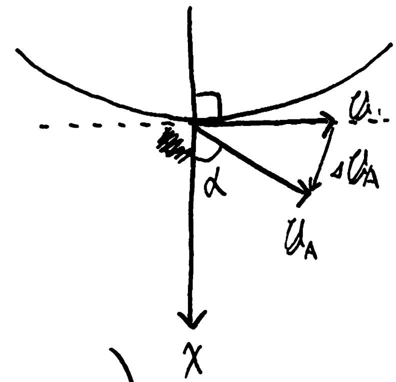
# 3.2

$v_A = v_1 + \Delta v_A$ より、




$$
\begin{aligned}
\Delta v_A &= v_A - v_1 \\
&= \sqrt{(v_A - v_1)^2} \\
&= \sqrt{v_A^2 + v_1^2 - 2v_A \cdot v_1} \\
&= \sqrt{v_A^2 + v_1^2 - 2 v_1 v_A \sin\alpha} \quad \cdots (3.7)
\end{aligned}
$$

$\left( \because v_A \cdot v_1 = v_A v_1 \cos(\frac{\pi}{2} - \alpha) = v_A v_1 \sin\alpha \right)$

・A点での$h$と$E$を評価

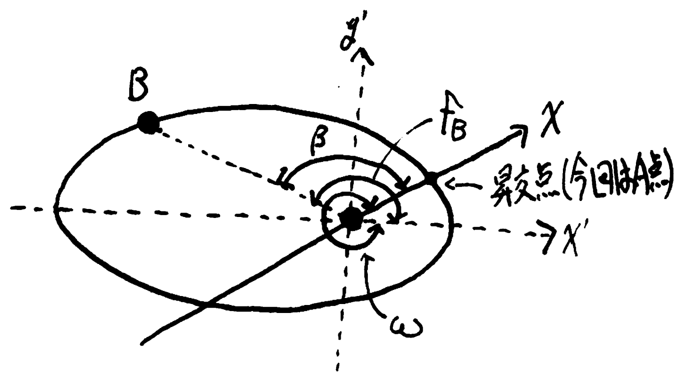
$$h = a_1 \cdot v_A \sin\alpha = \sqrt{\mu a(1-e^2)} \quad \cdots (3.8)$$

(∵2.90)　(∵2.69)

$$E = \frac{1}{2}v_A^2 - \frac{\mu}{a_1} = -\frac{\mu}{2a} \quad \cdots (3.9)$$

(∵2.16)　(∵2.68)

L・A点での離心積分を評価




(2.53)より

A点では$y=0$

$$v_A \cos\alpha = -\frac{\mu}{h}(e\sin\omega + 0)$$

$$\Rightarrow e\sin\omega = -\frac{h}{\mu} v_A \cos\alpha \quad \cdots (3.11)$$

(2.54)より

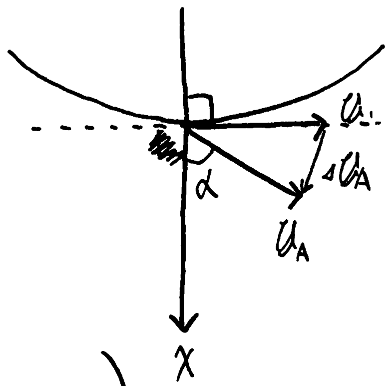
A点では$x=r$

$$v_A \sin\omega = \frac{\mu}{h}\left(e\cos\omega + \frac{r}{r}\right)$$

$$\Rightarrow e\cos\omega = \frac{h}{\mu} v_A \sin\omega - 1 \quad \cdots (3.10)$$

# 3.2

$v_A = v_1 + \Delta v_A$ より、




$$\Delta v_A = v_A - v_1$$
$$= \sqrt{(v_A - v_1)^2}$$
$$= \sqrt{v_A^2 + v_1^2 - 2 v_A \cdot v_1}$$
$$= \sqrt{v_A^2 + v_1^2 - 2 v_1 v_A \sin\alpha} \quad \left(\because v_A \cdot v_1 = v_A v_1 \cos\left(\frac{\pi}{2} - \alpha\right) = v_A v_1 \sin\alpha\right)$$
$$\cdots (3.7)$$

