The relevant appendices can be found under 'Related Documents' for each experiment at this link:
http://sky.campus.mcgill.ca/Exp/exp.a5w#

Compton Scattering

Updated in August 2018

Warnings:

- This experiment involves a strong radioactive source. Before starting the manipulations, you should get safety instructions from an instructor or a technician. In particular, do not look at the source from the wall;
- Do not move the lead surrounding the source without being assisted by an instructor;
- The detector accepts positive bias voltages of up to 950V. Apply and reduce the bias to the detector slowly;
- Never put the detector at angles close to 0° except if you put an absorber in front of it. The source is very strong, such that the phototube of the detector could be damaged due to the high counting rate at such angles.

Contents

1 Introduction

In this experiment, we will use a NaI scintillation detector with a photomultiplier to demonstrate the Compton effect and the quantization of light. Then, we will verify the Klein-Nishina scattering cross section formula, which represents the probability of a photon to be scattered from the target at a specific angle.

2 Theory

In the early 1900s, physicists were stupefied to find that the wavelength of scattered photons varied with the scattering angle. Arthur H. Compton was particularly interested in the subject and was struggling to find out the roots of this phenomenon. In 1922, he came up with the right interpretation of the results; he showed that the collision between a photon and an electron behaves like any classical collision between two objects and came up with the formula describing the change in the photon energy. This is now known as the *Compton effect* or as *Compton scattering*. For this discovery, he was awarded the Nobel Prize in Physics in 1927.

2.1 Frequency Shift

From Planck and Einstein, it is known that the energy E of a photon is $E = h\nu$ and that its momentum p is $p = \frac{h\nu}{c}$, where h is Planck's constant, ν is the frequency of the photon and c is the speed of light. In a collision between a photon and an electron at rest, using the law of conservation of energy and momentum and the relativistic equation $E^2 = p^2c^2 + (mc^2)^2$, the loss of energy of the photon as a function of the scattered angle θ is given by

$$\frac{1}{E} - \frac{1}{E_0} = \frac{1}{m_e c^2}(1 - \cos\theta) \tag{1}$$

where m_e is the rest mass of the electron. We can rewrite this equation as a change in wavelength λ. It is known as *Compton's equation*; a full derivation is given in Appendices A and E.

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta) \tag{2}$$

The change of wavelength is thus independent of the incident photon wavelength in Compton scattering. Since the Compton wavelength of the electron $(\frac{h}{m_e c})$ is 0.00243nm, we need incident light in that range to observe a significant change.

2.2 Cross Section

In nuclear and particle physics, the cross section σ represents the likelihood of interaction between particles. In our experiment, we are interested in the differential cross section with respect to the infinitesimal solid angle $d\Omega$ (written as $d\sigma/d\Omega$), which gives us the probability of finding a scattered photon at a specific angle.

Theoretical Predictions

From the definition of the cross section we have

$$\frac{d\sigma}{d\Omega} = \frac{\text{\# of particles detected at a given angle/unit solid angle}}{\text{\# of incident particles/unit area}}. \tag{3}$$

The same equation, written in radiation form, is

$$\frac{d\sigma}{d\Omega} = \frac{\text{energy radiated/(unit time} \cdot \text{unit solid angle)}}{\text{incident energy/(unit area} \cdot \text{unit time)}} \tag{4}$$

which, for our experiment, classically yields the *Thomson cross section*[1] equation:

$$\frac{d\sigma}{d\Omega} = r_0^2 \left(\frac{1 + \cos^2\theta}{2} \right) \tag{5}$$

where $r_0 = 2.818 \times 10^{-13}$ cm, the classical radius of the electron. However, equation (5) does not match experimental results since it does not take into account relativistic and quantum effects. Moreover, it was shown experimentally that there should be a dependence on frequency in the cross section. Taking these facts into consideration, we obtain the *Klein-Nishina formula*[2]:

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} \frac{1 + \cos^2\theta}{(1 + \alpha_0(1 - \cos\theta))^2} \left[1 + \frac{\alpha_o^2(1 - \cos\theta)^2}{(1 + \cos^2\theta)(1 + \alpha_0(1 - cos\theta))} \right] \tag{6}$$

where $\alpha_0 = \frac{E_0}{m_e c^2}$. Notice that we get the Thomson formula, multiplied by some factors.

