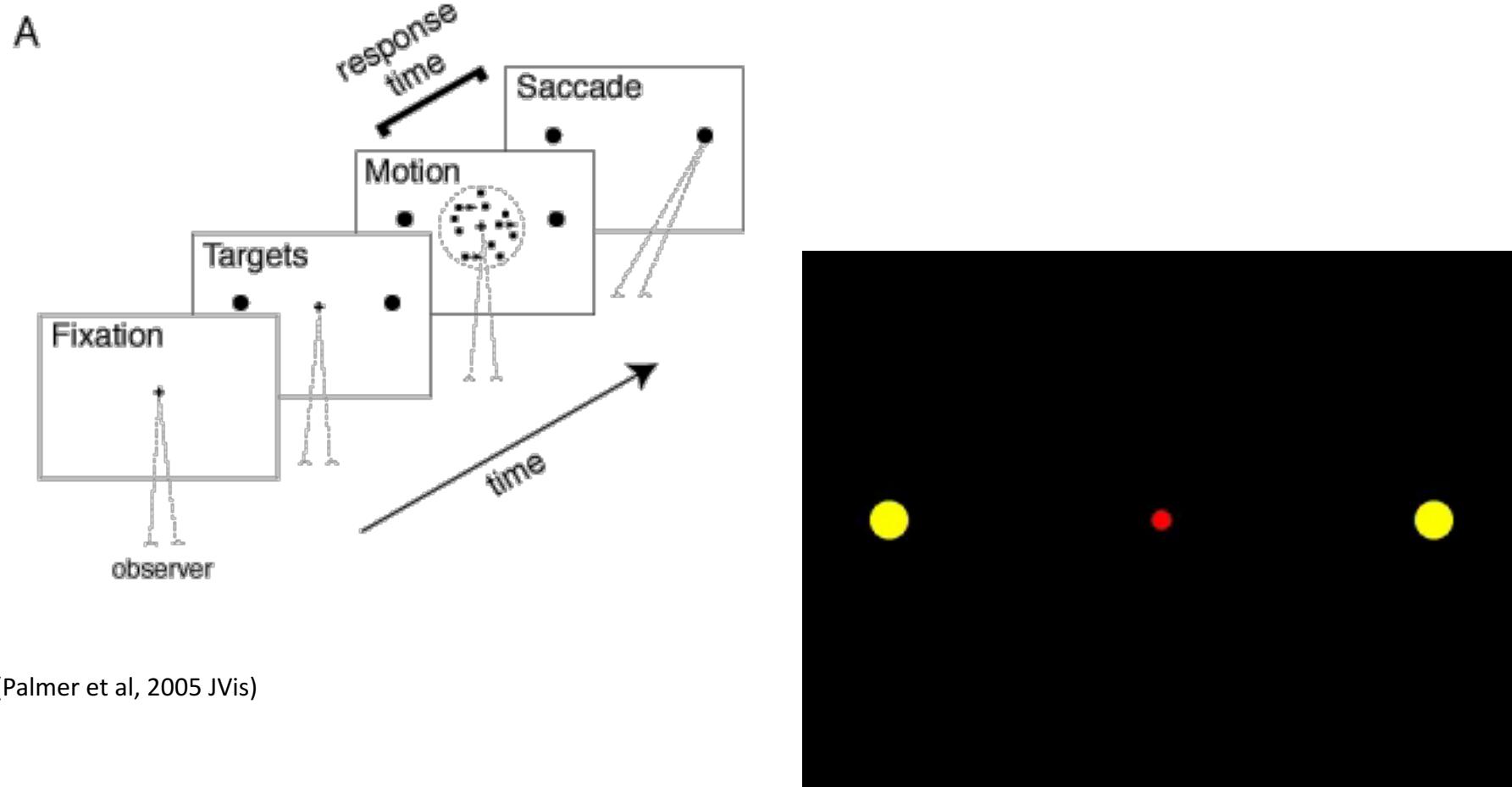


# Accumulator Models of choice

Leendert van Maanen



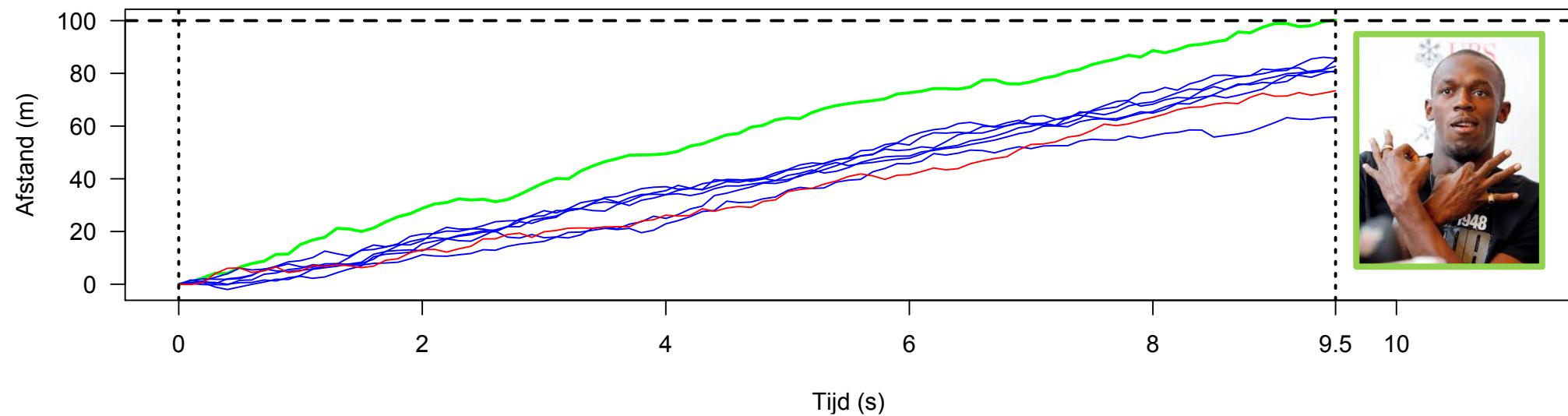
# Two-alternative forced choice (2AFC) task





ZDF | WM09







WL

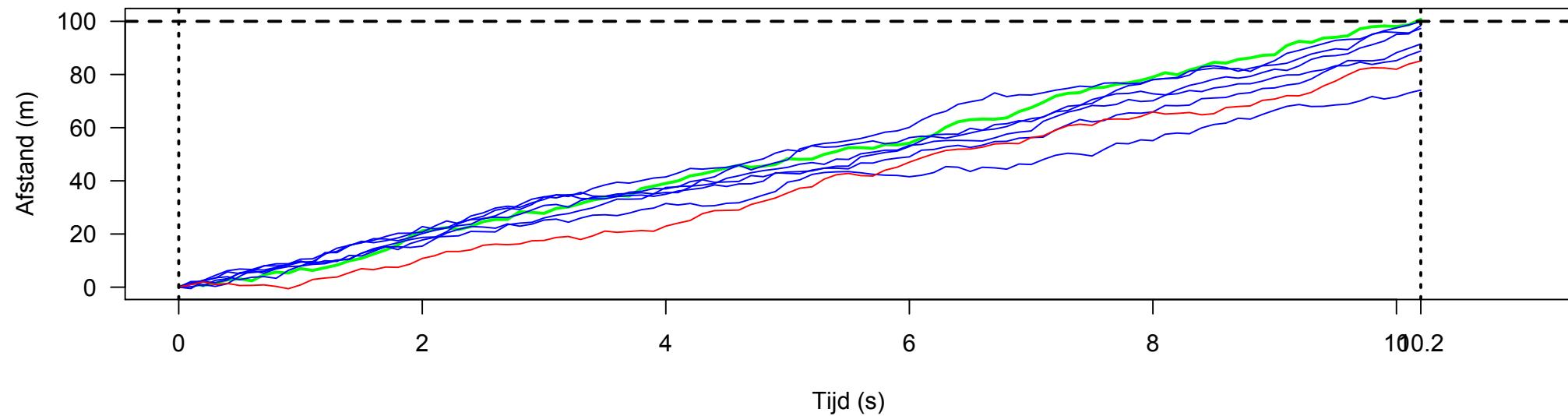
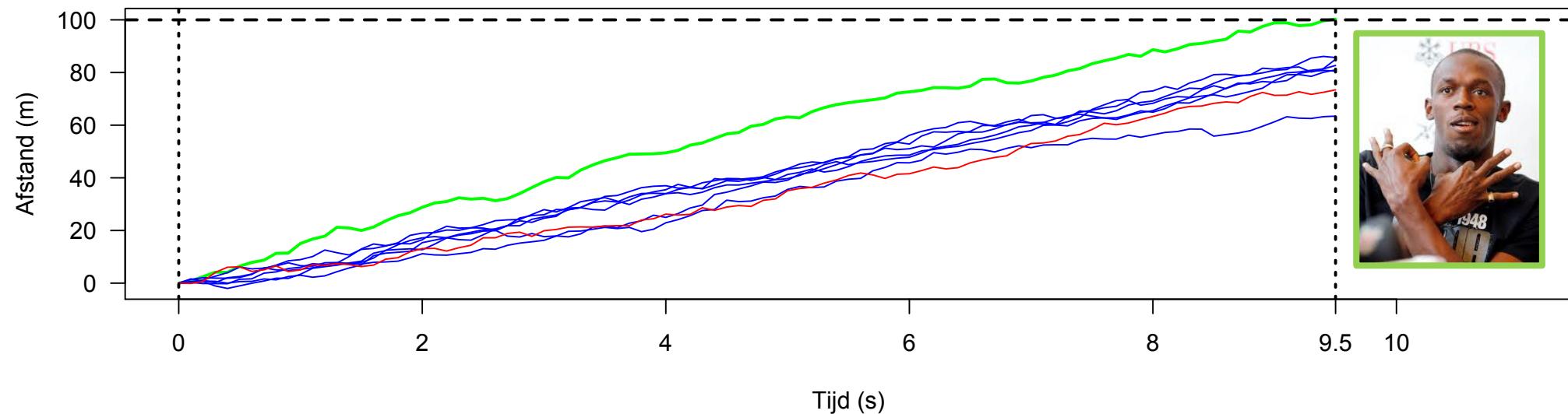
9.86

MR

9.76

100m M

0.0



# Overview

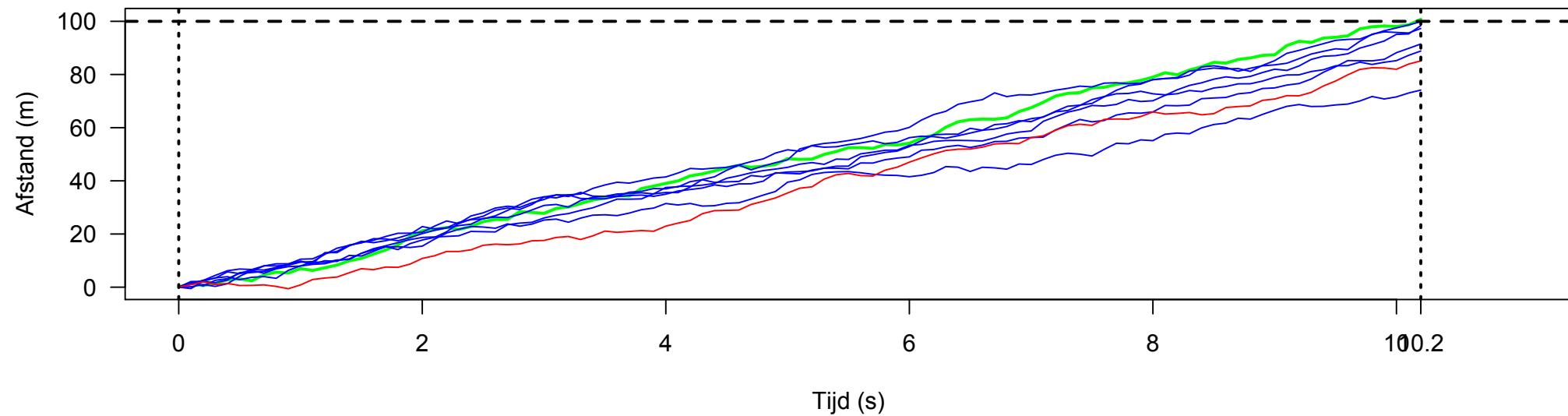
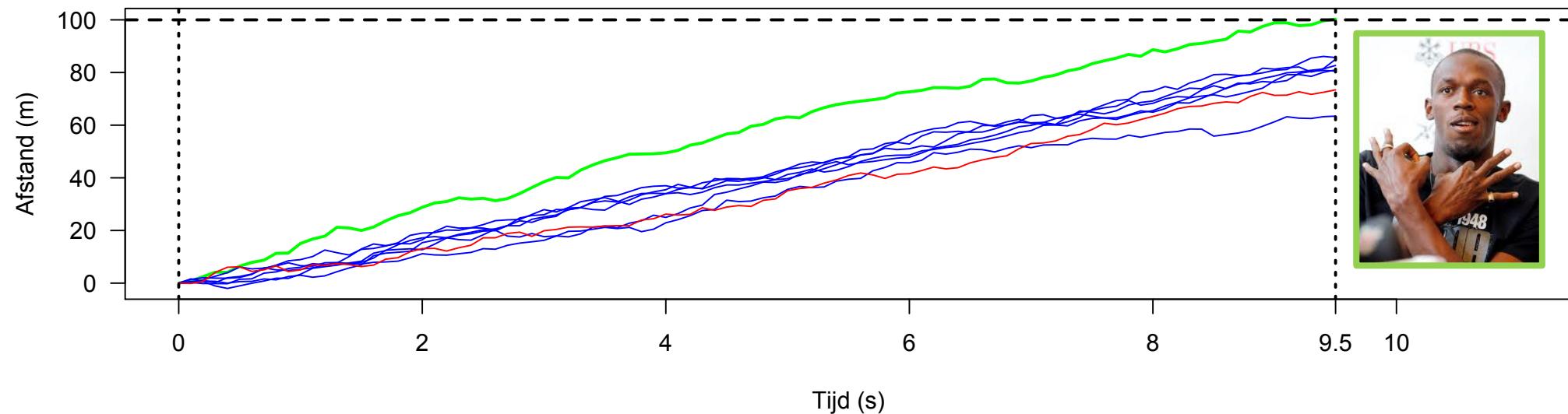
- One implementation: Linear Ballistic Accumulator
- Estimating parameters
- Relationship with neuroscience
- Another implementation: The (EZ) diffusion decision model

