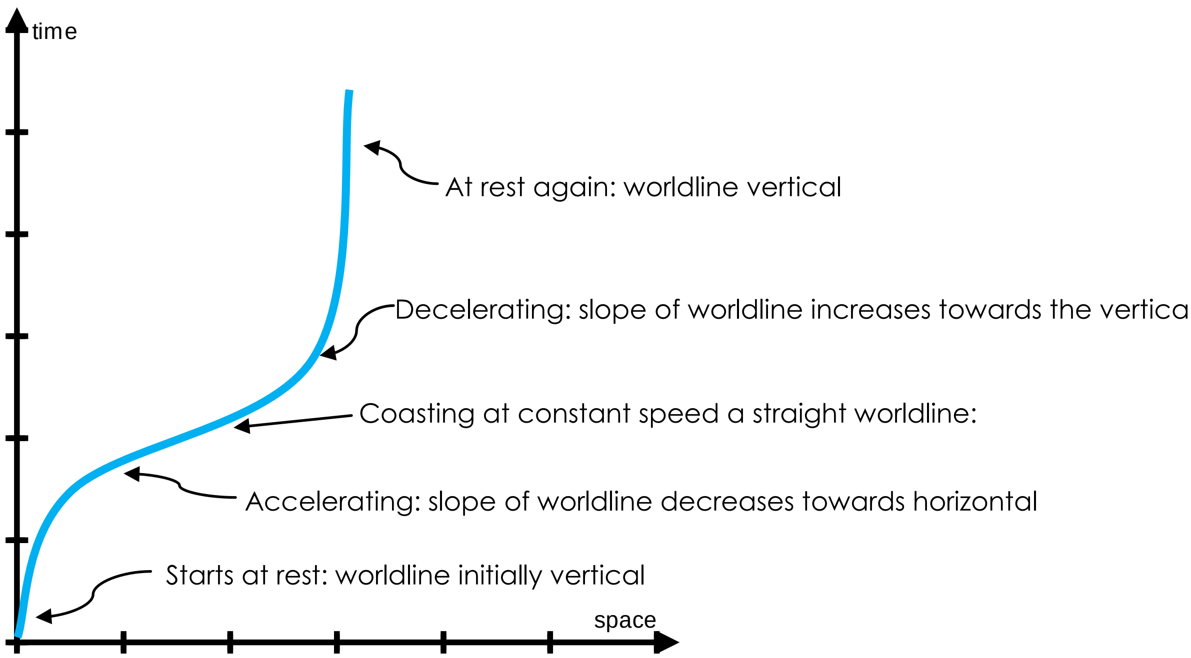
Special Relativity Solutions 2:

