Heterogeneous Impact of the Global Financial Cycle*

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Abstract

I develop a dynamic heterogeneous-country model of the world economy to study the distributional impact of capital flight episodes. The dominant country acts as a global intermediary, borrowing from regular countries to finance investment in their risky assets. Wealth heterogeneity between regular countries arises naturally due to idiosyncratic shocks. A single global factor that combines the intermediary's wealth and risk-taking capacity determines capital inflows and risk premia in every country. I express country-specific loadings on this factor as a function of their wealth in closed form. A shock to the intermediary's risk-taking capacity generates global capital flight. Investors from rich countries retrench and replace the falling demand from abroad. These countries see offsetting gross flows, and their risky assets appreciate together with the global safe asset. In poor countries, markets are not deep enough, so prices adjust instead of quantities, and risk premia rise. Capital flight episodes redistribute wealth from poor to rich countries. I show that the differences in the synchronization of gross flows between advanced economies and emerging markets are consistent with data.

Key Words: capital flows, risk premium, global financial cycle, heterogeneity, retrenchment

JEL Classification Numbers: F32, F44, G12, G15

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1 Introduction

This paper presents a dynamic heterogeneous-country model of the world economy to study the distributional impact of the global financial cycle. The model economy is centered around a dominant country that occupies a special place in the world's financial system. The other countries differ in wealth, which endogenously makes them unequally exposed to global shocks that drive capital flows and prices of risky assets. I study this heterogeneity and its general equilibrium implications for aggregate variables.

The global financial cycle is a co-movement between asset prices and international capital flows around the world. Prices of risky assets and capital flows tend to rise in booms and fall in downturns. A large and growing empirical literature studies factors driving variation in these variables and finds strong common components that correlate with measures of global risk-taking capacity and financial conditions.

There is substantial heterogeneity in exposure to the global financial cycle across countries. First, the US is different from the rest of the world. It is at the center of the global financial system, issuing safe assets and investing in risky assets abroad. Gourinchas and Rey (2022) find that the US earns a 2% excess return on its external position. It takes substantial losses on this position in crises, essentially acting as an insurer for the rest of the world.

A facet of heterogeneity that so far has received less attention is within the rest of the world, between advanced economies and emerging markets. This is what I focus on. The main motivating fact is that capital flows in advanced economies are more synchronized with the global financial cycle. Does it make them more or less exposed to the cycle than emerging markets? And how does exposure to capital flows shape the distribution of gains and losses in global downturns?

I first investigate synchronization patterns using data on gross financial flows. One type of gross flows is gross outflows, defined as net purchases of foreign assets. The other type is gross inflows, defined as net purchases of domestic assets by foreigners. Both types are highly cyclical and large in magnitude. Forbes and Warnock (2012) and Forbes and Warnock (2021) document large swings in gross inflows and outflows that partly offset each other and result in relatively small net flows. They show that massive repatriation of assets by domestic investors during episodes of capital flight often limits net outflows, making sudden stops seem less widespread.

I specifically focus on the role played by domestic investors in a country's response to global shocks. In booms, domestic investors increase their foreign holdings. In downturns, they actively liquidate them to buy domestic assets from foreign investors who want to leave the country. This process is referred to as "retrenchment". Caballero and Simsek (2020) and Jeanne and Sandri (2023) theoretically show the importance of retrenchment for domestic stabilization as local investors replace the foreign demand for risky assets in crises and support asset prices. Aggregate data on private flows suggest that retrenchment is characteristic of advanced economies.

I show that the dominant component in capital flows explains a higher share of variation in gross outflows from advanced economies. This dominant component closely follows indicators of the global financial cycle such as VIX, the asset price factor from Miranda-Agrippino et al. (2020), the intermediary leverage factor from He et al. (2017), and the treasury basis from Jiang et al. (2021). Gross outflows are more strongly correlated with all these in advanced economies.

This means that outward investment booms and retrenchment in advanced economies are more tightly timed to the global cycle. Having established this fact, I show that the magnitudes are larger in advanced economies too. Normalizing outflows by outstanding positions, I show that not only is the aggregate component in advanced economies more correlated with the global factors, but it is also more volatile. Position-adjusted outflows in emerging markets are more volatile overall, but idiosyncratic variation dominates.

Taken together, these facts show better synchronization of outflows from advanced economies with the global financial cycle. In downturns, investors from advanced economies retrench and rebalance their portfolios more actively than those from emerging markets.

The purpose of the model is to study the general equilibrium implications of this heterogeneity for asset prices, risk sharing, and macro adjustment to external shocks around the world. I construct a multi-country exchange economy with trade in risky assets and a global safe asset. Wealth heterogeneity between countries is persistent and arises endogenously. The economy is hit by global shocks that generate aggregate movements in asset prices and financial flows.

The world consists of a continuum of regular countries and a large special country, the US. Every regular country has a saver and issues a risky asset with country-specific idiosyncratic risk. The model takes seriously the interpretation of the US as a global banker. This dominant country hosts a global intermediary that invests in risky assets from all regular countries, financing its investments by issuing short-term riskless debt to savers from those countries. Markets are segmented: local savers can only hold this short-term debt and their domestic assets. They cannot issue their own riskless debt either. The intermediary is fully responsible for trading assets across borders. It also has access to a long-term riskless asset issued and traded within the US.

The intermediary's risk-taking capacity is limited by ambiguity concerns. It takes a cautious approach to idiosyncratic country-specific risks and does not take full advantage of having access to a continuum of assets. As a result, it cannot fully insure regular countries. Idiosyncratic shocks create persistent wealth heterogeneity and non-degenerate wealth distribution. Regular countries continually evolve, moving up and down the distribution as time progresses.

Asset prices and returns differ between rich and poor countries. In rich countries, high wealth translates into high prices and low excess returns. Domestic investors keep a small portion of their wealth in risky assets, facing little risk and relying more on safe foreign investments. If a country becomes poorer after a spell of bad shocks, total wealth in the market falls, depressing prices and raising returns. Local investors increase the weight on their domestic assets. At some point, they hit the constraint that prevents them from issuing their own riskless debt to buy even more. I associate these constrained high-yielding countries with emerging markets, and rich unconstrained countries with advanced economies.

The intermediary earns a risk premium on its investments. It actively trades with local savers, holding a portfolio that is skewed toward poorer countries. Once a country grows richer, its assets appreciate, and the intermediary sells them for capital gains, buying riskier assets that trade at a larger discount. This happens even without aggregate shocks, as idiosyncratic shocks move regular countries around the wealth distribution.

The model is parsimonious enough to summarize the intermediary's demand for risky assets, and hence international capital flows, by one factor that combines its net worth and fundamental risk-taking capacity coming from ambiguity concerns. Along with the global interest rate, this factor determines returns on risky assets around the world, with heterogeneous loadings that depend on wealth. The returns can be expressed in closed form. Rich countries are less sensitive to foreign demand since local savers have large portfolios and quickly adjust them when spreads move. Poor countries are less elastic since their investors do not have enough savings to absorb fluctuations in foreign demand and cannot issue assets to do it with leverage.

This creates heterogeneity in responses to shocks to the intermediary's risk-taking capacity. When the intermediary loses the appetite for risk and seeks to deleverage, rich countries exhibit large offsetting gross flows since investors in these countries use their savings to buy domestic assets that it sells. Retrenchment compensates for foreign demand and stabilizes domestic asset markets. Poor countries instead see a rise in risk premia and a fall in prices. This is further exacerbated by the portfolio constraint they face: unable to issue their own safe assets, investors from poor countries cannot borrow to buy what the intermediary seeks to sell. The resulting heterogeneity in domestic demand elasticities is consistent with my stylized facts: flows in advanced economies are better synchronized with the global financial cycle.

This has important general equilibrium implications. First, falling asset prices make everyone want to consume less. However, without a shock to aggregate output, consumption cannot go down, so the interest rate falls to boost it. This revalues assets. In particular, risky assets issued by rich countries appreciate, since risk premia in these countries do not rise. Retrenchment effectively makes them load negatively on the intermediary's risk-taking capacity. They endogenously behave like a good substitute for safe assets, even though dividends have the same volatility in all regular countries, whether they are rich or poor.

Second, the safe asset issued in the special country appreciates the most. In fact, its price increases so much that the wealth share of the global intermediary, who holds it, increases. This is an important coincidence highlighted by Dahlquist et al. (2022). The special country takes losses on its position in risky assets around the world, but these losses are more than covered by the gains it makes on its domestic holdings. The hegemon becomes relatively richer in downturns.

Risk-sharing arrangements in the model have multiple aspects. First, unsurprisingly, the intermediary provides insurance to poor countries by absorbing part of the losses on their risky assets. This is consistent with Maggiori (2017) and Gourinchas and Rey (2022), who find sizeable wealth transfers from the hegemon to the rest of the world in crises. These wealth transfers come in the form of valuation adjustments, as poor countries are inelastic and do not rebalance through gross flows. Valuation gains then contribute to the reversal of the hegemon's net foreign assets back to normal, as shown empirically by Gourinchas et al. (2019).

Second, and this is particular to a multi-country setup, rich countries provide insurance to the global intermediary. Their assets appreciate on impact, supporting the intermediary's net worth and thereby limiting the fall in foreign demand for everyone. Moreover, asset markets in advanced economies are elastic, meaning that domestic investors in these countries let the intermediary realize the capital gains on impact by buying back their assets at new high prices.

Finally, the safe asset in the special country supports other assets by keeping the intermediary's net worth from falling. If its net worth fell, foreign demand everywhere would weaken even more, further depressing asset prices, especially in emerging markets. The rise in the special country's wealth share thus limits regressive wealth redistribution within the rest of the world.