# Overview

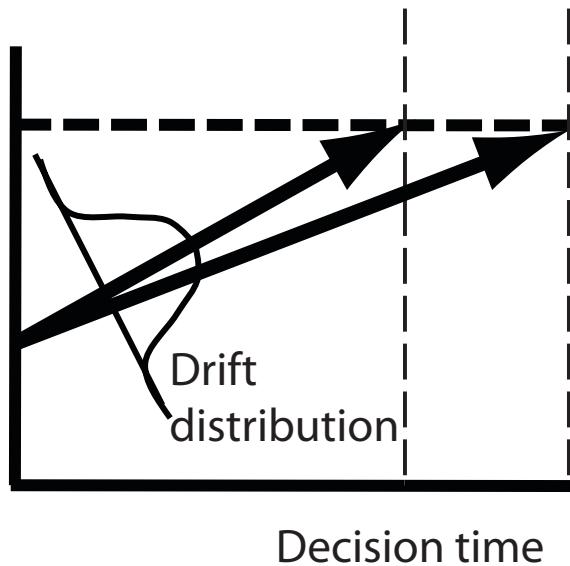
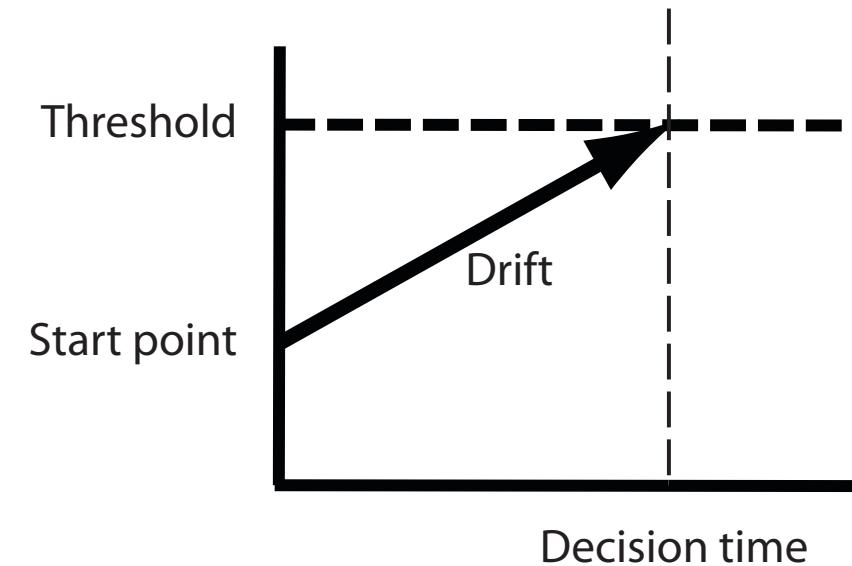
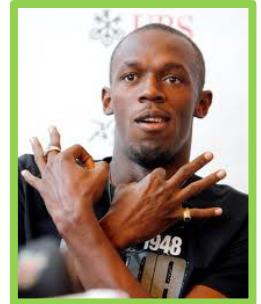
- **One implementation: Linear Ballistic Accumulator**
- Estimating parameters
- Relationship with neuroscience
- Another implementation: The (EZ) diffusion decision model

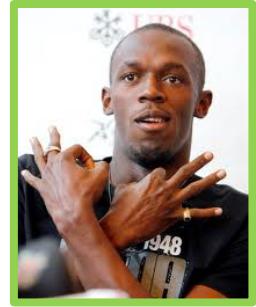
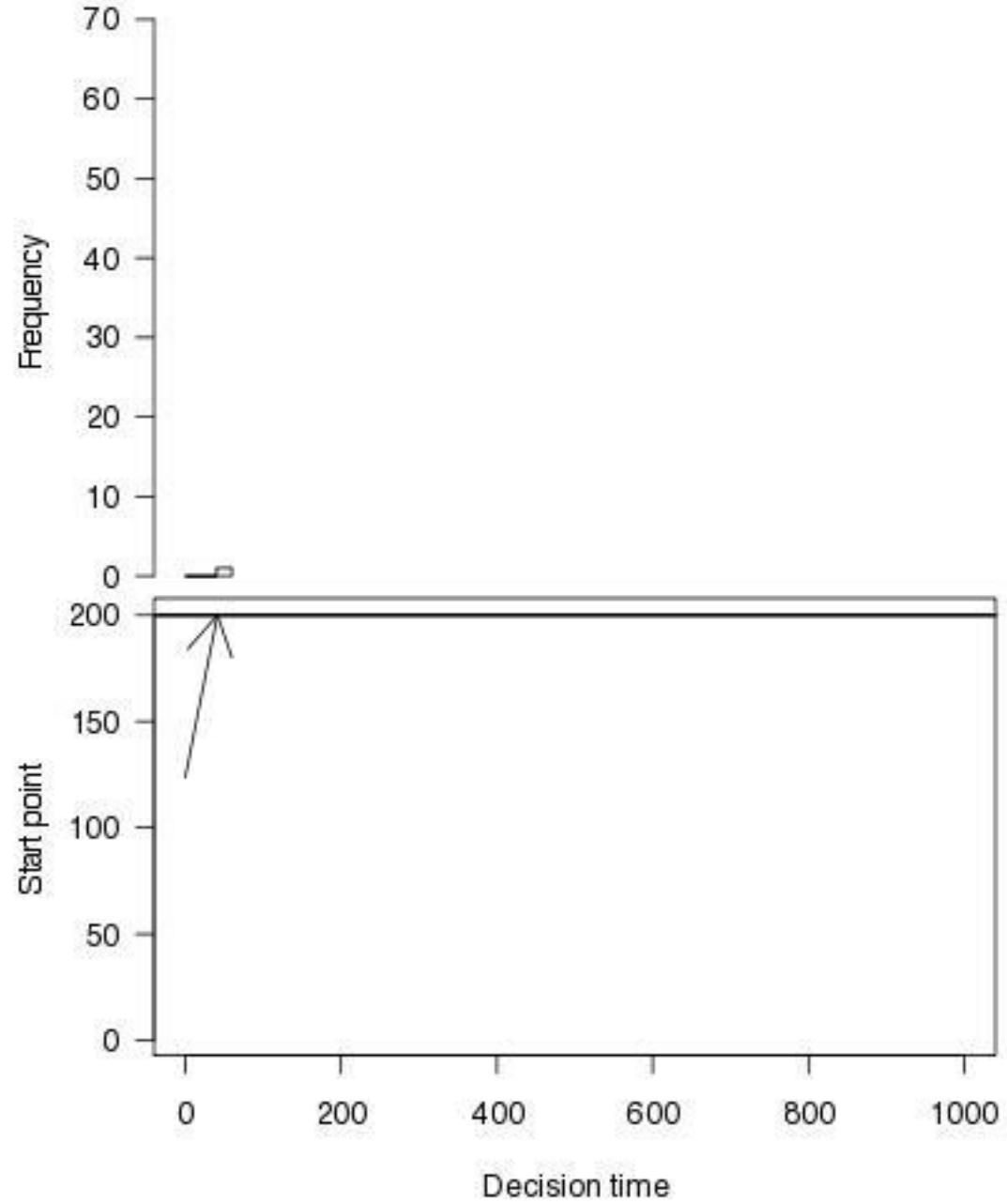
# How to quantify a race?

- Sequential Probability Ratio Test (SPRT)
- **Diffusion Decision model (DDM) (EZ diffusion model)**
- **Linear Ballistic Accumulator model (LBA)**
- Leaky Competitive Accumulator model (LCA)
- Retrieval by Accumulating Evidence in an Architecture (RACE/A)
- ...

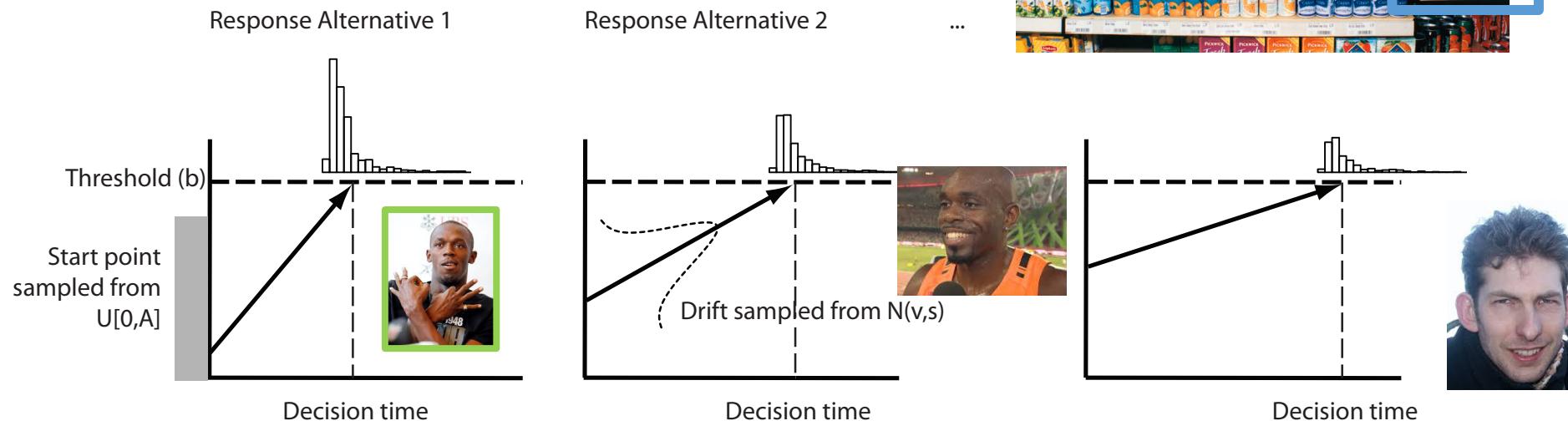
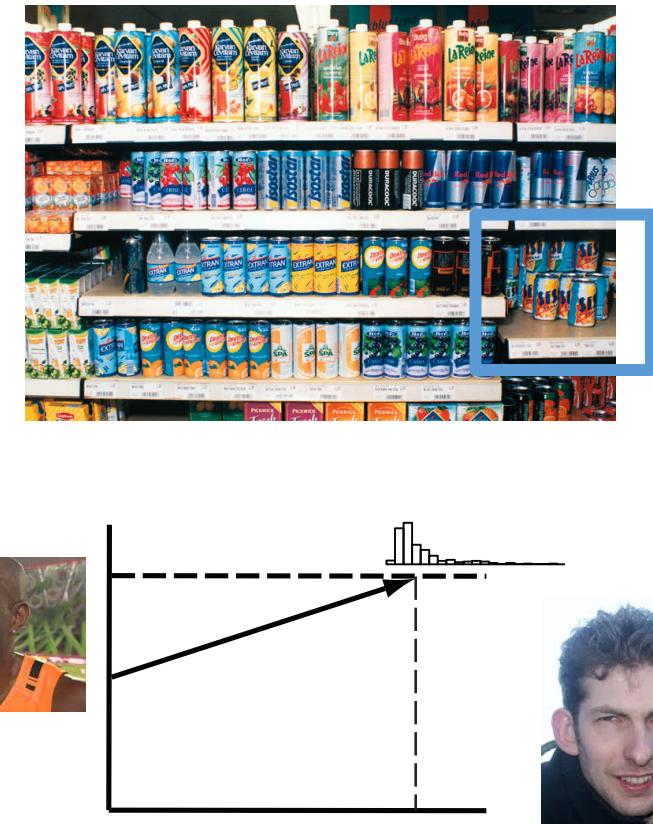


# Linear Ballistic Accumulator





# Linear Ballistic Accumulator



- Every option has its own accumulator
- Fastest accumulator wins (that option gets selected)

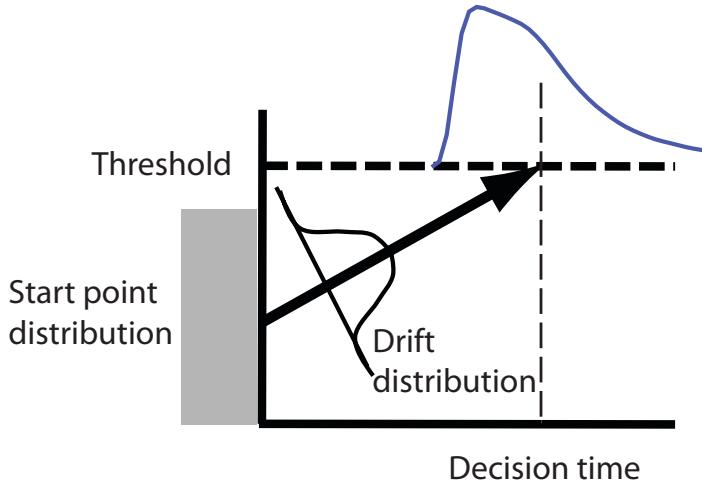
$$f_i(t) = \frac{1}{A} \left[ -v_i \Phi\left(\frac{b - A - tv_i}{ts}\right) + s\phi\left(\frac{b - A - tv_i}{ts}\right) + v_i \Phi\left(\frac{b - tv_i}{ts}\right) - s\phi\left(\frac{b - tv_i}{ts}\right) \right]$$

# Overview

- One implementation: Linear Ballistic Accumulator
- **Estimating parameters**
- Relationship with neuroscience
- Another implementation: The (EZ) diffusion decision model

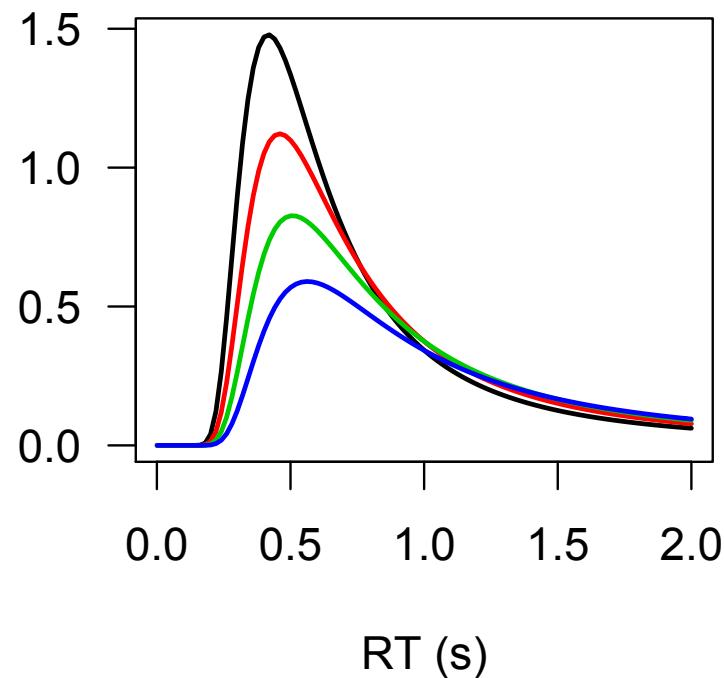
# Model fitting

- What are the optimal parameters
  - Startpoint
  - Threshold
  - Drift rate
  - Non-decision time
- Optimal: Least **unexplained** variance

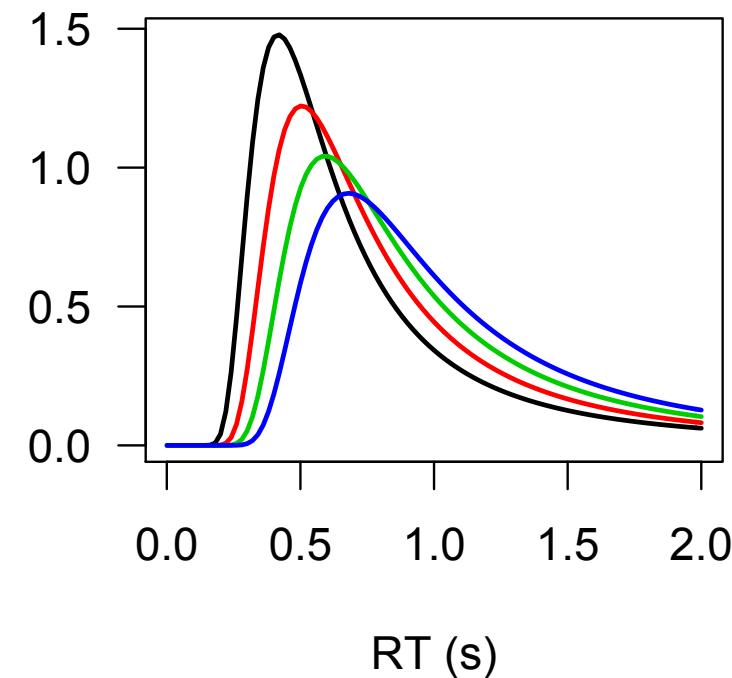


$$f_i(t) = \frac{1}{A} \left[ -v_i \Phi\left(\frac{b - A - tv_i}{ts}\right) + s \phi\left(\frac{b - A - tv_i}{ts}\right) + v_i \Phi\left(\frac{b - tv_i}{ts}\right) - s \phi\left(\frac{b - tv_i}{ts}\right) \right]$$

