Bayesian Inference

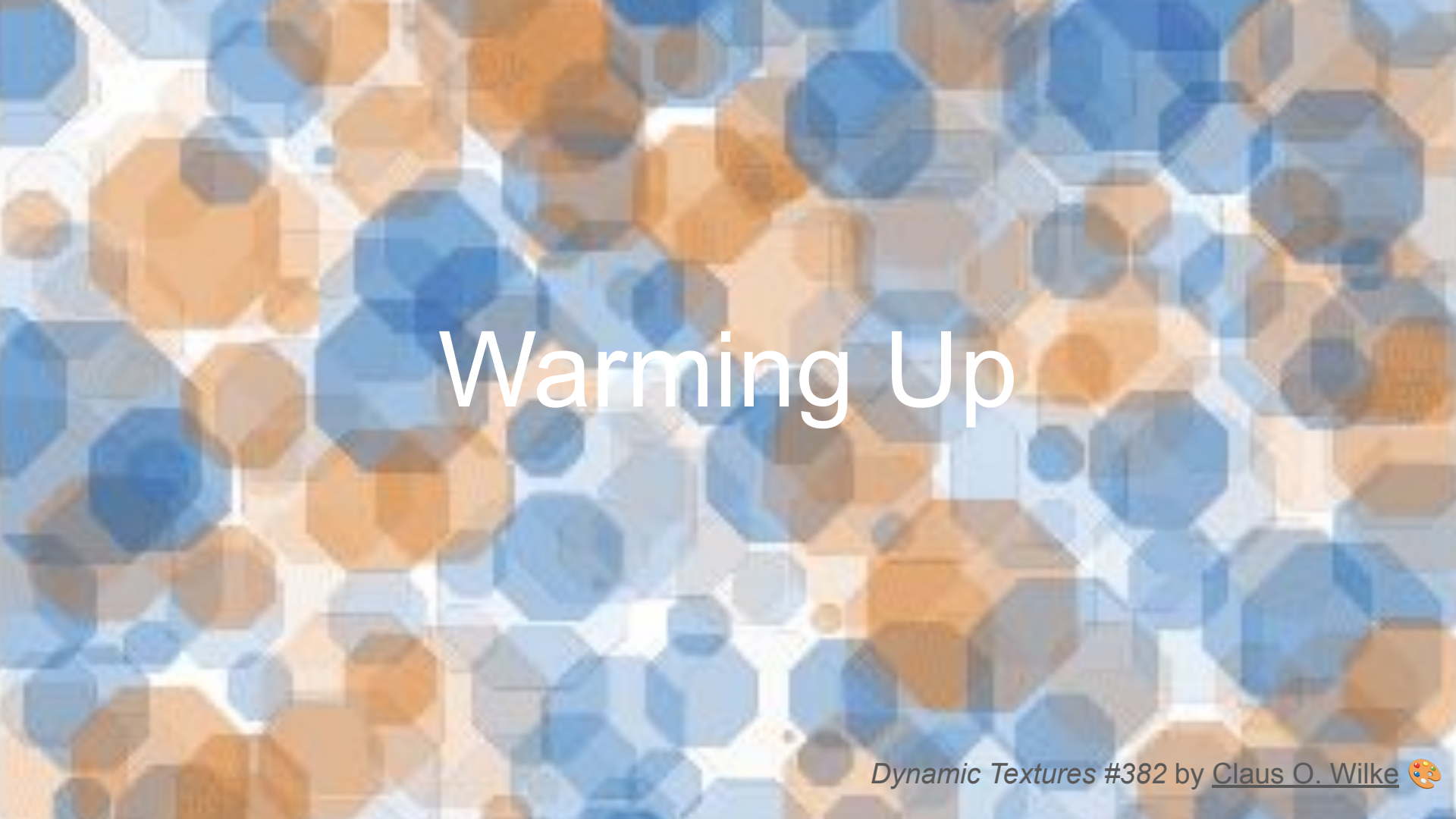
“Absence of evidence is not evidence of absence.”
— Martin Rees

👨‍🏫 Statistical Reasoning Lecture #6
Alexander Savi, 2024
📚 Dienes, Ch. 4

Dynamic Textures #382 by Claus O. Wilke 🎨

📢 Announcements

Warming Up



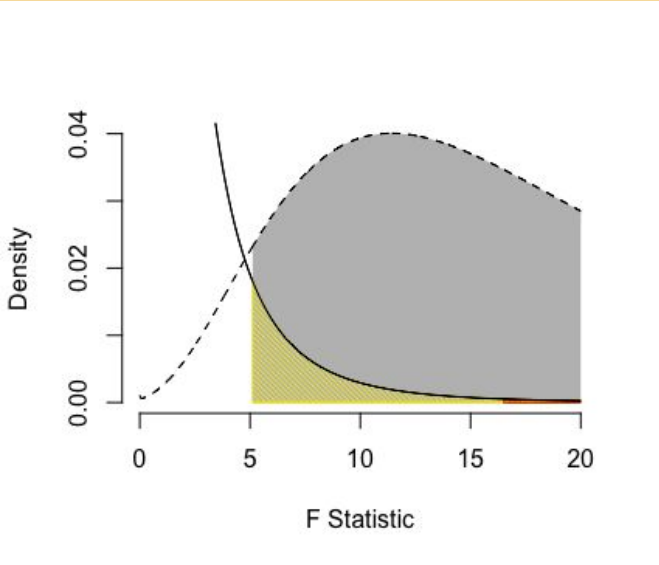
Dynamic Textures #382 by Claus O. Wilke 🎨

📰 News

— NRC (Sep. 23, 2024)

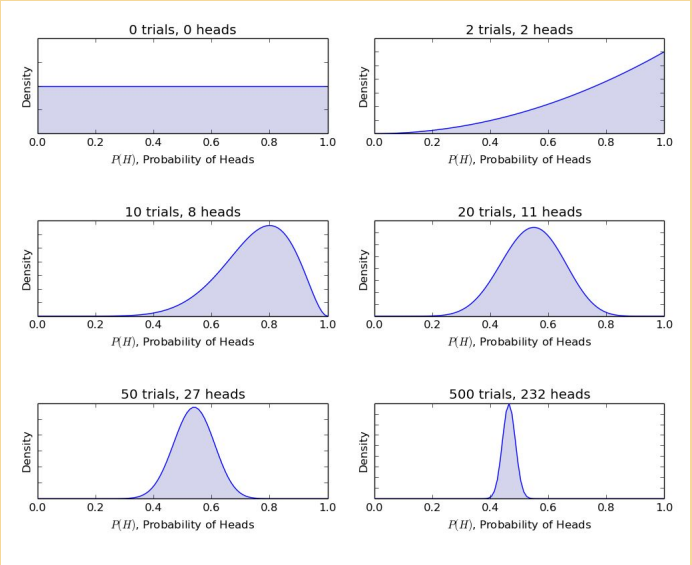
🔁 Recap

🏛 Frequentist inference



$p(D \mid H_0)$

🏛 Bayesian inference



$p(H \mid D)$

🔮 Overview

Topics

Probabilities & distributions
Frequentist inference
Multiple linear regression
Factorial ANOVA
Nonparametric inference
| Bayesian inference

Learning goals

Understand Bayes theorem

Perform Bayesian hypothesis testing

Perform Bayesian parameter estimation

🎁 Brand new probability distribution

Bayes Theorem

Bayes theorem

D(ata): 📋 (positive ADHD test result)
H(ypothesis): 🤯 (ADHD)

💭 What's the probability of $H_{🤯}$?