Can the shock to the intermediary's risk-taking capacity explain large swings in aggregate wealth and consumption given that it generates sizeable financial flows? The answer in the model is no. Its impact on asset prices is largely redistributive since it does not destroy output. Foreign investment recedes, but in advanced economies, where it falls the most, retrenchment actually supports asset prices. To get a more complete picture, I compare this shock to dividend shocks that hit output with a persistent decline.

Dividend shocks impact the prices of all assets and, through that, the intermediary's net worth. The interest rate rises to induce a decrease in consumption, further depressing prices. This time, they fall in all countries. The difference between rich and poor countries is that in rich countries prices only respond to rising interest rates, while the short-run impact of dividends and foreign demand is neutralized by low effective discount rates of the rich local investors. In poor countries, prices react to both foreign demand and dividends, since local investors have higher effective discount rates, expecting to grow out of the left tail of the wealth distribution. Financial flows are muted since the shock does not create new disagreements on risk-return profiles relative to the steady state, so there is not much reason for trade on impact.

Importantly, results are qualitatively very similar between the shock to output in regular countries and in the special country. This is because the intermediary's net worth is affected in both cases. Since the intermediary is exposed to all assets, the movements in its net worth transmit to all countries regardless of origin. Contagion is the reverse side of risk-sharing in the model. US output shocks drive prices of risky assets everywhere and are quantitatively as important as shocks to its risk-taking capacity.

Related literature. A large literature explores global drivers of international capital flows and asset prices. Miranda-Agrippino and Rey (2022) provide a comprehensive review. The dominant global factor in a large panel of risky asset prices has been extracted by Miranda-Agrippino et al. (2020) and more recently updated by Miranda-Agrippino and Rey (2020). Habib and Venditti (2019) find a similar global component driving stock prices around the world. Jordà et al. (2017) document co-movement between risk-premia across the world.

Similarly strong co-movement has been documented for capital flows. Barrot and Serven (2018) identify common components in gross capital flows and show that these common components are strongly related to aggregate variables such as VIX, US dollar exchange rate, and interest rates. The main one is strongly correlated with the dominant factor in risky asset prices. Davis et al. (2021) show that these factors also explain a large share of variation in net flows.

Part of this literature deals with heterogeneity between advanced economies and emerging markets. Barrot and Serven (2018) and Cerutti et al. (2019) show that flows in advanced economies are more responsive to common factors. Loadings reported in Miranda-Agrippino and Rey (2022) largely support this conclusion. I perform a simplified version of their factor extraction exercise to illustrate these already known differences in synchronization. In addition to showing that global factors explain a larger share of variation in advanced economies, I find that the aggregate component of flows, properly adjusted for size, in this group is larger in magnitude. This stylized fact is at the heart of the model, which is built to generate more elastic asset markets in advanced economies and explore the implications of this heterogeneity.

Predictions of my model concern the distributional consequences of global financial shocks. The literature studying distributions of returns and flows includes Chari et al. (2020), Gelos et al. (2022), and Eguren Martin et al. (2021). Chari et al. (2020) show the outsized effect of risk-off episodes on the worst realizations, the left tail. This is the response to a shock to risk-taking capacity in my model: the left tail of returns shifts significantly further, while the average stays very close to normal times.

Another strand of literature documents the special position of the US in the global financial system. Gourinchas et al. (2019) review evidence on various dimensions of its dominance. Gourinchas and Rey (2022) find that the US earns significant net returns on its net foreign asset position. Similarly to this gap between the US and the rest of the world, there is heterogeneity within the latter. Adler and Garcia-Macia (2018) show substantial differentials in returns on net foreign asset positions between advanced economies and emerging markets. My model assumes a special position of the US but generates the differences between other economies endogenously.

I contribute to the theoretical literature on the global cycle. The most closely related papers are Caballero and Simsek (2020), Davis and Van Wincoop (2021), and Dahlquist et al. (2022). Caballero and Simsek (2020) show how retrenchment stabilizes domestic asset markets. In their model, many countries invest in each other's risky assets. Liquidity shocks to a country's asset trigger fire sales by foreign investors. Local investors then use their foreign holdings to pick up the unwanted asset and support its price. I build on this mechanism, which is also present in Jeanne and Sandri (2023). My model is a dynamic version of Caballero and Simsek (2020) with an intermediary at the center, with endogenously arising differences in wealth and returns, and with an aggregate capital flight. I calibrate it to reproduce empirical patterns in capital flows and study how retrenchment shapes aggregate and distributional consequences of global shocks.

Davis and Van Wincoop (2021) construct a multicountry model to generate gross flows after a shock to global risk aversion, which is also what drives dynamics in my model. Davis and Van Wincoop (2021) show the importance of within-country heterogeneity for generating trades in equilibrium. I build on their insights about the disagreement between market participants that lead to non-trivial gross flows. However, I model segmented markets and a global intermediary instead of different preferences within countries. Each country has one saver that trades with the intermediary and has no direct access to other countries. Heterogeneity between regular countries in my model is all ex-post and arises endogenously, and the model can account for the most leveraged country, the US, simultaneously taking losses on foreign assets and becoming richer.

Dahlquist et al. (2022) build a multicountry model with home bias in consumption and time-varying appetites for risk coming from deep habits in preferences. They demonstrate how an adverse output shock in a large country leads to an appreciation of its currency and, consequently, an increase in its wealth share, as its stock prices fall by less than foreign stocks when adjusted for the exchange rate. I arrive at regressive redistribution in downturns through capital flows instead. In relative terms, rich countries become richer because they are able to compensate for the falling demand from abroad. In absolute terms, they become richer because the shock to risk-taking capacity decreases the interest rate, while risk premia are held down by retrenchment, and asset prices have to rise. This happens in rich regular countries as much as in the special one.

Farboodi and Kondor (2022) study heterogeneous global dynamics with imperfect information about asset quality. In their model, shocks determine what investors learn about firms. In bad times, they flee from emerging markets to advanced economies, inducing a recession in the former and stabilizing output in the latter. My model generates similar outcomes in a different setup.

Fu (2023) models joint determination of capital flows and exchange rates. He shows empirically that currency betas on global returns are low for countries where domestic investors have a higher propensity to retrench than foreign ones. In his model, net flows in risky assets have to be financed through intermediaries with frictions similar to those in Gabaix and Maggiori (2015). Countries with flightier investors face net risky inflows in downturns, and their currencies appreciate. The resulting link between retrenchment and cyclicality of returns is similar to that in my model.

The role of intermediaries in my paper is similar to that in Gourinchas et al. (2022). In their model, intermediaries are international arbitrageurs that trade assets in two otherwise disjoint markets. The intermediary in my model also has effective mean-variance preferences derived from ambiguity concerns. I also relate these preferences to a VAR-type constraint, which Gourinchas et al. (2022) mention as a possible microfoundation. I show how a single institutional constraint can replicate the portfolio choice of an intermediary with ambiguity concerns.

Models with the US as a special country include, among others, Maggiori (2017), Farhi and Maggiori (2018), Jiang et al. (2020), Kekre and Lenel (2021), and Sauzet (2023). In Maggiori (2017), the US faces laxer financial constraints. In Farhi and Maggiori (2018), it realizes it has monopoly power over issuing safe debt. In Jiang et al. (2020) and Kekre and Lenel (2021), the dollar carries a convenience yield. Kekre and Lenel (2021) study flight to safety caused by a shock to this convenience yield in a model with nominal frictions and investment. Sauzet (2023) generates a rise in global risk premia and an appreciation of the dollar in times of stress with general recursive preferences and a less risk-averse US.

Aversion to ambiguity in my model is built on the large theoretical literature dating back to Anderson et al. (2000), Hansen and Sargent (2001), and Chen and Epstein (2002). More recently, Ilut and Saijo (2021) use a similar specification in a model with a continuum of firms to generate empirically relevant co-movements and cyclical patterns in a business cycle model.

On the technical side, I use heterogeneous-agent tools that have mostly been used to model separate countries. This allows me to use insights from recent work in the HANK literature. I employ methods from Kaplan et al. (2018) to analyze non-linear solutions for aggregate one-time unanticipated shocks. Sequence-space methods of Auclert et al. (2021) and insights from Bhandari et al. (2023) adapted to continuous time allow me to speed up computations and linearize the model for estimation. These tools have been underused in the international context so far. I believe that they can generate substantial progress in studying the distributional effects of global shocks and aggregate implications of international heterogeneity.

Layout. The plan of the paper is as follows. Section 2 shows stylized facts that motivate the model. Section 3 presents the model, Section 4 describes equilibrium and calibration. Section 5 describes the shock to risk-taking capacity. Finally, Section 6 describes the shock to output.

2 Capital Flows Across Countries

Before setting up the model, I briefly discuss distributional patterns of capital outflows. "Outflows" are net purchases of foreign assets by domestic investors. "Inflows" are net purchases of local assets by foreign investors. Both are "gross" flows, even though they can be negative. I zoom in on outflows to focus on the role of domestic investors in shaping international heterogeneity.

The main point this section makes is that private outflows from advanced economies are more synchronized with the global financial cycle, both in terms of correlation and in terms of magnitude. Investors from advanced economies time their retrenchment to downturns more precisely and retrench more actively than those in emerging markets.

