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Abstract

Optical systems, due to the wave nature of light, are complex and difficult to model mathematically. One technique to simplify these systems is known as Fourier optics, allowing the use of common aspects of Fourier analysis. This model is based on the Fraunhofer approximation which requires the resulting image to be viewed at an infinite distance from the object. This approximation, and thus the Fourier optics model, can be used when a converging lens projects this infinite image to a focal point. Throughout this lab we test the validity of Fourier optics by analyzing the image produced by different diffracting objects. We first verified the Fraunhofer approximation with both a single slit object and a diffraction grating. Using Fourier analysis we found the width of the single slit to be 0.075 ± 0.002 cm, which is consistent with the measured value of 0.10 ± 0.05 cm. This was also preformed with a multi-slit grating, giving a theoretical value of 0.034 ± 0.002 cm which is in agreement with the actual value of 0.03 ± 0.01 cm. We next looked at a common tool of Fourier analysis, the convolution theorem. By showing that the aperture function of the diffraction grating is an convolution between a rectangular function and the sum of delta functions, we showed the authority of the convolution theorem in this system. In all we were able to verify that Fourier analysis is a simple and powerful tool when analyzing complex optical systems.

Contents

1 Introduction

Fourier analysis is a ubiquitous tool used throughout physics and engineering. Its success is based on its ability to simplify common differential equations, such as the wave equation. One particular use of Fourier transforms is modeling the effect of optical setups [1]. In this lab we will be confirming the accuracy of this theory through its ability to model the diffraction pattern caused by various images.

Light exhibits interference properties due to its wave structure. Quantitatively light can be represented by a complex function, giving it an amplitude and a phase [2]. When two waves combine, depending on the phase of both waves, there can either be constructive or destructive interference. We are using monochromatic laser light thus if the path length between two waves is an integer multiple of the wavelength there will be constructive interference.

We will be analyzing the diffraction patterns from multiple different objects. In this context "object" refers to a partially transparent image such as a mesh pattern or grating. These diffraction patterns result in the interference of many waves traveling through different openings in the object. The propagation of light through the object can be modeled using the Huygens-Frensnel Principle [2]. It states that each point on a wavefront acts as a source of light. The resulting diffraction pattern can then be found by the interference of each one of these new sources of light. Equation 1 describes the wavefront at a distance z from the object [3]. To find the diffraction pattern, the intensity, one can take the square of this function.

$$A'(x', y') = \frac{z}{i\lambda} \iint_{-\infty}^{\infty} A(x, y) \frac{e^{ikr}}{r^2} dxdy. \tag{1}$$

Here $A(x, y)$ is the aperture function, a 2D function describing the object. The aperture function fluctuates between 0 and 1, being 0 at points where the object is opaque and 1 where is is fully transparent. Here r is the distance from (x, y) on the object to (x', y') on the produced image, λ is the wavelength of the light.

The Frensnel approximation can be used to simplify the diffracted wavefront equation [3]. Taking the observation plane to be far from the object, we can expand r using a Taylor series.

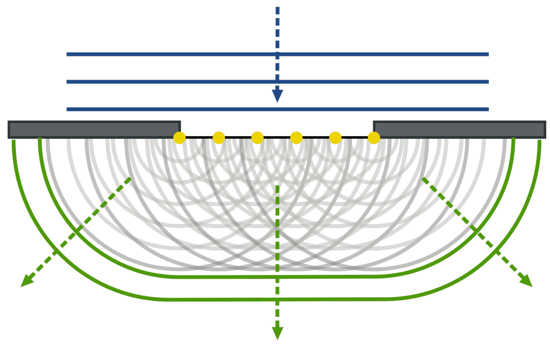


Figure 1: Visualization of Huygens Principle.

$$r \approx z + \frac{(x - x')^2}{2z} + \frac{(y - y')^2}{2z}. \tag{2}$$

We take a further approximation, known as the Fraunhofer approximation, by taking the observation plane to be at infinity [2]. This allows the second term in equation 2 to be omitted. Updating equation 1 with these approximations gives equation 3.

$$A^{'}(\nu_x, \nu_y) = C \iint_{-\infty}^{\infty} A(x, y) e^{i2\pi(x\nu_x + y\nu_y)} dxdy. \tag{3}$$

Experimentally, the square of equation 3 is the intensity of the signal which is observed thus the phases can be replaced with a constant, C. It is given by: $C = \frac{1}{i\lambda z} e^{ikz} e^{i\pi\lambda(\nu_x^2 + \nu_y^2)}$, where k is the wavenumber $k = 2\pi/\lambda$ and λ is the wavelength of light. Here $\nu_x = \frac{x'}{z\lambda}$ and $\nu_y = \frac{y'}{z\lambda}$, which are variables of spatial frequency. This leaves the mathematical representation of the intensity as the familiar Fourier transform of the aperture function [3]. These approximations allow the modelling of optical systems using Fourier analysis. Fourier optics makes use the the spatial frequency domain and space in contrast to frequency and time use in common Fourier analysis.

One practical application of Fourier analysis which was implemented in this lab is the convolution theorem. This states that a convolution in the frequency domain is equivalent to direct multiplication the the spatial domain as seen in equation 5. It is also true that a convolution in the spatial domain is a multiplication in the frequency domain [4]. In our application this theorem is used to simplify the aperture function of certain objects, as it is often defined as a multiplication in Fourier space and a convolution in real space.

$$F\{f \cdot g\} = F\{f\} * F\{g\}. \tag{4}$$

Here the dot represents multiplication where as the star is a convolution. The convolution operator is defined below.

$$(f * g)(t) = \iint_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau. \tag{5}$$

2 Experimental Set-up

In order to observe the Fourier model of light we need to observe the diffraction pattern at an infinite distance, allowing the Fraunhofer approximation to be applied. A converging lens allows the signal at a near infinite distance to be directed to a focal point, thus converging lenses are essential. The experimental setup is shown in figure 1.