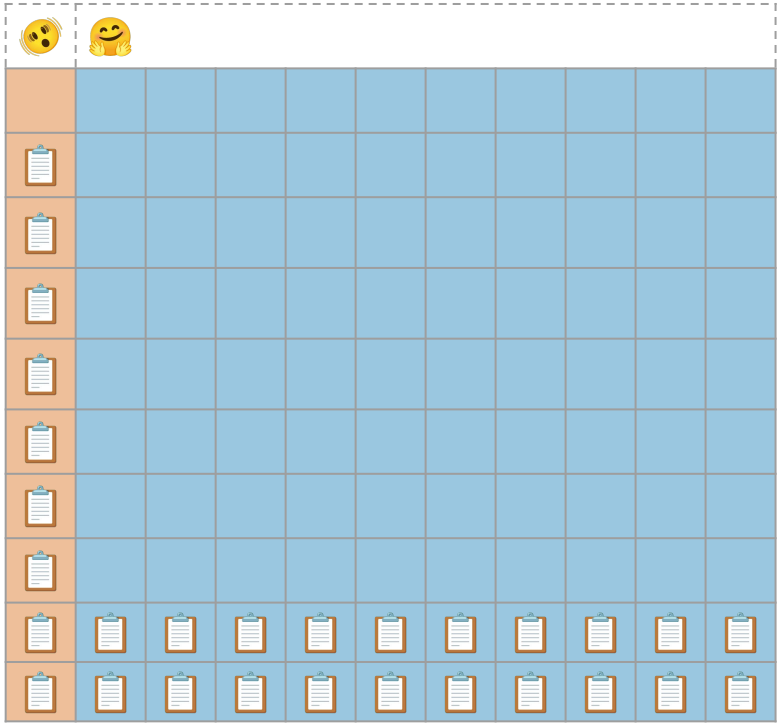
- High?
- Low?
- Why?

Bayes theorem

D(ata): 📋 (positive ADHD test result)
H(ypothesis): 😯 (ADHD)

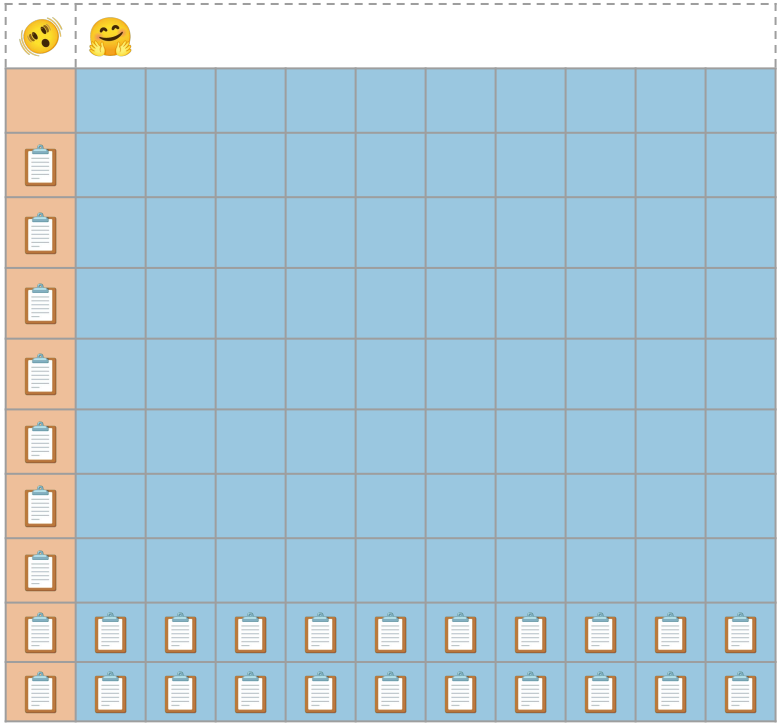
💭 What's the probability of H😯?

- High?
- Low?
- Why?



Base rate

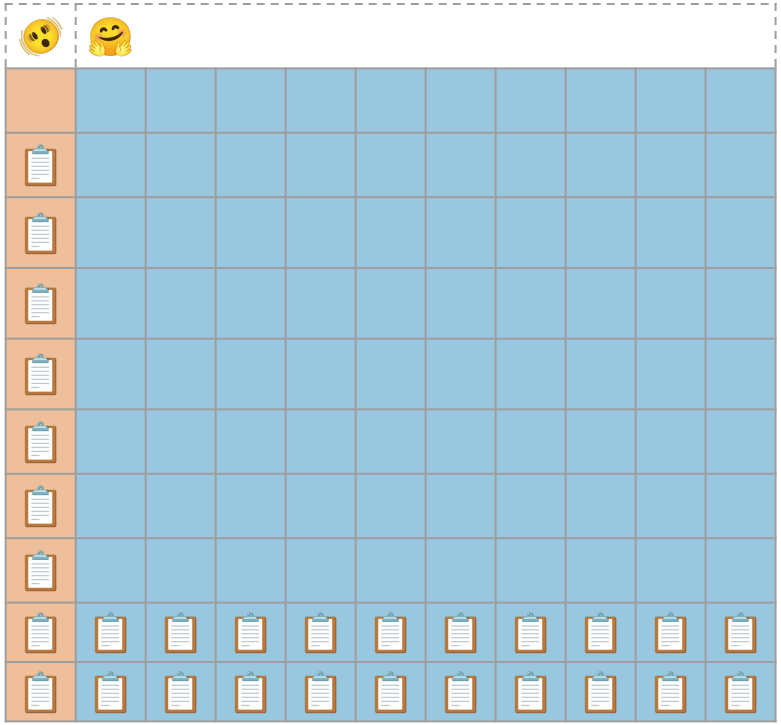
> It is the proportion of individuals in a population who have a certain characteristic or trait. — Wikipedia



Base rate fallacy

> [A] type of fallacy in which people tend to ignore the base rate (e.g., general prevalence) in favor of the individuating information (i.e., information pertaining only to a specific case). — Wikipedia

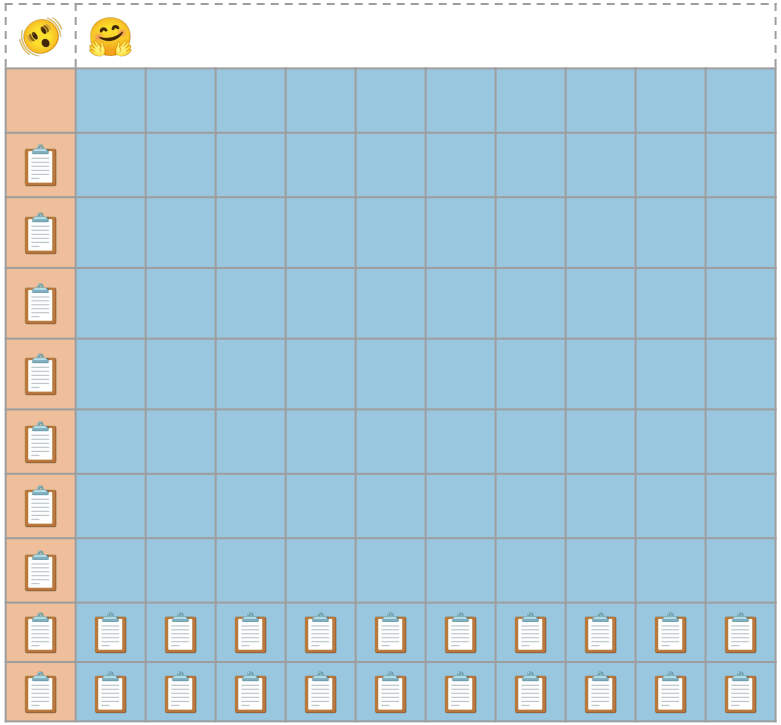
💭 Out of the number of people who test positive, how many have ADHD? (9/29 ≈ .3)



💡 Keith Devlin on base rates (Edge.org).

