Math Primer Exercises 3:
IMAGINARY & COMPLEX NUMBERS

[easy] Plot z, z^2, z^3, z^4, z^5 when $z = 1 + i$

[medium] Write $\sin(\theta)$ and $\cos(\theta)$ in terms of e.

Hint: use $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ and $e^{-i\theta} = \cos(-\theta) + i\sin(-\theta)$.

[hard] Using $e^{i\theta} = \cos(\theta) + i\sin(\theta)$, show that $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ and that $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

Hint: $\cos(\theta)$ is the real component and $\sin(\theta)$ is the imaginary component of the complex number $e^{i\theta}$.

[hard] For those of you with some Calculus experience:

Show that $\frac{d}{d\theta}(\sin(\theta)) = \cos(\theta)$ and $\frac{d}{d\theta}(\cos(\theta)) = -\sin(\theta)$ by doing the derivative of $e^{i\theta}$ and remembering that $\cos(\theta)$ is the real component and $\sin(\theta)$ is the imaginary component.

[hard] For those of you with more Calculus experience:

Determine $\int e^{-x}\cos(x)$ by using $e^{i\theta} = \cos(\theta) + i\sin(\theta)$, and recalling that $\cos(\theta) = \mathrm{Re}\left(e^{i\theta}\right)$ (the real component of $e^{i\theta}$).