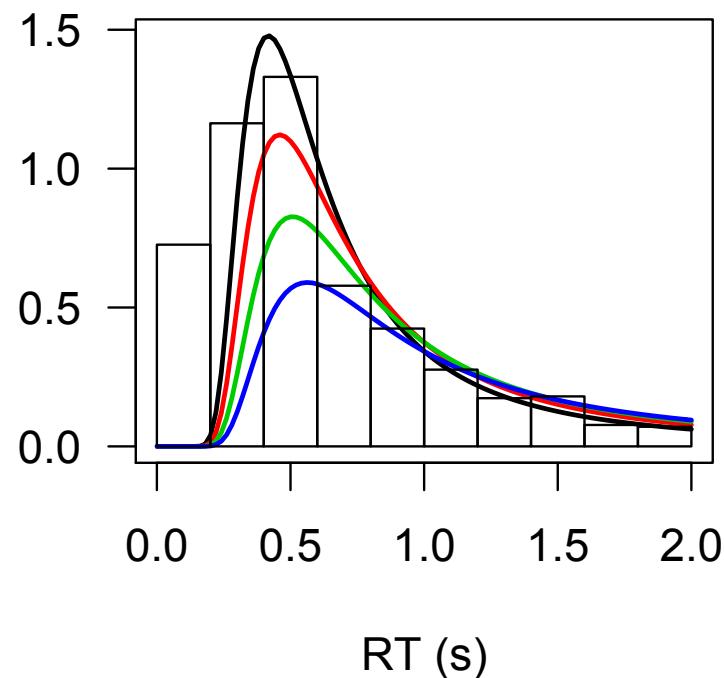
**Drift change**



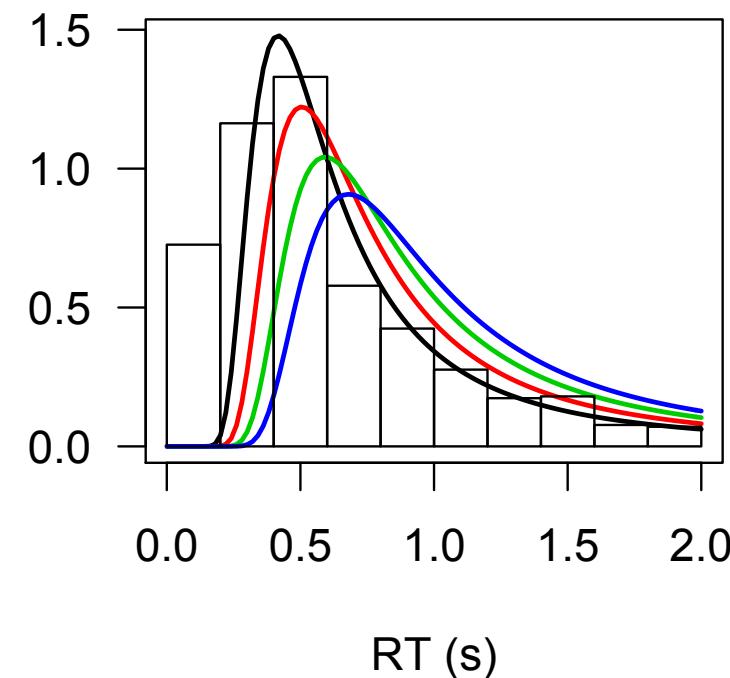
**Threshold change**



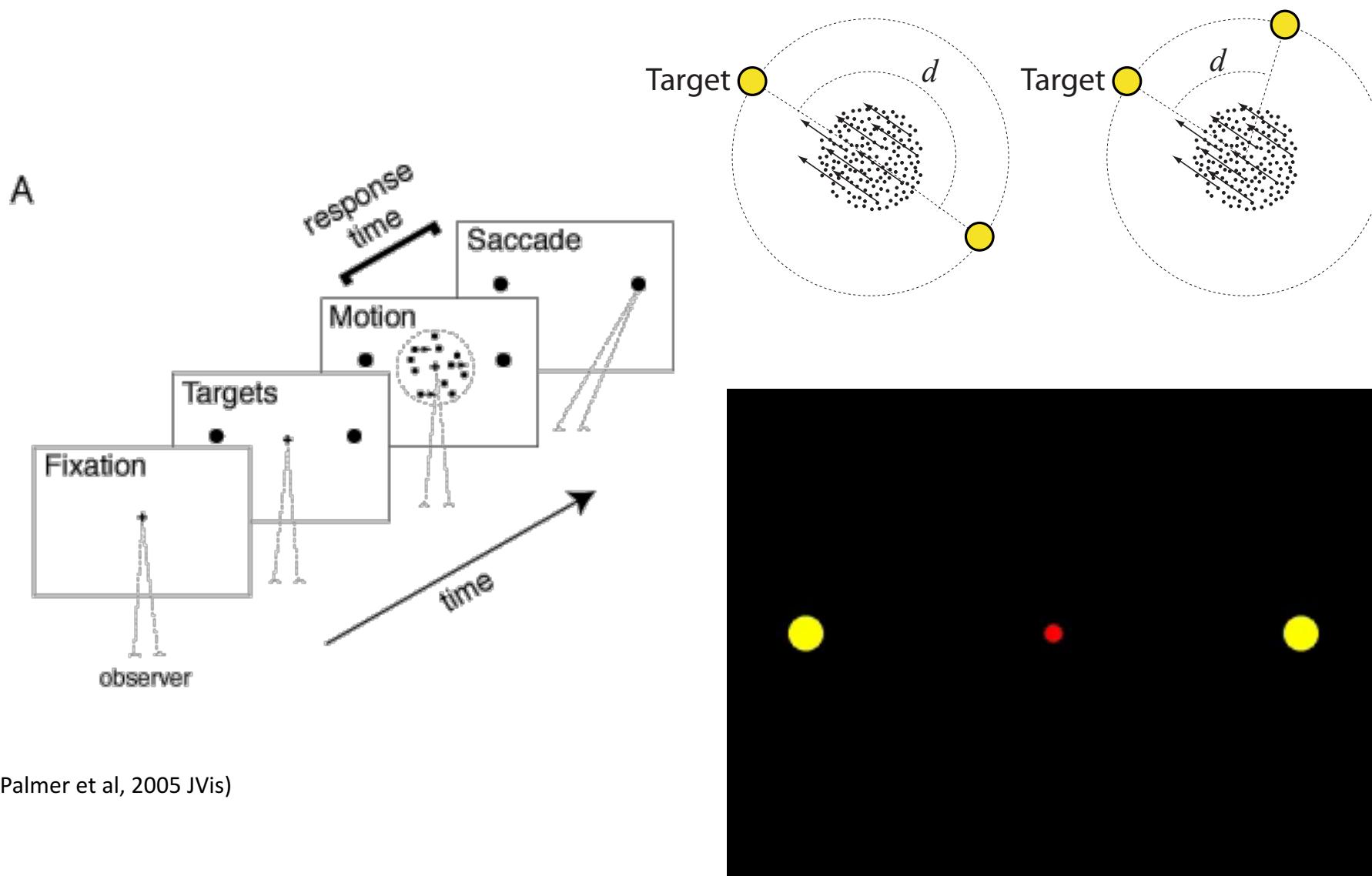
**Drift change**



**Threshold change**

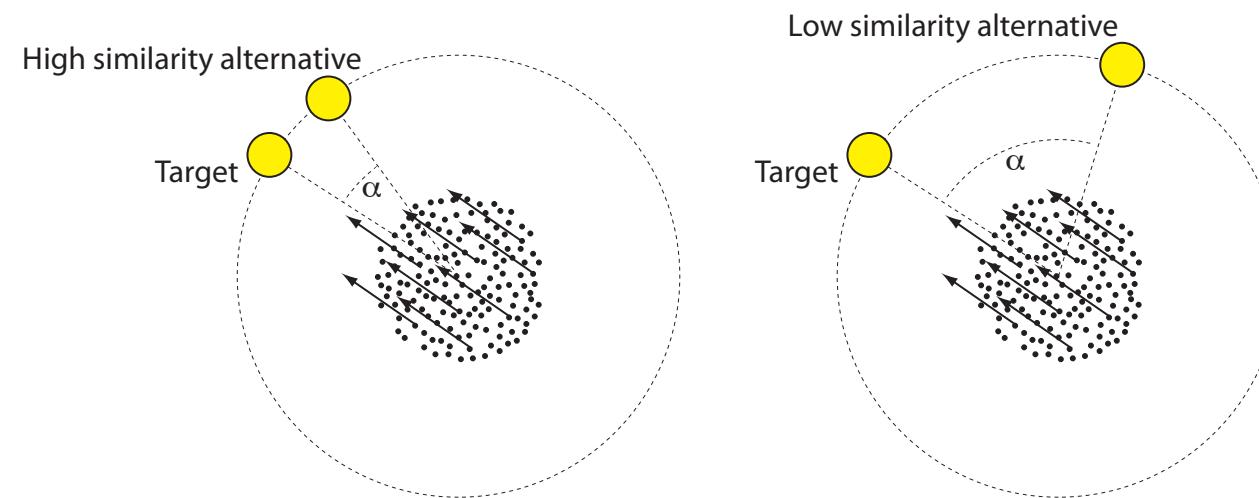


# Example: effect of target location in 2AFC

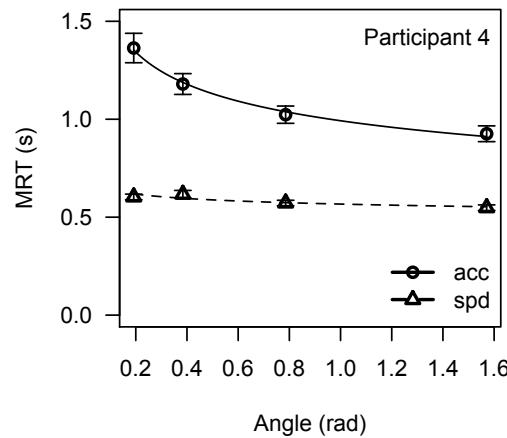
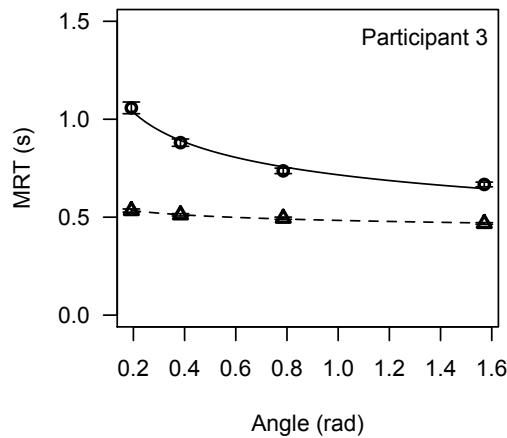
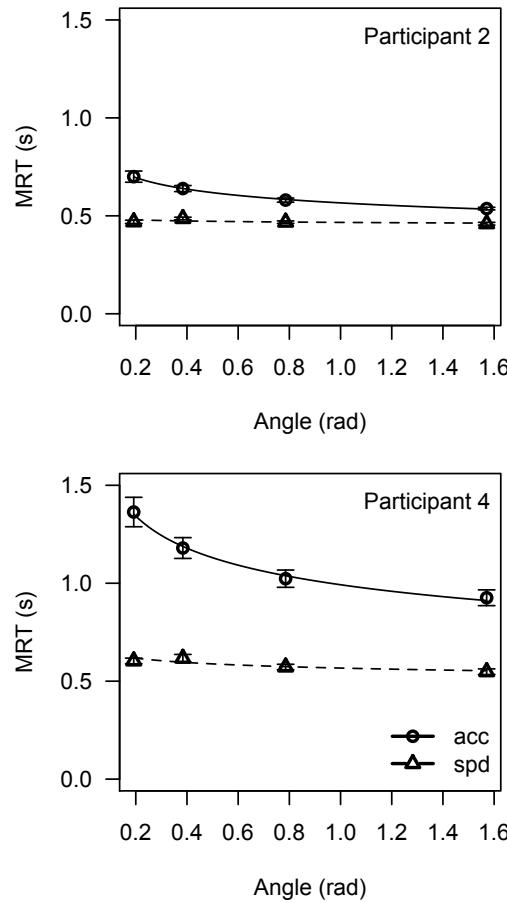
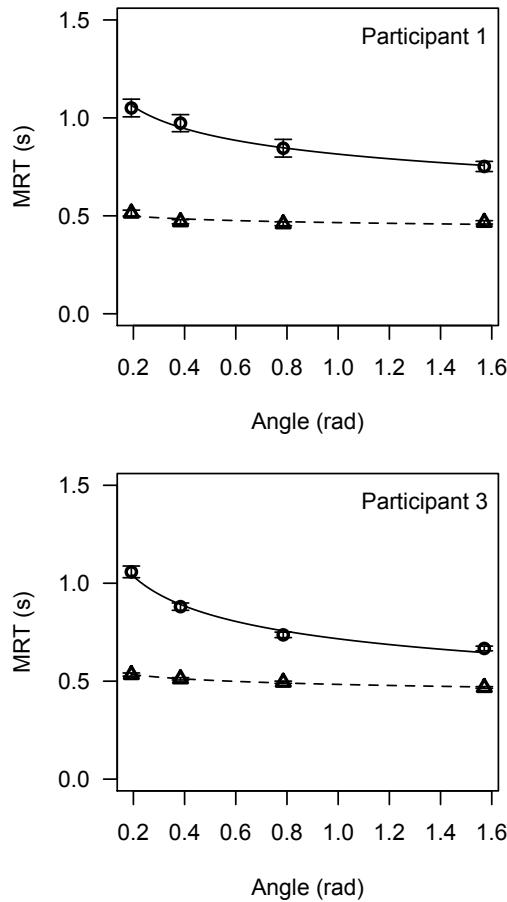


# Example: effect of target location in 2AFC

- Binary choice
- Two instructions (blocks): “Focus on speed” vs “Focus on Accuracy”
- Target location varied

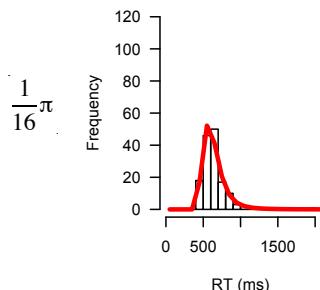


# Mean response times

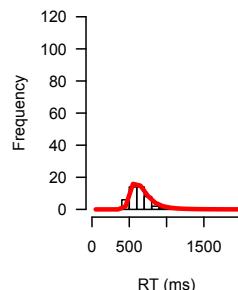


# Speed

Correct

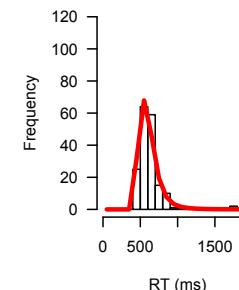


Error

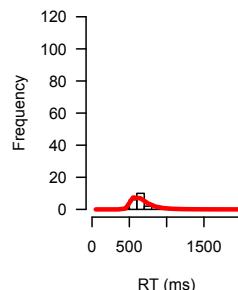


# Accuracy

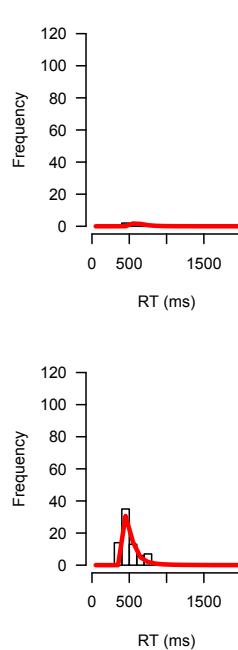
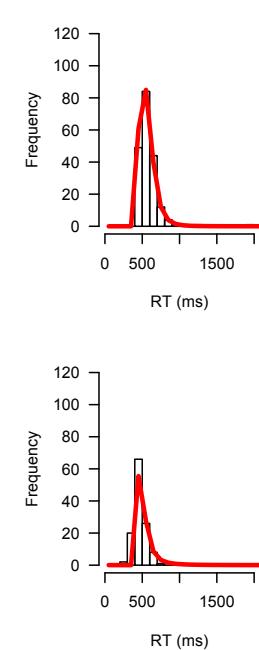
Correct