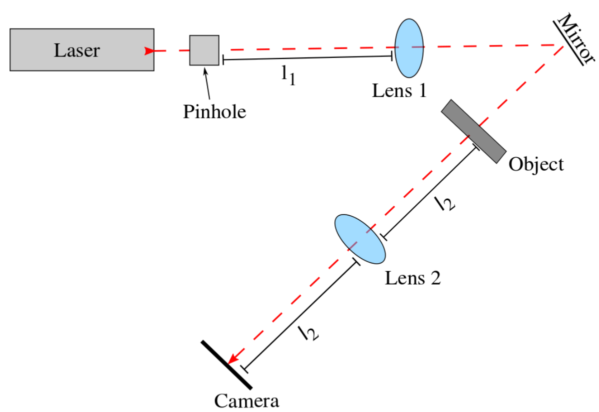


Figure 2: Experimental set-up for the frequency analyzer. The laser beam is incident upon a 10X microscope lens. Lens 1 and lens 2 are two converging lenses with focal lengths l_1 and l_2 respectively. In this particular case, $l_1 = 30$ cm and $l_2 = 50$ cm. The object consisted of a variety of different diffraction gratings.

A HeNe laser with wavelength 632.8 nm is incident on a pinhole which focuses the light. A converging lens is then used to collimate the light such that plane waves will be incident on the object. This is achieve by placing converging lens 1 at a distance l_1 from the pinhole, where l_1 is the focal length of lens 1. The mirror is then used to change the direction of light to fit our spatial constraints. The plane wave light is the incident on the object, causing a diffraction pattern. Lens 2 is placed its focal length both from the object and the camera, allowing the image of the diffraction pattern at infinity to be projected onto the camera. A Canon ESO Rebel T5 camera was used without the lens to take high-quality digital images of the diffraction patterns.

3 Analysis

All images taken during the experiment were analysed using python's skimage package. With this tool, each pixel has an associated relative intensity value. In order to obtain a functional representation of the diffraction pattern the intensity values were averaged along the vertical lines of the photo. The resulting data, shown in 3, is the average intensity along the vertical axis as a function of the horizontal axis. The top image is the imported photo, and the bottom graph the average intensity. The image in figure 3 was taken from a single slit object.

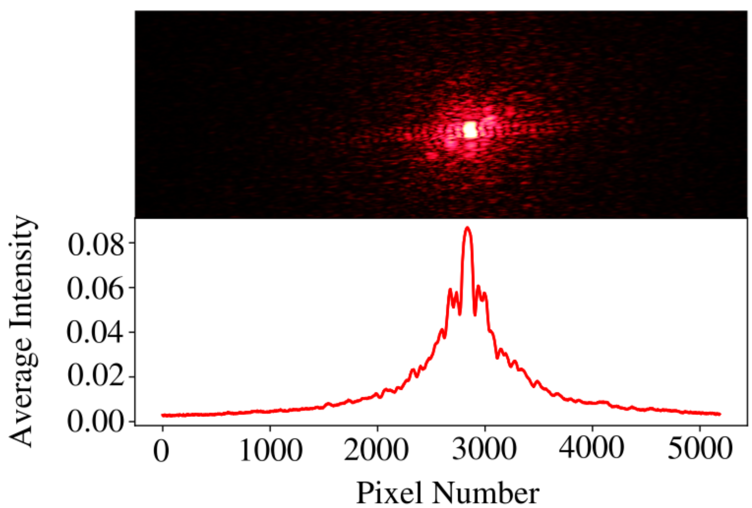


Figure 3: TOP: Sample photograph taken for a single-slit aperture of width 0.10 ± 0.05 cm. BOTTOM: plot of the average intensity of the columns of pixels in the image.

In order to perform any further analysis on the intensity plots, an estimation of the uncertainty of our measurements was necessary. The uncertainty comes from random scattering of the laser beam through dust and minor scratches on the mirror. This can be seen in figure 3 as the dispersed red dots of light. In order to quantify this uncertainty, 10 different images taken with the same single slit were compared. Before each image was taken, the mirrors and lenses of the set-up were wiped down with a kimtech optical wipe. Since the set-up was otherwise unchanged, all images should be the same. In particular, the maximum intensity of each image should remain constant. The uncertainty on the intensity was thus obtained by plotting the maximum intensity value of each image and fitting them to a constant value. The uncertainty on intensity was manipulated until the expected χ^2 of approximately 1 was obtained. This plot with its residuals is shown in figure 4. The uncertainty on intensity measurements was estimated to be ± 0.003 for a reduced $\chi^2 = 1.04 \pm 0.49$. Similarly this analysis was preformed to get an uncertainty on the spatial frequency. We found an uncertainty of ± 0.002 with a reduced $\chi^2 = 0.96 \pm 0.32$.

