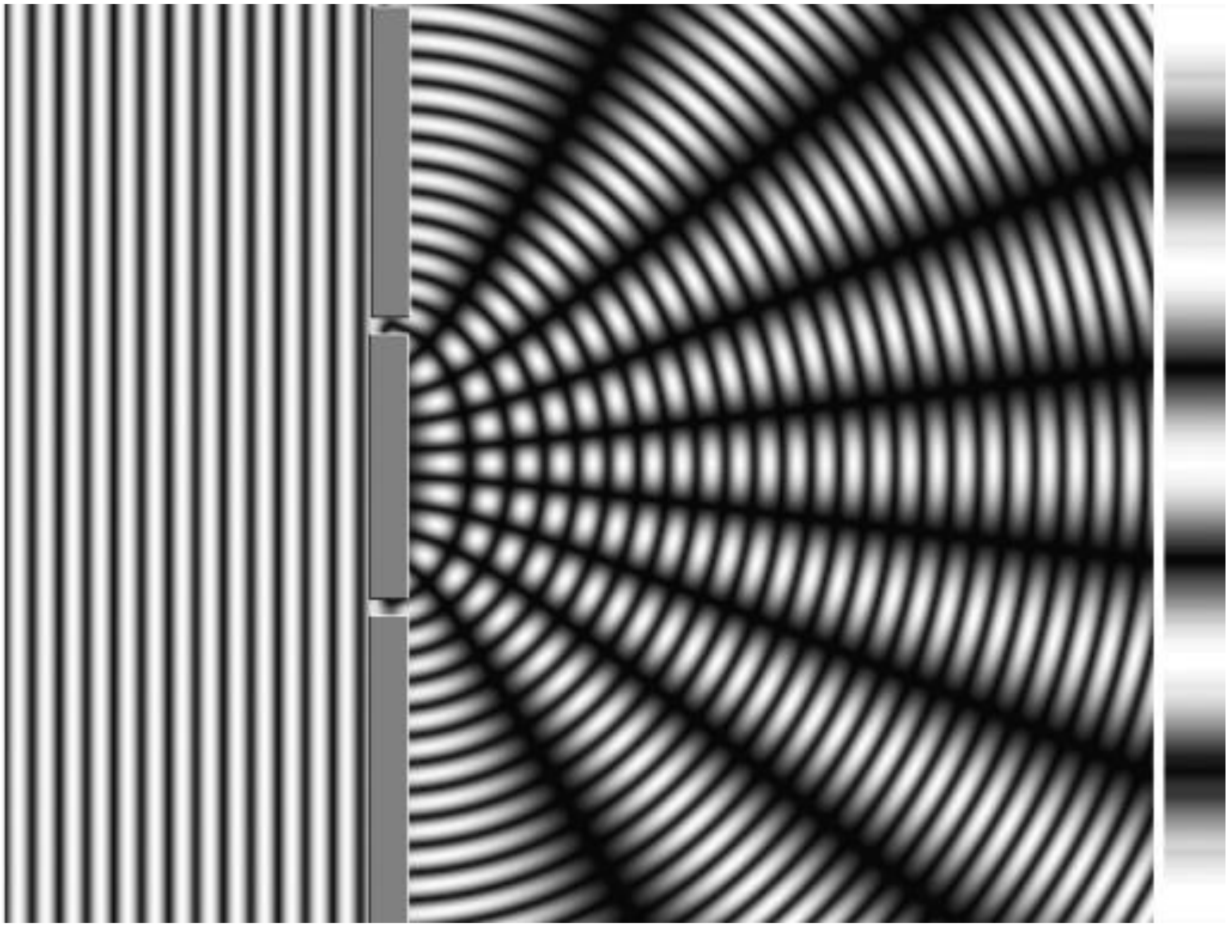




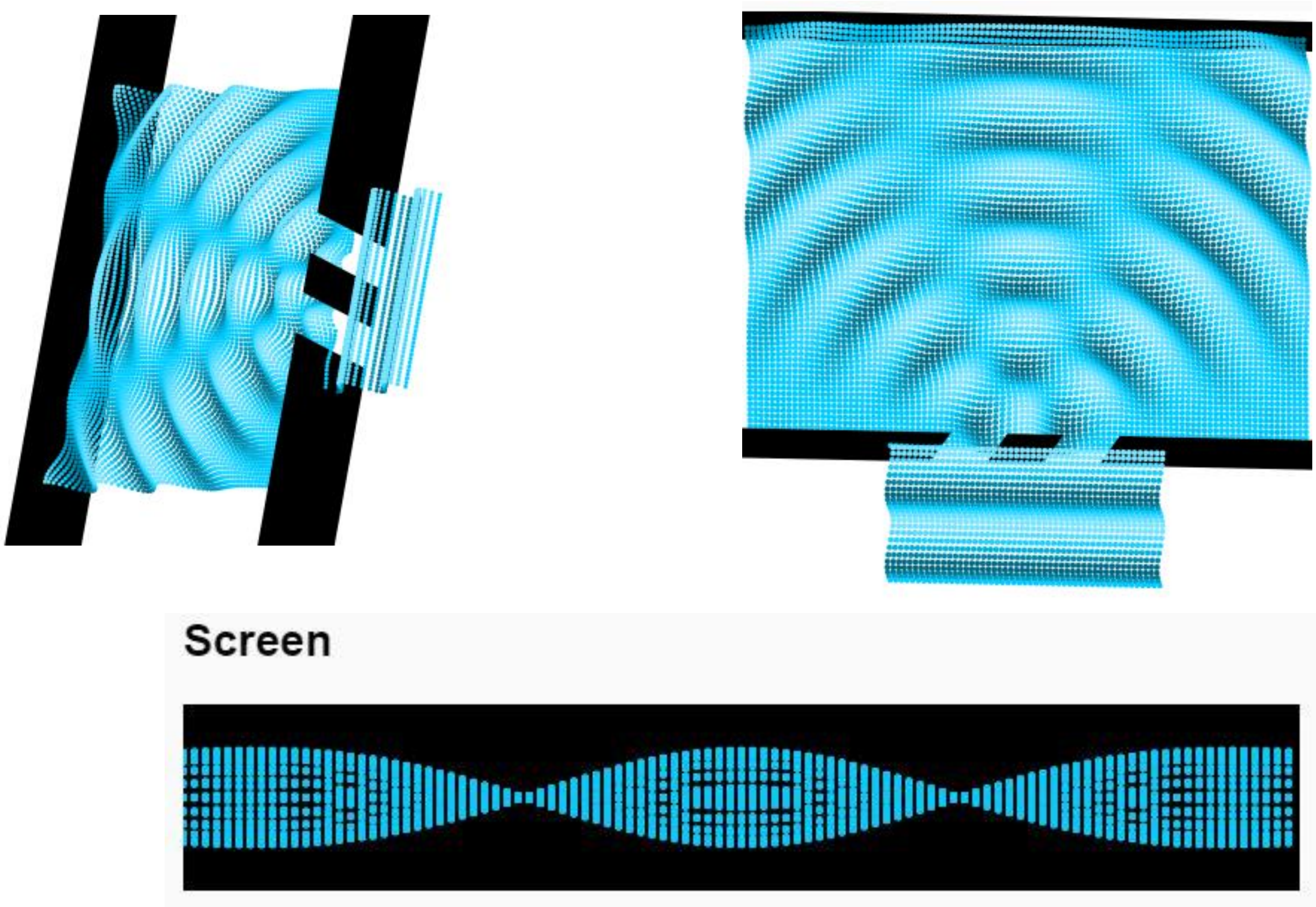
CONSTRUCTIVE

WAVES Interfere

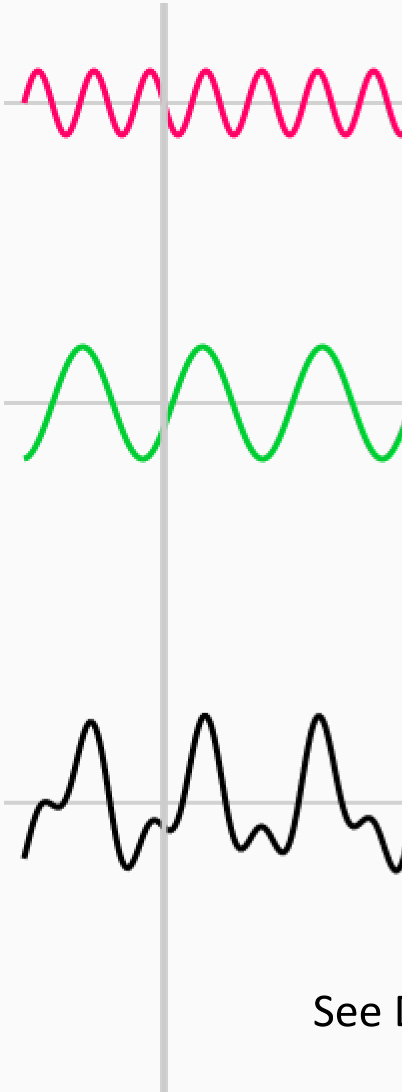




DESTRUCTIVE


Screen

Make your own waves and interference patterns!



Wave 1

Amplitude = 16 Wavelength = 28 Phase = 0°

Amplitude

Wavelength

Phase

Wave 2

Amplitude = 28 Wavelength = 60 Phase = 86°

Amplitude

Wavelength

Phase

Resultant Wave

Measure amplitude by sliding slider

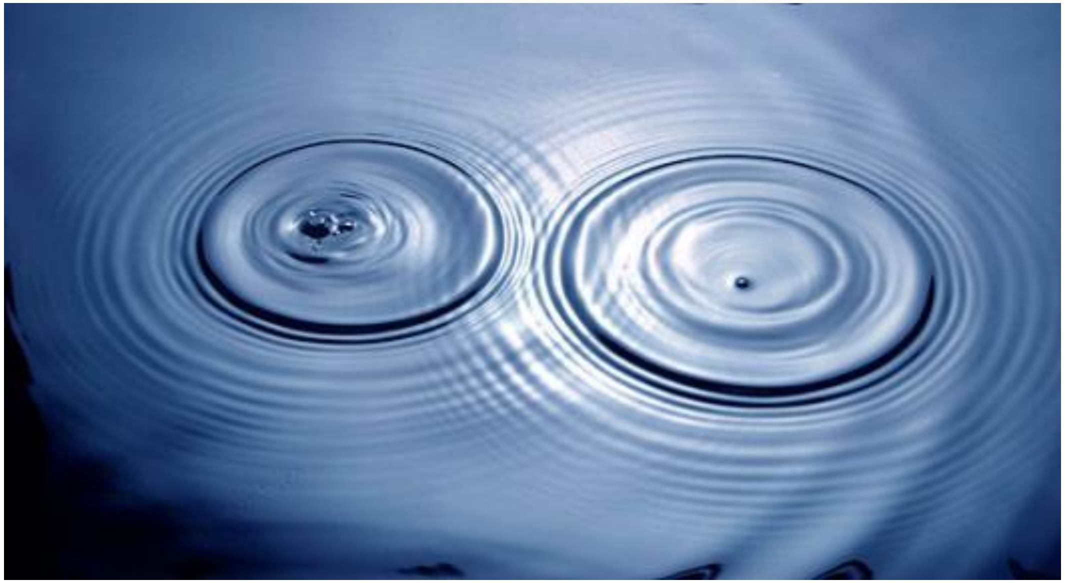
Wave 1 Amplitude = 0
Wave 2 Amplitude = -12.3
Resultant Amplitude =-12.3

See Demo folder in D2L!

Wave Model of Nature

What is a wave?

- Non-localized
- Spread out
- Able to interfere and pass through other waves

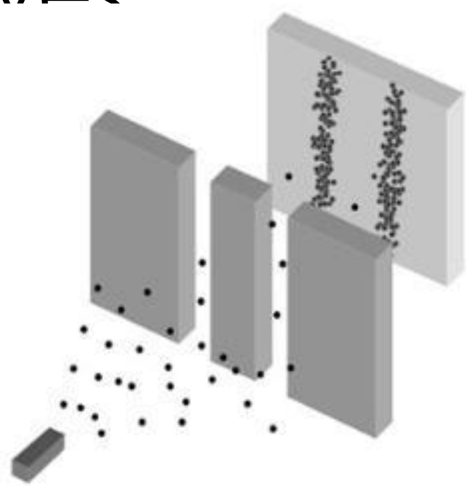


Summary

Particles

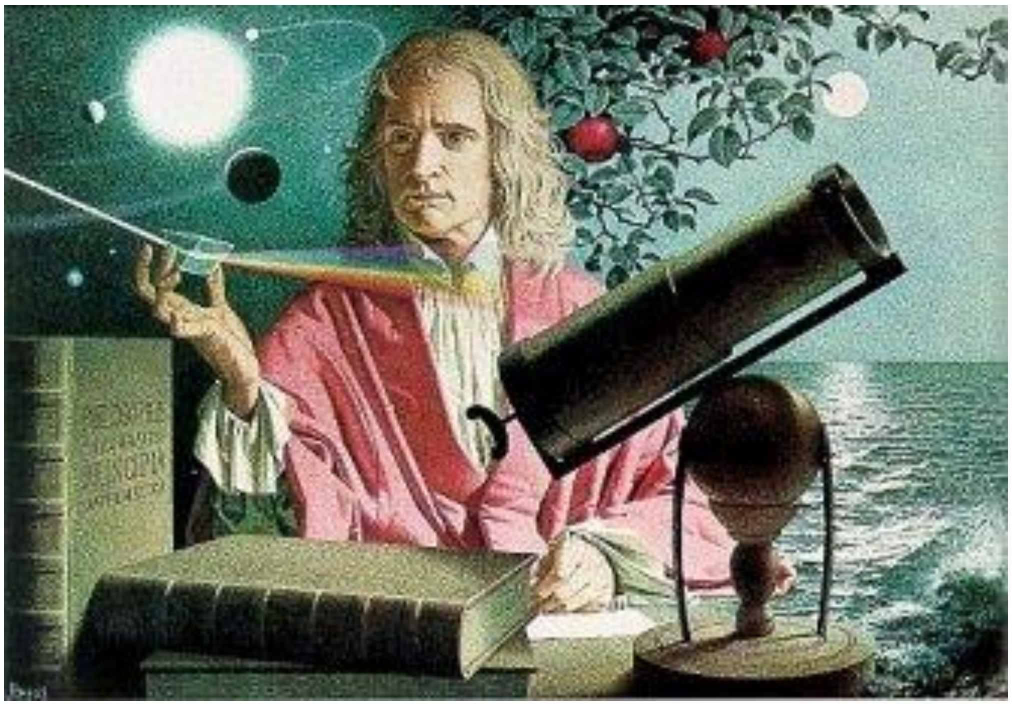
- Localized
- One place at a time
- Can bounce off other particles

Waves



- Non-Localized
- Spread out
- Can pass through other waves

NEWTON'S MODEL - *Light is a Particle*

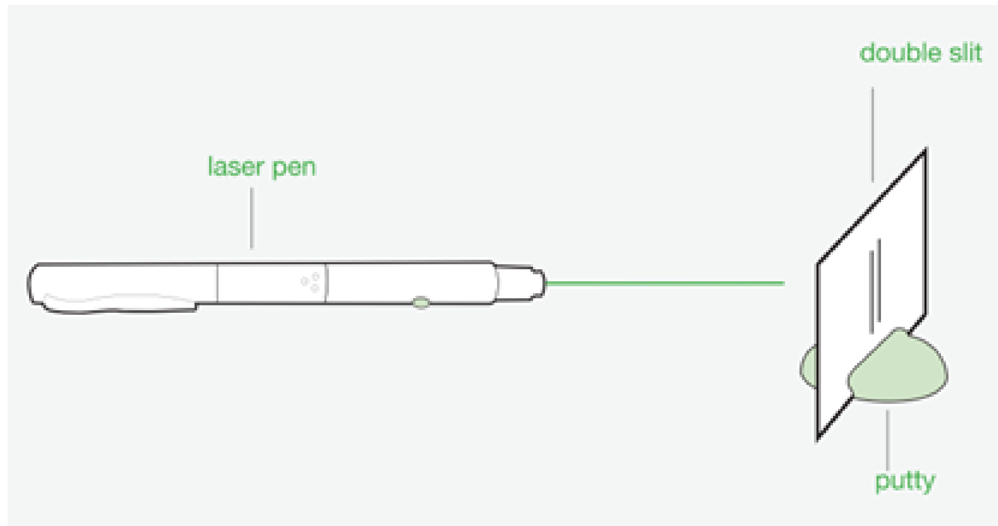



Google earth

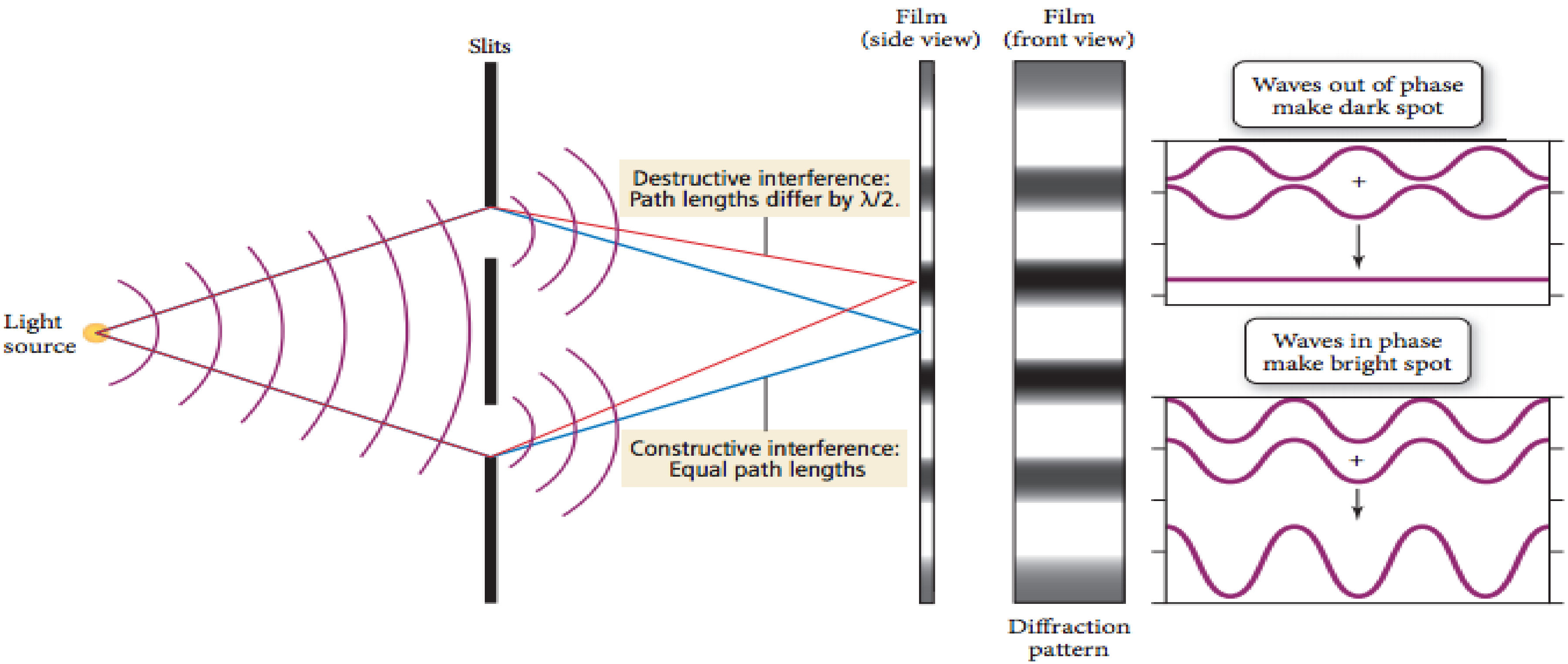
HUYGEN'S MODEL - Light is a Wave

What is light? A particle or a wave?