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$$h = a_1 \cdot v_A \sin\alpha = \sqrt{\mu a (1 - e^2)} \quad \cdots (3.8)$$
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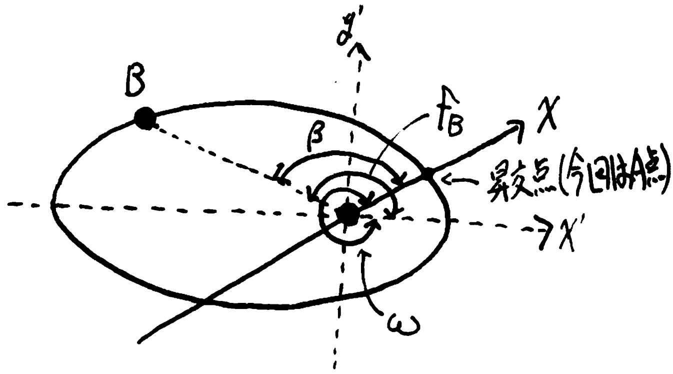
L・A点での離心積分を評価




(2.53)より

A点では $y=0$

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$$\Rightarrow e \sin\omega = -\frac{h}{\mu} v_A \cos\alpha \quad \cdots (3.11)$$

(2.54)より

A点では $x=r$

$$v_A \sin\omega = \frac{\mu}{h}\left(e \cos\omega + \frac{r}{r}\right)$$
$$\Rightarrow e \cos\omega = \frac{h}{\mu} v_A \sin\omega - 1 \quad \cdots (3.10)$$

○Bについて

楕円軌道の方程式(2.56)より、

$$a_2 = \frac{h^2}{\mu(1+e\cos f_B)} \quad (\because 3.8) \quad \cdots (3.12)$$

前ページの図を参考に、$\beta, f_B, \omega$ の関係を書くと、

$$\omega + (f_B - \beta) = 2\pi$$

$$\beta = f_B + \omega - 2\pi \quad \cdots (3.13)$$

遷移軌道TのB点における速度を離心積分(2.53)(2.54)より求める。座標は前ページの図の$(x, y)$座標で考える。
すると、

$$x = r\cos\beta = r\cos(f_B+\omega) \qquad y = r\sin\beta = r\sin(f_B+\omega) \quad (\because 3.13)$$

となるので、これらを用いて $v_{Bx}$, $v_{By}$ を求めると、

$$v_{Bx} = -\frac{\mu}{h}\left\{e\sin\omega + \frac{r}{r}\sin(f_B+\omega)\right\}$$

$$= -\frac{\mu}{h}\left\{\sin(f_B+\omega) + e\sin\omega\right\} \quad \cdots (3.14)$$

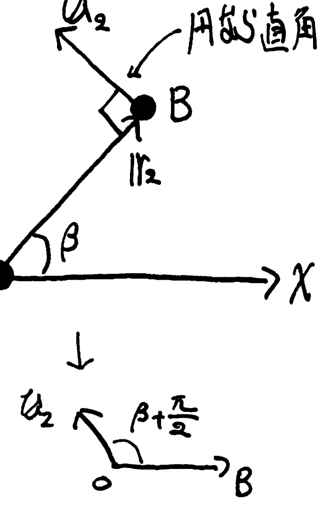
$$v_{By} = \frac{\mu}{h}\left\{e\cos\omega + \frac{r}{r}\cos(f_B+\omega)\right\}$$

$$= \frac{\mu}{h}\left\{\cos(f_B+\omega) + e\cos\omega\right\} \quad \cdots (3.15)$$