Bayes theorem

$p(H \mid D) = (p(H) \times p(D \mid H)) / p(D)$

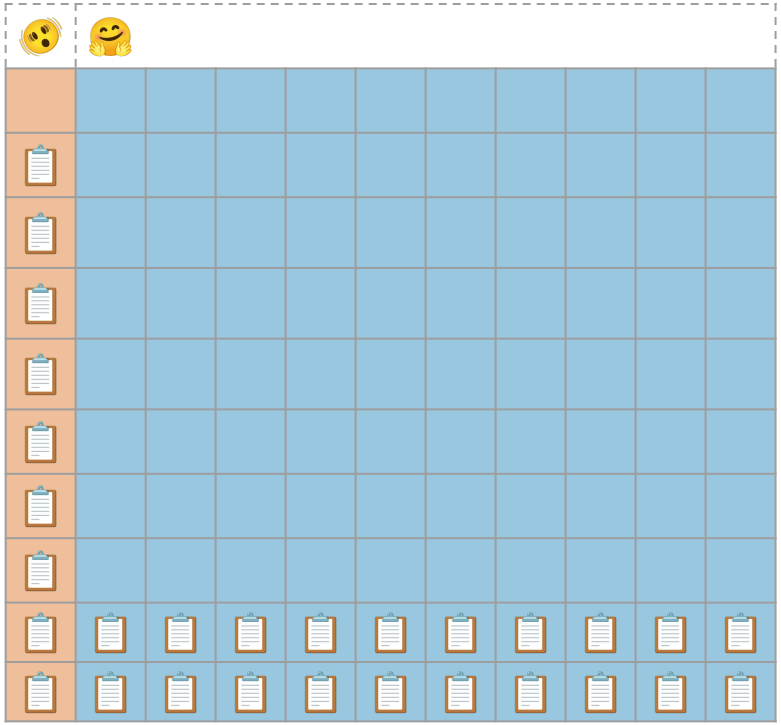


Prior

p(H | D) = (**p(H)** × p(D | H)) / p(D)

p(H) = 🟧 / (🟧 + 🟦) = 10 / (10 + 100) = .09

The probability of ADHD.



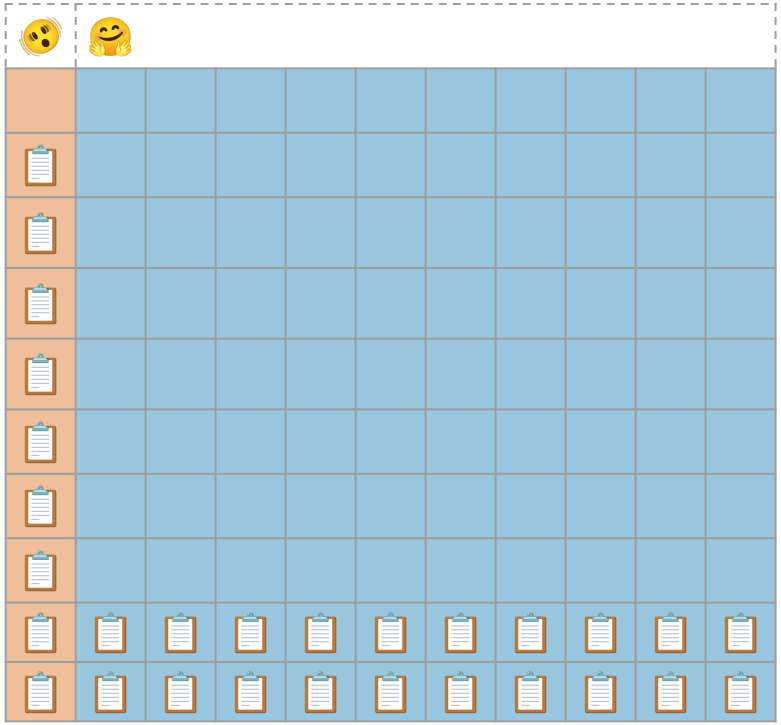
Likelihood

p(H | D) = (p(H) × **p(D | H)**) / p(D)

p(H) = 🟧 / (🟧 + 🟦) = 10 / (10 + 100) = .09

p(D | H) = 📋 / 🟧 = 9 / 10 = .9

The probability of a positive test result, given ADHD.



Prior × likelihood

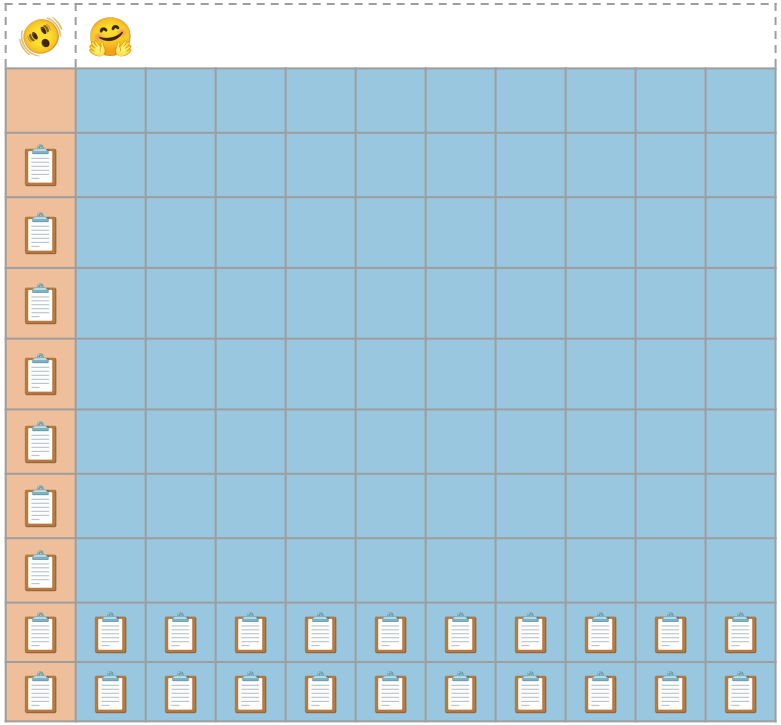
p(H | D) = (**p(H) × p(D | H)**) / p(D)

p(H) = 🟧 / (🟧 + 🟦) = 10 / (10 + 100) = .09

p(D | H) = 📋 / 🟧 = 9 / 10 = .9

p(H) × p(D | H) = 📋 / (🟧 + 🟦) = .09 × .9 = .08

The probability of a positive test result *and* ADHD.