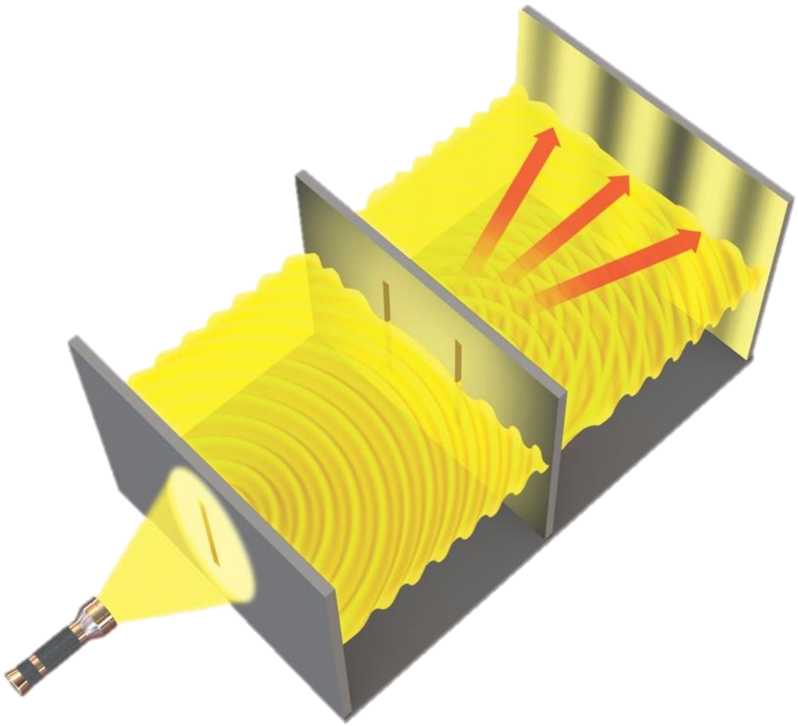
Sketch your Prediction



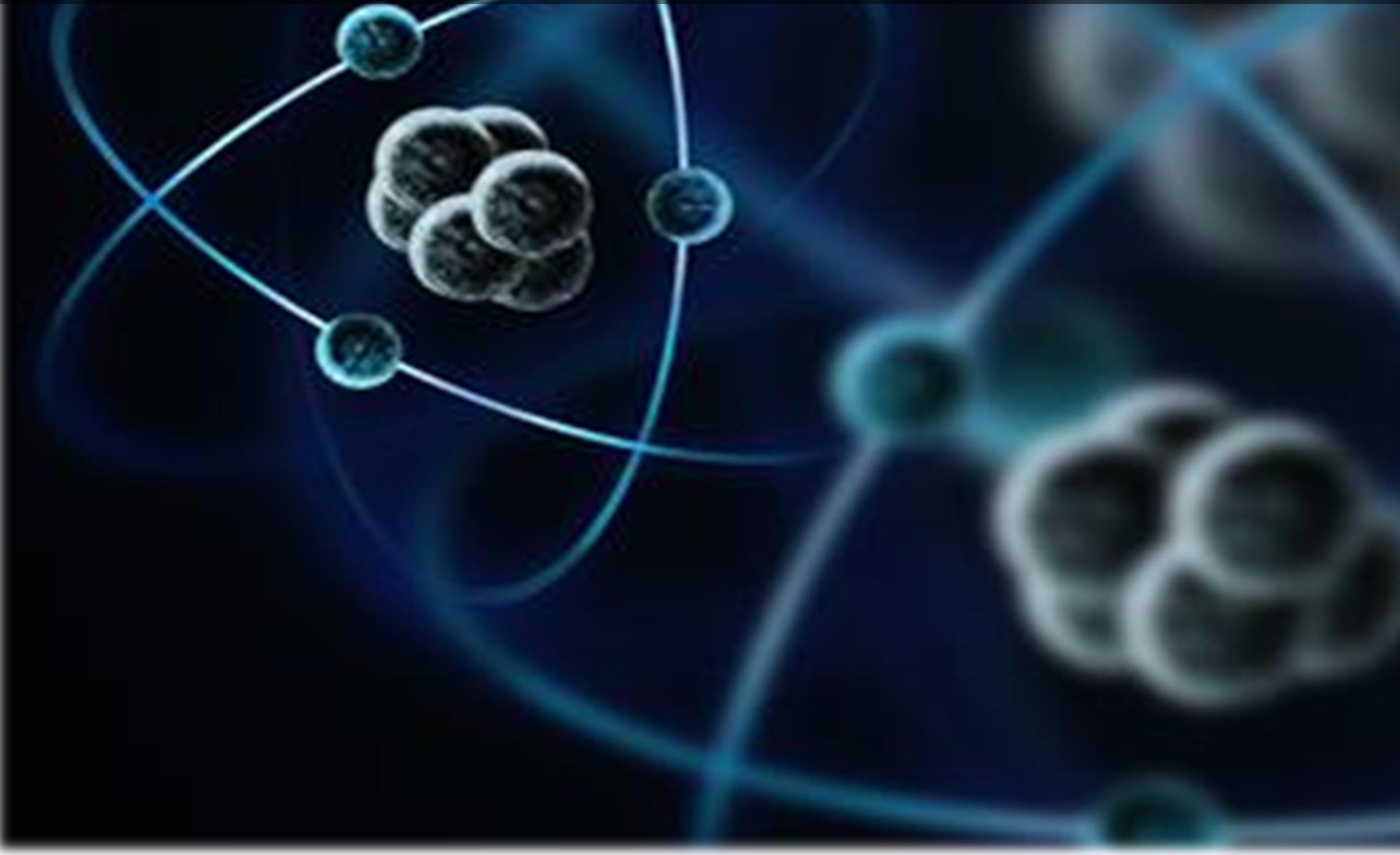
Light Seems like a Wave



YOUNG'S EXPERIMENT- Light is a Wave



ELECTRONS



Electrons

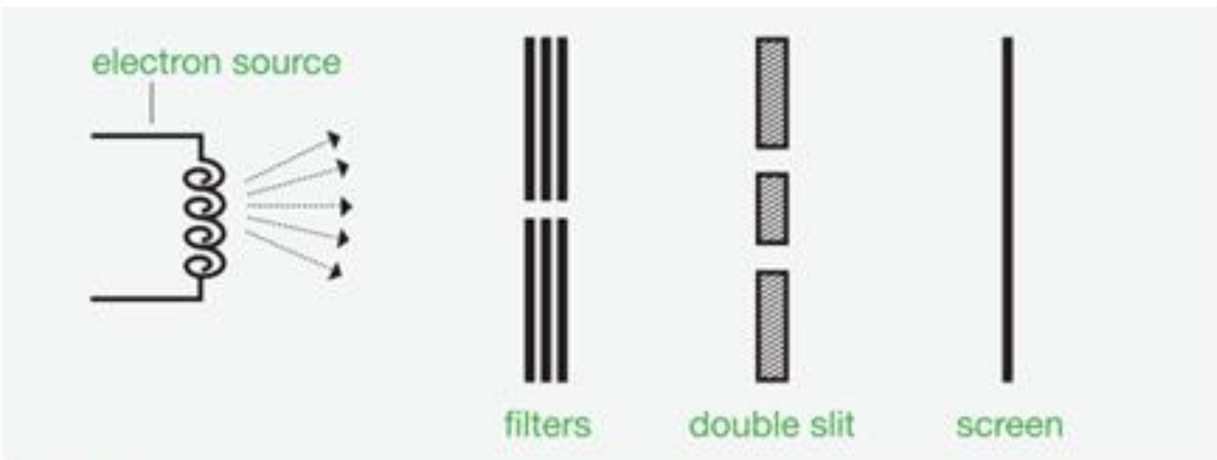
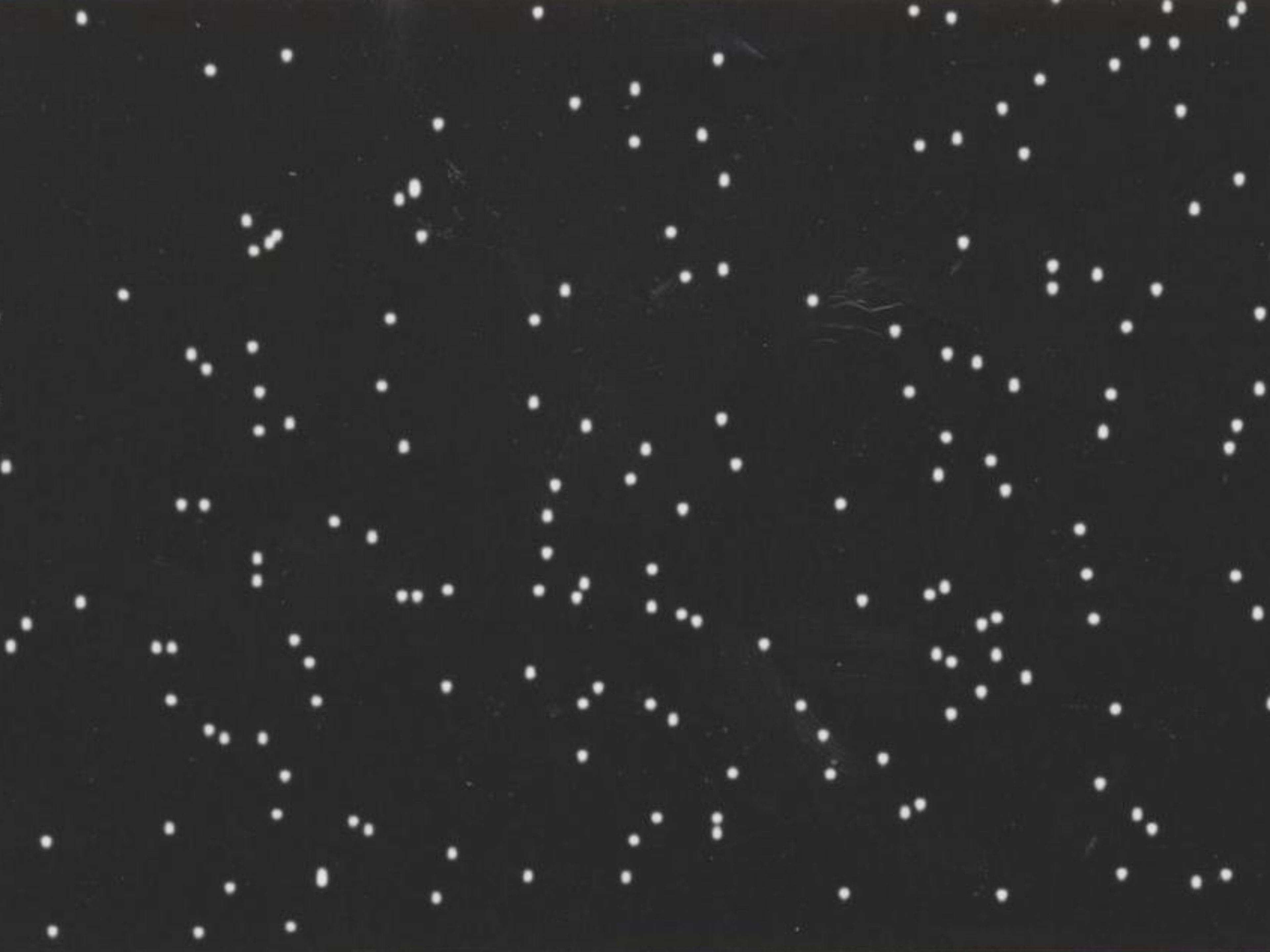
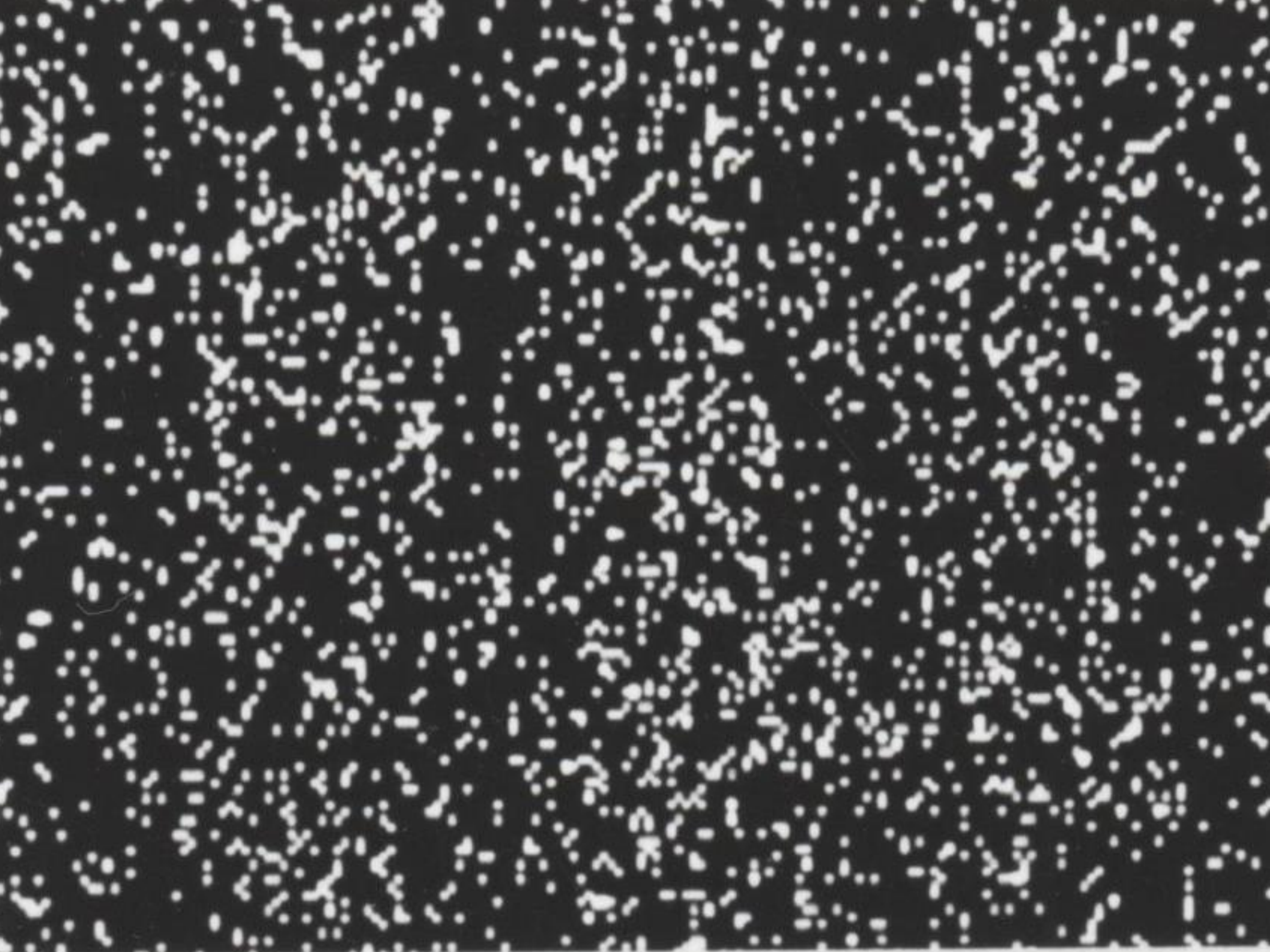


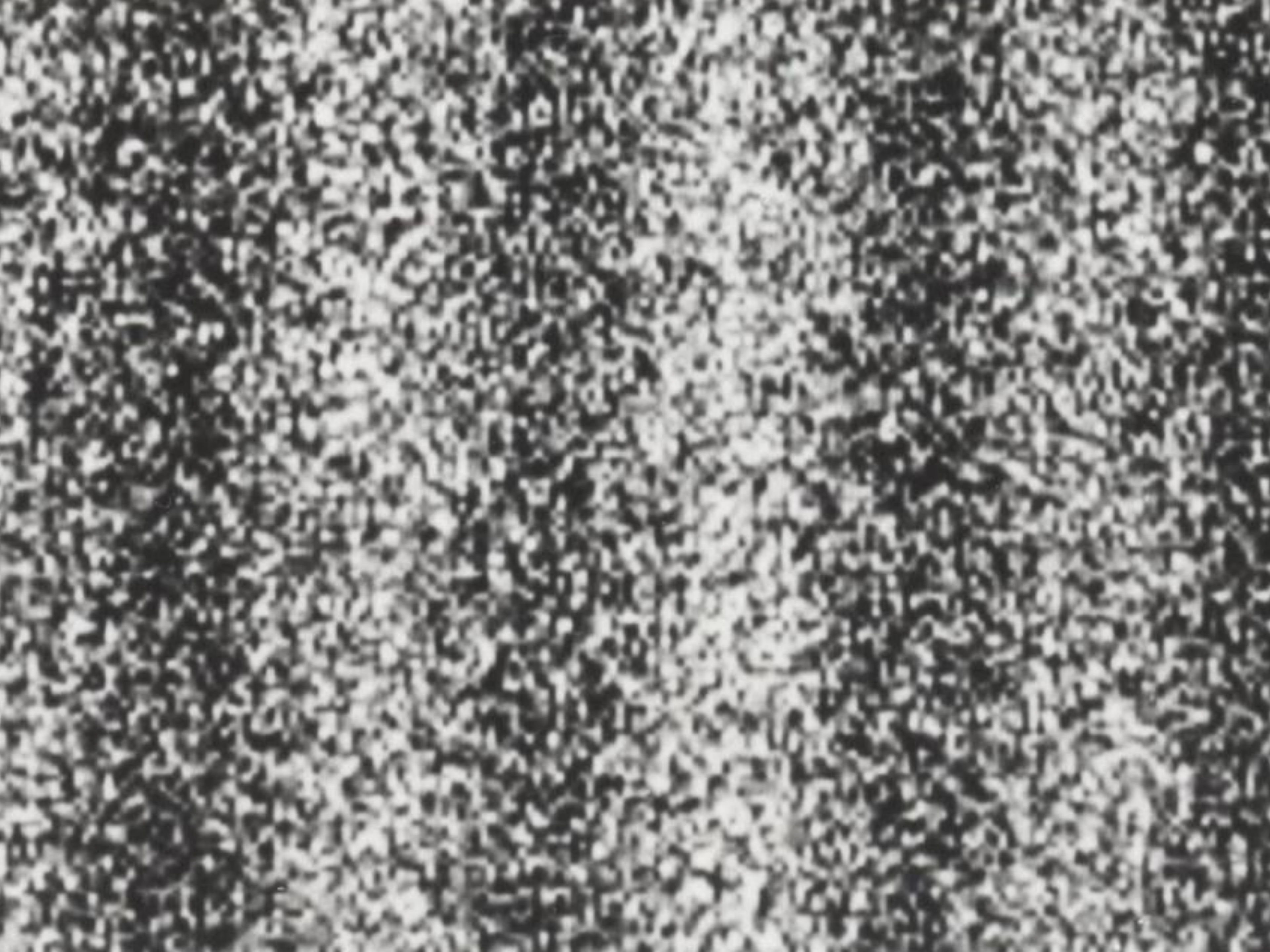
Figure 3 Experiment set-up (not to scale)

Electron 2-slit experiment:

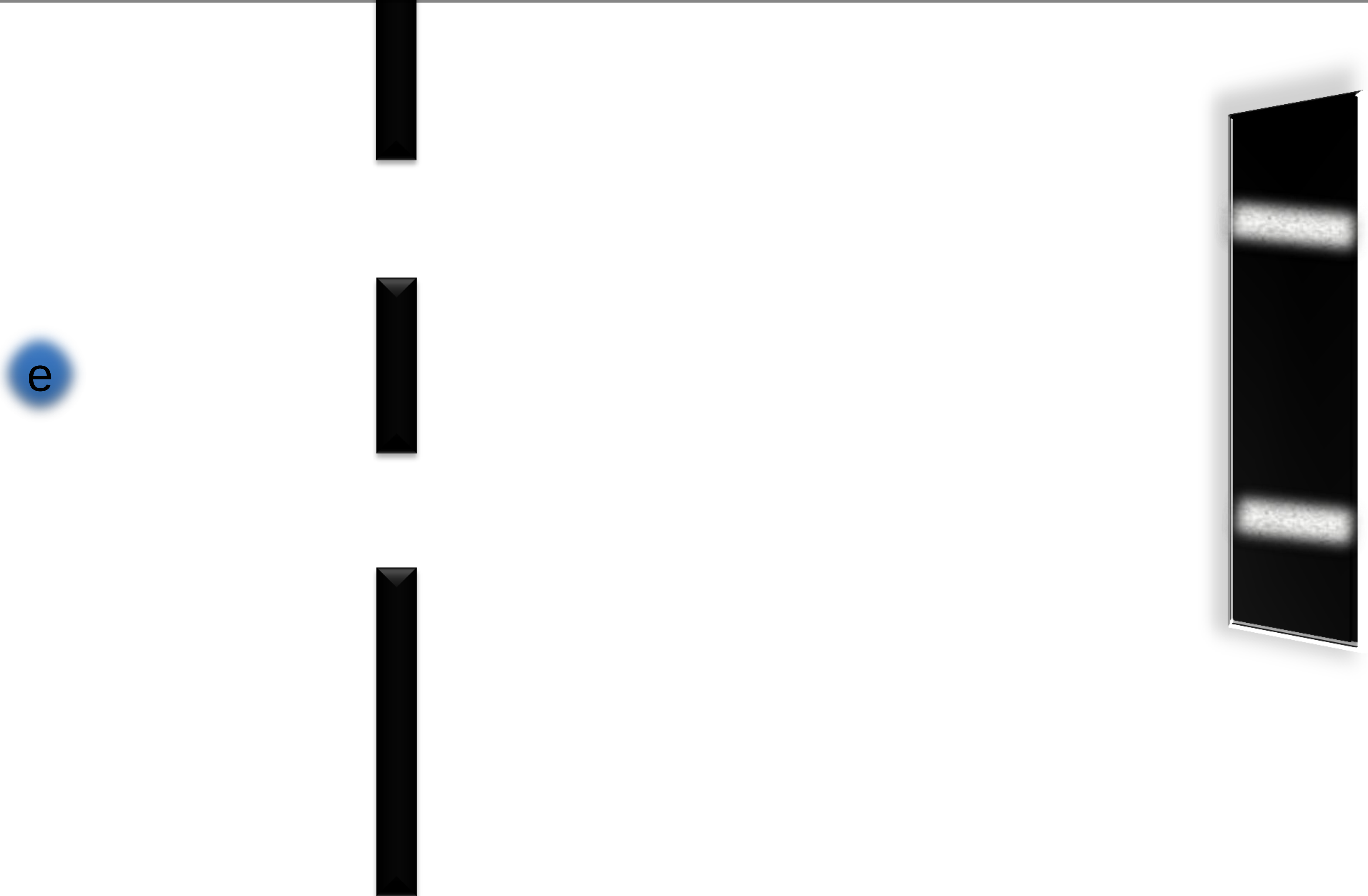
From: Dr. B. King



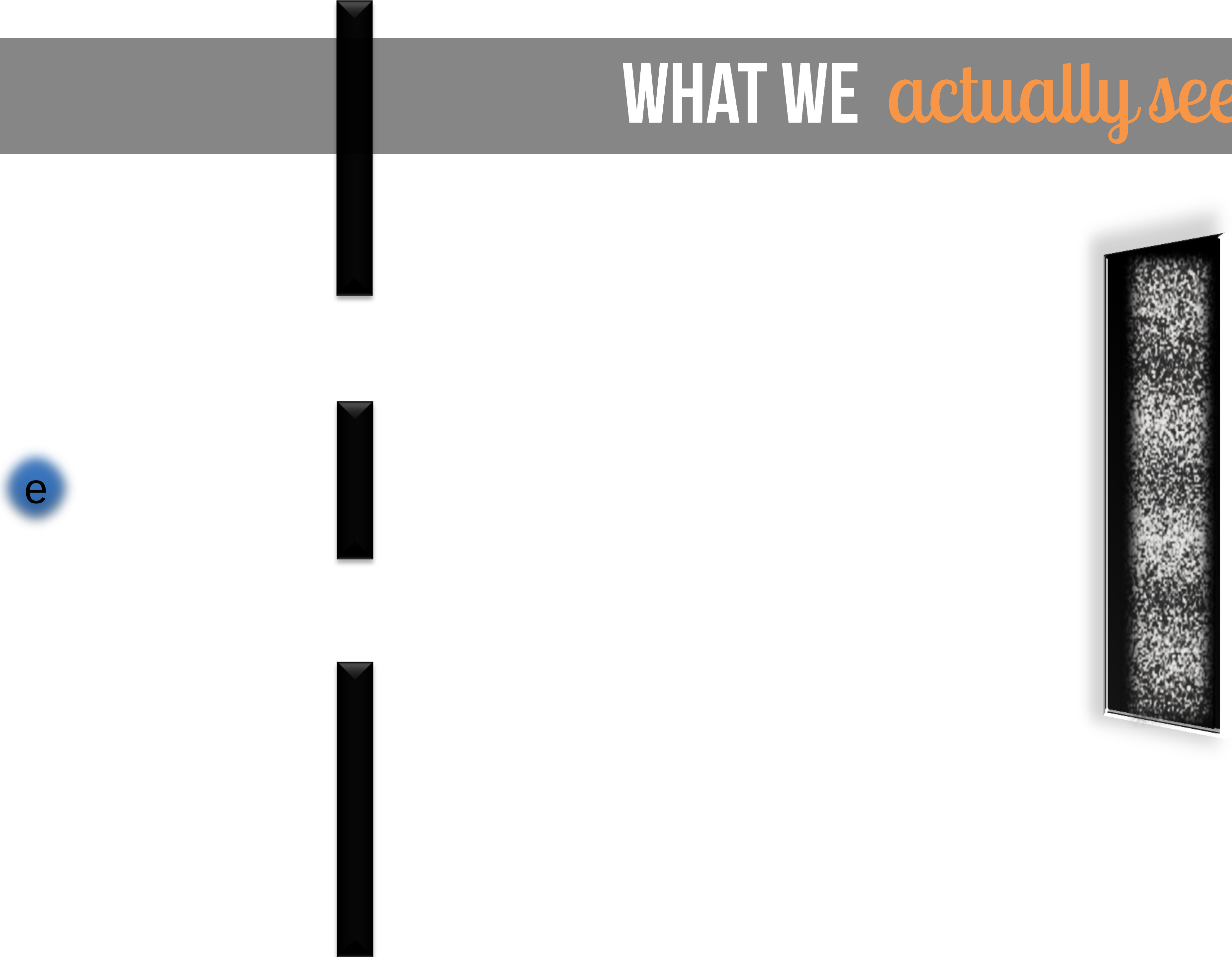




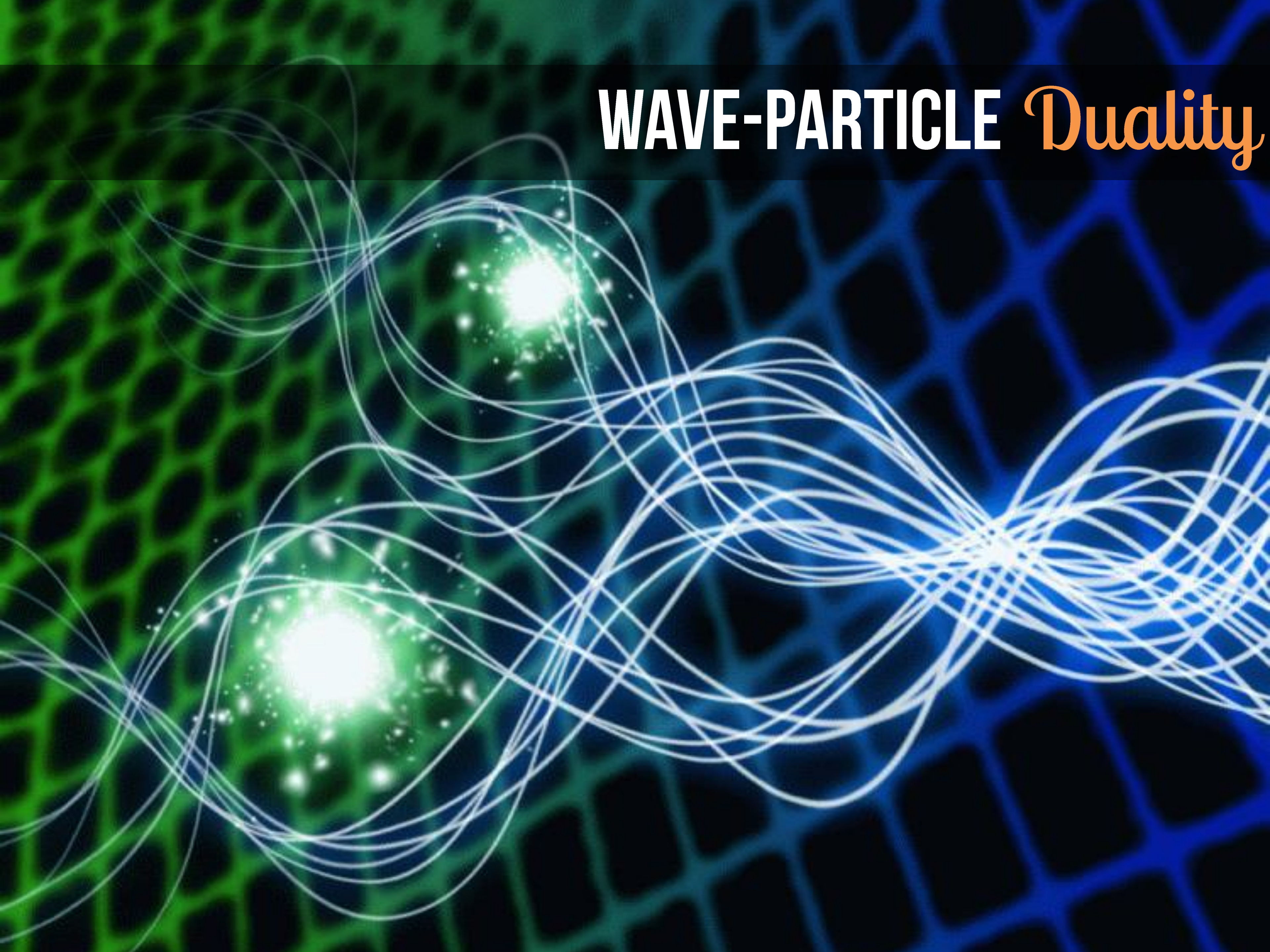
WHAT WE *expected*



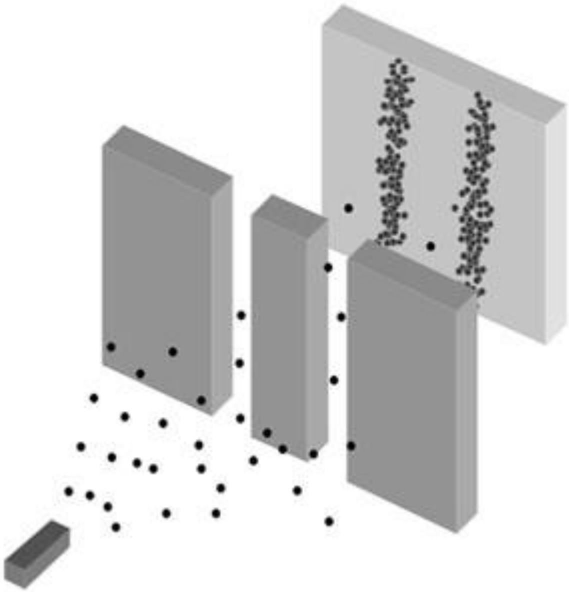
WHAT WE actually see



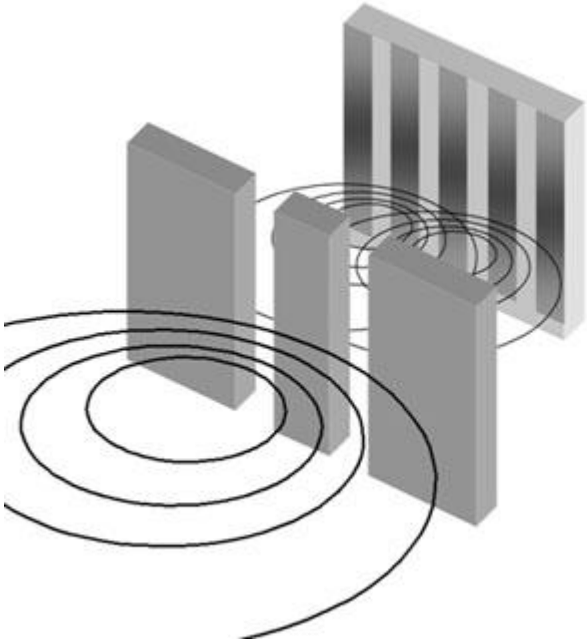
WAVE-PARTICLE Duality



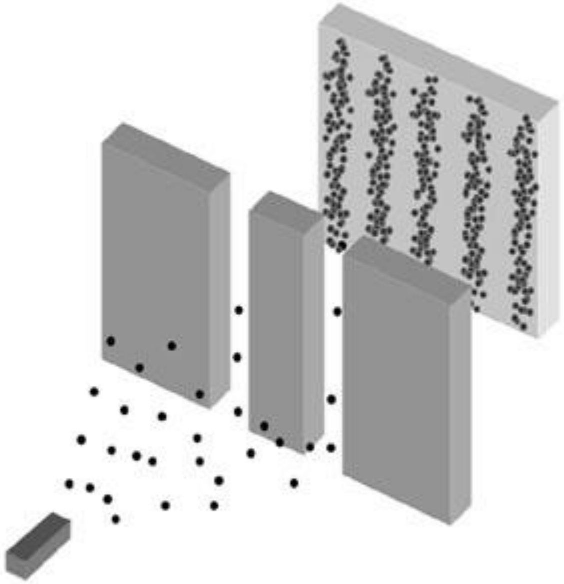
Particles



Waves



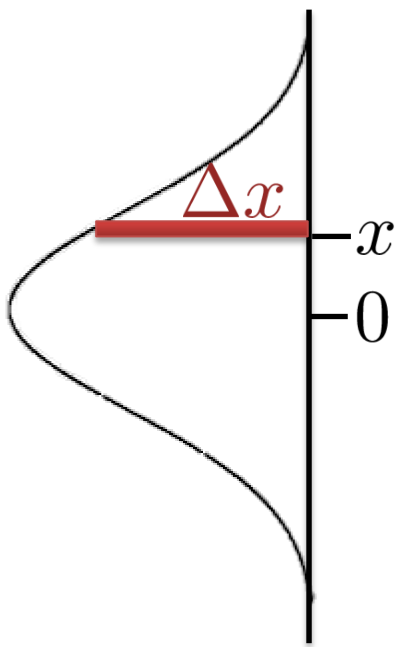
Electrons



The Beginning of the End

- Prior to the late 19^{th} century, we viewed the universe as:
  - divisible into particles (matter) and waves (light)
  - smooth on small scales (no structure on scales smaller than atoms)
  - deterministic, so that if you know the state of a system *now*, you can know its state at any time by using the laws of kinematics and dynamics

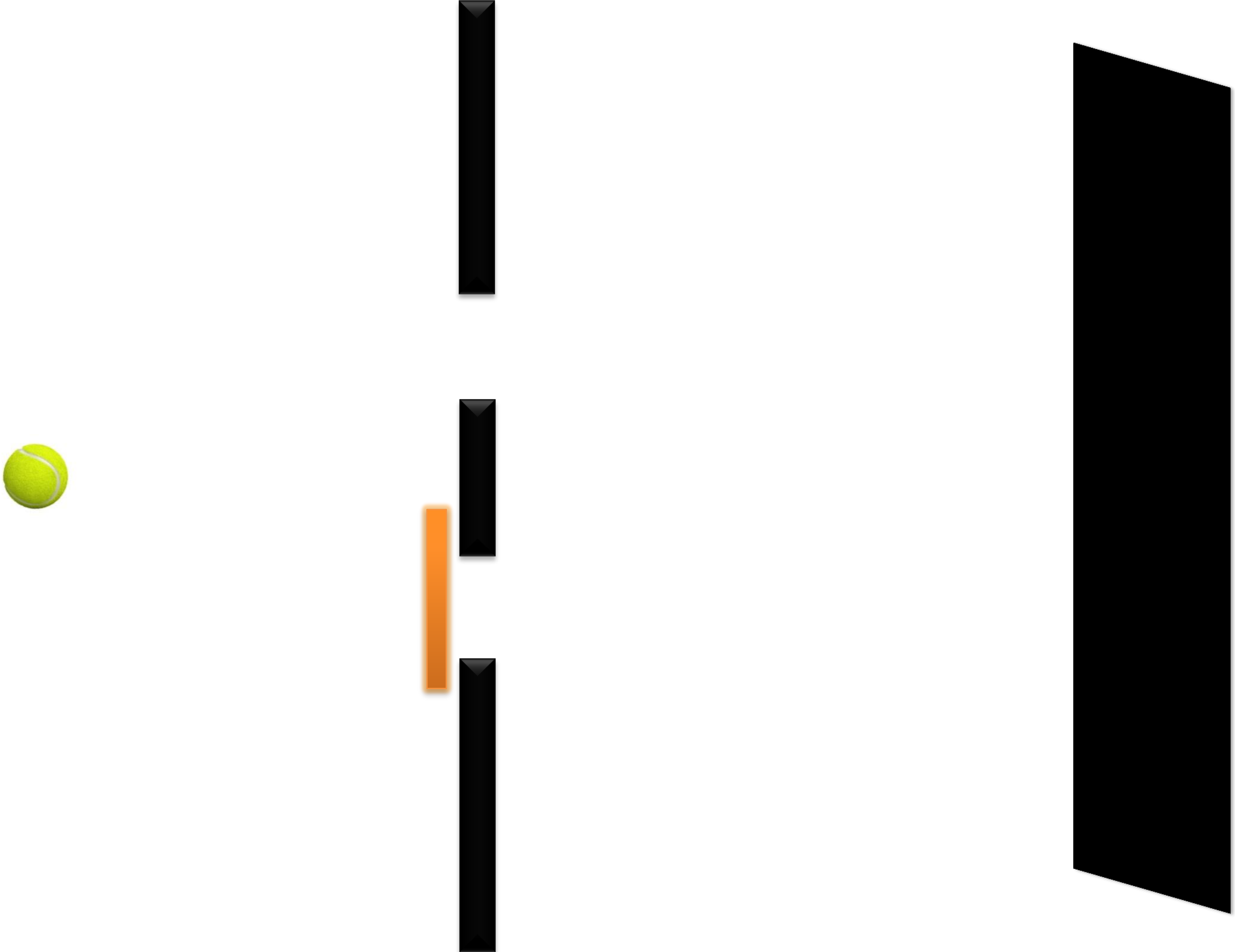
Probability

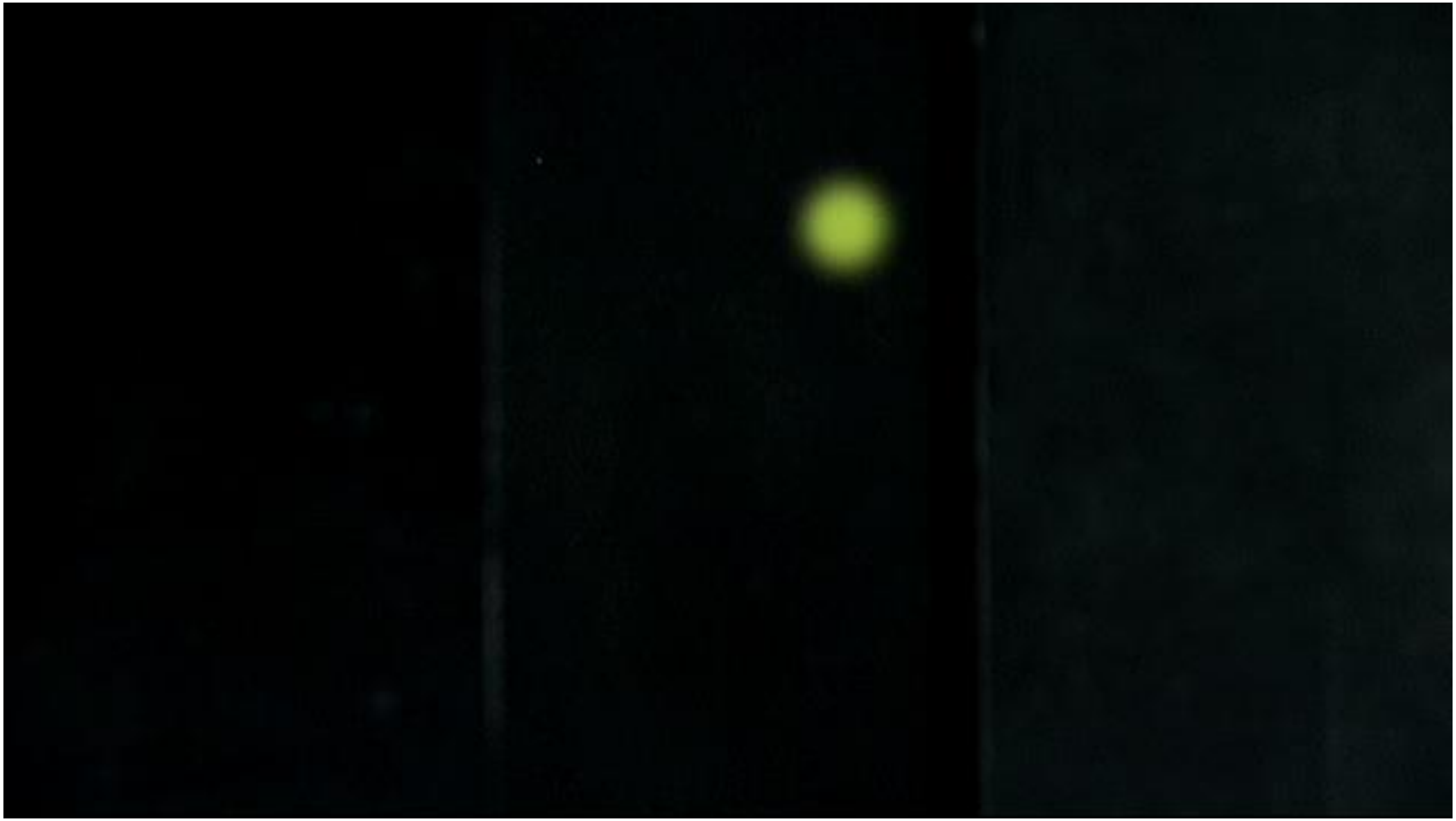


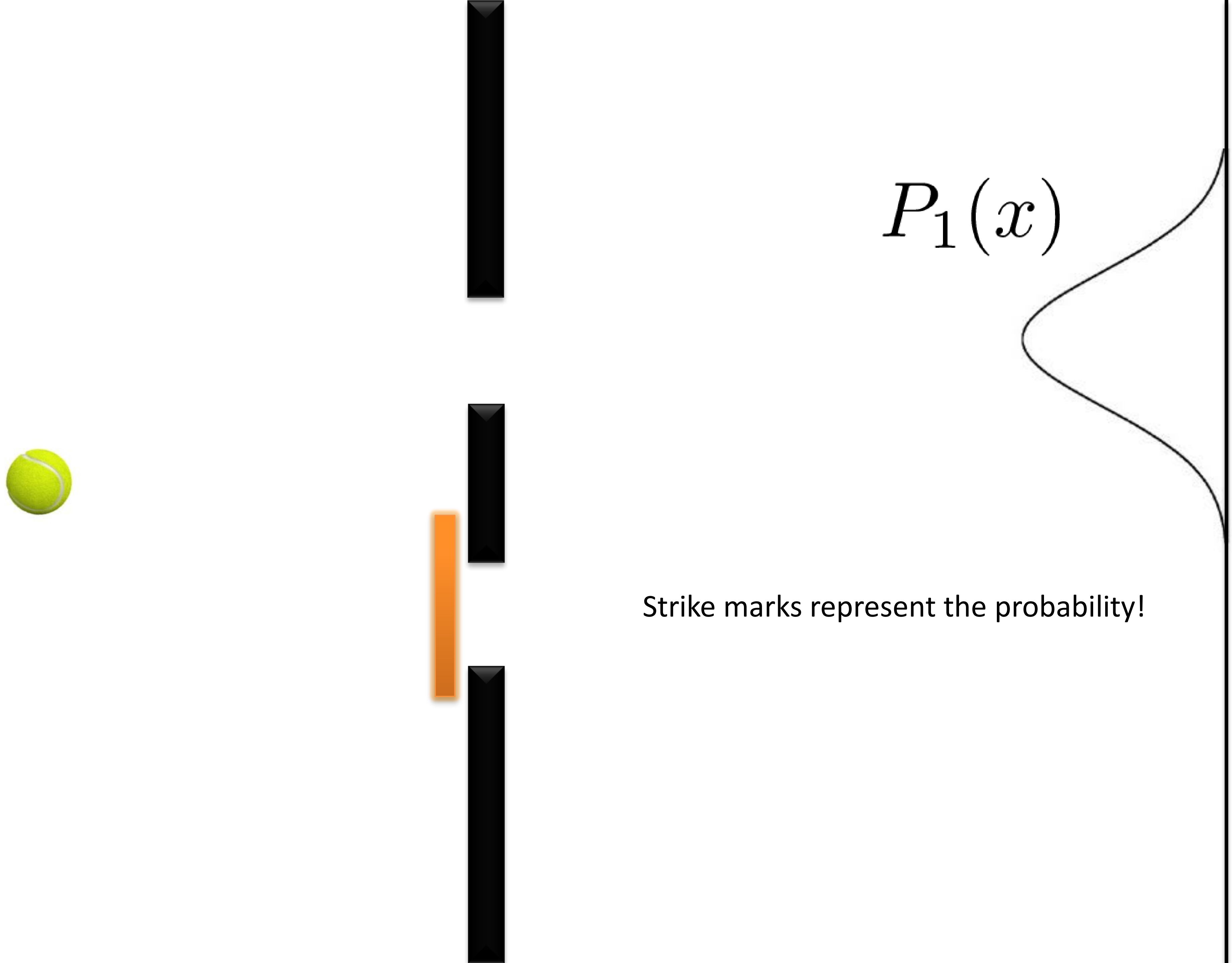
$$P(x)\Delta x$$

Probability a particle will hit a region with width Δx centered at pt. x

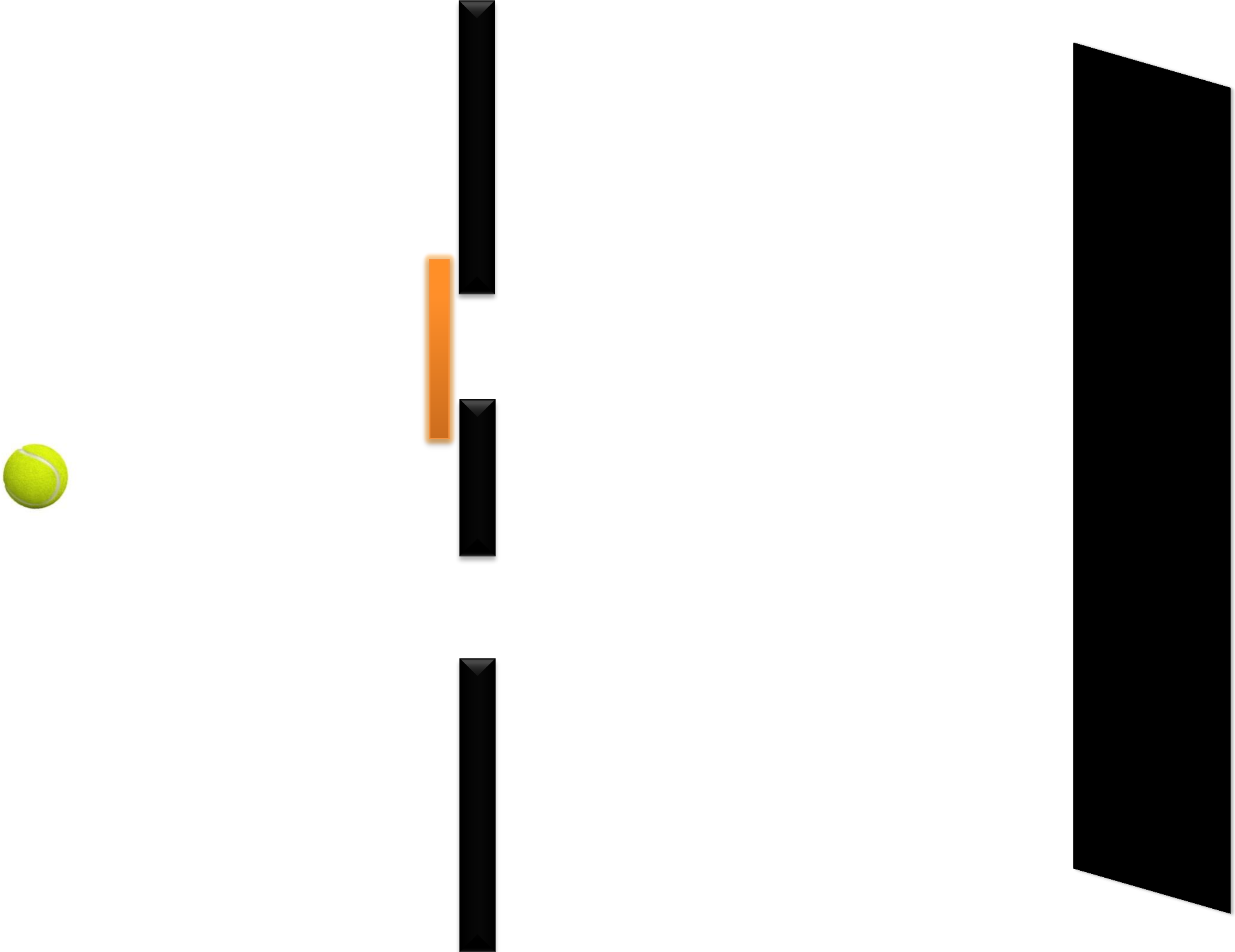
The sum of all such rectangles is the total area under the curve P(x). It represents the total probability that any given particle will hit somewhere on the screen, which must be 1 (100%).