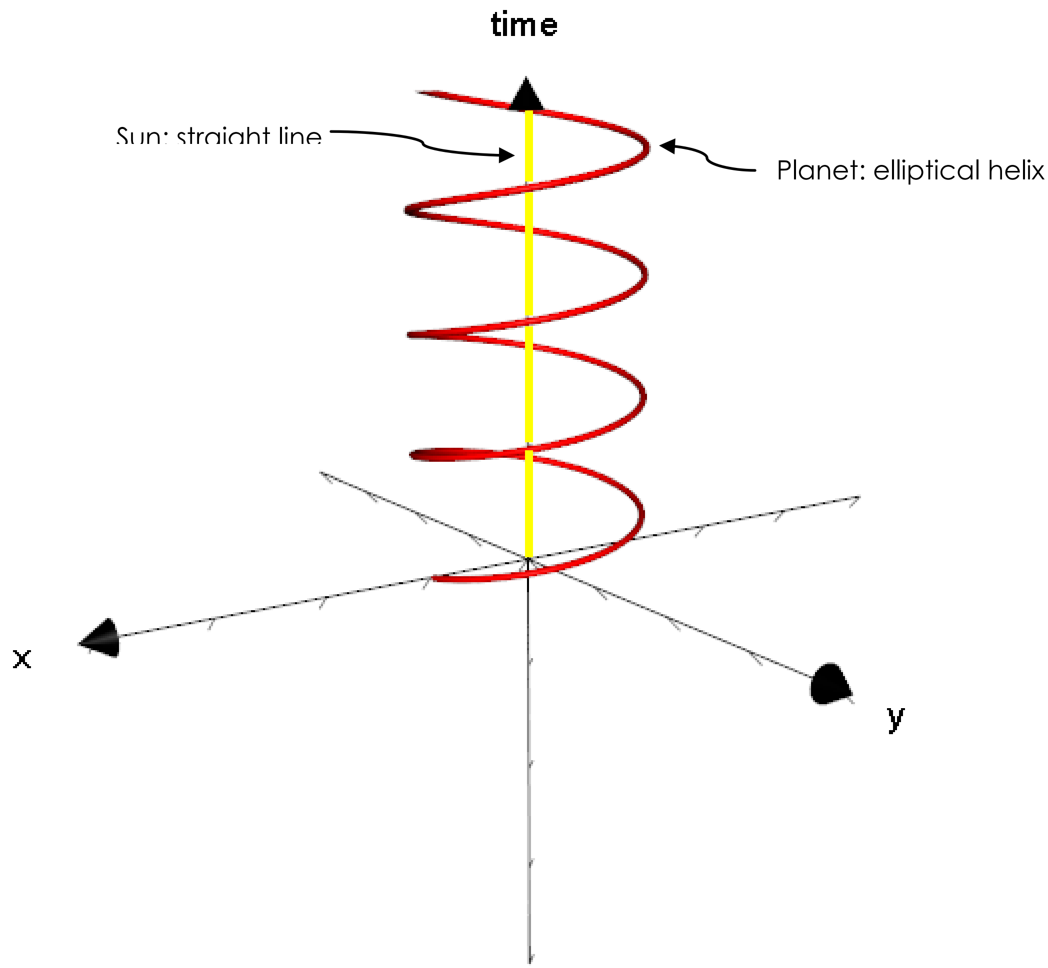
SPACETIME DIAGRAMS (II)

In this exercise you will sketch various spacetime diagrams. These are just rough sketches—they need not be drawn with a ruler.

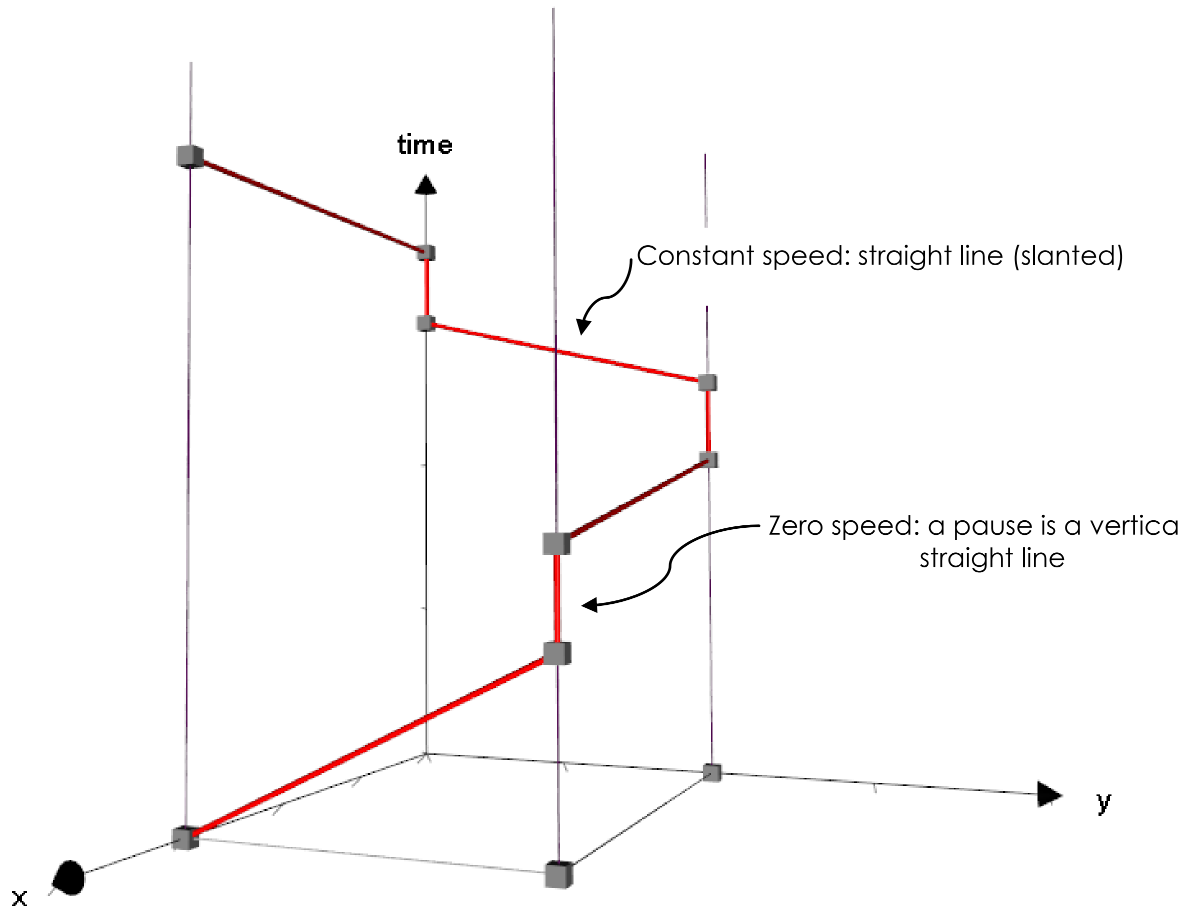
1. Sketch the spacetime diagram of a car, starting from rest, accelerating to a maximum speed, coasting at a constant speed for some time, and then decelerating to rest again.



