Statistical Reasoning

"The most important questions of life are, for the most part, really only problems of probability."
— Pierre-Simon, Marquis de Laplace

👨‍🏫 Statistical Reasoning Lecture #1
Alexander Savi, 2024
📚 Whitlock/Schluter, Ch. 2; Dienes, Ch. 3 to p. 59

Curled by Danielle Navarro 🎨

Warming Up

Course Introduction

Curled by Danielle Navarro 🎨

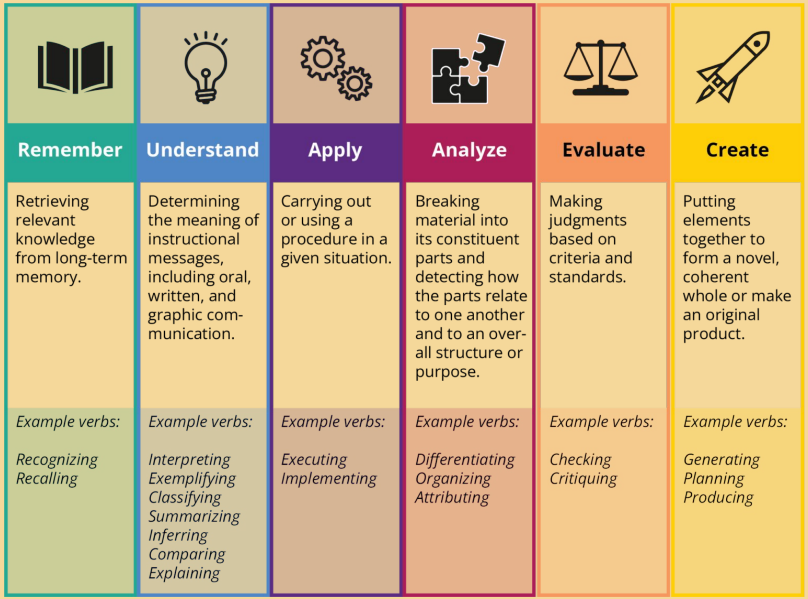
Statistical reasoning

Statistical [reasoning, literacy, thinking].

Deals with inference (or prediction) when there is variability, probability, randomness, uncertainty, …

- Remember: vocabulary
- Understand: e.g., confidence intervals
- Apply: statistical procedures, R skills
- Analyze: e.g., frequentist inference
- Evaluate: fallacies, generalizability, other's work
- Create: e.g., simulations, new knowledge

💡 Bloom's taxonomy can be applied to any domain, e.g., systems thinking.



Remember	Understand	Apply	Analyze	Evaluate	Create
Retrieving relevant knowledge from long-term memory.	Determining the meaning of instructional messages, including oral, written, and graphic com-munication.	Carrying out or using a procedure in a given situation.	Breaking material into its constituent parts and detecting how the parts relate to one another and to an over-all structure or purpose.	Making judgments based on criteria and standards.	Putting elements together to form a novel, coherent whole or make an original product.
Example verbs: *Recognizing* *Recalling*	*Example verbs:* *Interpreting* *Exemplifying* *Classifying* *Summarizing* *Inferring* *Comparing* *Explaining*	*Example verbs:* *Executing* *Implementing*	*Example verbs:* *Differentiating* *Organizing* *Attributing*	*Example verbs:* *Checking* *Critiquing*	*Example verbs:* *Generating* *Planning* *Producing*

Bloom's taxonomy (cognitive domain). Illustration by Utrecht University

Climate change

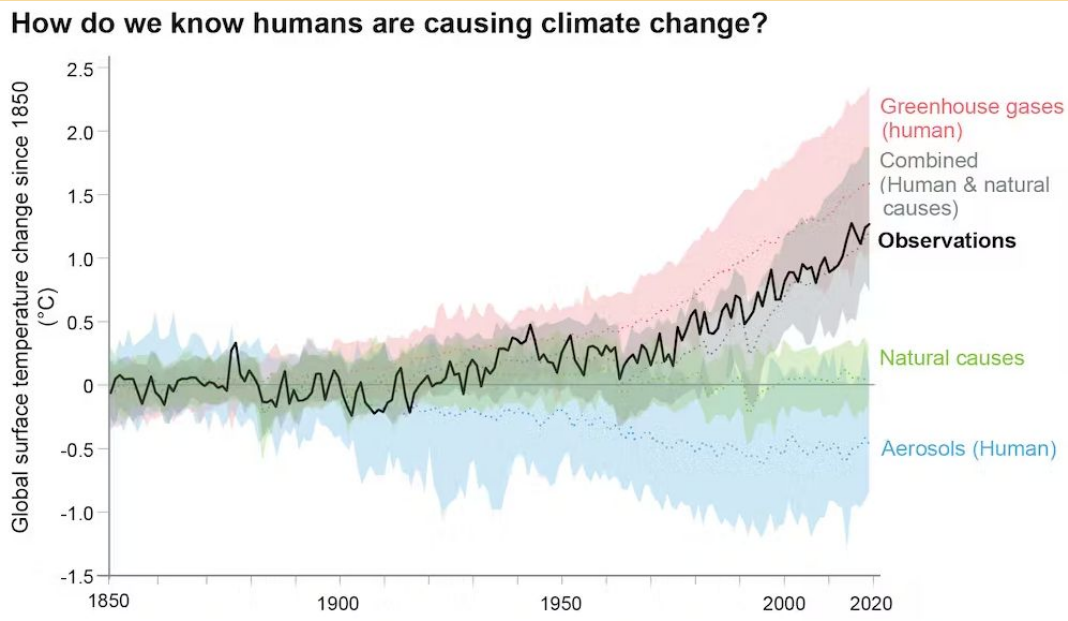


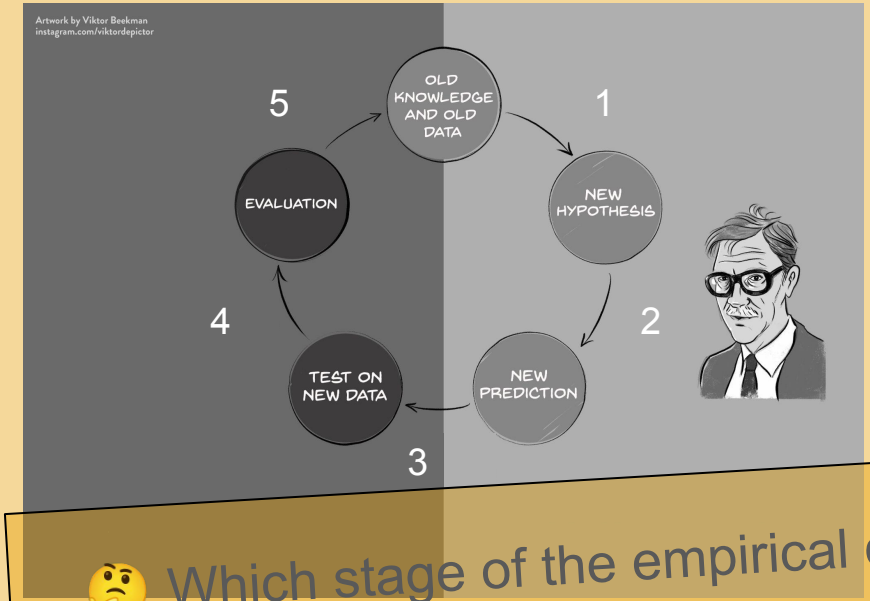
Illustration by IPCC (in [The Conversation])

Evolution

Illustration by Daniel Hertzberg (in Quanta)

💡 More popular science in Nautilus & Undark

Scientific method



🤔 Which stage of the empirical cycle requires statistical reasoning?