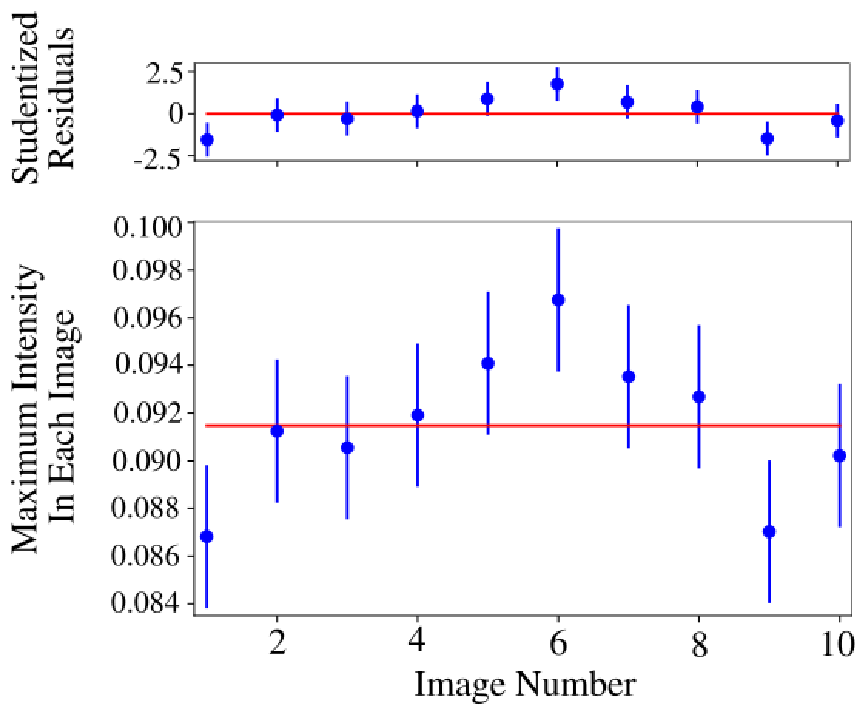


Figure 4: Plot comparing the maximum intensity in 10 different images. Each image was taken from the same single slit aperture and using the same experimental-set. The maximum intensities are fit to a constant line, with reduced $\chi^2 = 1.04 \pm 0.49$ when an uncertainty of ± 0.003 was introduced. The residuals plot of this fit is shown above.

3.1 Verifying Fraunhofer Approximation

Using the layout outlined in the experimental setup section we first analyzed a single-slit object and a multi-slit diffraction grating to verify the Fraunhofer approximation. We performed two seperate analyses to confirm that this setup can be modeled with a Fourier transform. First we found the width of the single slit using Fourier analysis and compared it to the measured value found with a travelling microscope. Secondly, we further verified the

Fraunhofer approximation by comparing the calculate slit width of the multi-slit object and the measured value.

For a single slit, the aperture function is known to be a rectangle function, given by equation 6.

$$\mathrm{rect}(\frac{x}{w}) = \begin{cases} 0 & |x| \geq \frac{w}{2} \\ 1 & |x| \leq \frac{w}{2} \\ 1/2 & |x| = \frac{w}{2}. \end{cases} \tag{6}$$

Here, x is a position along the grating and w is the width of the slit. According to Fraunhofer's approximation, the observed diffraction pattern seen from such an aperture function should be equal to the square of the aperture function's fourier transform. This gives an equation for the intensity of the diffraction pattern as a function of the position along the screen. The intensity of the diffraction pattern for a single slit is given by equation 7 [5].

$$\mathrm{I(x)} = I_0[\frac{\sin(\pi a x/\lambda D)}{(\pi a x/\lambda D)}]^2. \tag{7}$$

In equation 7, x is the distance from the brightest spot, a is the width of the slit, D is the focal length of the converging lens, λ is the wavelength of the laser and I_0 is the intensity when $x = 0$. Where the width of the slit is given by equation 8 [6].

$$\mathrm{a} = \frac{D\lambda}{x_m}. \tag{8}$$

Here D and λ are defined as before, and x_m is the distance from the maximum intensity point to the first minimum. The value of x_m was found from our data using the relation that 1 pixel = 0.00000429 m, for this specific camera. Figure 5 shows the resulting single-slit intensity versus distance plot centered at the maximum peak. From the experimental equipment $D = 0.3$ m and $\lambda = 632.8$ nm. From these values a slit width was calculated to be $a = 0.075 \pm 0.002$ cm. In comparison the width determined by the traveling microscope is $a = 0.10 \pm 0.05$ cm. Since both values of a agree with one another, we conclude that the Fraunhofer approximation is valid in our set-up.

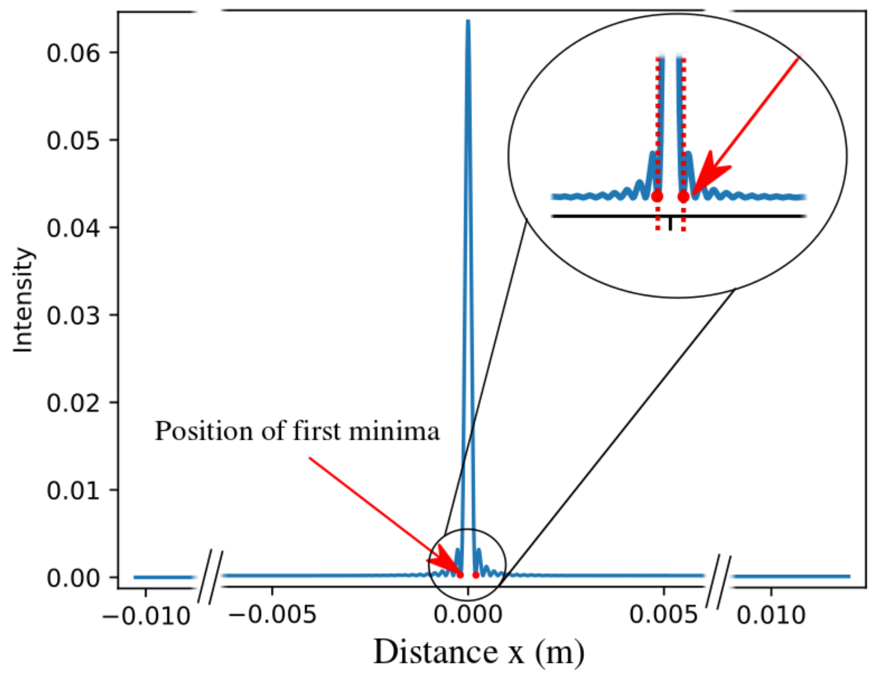


Figure 5: Intensity versus distance x from the center of the diffracting grating of a single slit diffraction. The two red dots indicate the position of the first minimum of the intesity. The two positions are determined to be 0.025 cm away from the origin.