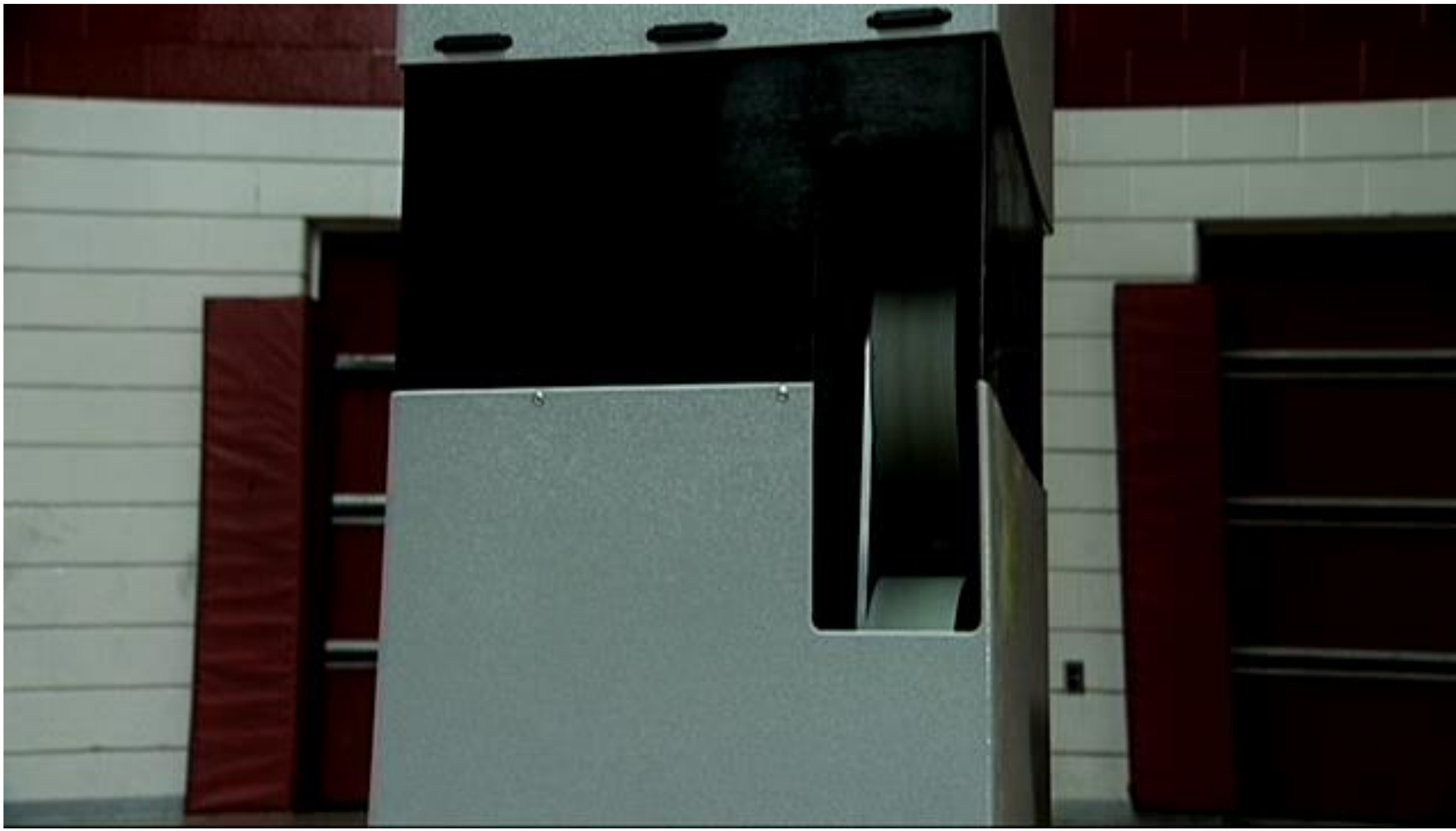


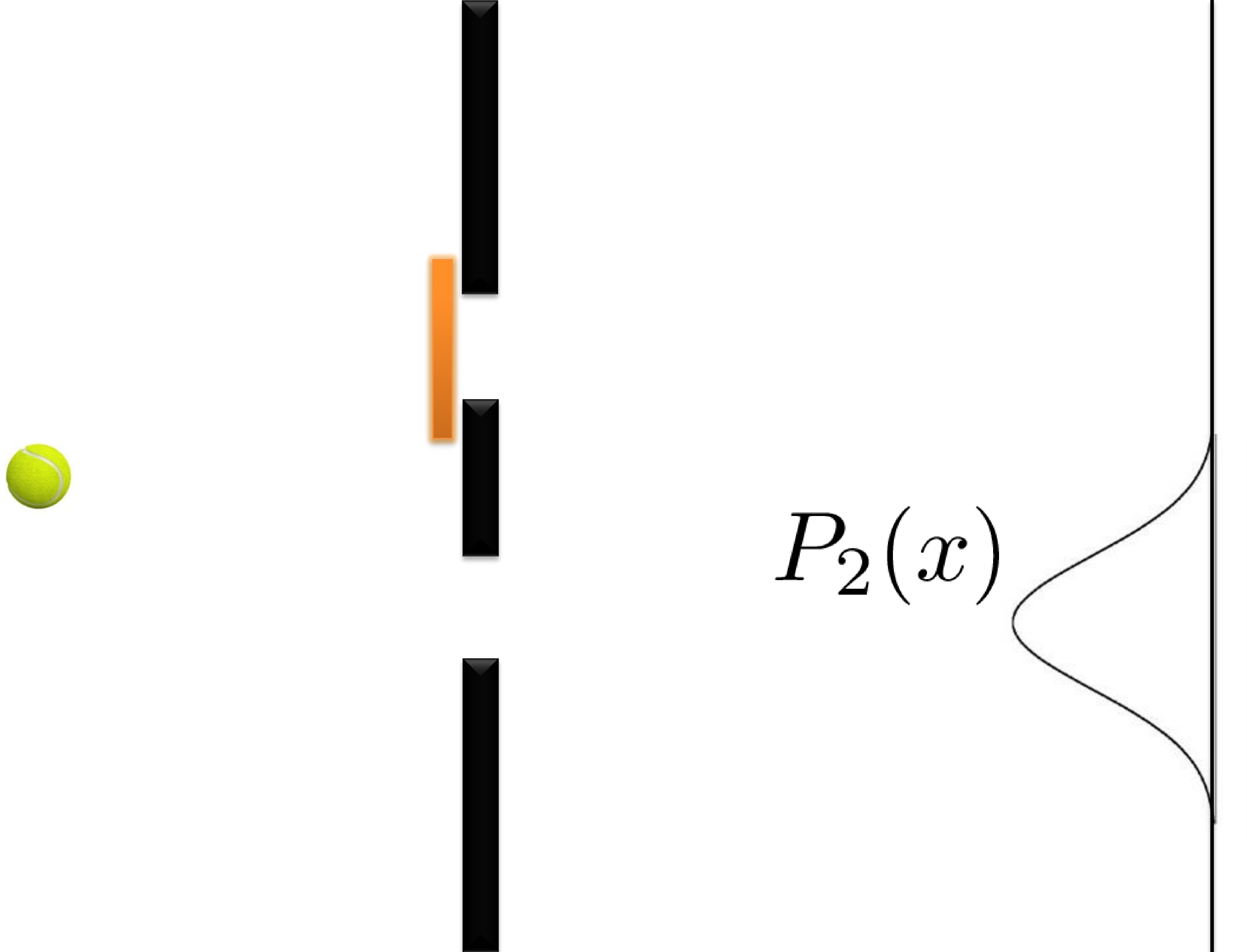




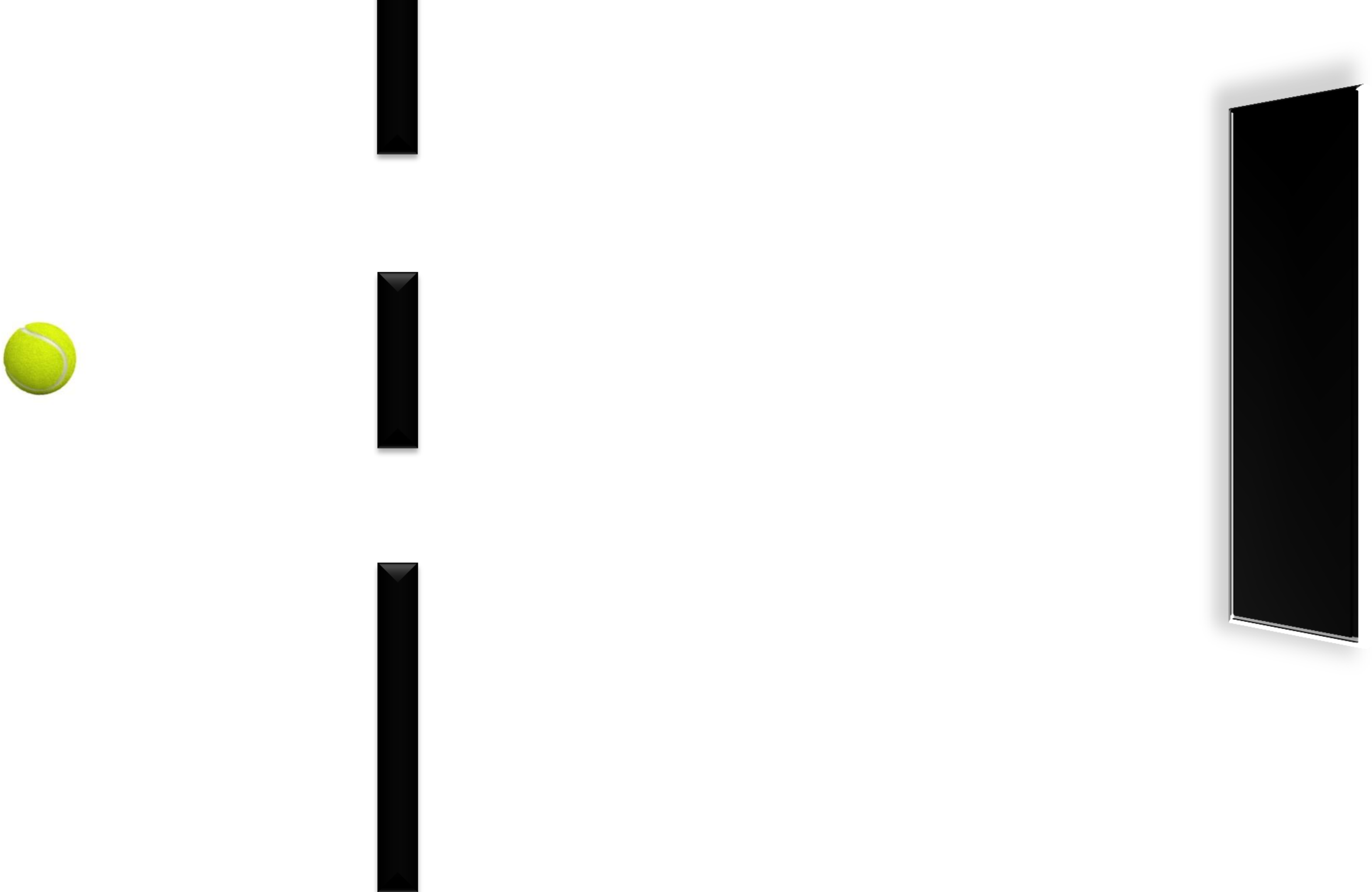
Strike marks represent the probability!