Experimental Implications

Considering the apparatus we have at hand, we can use the definition of the differential cross section given in equation (3) to get the following relation:

$$\frac{d\sigma}{d\Omega} = \frac{I}{(\Delta\Omega) N I_0} \tag{7}$$

where I is the number of scattered photons that hit the detector during a given time, N is the total number of electrons in the target, I_0 is the flux density at the target and $\left[\Delta\Omega = (\text{crystal area})/r^2\right]$ is the detector solid angle, r being the distance from the target to the detector. The number of electrons in the target can be obtained by the following formula:

$$N = \frac{V \rho Z N_A}{M} \tag{8}$$

where V is the volume of the target, ρ its density, Z its atomic number and M its molar mass, and N_A is Avogadro's number.

We now need to obtain I_0 to be able to correlate I and $d\sigma/d\Omega$ through equation (7). It can be somewhat hard to measure; the problem lies in the fact that we cannot put the detector at 0° due to the high intensity of the radiation beam at this angle. A first solution could be to check when the source was made. The decay constant λ can be obtained with the simple relation $\lambda = \ln(2)/t_{1/2}$, where $t_{1/2}$ is the half-life of the source. Since 1 Ci corresponds to 3.7×10^{10} disintegrations per second, we have the photon density at the target:

$$I_0 = \frac{3.7 \times 10^{10} \times R_0}{4\pi r^2} e^{-\lambda t} \tag{9}$$

where R_0 was the initial radiation (in curie), t is the time elapsed since the source was made (in seconds) and r is the source-target distance (in meters). A second solution is to place absorbers in front of the detector at 0°. The value of I_0 can be extrapolated from the perpendicular collimated beam radiation attenuation relation:

$$I = I_0 e^{-(\mu/\rho)x} \tag{10}$$

where x is the thickness of the absorber, μ is the mass attenuation coefficient and ρ is the absorbers' density. A comparison between the yield and the Klein-Nishina formula may then be done with a renormalization of the data.

3 Equipment

The experimental setup is shown in figure (1).

[1]A detailed derivation can be found in Appendix A, p.372-375.

[2]The derivation requires the use of Quantum Electrodynamics and can be found in F. Gross, *Relativistic Quantum Mechanics and Field Theory*, Section 10.5, Wiley, New York, 1993.

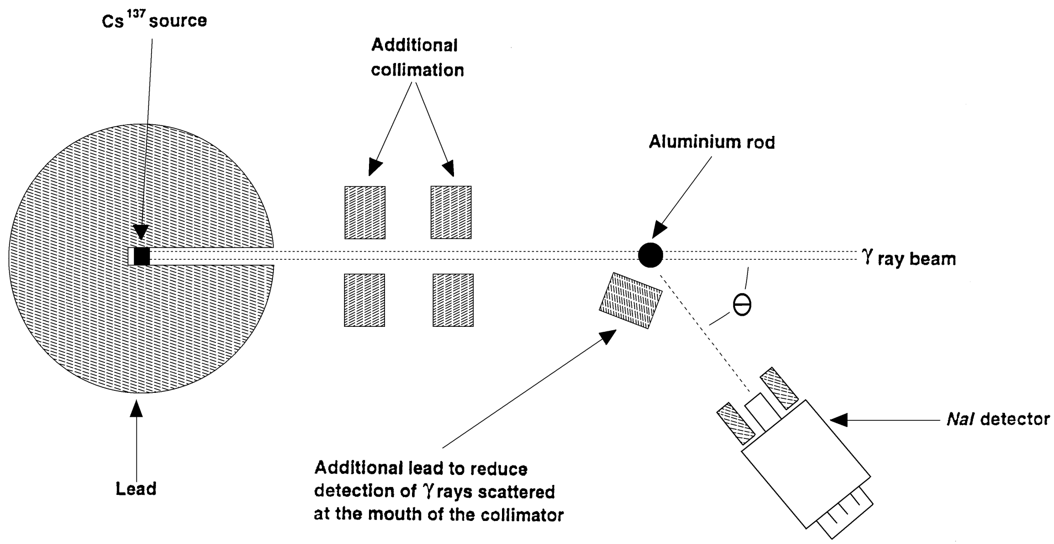

Figure 1: Experimental Setup. Do not move the lead without permission.