円軌道$C_2$のB点における速度成分は

$$v_{2x} = v_2\cos\left(\beta+\frac{\pi}{2}\right) = -v_2\sin\beta \quad \cdots (3.16)$$

$$v_{2y} = v_2\sin\left(\beta+\frac{\pi}{2}\right) = v_2\cos\beta \quad \cdots (3.17)$$



$$v_2 = v_B + \Delta v_B$$

$$\Rightarrow \Delta v_B = \sqrt{(v_2 - v_B)^2}$$

$$= \sqrt{(v_{2x} - v_{Bx})^2 + (v_{2y} - v_{By})^2} \quad \cdots (3.18)$$

(3.14)〜(3.17)を代入

$$\Delta v_B = \left\{ \left[ -v_2 \sin\beta + \frac{\mu}{h}\left\{ \sin(\underbrace{f_B + \omega}_{=\beta\ (\because 3.13)}) + e \sin\omega \right\} \right]^2 + \left[ v_2 \cos\beta - \frac{\mu}{h}\left\{ \cos(\underbrace{f_B + \omega}_{=\beta}) + e \cos\omega \right\} \right]^2 \right\}^{\frac{1}{2}}$$

$$= \left[ \left\{ \left(\frac{\mu}{h} - v_2\right)\sin\beta + \frac{\mu}{h} e \sin\omega \right\}^2 + \left\{ \left(v_2 - \frac{\mu}{h}\right)\cos\beta - \frac{\mu}{h} e \cos\omega \right\}^2 \right]^{\frac{1}{2}}$$

$$= \left[ \left(\frac{\mu}{h} - v_2\right)^2 \sin^2\beta + \left(\frac{\mu}{h} e\right)^2 \sin^2\omega + 2\left(\frac{\mu}{h} - v_2\right)\frac{\mu}{h} e \sin\beta \sin\omega \right.$$
$$\left. + \left(\frac{\mu}{h} - v_2\right)^2 \cos^2\beta + \left(\frac{\mu}{h} e\right)^2 \cos^2\omega + 2\left(\frac{\mu}{h} - v_2\right)\frac{\mu}{h} e \cos\beta \cos\omega \right]^{\frac{1}{2}}$$

$$= \left[ \left(\frac{\mu}{h} - v_2\right)^2 + \left(\frac{\mu}{h} e\right)^2 + 2\left(\frac{\mu}{h} - v_2\right)\frac{\mu}{h} e \cos(\underbrace{\beta - \omega}_{=f_B\ (\because 3.13)}) \right]^{\frac{1}{2}}$$

$$= \sqrt{\left(\frac{\mu}{h} - v_2\right)^2 + \left(\frac{\mu}{h} e\right)^2 + \frac{2\mu}{h} e \cos f_B \left(\frac{\mu}{h} - v_2\right)} \quad \cdots (3.19)$$

(3.12)より

$$\frac{1}{a^2} = \frac{\mu}{h^2}(1 + e\cos f_B) \rightarrow \frac{h}{a^2} = \frac{\mu}{h} + \frac{\mu}{h} e \cos f_B \rightarrow \frac{\mu}{h} e \cos f_B = \frac{h}{a^2} - \frac{\mu}{h} \quad \cdots (3.20)$$

$(3.10)^2 + (3.11)^2$

$$
\begin{aligned}
&e^2 \cos^2\omega = 1 + \frac{h^2}{\mu^2} v_A^2 \sin^2\alpha - \frac{2h}{\mu} v_A \sin\alpha \\
+)\ &e^2 \sin^2\omega = \frac{h^2}{\mu^2} v_A^2 \cos^2\alpha \\
\hline
&e^2 = 1 + \frac{h^2}{\mu^2} v_A^2 - \frac{2h}{\mu} \underbrace{v_A \sin\alpha}_{= \frac{h}{a_1}\ (\because 3.8)}
\end{aligned}
$$

$$e^2 = 1 + \frac{h^2}{\mu^2} v_A^2 - \frac{2h^2}{\mu a_1}$$

$$\downarrow \times \frac{\mu^2}{h^2}$$

$$\frac{\mu^2}{h^2} e^2 = \frac{\mu^2}{h^2} + v_A^2 - \frac{2\mu}{a_1}$$

$$\left(\frac{\mu}{h} e\right)^2 = \frac{\mu^2}{h^2} - 2\frac{\mu}{a_1} + v_A^2 \quad \cdots (3.21)$$