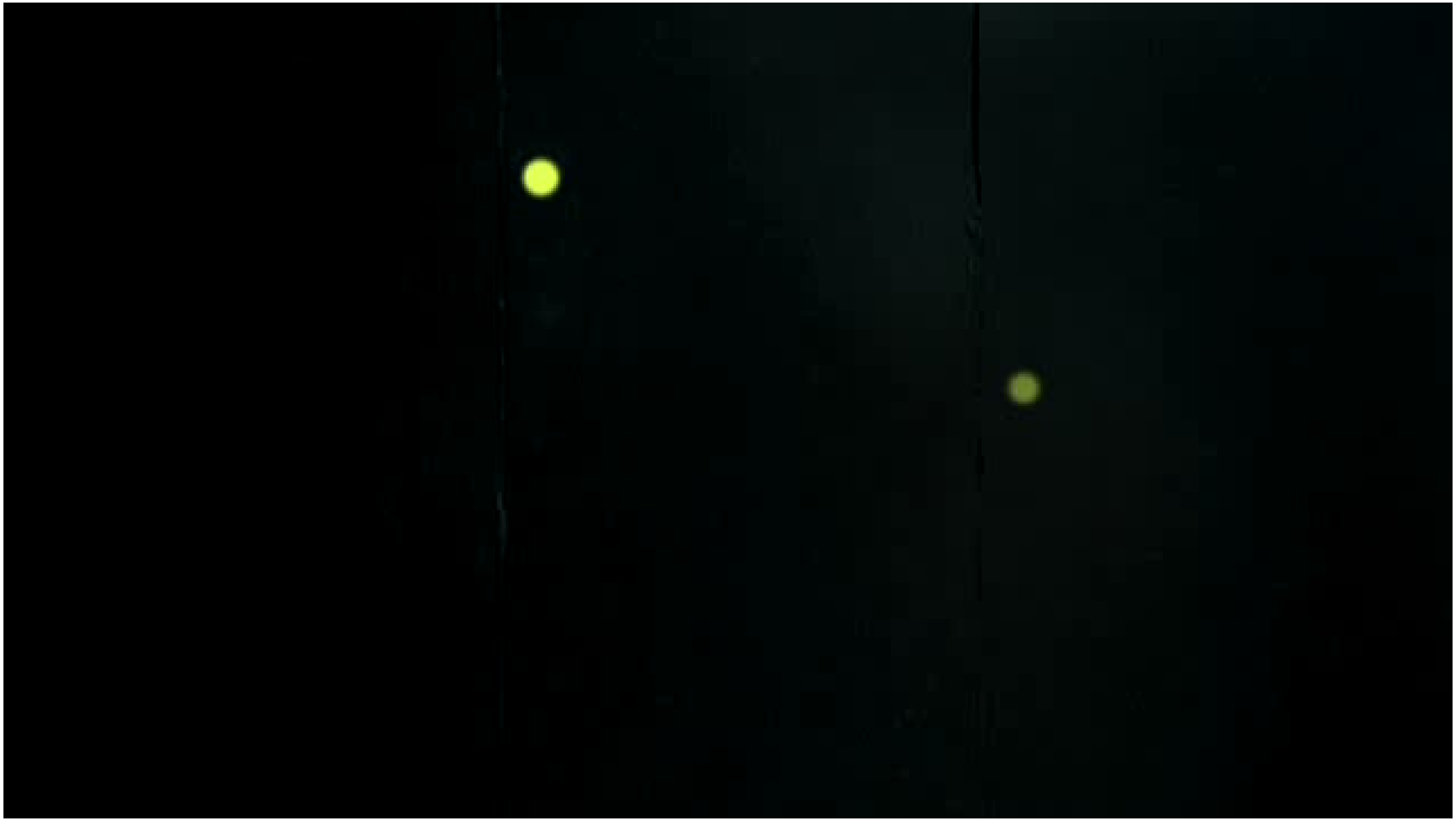


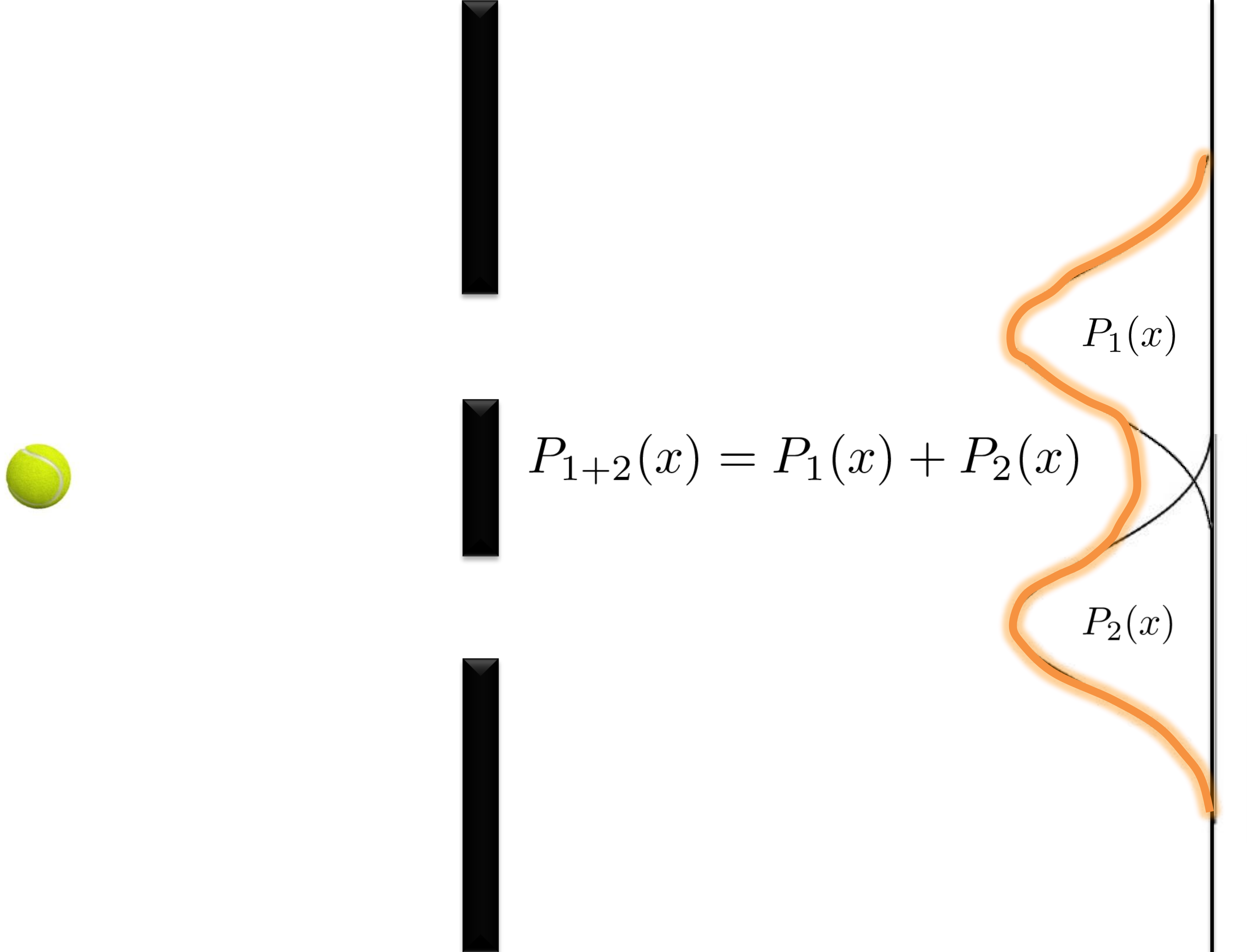

$P_2(x)$

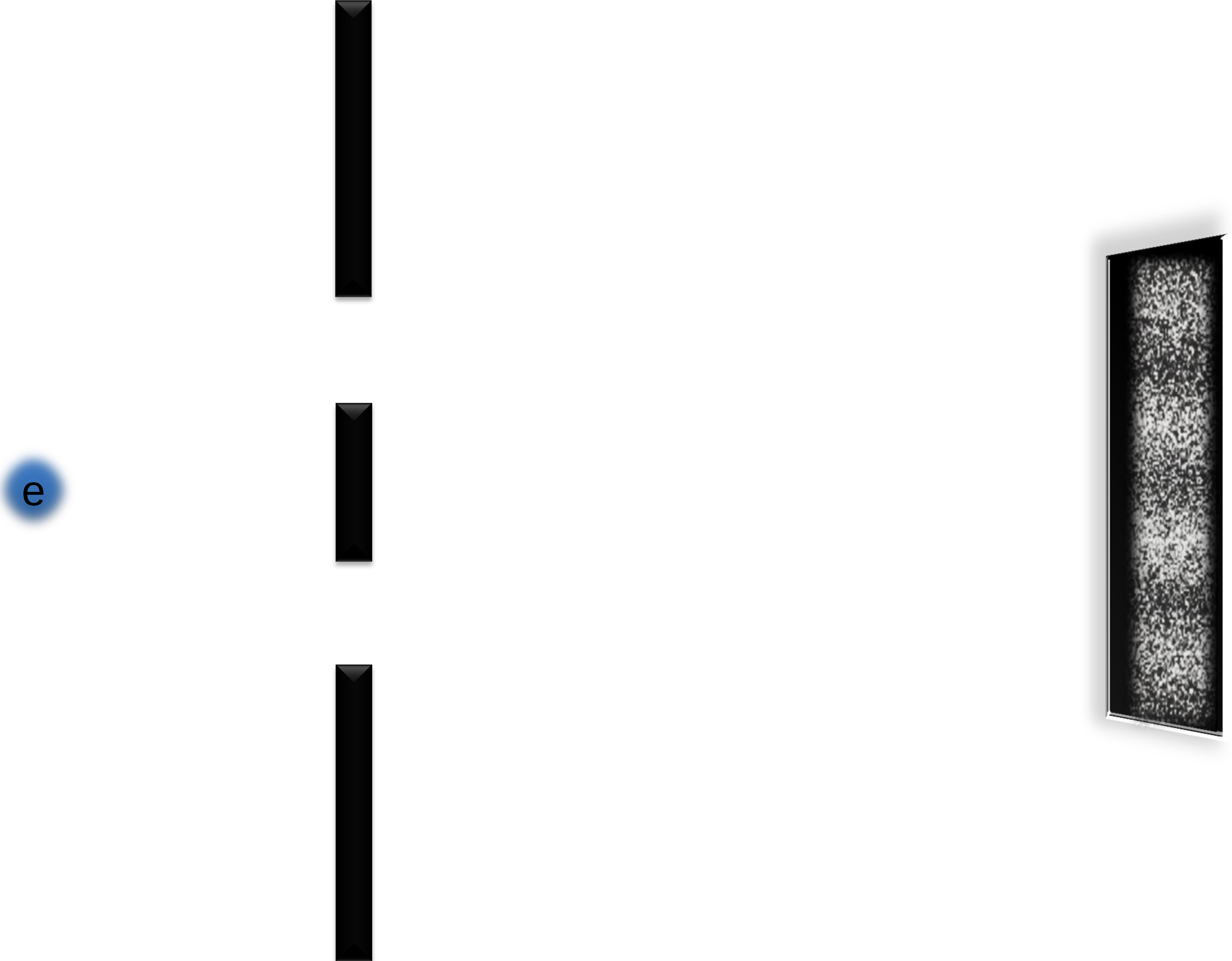
FIRE *tennis balls* AT TWO SLITS



TENNIS BALLS ARE localized




$P_{1+2}(x) = P_1(x) + P_2(x)$
$P_1(x)$
$P_2(x)$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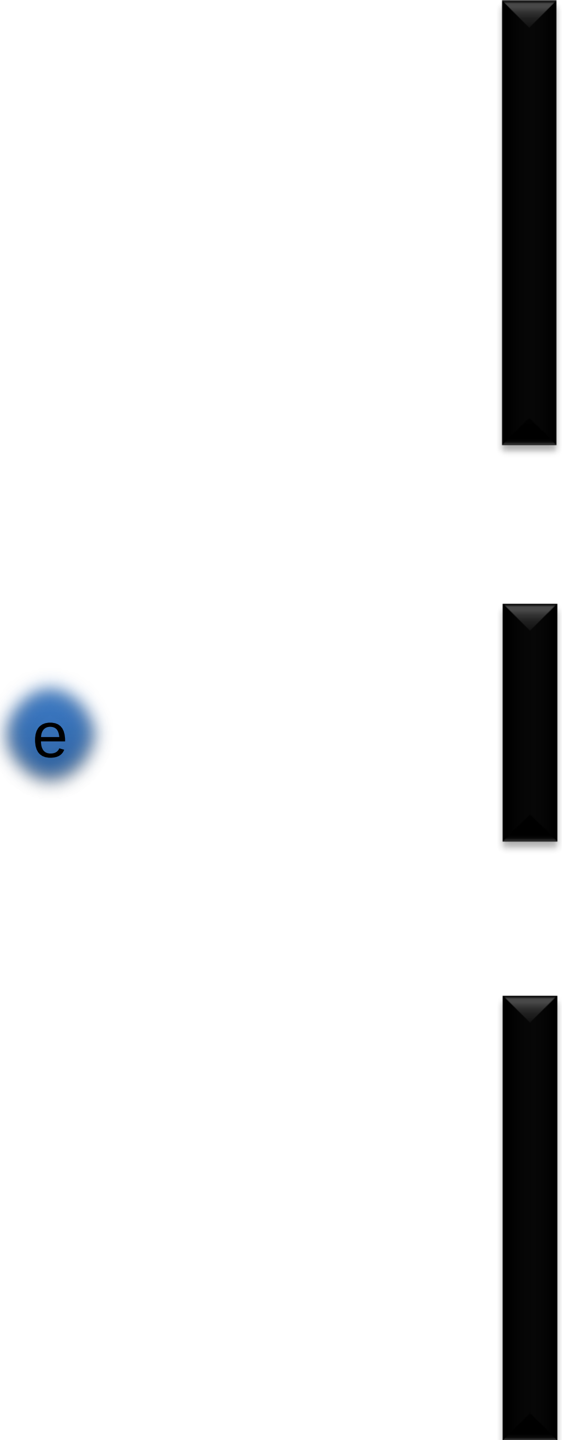
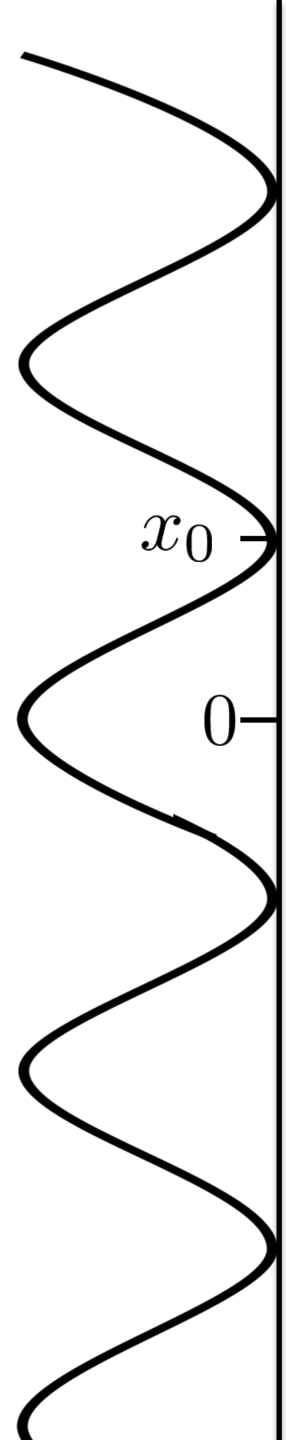


$$P_{1+2}(x) \neq P_1(x) + P_2(x)$$



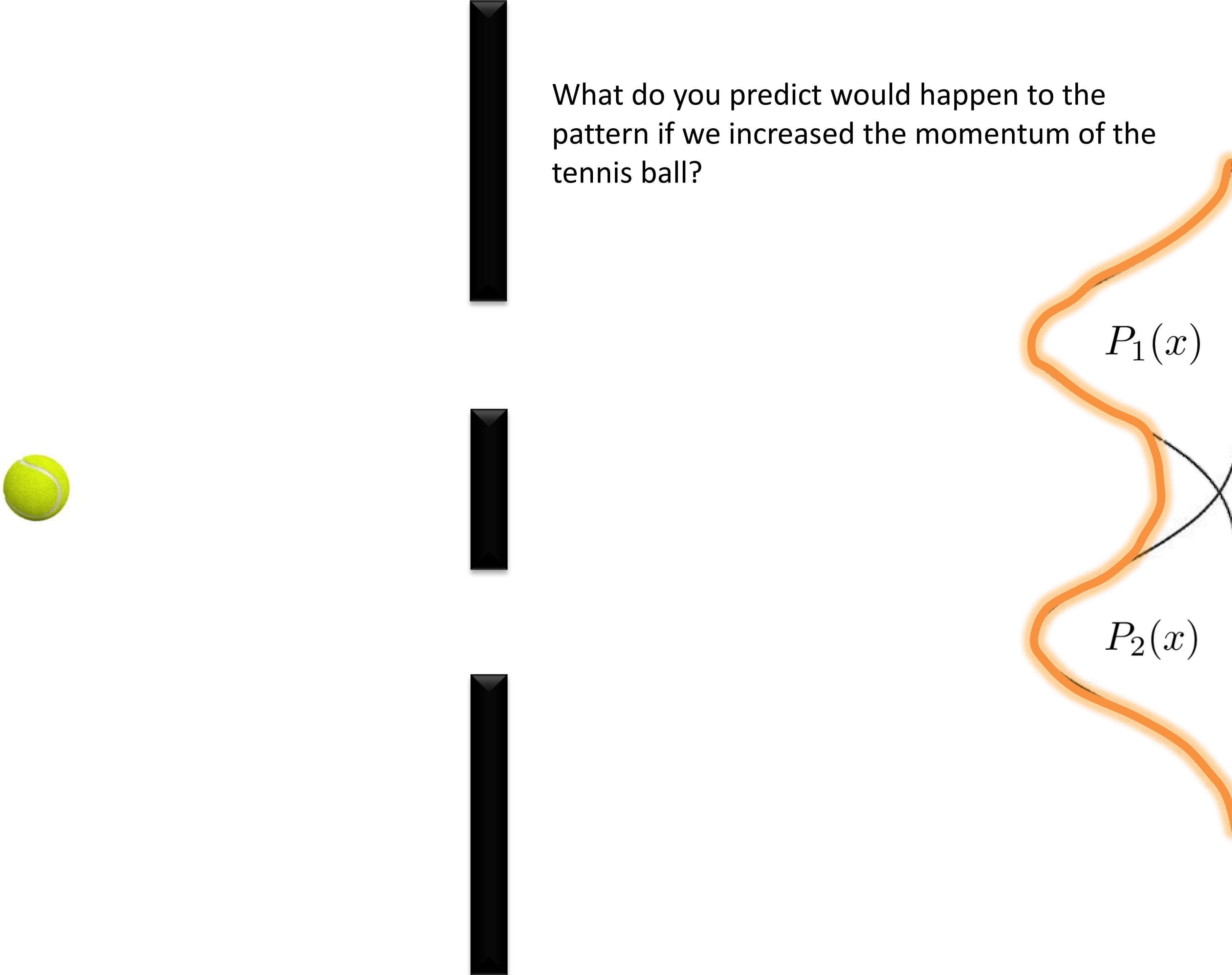
There are places like x_0! When only 1 slit is open electrons hit there, but when both slits are open no electrons hit there!!!

How can this be???

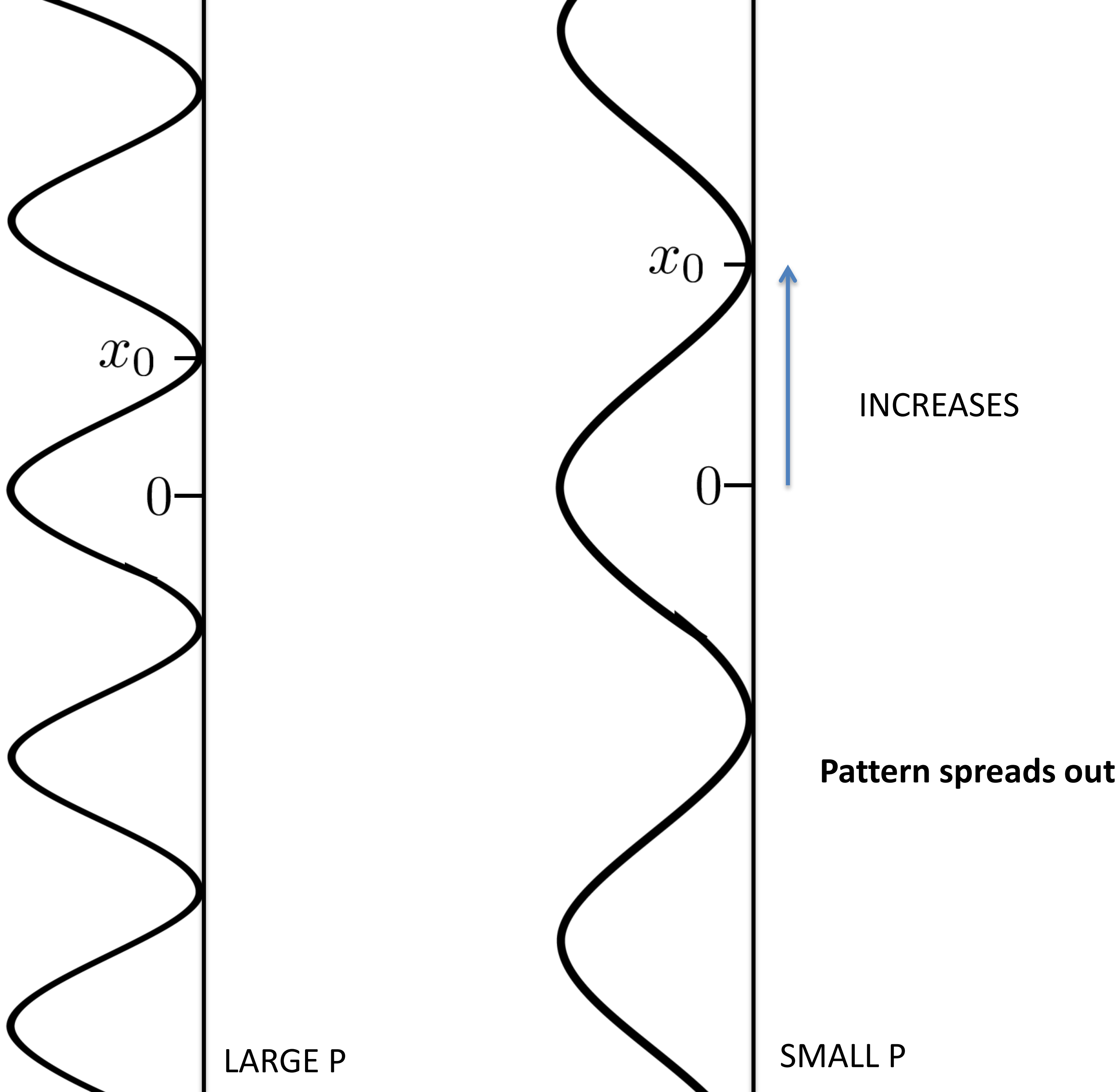
Can we explain electron behaviour with classical physics?

- Ricochet
- Non-contact force at slit edges
- Collisions

- Experiments show same results even when electrons are much smaller than slit, neutrons are employed and electrons are fired one at a time!

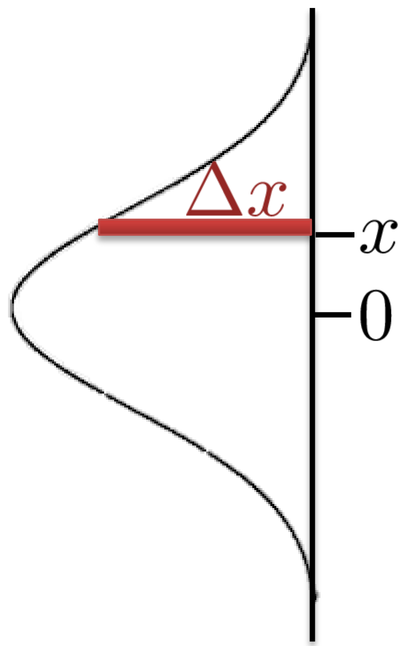


What do you predict would happen to the pattern if we increased the momentum of the tennis ball?


x_0
0
LARGE P
x_0
0
INCREASES
Pattern spreads out!
SMALL P

Probability Pattern

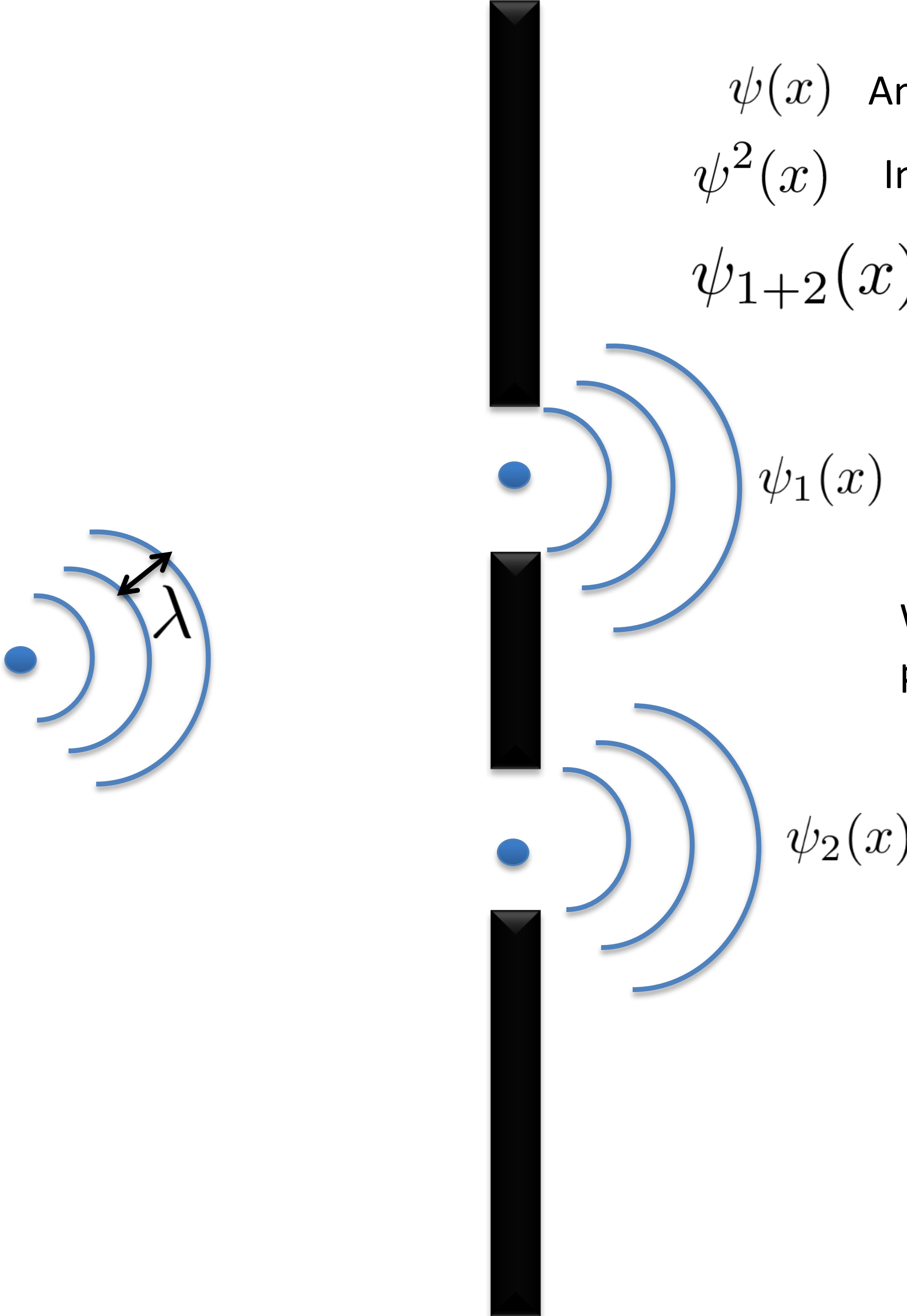
- As the momentum of the electrons is reduced, the probability pattern P(x) spreads out, but it must spread out in such a way the the total area under it remains equal to one.