Marginal likelihood

p(H | D) = (p(H) × p(D | H)) / **p(D)**

p(H) = 🟧 / (🟧 + 🟦) = 10 / (10 + 100) = .09

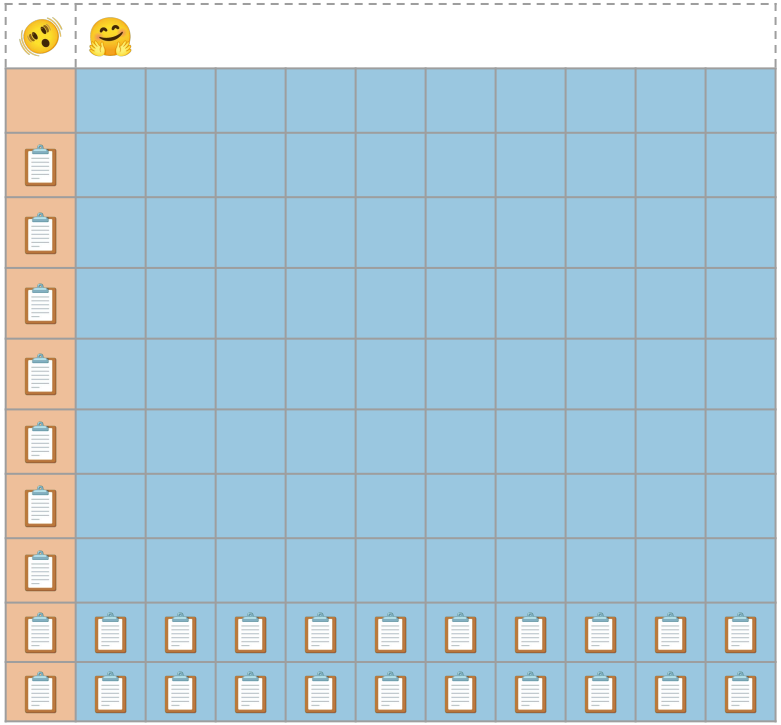
p(D | H) = 📋 / 🟧 = 9 / 10 = .9

p(H) × p(D | H) = 📋 / (🟧 + 🟦) = .09 × .9 = .08

p(D) = (p(H) × p(D | H)) + (p(¬H) × p(D | ¬H)) =

.08 + .18 = .26

The probability of a positive test result.



Posterior

p(H | D) = (p(H) × p(D | H)) / p(D) = .08 / .26 ≈ .3

p(H) = 🟧 / (🟧 + 🟦) = 10 / (10 + 100) = .09

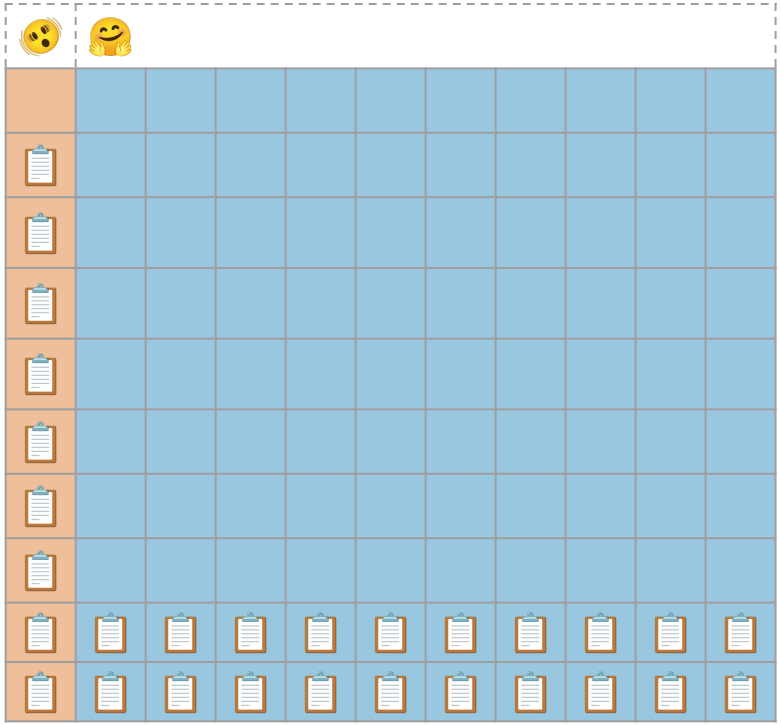
p(D | H) = 📋 / 🟧 = 9 / 10 = .9

p(H) × p(D | H) = 📋 / (🟧 + 🟦) = .09 × .9 = .08

p(D) = (p(H) × p(D | H)) + (p(¬H) × p(D | ¬H)) =

.08 + .18 = .26

The probability of someone with ADHD, given a positive test result.



Posterior

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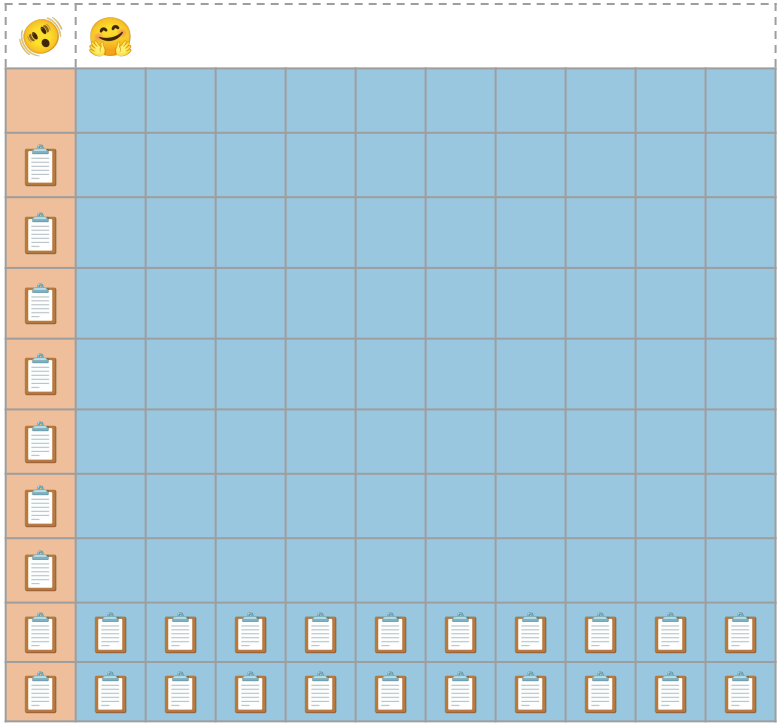
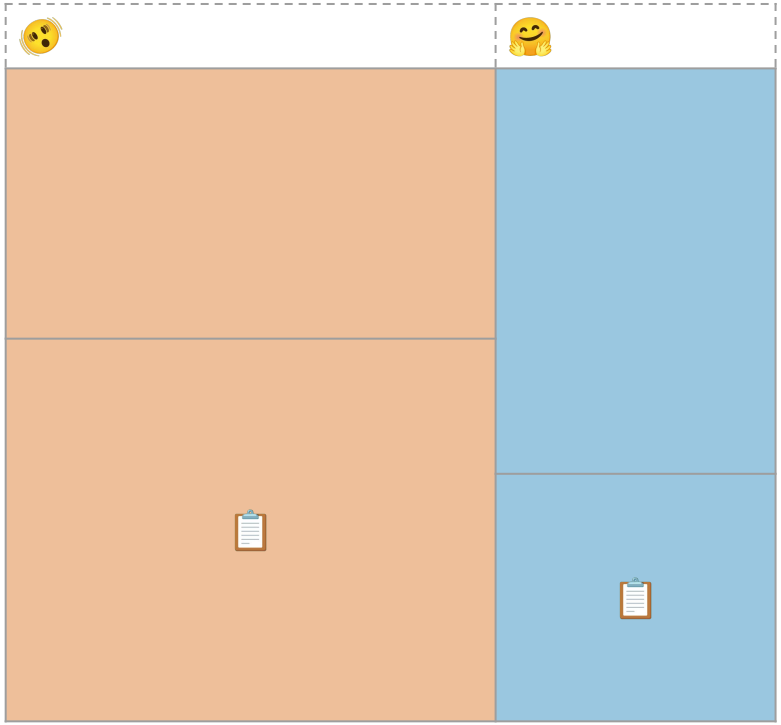
.08 + .18 = .26