The data come from the Balance of Payments and International Investment Positions statistics provided by the IMF. I supplement these with GDP data from the World Bank. See Appendix A for details. Following the detrending procedure from Forbes and Warnock (2012) and Forbes and Warnock (2021), I construct the following variables:

$$a_{it} = \sum_{s=t-3}^{s=t} FA_{i,s} - \sum_{s=t-7}^{s=t-4} FA_{i,s} \tag{1}$$

Here t is a quarter, i is a country, and $FA_{i,t}$ records net purchases of foreign assets by i. Forbes and Warnock (2012) and Forbes and Warnock (2021) use similar measures to detect extreme capital flow episodes such as stops, surges, flight, and retrenchment. The size-adjusted versions are

$$\overline{a}_{it} = \sum_{s=t-3}^{s=t} \frac{FA_{i,s}}{A_{i,s-1}} - \sum_{s=t-7}^{s=t-4} \frac{FA_{i,s}}{A_{i,s-1}} \tag{2}$$

$$\underline{a}_{it} = \sum_{s=t-3}^{s=t} \frac{FA_{i,s}}{GDP_{i,s-1}} - \sum_{s=t-7}^{s=t-4} \frac{FA_{i,s}}{GDP_{i,s-1}} \tag{3}$$

Here A_{it} is the total stock of i's external assets in quarter t. Both FA_{it} and A_{it} are measured in dollars and exclude reserves and FDI. Private outflows do not fully describe adjustment to shocks, since countries regularly deploy reserves, often at a large scale. These interventions and other operations not driven by profit maximization are beyond the scope of my paper.

Note that $FA_{i,t}$ is different from the total change in position $A_{i,t} - A_{i,t-1}$ because the latter includes valuation effects. The variables a_{it}, $\overline{a}_{it}$, and $\underline{a}_{it}$ are constructed to account for trades, not price changes. I interpolate GDP from annual data linearly. Equation (1) calculates a detrended version of changes in assets due to trades, equation (2) calculates percentage changes due to trades, and equation (3) scales trades by the size of the economy.

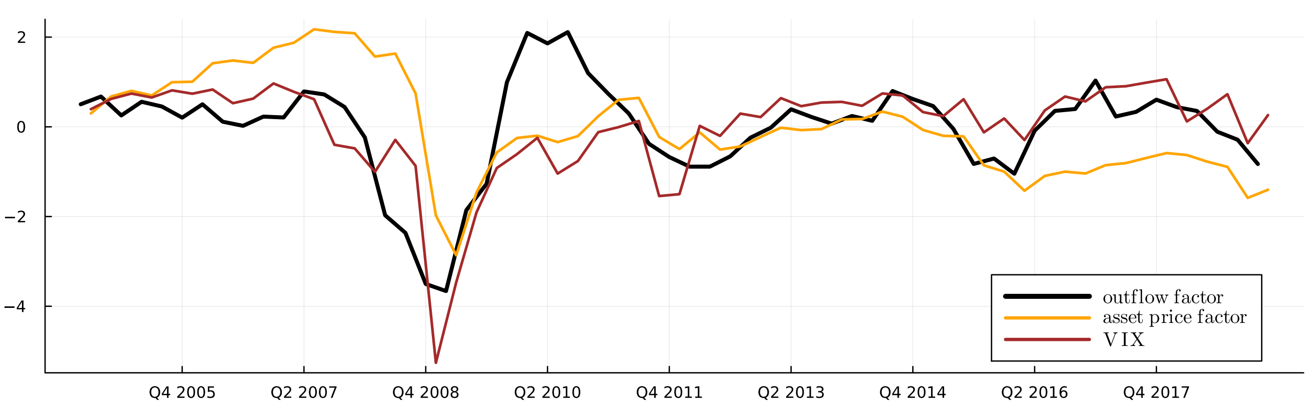


Figure 1: Principal component f_t of outflows a_{it}, asset price factor from Miranda-Agrippino et al. (2020), and minus VIX, all normalized to have zero mean and unit standard deviation.

To capture aggregate flows, I extract the principal component f_t, which I call the outflow factor, from a balanced subpanel of a_{it} covering the last 20 years. Figure 1 shows f_t along with the risky asset price factor computed by Miranda-Agrippino et al. (2020) and VIX. Capital flows have been documented to co-move with global measures of appetites for risk. Outflows from countries that load more on the outflow factor should be more aligned with these measures and more closely follow the global financial cycle. These more exposed countries turn out to be advanced economies.

Fact 1: outflows from advanced economies are more correlated with global factors. The principal component f_t of capital outflows explains more variation in outflows a_{it} from advanced economies than from emerging markets. Figure 2 shows these shares for individual countries in the sample. The average over advanced economies is 29% and that over emerging markets is 8%. Outflows from advanced economies appear to be more tightly connected to global factors than those from emerging markets. Investors from advanced economies increase foreign holdings during upticks in global outflows and retrench during downturns.

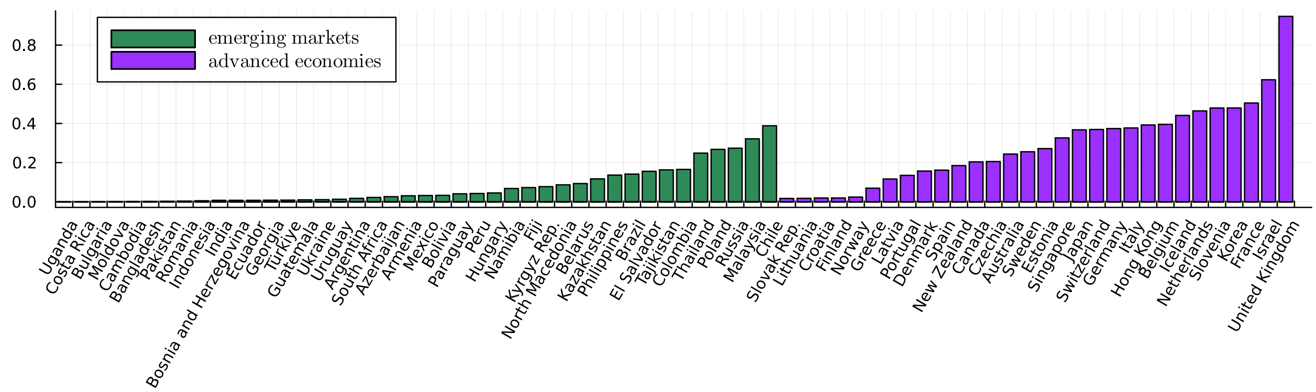


Figure 2: Share of time-series variation in a_{it} explained by f_t for a given country i.

Another way to illustrate the differences is to average outflows from each group. Consider the variables a_t^{AE}, $\overline{a}_t^{AE}$, and $\underline{a}_t^{AE}$ that average a_{it}, $\overline{a}_{it}$, and $\underline{a}_{it}$ over i corresponding to advanced economies. Similarly, a_t^{EM}, $\overline{a}_t^{EM}$, and $\underline{a}_t^{EM}$ average outflows from emerging markets. I project each of these variables on f_t and calculate the share of variation that f_t captures:

$$x_t = \alpha + \beta f_t + \epsilon_t \tag{4}$$

Here $x_t \in \{a_t^{AE}, \overline{a}_t^{AE}, \underline{a}_t^{AE}, a_t^{EM}, \overline{a}_t^{EM}, \underline{a}_t^{EM}\}$. The share of variation captured by f_t is the R-squared.

In advanced economies, f_t explains 93% of variation in average outflows, 74% in average outflows to assets, and 85% in average outflows to GDP. In emerging markets, it is 31%, 10%, and 45%, respectively. These numbers are much closer to one than individual shares from Figure 2, since averaging partly offsets idiosyncratic shocks. But even with idiosyncratic variation partly purged, they are still significantly different between the two groups.

The dominant factor in capital flows is statistically related to measures of global risk appetite, as has been shown by a large literature reviewed in Miranda-Agrippino and Rey (2022). This means that outflows from advanced economies co-move with measures or risk appetites more closely since they are more tightly connected to f_t. Table 1 confirms this by showing correlations between within-group averages and time series that have been associated with the global financial cycle.

Table 1: Correlation between aggregate series and averages $\{a_t^{AE}, \overline{a}_t^{AE}, \underline{a}_t^{AE}, a_t^{EM}, \overline{a}_t^{EM}, \underline{a}_t^{EM}\}$

	a_t^{AE}	a_t^{EM}	$\overline{a}_t^{AE}$	$\overline{a}_t^{EM}$	$\underline{a}_t^{AE}$	$\underline{a}_t^{EM}$
outflow factor f_t	0.95	0.24	**0.86**	0.29	0.90	0.62
VIX (negative)	0.36	0.20	**0.38**	0.15	0.40	0.30
asset price factor, Miranda-Agrippino and Rey (2020)	0.42	0.46	**0.32**	0.04	0.49	0.29
intermediary factor, He et al. (2017)	0.24	0.01	**0.21**	-0.16	0.27	0.24
treasury basis, Jiang et al. (2021)	0.27	0.03	**0.27**	0.00	0.28	0.16

The asset price factor from Miranda-Agrippino and Rey (2020) is a dominant component that they extract from 858 series of risky asset prices around the world. The intermediary factor from He et al. (2017) traces the capital ratios of financial intermediaries, is highly cyclical, and explains variation in expected returns on large classes of assets. The treasury basis from Jiang et al. (2021) is a measure of convenience yield on short-term US treasury bonds that rises with demand for these safe assets.

The differences in correlations between advanced economies and emerging markets are noticeable in most cases and particularly salient when flows are measured relative to outstanding positions (the highlighted column). Outflows from advanced economies do seem to be more strongly synchronized with measures of global risk appetites. But are they larger in magnitude?