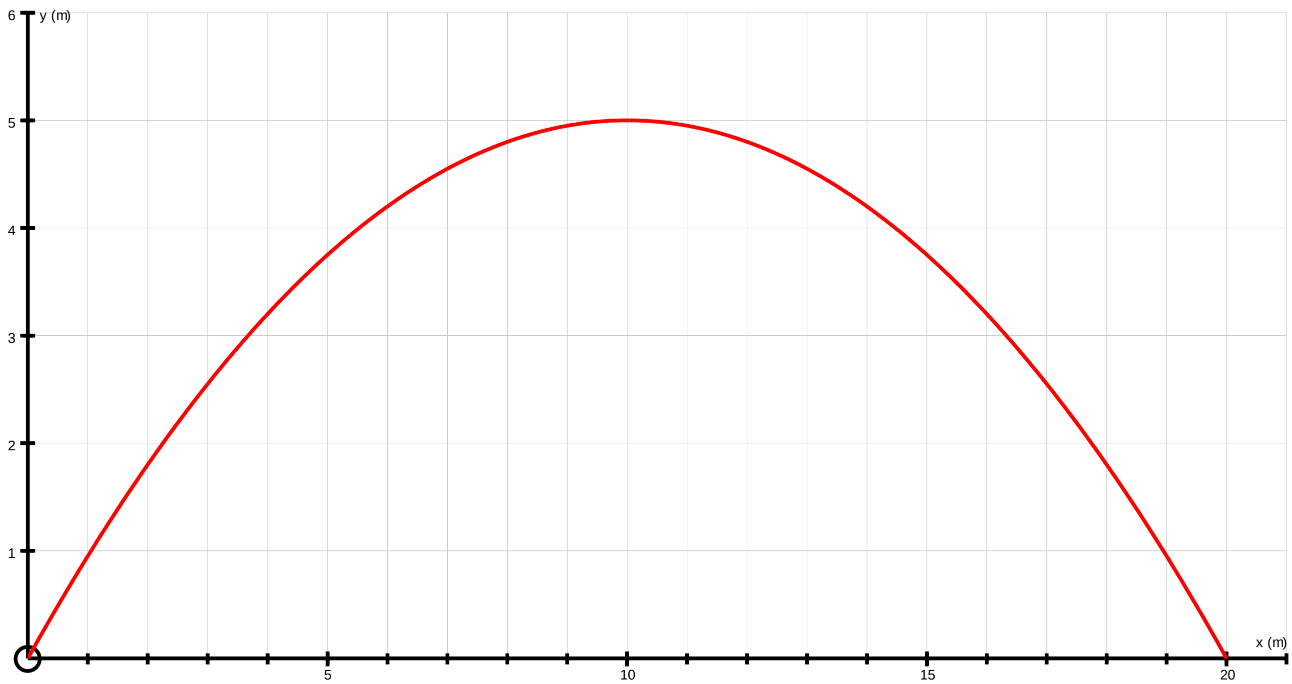
2. Sketch the spacetime diagram of a planet orbiting the Sun, assume that the planet mass is much less than the mass of the Sun. (Show both the planet and the Sun in the diagram.)