$\psi(x)$ Amplitude of oscillation at x

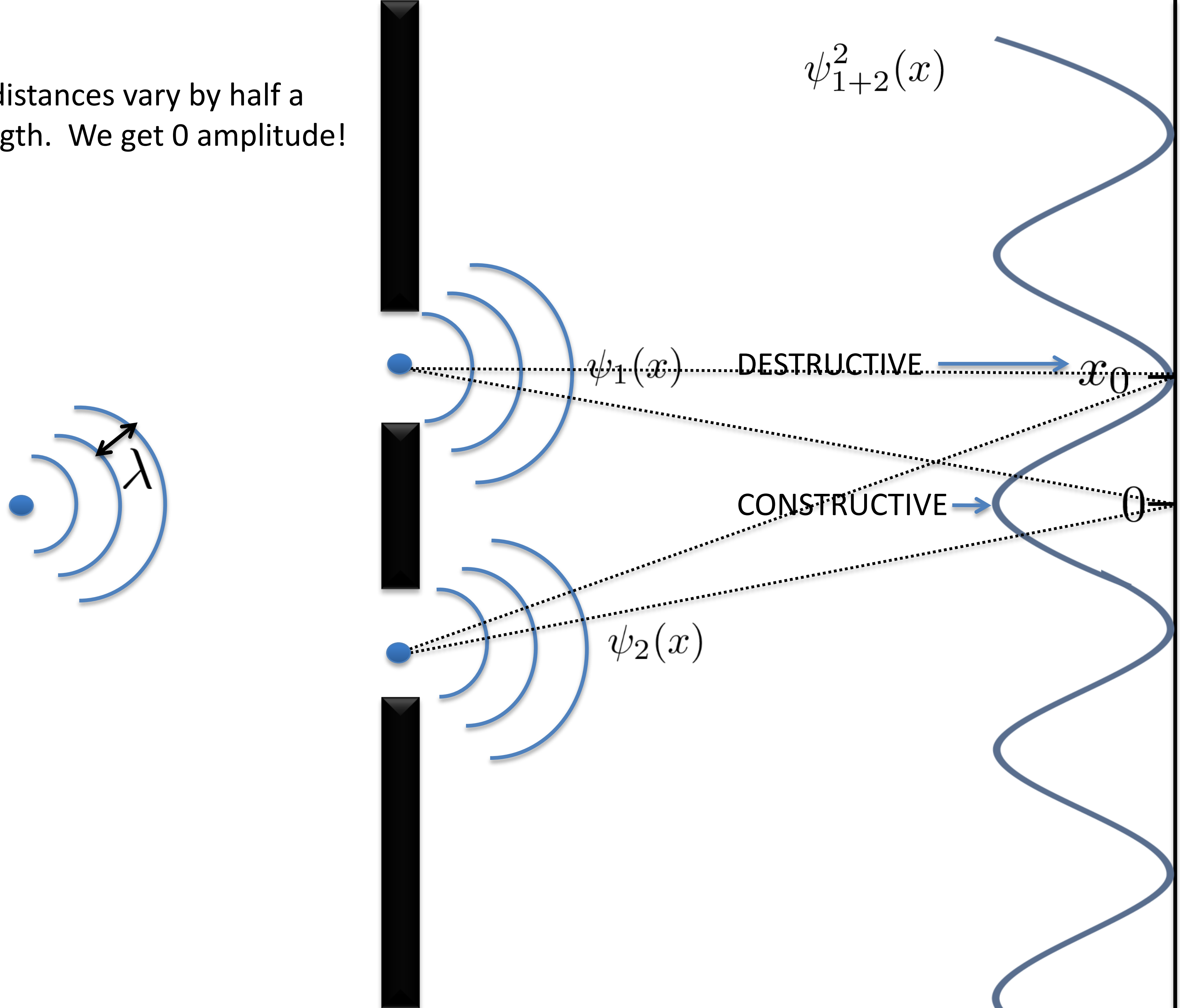
$\psi^2(x)$ Intensity of wave at x

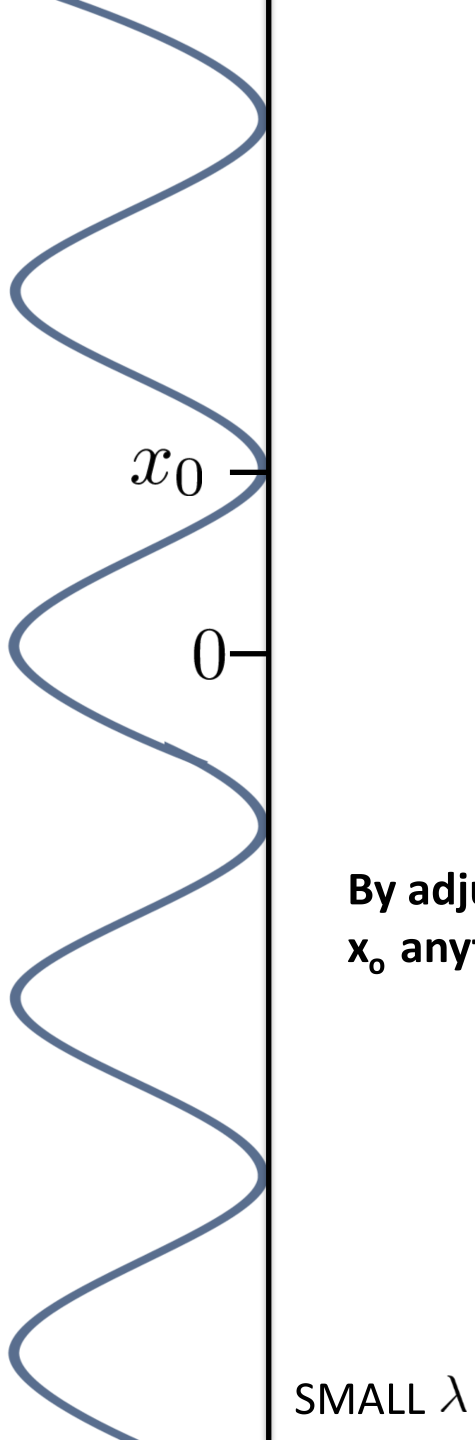
$$\psi_{1+2}(x) = \psi_1(x) + \psi_2(x)$$



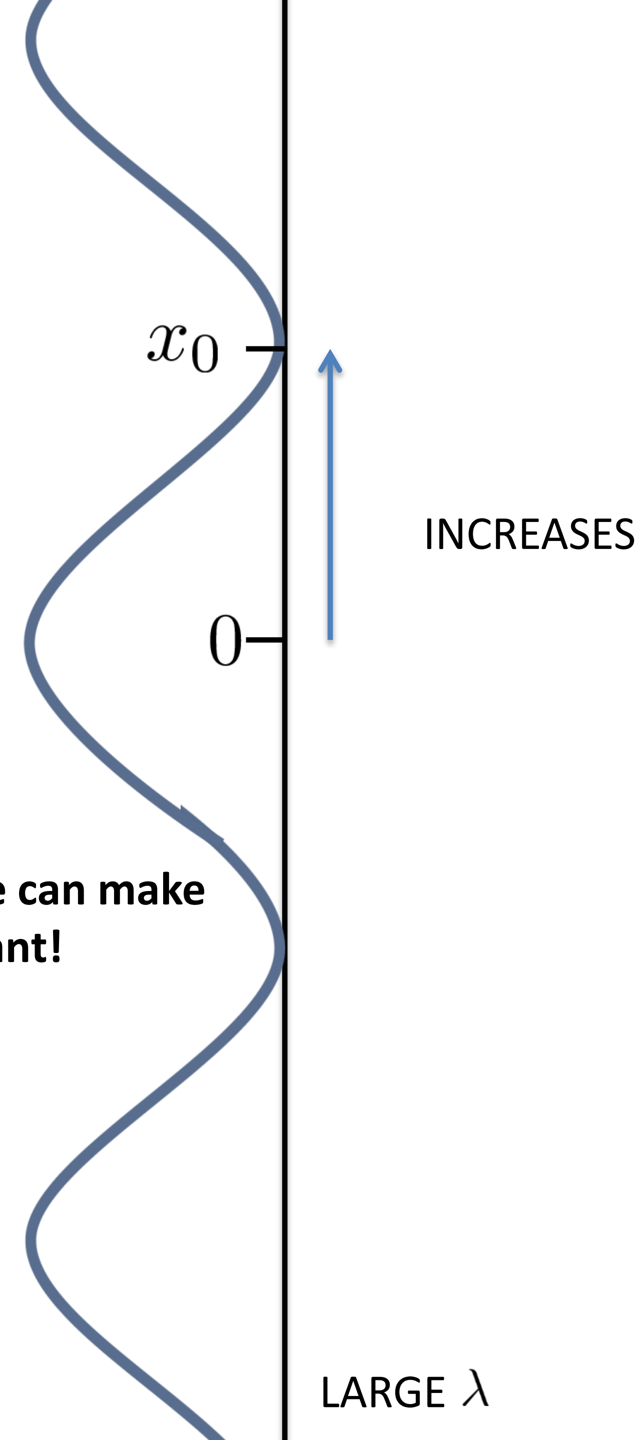
What will the wave intensity pattern $\psi^2_{1+2}(x)$ look like?

At $x=x_o$ distances vary by half a wavelength. We get 0 amplitude!





By adjusting λ we can make x_o anything we want!



- Detailed measurements show that given any double slit experiment with electrons of momentum p, a double slit experiment with waves can be set up such that $\psi^2(x)$ **exactly matches** the probability pattern $P(x)$!

Electrons are similar to Waves!

- Wave intensity is similar to electron probability $\psi^2(x) = P(x)$

- There seems to an inverse relationship between wavelength and momentum!

- We find $\lambda = \dfrac{h}{p}$

h is a proportionality constant!

How could you determine h?

- Experiment
  - Electrons with momentum p produce a pattern
  - Repeat with waves and find λ such that
    $\psi^2(x) = P(x)$
  - Given these values find $h = \lambda p$
- h is the same regardless of particle (electron, photons, neutrons) and no matter the momentum!

Planck's Constant

- We have a new fundamental constant!

$$h = 6.626068 \times 10^{-34} m^2 kg/s$$

Adding Wave Amplitudes

We saw that for e- $P_{1+2}(x) \neq P_1(x) + P_2(x)$

Let's see why! We know from waves that wave amplitudes **add**!

$$\psi_{1+2}(x) = \psi_1(x) + \psi_2(x)$$

But *Particle Probabilities = Wave Intensity* (**square** of the amplitude)

$$P_{1+2}(x) = \psi_{1+2}^2(x)$$

$$\begin{aligned} P_{1+2}(x) &= [\psi_1(x) + \psi_2(x)]^2 \\ &= \psi_1^2(x) + \psi_2^2(x) + 2\psi_1(x)\psi_2(x) \\ &= P_1(x) + P_2(x) + 2\psi_1(x)\psi_2(x) \end{aligned}$$

CROSS TERM!

The cross term accounts for the difference!

Replacing F=ma

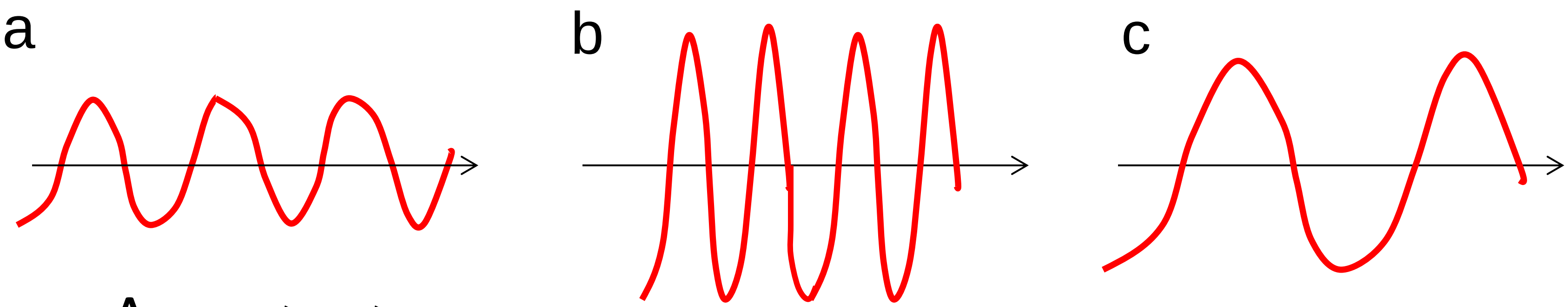
The double slit experiment shows us that particles like electrons do not obey F=ma from Newtonian mechanics.

Instead they obey wave mechanics and the de Broglie relationship

$$F = ma \longrightarrow \lambda = \frac{h}{p}$$

"What path does the particle follow?" $\longrightarrow$ *"What is the probability the particle will hit here or there on the screen?"*

Three particles with equal mass are associated with the following matter waves. Rank, from largest to smallest, the speed of these particles.



A. $v_a > v_b > v_c$

B. $v_b > v_c > v_a$

C. $v_a > v_c > v_b$

D. $v_b > v_a > v_c$

E. $v_c > v_a > v_b$

A bullet is fired from a rifle. The end of the rifle is a circular aperture. Is diffraction a measurable effect?

A. No, because only charged particles have a de Broglie wavelength.
B. No, because a circular aperture never causes diffraction.
C. No, because the de Broglie wavelength of the bullet is too large.
D. No, because the de Broglie wavelength of the bullet is too short.
E. Yes

Photons

We see waves behaving like particles!

De Broglie Relation For Photons

- Einstein's famous equation $E = mc^2$ expresses the equivalence of mass and energy for particles at *rest*.

- For particles with motion (momentum, p), the full equation reads:

$$E = \sqrt{m^2c^4 + p^2c^2}$$

- Photons are moving but have zero rest mass, so the equation reduces to $E = cp$

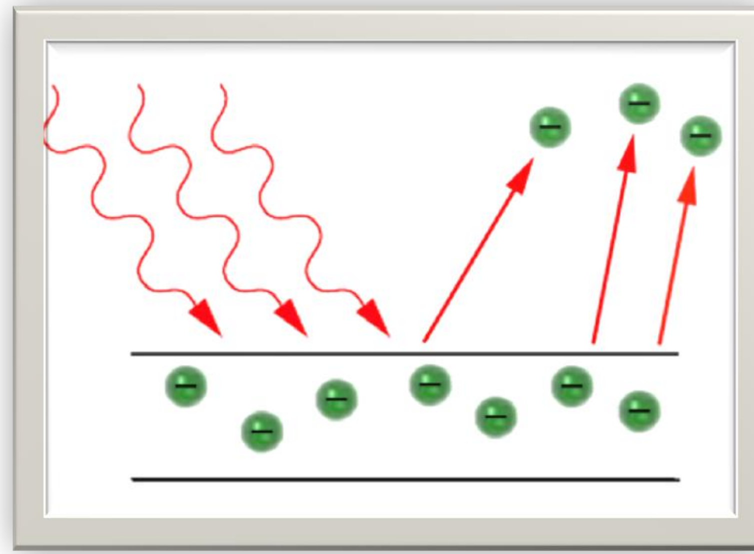
- **Photons have momentum!**

De Broglie Relation For Photons

- Combine $E = cp$ with the de Broglie relation and simplify using the universal wave equation $c = f \times \lambda$ to find the relation between the **energy** of a photon and its **frequency**.

- Which colour of photon requires more energy to produce: red (low-frequency) or blue (high-frequency)? Why?

EINSTEIN'S IDEA - Photons



SUMMARY

Sand and tennis balls are *classical* particles that obey Newton's law: $F = ma$. A force is needed to deflect their trajectory.

Electrons are *quantum* particles that obey the de Broglie relation: $\lambda = h/p$. As happens in *wave diffraction*, just passing through a hole can deflect their trajectory!

Electrons also exhibit *wave interference*. A single electron passing through two or more holes interferes *with itself*!

Photons are also quantum particles that obey the de Broglie relation: $\lambda = h/p$. This relation can be rewritten as $E = hf$.

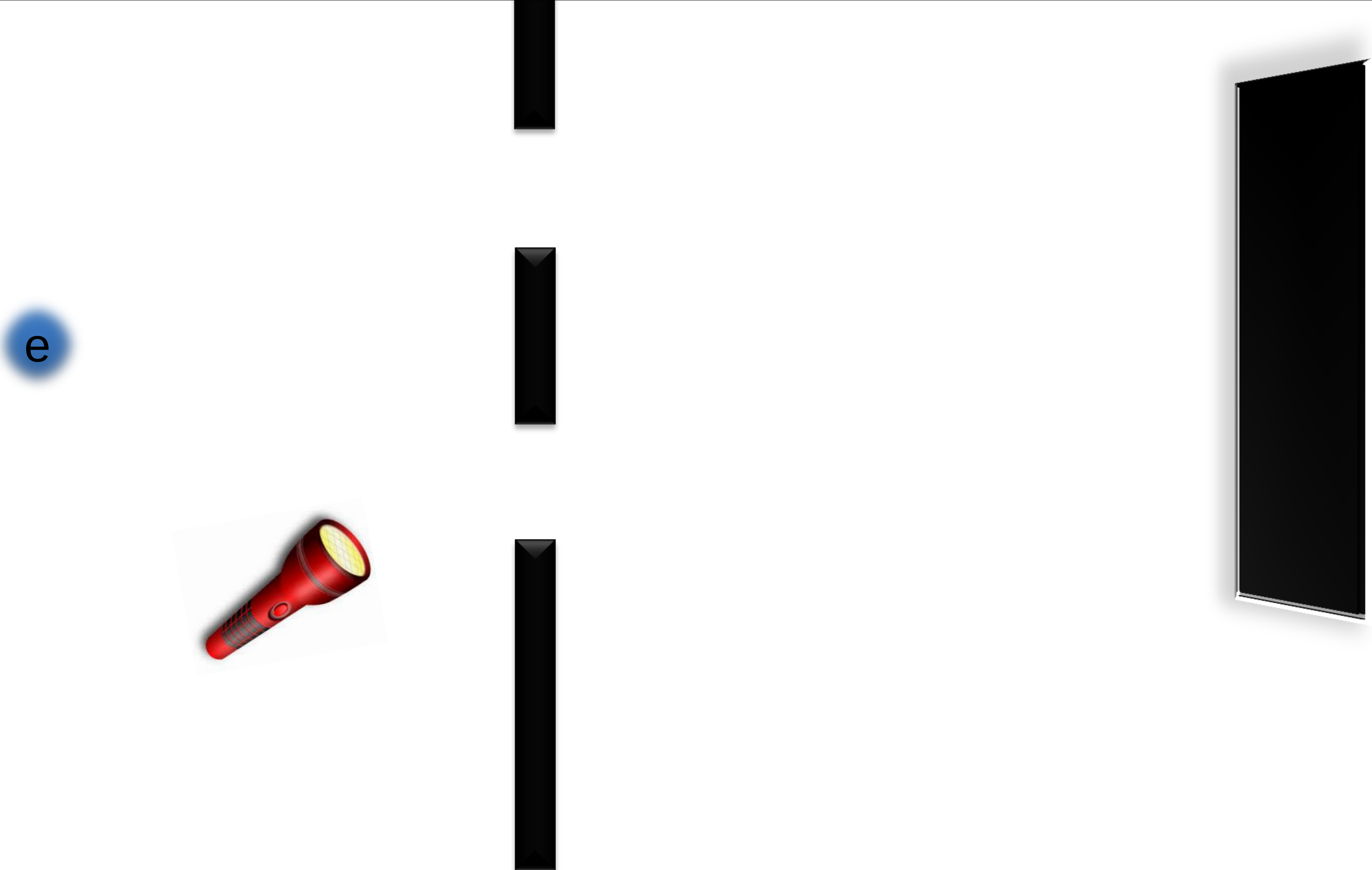
All matter and radiation is made of quantum particles that obey the *same* universal law: $\lambda = h/p$.

We can think of quantum particles as **particles behaving like waves** (e.g., electrons diffracting and interfering), or as **waves behaving like particles** (e.g., light hitting the screen as individual particles).

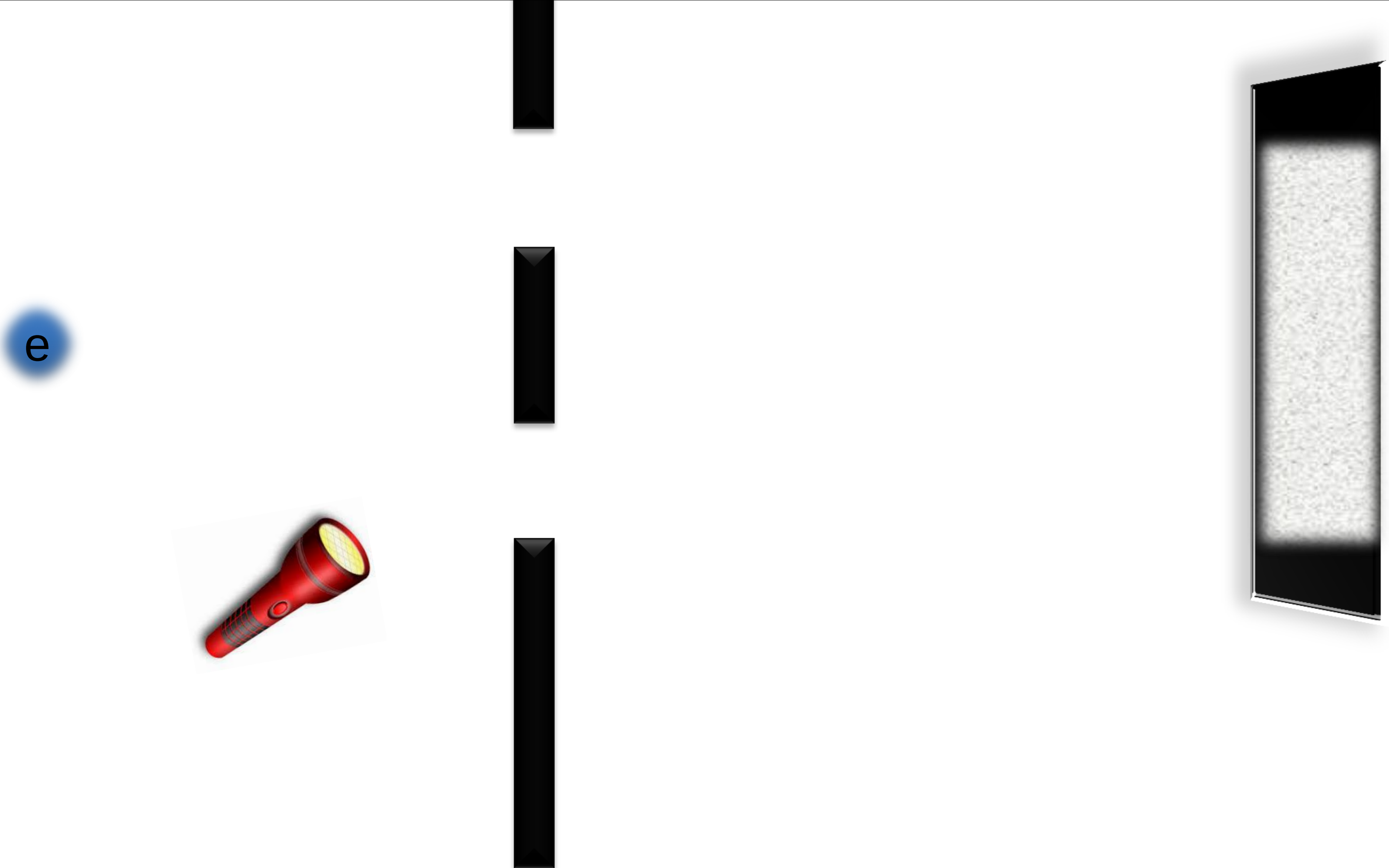
This *wave-particle duality* represents one of the most important discoveries (and mysteries!) in all of science.

WHAT IS THE ELECTRON *doing?*

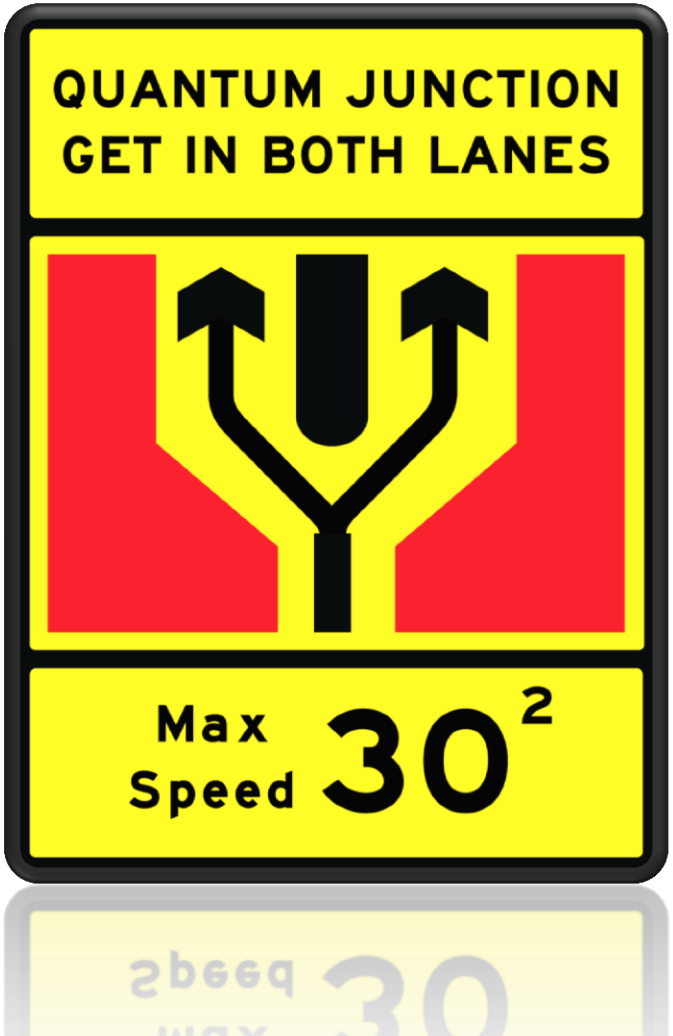
LET'S HAVE A look!