Fact 2: aggregate component of outflows relative to outstanding positions is more volatile in advanced economies. Measuring magnitudes requires an adjustment for size. Flows relative to outstanding positions and relative to GDP, $\overline{a}_{it}$ and $\underline{a}_{it}$, are two natural candidates. The variance of these size-adjusted flows can be decomposed into time-series dispersion of the averages $\overline{a}_t$ or $\underline{a}_t$ and cross-sectional dispersion around these averages. Taking flows relative to outstanding positions, the variation decomposes into aggregate and idiosyncratic parts as

$$\mathbb{V}[\overline{a}_{it}] = \underbrace{\mathbb{V}[\mathbb{E}[\overline{a}_{is}|s=t]]}_{\text{aggregate}} + \underbrace{\mathbb{E}[\mathbb{V}[\overline{a}_{is}|s=t]]}_{\text{idiosyncratic}} \tag{5}$$

Table 2 shows this decomposition conditional on advanced economies and emerging markets. Importantly, the aggregate component is larger in advanced economies. Not only are the average outflows from advanced economies more tightly connected to aggregates, but they are also more volatile. Fact 1 shows that investors in advanced economies time their purchases of foreign assets to the global financial cycle. Fact 2 now additionally shows that they rebalance portfolios

Table 2: Decomposition of variance in $\overline{a}_{it}$ in advanced economies and emerging markets. The sample spans Q1 2003 through Q4 2022 and contains 28 advanced economies and 47 emerging markets on average.

	standard deviation of $\overline{a}_{it}$	total variance	aggregate	idiosyncratic
advanced economies	0.10	0.0107	**0.0020**	0.0087
emerging markets	0.21	0.0456	**0.0012**	0.0444

more actively, as measured by flows relative to outstanding positions. The same applies to busts: advanced economies retrench in crises and do it more actively than emerging markets.

Figure 3 summarizes the patterns of synchronization in the two groups. Position-adjusted outflows in advanced economies follow the outflow factor more closely on average and are less dispersed around it. This cross-sectional dispersion is shown by the error band. In emerging markets, the average outflows are less synchronized with the outflow factor, idiosyncratic dispersion dominates, and the average component is less volatile than that in advanced economies.

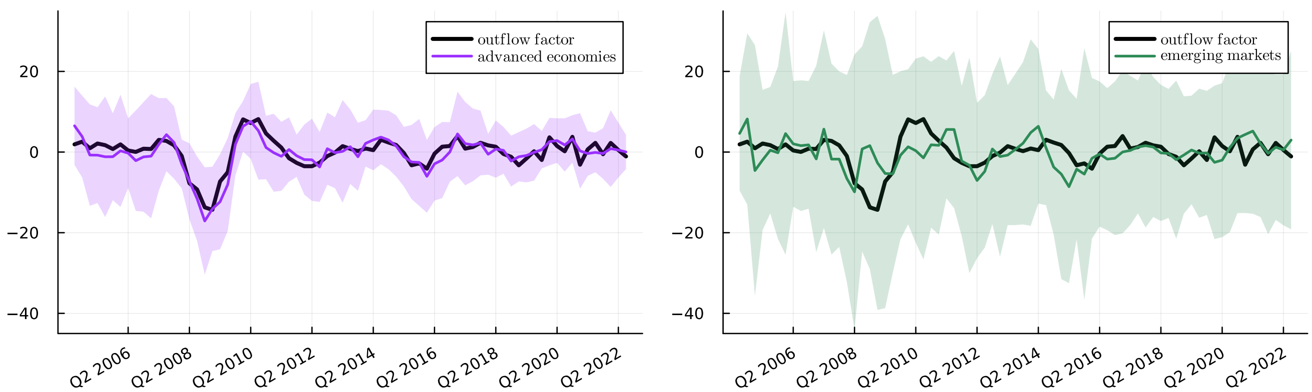


Figure 3: Aggregate outflows from advanced economies and emerging markets as a percentage of the outstanding position. Black line: outflow factor f_t normalized to have the average standard deviation between $\overline{a}_{it}^{AE}$ and $\overline{a}_{it}^{EM}$. Error bands show cross-sectional standard deviations of $\overline{a}_{it}$ in the two groups for each quarter t.

These results are stronger for GDP-adjusted flows, as I show in Appendix A. The reason is that advanced economies have more external assets relative to GDP, so flows as a share of GDP are more volatile in both aggregate and idiosyncratic components. Flows relative to positions are more informative because they quantify investor activity relative to their wealth. The fact that wealth is not equally comparable with output in all economies speaks to the economics of long-run asset accumulation. The fact that investors in advanced economies, on aggregate, move larger shares of their portfolio with the global financial cycle, is instead more informative about their demand for risky assets and the distribution of short-run demand elasticities. These observations motivate the model I outline in the next section.

3 Model

Time is continuous and runs forever. There is no aggregate uncertainty. The world consists of a unit measure of regular countries indexed by $i \in [0, 1]$ and a large special country. Each of the regular countries has a Lucas Jr (1978) tree. Output is homogeneous across countries and stochastic. Cumulative yield y_{it} of a country i evolves as $dy_{it} = \nu_t dt + \sigma dZ_{it}$. The deterministic rate ν_t is a function of time only, and the volatility σ is constant. The random increments dZ_{it} are standard Brownian. They are independent across countries, and the total output coming from the regular countries is constant.

I use hats in the notation for the special country throughout the paper. The special country has a measure $\hat{q}$ of trees that are pooled together in a fund. The random components of their yields wash out, so the total output over dt in the special country is $\hat{q}\hat{\nu}_t dt$, where $\hat{\nu}_t$ is potentially different from ν_t. Shares of these trees can only be traded as one, in a bundle with equal weights. I will refer to this fund as the special country's tree for convenience.

The special country has a representative saver who holds shares of this domestic tree and provides intermediation services to regular countries. Specifically, it takes deposits from savers in regular countries and buys shares of their trees. The state variables of an individual saver in the special country are her wealth $\underline{\hat{w}}_t$ and the aggregate wealth of the whole country is $\hat{w}_t$. These two are the same in equilibrium, but the savers do not internalize this fact.

Representative savers in regular countries only invest in their own domestic trees and keep deposits with the global intermediary. Deposits are riskless and short-term, paying an interest rate $r_t dt$. The share price of i's tree is an endogenous stochastic process p_{it}. The instantaneous excess return on the shares is

$$dR_{it} = \frac{\nu_t dt + \sigma dZ_{it} + dp_{it}}{p_{it}} - r_t dt \tag{6}$$

It includes the dividend yield $\nu_t dt + \sigma dZ_{it}$ and capital gains dp_{it}. The wealth $\underline{w}_{it}$ of an individual saver from country i is her tree holdings and deposits with the global intermediary. The aggregate wealth of country i is w_{it}, which will be equal to $\underline{w}_{it}$ in equilibrium, although the savers do not internalize this fact. The evolution of $\underline{w}_{it}$ is

$$d\underline{w}_{it} = (r_t \underline{w}_{it} - c_{it})dt + \theta_{it}\underline{w}_{it} dR_{it} + \frac{\underline{w}_{it}}{w_{it}} \cdot \hat{\lambda}\hat{w}_t dt - \frac{\underline{w}_{it}}{w_{it}} \cdot \lambda w_{it} dt \tag{7}$$

Here c_{it} is the consumption rate, and the first term in equation (7) represents the consumption-savings trade-off. The second term represents the returns on the domestic tree, where θ_{it} is its portfolio share that the saver chooses. She operates under a portfolio constraint

$$\theta_{it} \leq \theta \tag{8}$$

This constraint can be interpreted as the inability to issue safe assets without a limit. If $\theta = 1$, the saver can fully invest their wealth in domestic shares but cannot borrow from the intermediary. If $\theta < 1$, the saver must keep at least $1 - \theta$ of her wealth on riskless deposits. If $\theta > 1$, the saver can borrow from the intermediary up to a limit of $(\theta - 1)\underline{w}_{it}$. Since the rest of her wealth is the tree holdings, this effectively means that borrowing is limited by $(\theta - 1)/\theta$ of their market value.

The third term in equation (7) represents wealth immigration from the special country. Savers in the special country die with intensity $\hat{\lambda}$, and their money is sent to one of the regular countries, where it is shared between the local savers in proportion to net worth. The destination country is chosen uniformly, so each country i has an influx of wealth $\hat{\lambda}\hat{w}_t dt$. In i, each saver gets a share $\underline{w}_{it}/w_{it}$ of this transfer, where w_{it} is the country i aggregate wealth.

Finally, the fourth term represents emigration to the special country. In regular countries, savers die with an intensity λ, and their wealth is sent to the special country. The total outflow of money from i is $\lambda w_{it} dt$. New savers are born to replace emigrants. They start with zero wealth and instantly get transferred a portion of everyone's savings so that everyone within the country has the same net worth. This redistribution from continuing savers is in proportion to their $\underline{w}_{it}$. Hence, conditional on continuing, they always make flow payments $\lambda\underline{w}_{it}dt$ to the newborns.

The sequence problem of the saver in the country i is

$$V_{it} = \max_{(c_{is},\theta_{is})_{s\geq t}} \mathbb{E}_t \left[\rho \int_t^\infty e^{\rho(t-s)} \log(c_{is}) ds\right] \tag{9}$$

subject to equation (7) and equation (8). Savers choose consumption rate and the portfolio share allocated to risky assets. They take the interest rate r_t, tree price p_{it}, and aggregate net worth w_{it} as given. The discount rate ρ here includes the death rate λ. Since everyone has the same $\underline{w}_{it}$, in equilibrium $w_{it} = \underline{w}_{it}$, and the evolution of the total wealth in country i is

$$dw_{it} = (r_t w_{it} - c_{it})dt + \theta_{it} w_{it} dR_{it} + (\hat{\lambda}\hat{w}_t - \lambda w_{it})dt \tag{10}$$

There is only idiosyncratic uncertainty in this economy, and only savers from country i and the special country have access to trading country i's tree. For this reason, the excess returns dR_{it} and price movements dp_{it} only load on dZ_{it} and can be written as

$$dR_{it} = \mu_{it}^R dt + \sigma_{it}^R dZ_{it} \tag{11}$$

$$dp_{it} = \mu_{it}^p dt + \sigma_{it}^p dZ_{it} \tag{12}$$

The drift and volatility terms $(\mu_{it}^R, \mu_{it}^p, \sigma_{it}^R, \sigma_{it}^p)$ are equilibrium objects that have to be solved for.