Illustration by Viktor Beekman

💡 Richard Feynman on Scientific Method (1964)

Scientific method in motion 🌊

💡 Complementary scientific methods:

- Hypothetico-deductive method
- Abductive method
- Generative method

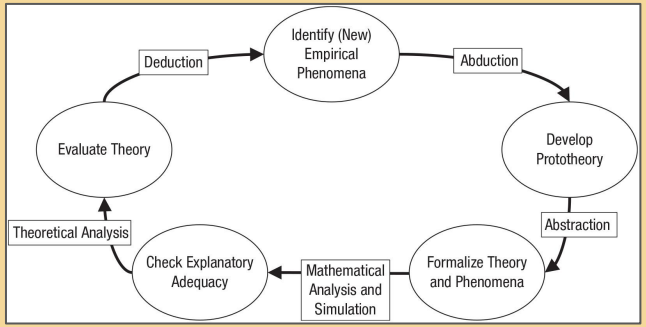


Illustration by Borsboom et al., 2021

Year 1: Methoden van Onderzoek en Statistiek

Universiteit van Amsterdam

CANVAS BY INSTRUCTURE

R Studio®

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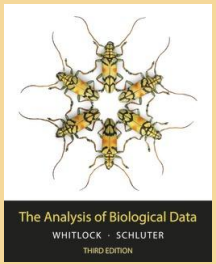

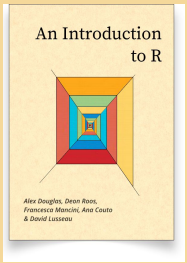

Topics

Samples & sampling distributions
Data visualizations
Confidence intervals
Probabilities
Bayes theorem
Test choice & research design
Various statistical tests

Year 2: (Wetenschapsfilosofie en) Statistisch Redeneren

UNIVERSITEIT VAN AMSTERDAM

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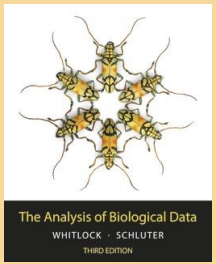
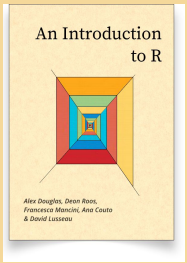
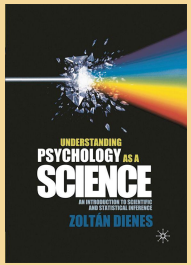






💡 Created with R package ggfx

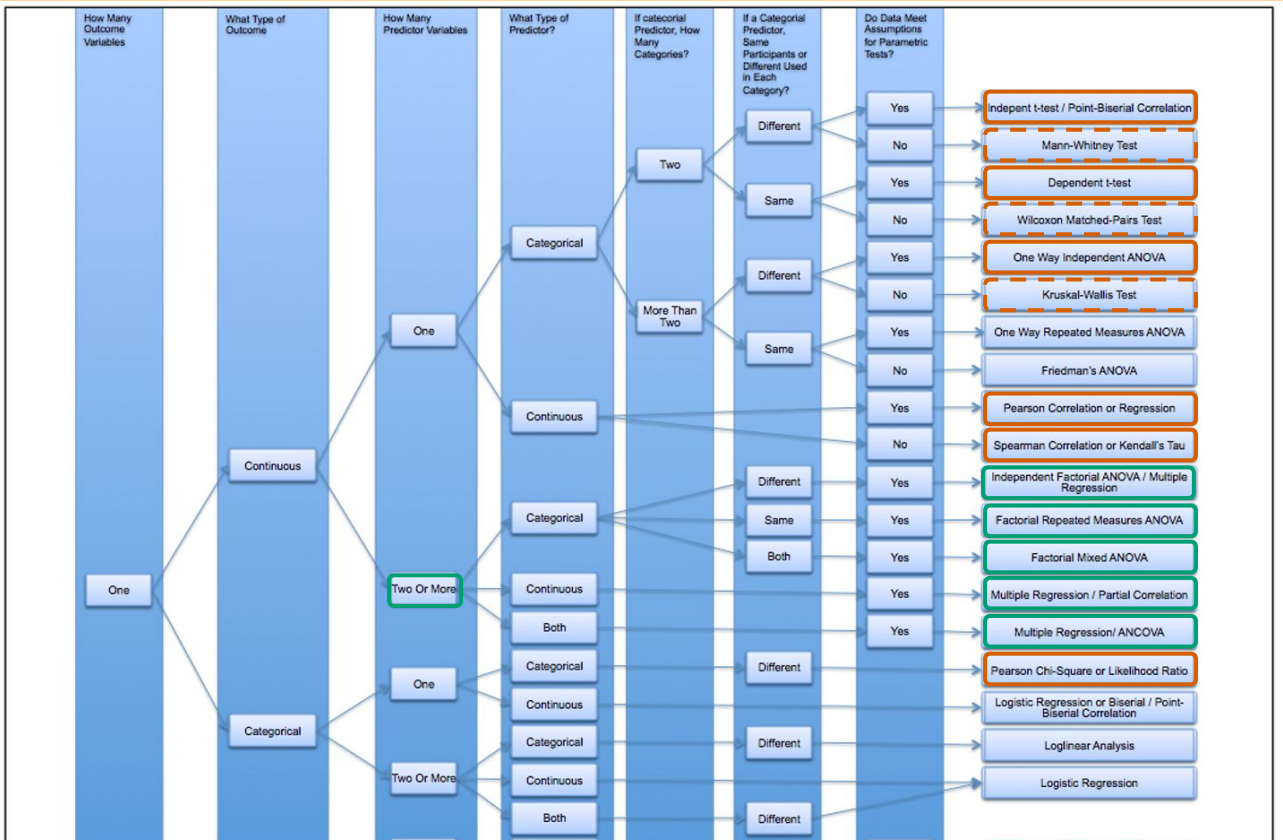
Learning goals

Knowledge and skills in statistical reasoning and inference.

- Conceptual understanding of statistical inference (frequentist, Bayesian and non-parametric statistics)
- Match statistical test to research design (and vice versa)
- Perform statistical tests with R and JASP
- Interpret test results from R and JASP

Foundation for follow-up courses and independent learning.

Statistical tests



Year 1

Binomial test
Chi-squared test
Student's *t*-test
One-way ANOVA
Correlation
Simple linear regression

Year 2

Multiple linear regression
Factorial ANOVA
Nonparametric inference
Bayesian inference

Module organization*

(*course manual is leading)

Lectures (web for reference)

1. Intro & probability refresher
2. Frequentist inference refresher
3. Multiple linear regression
4. Factorial ANOVA
5. Nonparametric inference
6. Bayesian inference
7. Outro

Team

Jonas van Nijnatten (co-coordinator), Saskia de Back and Anne Marijn Bruijn (teaching assistants), Alexander Savi (lecturer, coordinator)

Assignments (formative)

- Weekly, sufficient/insufficient, deadline every Sunday 23:55 (correct answers available after deadline)
- Pass with 5/7 sufficient assignments
- 3 attempts per assignment, 2 prior checks per attempt
- ~~Exemption for recidivists (if previously sufficient)~~
- Use decimal *points* (.)

Exam (summative)

- 80% of final grade (⅝ SR, ⅜ PhS)
- SR digital open book (PhS *not* open book)

Lecture organization

Sections

🤸 Warming up

🧗 Main content

🧘 Cooling down

⚠️ Don't look here!