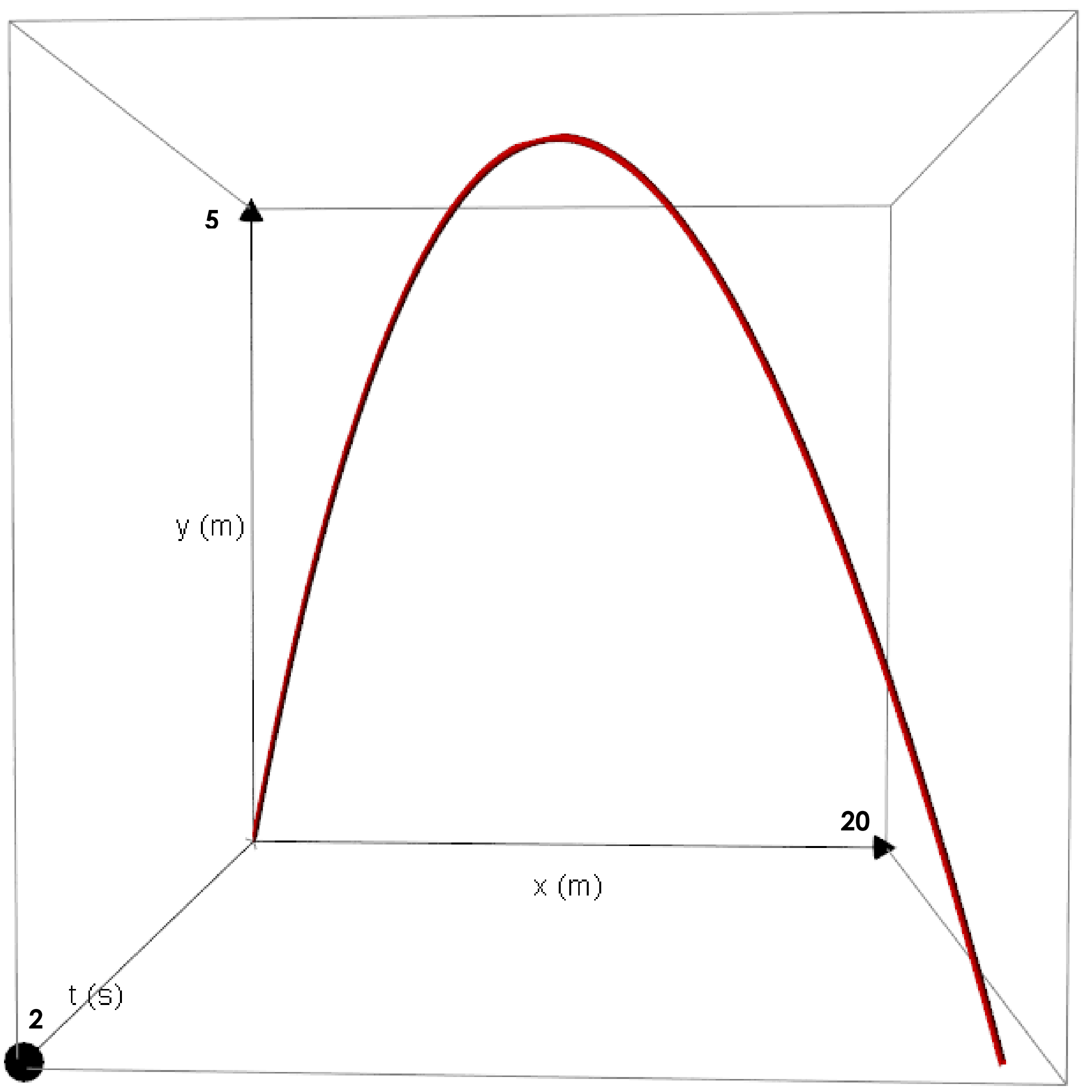
3. Sketch the spacetime diagram of **Bob**, who walks around the block at constant speed, pausing for a few moments at each corner.