3.1 Sources

In this experiment, a beam of 661.6KeV gamma rays is obtained from a 0.1Ci source of ^{137}Cs. This isotope has a half-life of 30.17yr. This is the strongest source used of all experiments in the Lab in Modern Physics courses, so you must be especially careful. Do not look at the source from the wall, and do a maximum of manipulations with the tongs. In particular, get help from an instructor before opening the lead collimator in front of the source.

In addition to this source, you will use ^{22}Na, ^{133}Ba and (another weaker) ^{137}Cs sources for the energy calibration procedure. Both of these sources are much less dangerous and can be safely handled by hand. Ask a technician to obtain the sources from the radioactive source locker when the time comes to use them. The ^{22}Na source has a relatively weak energy peak at 511KeV and the ^{133}Ba source has two strong main peaks at 81KeV and 356KeV.

3.2 Detector

This experiment uses a NaI scintillation detector. Its basic function is to interact with the incoming gamma-rays to obtain all of their energy; the energy is then re-emitted in the form of photons, which are then sent to a photomultiplier tube (PMT) which uses the photoelectric effect to convert the incoming photon in an electron by the means of a photocathode, which is then multiplied by hitting successive dynodes. The outcoming electrons result in an output current pulse. More information on the physics behind the scintillator and the PMT is given in Appendices B and C.

The efficiency of the detector depends substantially on the energy of the incoming gamma rays. For the 2"x2" detector used in this experiment, this is pictured in figure (2). You must take into account this relative efficiency when you will compare results at different scattering angles.

The photomultiplier used in this experiment accepts positive bias voltages up to 950V, which is controlled in MCAs program on the computer (maestro). Go to MCB properties, and select high voltage. Do not exceed the maximum voltage.