Regular items

📰 In the news
🌊 In motion

🔁 Repeat
🔀 Shuffle
🎲 Activity
🤔 Quiz

🔮 Clairvoyance
📦 Takeaways
🏅 Nail it

🦄 Superpowers

Explore on your own

💡 Inspiration

🧠 Learning theory

`R code`

Behavior

🕐 We need our time
🤫 Annoyance #1
🙋 During or after

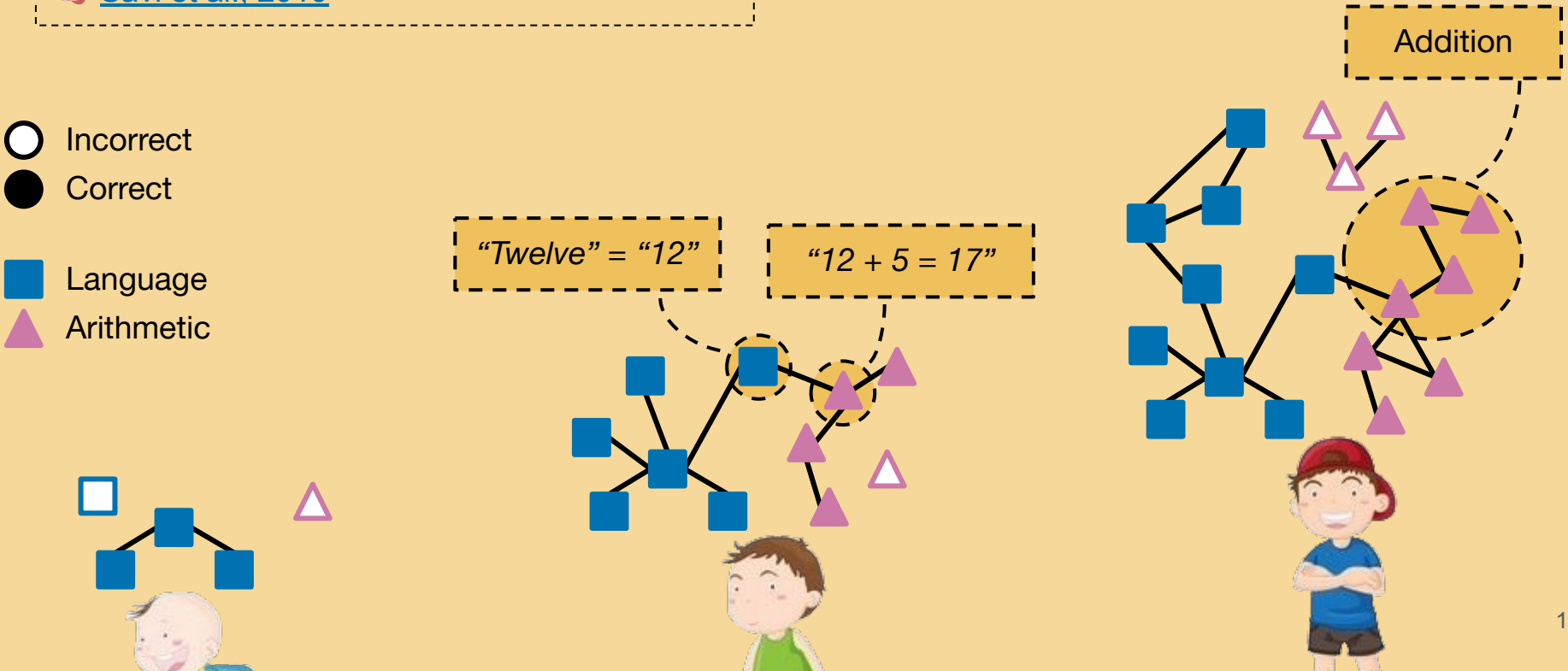
💡 Emoji science

The wiring of intelligence

🧠 Savi et al., 2019

Incorrect
Correct

Language
Arithmetic



🔮 Overview

Topics

🔁 Probabilities & distributions
- Sum rule
- Product rule
- Bernoulli distribution
- Binomial distribution

Frequentist inference
Multiple linear regression
Factorial ANOVA
Nonparametric inference
Bayesian inference

Learning goals

An introduction to the Statistical Reasoning module and a basic understanding of statistical reasoning and its role in the scientific method.

A basic understanding of probability and probability distributions.

Probabilities & Distributions

Skunk 🎲

Five rounds. The winner collects the most points in all rounds. Each round:

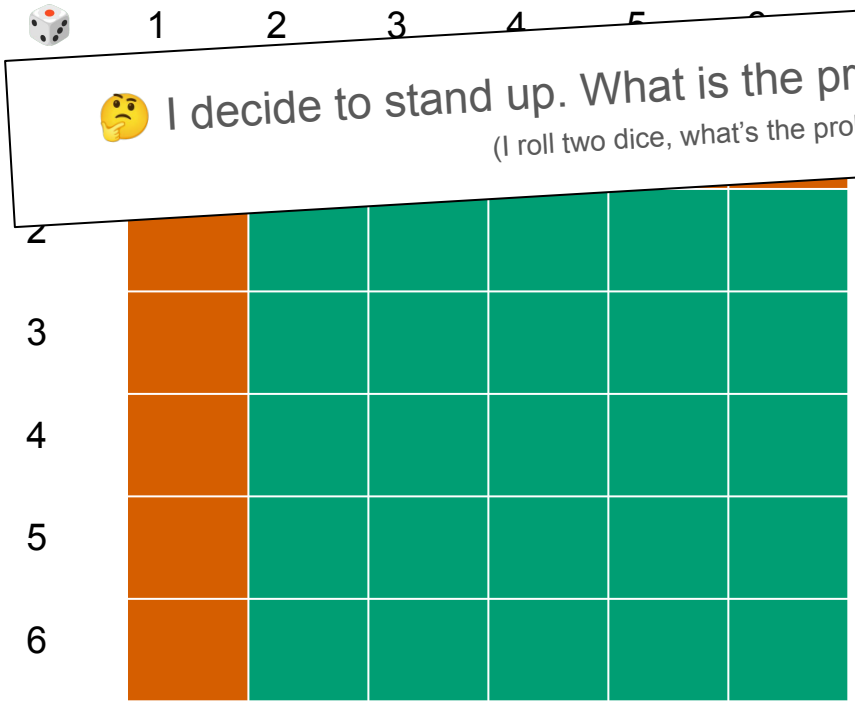
1. Decide to **sit down** 🪑 or **stand up** 🧍
   a. 🪑 forfeit this round and carry your score over to the next round
   b. 🧍 continue this round
2. I roll **two dice**
   a. ~~[1]~~ → **round continues**
      i. 🪑 wait for a new round
      ii. 🧍 add the numbers on the dice to your score
   b. [1] → **round ends**
      i. 🪑 join the new round
      ii. 🧍 erase your score for this round and join the new round

```
?sample
```

Probabilities

🤔 I decide to stand up. What is the probability I will lose my score for this round?

(I roll two dice, what's the probability of observing at least one [1]?)