To further confirm this result, a different aperture was analyzed; a periodic, multi-slit grating. Using a traveling microscope, the width of one slit was measured to be: $a = 0.03 \pm 0.01$ cm. Using the Fourier transform to calculate the width, one needs to measure x_m as the distance from the maximum peak to the first minimum. This is done with the same method as in the single slit analysis. From equation 8 and the diffraction pattern, the width of one slit was calculated to be $a = 0.034 \pm 0.002$ cm. Once again, this agrees with the directly measured result where $a = 0.03 \pm 0.01$ cm strengthening the conclusion that the Fraunhofer approximation holds. Hence, the experimental set-up can be used to explore properties of Fourier transforms.

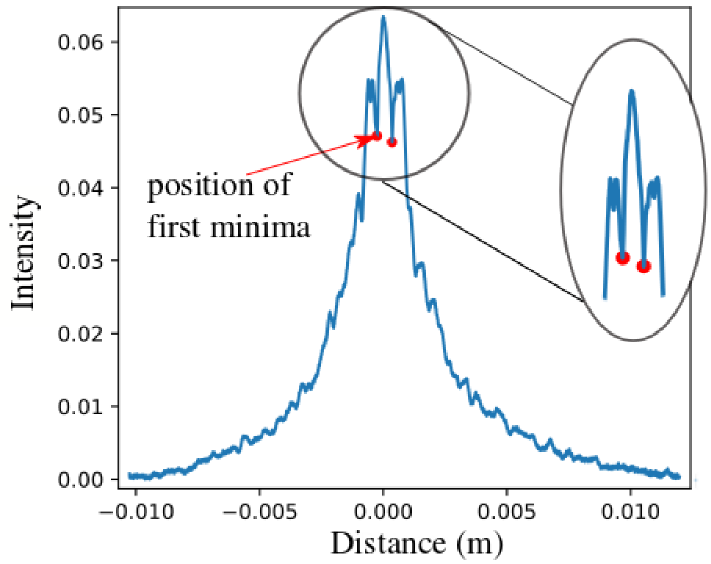


Figure 6: Intensity versus distance x from the center of the diffracting grating of a multi-slit diffraction. The two red dots indicate the position of the first minimum of the intesity. The two positions are determined to be 0.055 cm away from the origin.

3.2 Convolution Theorem

An invaluable tool in Fourier analysis is the convolution theorem. As outlined in the introduction, a convolution in the spatial domain is equivalent of multiplication in the Fourier domain. We can show this theorem is valid by further analyzing the diffraction grating used in the previous section. The aperture function used in equation 3 for a diffraction grating is the multiplication of a sum of delta functions, known as a comb function, and a rectangle function. The comb function is defined in equation 9 and the rectangle function in equation 6.

$$\text{comb} = \sum_{n=0}^{N} \delta(t - ns). \tag{9}$$

Where N is the number of slits in the grating and s is the distance between the two slits. As the aperture function is defined in Fourier space, this multiplication is equivalent to a convolution in real space. We can show that this is the case by dividing out the Fourier transform of the rectangle function from the intensity data. The Fourier transform of the rectangular function is given by equation 7. By dividing the intensity data by the intensity of a single slit, multiple delta functions were observed as expected. These delta functions are the Fourier transform of the comb function described above. Figure 7 shows the resulting sum of delta functions, showing the validity of the convolution theorem.

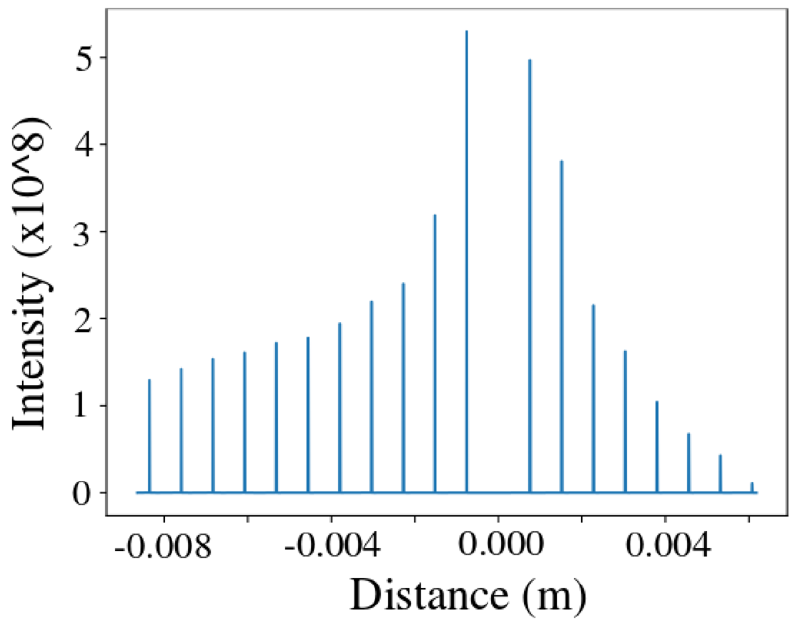


Figure 7: The resulted division of the intensity data by the intensity of a single slit. Those lines spiking up very high with narrow width indicate the characteristic of delta function. The spatial separation of those line are effectively similar, showing that the slits in the diffraction grating are evenly spaced.