(3.20)(3.21)を(3.19)へ代入

$$\Delta v_B = \left\{\left(\frac{\mu}{h} - v_2\right)^2 + \frac{\mu^2}{h^2} - 2\frac{\mu}{a_1} + v_A^2 + 2\left(\frac{h}{a_2} - \frac{\mu}{h}\right)\left(\frac{\mu}{h} - v_2\right)\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{\mu^2}{h^2} + v_2^2 - \frac{2\mu}{h} v_2 + \frac{\mu^2}{h^2} - 2\frac{\mu}{a_1} + v_A^2 + \frac{2\mu}{a_2} - \frac{2h}{a_2} v_2 - \frac{2\mu^2}{h^2} + \frac{2\mu}{h} v_2\right\}^{\frac{1}{2}}$$

$$= \left\{v_A^2 + v_2^2 - 2\underbrace{\frac{\mu}{a_1}}_{= v_1^2} + 2\underbrace{\frac{\mu}{a_2}}_{= v_2^2} - 2\frac{h}{a_2} v_2\right\}^{\frac{1}{2}}$$

$(\because 3.3)$ $\qquad h = a_1 v_A \sin\alpha \ (\because 3.8)$

$$= \sqrt{v_A^2 - 2v_2 \frac{a_1}{a_2} v_A \sin\alpha + 3v_2^2 - 2v_1^2} \quad \cdots (3.22)$$

(3.8)より

$$a_1^2 v_A^2 \sin^2\alpha = \mu a(1-e^2)$$

$$1-e^2 = \frac{a_1^2 v_A^2 \sin^2\alpha}{\mu a} \quad \cdots ①$$

(3.9)より

$$\frac{2a}{\mu} = \left(\frac{\mu}{a_1} - \frac{1}{2}v_A^2\right)^{-1}$$

$$a = \frac{\mu}{2}\left(\frac{\mu}{a_1} - \frac{1}{2}v_A^2\right)^{-1}$$

$$a^{-1} = \frac{2}{\mu}\left(\frac{\mu}{a_1} - \frac{1}{2}v_A^2\right)$$

(∵ 3.3)

$$= \frac{2}{\mu}\left(v_1^2 - \frac{1}{2}v_A^2\right) \quad \cdots ②$$

②を①へ代入

$$1-e^2 = \frac{1}{\mu}a_1^2 v_A^2 \sin^2\alpha \cdot \frac{2}{\mu}\left(v_1^2 - \frac{1}{2}v_A^2\right)$$

$$= \frac{2 a_1^2 v_A^2 \sin^2\alpha}{\mu^2}\left(v_1^2 - \frac{1}{2}v_A^2\right)$$

$\frac{a_1^2}{\mu^2} = \frac{1}{v_1^4}$ (∵ (3.3))

$$\Rightarrow e^2 = 1 + \frac{2 v_A^2 \sin^2\alpha}{v_1^4}\left(\frac{1}{2}v_A^2 - v_1^2\right) \cdots ③$$

(3.23)より.

$$1+e \geq \frac{a_2}{a}$$

(∵ $a_2 > a$ より $\frac{a_2}{a} - 1 > 0$)

$$e^2 \geq \left(\frac{a_2}{a} - 1\right)^2 \cdots ④$$

②.③を④へ代入

$$1 + \frac{2 v_A^2 \sin^2\alpha}{v_1^4}\underbrace{\left(\frac{1}{2}v_A^2 - v_1^2\right)}_{= E} \geq \left\{\frac{2a_2}{\mu}\underbrace{\left(v_1^2 - \frac{1}{2}v_A^2\right)}_{= -E} - 1\right\}^2$$