Using probability rules

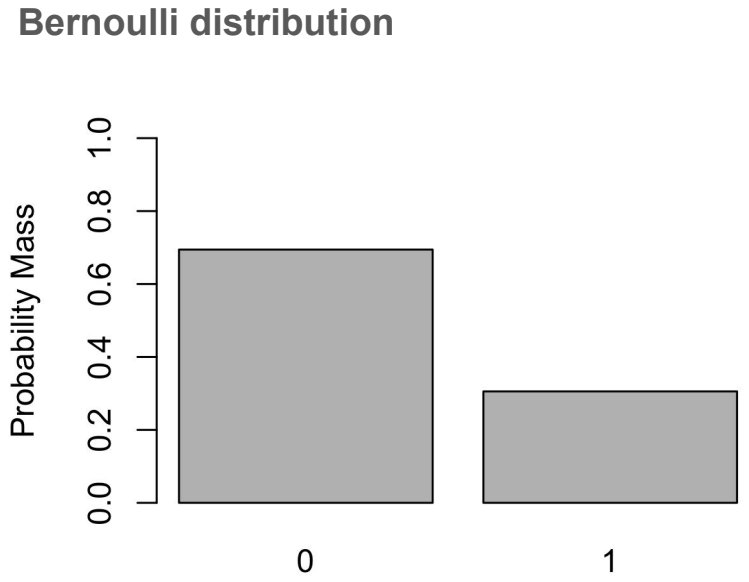
Event A: roll a 1 on first die
Event B: roll a 1 on second die

Sum rule: p(A or B) = p(A) + p(B) − p(A and B)
Product rule for independent events: p(A and B) = p(A) × p(B)

p(A or B) = 1/6 + 1/6 − (1/6 × 1/6) = 11/36 ≈ .3

Probability distributions

> " A probability distribution is the mathematical function that gives the probabilities of occurrence of different possible outcomes for an experiment.
> — Wikipedia

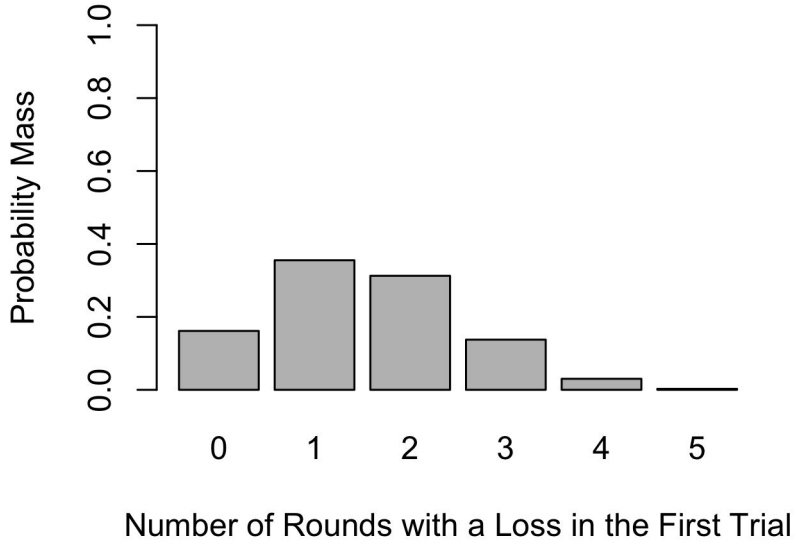


A special case of the binomial distribution, where *n* trials equals 1.

Binomial distribution

🤔 A strategy: in all five rounds, I sit down after the first roll. What is the probability distribution for losing these rounds (rolling a 1)?

- 🪙 Correct / Incorrect
- 🪙 Success / Failure
- 🪙 True / False
- 🪙 1 / 0
- 🪙 Yes / No (/ Maybe)
- 🪙 Head / Tail (/ Edge)
- 🪙 Female / Male (/ Intersex)



💡 Fair coins tend to land on the same side they started

Binomial distribution

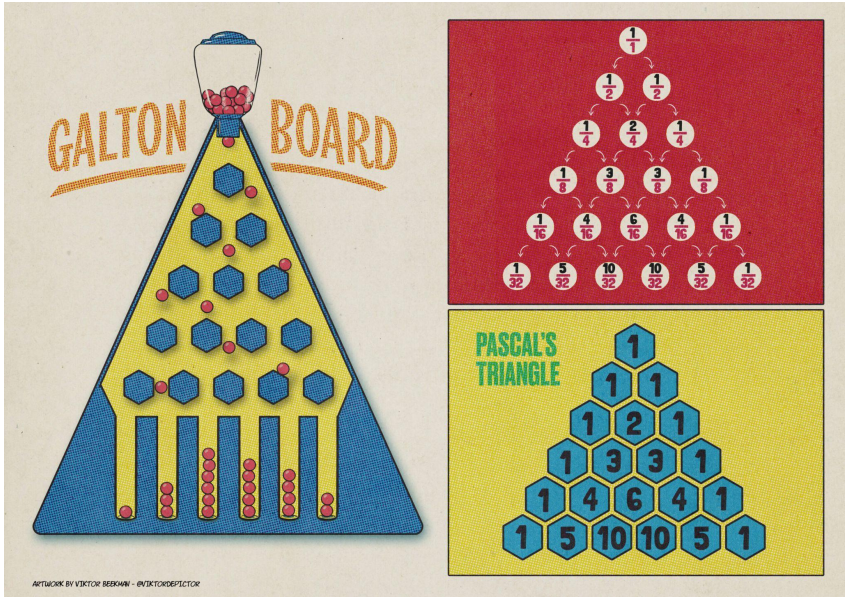


Illustration by Viktor Beekman

Video by [Wikipedia]

Binomial distribution

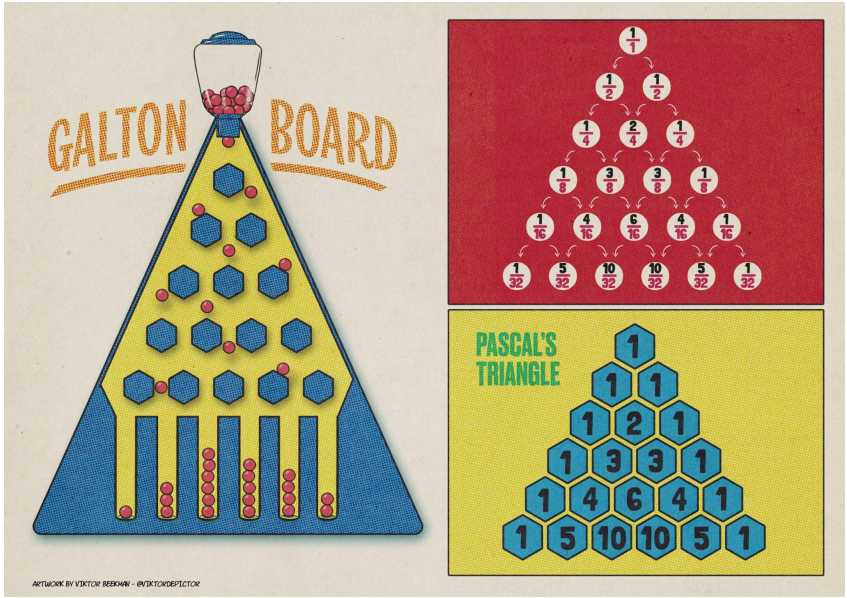


Illustration by Viktor Beekman

🤔 You use the aforementioned strategy. That is, each level represents the first trial of a round in Skunk. Where in the illustrations is the following shown?

