Problem 13a) Verify $(A \triangle B) \cup C = (A \cup C) \triangle (B \setminus C)$

Prerequisite notation:

Set Difference: $B \setminus C = B \cap \neg C$
Symmetric Difference: $A \triangle B = (A \cup B) \setminus (A \cap B)$ or $A \triangle B = (A \setminus B) \cup (B \setminus A)$

Start by working backwards from

$$(A \cup C) \triangle (B \setminus C) = ((A \cup C) \setminus (B \setminus C)) \cup ((B \setminus C) \setminus (A \cup C))$$
$$((A \cup C) \setminus (B \setminus C)) \cup ((B \setminus C) \setminus (A \cup C)) = ((A \cup C) \cap \neg (B \cap \neg C)) \cup (((B \cap \neg C) \cap \neg (A \cup C))$$

Switch to logic symbols and apply DeMorgan's Law where applicable

$$((A \vee C) \wedge \neg (B \wedge \neg C)) \vee ((B \wedge \neg C) \wedge \neg (A \vee C)) = ((A \vee C) \wedge (\neg B \vee C)) \vee ((B \wedge \neg C) \wedge (\neg A \wedge \neg C))$$

Apply associvativty law to right most equation (can drop the paranthesis). Apply Idempotence to $\neg C$

$$((A \vee C) \wedge (\neg B \vee C)) \vee ((B \wedge \neg C) \wedge (\neg A \wedge \neg C)) = ((A \vee C) \wedge (\neg B \vee C)) \vee (B \wedge \neg C \wedge \neg A)$$

Distrubute $(A \vee C)$ over $(\neg B \vee C)$

$$((A \vee C) \wedge (\neg B \vee C)) \vee (B \wedge \neg C \wedge \neg A) = (((A \vee C) \wedge \neg B) \vee ((A \vee C) \wedge C)) \vee (B \wedge \neg C \wedge \neg A)$$

Apply absorption to $((A \vee C) \wedge C))$

$$(((A \vee C) \wedge \neg B) \vee ((A \vee C) \wedge C)) \vee (B \wedge \neg C \wedge \neg A) = (((A \vee C) \wedge \neg B) \vee C) \vee (B \wedge \neg C \wedge \neg A)$$

Distribute $\neg Bover(A \vee C)$

$$(((A \vee C) \wedge \neg B) \vee C) \vee (B \wedge \neg C \wedge \neg A) = ((A \wedge \neg B) \vee (C \wedge \neg B) \vee C) \vee (B \wedge \neg C \wedge \neg A)$$

Apply absorption to $(C \wedge \neg B) \vee C$

$$((A \wedge \neg B) \vee (C \wedge \neg B) \vee C) \vee (B \wedge \neg C \wedge \neg A) = ((A \wedge \neg B) \vee C) \vee (B \wedge \neg C \wedge \neg A)$$

Use associative law to move around paranthesis. Distrube C over $(B \wedge \neg C \wedge \neg A)$

$$(A \wedge \neg B) \vee C \vee (B \wedge \neg C \wedge \neg A) = (A \wedge \neg B) \vee ((B \vee C) \wedge (\neg C \vee C) \wedge (\neg A \vee C))$$
$$(A \wedge \neg B) \vee ((B \vee C) \wedge (\neg C \vee C) \wedge (\neg A \vee C)) = (A \wedge \neg B) \vee ((B \vee C) \wedge (\neg A \vee C))$$

Use distrubtion law to move C from $((B \vee C) \wedge (\neg A \vee C))$

$$(A \wedge \neg B) \vee ((B \vee C) \wedge (\neg A \vee C)) = (A \wedge \neg B) \vee (B \wedge \neg A) \vee C$$

Switching back to set notation.

$$(A \cap \neg B) \cup (B \cap \neg A) \cup C = (A \setminus B) \cup (B \setminus A) \cup C = (A \triangle B) \cup C$$

Using the definition for set and symmetric difference, we have verified that

$$(A \cup C) \triangle (B \setminus C) = (A \triangle B) \cup C$$