[問題5.2]

(5.33)を(5.32)に代入

$$X=\left(a-\frac{1}{3}\epsilon a^2+\frac{1}{48}\epsilon^2a^3\right)\cos 2\tau$$
$$+\epsilon\left(a-\frac{1}{3}\epsilon a^2+\frac{1}{48}\epsilon^2a^3\right)^2\left(-\frac{1}{2}+\frac{1}{3}\cos 2\tau+\frac{1}{6}\cos 2\tau\right)$$
$$+\epsilon^2\left(a-\frac{1}{3}\epsilon a^2+\frac{1}{48}\epsilon^2a^3\right)^3\left(-\frac{1}{3}+\frac{29}{144}\cos 2\tau+\frac{1}{9}\cos 2\tau+\frac{1}{48}\cos 3\tau\right)$$

$$=\left(a-\frac{1}{3}\epsilon a^2+\frac{1}{48}\epsilon^2a^3\right)\cos 2\tau$$
$$+\epsilon\left(a^2-\frac{2}{3}\epsilon a^3\right)\left(-\frac{1}{2}+\frac{1}{3}\cos 2\tau+\frac{1}{6}\cos 2\tau\right)$$
$$+\epsilon^2\cdot a^3\cdot\left(-\frac{1}{3}+\frac{29}{144}\cos 2\tau+\frac{1}{9}\cos 2\tau+\frac{1}{48}\cos 3\tau\right)+O(\epsilon^3)$$

$$=a\cos 2\tau+\epsilon a^2\left(-\frac{1}{3}\cos 2\tau-\frac{1}{2}+\frac{1}{3}\cos 2\tau+\frac{1}{6}\cos 2\tau\right)$$
$$+\epsilon^2a^3\left\{\frac{1}{48}\cos 2\tau-\frac{2}{3}\left(-\frac{1}{2}+\frac{1}{3}\cos 2\tau+\frac{1}{6}\cos 2\tau\right)-\frac{1}{3}+\frac{29}{144}\cos 2\tau+\frac{1}{9}\cos 2\tau+\frac{1}{48}\cos 3\tau\right\}$$
$$+O(\epsilon^3)$$

$$=a\cos 2\tau+\epsilon a^2\left(-\frac{1}{2}+\frac{1}{6}\cos 2\tau\right)+\epsilon^2a^3\left\{\frac{1}{3}-\frac{1}{3}+\left(\frac{1}{48}-\frac{2}{9}+\frac{29}{144}\right)\cos 2\tau+\left(-\frac{1}{9}+\frac{1}{9}\right)\cos 2\tau\right.$$
$$\left.+\frac{1}{48}\cos 3\tau\right\}+O(\epsilon^3)$$

$$=a\cos 2\tau+\epsilon a^2\left(-\frac{1}{2}+\frac{1}{6}\cos 2\tau\right)+\frac{\epsilon^2a^3}{48}\cos 3\tau+O(\epsilon^3)$$

▨