- Number of ways to observe 3 rounds of losses (1)
- Probability of observing 0 rounds of losses (2)
- Probability of precisely this series of losses (0) and gains (1): 00110?
- Probability of observing 2 or fewer rounds with a loss (3)
- Lowest 5% number of rounds with a loss (4)

Binomial distribution

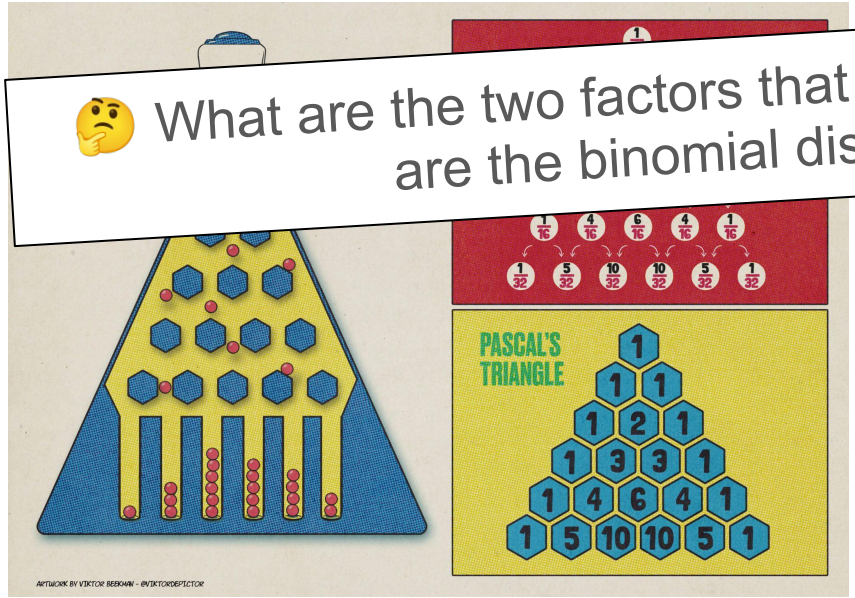
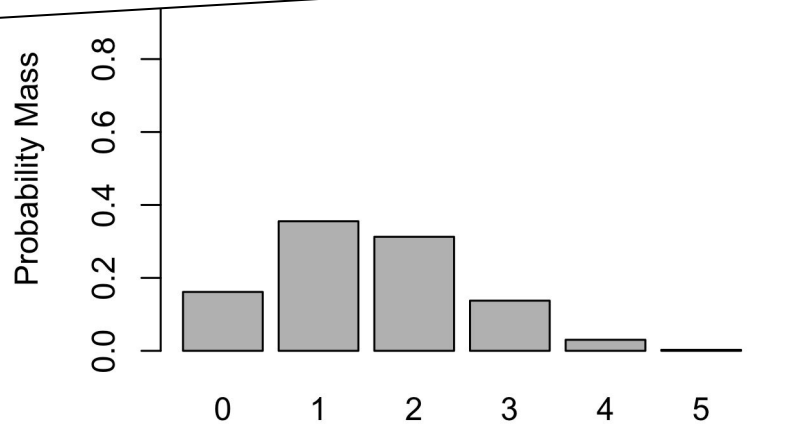


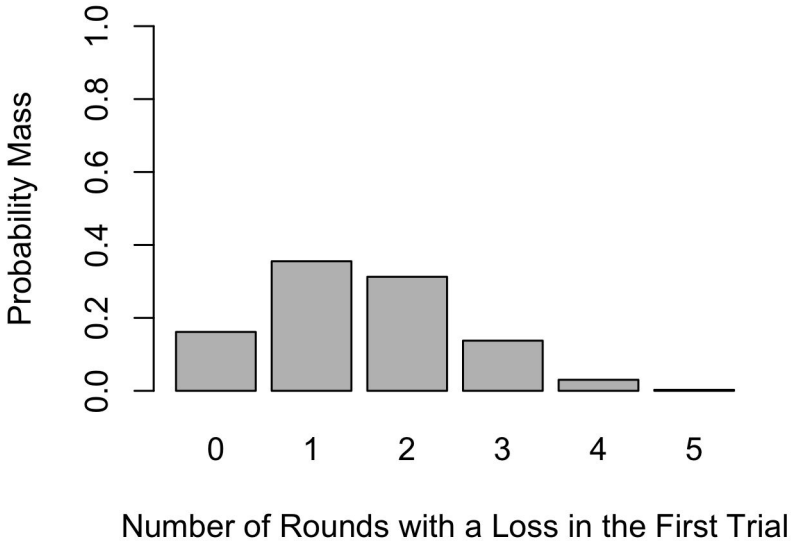
Illustration by Viktor Beekman

🤔 What are the two factors that determine these probabilities? What are the binomial distribution's parameters?

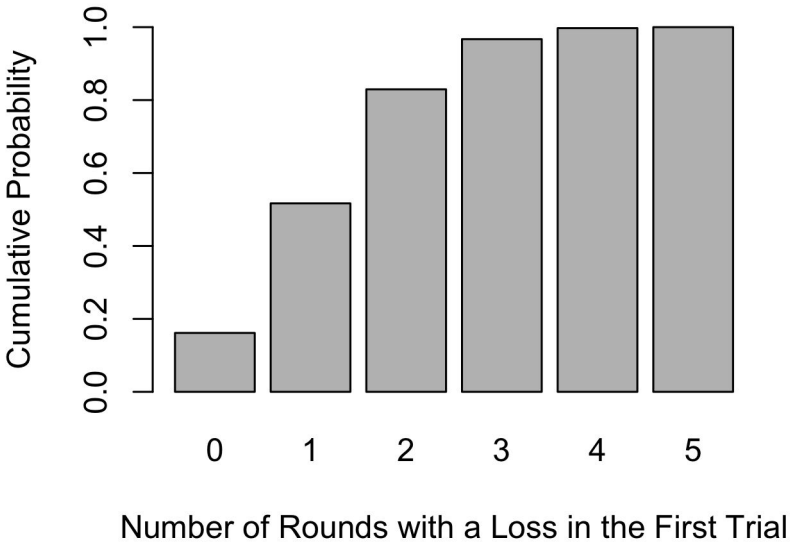


Binomial distribution

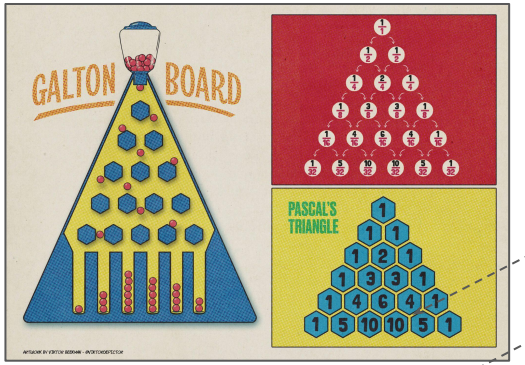
Probability mass distribution

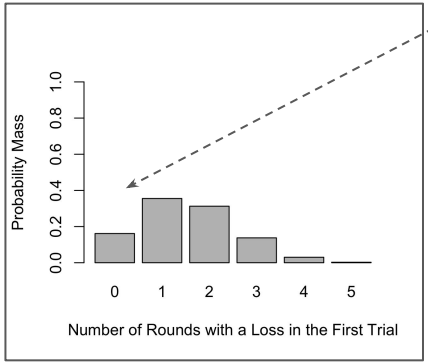


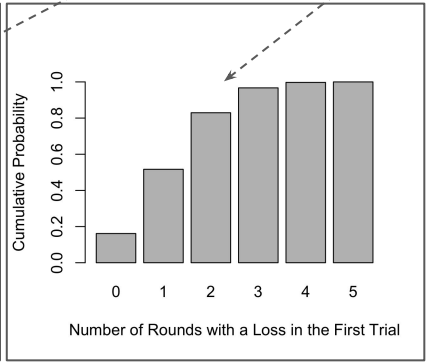
Cumulative probability distribution



Binomial distribution in R

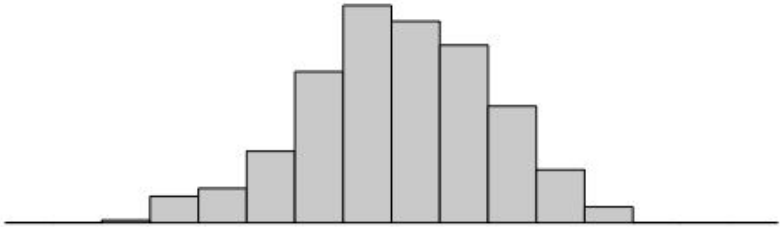







```
# (1) number of ways to observe 3 rounds of losses at the first trial
choose(n = n, k = 3)

# (2) probability of observing 0 rounds of losses at the first trial
dbinom(x = 0, size = n, prob = p)

# (3) probability of observing 2 or fewer rounds with a loss at the first trial
pbinom(q = 2, size = n, prob = p)

# probability of observing more than two rounds with a loss at the first trial
pbinom(q = 2, size = n, prob = p, lower.tail = FALSE)

# (4) lowest 5% number of rounds with a loss at the first trial
qbinom(.05, size = n, prob = p)

# highest 5% number of rounds with a loss at the first trial
qbinom(.05, size = n, prob = p, lower.tail = FALSE)

# sample 15 observations from the binomial distribution
rbinom(n = 15, size = n, prob = p)

# use the same functions for different distributions
_binom(); _norm(); _t(); _f()
```