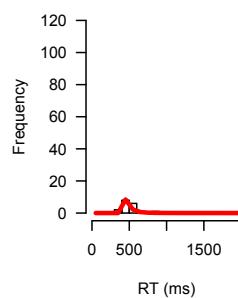
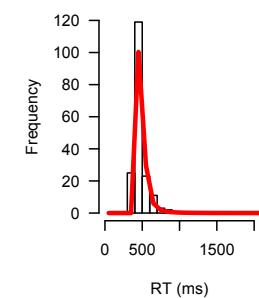
Error



1/4 $\pi$

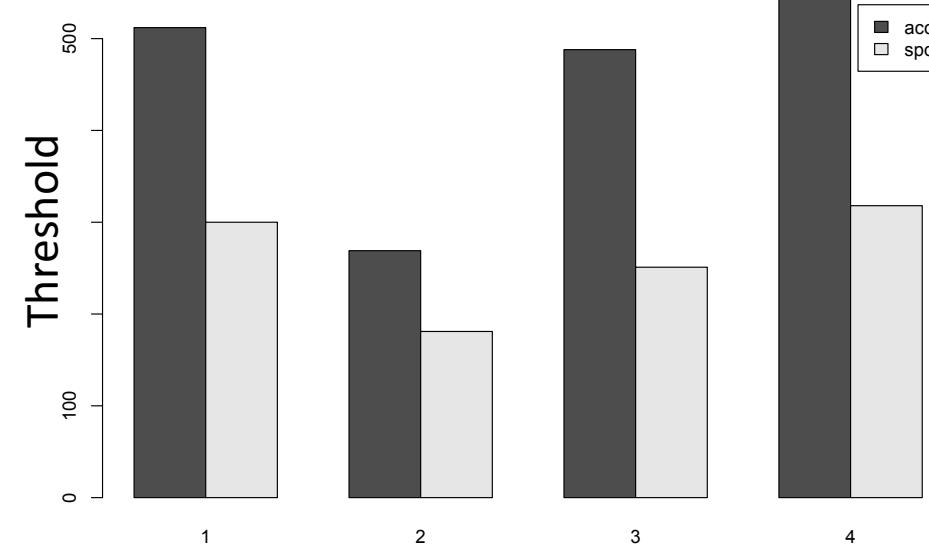
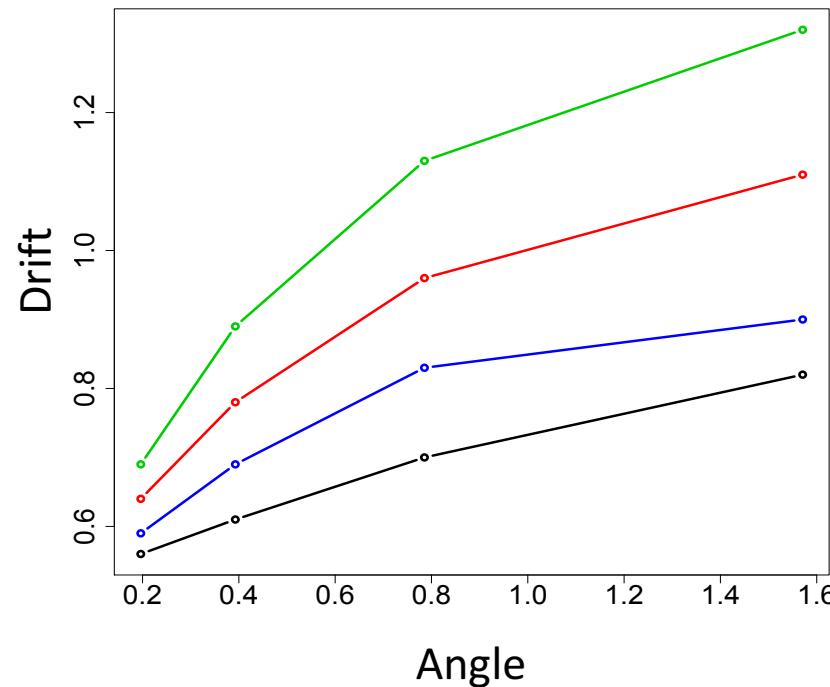
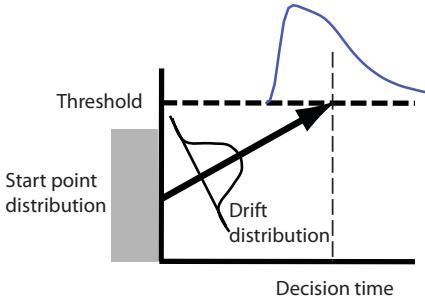


1/2 $\pi$

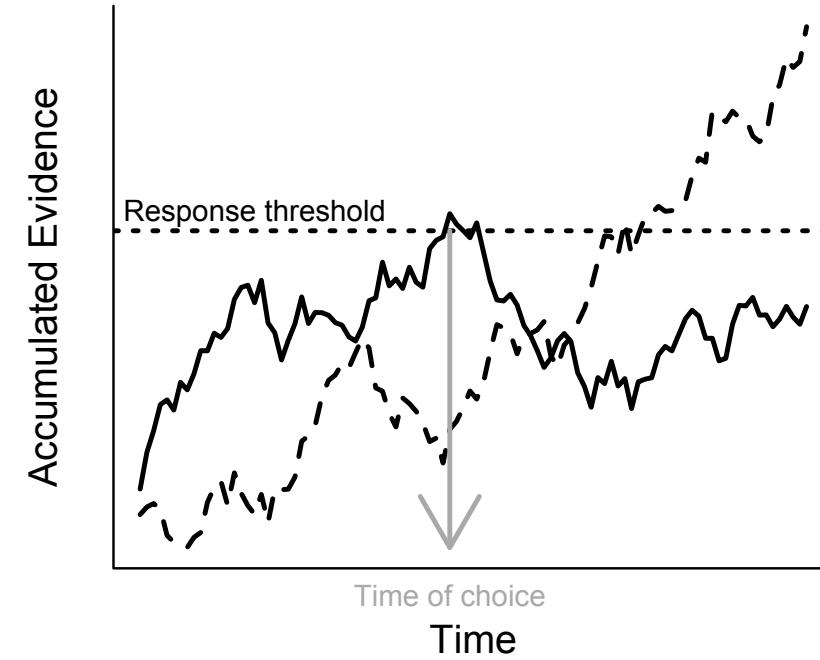
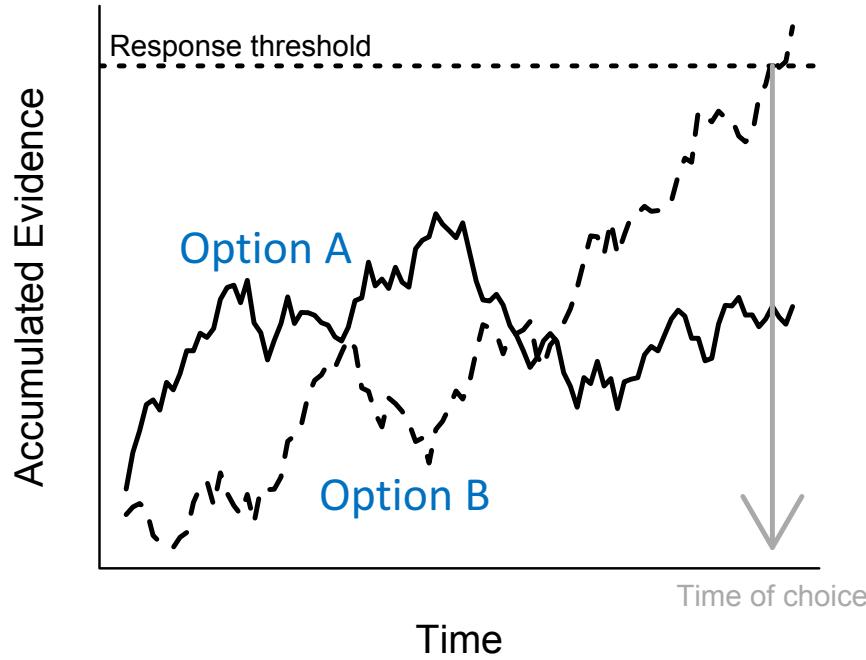


# LBA parameters

- Drift rate  $\sim$  angle
- Threshold  $\sim$  instruction (speed/accuracy)
- Other parameters did not vary across conditions

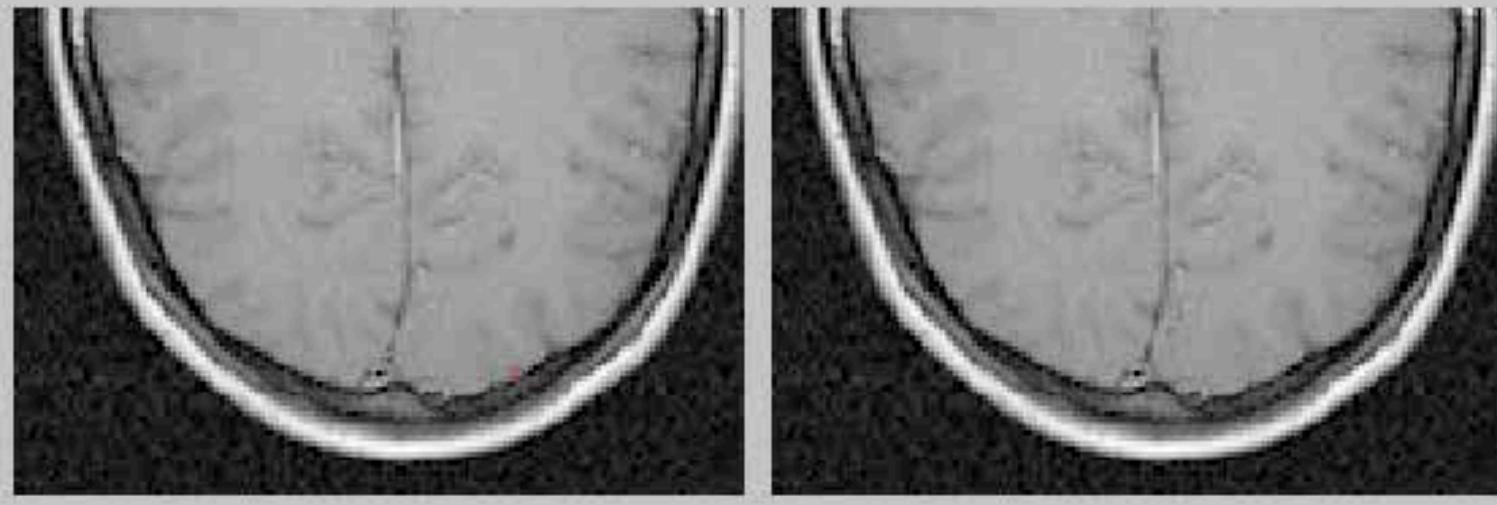
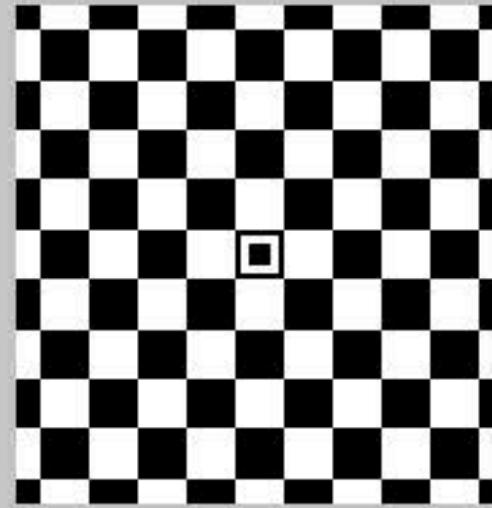
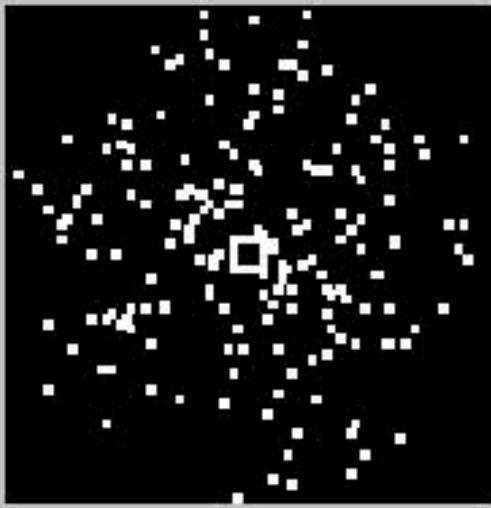


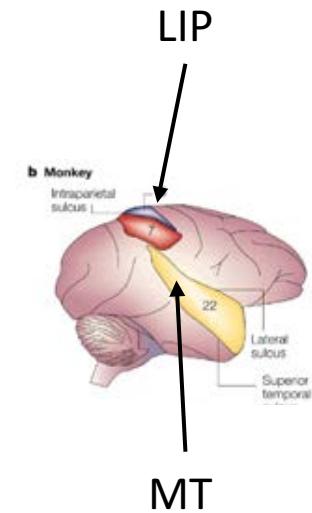
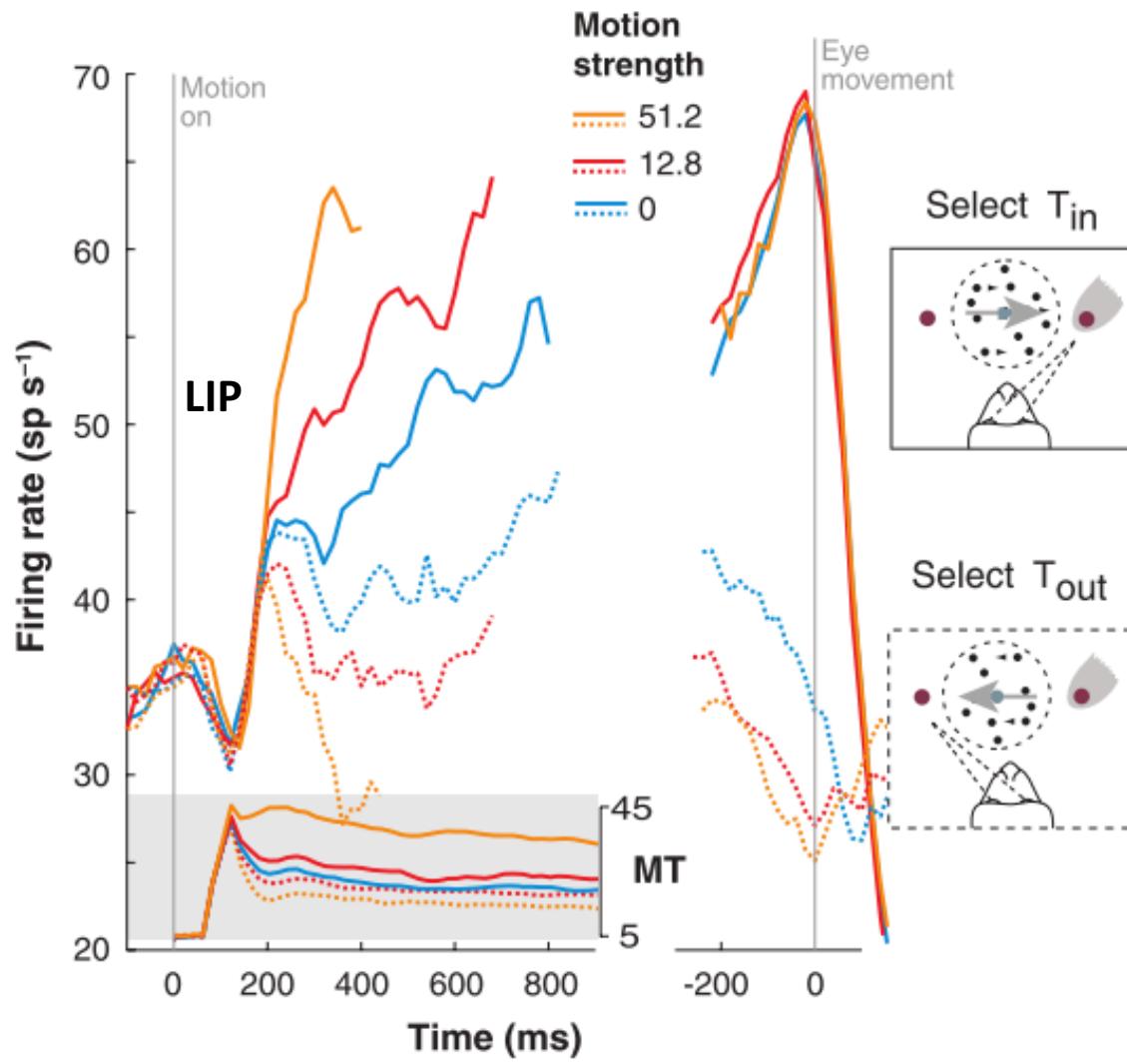
# Speed instruction: Lower threshold