🔁 Out of the number of people who test positive, how many have ADHD? (9/29 ≈ .3)



Bayes theorem

$p(H_{😮} \mid D) = p(H_{😮}) \times p(D \mid H_{😮}) / p(D) \approx .3$
$p(H_{🤗} \mid D) = p(H_{🤗}) \times p(D \mid H_{🤗}) / p(D) \approx .7$
(‘alternative’ hypothesis)

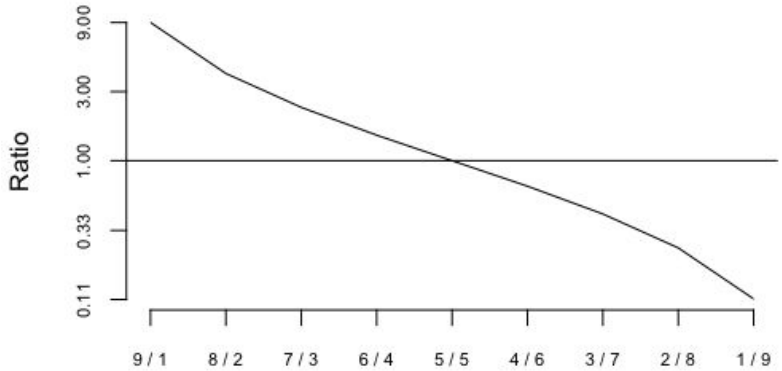


Bayesian Hypothesis Testing

Bayesian hypothesis testing

> The Bayes factor is a ratio of two competing statistical models represented by their evidence, and is used to quantify the support for one model over the other.
> — Wikipedia

🔁 Remember the *F*-ratio?



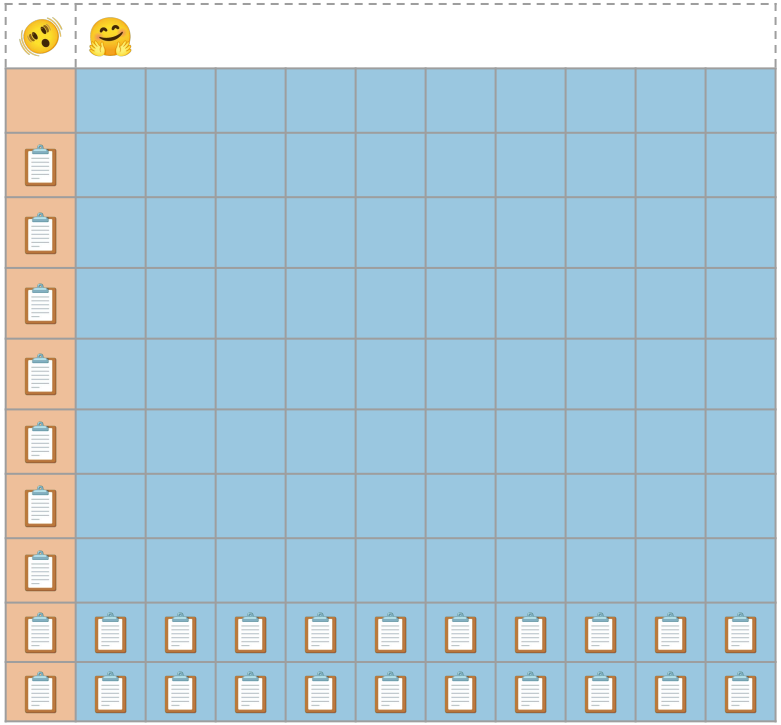
Bayes factor

K = p(D | $H_{🧐}$) / p(D | $H_{🤗}$) = ?

p(H | D) = (p(H) × **p(D | H)**) / p(D)
p(D | $H_{🧐}$) = 📋 / 🟧 = 9 / 10 = .9
p(D | $H_{🤗}$) = 📋 / 🟦 = 20 / 100 = .2

K = p(D | $H_{🧐}$) / p(D | $H_{🤗}$) = 4.5

- Continuous degree of evidence (vs. all-or-none)
- Monitor evidence during data collection
- Evidence of absence (data support a null effect) and absence of evidence (data are not informative)



Bayes factor interpretation

K	dHart	bits	Strength of evidence
$< 10^0$	< 0	< 0	Negative (supports M_2)
10^0 to $10^{1/2}$	0 to 5	0 to 1.6	Barely worth mentioning
$10^{1/2}$ to 10^1	5 to 10	1.6 to 3.3	Substantial
10^1 to $10^{3/2}$	10 to 15	3.3 to 5.0	Strong
$10^{3/2}$ to 10^2	15 to 20	5.0 to 6.6	Very strong
$> 10^2$	> 20	> 6.6	Decisive

— Wikipedia

$\log_{10} K$	K	Strength of evidence
0 to 1/2	1 to 3.2	Not worth more than a bare mention
1/2 to 1	3.2 to 10	Substantial
1 to 2	10 to 100	Strong
> 2	> 100	Decisive

— Wikipedia

🔥 Discussion on Andrew Gelman's blog. Mindless (Bayesian) statistics 🔒.

Bayes factor interp

K	dHart	bits	Str
$< 10^0$	< 0	< 0	Ne
10^0 to $10^{1/2}$	0 to 5	0 to 1.6	Ba
$10^{1/2}$ to 10^1	5 to 10	1.6 to 3.3	
10^1 to $10^{3/2}$	10 to 15	3.3 to 5.0	
$10^{3/2}$ to 10^2	15 to 20	5.0 to 6.6	
$> 10^2$	> 20	> 6.6	

Strength of evidence
t worth more than a bare mention
Substantial
Strong
Decisive

— Wikipedia

n Andrew Gelman's blog.
sian) statistics 🔒.