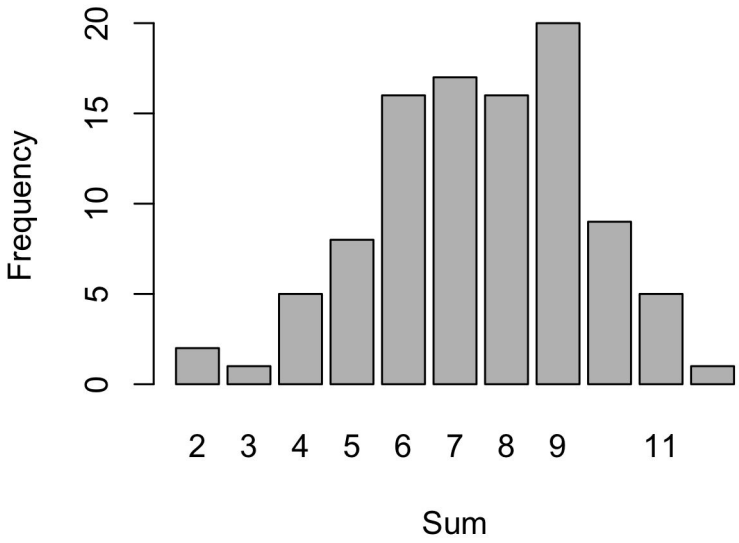
🦄 Simulation superpower



💡

```
# Galton board (do not run in RStudio)
library("animation")
n_rep <- 500
n <- 15
n_layers <- n + 2
animation::ani.options(interval = 0.05, nmax =
n_rep + n_layers)
animation::quincunx(balls = n_rep, layers =
n_layers)
```

🦄 Simulation superpower



🤔 If we change the probabilities,

- will the frequency distribution change?
- will the probability distribution change?

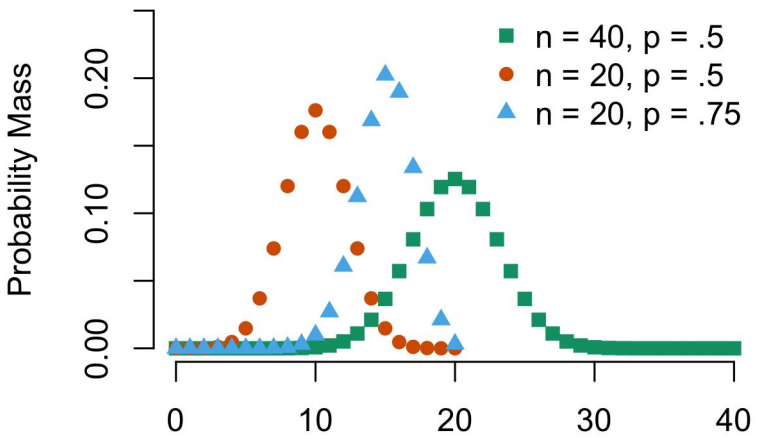
🤔 If we change the number of repetitions,

- will the frequency distribution change?
- will the probability distribution change?

```
n_rep <- 100  # e.g., persons
n <- 15  # e.g., items
dat <- rbinom(n = n_rep, size = n, prob = .5)
tab <- table(dat)
barplot(tab, xlab = "Sum", ylab = "Frequency")
```

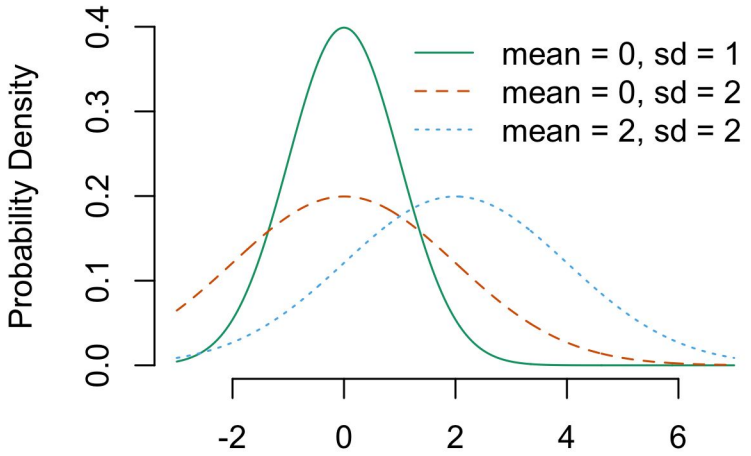
Probability distributions

Discrete probability (mass) distributions



Bernoulli distribution, Binomial distribution

Continuous probability (density) distributions



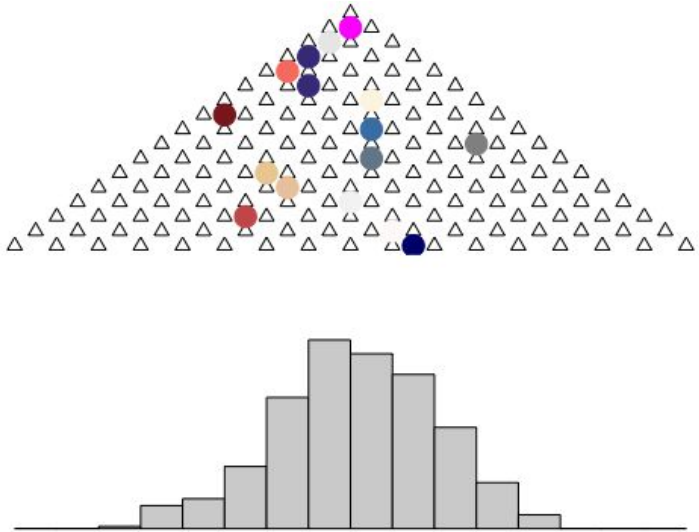
Normal/Gaussian distribution, Student's *t*-distribution, Gamma distribution, *F*-distribution, Beta distribution

Statistical inference

> *Statistical inference makes propositions about a population, using data drawn from the population with some form of sampling. — Wikipedia*

- Frequentist inference
- Bayesian inference
- Nonparametric inference

Frequentist inference



Statistical inference

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- Frequentist inference
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- Nonparametric inference