$E$ とおく

$$\vdots$$

$$1+2\frac{v_A^2}{v_1^4}\sin^2\alpha\cdot E \geq \frac{4a_2^2}{\mu^2}E^2+1+\frac{4a_2}{\mu}E$$

$$\frac{v_A^2}{v_1^4}\sin^2\alpha \leq \frac{2a_2^2}{\mu^2}\cdot E+\frac{2a_2}{\mu} \qquad (\because E<0)$$

$$\frac{v_A^2}{v_1^4}\sin^2\alpha \leq \frac{2}{v_2^4}\left(\frac{1}{2}v_A^2-v_1^2\right)+\frac{2}{v_2^2} \qquad (\because 3.3)$$

$$\left(\frac{1}{v_1^4}\sin^2\alpha-\frac{1}{v_2^4}\right)v_A^2 \leq -2\frac{v_1^2}{v_2^4}+\frac{2}{v_2^2}$$

$$\leq \frac{2}{v_2^2}\left(1-\frac{a_2}{a_1}\right) \qquad \left(\because (3.3)\text{より}\ \frac{v_1^2}{v_2^2}=\frac{a_2}{a_1}\right)$$

両辺に $\times v_2^4$

$$\left(\frac{v_2^4}{v_1^4}\sin^2\alpha-1\right)v_A^2 \leq 2v_2^2\left(1-\underbrace{\frac{a_2}{a_1}}_{=\frac{v_1^2}{v_2^2}}\right)$$

$$\leq 2\left(v_2^2-v_1^2\right)$$

両辺に $\times(-1)$

$$\left\{1-\left(\frac{a_1}{a_2}\sin\alpha\right)^2\right\}v_A^2 \geq 2\left(v_1^2-v_2^2\right) \quad \cdots (3.24)$$

$$\Delta v=\Delta v_A+\Delta v_B$$

$$=\underbrace{\sqrt{v_A^2+v_1^2-2v_1v_A\sin\alpha}}_{(\because 3.7)}+\underbrace{\sqrt{v_A^2-2v_2\frac{a_1}{a_2}v_A\sin\alpha+3v_2^2-2v_1^2}}_{(\because 3.22)} \quad \cdots (3.25)$$

$$\Delta v\left(\alpha=\frac{\pi}{2}\right)=\sqrt{v_A^2+v_1^2-2v_1v_A}+\sqrt{v_A^2-2v_2\frac{a_1}{a_2}v_A+3v_2^2-2v_1^2}$$

$$=\underbrace{v_A-v_1}_{(\because v_A>v_1)}+\sqrt{v_A^2-2v_2\frac{a_1}{a_2}v_A+3v_2^2-2v_1^2} \quad \cdots (3.26)$$

$\Delta v(\alpha=\frac{\pi}{2})$ を $v_A$ について微分

$$\frac{\partial(\Delta v(\alpha=\frac{\pi}{2}))}{\partial v_A} = 1 + \frac{v_A - v_2\frac{a_1}{a_2}}{\sqrt{v_A^2 - 2v_2\frac{a_1}{a_2}v_A + 3v_2^2 - 2v_1^2}} \quad \cdots (3.27)$$

(3.24)より

$$v_{A\min}^2 = \frac{2(v_1^2 - v_2^2)}{\left\{1 - \left(\frac{a_1}{a_2}\sin\frac{\pi}{2}\right)^2\right\}}$$

$$v_{A\min} = \sqrt{\frac{2(v_1^2 - v_2^2)}{1 - \frac{a_1^2}{a_2^2}}} = \sqrt{\frac{2a_2^2\left(\frac{\mu}{a_1} - \frac{\mu}{a_2}\right)}{(a_2 - a_1)(a_2 + a_1)}} = \sqrt{\frac{2\frac{\mu a_2^2}{a_1 a_2}(a_2 - a_1)}{(a_2 - a_1)(a_2 + a_1)}}$$