# Overview

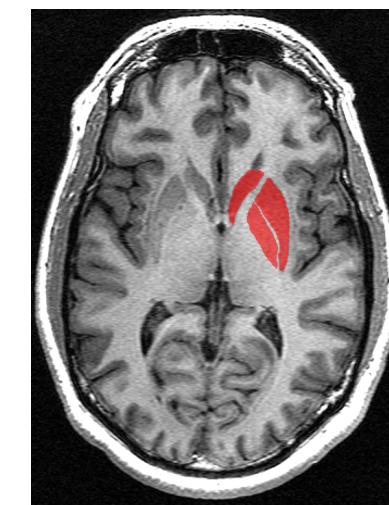
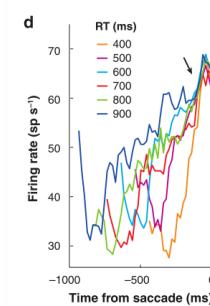
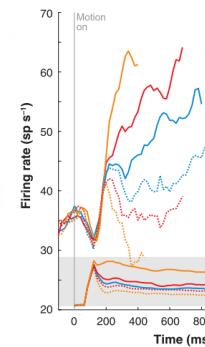
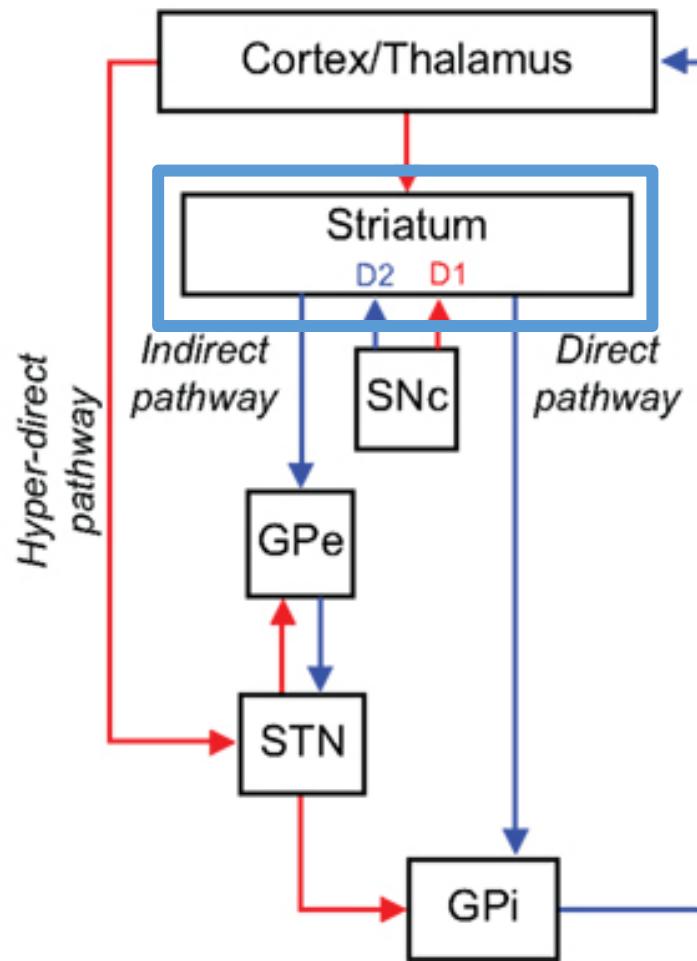
- One implementation: Linear Ballistic Accumulator
- Estimating parameters
- **Relationship with neuroscience**
- Another implementation: The (EZ) diffusion decision model





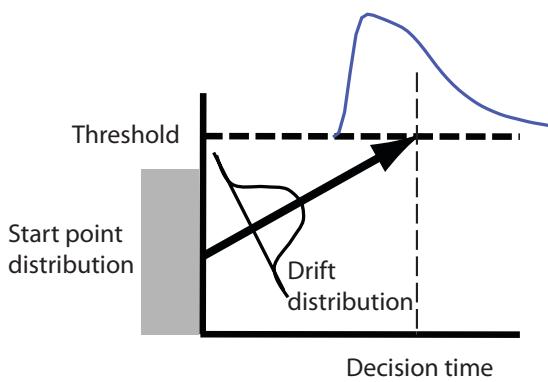
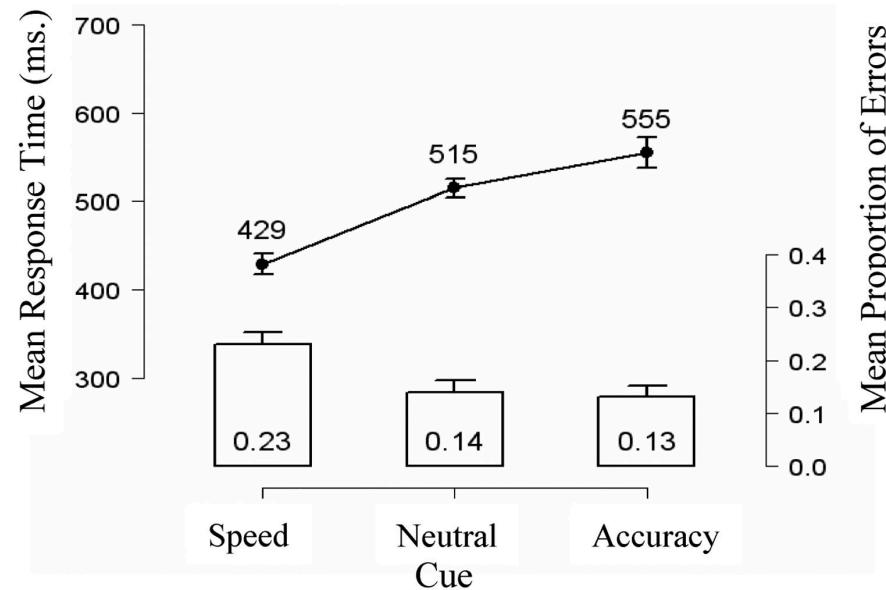
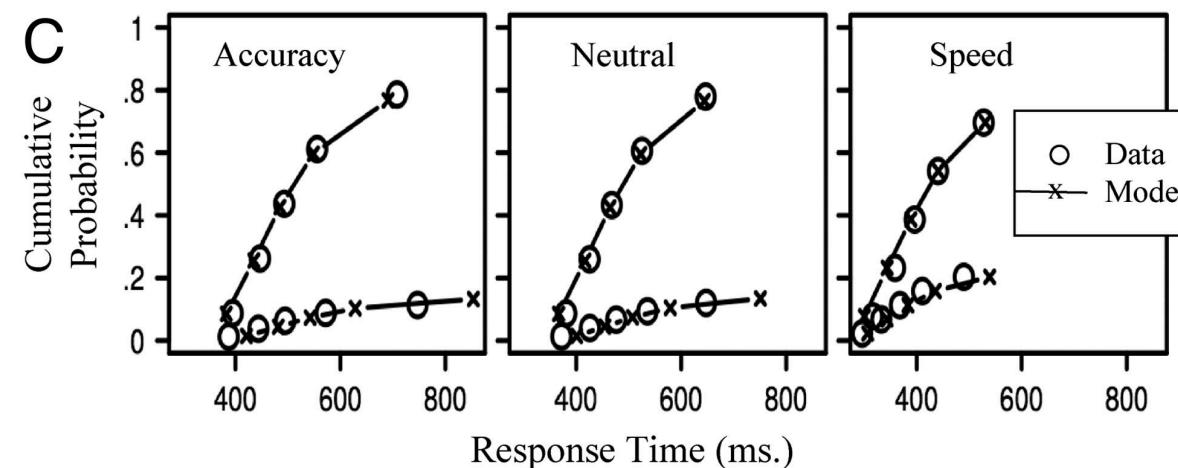
# Decision making network in the brain

Box and arrow model



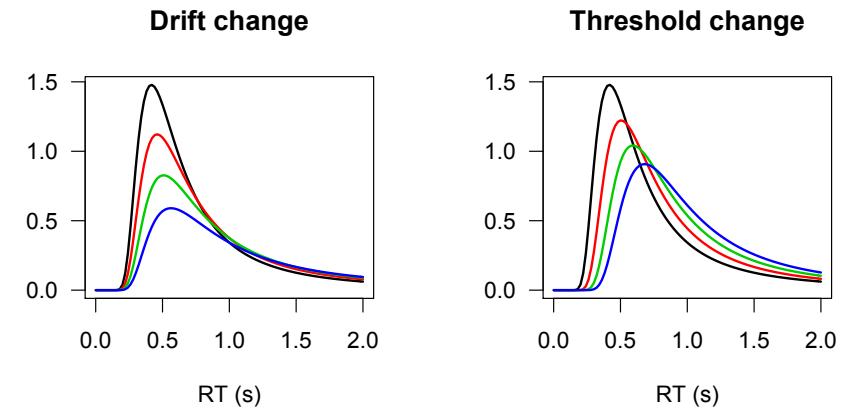
striatum

Time		Speed	Neutral	Accuracy
0-1500 ms	Jitter	+	+	+
4800 ms	Cue	SN	NE	AK
0-1500 ms	Jitter	+	+	+
1500 ms	Stimulus & Response			
350 ms	Feedback	too slow	in time correct	incorrect
		10s		

**A****C**

# Approach

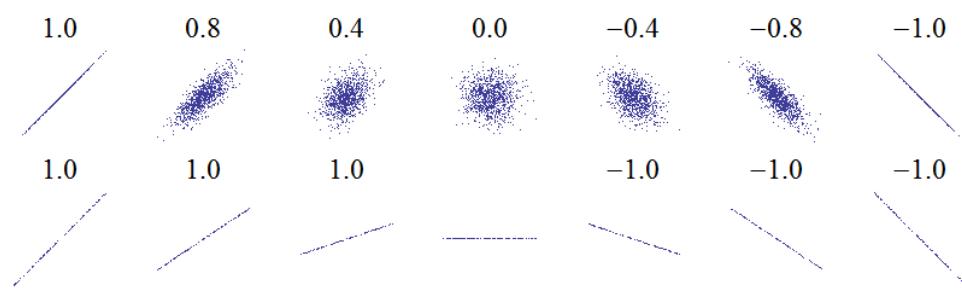
- Fit model for individual participants
  - Estimate the optimal set of parameters



- Extract average BOLD response per participant/condition
  - Or other relevant neurophysiological measure

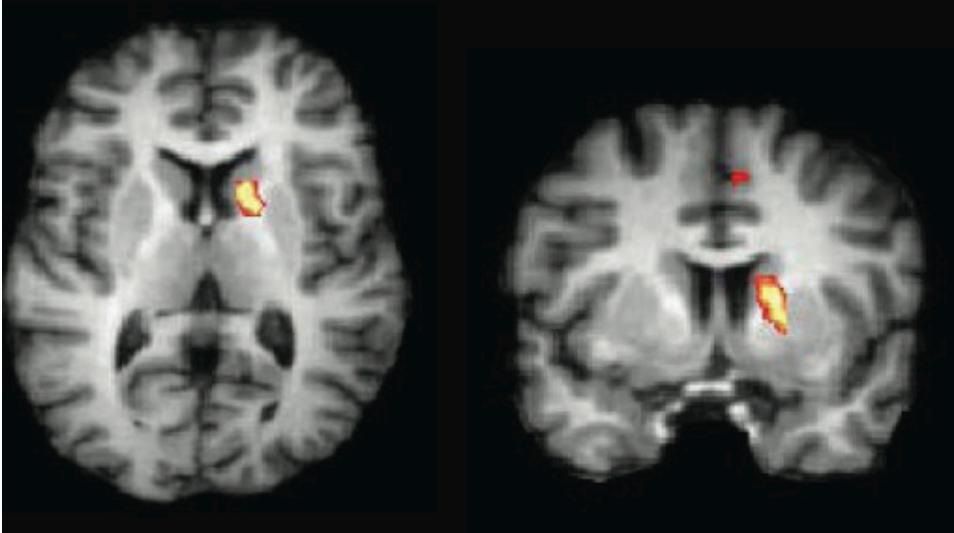


- Do these correlate?