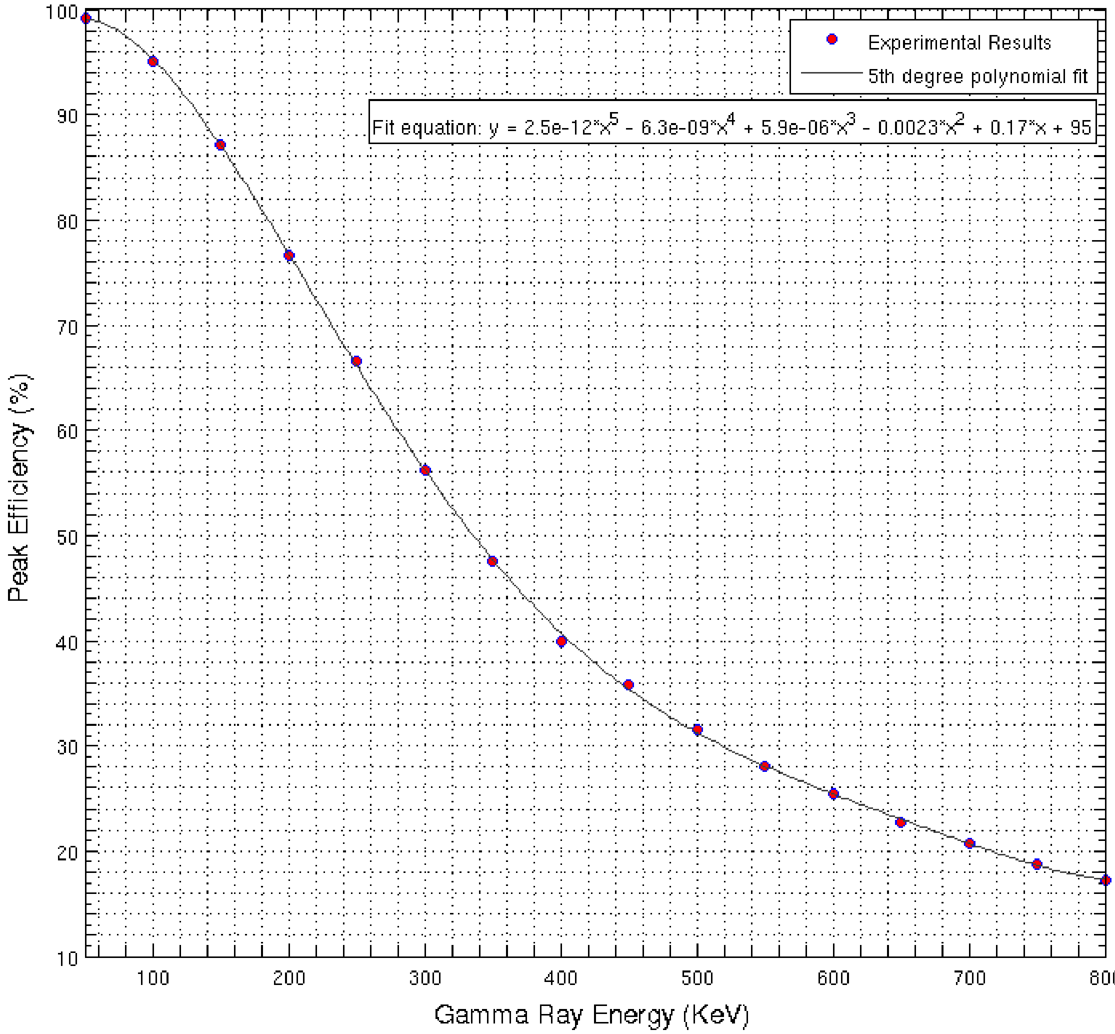

Figure 2: Efficiency of the NaI detector in function of the gamma ray energy

3.3 Scattering rods

You are given an aluminium rod, a copper rod and a stainless steel rod for this experiment. While the energy shift is the same for all rods, the absolute number of scattered photons differs as the density of the individual rods is different. In addition, their gamma ray mass attenuation coefficient is different. You may have to take into account this property if you expand the experiment; data on the mass attenuation coefficient on the diverse materials is given in Appendix D.

3.4 Electronics

3.4.1 Detector

In an usual counting experiment, you would have to use a detector connected to a pre-amplificator to get a steady signal, following which you would need another amplifier to get high enough voltage to use in a multi-channel analyser. Luckily, the Compton expriment makes use of a computer-controlled amplifier connected direcly to the multi-channel analyzer program. It is from Maestro that you can control the bias voltage applied to the detector. From now on, the brown instrument you attach to the detector will be refferd as the *preamp*.

Open up Maestro. Go in *Acquire - MCB Propreties*. From there, you can control the applied voltage from the *High voltage* tab, along with the amplification. As a good rule of thumb, for a bigger applied voltage, a same peak will be centered on a higher channel. The *Amplifier* tab also control the peak location. To prevent damage to the preamp, you want to always keep the voltage under 1100 volts, while adjusting the amplification to your needs. Generally, having 1.60 of amplification and 1000V is enough.