[easy] 4a. Sketch the parabolic trajectory of a baseball through space (from the pitcher to the back catcher) with the **x** axis as the horizontal axis and ***y*** axis as the vertical axis. Based on your experience, estimate reasonable numbers for the range (**x** distance) and maximum height (***y*** distance) of the ball.



4b. Sketch the same trajectory, except now in spacetime. That is, sketch the baseball's worldline in ***t***, ***x***, ***y*** coordinates.



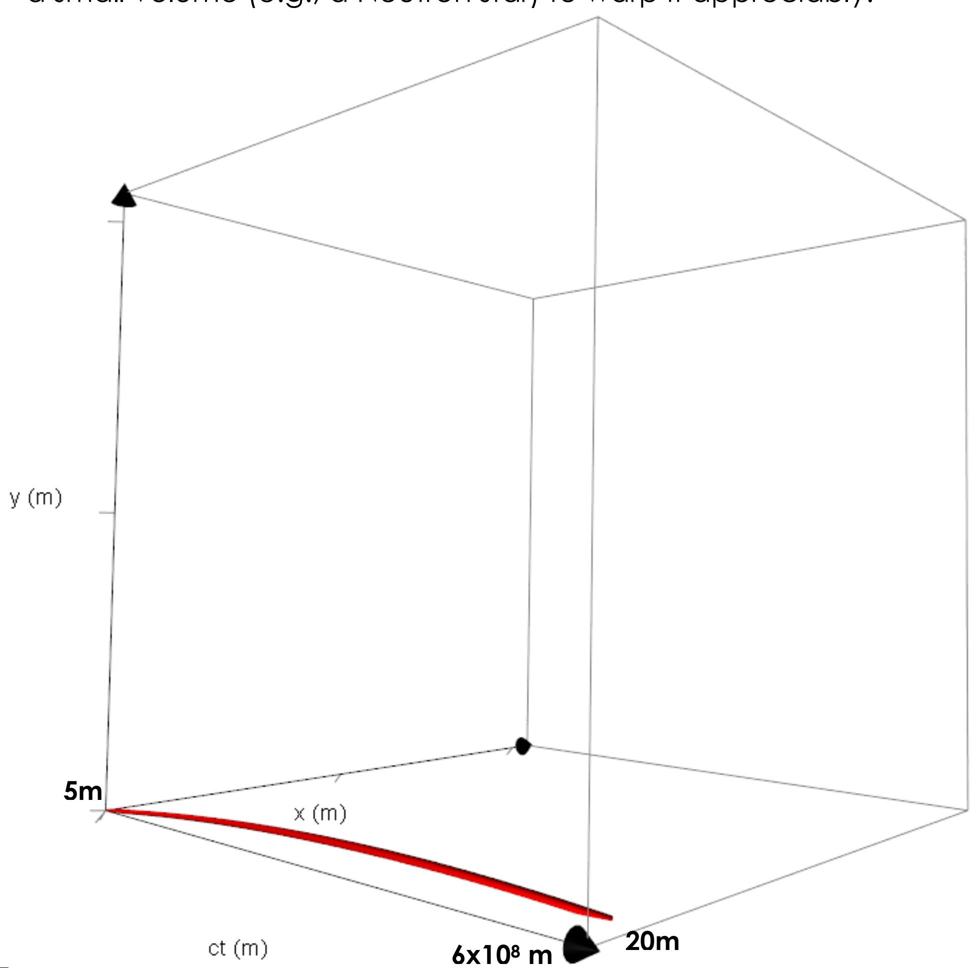
[medium] 5. Sketch the worldline in the more natural ***ct***, **x**, ***y*** coordinates, in which both space and time are treated on the same footing, and measured in the same units (metres). Notice that the curved parabola from **question *1b*** is now stretched to nearly a straight line. What do you think the slight curvature of this line might mean (physically or geometrically)?