The problem inequation (9) has a particularly simple solution because of log utility. Savers always consume a constant fraction of their wealth and choose a mean-variance portfolio whenever the constraint $\theta_{it} \leq \overline{\theta}$ allows for it. Given the time paths of the global interest rate r_t, the special country's wealth $\hat{w}_t$, and the drift and volatility of the excess return process $(\mu^R_{it}, \sigma^R_{it})$, consumption is $c_{it} = \rho \underline{w}_{it}$ and portfolio choice is

$$\theta_{it} = \min\left\{\overline{\theta}, \frac{\mu^R_{it}}{(\sigma^R_{it})^2}\right\} \tag{13}$$

Unit elasticity of intertemporal substitution leads to a constant consumption-wealth ratio. Unit relative risk aversion in addition leads to portfolio choice with no hedging motive.

It is useful to track the deposits and tree holdings of the savers separately. Denote deposits of country i by l_{it} and their holdings of their tree by h_{it}. By construction, the wealth of i is $w_{it} = l_{it} + p_{it}h_{it}$, and the risky share θ_{it} determines the split: $\theta_{it}w_{it} = p_{it}h_{it}$ and $(1-\theta_{it})w_{it} = l_{it}$.

Special country. The price of the special country's tree is $\hat{p}_t$. The instantaneous excess return is $d\hat{R}_t = (\hat{\nu}_t dt + d\hat{p}_t)/\hat{p}_t - r_t dt$, idiosyncratic shocks washing out. The net worth of the saver in the special country, which is also the global intermediary, consists of its domestic tree and its holdings of foreign trees less the deposits outstanding. The individual net worth $\underline{\hat{w}}_t$ evolves as

$$d\underline{\hat{w}}_t = (r_t\underline{\hat{w}}_t - \hat{c}_t)dt + \sum_i f_{it}\underline{\hat{w}}_t dR_{it} + \hat{f}_t\underline{\hat{w}}_t d\hat{R}_t + \frac{\underline{\hat{w}}_t}{\hat{w}_t} \cdot \lambda w_t dt - \frac{\underline{\hat{w}}_t}{\hat{w}_t} \cdot \hat{\lambda}\hat{w}_t dt \tag{14}$$

Consumption is $\hat{c}_t$ and $\hat{w}_t$ is the special country's aggregate wealth. The second and third terms are excess returns on trees in regular countries and at home. Positions $(f_{it}, \hat{f}_t)$ are akin to θ_{it} in the regular country's problem. Portfolios with a continuum of idiosyncratic returns require special care, so I write the total return as a sum without specifying its type. Below, I work out this problem for a finite number of countries and then let the number of countries grow to infinity.

The last two terms are the mirror image of wealth migration terms in equation (7): there is an inflow of $\lambda w_t dt$, where $w_t = \int_0^1 w_{it}di$ is the aggregate wealth of the regular countries, and an outflow of $\hat{\lambda}\hat{w}_t dt$. Again, as in regular countries, newborn savers immediately receive transfers from everyone else so that everyone's wealth is the same.

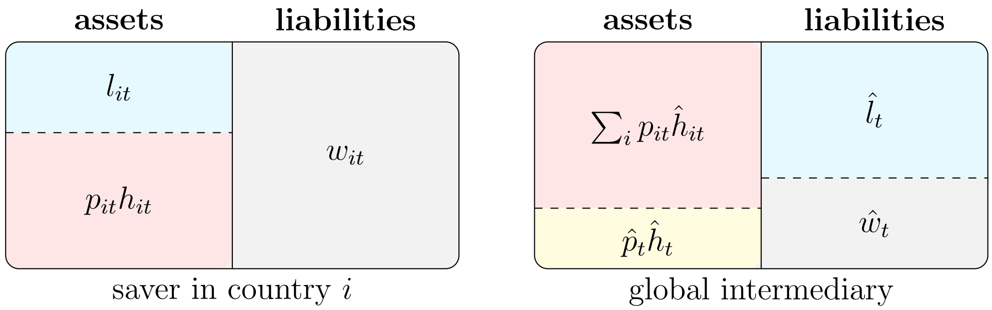
The individual wealth $\underline{\hat{w}}_t$ of the special country savers aggregates into $\hat{w}_t$ evolving as

$$d\hat{w}_t = (r_t\hat{w}_t - \hat{c}_t)dt + \sum_i f_{it}\hat{w}_t dR_{it} + \hat{f}_t\hat{w}_t d\hat{R}_t + (\lambda w_t - \hat{\lambda}\hat{w}_t)dt \tag{15}$$

To track the holdings of trees and deposits, denote by $\hat{l}_t$ the total deposits taken by the special country, and let $\hat{h}_t$ and $(\hat{h}_{it})$ be the holdings of trees. By construction, $\hat{w}_t = \hat{p}_t\hat{h}_t + \sum_i p_{it}\hat{h}_{it} - \hat{l}_t$. Positions $\hat{f}_t$ and (f_{it}) satisfy $\hat{f}_t\hat{w}_t = \hat{p}_t\hat{h}_t$ and $f_{it}\hat{w}_t = p_{it}\hat{h}_{it}$ for all i.

Figure 4 shows a crude example to account for all types of assets in the model. Regular country savers have no liabilities and only hold domestic shares and deposits. The intermediary holds domestic shares and shares of trees in all other countries. Deposits are its liabilities.

Figure 4: Schematic balance sheets of savers from a regular country i and the global intermediary.



Risk-taking capacity. The risk-taking capacity of the intermediary is affected by ambiguity concerns. It is unsure about the right probability measure for dZ_{it}, the random component of dividends in every country i. To account for possible misspecification, the intermediary considers other probability measures Q_i under which the process $\{\widetilde{Z}_{it}\}_{t\geq 0}$ with $d\widetilde{Z}_{it} = dZ_{it} + \xi_{it}dt$ is a Brownian motion instead of $\{Z_{it}\}_{t\geq 0}$. The adapted sequences $\{\xi_{it}\}_{t\geq 0}$ index these alternative measures Q_i. The evolution of an individual intermediary's net worth can be rewritten as

$$d\underline{\hat{w}} = (r_t\underline{\hat{w}}_t - \hat{c}_t)dt + \sum_i f_{it}\underline{\hat{w}}_t((\mu^R_{it} - \sigma^R_{it}\xi_{it})dt + \sigma^R_{it}d\widetilde{Z}_{it}) + \hat{f}_t\underline{\hat{w}}_t d\hat{R}_t + \frac{\underline{\hat{w}}_t}{\hat{w}_t}(\lambda w_t dt - \hat{\lambda}\hat{w}_t)dt \tag{16}$$

Each of the alternative measures Q_i is associated with a likelihood ratio M_{it} relative to the original measure. These likelihood ratios are equal to 1 at $t = 0$ and evolve as

$$dM_{it} = -\xi_{it}M_{it}dZ_{it} \tag{17}$$

The intermediary would like to consider scenarios with heavy losses but faces a penalty for deviating too much from the baseline measure. This penalty per unit of time for every i is proportional to the expected value of $\eta(w_{it})dm_{it}$, where $\eta(\cdot)$ is an increasing function of country i's wealth and dm_{it} is the increment of the log-likelihood ratio $m_{it} = \log(M_{it})$. The intermediary is less concerned about ambiguity when it comes to richer countries, so the penalty for choosing alternative measures is higher for high w_{it}. Fluctuations in w_{it} thus change the intermediary's attitude to i's assets given its risk-return profile, generating idiosyncratic capital flows. Hassan et al. (2021) provide evidence for dynamically changing perception of country-specific risk that affects investor preferences. In the terminology of Cerutti et al. (2019), this represents "pull" factors driving capital flows, which they show to be empirically important in emerging markets.

The problem of an individual intermediary is

$$\hat{V}_t = \max_{(\hat{c}_s,\hat{f}_s,(f_{is}))_{s\geq t}} \inf_{Q\in\mathcal{Q}} \mathbb{E}^Q_t\left[\hat{\rho}\int_t^\infty e^{\hat{\rho}(s-t)}\log(\hat{c}_s)ds + \frac{1}{2}\int_t^\infty e^{\hat{\rho}(s-t)}\hat{\gamma}_s\sum_i \eta(w_{is})dm_{is}\right] \tag{18}$$

subject to equation (16). Here $\mathcal{Q}$ is the set of alternative measures that can be represented by equation (17) and $\{\hat{\gamma}_t\}_{t\geq 0}$ is an adapted sequence of cost parameters.

These time-varying costs capture appetites for risk since they determine how far the intermediary is willing to go with alternative adverse scenarios. A low $\hat{\gamma}_t$ means low costs of considering models with large potential losses, so assets from the regular countries seem less attractive. For this reason, I call $\hat{\gamma}_t$ risk-taking capacity. The standard case is nested as $\hat{\gamma}_t = \infty$, which leads the intermediary to set $dm_{it} = 0$ and stick to the baseline probability measure.

The intermediary solves the minimization problem first, choosing an alternative measure for every country i and their product measure Q to account for losses suggested by models in the admissible set $\mathcal{Q}$. It then solves the usual maximization problem for given model adjustments, choosing consumption streams and portfolio composition. When $\hat{\gamma}_t$ and $\eta(w_{it})$ are constant, equation (18) is exactly the objective function from Hansen and Sargent (2001), and this case will correspond to the steady state in my model. In general, $\hat{\gamma}_t$ could be varying over time, generating temporary episodes of low risk-taking capacity.