Bayesian inference

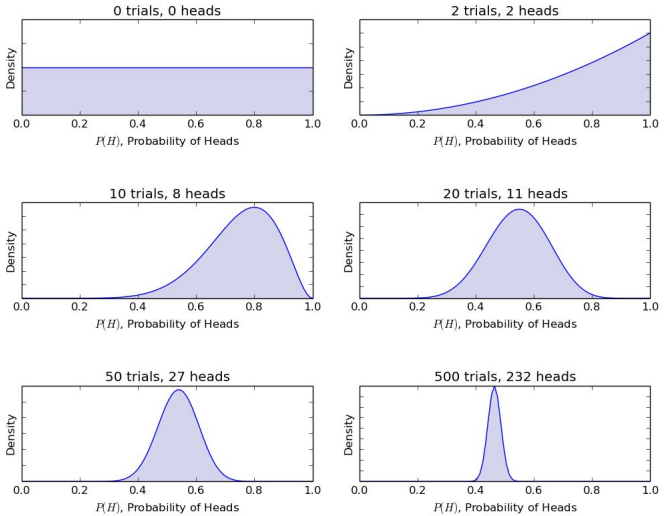


Illustration by QuantStart

Cooling Down

Takeaways

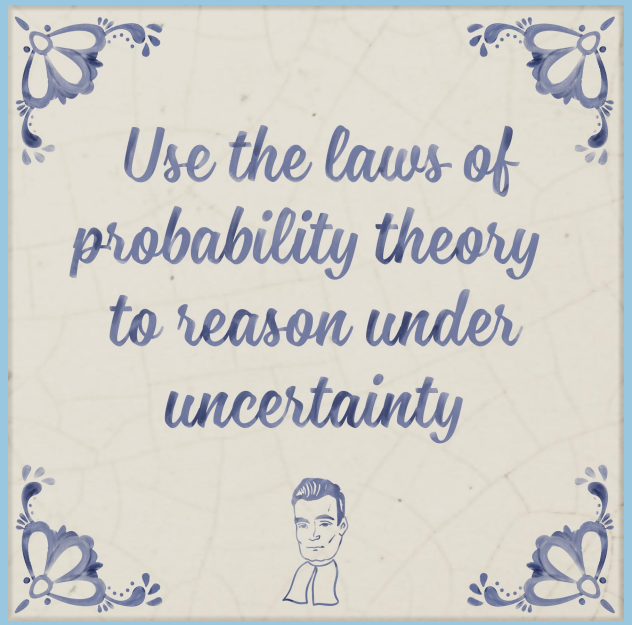


Illustration by Viktor Beekman

🥇 Nail it

Large individual differences

- Statistics
- Programming

Too easy?

- Check the lecture resources (“look here”)
- Do a simulation
- Do the challenge (“don’t look here”)
- Create a pub quiz question

Too hard?

- Check the lecture resources (“look here”)
- SMASH (statistics support hub)
- Cheat sheets: RStudio IDE, Base R (syntax comparison)

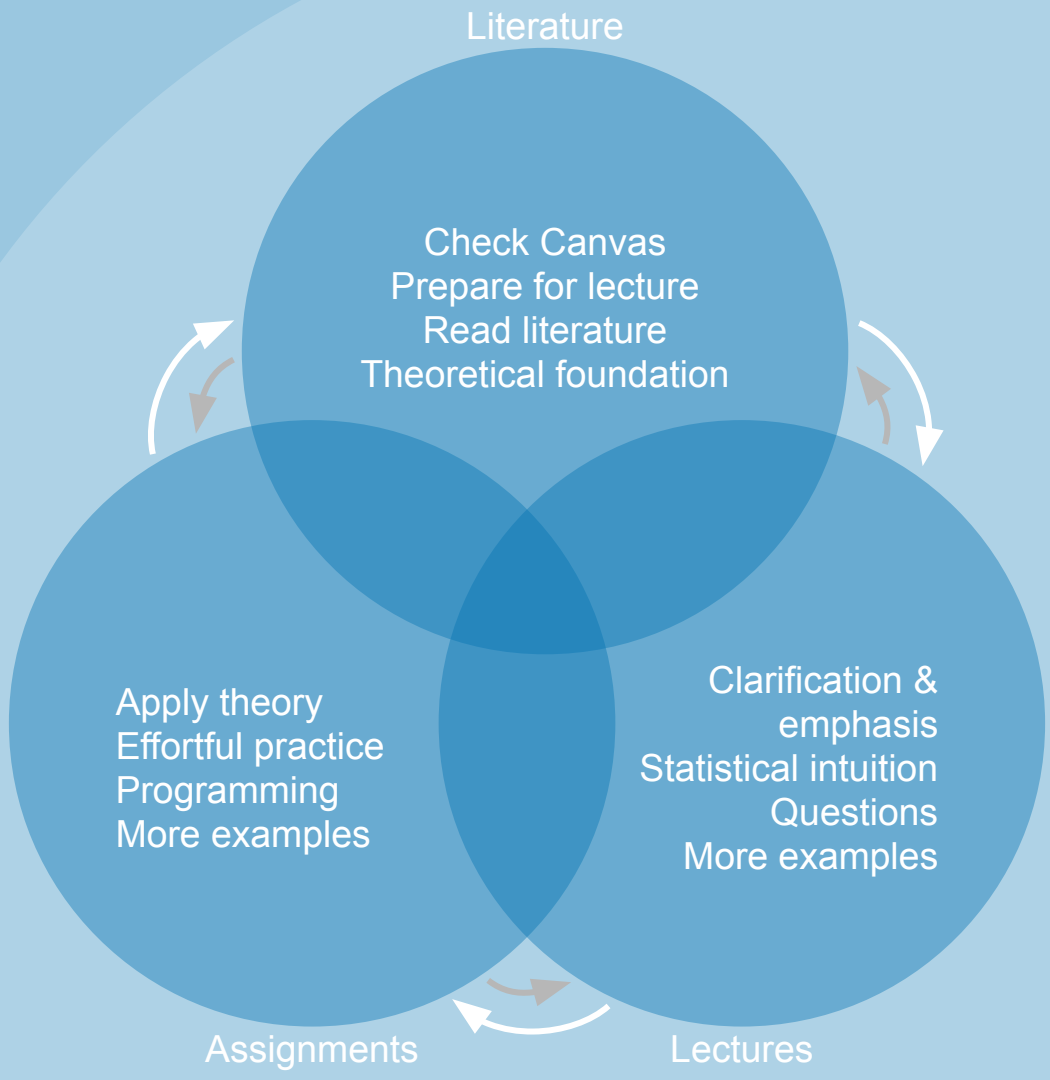
How to interact with the course

- Ask questions at the lectures
- Use Canvas *Discussions*
- Discuss & collaborate with peers
- Create your own Statistical Reasoning manual (in R or a text editor): exam tip!