4 Conclusion

Fourier analysis is a concise and powerful theory, its success coming from its ability to simplify complex systems. Throughout this lab we showed Fourier analysis, through the application of the Fraunhofer approximation, can be used to model optical systems. We verified the use of this approximation in a system with a converging lens using two different objects. The theoretical model, a Fourier transform, used agreed with the physical results in both methods. From there we explored a useful aspect of Fourier analysis, the convolution theorem. This theorem allows complex Fourier transforms to be split into simpler components. We showed that this theorem can be used to simplify the aperture function of a diffraction grating by proving it is the convolution of a rectangle function and a comb function. Overall, Fourier optics allows the interference of electromagnetic waves to be simply modeled, allowing the prediction of the diffraction pattern of light through different objects.

References

[1] J. W. Goodman, *Introduction to Fourier Optics.* W. H. Freeman, 2004. 1

[2] D. Jones, *Fourier Optics*, 2010. 1, 2

[3] J. W. Goodman, C. R. . C. Englewood, Ed. W. H. Freeman. 1, 2

[4] K. Howell, *The Transforms and Applications Handbook: Second Edition.*, 1st ed., A. D. Poularikas, Ed. Boca Raton: CRC Press LLC, 2000. [Online]. Available: http://dsp-book.narod.ru/TAH/ch02.pdf 2

[5] M. L. Boas, *Mathematical Methods in the Physical Sciences*, 3rd ed. Wiley, 2005. 6

[6] F. A. Jenkins and H. E. White, *Fundamentals of Optics.* McGraw-Hill College, 1976. 6

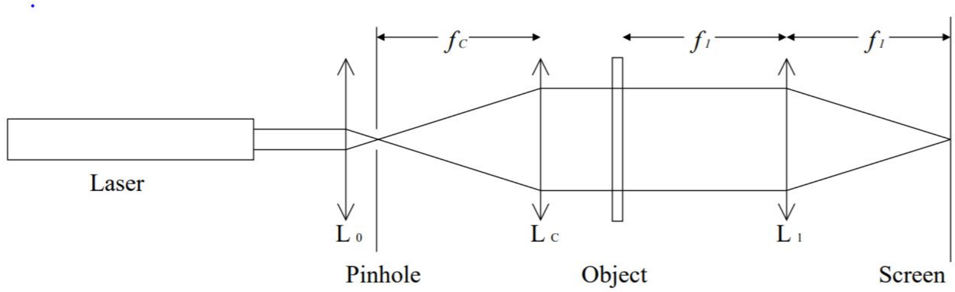
Fourier Optics Lab Book

Feb 4, 2019:

- Goal of the day:
  - Go over lab manual and get understanding of what the experiment is and what different steps we need to take - will allow us to manage our time and organise ourselves.
  - Go over equipment to make sure we have everything needed
  - Background research.
    - Starting point check out youtube video for better idea of what experiment does: https://www.youtube.com/watch?v=wcRB3TWIAXE

Fourier Optics Experiment Outline:

1. Frequency Analyzer:



- take photos of the resulting diffraction pattern
- the object is a diffraction grating which you can change
  - start with a simple grating and then try out more complicated ones to see what the resulting pattern looks like.
  - analyze the pattern to see if it agrees with theory

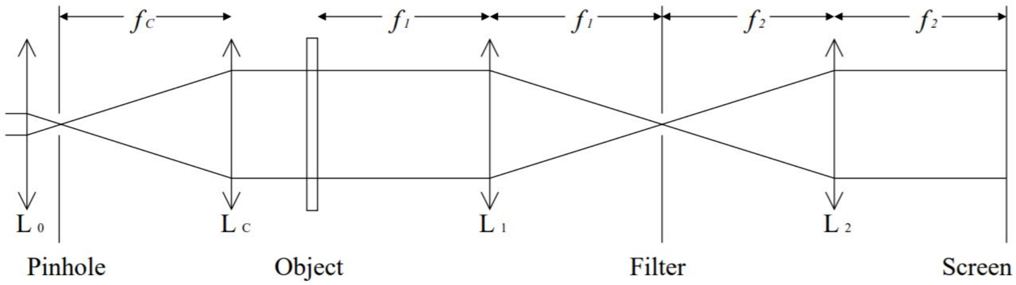
2. Uni-dimensional Periodic Functions:

- using any slides that present a periodic pattern will find spatial frequency using two methods
- one if using is traveling microscope to find the distance between gratings lines
- other is by measuring the diffraction grating and using equation 21
- try with other gratings (different directions, spherical)

3. Non-Periodic Functions:

- simple slit are non-periodic (pulse functions)
- figure out an analogous bandwidth theorem for position spatial frequencies
- show its valid by measure patterns of slits with different width

4. Inverse Fourier:



- test with this setup with any slide and verify it reconstructs

5. Spatial Filtering:

- want to be able to remove a part of the Fourier transform before transforming the diffraction pattern back to the image
- the zero frequency is found at the optical axis so a low pass filter only lets light near the origin pass where as a high pass filter only lets like radial away from the origin to pass

5.1 Uni-dimensional Filtering:

- build a high pass and low pass filter and test

5.2 Bi-dimensional Filtering:

- take a slide with different spatial frequencies (14, 20, 21) determine which regions have higher or lower spatial frequencies
- build a high pass and low pass filter
- test using image of a grid (slide 11) get only diagonal dots to pass

5.3 Image Processing:

- use cloud chamber simulation photograph (22) remove large tracks leaving only the curved ones using appropriate filters
- build filters which let you individually reconstruct the letters in the AB slide
- build filter which removes dots from photos of half-tone image (side 24)