$(\because 3.3)$

$$= \sqrt{\frac{\mu}{a_1}\frac{2a_2}{a_1 + a_2}} \quad \cdots (3.28)$$

(3.26)へ(3.28)を代入して $\Delta v$ の最小値を求める。
ここで、(3.3)も代入して式を整理する。

$$\Delta v_{\min} = v_{A\min} - v_1 + \sqrt{v_A^2 - 2v_2\frac{a_1}{a_2}v_{A\min} + 3v_2^2 - 2v_1^2}$$

$$= \sqrt{\frac{\mu}{a_1}\frac{2a_2}{a_1+a_2}} - \sqrt{\frac{\mu}{a_1}} + \underbrace{\sqrt{\frac{\mu}{a_1}\frac{2a_2}{a_1+a_2} - 2\sqrt{\frac{\mu}{a_2}}\frac{a_1}{a_2}\sqrt{\frac{\mu}{a_1}\frac{2a_2}{a_1+a_2}} + 3\cdot\frac{\mu}{a_2} - 2\frac{\mu}{a_1}}}_{=⑤}$$

⑤とおいて別に計算する

$$⑤ = \left[\frac{\mu}{a_1}\frac{2a_2}{a_1+a_2} - 2\frac{\mu}{a_2}\sqrt{\frac{2a_1}{a_1+a_2}} + 3\frac{\mu}{a_2} - 2\frac{\mu}{a_1}\right]^{\frac{1}{2}}$$

$$= \left[\frac{\mu}{a_1}\left(\frac{2a_2}{a_1+a_2} - 2\frac{a_1+a_2}{a_1+a_2}\right) - 2\frac{\mu}{a_2}\sqrt{\frac{2a_1}{a_1+a_2}} + 3\frac{\mu}{a_2}\right]^{\frac{1}{2}}$$

$$= \left[\frac{\mu}{a_1}\cdot\frac{(-2a_1)}{a_1+a_2} - 2\frac{\mu}{a_2}\sqrt{\frac{2a_1}{a_1+a_2}} + 3\frac{\mu}{a_2}\right]^{\frac{1}{2}}$$

$$= \left[\frac{\mu}{a_2}\left\{3 - 2\sqrt{\frac{2a_1}{a_1+a_2}} - \underbrace{\frac{2a_2}{a_1+a_2}}_{=2\frac{(a_2+a_1-a_1)}{a_1+a_2} = 2 - \frac{2a_1}{a_1+a_2}}\right\}\right]^{\frac{1}{2}}$$

$$⑤ = \left[\frac{\mu}{a_2}\left\{1-2\sqrt{\frac{2a_1}{a_1+a_2}}+\frac{2a_1}{a_1+a_2}\right\}\right]^{\frac{1}{2}}$$

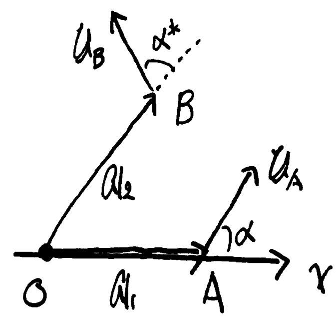
$$=\left[\frac{\mu}{a_2}\left\{1-\sqrt{\frac{2a_1}{a_1+a_2}}\right\}^2\right]^{\frac{1}{2}}$$

$$=\sqrt{\frac{\mu}{a_2}}\left(1-\sqrt{\frac{2a_1}{a_1+a_2}}\right)$$

以上より

$$\Delta v_{min}=\sqrt{\frac{\mu}{a_1}}\left(\sqrt{\frac{2a_2}{a_1+a_2}}-1\right)+\sqrt{\frac{\mu}{a_2}}\left(1-\sqrt{\frac{2a_1}{a_1+a_2}}\right) \quad \cdots (3.29)$$

