4.2 + [問題4.2]

(4.20)(4.21), (4.23) に各々 $\dot{X}$, $\dot{Y}$, $\dot{Z}$ を掛けて和をとる

$$\dot{X}(\ddot{X}-2n'\dot{Y}) = -\frac{\partial U^*}{\partial X}\cdot\dot{X}$$

$$\dot{Y}(\ddot{Y}+2n'\dot{X}) = -\frac{\partial U^*}{\partial Y}\cdot\dot{Y}$$

$$+)\qquad \dot{Z}\cdot\ddot{Z} = -\frac{\partial U^*}{\partial Z}\cdot\dot{Z}$$

$$\underbrace{\dot{X}\ddot{X}+\dot{Y}\ddot{Y}+\dot{Z}\ddot{Z}}_{=\frac{1}{2}\frac{d}{dt}(\dot{X}^2+\dot{Y}^2+\dot{Z}^2)} = -\frac{\partial U^*}{\partial X}\frac{dX}{dt}-\frac{\partial U^*}{\partial Y}\frac{dY}{dt}-\frac{\partial U^*}{\partial Z}\frac{dZ}{dt} = -\frac{dU^*}{dt} \quad \cdots (4.24)$$

(4.24)を t で積分

$$\underbrace{\frac{1}{2}(\dot{X}^2+\dot{Y}^2+\dot{Z}^2)+U^* = \mathrm{Const}}_{\text{ヤコビ積分(円制限3体問題の保存量)}} \quad \cdots (4.25)$$

(4.25)へ(4.26)を代入して、

$$\frac{1}{2}(\dot{X}^2+\dot{Y}^2+\dot{Z}^2)-\frac{1}{2}n'^2(X^2+Y^2)-\frac{GM_1}{r_1}-\frac{GM_2}{r_2} = \mathrm{Const} \quad \cdots ①$$

として、これを慣性系 (ξ, η, ζ) で表現する
まずは(4.6)(4.7)より、$\dot{X}$, $\dot{Y}$ を求める

$$X = \xi\cos\theta + \eta\sin\theta \quad \cdots (4.6)$$

$$\dot{X} = \dot{\xi}\cos\theta + \dot{\eta}\sin\theta - n'\xi\sin\theta + n'\eta\cos\theta \quad \cdots ②$$

一応 $\dot{Z}$ も

$$Z = \zeta \quad \cdots (4.5)$$

$$\dot{Z} = \dot{\zeta} \quad \cdots ④$$

$$Y = -\xi\sin\theta + \eta\cos\theta \quad \cdots (4.7)$$

$$\dot{Y} = -\dot{\xi}\sin\theta + \dot{\eta}\cos\theta - n'\xi\cos\theta - n'\eta\sin\theta \quad \cdots ③$$

②, ③, ④より.

$$\dot{X}^2+\dot{Y}^2+\dot{Z}^2$$

$$= \dot{\xi}^2\cos^2\theta + \dot{\eta}^2\sin^2\theta + n'^2\xi^2\sin^2\theta + n'^2\eta^2\cos^2\theta$$

$$+2\dot{\xi}\dot{\eta}\sin\theta\cos\theta - 2n'\dot{\xi}\xi\sin\theta\cos\theta + 2n'\dot{\xi}\eta\cos^2\theta - 2n'\xi\dot{\eta}\sin^2\theta + 2n'\dot{\eta}\eta\sin\theta\cos\theta - 2n'^2\xi\eta\sin\theta\cos\theta$$

$$+\dot{\xi}^2\sin^2\theta + \dot{\eta}^2\cos^2\theta + n'^2\xi^2\cos^2\theta + n'^2\eta^2\sin^2\theta$$

$$-2\dot{\xi}\dot{\eta}\sin\theta\cos\theta + 2n'\dot{\xi}\xi\sin\theta\cos\theta + 2n'\dot{\xi}\eta\sin^2\theta - 2n'\xi\dot{\eta}\cos^2\theta - 2n'\dot{\eta}\eta\sin\theta\cos\theta + 2n'^2\eta\sin\theta\cos\theta$$

$$+\dot{\zeta}^2$$

$$= \dot{\xi}^2 + \dot{\eta}^2 + \dot{\zeta}^2 + n'^2\xi^2 + n'^2\eta^2 + 2n'\dot{\xi}\eta - 2n'\xi\dot{\eta} \quad \cdots ⑤$$

$$X^2+Y^2$$

$$= \xi^2\cos^2\theta + \eta^2\sin^2\theta + 2\xi\eta\sin\theta\cos\theta + \xi^2\sin^2\theta + \eta^2\cos^2\theta - 2\xi\eta\sin\theta\cos\theta$$

$$= \xi^2 + \eta^2 \cdots ⑥$$

⑤, ⑥を①へ代入する

$$\frac{1}{2}(\dot{\xi}^2+\dot{\eta}^2+\dot{\zeta}^2) + \frac{1}{2}n'^2(\xi^2+\eta^2) + n'(\dot{\xi}\eta - \xi\dot{\eta}) - \frac{1}{2}n'^2(\xi^2+\eta^2) - \frac{GM_1}{r_1} - \frac{GM_2}{r_2} = \mathrm{const}$$

$$\therefore \frac{1}{2}(\dot{\xi}^2+\dot{\eta}^2+\dot{\zeta}^2) - n'(\xi\dot{\eta} - \eta\dot{\xi}) - \frac{GM_1}{r_1} - \frac{GM_2}{r_2} = \mathrm{const} \quad \cdots (4.27)$$