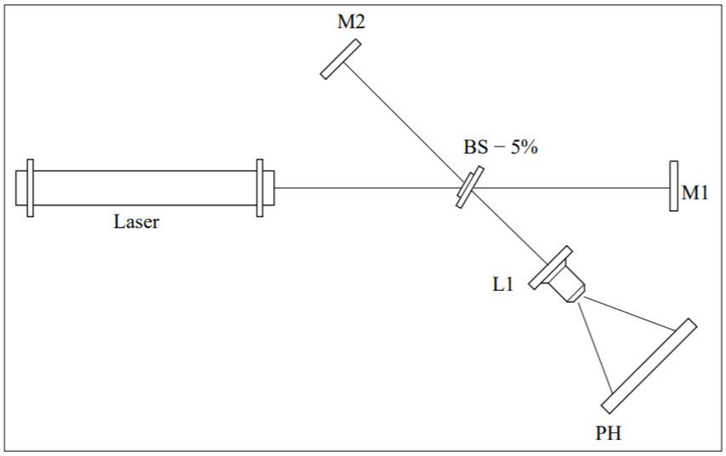
5.4 Supplemental Activities: *If time permits

- do things you find in textbooks
- prove the convolution theorem for Fourier transform

6. Holography:

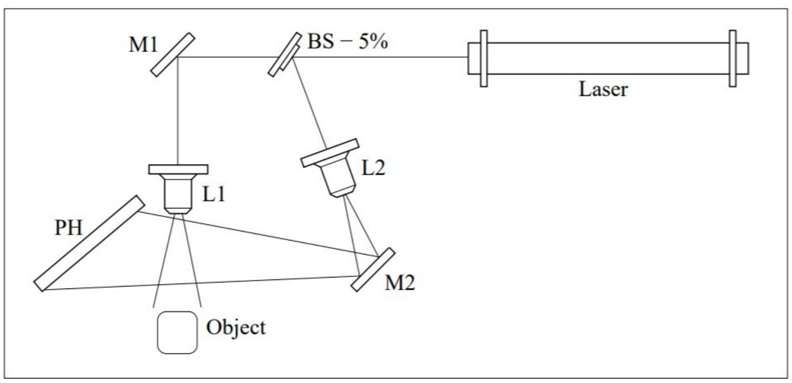
- a hologram is able to reconstruct a wavefront, we will use holographic film to do this

6.1 Michelson Interferometer:



- want system to be as stable as possible

6.2 Transmission Hologram:



Feb 7, 2019:

Goal of the Day: Perform the first set-up and test it out with different gratings to gain a good understanding of how to position the different lenses and mirrors.

- Began initial setup of fourier transform
- Was able to get all but the camera set up

- CAREFUL to make sure the microscope lens is in the good way - resulting beam should not look like a tiny dot!

Feb 11, 2019:

Goal: Figure out how to take good photos of the diffraction pattern and being image analysis process.

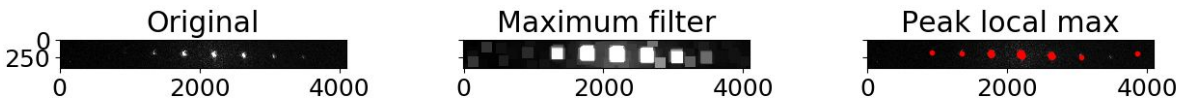
- Played with camera settings to get best photos
- EQ1409 (grating?)
    - Found best results with 1s shutter speed, F13, ISO800 (100-0670, 100-0669)

EQ1408:
- 0.8s, F13, ISO800 (100-0675)

EQ1415:
- 0.8s, F13, ISO800 (100-0680)

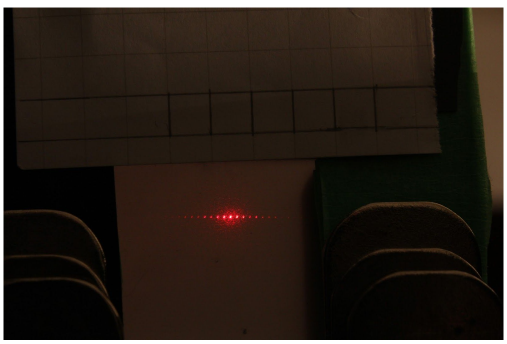
Began initial image processing:

- able to identify dot position and trying find line which they all lie on
- plan to then get brightness value along that line which we can then fit with gaussian

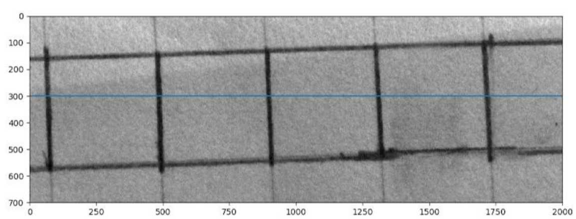


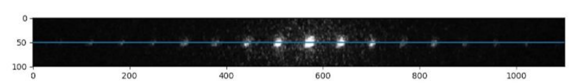
Feb 13, 2019:

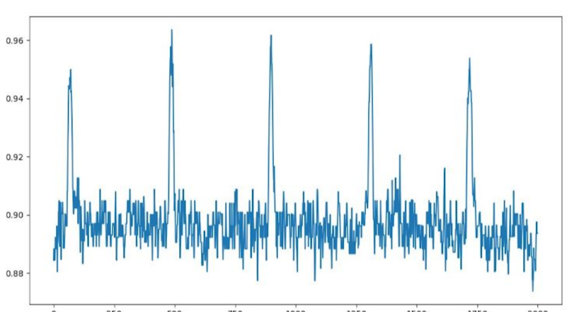
-needed to retake photos to have graph paper in view to get scale for analysis
-was able to get proper lighting to see both graph paper and the diffraction pattern

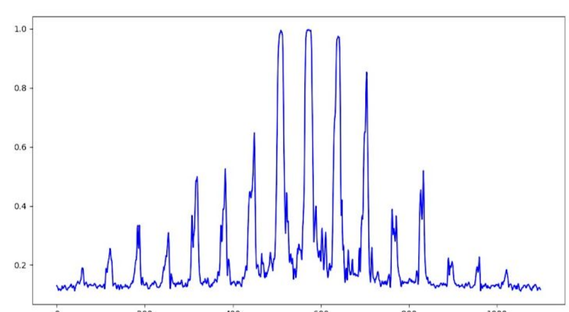


-was able to create histogram like plot to show pixel brightness along certain line