The increment of the log-likelihood ratio dm_{it} can be rewritten as

$$dm_{it} = -\xi_{it} dZ_{it} - \frac{1}{2}\xi_{it}^2 dt \tag{19}$$

Since $dZ_{it} = d\widetilde{Z}_{it} - \xi_{it}dt$, where $d\widetilde{Z}_{it}$ is a standard Brownian increment under Q_i, the expectation of equation (19) under Q_i is $\mathbb{E}_t^Q[dm_{it}] = \xi_{it}^2 dt/2$. The problem in equation (18) transforms into

$$\hat{V}_t = \max_{(\hat{c}_s, \hat{f}_s, (f_{is}))_{s\geq t}} \inf_{((\xi_{it}))_{s\geq t}} \mathbb{E}_t^{Q(\xi)} \left[\int_t^\infty e^{\hat{\rho}(s-t)} \left(\hat{\rho}\log(\hat{c}_s) + \frac{\hat{\gamma}_s}{2} \sum_i \eta(w_{is})\xi_{is}^2 \right) ds \right] \tag{20}$$

subject to equation (16). This formulation acknowledges that the choice of measure $Q \in \mathcal{Q}$ is equivalent to selecting drift corrections (ξ_{it}).

Fix the number of regular countries at n. Consumption of the intermediary is $\hat{c}_t = \hat{\rho}\underline{\hat{w}}$, and portfolio weights and drift correction for each country i are

$$f_{it}^{(n)} = \frac{\hat{\gamma}_t \eta(w_{it})}{1 + \hat{\gamma}_t \eta(w_{it})} \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{21}$$

$$\xi_{it}^{(n)} = \frac{1}{\hat{\gamma}_t \eta(w_{it})} \cdot f_{it}^{(n)} \sigma_{it}^R = \frac{1}{1 + \hat{\gamma}_t \eta(w_{it})} \cdot \frac{\mu_{it}^R}{\sigma_{it}^R} \tag{22}$$

The intermediary's portfolio weight of each risky asset is proportional to the mean-variance ratio. It also increases in the risk-taking capacity $\hat{\gamma}_t$ and the country's weight $\eta(w_{it})$. Whenever $\hat{\gamma}_t$ is finite, ambiguity aversion attenuates portfolio weights, as the multiplier before the mean-variance ratio is between 0 and 1. The drift correction is proportional to the Sharpe ratio, meaning that the intermediary takes a more cautious view of high-yielding countries.

To take the continuous limit, I let the number of countries n go to infinity. Payoffs in the limit have to include integrals over $i \in [0,1]$, and portfolio weights f_{it} in the limit become a density:

$$f_{it} = \lim_{n\to\infty} n f_{it}^{(n)} \tag{23}$$

To ensure that this limit exists, I let the risk-taking capacity $\hat{\gamma}_t$ decrease as n diverges: $\hat{\gamma}_t = \gamma_t/n$. It follows from equation (21) that the limit is

$$f_{it} = \gamma_t \eta(w_{it}) \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{24}$$

Why does risk-taking capacity have to go to zero as the number of countries grows? If the intermediary's aversion to uncertainty does not rise as it gets access to a continuum of uncorrelated returns, it will take infinite positions as the law of large numbers wipes out all the risk. The limiting density in equation (24) does not exist unless $\hat{\gamma}_t$ decreases at least as fast as γ_t/n.

Another way to look at it is to account for total payoffs. Increasing n indefinitely gives the intermediary access to more and more uncorrelated assets, so without a commensurate rise in aversion to uncertainty the intermediary's portfolio would blow up. It would borrow and invest without an upper bound, and its wealth would not have well-defined dynamics in the limit.

The drift corrections approach the Sharpe ratio:

$$\xi_{it} = \lim_{n\to\infty} \xi_{it}^{(n)} = \frac{\mu_{it}^R}{\sigma_{it}^R} \tag{25}$$

Finally, the wealth of the special country in the limit follows

$$d\hat{w}_t = (r_t \hat{w}_t - \hat{c}_t)dt + \int_0^1 f_{it}\hat{w}_t dR_{it} di + \hat{f}_t \hat{w}_t d\hat{R}_t + (\lambda w_t - \hat{\lambda}\hat{w}_t)dt \tag{26}$$

Borovička et al. (2023) similarly take a double limit to study aversion to ambiguity with recursive preferences. In their case, uncertainty vanishes at the same speed as the aversion to ambiguity increases, delivering a tractable limit.

Appendix C.1 reverses the order and sets up the problem with a continuum of assets instead of solving a finite-country problem and taking the limits of the solutions. The results are the same. Solving the finite-country problem first has a technical advantage: alternative measures chosen by the intermediary are absolutely continuous with respect to the true measure. With a continuum of countries, this does not hold, as all aggregates become deterministic and measures degenerate.

Ilut and Saijo (2021) use the same continuous limit in a model with a large portfolio of firms. They explain the effect of idiosyncratic uncertainty on an ambiguity-averse decision-maker and relate it to work on laws of large numbers by Marinacci (1999) and Epstein and Schneider (2003).

Global factor. The intermediary is the only foreign investor from the perspective of every regular country. Hence, total foreign holdings of a country i's tree are

$$f_{it}\hat{w}_t = \varphi_t \cdot \eta(w_{it}) \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{27}$$

Here φ_t is the global factor that combines the intermediary's risk-taking capacity and net worth:

$$\varphi_t = \gamma_t \hat{w}_t \tag{28}$$

Importantly, it fluctuates both exogenously when there is a shock to γ_t and endogenously when the net worth of intermediaries changes in equilibrium. If the intermediary's wealth takes a hit, this will have the same effect on its demand for foreign assets as a decrease in γ_t.

The other part of equation (27) includes i-specific variables that fluctuate together with country i's wealth. Total foreign holdings hence respond to both global and local shocks in equilibrium, generating capital flows even in steady state, when all aggregates are constant.

VAR-type constraints. Ambiguity concerns are a useful modeling tool. They introduce non-trivial portfolio choice into the intermediary's problem and effectively limit its risk-taking capacity. The intermediary cannot leverage its access to a fully diversified set of assets and fully insure all other countries. Moreover, its demand for risky assets takes a simple mean-variance form as if it were allocating its wealth to a finite set of independent factors. Shocks to the penalty parameter γ_t generate sizeable gross flows, as I will show in Section 5.

However, this theory of limited risk-taking capacity is by no means unique. There is another way to model intermediaries that have mean-variance demand for each asset in a continuum. To arrive at the same portfolio choice without model misspecification, one could instead impose a VAR-type constraint on the intermediary. This is less mathematically intensive but requires a regulatory or institutional justification. I show the equivalence in Appendix C.2.

Consider an intermediary without any freedom to choose model corrections that simply solves

$$\max_{(\hat{c}_s, \hat{f}_s, (f_{is}))_{s \geq t}} \mathbb{E}_t \left[\int_t^\infty e^{\hat{\rho}(s-t)} \log(\hat{c}_s) ds \right] \tag{29}$$

subject to the normal budget constraint

$$d\underline{\hat{w}}_t = (r_t \underline{\hat{w}}_t - \hat{c}_t)dt + \int_0^1 f_{it}\underline{\hat{w}}_t dR_{it} di + \hat{f}_t \underline{\hat{w}}_t d\hat{R}_t + \left(\lambda \frac{w_t}{\hat{w}_t} - \hat{\lambda} \right) \underline{\hat{w}}_t dt \tag{30}$$

and the following regulatory constraint:

$$\int_0^1 \eta(w_{it})^{-1} \mathbb{V}_t[f_{it} dR_{it}] di \leq \gamma_t \int_0^1 \mathbb{E}_t[f_{it} dR_{it}] di \tag{31}$$

The relationship in equation (31) caps the total amount of idiosyncratic uncertainty in the intermediary's portfolio by a multiple of expected profits from these investments. This idiosyncratic uncertainty does not create real randomness to aggregate payoffs, but the constraint ignores the law of large numbers and deals with managing country-specific risk. If the function $\eta(w_{it})$ is increasing, the intermediary needs more provisions for poorer countries conditional on risk-return profiles, making it more costly to gain exposure to their idiosyncratic risk.

4 Equilibrium

This section defines equilibrium and describes the steady state. The equilibrium processes must satisfy four types of market clearing conditions: for shares of regular country trees, shares of the special country tree, deposits, and consumption goods. Individual wealth of representative savers must also agree with aggregate wealth in their respective countries.

Definition 1. Given a path of $(\gamma_t, \nu_t, \hat{\nu}_t)_{t\geq 0}$, an equilibrium is a collection of price processes $(r_t, (p_{it}), \hat{p}_t)_{t\geq 0}$, wealth processes $((\underline{w}_{it}), (w_{it}), \underline{\hat{w}}_t, \hat{w}_t)_{t\geq 0}$, consumption processes $((c_{it}), \hat{c}_t)_{t\geq 0}$, and processes for asset holdings $((h_{it}), (\hat{h}_{it}), (l_{it}), \hat{h}_t, \hat{l}_t)_{t\geq 0}$ such that all agents optimize and

- aggregate wealth process agrees with individual wealth: $w_{it} = \underline{w}_{it}$ for all i and $\underline{\hat{w}}_t = \hat{w}_t$ a.e.
- markets for regular country trees clear: $h_{it} + \hat{h}_{it} = 1$ for all $i \in [0, 1]$
- deposit market clears: $\int_0^1 l_{it} di = \hat{l}_t$
- market for the special country tree clears: $\hat{h}_t = \hat{q}$
- market for consumption goods clears: $\int_0^1 c_{it} di + \hat{c}_t = \nu_t + \hat{q}\hat{\nu}_t$

The last market clearing condition is automatically satisfied once all other market clearing conditions and budget constraints hold.