3.4.2 The Multichannel Analyzer - Maestro

Once started, Maestro can perform the following tasks:

- **(Alt-1) Acquire**: This starts the data acquisition. You should see counts accumulating in the bins of the graph. Pressing **(Alt-2)** stops the acquisition.
- **(Alt-3) Clear**: This resets all bins to zero counts for a new data acquisition.
- **Scaling the graph**: You may scale the graph in both directions by using the "Display" menu.
- **Saving and loading datasets**: You may save your data acquisition in MAESTRO's native file format (.chn) and load it back later through the "File" menu..
- **Region of Interest (ROI)**: The ROI functions allow you to select a specific region of the spectrum to obtain its total number of counts. You will use them multiple times during the experiment. Select a region with the mouse, right-click and select "Mark ROI". The region will become highlighted, and the bottom of the window will show the number of counts in the region when the cursor is above it. The "net area" uses the rest of the spectrum to do an approximate removal of the background noise - be very wary of using the value it returns.
- **Job control**: The computer may divide a data acquisition in multiple segments of a given time, as you will have to do when measuring the activity of the source as a function of time. MAESTRO does this by using code stored in a .job file, which you may use through the "Job Control" function in the "Services" menu. You will need to copy this piece of code [3] in a text editor, replace the text in caps by the values you want to use and save it as a .job file:

```
; Batch Process to do three acquistions and save to disk
; January 8, 2010
; Paste this code into a file called loop.job
; Replace text in caps by the value you want to use
; You may change the filename to prevent overwriting,
; however keep the ??? as they represent the iteration number of the loop.
set_detector 1
stop
set_preset_clear
set_preset_live TIME_ACQUISITION_PERIOD_IN_SECONDS
loop NUMBER_OF_ITERATIONS
        clear
        start
        wait
        stop
        save "file???.chn"
end_loop
```

[3]Which you may copy and paste for your convenience at http://www.ugrad.physics.mcgill.ca/wiki/index.php/Alpha_Decay#Maestro_Software

You may then follow the aforementioned steps to start the acquisition.

You will notice that two time measurements appear on the display - the “elapsed” time and the “real” time. The latter is simply synchronized with the computer clock and returns the total number of seconds of data acquisition. The former also accounts for the “dead time”. To illustrate dead time, suppose that you take a picture with a digital camera with the flash on; the camera will need a minimum time to take another picture with the flash as it needs to recharge. The computer, here, needs an analogous minimal time before it can register a second pulse - a dead time interval corresponding to that minimal time, during which the computer cannot register a second pulse, is then taken by the computer at every incoming pulse. The “elapsed” time removes these time intervals in order to get accurate counting rates in spite of this phenomenon - it is significant in experiments with very fast counting rates (such as the Compton experiment), and not-so-significant for experiments with lower counting rates (such as the Rutherford experiment or Alpha decay experiment).

3.4.3 Data extraction

If at any point, you desire to do computational analysis on the Maestro files, you may use the utility provided to the *Alpha Decay* experiment to extract data to Matlab. First obtain the function from: `http://www.ugrad.physics.mcgill.ca/wiki/index.php/Alpha_Decay#Spectrum_Tools`
You need to save your Maestro files as .Chn files. You can then feed it in the downloaded function. In your case, you are only interested with *get_spectrum.m*. The Matlab program can be found on various school computers, including the ones in the physics departement.

4 Procedure

4.1 Energy Calibration

In order to relate the channel counts to the energy levels, you will need to obtain peaks of known energy on Maestro. Put the calibration sources upright about 10cm in front of the detector and obtain a spectrum for each of them. Start with the ^{137}Cs source as it is the one of interest in this experiment. The amplifier gain should be adjusted so that its peak is near the end of the spectrum, but still fully visible. You should then see a single peak for the ^{22}Na source and two strong peaks for the ^{133}Ba source. For the latter, an even stronger third peak may appear at a slightly lower energy than the 81KeV peak – this is an X-ray emission due to electron capture in the decay process. Record the channel number of each peak, associate them with their respective energies and perform a linear fit to obtain the channel number/energy relation.

The bias applied to the detector and the amplifier gain are now set for the rest of the experiment; shall they be changed, the calibration will be void.

4.2 Determining the True Zero Angle

There might be a misalignment between the gamma-ray beam axis and the rotatable part of the detector, such that the true zero angle is not the same as the reading’s zero angle. You will need to account for that by taking measurements on both sides of the indicated 0° position, then by adjusting the zero angle so that the distribution is symmetrical about it. This might be done by fitting the same function on both sides of the distribution, then by finding the value of the angle at the intercept of the fits.