Illustration by Viktor Beekman, Eric-Jan Wagenmakers

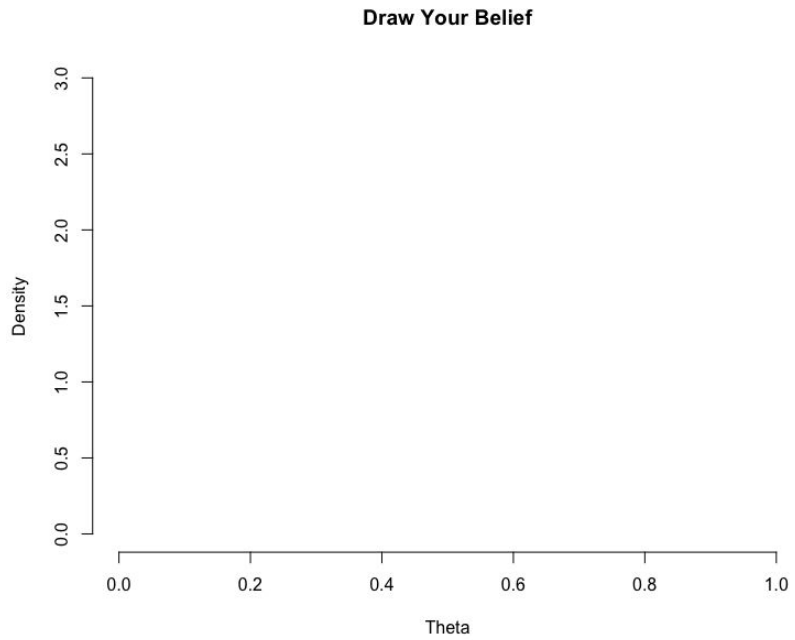
Bayesian Parameter Estimation

🎲 Bayesian parameter estimation

🪙: theta = p(heads)

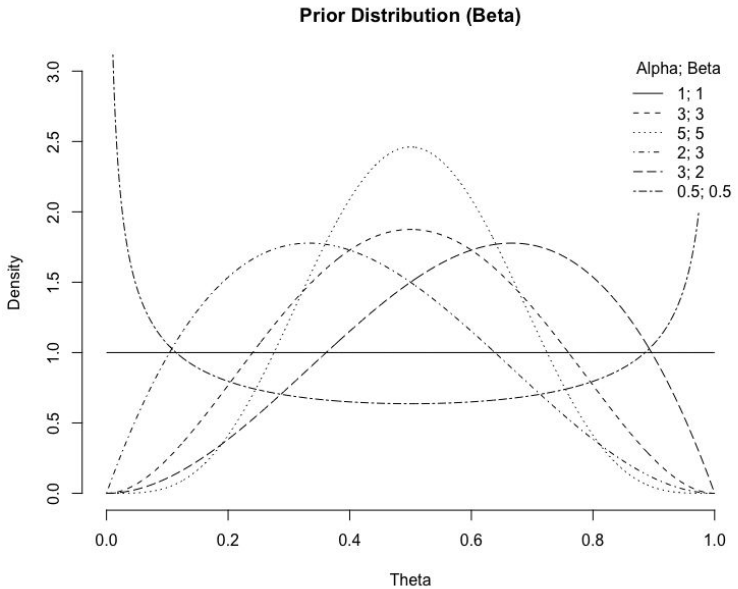
“Which values of theta *are good estimates (H), given my data (D)?”*

p(H | D) = (p(H) × p(D | H)) / p(D)



💡 How to get fair results from a biased coin?

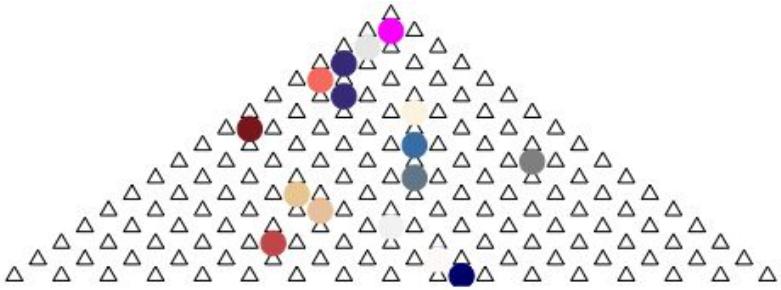
Prior distribution p(H)



- Prior belief/information
- Uninformative/informative
- Weakly/strongly informative
- Skeptical
- Point-valued

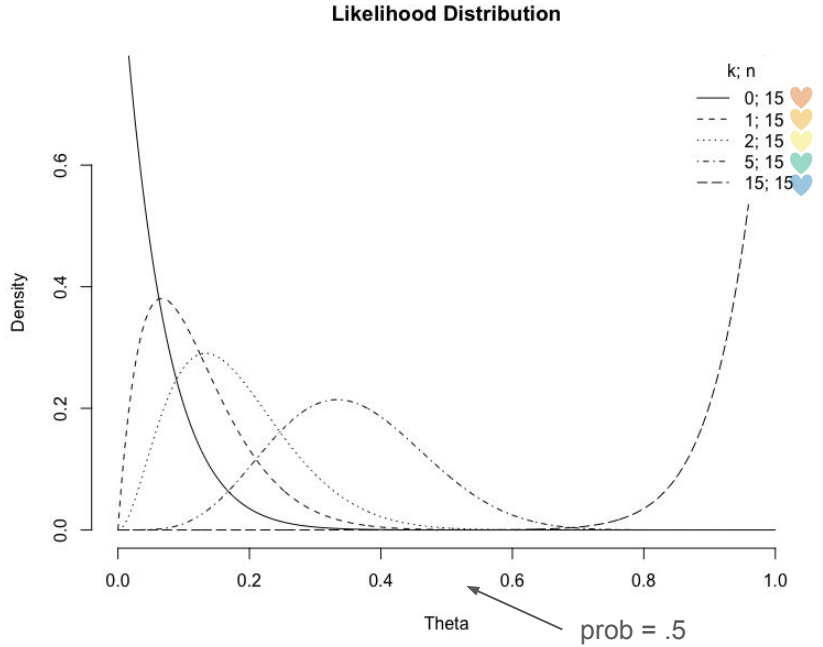
💡 Beta distribution.

Likelihood distribution p(D | H)



n = 15; prob = .5

```
dbinom(x = k, size = n, prob = seq(0, 1,
.01))
```



Posterior distribution p(H | D)

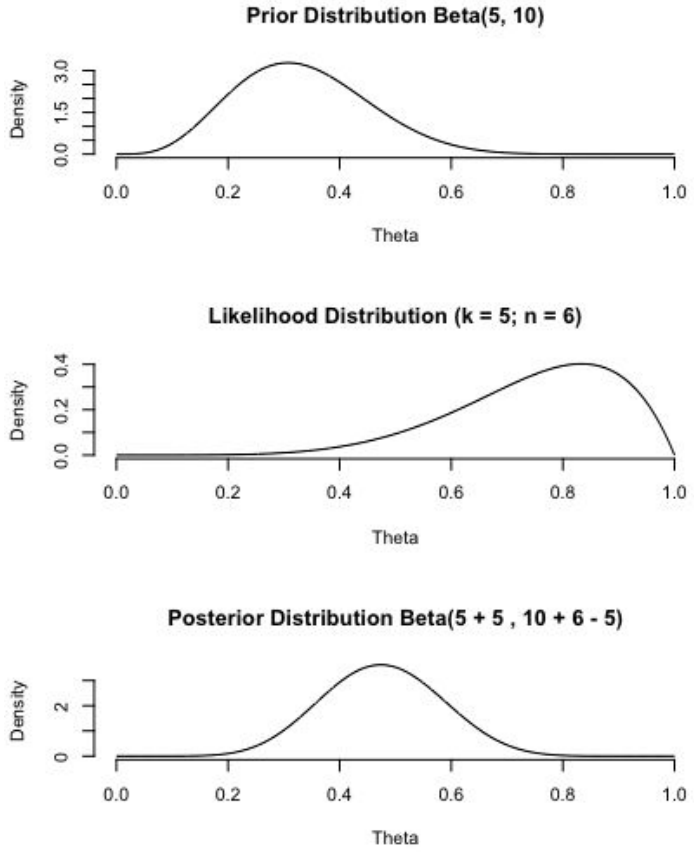
p(H | D) = (p(H) × p(D | H)) / p(D)

p(D) (marginal likelihood distribution): “integral of doom” 💀

But, beta distribution is *conjugate prior* of binomial distribution:

For a prior theta~Beta(a,b) and data *k* and *N*, the posterior is theta~Beta(a+k,b+N−k).

