

4.2 + [問題4.2]

4.2-①

(4.20) (4.21), (4.23) に各々 $\dot{X}, \dot{Y}, \dot{Z}$ を掛け1和をとる

$$\dot{X}(\ddot{X} - 2n'\dot{Y}) = -\frac{\partial U^*}{\partial X} \cdot \dot{X}$$

$$\dot{Y}(\ddot{Y} + 2n'\dot{X}) = -\frac{\partial U^*}{\partial Y} \cdot \dot{Y}$$

$$+) \quad \dot{Z} \cdot \ddot{Z} = -\frac{\partial U^*}{\partial Z} \cdot \dot{Z}$$

$$\underbrace{\ddot{X}\dot{X} + \ddot{Y}\dot{Y} + \ddot{Z}\dot{Z}}_{\text{"}} = -\frac{\partial U^*}{\partial X} \frac{dX}{dt} - \frac{\partial U^*}{\partial Y} \frac{dY}{dt} - \frac{\partial U^*}{\partial Z} \frac{dZ}{dt} = -\frac{dU^*}{dt} \quad \dots (4.24)$$

(4.24) を t で積分

$$\frac{1}{2}(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) + U^* = \text{const} \quad \dots (4.25)$$

\hat{L} ヤコビ積分 (円制限3体問題の保存量)

(4.25) \wedge (4.26) を代入して.

$$\frac{1}{2}(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) - \frac{1}{2}n'^2(X^2 + Y^2) - \frac{GM_1}{r_1} - \frac{GM_2}{r_2} = \text{const} \quad \dots ①$$

として、これを慣性系 (ξ, η, ζ) に表現する

まずは (4.6) (4.7) より、 \dot{X}, \dot{Y} を求める

$$X = \xi \cos 2\theta + \eta \sin 2\theta \quad \dots (4.6)$$

$$\dot{X} = \dot{\xi} \cos 2\theta + \eta \sin 2\theta - n' \xi \sin 2\theta + n' \eta \cos 2\theta \quad \dots ②$$

$$Y = -\xi \sin 2\theta + \eta \cos 2\theta \quad \dots (4.7)$$

$$\dot{Y} = -\dot{\xi} \sin 2\theta + \eta \cos 2\theta - n' \xi \cos 2\theta - n' \eta \sin 2\theta \quad \dots ③$$

→ 底 \dot{Z} と

$$Z = \zeta \quad \dots (4.5)$$

$$\dot{Z} = \dot{\zeta} \quad \dots ④$$

②, ③, ④を).

$$\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2$$

$$\begin{aligned}
 &= \dot{\xi}^2 a^2 \theta + \eta^2 b^2 \theta + n'^2 \xi^2 b^2 \theta + n'^2 \eta^2 a^2 \theta \\
 &\quad + 2\dot{\xi}\dot{\eta} ab \theta - 2n'\dot{\xi}\dot{\xi} ab \theta + 2n'\dot{\xi}\dot{\eta} a^2 \theta - 2n'\dot{\xi}\dot{\eta} b^2 \theta + 2n'\dot{\eta}\dot{\eta} ab \theta \\
 &\quad - 2n'^2 \dot{\xi}\dot{\eta} ab \theta \\
 &\quad + \dot{\xi}^2 b^2 \theta + \eta^2 a^2 \theta + n'^2 \xi^2 a^2 \theta + n'^2 \eta^2 b^2 \theta \\
 &\quad - 2\dot{\xi}\dot{\eta} ab \theta + 2n'\dot{\xi}\dot{\xi} ab \theta + 2n'\dot{\xi}\dot{\eta} b^2 \theta - 2n'\dot{\xi}\dot{\eta} a^2 \theta - 2n'\dot{\eta}\dot{\eta} ab \theta \\
 &\quad + 2n'^2 \dot{\eta}\dot{\eta} ab \theta \\
 &\quad + \dot{\zeta}^2 \\
 &= \dot{\xi}^2 + \eta^2 + \dot{\zeta}^2 + n'^2 \xi^2 + n'^2 \eta^2 + 2n'\dot{\xi}\dot{\eta} - 2n'\dot{\xi}\dot{\eta} \dots \textcircled{5}
 \end{aligned}$$

$$X^2 + Y^2$$

$$\begin{aligned}
 &= \xi^2 a^2 \theta + \eta^2 b^2 \theta + 2\dot{\xi}\dot{\eta} ab \theta + \xi^2 b^2 \theta + \eta^2 a^2 \theta - 2\dot{\xi}\dot{\eta} ab \theta \\
 &= \xi^2 + \eta^2 \dots \textcircled{6}
 \end{aligned}$$

⑤, ⑥を①へ代入する

$$\begin{aligned}
 &\frac{1}{2}(\dot{\xi}^2 + \eta^2 + \dot{\zeta}^2) + \frac{1}{2}n'^2(\xi^2 + \eta^2) + n'(\dot{\xi}\dot{\eta} - \dot{\xi}\dot{\eta}) - \frac{1}{2}n'^2(\xi^2 + \eta^2) - \frac{GM_1}{r_1} - \frac{GM_2}{r_2} = \text{const} \\
 \therefore &\frac{1}{2}(\dot{\xi}^2 + \eta^2 + \dot{\zeta}^2) - n'(\dot{\xi}\dot{\eta} - \eta\dot{\xi}) - \frac{GM_1}{r_1} - \frac{GM_2}{r_2} = \text{const} \dots (4.27)
 \end{aligned}$$