You may also simply superpose data from both sides of the zero angle, and see if they differ. As you can see, both methods involve extracting the data from Maestro. Refer to section 3.4.3 to easily import data into Matlab.

4.3 Frequency Shift

Verify the Compton Scattering law (equation (1)) using different values of θ which you should keep over 15° to protect the detector and as the contribution from the non-scattered beam is too substantial below this value.

For each angle, you should use Maestro to first add counts over a certain period of time (400s is reasonable) with the target in place, then to subtract counts over the same period of time with the target removed. Up to statistical fluctuations, this will remove the background spectrum from the Compton spectrum.

A plot of the shift of the inverse energy against $(1-\cos(\theta))$ should be linear. From the slope of the linear fit, then calculate the rest mass m_e of the electron.

4.4 Differential Cross Section

This uses the same dataset. With the ROI functions, select a part of the peak (be consistent with your choice – you may choose to take the full width at half the maximum of the peak, for example) to obtain its total number of counts. After dividing by the time to obtain the counting rate, you may verify the Klein-Nishina formula. Do not forget to use the efficiency curve to correct the yield for the various energies. Additionally, remember that, at this point, the Klein-Nishina formula will need to be renormalized to fit your data as you did not measure the absolute cross-section, which would require a determination of the beam flux and of the absolute efficiency of the detector.

4.5 Measurement of the Flux Density

You will notice a bunch of concrete bricks of similar dimensions and composition around the experiment. Those are not simply present for inconvenience. The flux density of the source at the detector can be computed by putting a number of bricks in front of the detector at the zero angle (without any target) and by obtaining the counting rate for four, three, two and one (not zero, as it may fry the detector!) bricks. From equation (10), you may then obtain I_{D_O} (which replaces I_O, the flux density at the target) by fitting an exponential distribution and extrapolating for a case without attenuation (no bricks). Again, note that, after correcting for the detector efficiency (see figure (2), I_{D_O} is the flux **over the surface of the detector**. What you want is the flux density (in gamma-ray per area per unit time) **at the target**. The latter can be obtained by a simple manipulation based on the geometry of the situation:

$$I_0 = \frac{I_{D_O}}{r_{TS}^2 d\Omega_{detector}} \tag{11}$$

Where r_{TS} is the distance from the target to the source.

Since you are given the initial activity of the source, its age and the solid angle of the rod, you may also compute the flux density geometrically, as discussed above; you may compare the two values.

4.6 (Optional) Absolute Number of Counts for Various Targets

You may repeat the experiment with the other rods to verify how the different gamma ray mass attenuation coefficients for the different materials influence the normalization factor you must apply when comparing to the Klein-Nishina formula. As mentioned, data on the mass attenuation coefficients is given in Appendix D.

5 Goals

In short, you must:

- Calibrate the energy scale of the MCA;
- Verify the relation for the Compton scattering and compute the rest mass of the electron;
- Compare the results for the differential cross section with the (renormalized) Thomson and Klein-Nishina formulas;
- Obtain the flux density at the detector of the radioactive source by two different means;

- (optional) Compare the values for the absolute number of counts with the different targets;
- (optional) You may try to see if the two steel rods are made of a different alloy;

Good luck!

References

[1] Jerome L. Duggan. *Experiment 10: Compton Scattering.* EG&G ORTEC, `http://www.ortec-online.com/application-notes/an34/an34-content.htm`.

[2] Gordon Gilmore. *Pratical Gamma-Ray Spectrometry.* John Wiley & Sons Inc. England, 2008.

[3] Glenn F. Knoll. *Radiation Detection and Measurement.* John Wiley & Sons Inc., New York, 1979.

[4] Adrian C. Melissinos and Jim Napolitano. *Experiments in Modern Physics.* Academic Press, USA, 2^{nd} edition, 2003.

[5] Paul A. Tipler and Ralph A. Llewellyn. *Modern Physics.* W. H. Freeman and Company, New York, 5^{th} edition, 2008.

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