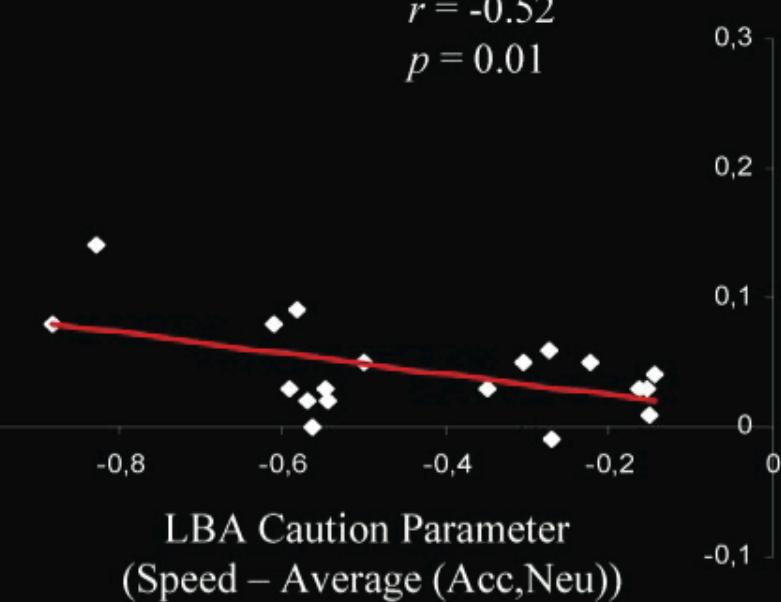
**A**

Anterior Striatum (16, 7, 5)

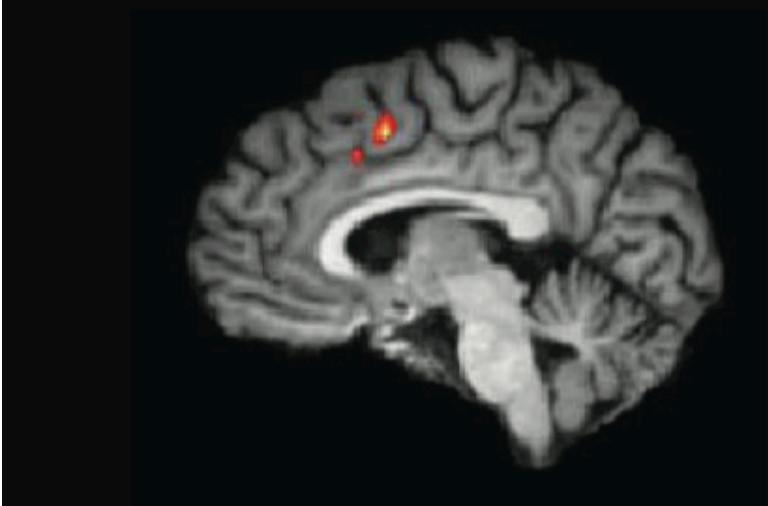
**B**

$$r = -0.52$$
$$p = 0.01$$

% Signal Change Striatum

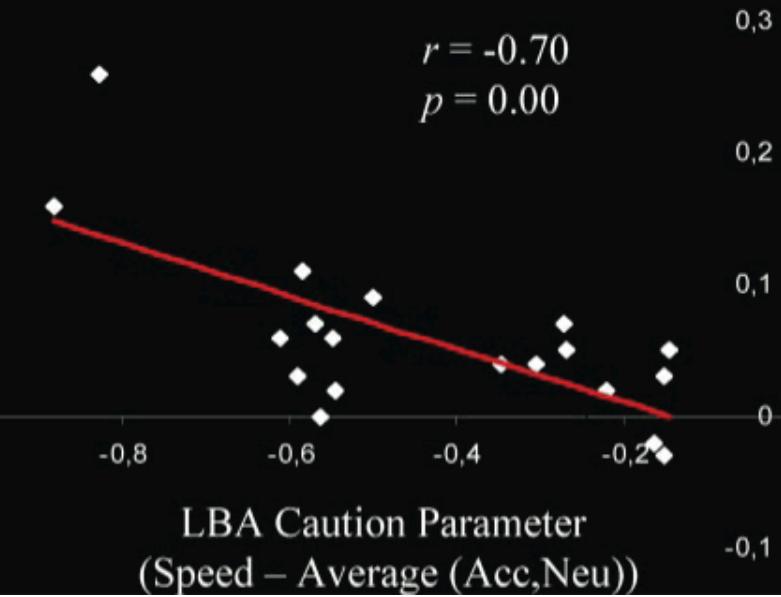


Pre-SMA (4, 5, 45)

**C**

$$r = -0.70$$
$$p = 0.00$$

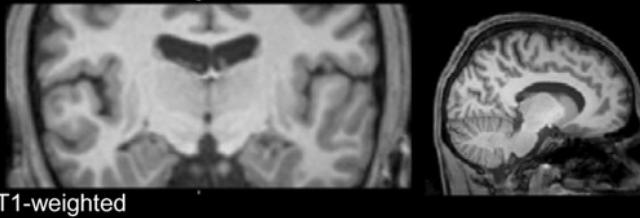
% Signal Change Pre-SMA



A

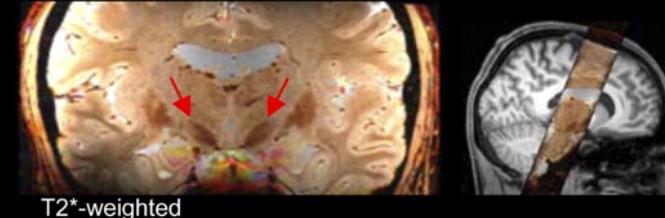
**3T structural MRI**

(1 mm<sup>3</sup> isotropic)

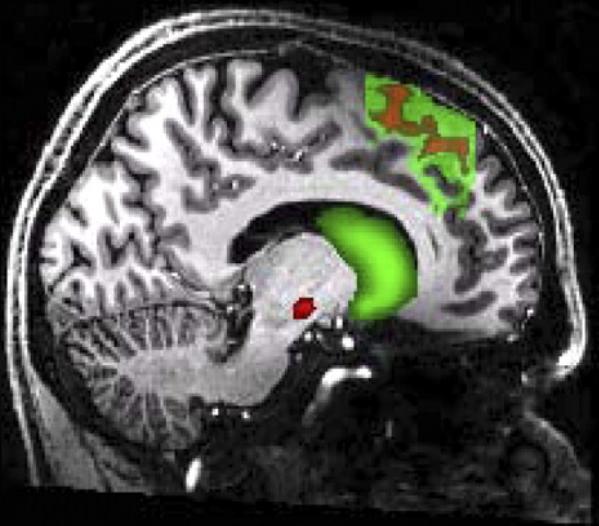


**7T structural MRI**

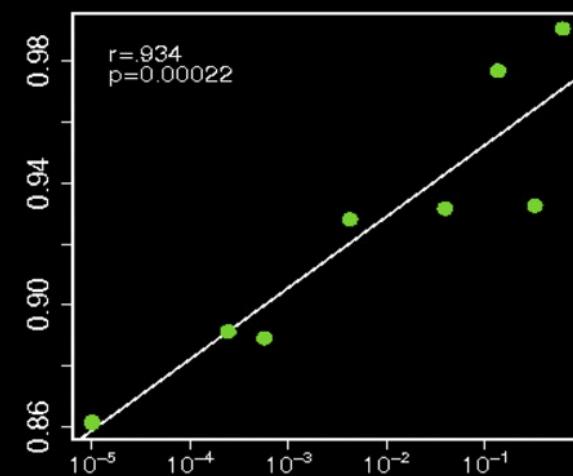
(0.5 mm<sup>3</sup> isotropic)



B



Tract Strength  
Right Pre-SMA – Right Striatum

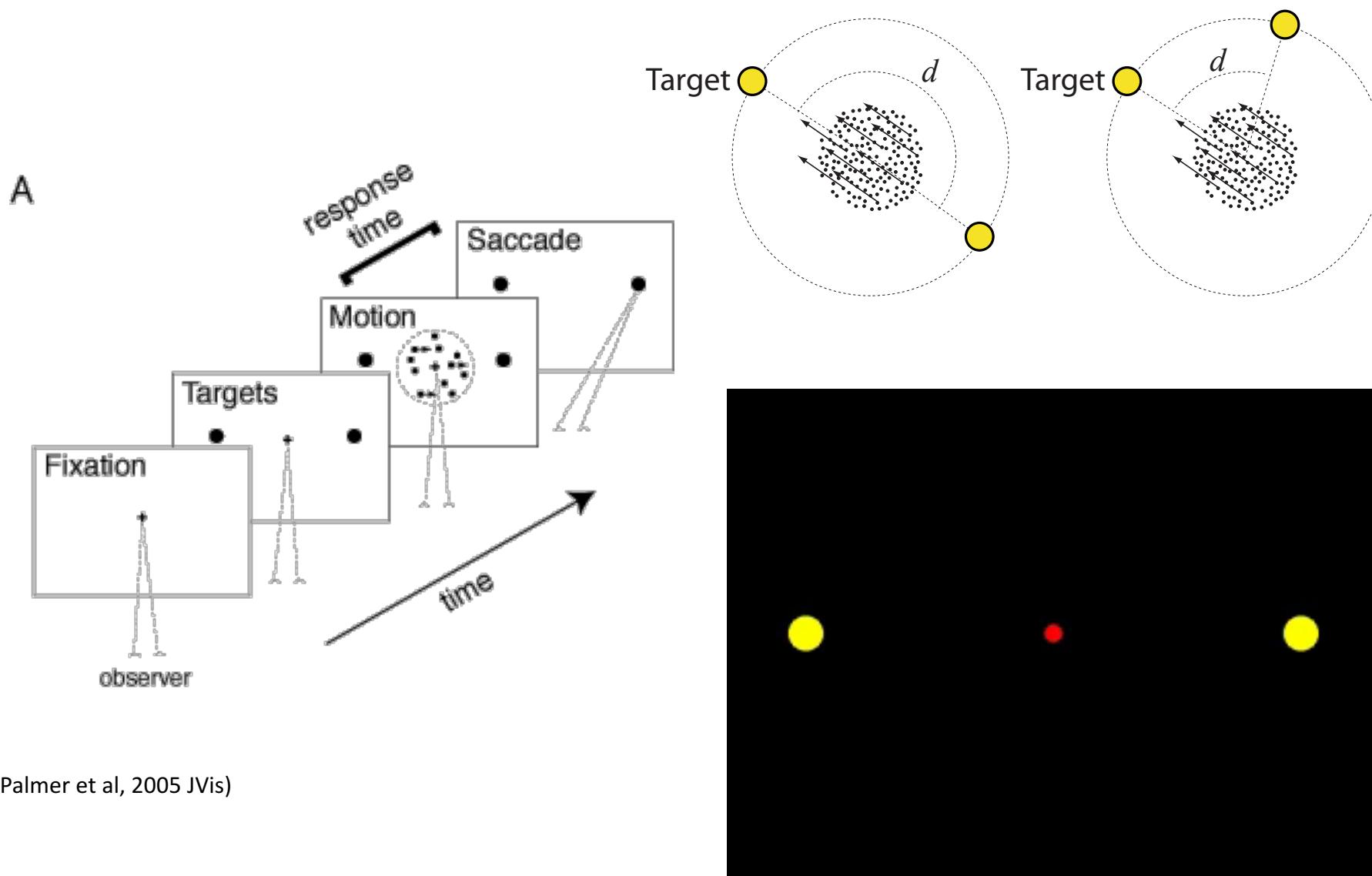


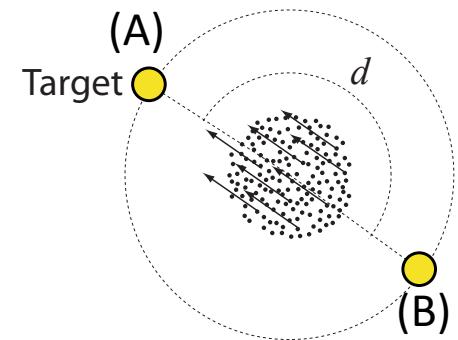
LBA Caution Parameter  
(Accuracy - Speed)

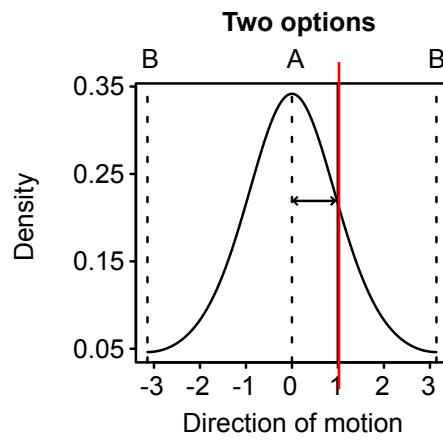
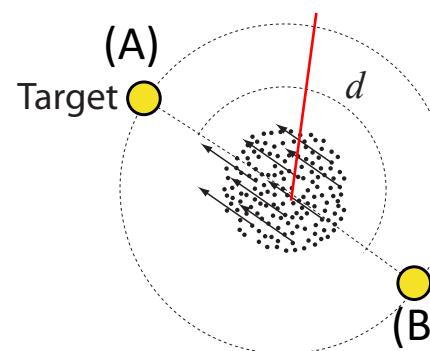
# Overview

- One implementation: Linear Ballistic Accumulator
- Estimating parameters
- Relationship with neuroscience
- **Another implementation: The (EZ) diffusion decision model**

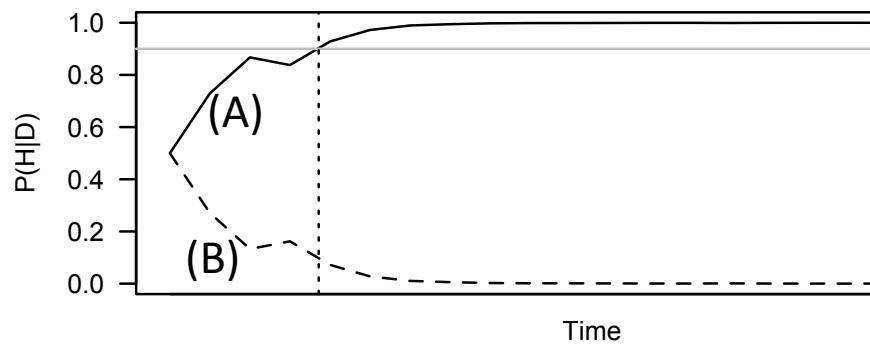
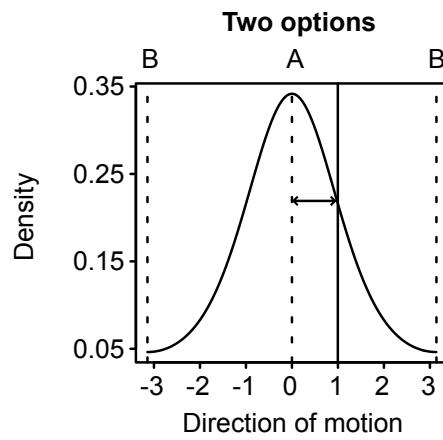
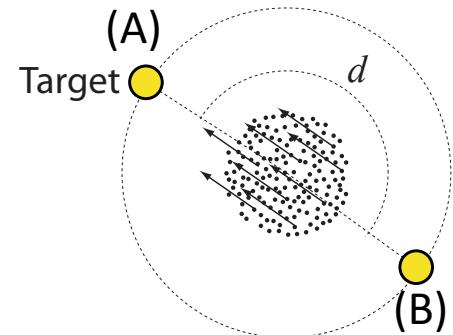
# Example: effect of target location in 2AFC