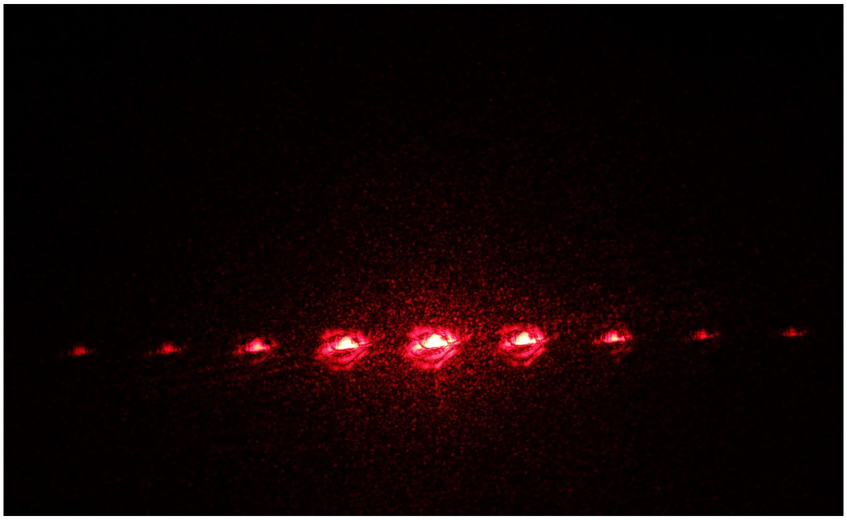




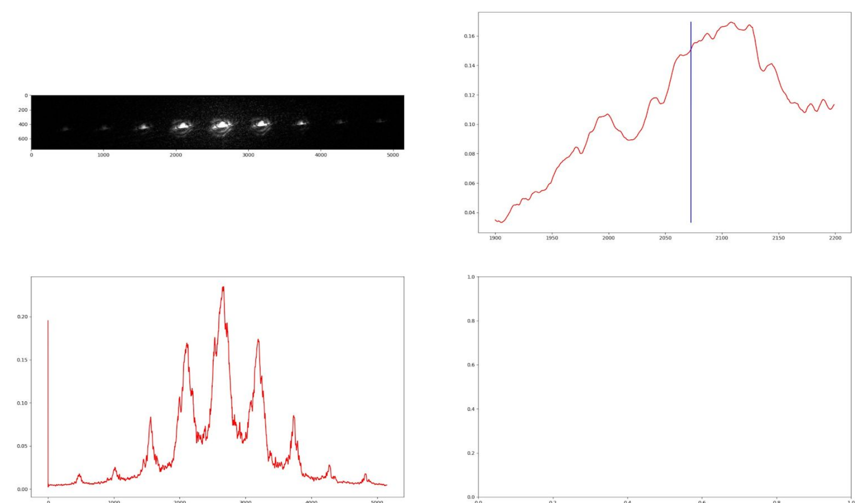
-made plots for both the graph paper and the diffraction pattern with the goal to get a scale from the graph paper then to fit the peaks to get coordinates of the diffraction pattern peaks

Feb 18, 2019:

-talked to Professor Sankey and he recommended to realign our setup to reduce systematic noise



-we got new photos of the diffraction batter by aligning the camera with the optical axis and placing it at the focal point instead of taking photos of the pattern on the screen



-this is the histogram of this new diffraction pattern
-tried out a few techniques to get a rough peak position

- -local max
- -weighted average of x-values

-we can then use these rough estimations to get an error on our data, which we can use to fit the theoretical model

Feb 22, 2019:

-goal today is to get photos of more images
-we will then have them on record for further analysis

EQ1408:

- Narrow grating horizontal
- 1/320s, F13, ISO800 (100-0700)

EQ1402: (edit feb 27th: I dont think we should use this since we are unsure which aperture the light went through)

- 1/200s, F13, ISO800 (100-0675)
- Thin slit (multiple slits with different widths)

EQ1407:

- Narrow grating horizontal (larger spaces than EQ1408
- 1/1250s, F13, ISO800 (100-0711)

EQ1413:

- Grid
- 1/1250s, F13, ISO800 (100-0712)

EQ1418: Single slit

- Large horizontal slit (turned vertical)
- 1/1250s, F13, ISO800 (100-0718)

EQ 1405:

- Square aperture approx. 2mm
- 1/160s , F13, ISO800 (100-0725)

EQ 1424:

- Lines of variable spacing
- 1/500s , F13, ISO800 (100-0727)

EQ 1405 (square) & EQ 1408 (thin multiple slit, lines horizontal):

- 1/250s, F13, ISO800 (100-0729)

EQ 1418 (single slit - horizontal) & EQ 1413 (mesh/grid): not giving anything really

EQ 1408 (multiple slits, vertical) & EQ 1407 (multiple slits horizontal):

- 1/800s , F13, ISO800 (100-0731)

UPDATED OUTLINE OF REPORT:

1. **Showing that set-up is indeed FT**: With unidimensional periodic functions test out the fourier transform properties of our optical set up:
    a. A periodic grating will only (or should only) have 1 frequency in fourrier space
    b. Measure this frequency using a traveling microscope as a lines/m measure
    c. Using the optical set-up take photos of the grating. Analyse them to obtain a measure of the frequency of the grating (nu = x' / lambda * f) . This will test the theory.
    d. If possible, compute the FT of the aperture function and try to fit it to the resulting intensity data from the image. ***MAYBE NOT NECESSARY
    e. For this we can take 3 different gratings:
        i. EQ 1408 (mulitple slit)
        ii. EQ 1407 (multiple slit) larger spaced slits
        iii. EQ 1413 (mesh pattern - 2 frequencies 1 for x and 1 for y which should be the same)