I will consider symmetric equilibria in which the identity i of a regular country does not matter conditional on its state variable w_{it}. The logic is as follows. Suppose that two different countries with equal wealth have the same local properties of returns. For the intermediary, their portfolio weights will then be the same. Domestic savers in these countries will also make equivalent choices. Local properties of returns will hence be the same in equilibrium.

With this in mind, below, I drop the subscripts and refer to regular countries using their current wealth w. This step could not be done before solving the intermediary's problem because countries with different identities provide independent returns to the intermediary and have to be accounted for separately, even if their local dynamics are the same. But now, with portfolio choice given by equation (24), I can work directly with prices $p(w, t)$ and portfolio choice functions $\theta(w, t)$ and $f(w, t)$ that map into holding functions $h(w, t)$ and $\hat{h}(w, t)$.

Rewriting the market clearing conditions for each regular country,

$$1 = h(w,t) + \hat{h}(w,t) = \frac{\theta(w,t)w}{p(w,t)} + \frac{f(w,t)\hat{w}(t)}{p(w,t)} \tag{32}$$

Now plugging the expressions for optimal portfolio choice into equation (32),

$$p(w,t) = w \min\left\{\overline{\theta}, \frac{\mu_R(w,t)}{\sigma_R(w,t)^2}\right\} + \varphi(t)\eta(w)\frac{\mu_R(w,t)}{\sigma_R(w,t)^2} \tag{33}$$

Rearranging,

$$\frac{\mu_R(w,t)}{\sigma_R(w,t)^2} = \max\left\{\frac{p(w,t)}{\varphi(t)\eta(w)+w}, \frac{p(w,t)-\overline{\theta}w}{\varphi(t)\eta(w)}\right\} \tag{34}$$

This equation shows how risk-return profiles $(\mu_R(w,t), \sigma_R(w,t))$ are determined by available cash in the market given by local wealth w and intermediary's demand $\varphi(t)\eta(w)$, where the global factor $\varphi(t) = \gamma(t)\hat{w}(t)$ is common to all markets. Excess return μ_R is equal to the quantity of risk σ_R^2 time the price of risk, which is on the right-hand side.

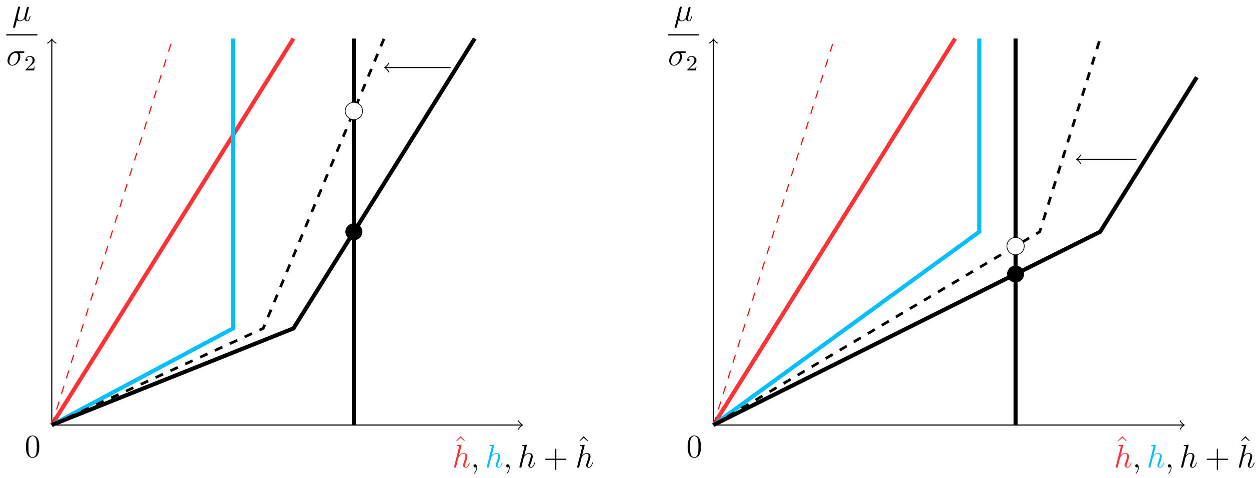
The max operator here shows that the local savers can be constrained or unconstrained in their portfolio choice. When the portfolio constraint is slack, the price of risk is given by the ratio of the total supply of assets $p(w,t)\cdot 1$ divided by the total demand $\varphi(t)\eta(w)+w$. Both local and foreign investors are marginal. When the constraint binds for local savers, only the intermediary is marginal, so the price of risk is the residual supply $p(w,t)-\overline{\theta}w$ divided by its demand $\varphi(t)\eta(w)$.

Elastic and inelastic markets. Markets in countries with different wealth w will react to changes in the global factor $\varphi(t)$ in different ways. Consider first a notion of partial equilibrium in which only returns respond, while asset prices $p(w,t)$ and wealth levels w do not change yet. Suppose there is a change in $\varphi(t)$. The elasticity of the mean-variance ratio in equation (34) to $\varphi(t)$ is

$$\epsilon_{mv,\varphi} = -1 + \delta_{\text{slack}}(w,t)\frac{w}{w+\varphi(t)\eta(w)} \tag{35}$$

In constrained countries, the intermediary is the only marginal investor. Its demand fully determines required returns, so the elasticity is minus one. In unconstrained countries, both investors are marginal. A fall in foreign demand is partly accommodated by trade, as local investors buy from the intermediary. This keeps returns from rising one-to-one with the global factor.

The less elastic market is illustrated on panel (a) of Figure 5. The red line depicts demand from the intermediary, local demand is in blue, and their horizontal sum in black is the overall demand. This line has a kink where the mean-variance ratio becomes so high that local savers hit their portfolio constraint. The supply line is vertical, and the supply-demand intersection is in the region where domestic demand is fully inelastic.



(a) Constrained savers

(b) Unconstrained savers

Figure 5: Supply and demand for country i's trees as a function of the mean-variance ratio μ_R/σ_R^2 for fixed w and p. Supply is vertical. Demand $\hat{h}$ from the intermediary in red, from the local savers h in blue. The total demand is a horizontal sum of the two. Local savers are constrained on panel (a) and unconstrained on panel (b). Dashed lines show shifted curves after foreign demand is partially withdrawn. Equilibria move from the black to the white dots.

If a negative shock to $\varphi(t)$ leads to withdrawal of foreign demand, total demand rotates left. Holding constant w and $p(w,t)$, the new supply-demand intersection features a higher mean-variance ratio. The blue line is vertical at this point, so local investors cannot increase their holdings. Since there is nobody to sell to, foreign investors have to accept the old quantities, and excess returns adjust to make it happen.

The more elastic case is illustrated on panel (a) of Figure 5. The fall in foreign demand still leads to an increase in required excess returns, but this time the local savers are unconstrained and can compensate for capital flight. There is trade as they increase their holdings in response to wider spreads, and excess returns do not move as much.

Importantly, assuming that $\eta(0) > 0$, very small countries ($w \to 0$) behave in the same way whether the constraint is slack or binds. Local investors cannot buy much of the risky assets from the intermediary if their wealth approaches zero. In the limit, quantities cannot adjust at all, and expected returns absorb the fall in $\varphi(t)$ in full.

General equilibrium. How far is Figure 5 from general equilibrium? First, higher promised returns come from future price growth, which requires asset prices $p(w,t)$ to fall in equilibrium. This revalues portfolios and impacts wealth w and $\hat{w}$. Consumption and interest rates respond to changes in wealth, which further revalues assets, including the safe asset issued in the special country that is not affected by $\varphi(t)$ in the first place. Moving interest rates also change the reference point for excess returns and hence affect the level of returns directly.

Capital flows offer another general equilibrium perspective. When local investors buy domestic assets from the intermediary, they run down their foreign holdings. Since all unconstrained countries do this simultaneously, a fall in $\varphi(t)$ leads to a wave of global retrenchment. This is reflected in the shrinking balance sheets of global intermediaries, who now raise less funding to finance their risky investments. This deleveraging affects short-term rates and launches a second round of price changes and wealth revaluation.

The main insights on heterogeneity from Figure 5 survive, though. Constrained markets show very limited trading activity, and adjustment happens through prices. Unconstrained markets see large trading volumes and, because of that, limited price movements. The main value added by closing the model is that it pins down absolute levels of local responses in addition to differences between rich and poor countries that are visible in partial equilibrium already.

To recover prices, define equilibrium drift and volatility functions $(\mu_w, \mu_p, \mu_R, \sigma_w, \sigma_p, \sigma_R)$ as

$$dw = \mu_w(w,t)dt + \sigma_w(w,t)dZ \tag{36}$$

$$dp = \mu_p(w,t)dt + \sigma_p(w,t)dZ \tag{37}$$

$$dR = \mu_R(w,t)dt + \sigma_R(w,t)dZ \tag{38}$$

Here dZ are increments of the country-specific Brownian motions. Using equation (6),

$$\mu_R(w,t) = \frac{\mu_p(w,t) + \nu(t)}{p(w,t)} - r(t) \tag{39}$$

$$\sigma_R(w,t) = \frac{\sigma_p(w,t) + \sigma}{p(w,t)} \tag{40}$$

Expected excess returns include the drift in prices $\mu_p(w,t)$ and expected dividends $\nu(t)$. The volatility of returns includes an exogenous part, which is the dividend volatility, and an endogenous part, which is the volatility of prices. Plugging this into market clearing and using Itô's lemma leads to a partial differential equation for $p(w,t)$. A related partial differential equation determines the evolution of the regular country wealth distribution $G(w,t)$ with the associated density $g(w,t)$.