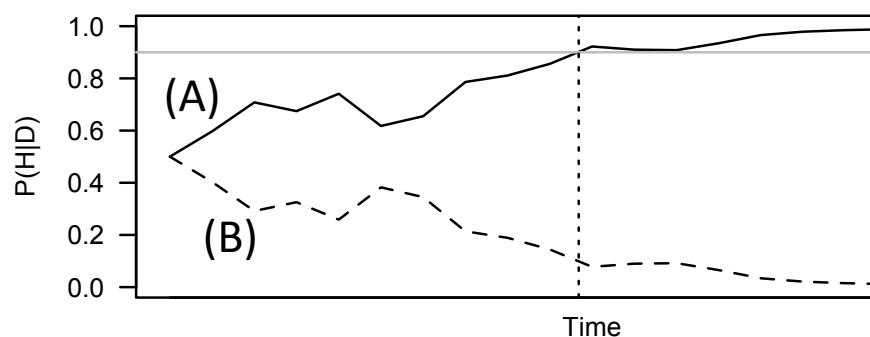
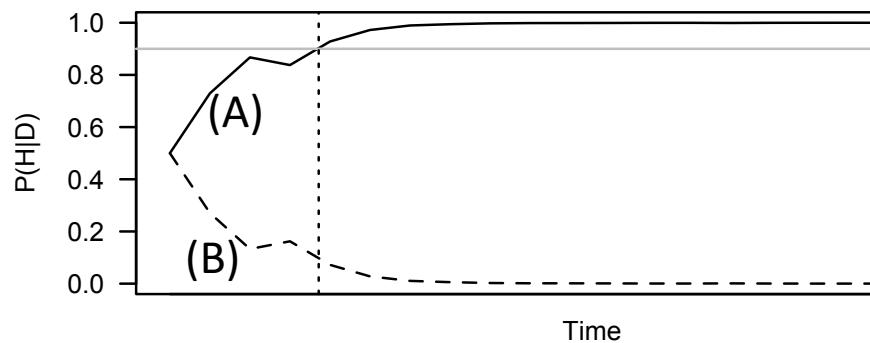
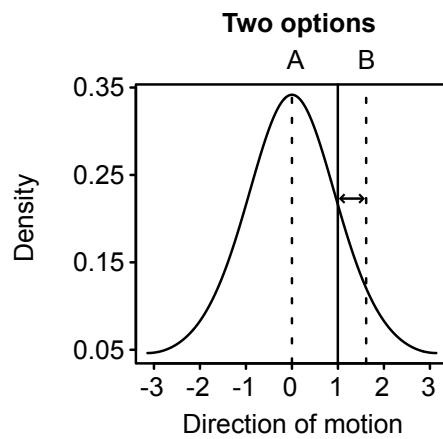
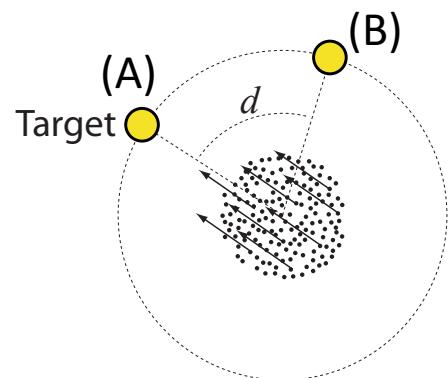
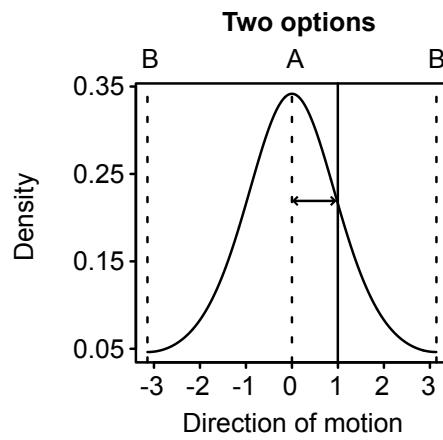
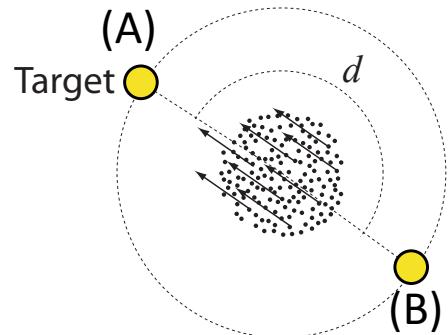






$$P(H_i|D) = \frac{P(D|H_i)P(H_i)}{\sum_j P(D|H_j)P(H_j)}$$

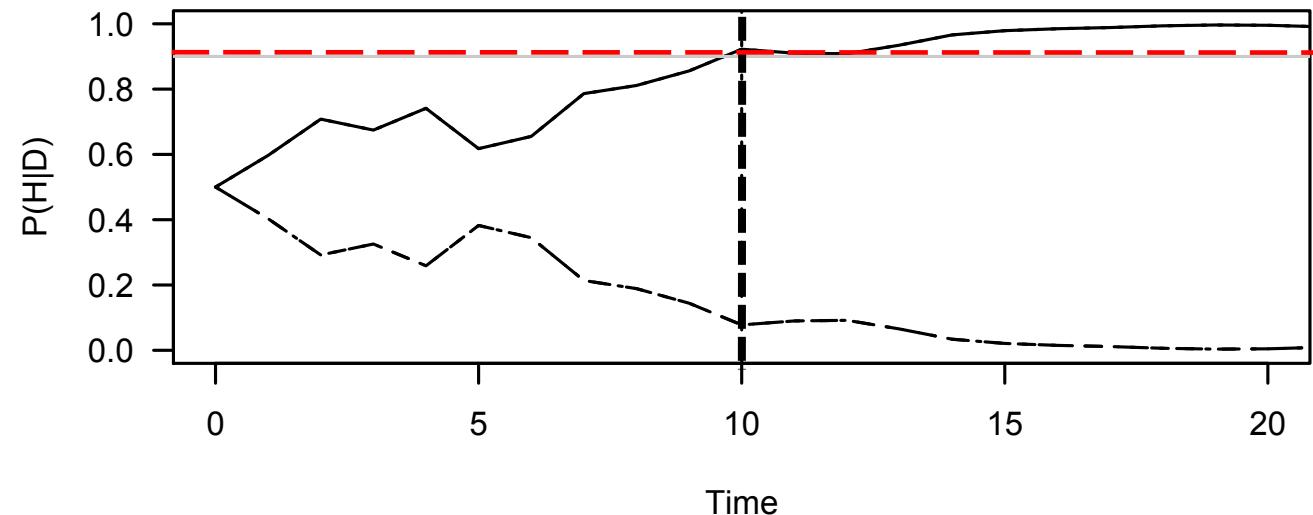




# Sequential Probability Ratio Test (SPRT)

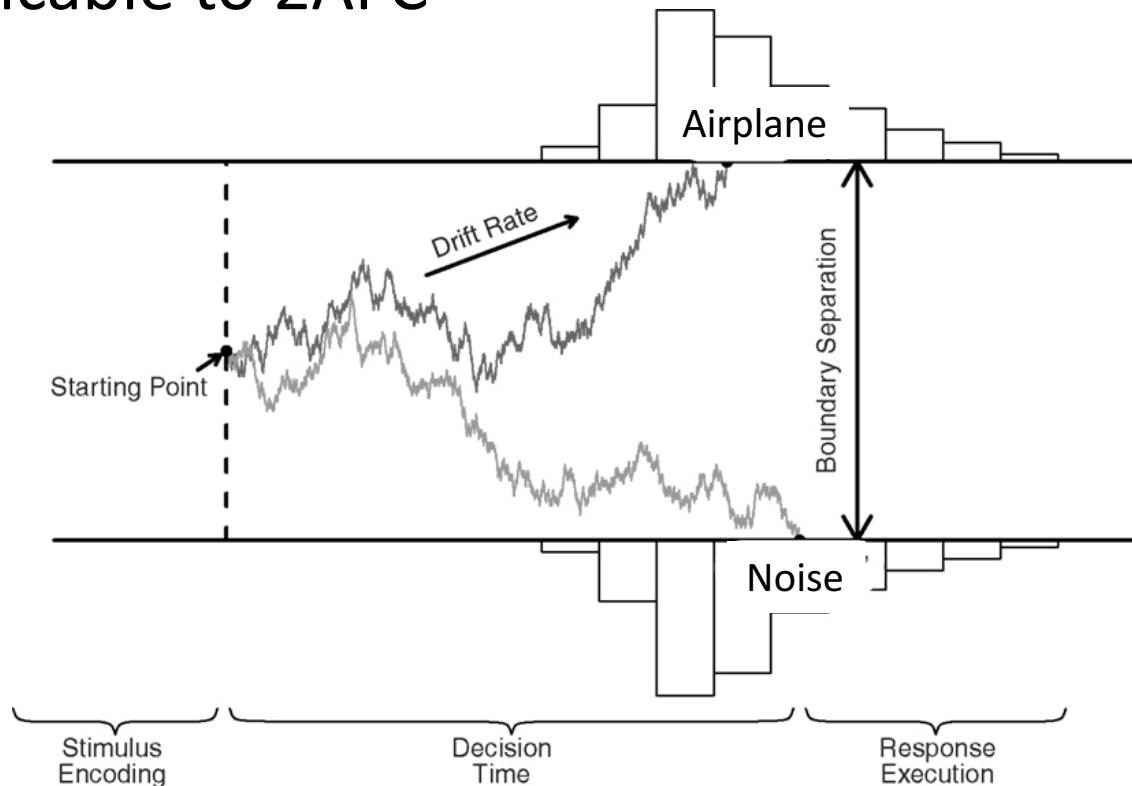
- “Optimal” procedure for binary choices
  - Guarantees minimal mean response time
    - For a specific critical threshold  $P(H|D)$

$$P(H_i|D) = \frac{P(D|H_i)P(H_i)}{\sum_j P(D|H_j)P(H_j)}$$

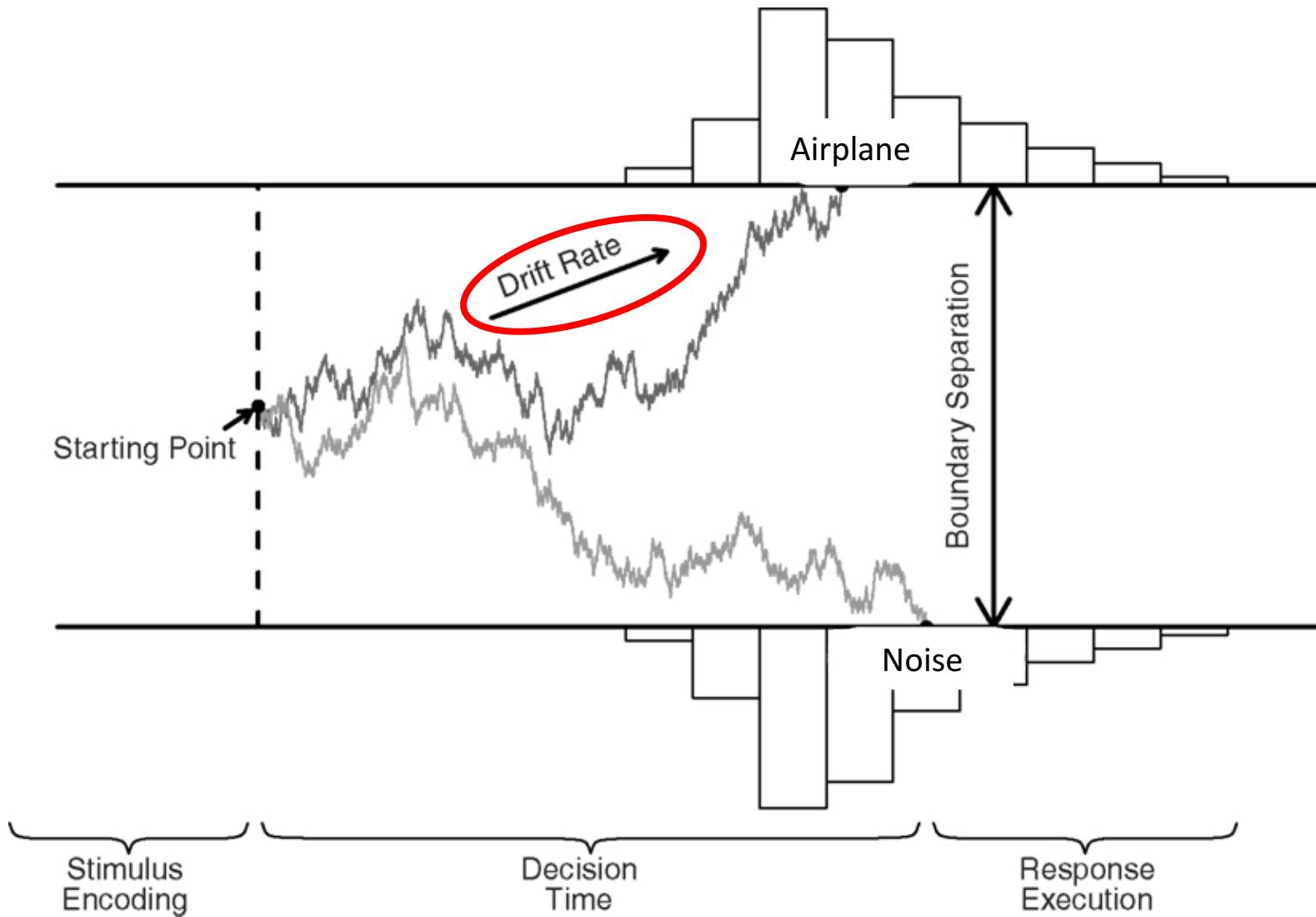


# Diffusion Decision Model

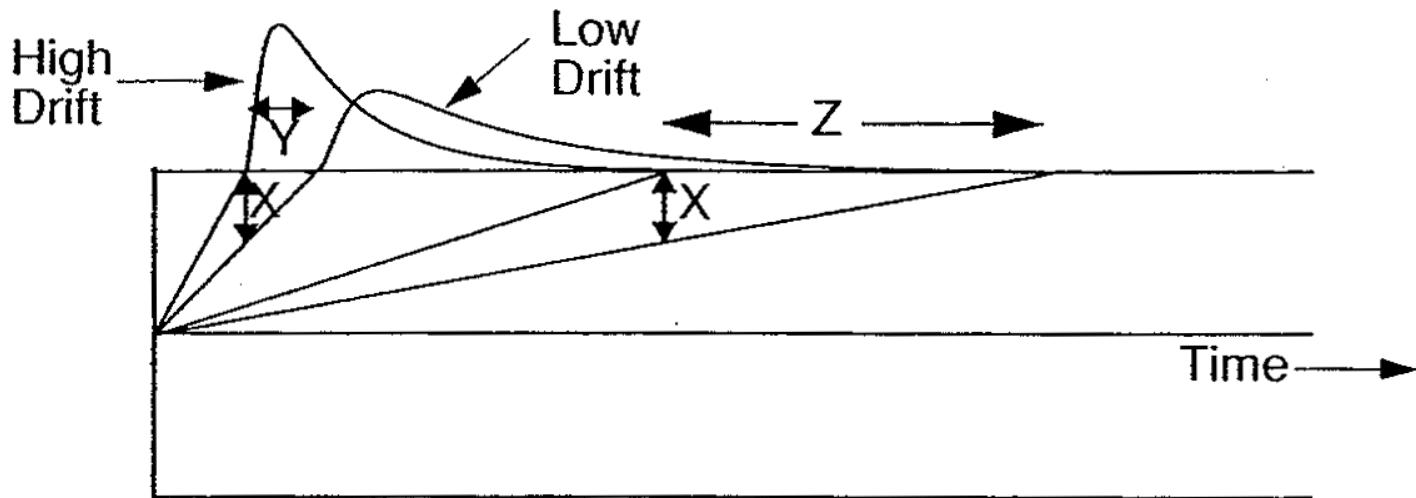
- “implementation” of SPRT
- Evidence FOR option 1 = evidence AGAINST option 2
- By design only applicable to 2AFC