INTERFERENCE PATTERN *disappears!*



QUANTUM Superposition



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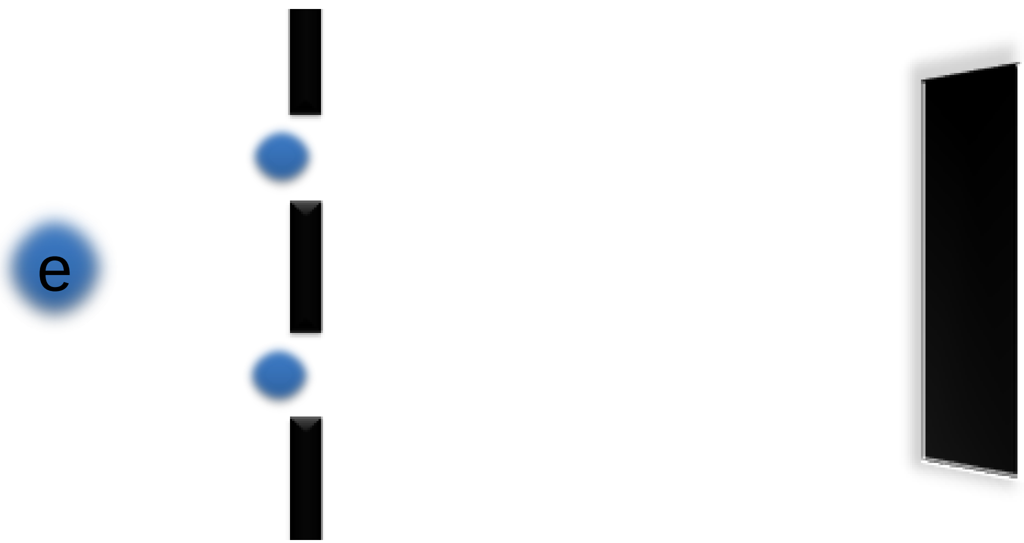

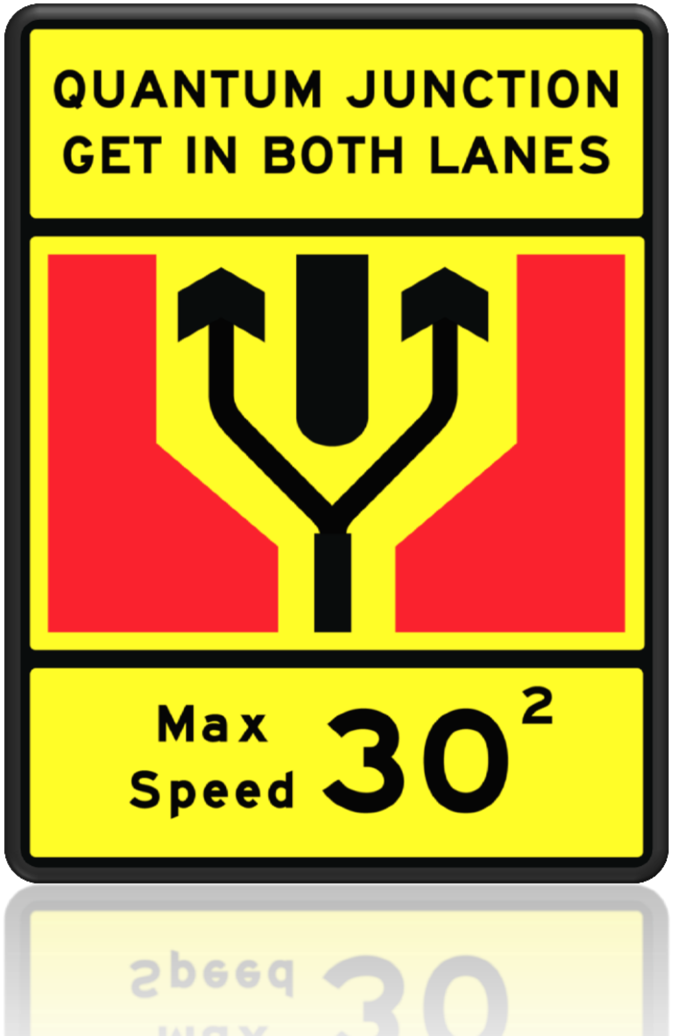

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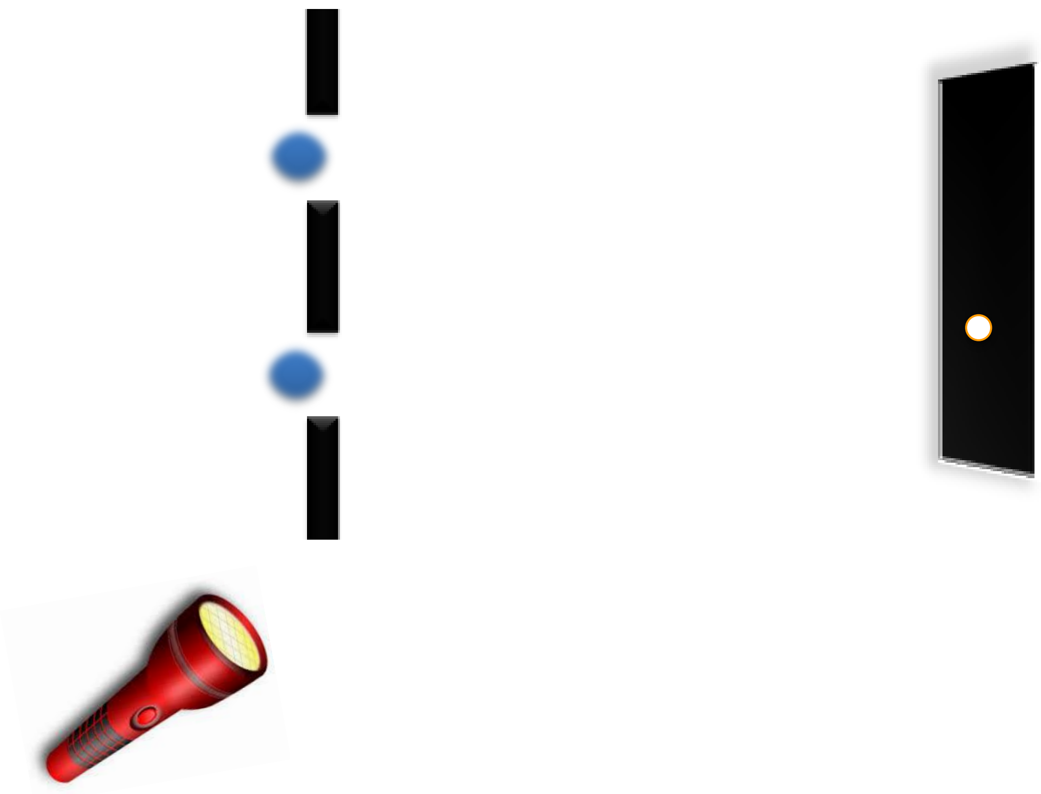
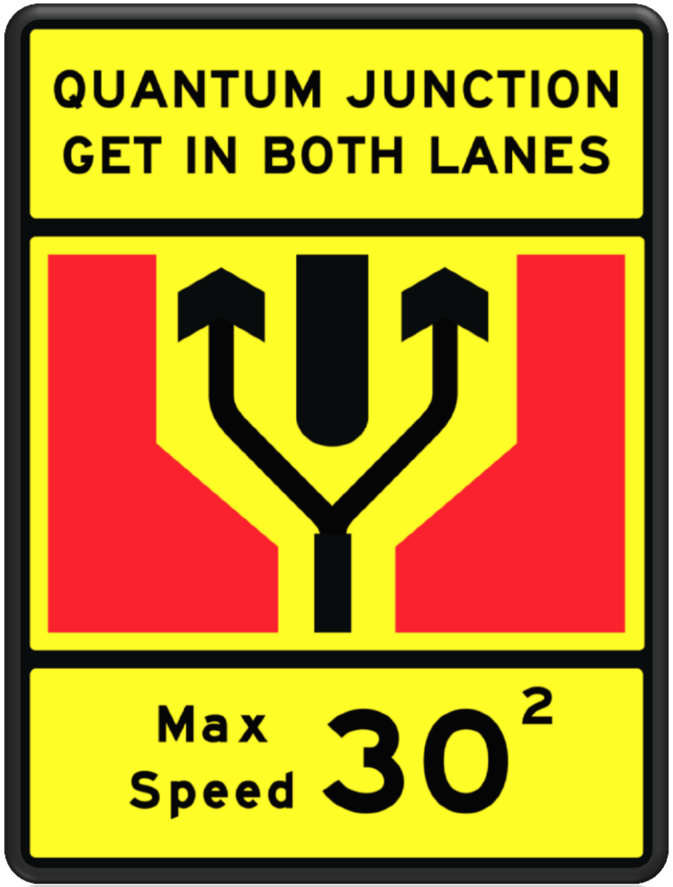


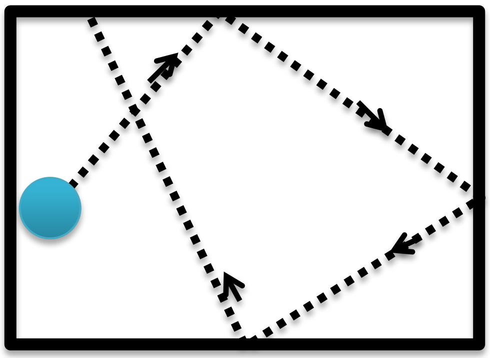


Quantum Interpretations

- Collapse
- Pilot Wave
- Many Worlds
- Copenhagen

- See D2L page

Classical Particle in a Box



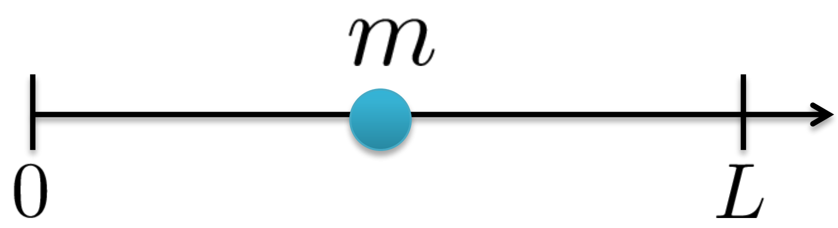
$$F = ma$$

Bounce in Perpetual Motion

But how does a quantum particle behave with this new quantum law $\lambda = \frac{h}{p}$?

Trap a Particle in a 1D Box

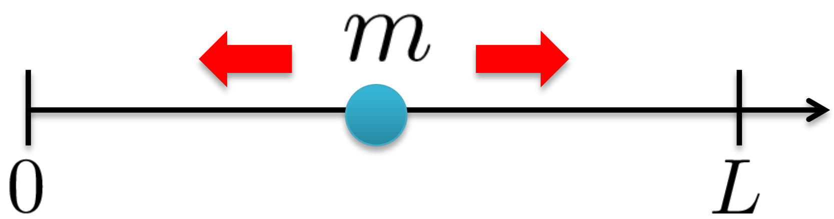
Suppose the particle as a definite energy E



What can be said about the particle's momentum?

Trap a Particle in a 1D Box

What can be said about the particle's momentum?



$$E = \frac{1}{2}mv^2 \quad \longrightarrow \quad E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\cancel{p = \sqrt{2mE}}$$

$$p = \pm\sqrt{2mE}$$

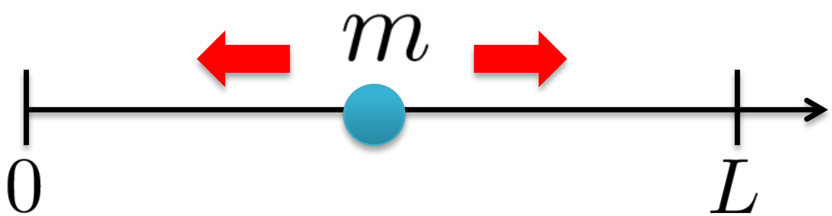
(right/left)

Classically, the particle behaves like a lump either moving left or right.

Trap a Particle in a 1D Box

What do we learn by using the de Broglie relation?

$$\lambda = \frac{h}{p}$$



$$p = \pm\sqrt{2mE}$$

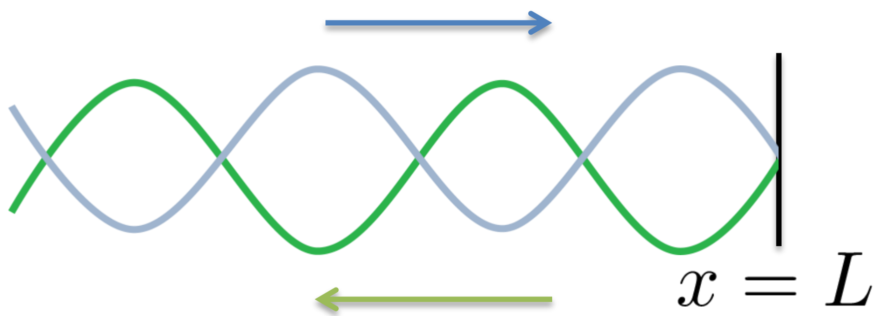
(right/left)

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$|p|$

What happens when an incident wave strikes a barrier?

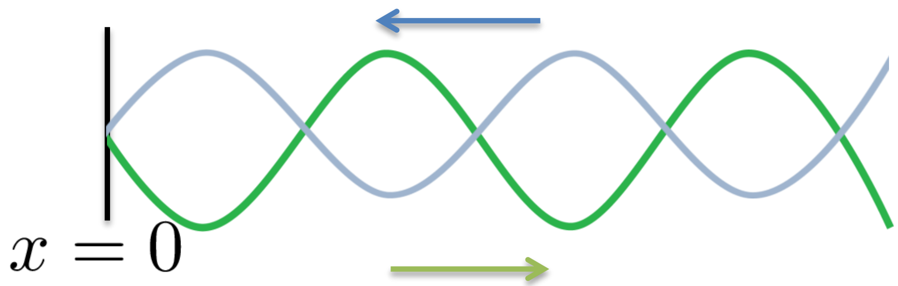
$$p = +\sqrt{2mE}$$



Reflected wave: $p = -\sqrt{2mE}$

What happens when an incident wave strikes a barrier?

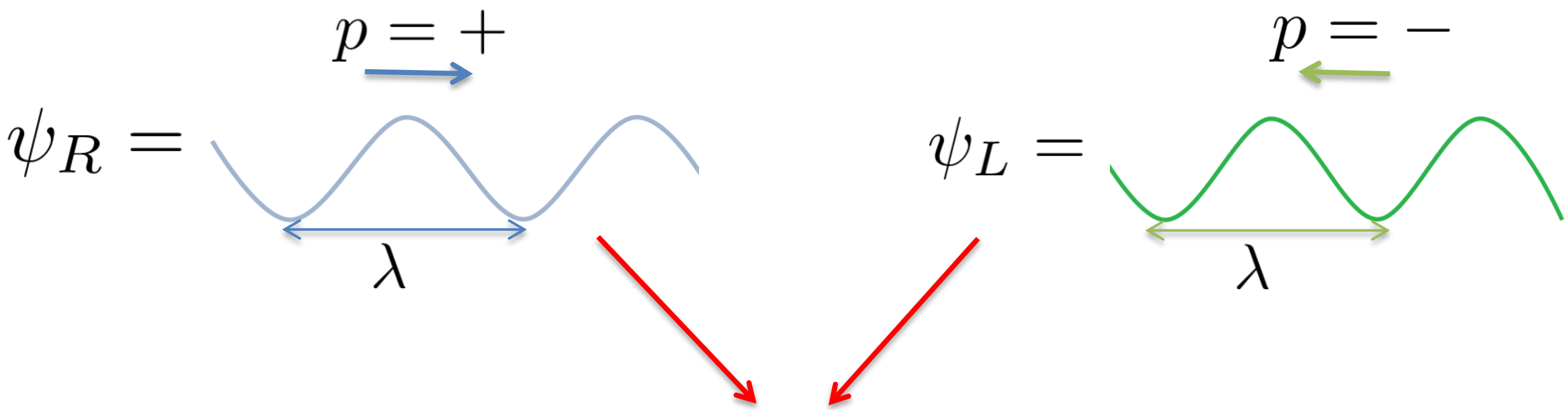
$$p = -\sqrt{2mE}$$



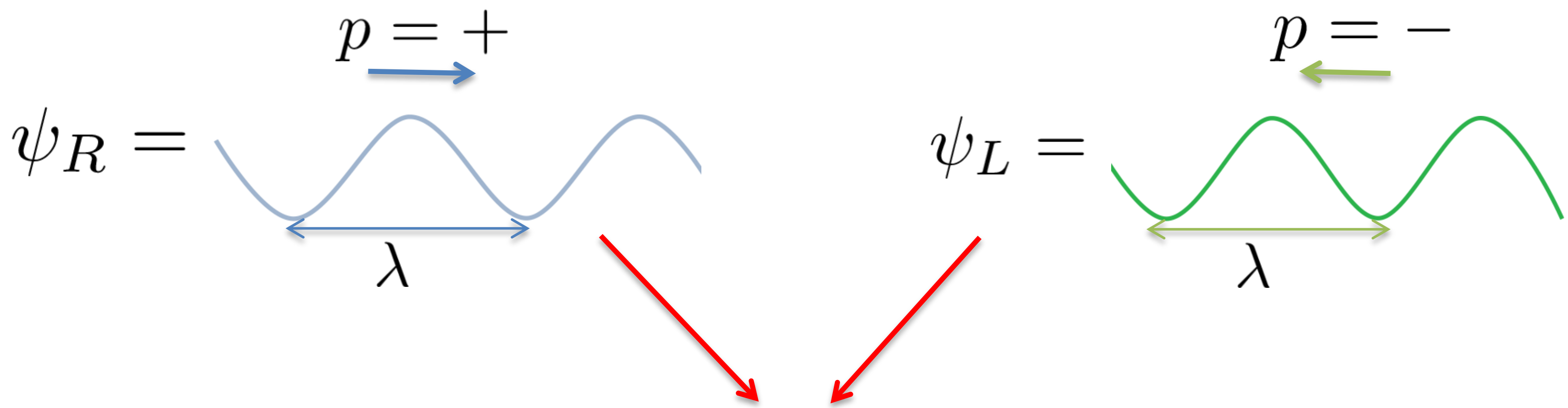
Reflected wave: $p = +\sqrt{2mE}$

The quantum view shows that the particle is not localized to one spot.

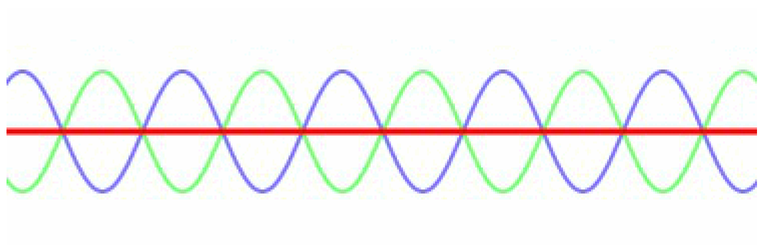
The wave spreads throughout the box with the right-moving wave being the source of the left moving wave and vice-versa.



$$\psi = \psi_L + \psi_R$$



$$\psi = \psi_L + \psi_R$$



Standing Wave: Two traveling waves (left & right) which are present simultaneously in the same space. The two waves are in ***superposition***.

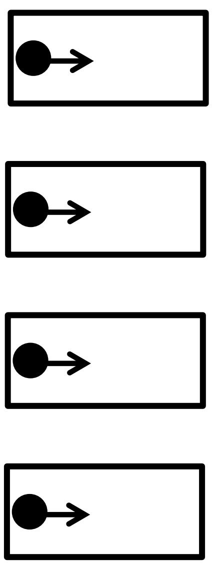
At any given instant the particle is neither moving left nor right. It is moving both ways at the same time!

$$\psi = \psi_L + \psi_R$$

At any given instant the particle is neither moving left nor right. It is moving <u>both</u> ways at the same time!

Weird!

<u>Classically</u> Lay out many boxes with particles all in the same initial state.



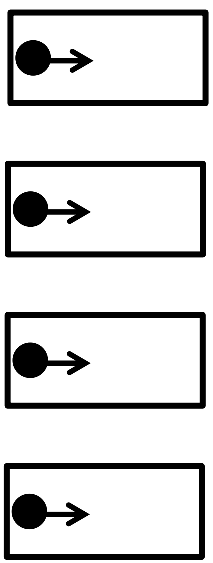
If we measure some time later they will all be in the same state or place.

$$\psi = \psi_L + \psi_R$$

At any given instant the particle is neither moving left nor right. It is moving <u>both</u> ways at the same time!

Weird!

<u>**Quantum**</u> Lay out many boxes with particles all in the same initial state.



If we measure some time later we get perfectly random result! 50/50 odds for either left or right moving particle.

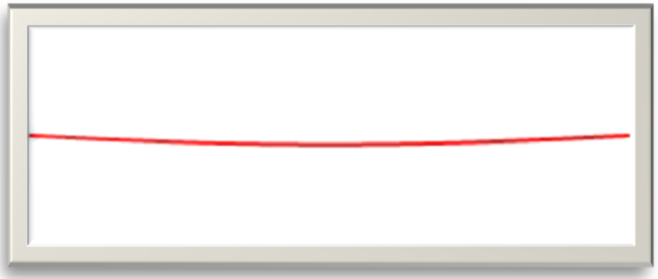
Collapse of the Wave Function

- Double slit experiment: particle behaves like it is passing through both slits at once.
- If we measure we force it to decide where it is.
- Not a physical wave, but a mathematical wave that describes the *probability* we will observe the particle here or there.
- When we measure, the wave describing the state of the particle goes from spread out to *localized*.