角運動量積分




$$\boldsymbol{a}_1\times\boldsymbol{v}_A=\boldsymbol{a}_2\times\boldsymbol{v}_B$$

$$\Rightarrow a_1 v_A \sin\alpha = a_2 v_B \sin\alpha^* \quad \cdots (3.30)$$

エネルギー積分

$$\frac{1}{2}v_A^2-\frac{\mu}{a_1}=\frac{1}{2}v_B^2-\frac{\mu}{a_2} \quad \cdots (3.31)$$

(3.28)を(3.31)へ代入

$$\frac{1}{2}\frac{\mu}{a_1}\frac{2a_2}{a_1+a_2}-\frac{\mu}{a_1}=\frac{1}{2}v_B^2-\frac{\mu}{a_2}$$

$$\Rightarrow v_B^2=\frac{\mu}{a_1}\frac{2a_2}{a_1+a_2}-\frac{2\mu}{a_1}+\frac{\mu}{a_2}$$

$$=\cancel{\frac{\mu}{a_1a_2(a_1+a_2)}\left(2a_2^2-2a_1a_2-2a_2\right.}$$

$$=\frac{\mu}{a_1}\frac{2a_2}{a_1+a_2}\cdot\frac{a_2}{a_2}-\frac{2\mu}{a_1}\frac{a_2(a_1+a_2)}{a_2(a_1+a_2)}+\frac{2\mu}{a_2}\cdot\frac{a_1(a_1+a_2)}{a_1(a_1+a_2)}$$

$$=\frac{\mu\cdot 2a_1^2}{a_1a_2(a_1+a_2)}$$

$$=\frac{\mu}{a_2}\frac{2a_1}{a_1+a_2}$$

$$\therefore v_B=\sqrt{\frac{\mu}{a_2}\frac{2a_1}{a_1+a_2}} \quad \cdots (3.32)$$

(3.32)を(3.30)へ代入

$$a_1 v_A \sin\alpha = a_2 v_B \sin\alpha^*$$

$$\rightarrow \sin\alpha^* = \frac{a_1}{a_2}\cdot\frac{v_A}{v_B} \quad (\because \alpha = \frac{\pi}{2})$$

$$= \frac{a_1}{a_2}\frac{\sqrt{\frac{\mu}{a_1}\frac{2a_2}{a_1+a_2}}}{\sqrt{\frac{\mu}{a_2}\frac{2a_1}{a_1+a_2}}}$$

$$= 1$$

$$\therefore \quad \alpha^* = \frac{\pi}{2}$$

(3.10)に $\omega = 0$、$\alpha = \frac{\pi}{2}$ をいれると.

$$e = -1 + \frac{h}{\mu} v_A$$

$$= -1 + \frac{a_1 v_A^2}{\mu} \quad (\because h = a_1 v_A \cos 0 = a_1 v_A)$$

$$= -1 + \frac{a_1}{\mu}\frac{\mu}{a_1}\frac{2a_2}{a_1+a_2} \quad (\because 3.28)$$

$$= \frac{a_2 - a_1}{a_1 + a_2}$$

これを(3.20)に代入

$$\frac{\mu}{h} e \cos f_B = \frac{h}{a_2} - \frac{\mu}{h}$$

$$\cos f_B = \frac{1}{e}\left(\frac{h^2}{\mu a_2} - 1\right)$$

$$= \frac{a_1+a_2}{a_2-a_1}\left(\frac{a_2^2 v_B^2}{\mu a_2} - 1\right)$$

$$= \frac{a_1+a_2}{a_2-a_1}\left(\frac{a_2}{\mu}\cdot\frac{\mu}{a_2}\frac{2a_1}{a_1+a_2} - 1\right)$$

$$= \frac{a_1+a_2}{a_2-a_1}\left(\frac{a_1-a_2}{a_1+a_2}\right)$$

$$= -1$$

$$\therefore \quad f_B = \pi$$