2. **Bandwidth Theorem:** Use non-periodic functions. Determining bandwidth theorem that relates spatial frequency (nu) to spacial distance (x) on aperture.
    a. Example of non-periodic function is: single slit, any non-symmetric slide
    b. For this we need also 3 gratings (so we can compare the results amongst different)
        i. EQ1402 (single slit)
        ii. EQ1424 (varying spaces with mountain pattern)
        iii. EQ1405 (Square)

3. **Convolution Theorem**:
    a. To convolve 2 different gratings we need to make the light pass through both of them.
    b. Convolution theorem: conv. In real space = multiplication in FS
    c. To show the convolution theorem, we can:
        i. Calculate the FT of the aperture functions of the two gratings in question
        ii. Mulitiply the two FT and square them (call this function F(v))
        iii. Observe on our equipment the diffraction pattern of light going through the two different gratings
        iv. Fit F(v) to the intensity curve from our experiment - should be a good fit!
        v. For this take 2 different "convolutions":
            1. EQ 1405 (square) & EQ 1408 (thin multiple slit)
            2. EQ 1408 (multiple slits, vertical) & EQ 1407 (multiple slits horizontal)

d. OR: take into account the pinhole as another grating (single slit) and improve our fits by convolving them.

Feb 25, 2019:

Re-thinking the experiment and analysis methods = more research and understanding of what is going on. New plan:

- Our ideas on how to prove convolution theorem are wrong.
    - The convolution happens on the diffraction grating itself where the aperture function is a convolution between delta functions and square functions (width of the slits). -> cite textbook!
        - All the images taken last time where we superimposed gratings are not good for use.
    - The envelope we see over the interference pattern of our gratings in the data should be coming from this convolution. How to test this hypothesis?
        - Measure the grating parameters using traveling microscope
        - <u>Find an error estimate on the data</u> and fit a theoretical model (*theoretical model:* we can represent the aperture function as a general mathematical expression. Then, if we take the fourier transform of this function and square it we should get the intensity function to fit to our data). See if we can retrieve the same parameters (ie compare the fit parameters to the real grating parameters)
        - This will show that :
            1. the intensity pattern we observe is the square of the FT of the aperture function.
        - To show that the convolution theorem holds we know that:
            1. The aperture function for a simple grating is a convolution of delta functions and square functions. In Fourier Space this should be a multiplication of the FT of the delta function and the FT of the square function (by the convolution theorem). Then if we divide our data by the FT of the square function, we should get back the FT of a sum of delta functions - this will simply be the interference pattern of the multi-slit diffraction grating!
    - With this, we can show that both the convolution theorem holds AND that the experimental set up we have does indeed allow us to measure the Fourier Transform of the aperture function.
- Bandwidth Theorem:
    - We have 2 different non-periodic gratings (single slit and square aperture) that we can use. We will use the single slit grating to derive a bandwidth theorem relating x and nu_x and verify it with the square aperture data.

Feb 27, 2019:

Measuring the parameters on our gratings: using microscope EQ1553
** All slides have dimension:

EQ1418: (Single Slit)
- Width of the slit: [**13.10** +/- 0.05 cm] - [**13.00** cm +/- 0.05 cm] = 0.1 +/- E cm

error estimate to be done for report for the sum of two measurements check textbook!*
*** will be the same for all the measurements made in this section. Error calculation will be done later.*

EQ1408: (Multiple Slits Grating thin lines)
- Width of the slits: [**13.10** +/- 0.05 cm] - [**13.075** +/- 0.05 cm] = 0.025 +/- E cm
- Width of the dark lines: too thin to measure with the microscope

EQ1407: (Multiple Slit Grating with thicker lines)
- Width of the slits: [**13.10** +/- 0.025 cm] - [**13.0375** +/- 0.052 cm] = 0.0625 +/- E cm
- Width of the dark lines: [**13.0375** +/- 0.025 cm] - [**13.00** +/- 0.025 cm] = 0.0375 +/- E cm

EQ1405: (Square Aperture)
- Side of square: [**13.20** +/- 0.025 cm] - [**13.00** +/- 0.025 cm] = 0.20 cm +/- E cm

EQ1413: (Grid)
- Width of the slits: [**13.2** +/- 0.025 cm] - [**13.15** +/- 0.025 cm] = 0.05 cm +/- E
- Width of the dark lines: [**13.15** +/- 0.025 cm] - [**13.10** +/- 0.025 cm] = 0.05 cm +/- E

Error estimation:
- Statistical error in our measurements come from the lenses:
    - The amount of dust collected on them, minor scratches etc.
    - Take 10 photos of the same grating pattern with minor modifications to set-up (such as wiping down lenses or shifting mirrors slightly)
    - Measure the deviation between all these photos. This should be a good approximation for the error on all of our data.
        - The maximum intensity of each photo when analysed should be the same (since all 10 photos will be taken with the same exact settings). The only difference between the maximum intensity would thus be due to the statistical errors of our experiments.
        - To estimate the uncertainty, we can then take the maximum intensity value of each photo and plot it. We can fit this plot with a horizontal line and force the error bars on intensity until we get a reduced Chi^2 ~ 1 (like in the tutorial). This uncertainty should be the standard deviation of the maximum intensity.

- Grating used: Single Slit EQ 1418
- **EQ1418 error estimation picture numbers:**
    - Picture #: 0739 ; 0741 ; 0742 ; 0743 ; 0744 ; 0745 ; 0747 ; 0748 ; 0749 ; 0750
    - Settings on camera: 1/1250s, F13, ISO800