Repeat! Iterative!



JASP

How to use JASP: Lots of resources.

Cooling Down

Dynamic Textures #382 by Claus O. Wilke 🎨

Takeaways

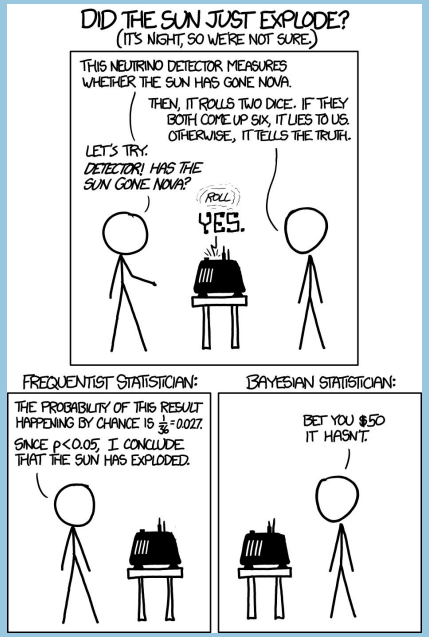
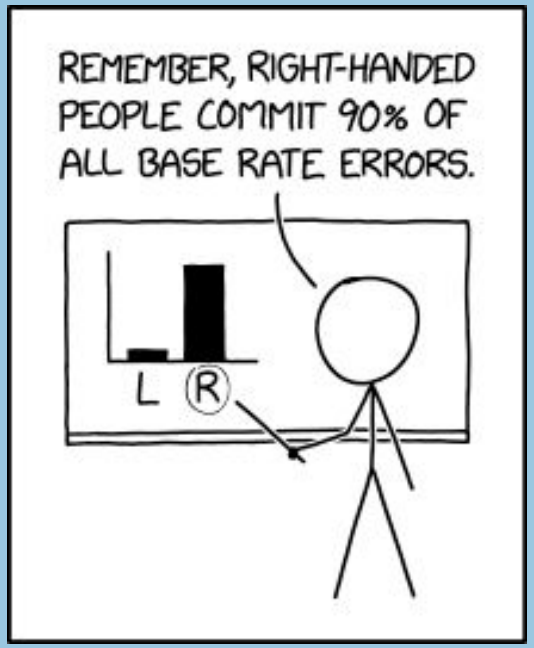


Illustration by Randall Munroe (wtf / 🔥)

Takeaways



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Takeaways

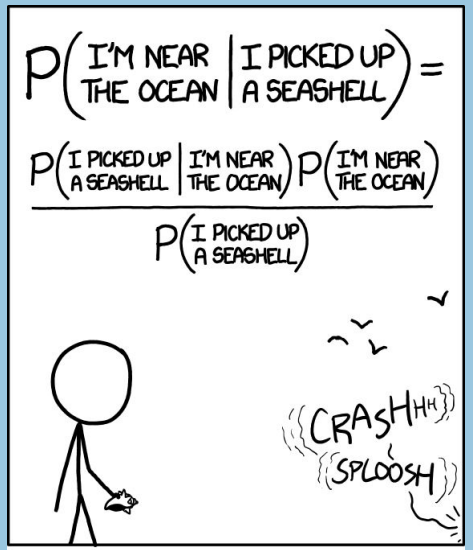

Illustration by Randall Munroe (wtf)

Takeaways

Illustration by Viktor Beekman

Takeaways

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Takeaways

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SAGE journals

Bayesians Caught Smuggling Priors Into Rotterdam Harbor

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Amsterdam, April 13, 2011. A group of international Bayesians was arrested today in the Rotterdam harbor. According to Dutch customs, they were attempting to smuggle over 1.5 million priors into the country, hidden between electronic equipment. The arrest represents the largest capture of priors in history.

"This is our biggest catch yet. Uniform priors, Gaussian priors, Dirichlet priors, even informative priors, it's all here," says customs officers Benjamin Roosken, responsible for the arrest. "There are priors for memory experiments, intelligence tests, flanker tasks, meta-analyses, political preference, everything! God only knows what would have happened if this had gotten through. We're pretty lucky to catch them too. The chance of being in the right place, given the right time, if you take into account the number of arrests, divided by the number of successful arrests every year, it's pretty slim. We're very glad indeed."

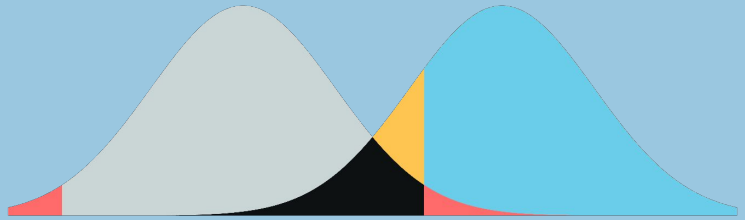
Sources suggest that the shipment of priors was going to be introduced into the Dutch scientific community by "white-washing" them. "They are getting very good at it. They found ghost-journals with fake articles, refer to the papers where the priors are allegedly based on empirical data, and before you know it, they're out in the open. Of course, when you look up the reference, everything is long gone," says Roosken.

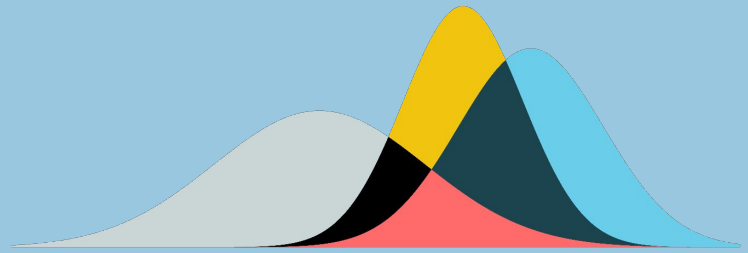
Until recently, the Dutch government adopted a lenient, pragmatic approach toward priors. As an anonymous source states, "It was quite simple. Scientists were allowed to *use* priors, but not to create them at home. It may sound a bit counterintuitive, but it worked quite well, for a while at least." However, according to critics, this policy created an uncontrollable backdoor industry.

The discovery of international smuggling rings has caused the government to revise its strategy and crack down hard on illegal trade. The capture of the smuggling ring symbolizes a new, tough stance on priors. "We will not stand for this unjustified and illegal use of priors any longer," says Roosken. If found guilty, the defendants may face 12 years in prison (95% CI [10.2, 13.8], $p < .01$).