Standing Waves

What is the *smallest* possible box?

What is the *smallest* possible box?

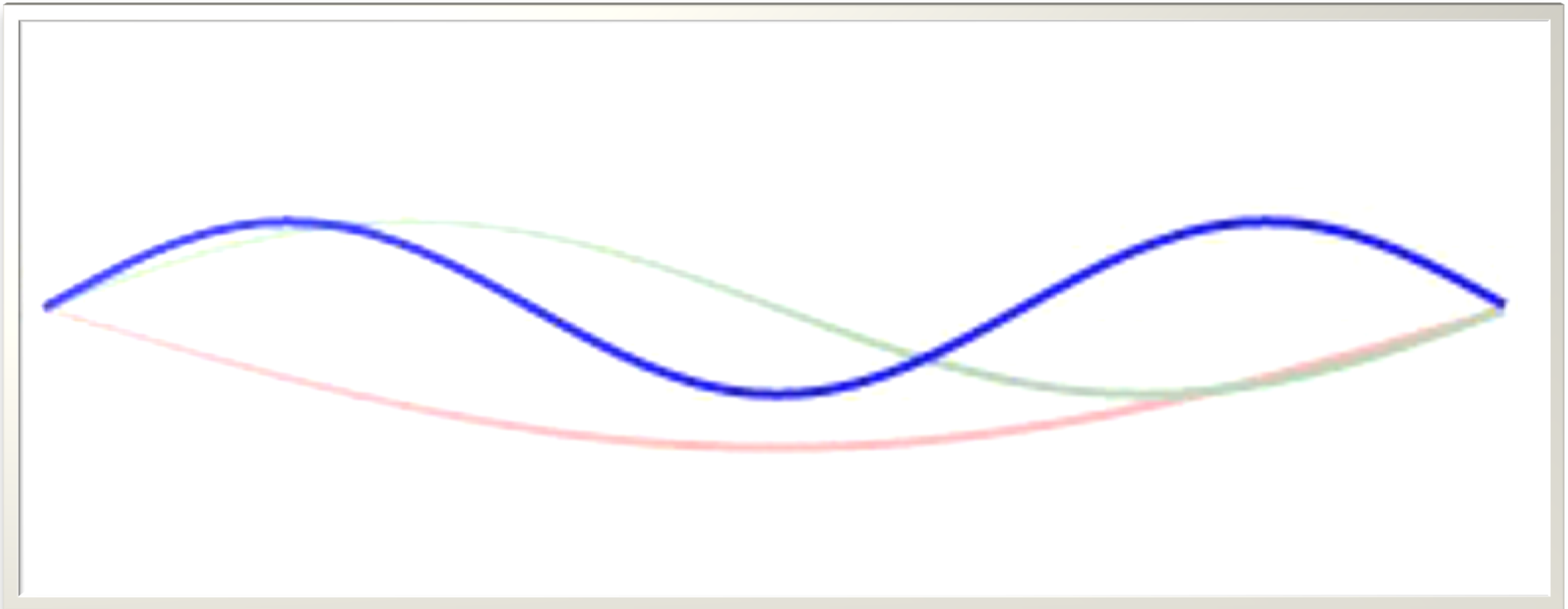


$$L = \frac{\lambda}{2}$$

What is the *next largest* box?

$$L = \frac{\lambda}{2} \qquad L = \lambda$$

What is the rule?



$$L = \frac{\lambda}{2} \qquad L = \lambda \qquad L = \frac{3\lambda}{2}$$

- Allowed box sizes: $L = \frac{n\lambda}{2}$ $\qquad n = 1, 2, 3, ...$

- Or, the allowed wavelengths are: $\lambda = \frac{2L}{n}$

- This is the same as saying only certain values of momenta are possible: $p = \frac{hn}{2L}$

- What does this mean for the energy?

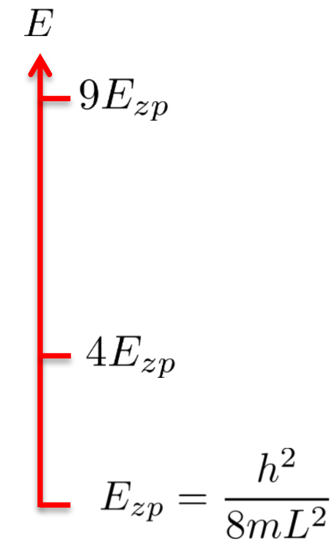
$$E = \frac{p^2}{2m} = \frac{h^2 n^2}{8mL^2}$$

Particle in a Box

Classical

- Continuum of possible energies
- Lowest energy is zero

Quantum



- Discrete set of energies
- Zero point energy

$$E = \frac{h^2 n^2}{8mL^2} = E_{zp} n^2$$

Quantization

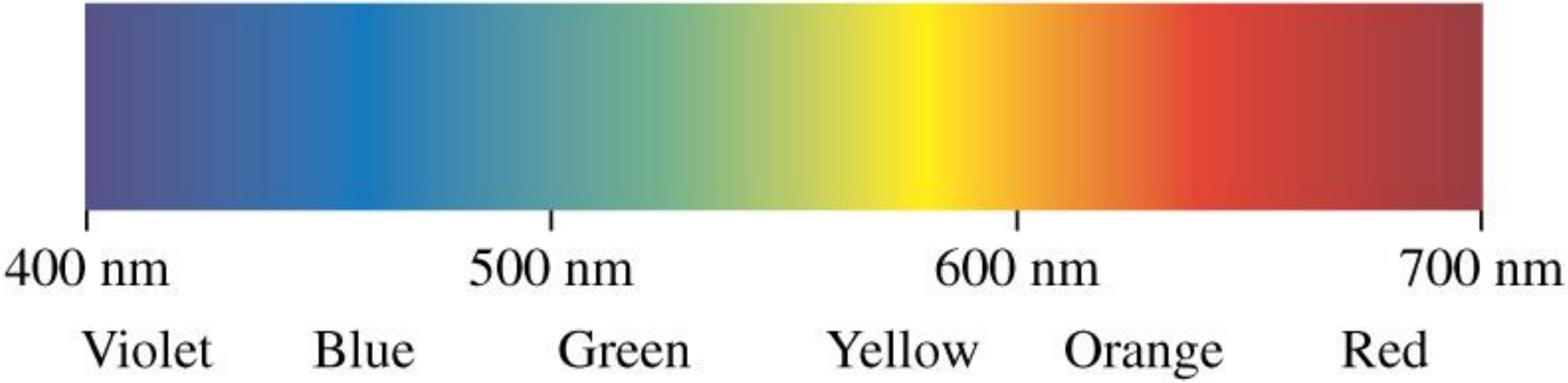
- A quantity is quantized if its possible values are limited to a discrete set.

Examples:

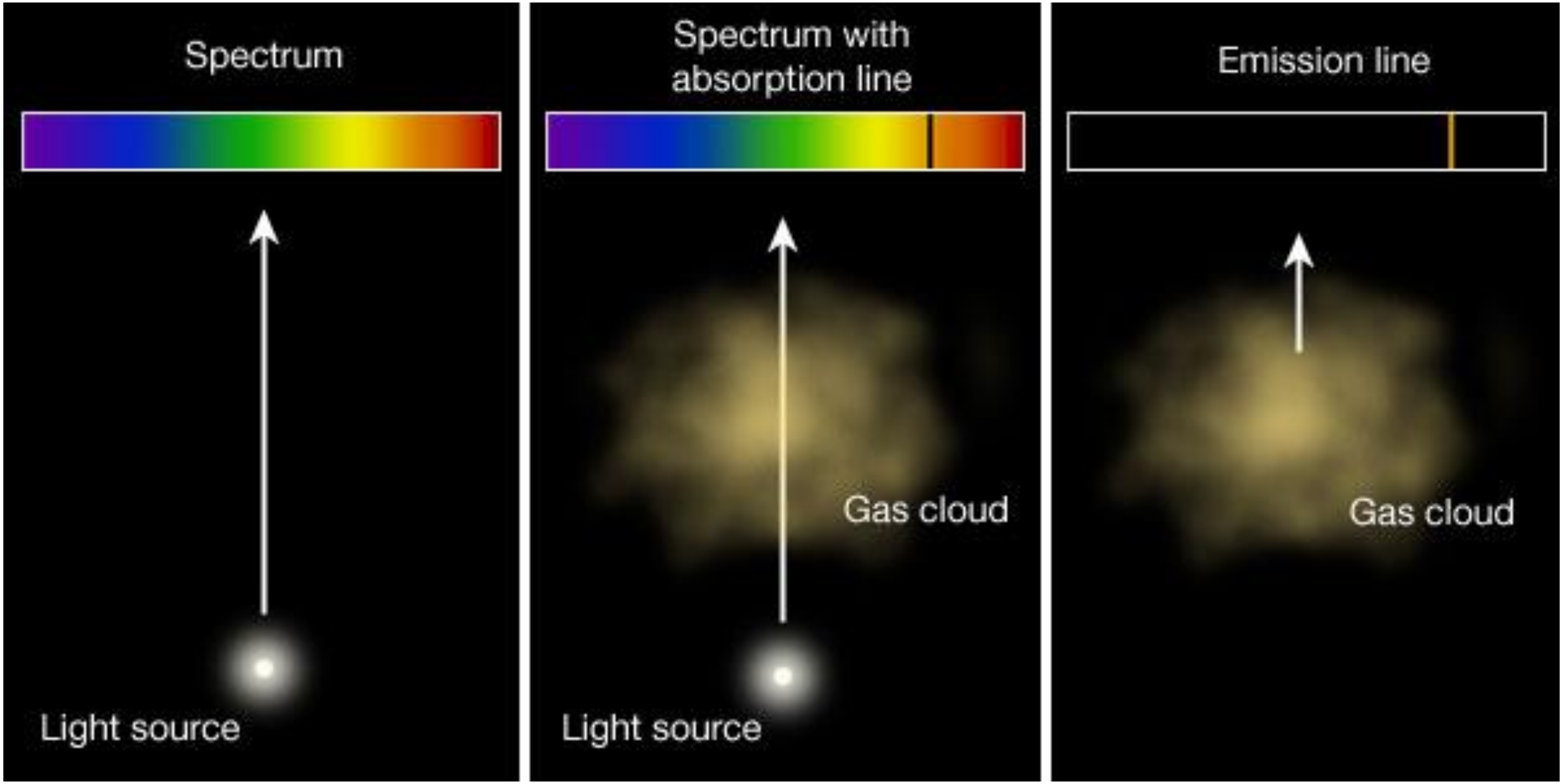
- Number of chairs in the room.
- Frequency of standing wave on a stretched string.

- A blackbody, such as the Sun or an incandescent light bulb, emits a ***continuous spectrum*** (some light at every wavelength)

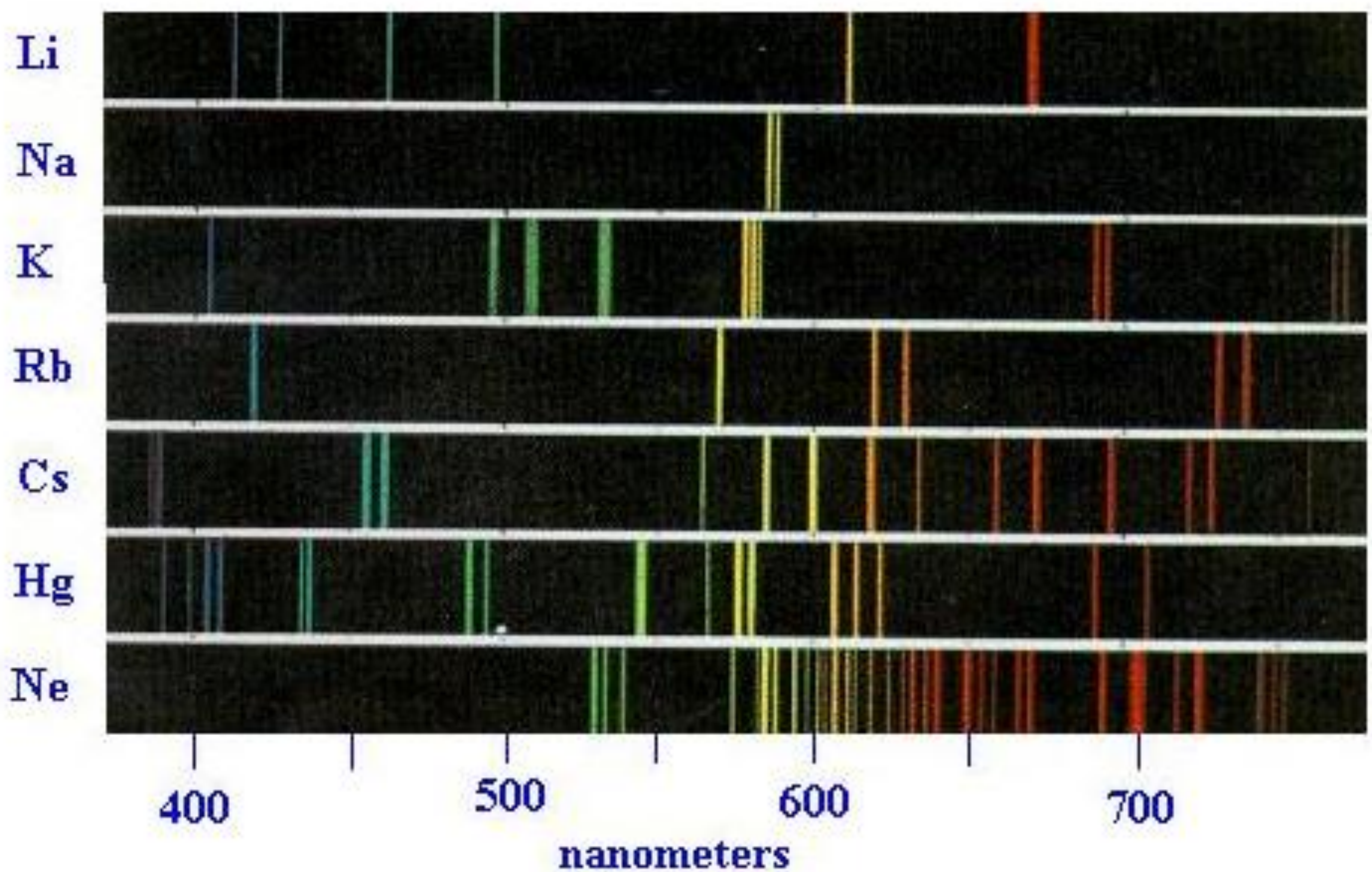
(a) Incandescent light bulb



Absorption & Emission



- Different emission pattern for different atoms.



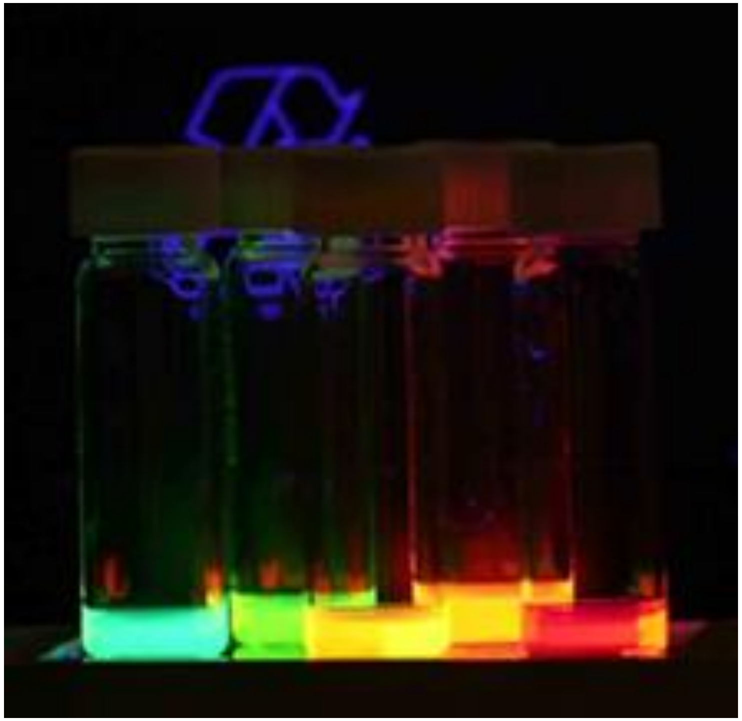
Zero Point Energy

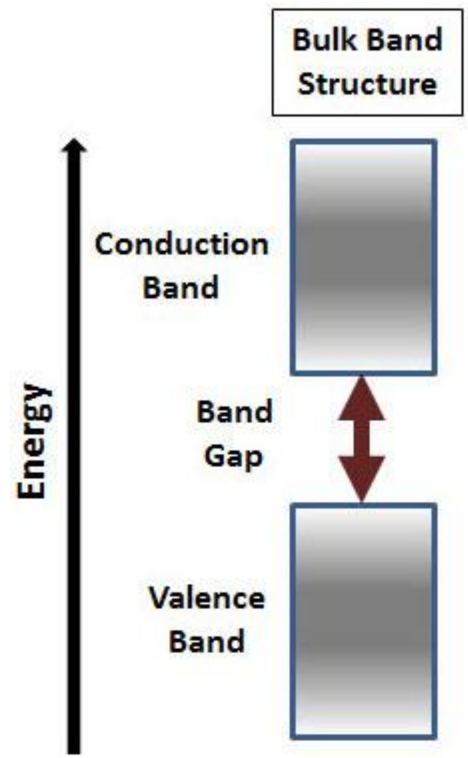
$$E_{zp} = \frac{h^2}{8mL^2}$$

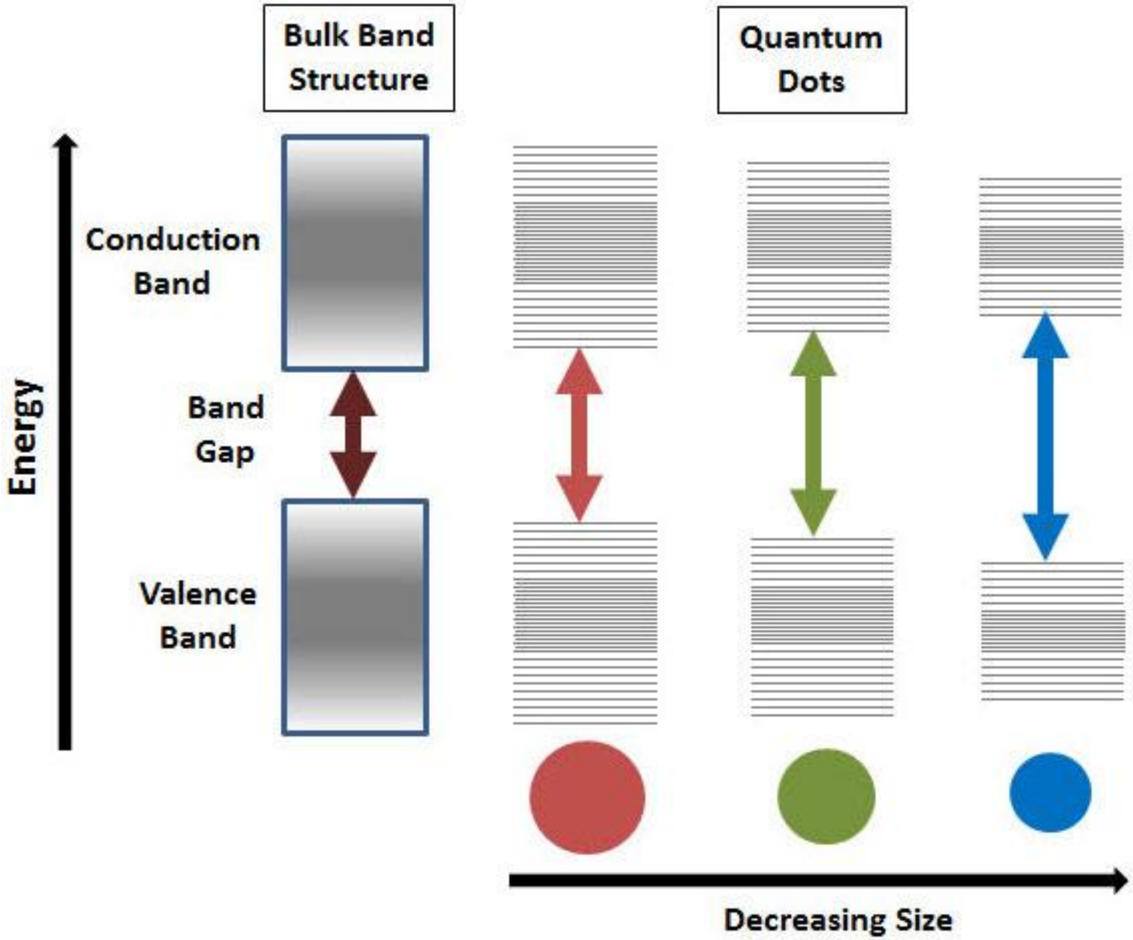
What happens when we squeeze the box?

As we squeeze the box and make L smaller the zero point energy goes up!

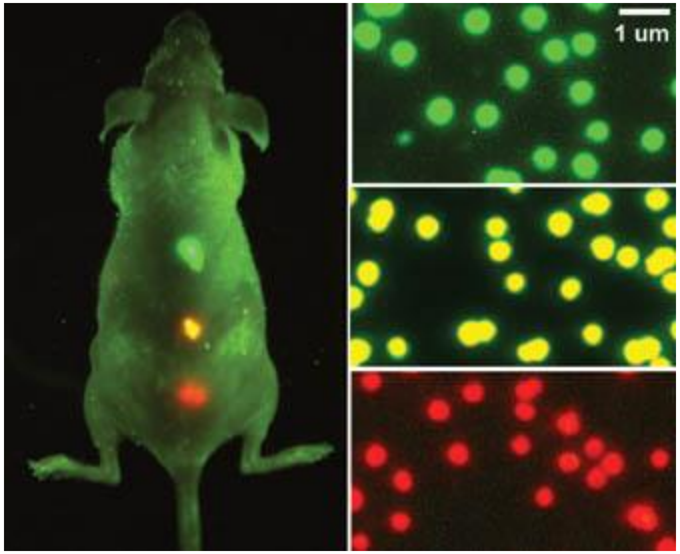
Quantum Claustrophobia




Bulk Band
Structure
Energy
Conduction
Band
Band
Gap
Valence
Band



Bulk Band Structure
Quantum Dots
Energy
Conduction Band
Band Gap
Valence Band
Decreasing Size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