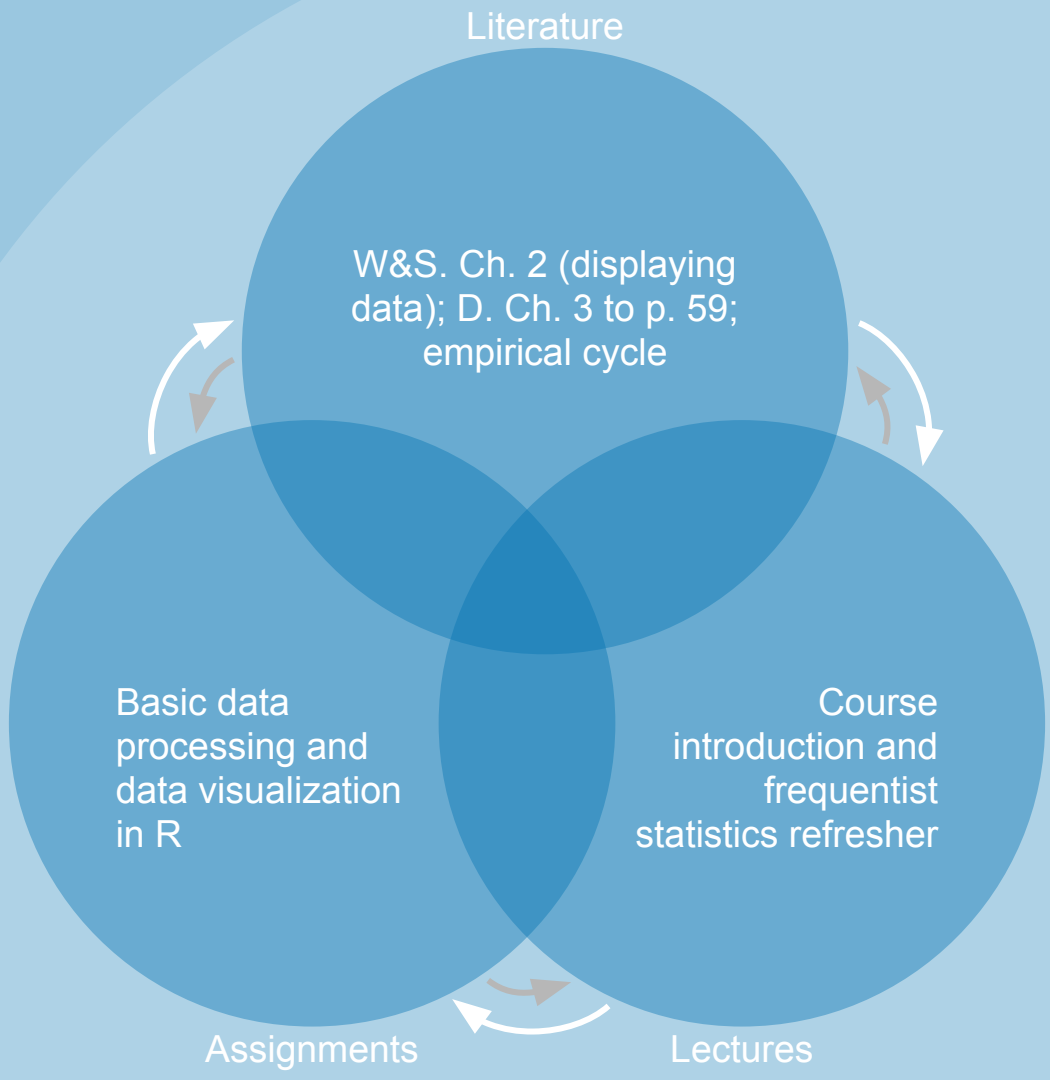
How to influence the course

- Approach us at the lectures
- Fill in course evaluation form
- Give (anonymous) feedback

🥇 Nail it


Literature
Course
Check Canvas
Prepare for lecture
Read literature
Theoretical foundation
Apply theory
Effortful practice
Programming
More examples
Clarification & emphasis
Statistical intuition
Questions
More examples
Discuss & collaborate with your peers
Assignments
Lectures

🥇 Nail it


Literature
W&S. Ch. 2 (displaying data); D. Ch. 3 to p. 59; empirical cycle
Basic data processing and data visualization in R
Course introduction and frequentist statistics refresher
Assignments
Lectures

🥇 Nail it

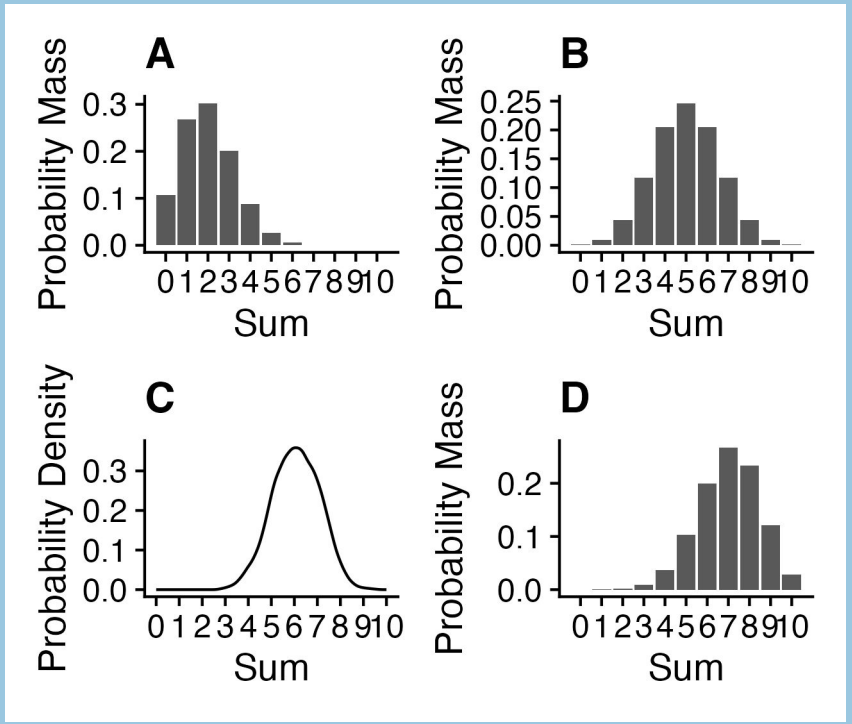
> No one can teach you to play the guitar.
>
> But they can help you learn.
>
> — Dan Morgan, 1965, Guitar

🥇 Exam(ple) question

Alexander gooit tien keer een munt op en noteert de som van het aantal keer kop. Hij herhaalt dit honderd keer. De munt is oneerlijk en valt vaker op kop dan op munt.

Wat is de meest waarschijnlijke verdeling van de gevonden waarden?

A.
B.
C.
D.



👩‍💻 Take-home assignments

📆 Weekly assignment

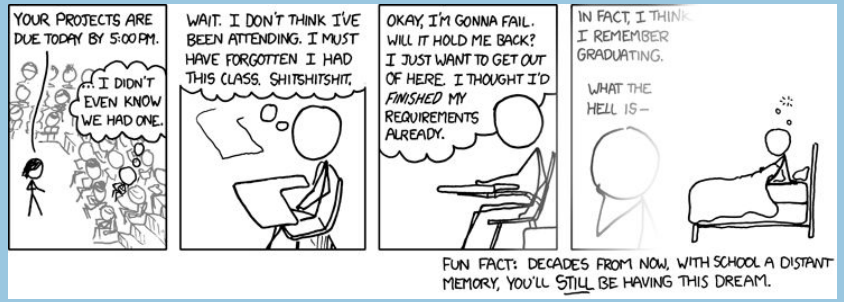


Illustration by Randall Munroe (wtf)

🍻 Pub quiz

Create an *informative* four-choice question about the content of today's lecture.

An informative question has a large spread in responses across answer options.

Clarify answer options (which are (in)correct and why).

Illustration adapted from Snippets.com

🔮 Overview

Topics

Probabilities & distributions
- Sum rule
- Product rule
- Bernoulli distribution
- Binomial distribution

🔁 Frequentist inference

Multiple linear regression

Factorial ANOVA

Nonparametric inference

Bayesian inference

Illustration by Jennifer Cheuk

🧰 Look here!

Statistical reasoning

- The joy of stats (Hans Rosling)

Probability distributions

- Probability distributions (Seeing Theory)
- Binomial distribution (3Blue1Brown)
- Probability density functions (3Blue1Brown)
- List of probability distributions (Wikipedia)
- Buy a probability distribution

⚠️ Don’t look here!

What is the best strategy for Skunk? Support your answer with calculations or simulations.

Share your attempt with the instructor (and tell which hints you used).

Additional challenge: Add your solution to your personal course manual and create that manual using R Markdown.

Hints (select and copy/paste the invisible text below to reveal it)

0.

1.

2.

3.

✍️ Colophon

Slides
alexandersavi.nl/teaching/

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