[Kievit, 2011](#) 🔒

Takeaways



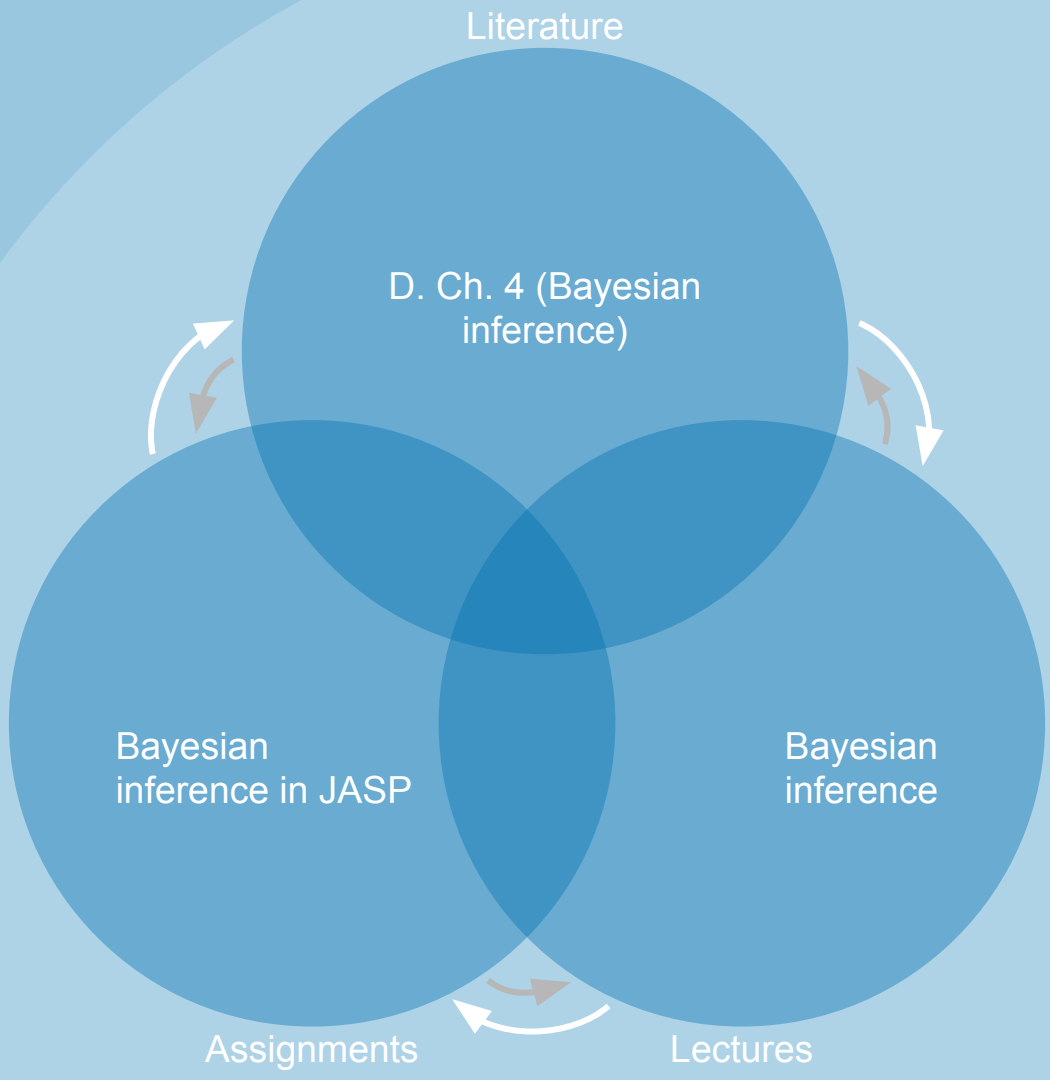


Illustrations by [Kristoffer Magnusson]

Frequentist Significance Testing

Bayesian Inference

🥇 Nail it


Literature
D. Ch. 4 (Bayesian inference)
Bayesian inference in JASP
Bayesian inference
Assignments
Lectures

🥇 Exam(ple) question

Sir Ronald Fisher test positief op Covid en vraagt zich nu af hoe hoog de kans is dat hij ook daadwerkelijk besmet is met het virus. Hij heeft een aantal feiten opgezocht.

- Van alle mensen test 1 op de 5 positief.
- Van de mensen die besmet zijn met Covid test 9 op de 10 positief.
- 1 op de 1000 mensen heeft Covid.

Wat is nu de kans dat Sir Ronald Fisher besmet is met het Covid virus?

A. .001
B. .0045
C. .2
D. .9

👩‍💻 Take-home assignments

📆 Weekly assignment

🍻 Pub quiz

Create an *informative* four-choice question about the content of today's lecture.

An informative question has a large spread in responses across answer options.

Clarify answer options (which are (in)correct and why).

Illustration adapted from Snippets.com

🔮 Overview

Topics

Probabilities & distributions
Frequentist inference
Multiple linear regression
Factorial ANOVA
Nonparametric inference
Bayesian inference

Illustration by Jennifer Cheuk

💯 Exam*

(*course manual is leading)

❓ Questions (SR)

- Open and closed
- Practical and theoretical (learning goals)
- R, JASP output (no access to JASP)
- No follow-up (like in weekly assignments)
- From literature, lectures, assignments
- Statistics exam, not a programming exam

💯 Points (see course manual)

- Grade = .8 × Exam + .2 × PhS Assignment
- Exam = ⅝ × SR + ⅜ × PhS (must be ≥5.5)
- Correction for guessing (applied after exam)

📚 Resources at the exam

- R, RStudio
- Course literature (.pdf)
- Lecture slides (.pdf, SR only, broken links)
- Weekly assignments and model solutions (.pdf)
- Scrap paper, if needed

🎒 What to bring

- Student ID card, **UvAnetID credentials**
- Pen (no calculator, use R)
- Water, snack

🖥️ At the exam

- Come early
  - Take into account delays (traffic, etc.)
  - Visit the restroom prior to the exam
  - Login may take up to 6 minutes
- Don't switch computers
- Open the resources folder, RStudio, SOWISO exam (*'Start'*, requires password)
- Late arrivals allowed during first 30 minutes (but same end time for everyone)
- Not allowed to leave during first 30 minutes
- Allowed to leave during last 15 minutes
- When leaving: leave scrap paper behind and be quiet

⛑️ Examination ICT can fail in unimaginable ways

🧰 Look here!

Bayes theorem

- Explanation video (3Blue1Brown)
- Product rule (prior × likelihood) (3Blue1Brown)
- Interactive visualization (Seeing Theory)
- Redefining Bayes rule (3Blue1Brown)

Prior and posterior

- Interactive visualization (Seeing Theory).

Likelihood

- Interactive visualization (Seeing Theory).

Bayesian inference

- Web simulation (Kristoffer Magnusson)
- Brief introduction (Johnny van Doorn)

Bayesian meets frequentist

- Side-by-side Shiny app (John Kruschke)
- Video tutorial for Shiny app (Eero Liski)

👶 Bayesian thinking for toddlers (Eric-Jan Wagenmakers)

⚠️ Don’t look here!

Hints (select and copy/paste the invisible text below to reveal it)

0.

1.

2.

3.

✍️ Colophon

Slides
alexandersavi.nl/teaching/

License