# Diffusion Decision Model

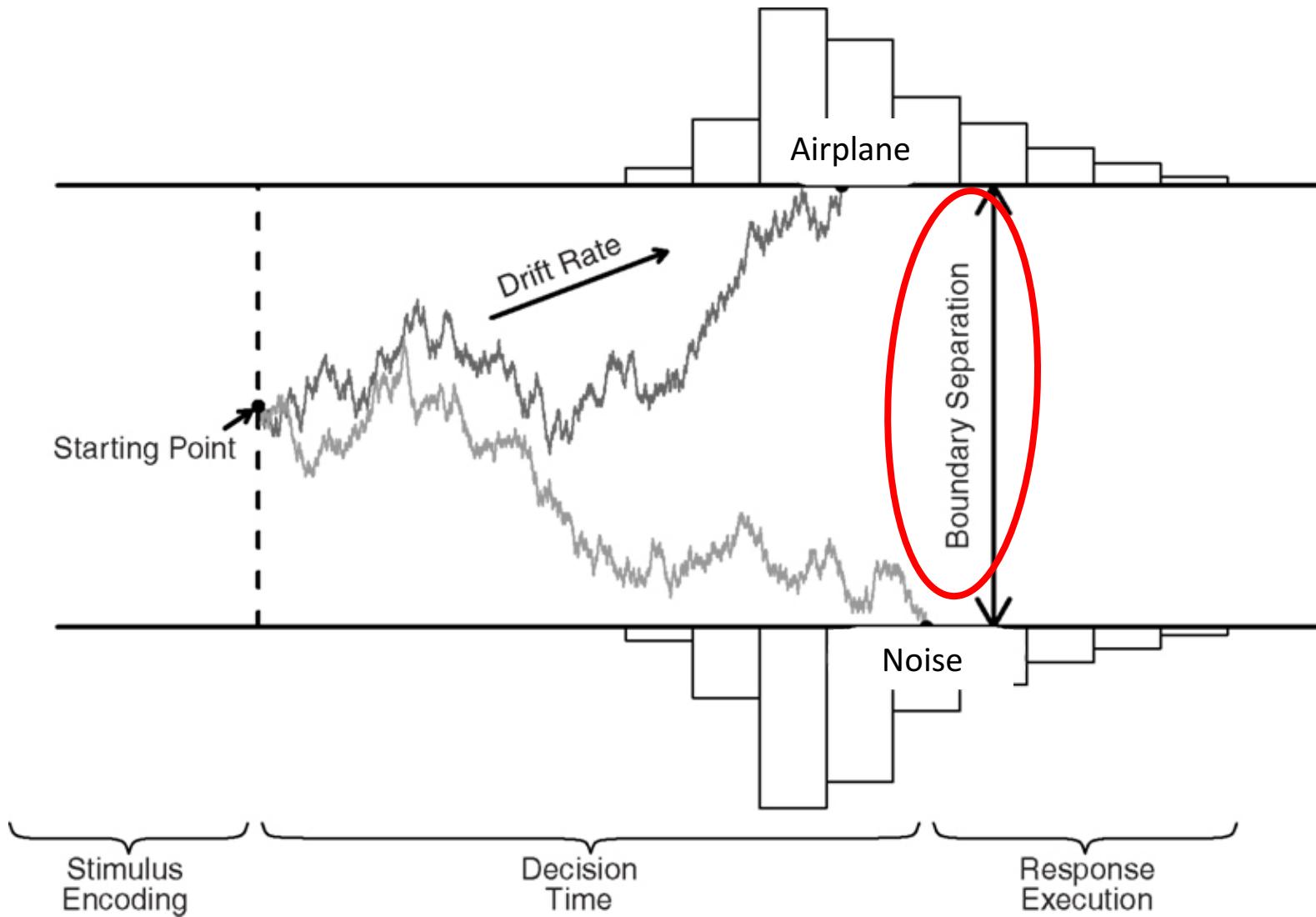


# Lower drift rate

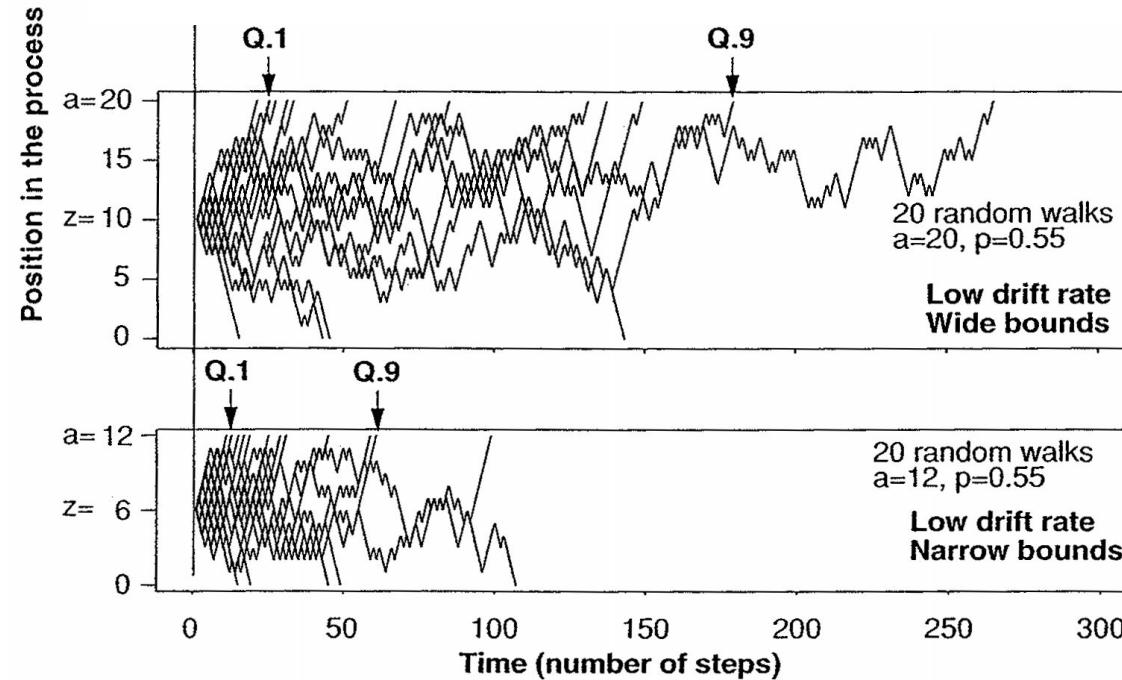


- Slower responses (effect is larger in tail of RT distribution)
- More errors

# Diffusion Decision Model

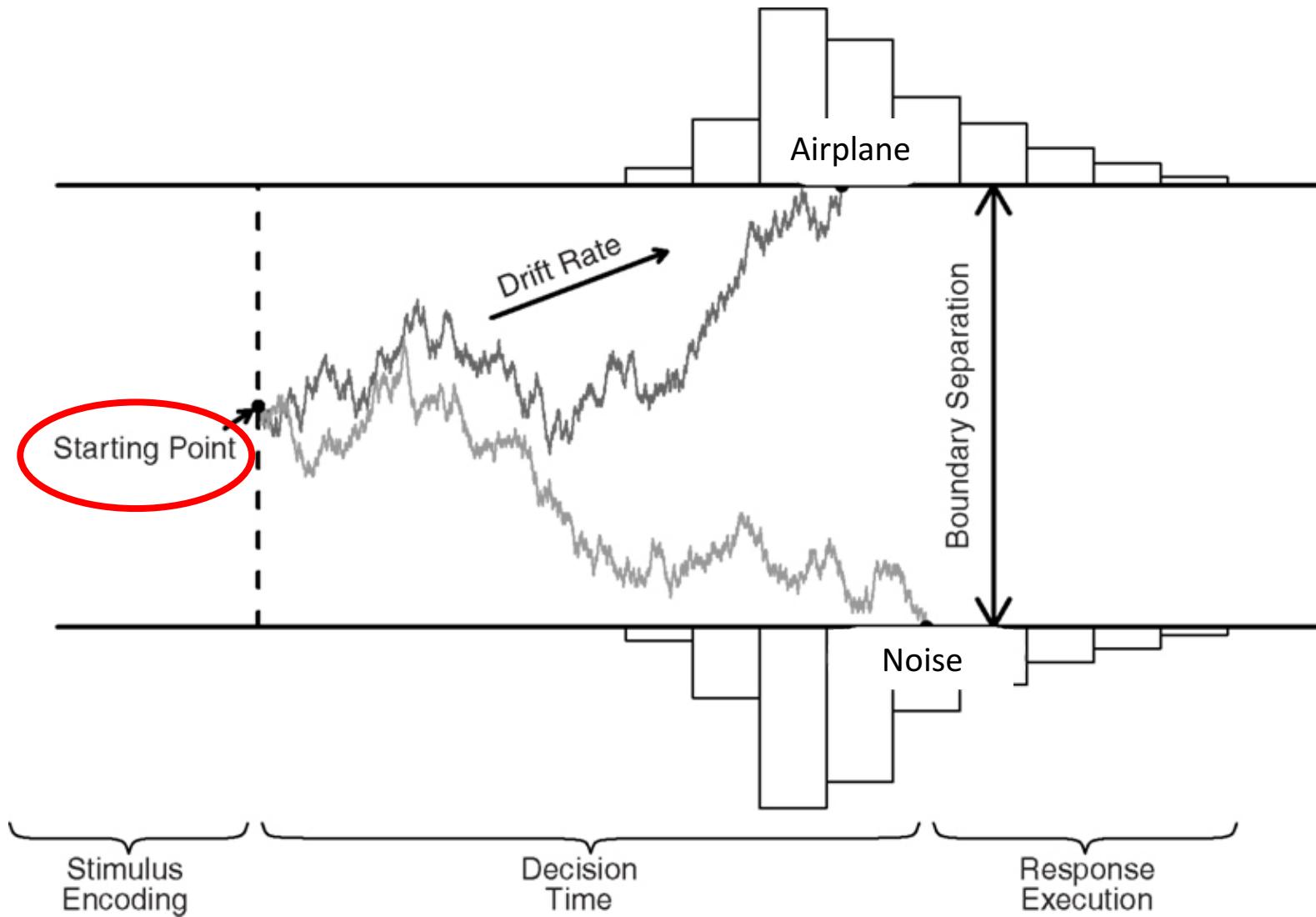


# Lower thresholds

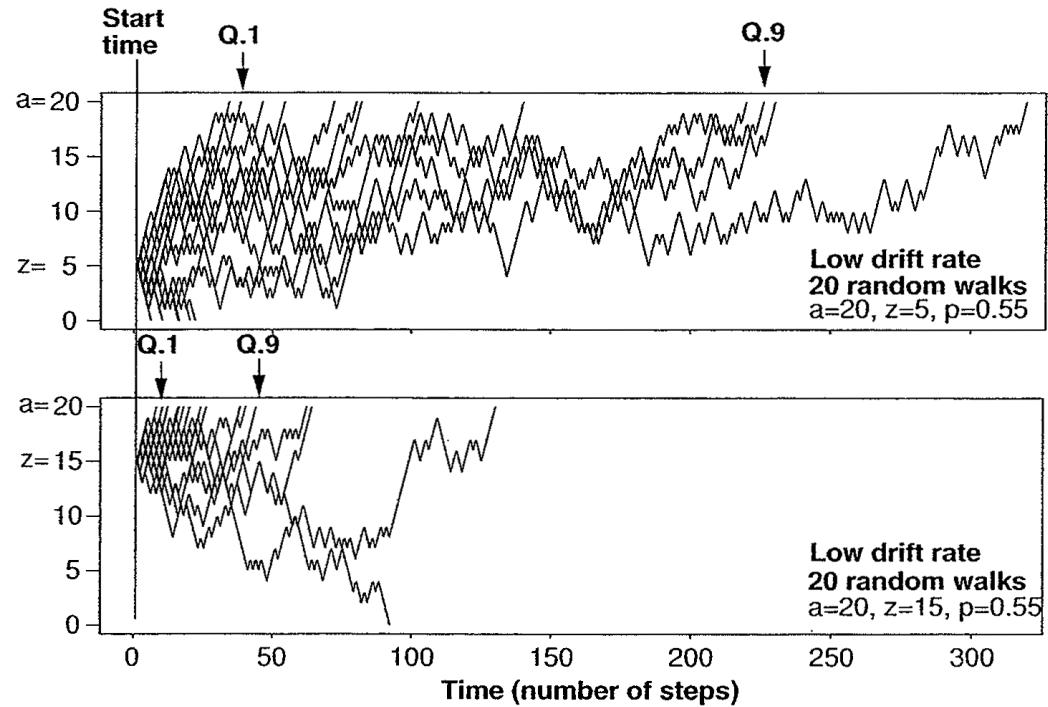


- Faster responses
- More errors

# Diffusion Decision Model



# Lower start point



<- Correct!

<- Error!

<- Correct!

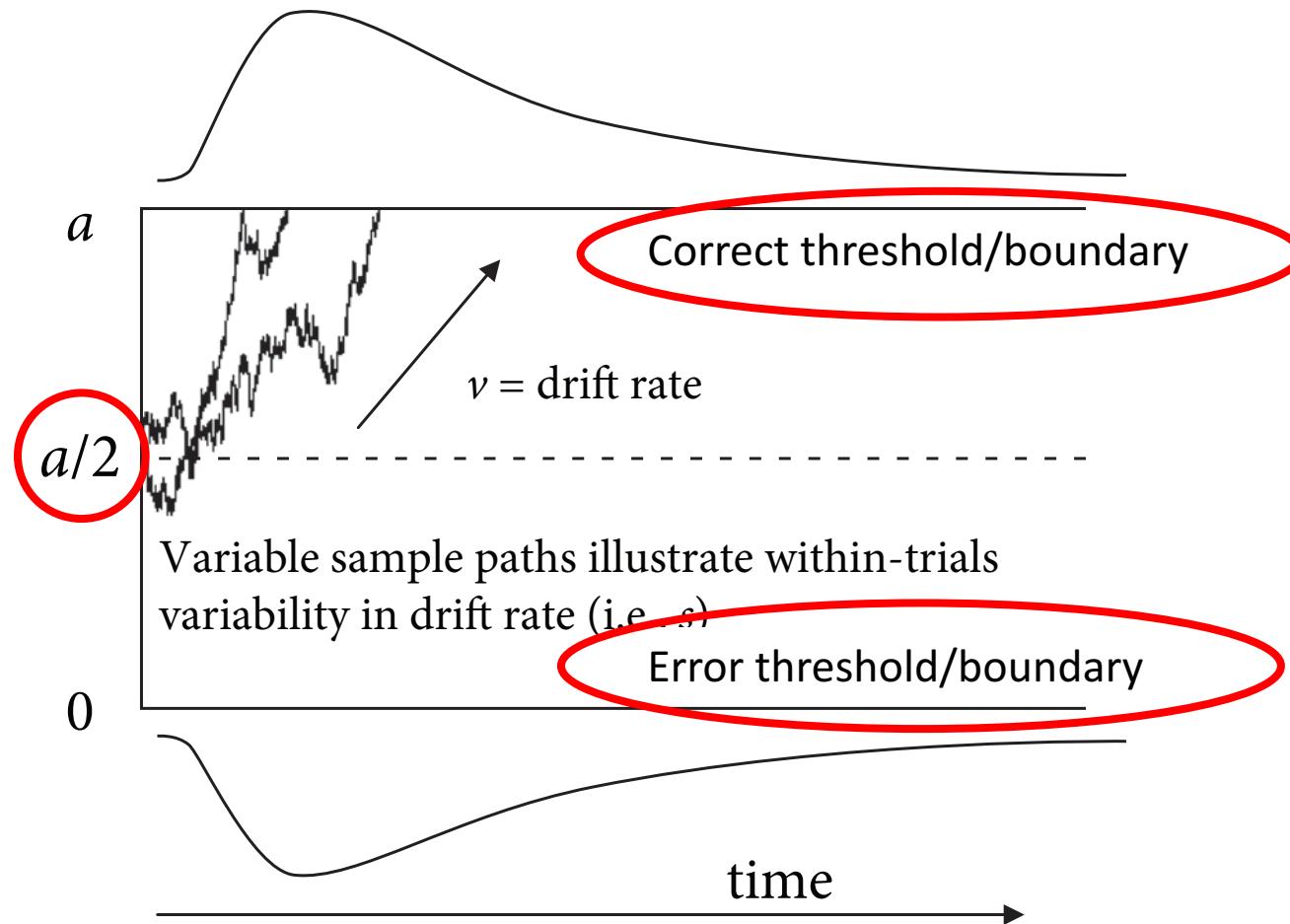
<- Error!

- Slower correct responses
- Faster errors
- More errors

# Lower non-decision time

- $NDT = T_{SE} + T_{RE}$
  - $RT = DT + NDT$
- 
- Faster responses
  - No change in error rate

# EZ Diffusion Decision Model (Thursday)



As preparation, read Wagenmakers, E.-J., van der Maas, H. L. J., & Grasman, R. P. P. P. (2007). [An EZ-diffusion model for response time and accuracy](#). *Psychonomic Bulletin & Review*, 14, 3-22

# Questions?

See you on thursday