Let's first determine the scaling on the ct axis

$$ct = \left(3\times10^{8}\,\frac{m}{s}\right)(2s)$$
$$= 6\times10^{8}\,m$$

This number is much much larger than 5m, or even 20m!

When we set the **ct** scale as $6\times10^{8}m$ then the **x** and **y** axes almost don't register. The parabola has been stretched to almost a straight line in spacetime. The slight deviation from the straight line (in this diagram) is a measure of how much the mass of the Earth warps the spacetime around it. As you can see the Earth provides very little warping. It takes a lot of mass in a small volume (e.g., a Neutron star) to warp it appreciably.



[hard] 6. Consider the simplest case of a baseball thrown straight upwards, reaching a maximum height ***h***, and then falling back to the ground. Using Newtonian kinematics, determine the elapsed time. Sketch the baseball's parabolic worldline in ***t***, ***y*** coordinates, and then the corresponding stretched parabola in ***ct***, ***y*** coordinates. Mathematically approximate the stretched parabola as an arc of a large circle of radius ***R*** (with both curves having the same height and chord). Assuming that ***h*** is very small compared to ***R***, make a reasonable approximation for ***R*** as a function of ***c*** and ***g*** (freefall acceleration near Earth's surface, $9.8\frac{\text{m}}{\text{s}^2}$). Observe that ***R*** is independent of **h**. Based on your answer in **question 2**, why must this be so?

First let's calculate the elapsed time for a ball to fall from a height **h**.

$$y_f = y_i + v_{yi}t + \frac{1}{2}at^2$$

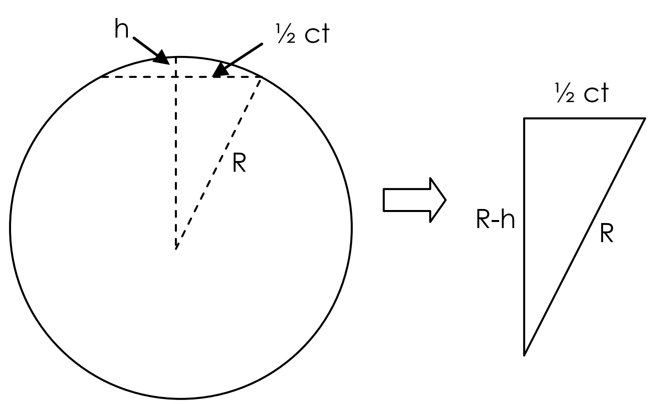
$$0 = h + 0 - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}}$$

The total amount of time for the ball's journey from being tossed to being caught is twice this, or $t = 2\sqrt{\frac{2h}{g}}$.

The sketches are almost identical to the sketches in question 1b and 2 except for the 5 and 20 axes markers.

Now if we approximate the stretched parabolic trajectory as an arc of a circle of radius **R**,



Then applying the **Pythagorean Theorem** to this right triangle

$$R^2 = (R-h)^2 + \left(\frac{1}{2}ct\right)^2$$

$$R^2 = R^2 - 2Rh + h^2 + \frac{1}{4}(ct)^2$$

$$0 = -2Rh + h^2 + \frac{1}{4}(ct)^2$$

Now we will apply the fact that **R>>h** . Therefore the term h^2 can be ignored since it too small to play any significant role compared to $(ct)^2$ and $-2Rh$.

Therefore

$$0 = -2Rh + \frac{1}{4}(ct)^2$$

$$2Rh = \frac{1}{4}(ct)^2$$

$$R = \frac{1}{8h}(ct)^2$$

Substituting our previously determined $t = 2\sqrt{\frac{2h}{g}}$

$$R = \frac{1}{8h}\left(c\left(2\sqrt{\frac{2h}{g}}\right)\right)^2$$

$$= \frac{c^2}{g}$$

$$= \frac{\left(3\times10^8 \frac{m}{s}\right)^2}{9.8\frac{m}{s^2}}$$

$$\approx 10^{16} m$$