Proposition 1. The prices $p(w,t)$ and density $g(w,t)$ solve the following system:

$$r(t)p(w,t) - \partial_t p(w,t) = y(w,t) + \mu_w(w,t)\partial_w p(w,t) + \frac{1}{2}\sigma_w(w,t)^2 \partial_{ww} p(w,t) \tag{41}$$

$$\partial_t g(w,t) = -\partial_w[\mu_w(w,t)g(w,t)] + \frac{1}{2}\partial_{ww}[\sigma_w(w,t)^2 p(w,t)] \tag{42}$$

subject to suitable boundary conditions. The risk-adjusted payoff function $y(w,t)$ is given by

$$y(w,t) = \nu(t) - \left(\frac{\sigma}{1-\epsilon(w,t)\theta(w,t)}\right)^2 \max\left\{\frac{1}{w+\varphi(t)\eta(w)}, \frac{1}{\varphi(t)\eta(w)}\left(1 - \frac{\overline{\theta}w}{p(w,t)}\right)\right\} \tag{43}$$

where $\epsilon(w,t) = w/p(w,t) \cdot \partial_w p(w,t)$ is the wealth elasticity of price.

The content of this proposition is that the price $p(w,t)$ solves a standard backward equation (41) akin to Kolmogorov backward equations that usually describe value functions. The discount rate is the interest rate, and the reward function $y(w,t)$ is the tree yield adjusted for risk. The distribution of wealth solves a standard forward equation (42), and these two equations constitute a coupled system. The feedback loop between them goes through the interest rate, which clears the global deposit market and therefore depends on the distribution of wealth, and the global factor $\varphi(t)$, which includes $\hat{w}(t)$, the special country's net worth that depends on the distribution of prices.

A slight complication in solving the coupled system is that the payoff function depends on $p(w,t)$ and its derivative in a non-linear way. On top of that, one needs to know $p(w,t)$ to compute the drift and volatility of wealth. This turns the problem posed by Proposition 1 into finding a fixed point. I describe my solution algorithm in Appendix F.

Steady state. Suppose all exogenous sequences $(\gamma(t), \nu(t), \hat{\nu}(t))_{t\geq 0}$ are constant. Then, there is an invariant wealth distribution $G(w)$ and invariant versions of price, return, and portfolio weight functions. Notation for these invariant versions omits the time argument.

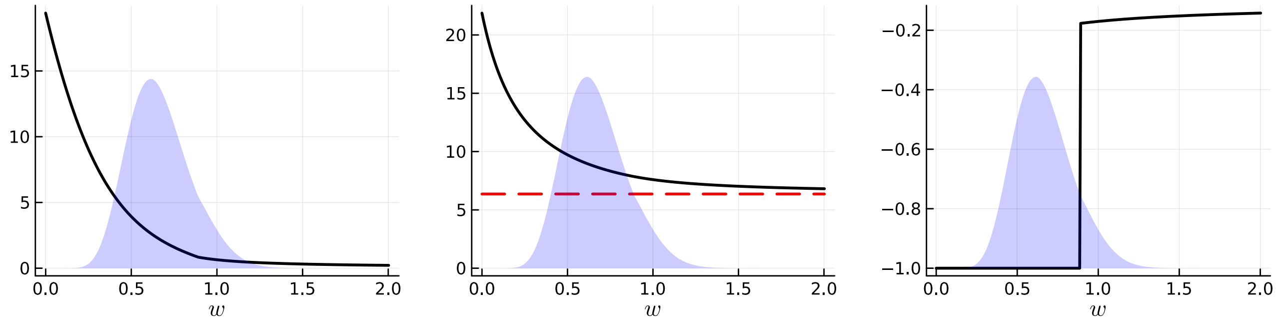
In Appendix D, I describe a benchmark with unlimited risk-taking capacity, $\gamma = \infty$. In this case, the intermediary fully insures regular countries by holding all existing trees. Regular countries are not exposed to idiosyncratic shocks, so the wealth distribution $G(w)$ is degenerate. The distribution of wealth between them and the special country is determined by time preferences and migration rates. The history of idiosyncratic shocks starts to matter when γ becomes finite. The intermediary's demand for risk gains elasticity, the interest rate falls, and regular countries hold non-trivial shares of their own trees. Wealth distribution between them becomes non-degenerate.

Rich and poor countries are qualitatively different. There is a threshold $\tilde{w}$ such that the local portfolio constraint $\theta(w) \leq \overline{\theta}$ binds in regular countries with $w < \tilde{w}$. In poor countries, prices are depressed by low wealth in the market. This renders expected returns high, so domestic investors want to hold a high share of their portfolio in the tree and hit the limit $\overline{\theta}$.

As a country grows rich, it outgrows the constraint. Tree price rises together with wealth, lowering expected returns and making the tree less attractive. In addition, the price is bounded from above by the price of the riskless asset, so in rich countries, even the whole tree would take up a small share of the portfolio. In the limit of infinite wealth, idiosyncratic risk contributes very little to wealth dynamics, and wealth simply drifts down because of consumption ($\rho > r$).

Figure 6 shows expected excess returns, dividend-price ratio, and the partial equilibrium elasticity of the mean-over-variance of returns to the global factor φ as functions of w. Panels (a) and (b) show that expected excess returns and the dividend-price ratio decline with wealth and approach safe asset benchmarks of 0 and r, respectively, when $w \to \infty$. In the richest countries, investors are so large relative to supply that the effect of idiosyncratic shocks on wealth is negligible.

Steady-state capital flows. The steady state features non-trivial capital flows. They happen as a result of idiosyncratic shocks that move countries across the wealth distribution. Local investors



(a) Expected excess returns (b) Dividend to price ratio and r (c) Elasticity of μ_R/σ_R^2 to φ

Figure 6: Panel (a): expected excess returns in regular countries. Panel (b): dividend-to-price ratio (solid), and the steady-state interest rate (dashed). On both panels, all in percentage points. Panel (c): partial equilibrium elasticity of mean-over-variance of returns to the common factor φ from equation (35). The invariant distribution is plotted in the background.

change their portfolios as they become richer or poorer. The obverse of this is, of course, the trading done by the intermediary. Countries continually churn in its portfolio, even though total investment is constant in the steady state due to the law of large numbers.

Steady-state consumption of the intermediary can be written as

$$\hat{c} = \underbrace{\hat{\nu} + \nu \cdot \int \hat{h}(w)dG(w)}_{\text{dividends}} - \underbrace{r \cdot \int l(w)dG(w)}_{\text{interest payments}} + \underbrace{\int \mu_p(w)\hat{h}(w)dG(w)}_{\text{trading profit}} \tag{44}$$

The intermediary consumes the dividends it gets, pays out interest on deposits, and realizes profits from trading. These profits come from the drift in prices. Average drift is zero in the steady state, $\int \mu_p(w)dG(w) = 0$. The intermediary, however, takes positions $\hat{h}(w)$ that are skewed towards growing, high-yielding countries: $\int \mu_p(w)\hat{h}(w)dG(w) > 0$. As idiosyncratic shocks reshuffle the wealth distribution, these countries become richer, their assets appreciate, and the intermediary sells them to buy cheaper assets issued by countries that arrive to the left tail.

Calibration. Table 3 presents the steady-state targets and model fit. I choose the model parameters to approximate the following moments. First, I designate the US as the special country and target US wealth share of 32.3% from Credit Suisse (2022) and US share of GDP of 25.4% from the World Bank data. Second, Gourinchas and Rey (2022) estimate that the US gets an annual return of 2-3 percent on its external position. Third, Adler and Garcia-Macia (2018) estimate that emerging markets earn a 2.3*pp* lower real return on NFA compared to advanced economies. Finally, I make the model reproduce some moments of the empirical distribution of external assets relative to external liabilities. See Appendix E for details.

I further restrict output to be the same on average in every country, $\nu = \hat{\nu}$, and set $\hat{\lambda}$ to a value that induces zero net migration flows in the steady state. The steady state is determined by

Table 3: Steady-state calibration.

	model	target	source
aggregates:			
US wealth share	31.5%	32.3%	Credit Suisse (2022)
US output share	23.7%	22.8%	World Bank
average risk premium	2.62pp	2.5pp	Gourinchas and Rey (2022)
emerging market premium	2.22pp	2.3pp	Adler and Garcia-Macia (2018)
external assets to external liabilities:			
mean	1.071	1.075	IFS (IMF)
standard deviation	0.686	0.685	IFS (IMF)
q25	0.614	0.621	IFS (IMF)
q50	0.849	0.877	IFS (IMF)
q75	1.285	1.249	IFS (IMF)

$(\rho, \hat{\rho}, \lambda, \hat{\lambda}, \nu, \hat{\nu}, \overline{\theta}, \sigma, \gamma, \hat{q}, \zeta)$, where ζ sets the intermediary's weights:

$$\eta(w) = \zeta + (1 - \zeta)w \tag{45}$$

This affine specification has two properties. First, at $w \to 0$ it fans out to a constant, so the total demand for assets is bounded away from zero even in countries with vanishing wealth. These countries are completely taken over by the intermediary. The prices of their trees are always positive. Second, at $w \to \infty$ the intermediary's demand is proportional to that of the local investors, so the trees are not completely taken over by domestic agents even as they become infinitely rich. This prevents the rich countries from having to hold the entire supply of their risky assets, in which case fluctuations in foreign demand would mechanically have very little effect.

Finally, the net worth of one of the agents in the model can be normalized to one, as is usual in models with intermediaries. This would turn wealth shares rather than levels into state variables. Instead, I normalize the level of dividends to maximize computational convenience.

The portfolio constraint parameter $\overline{\theta} = 0.71$ means that not only are regular countries unable to borrow, but they also must hold some riskless debt. This can be associated with regulatory mandates on investment vehicles. The share of unconstrained countries in the steady state is about 12%, which is close to the number of advanced economies in the world.

Shock parameters. I estimate parameters of shocks to $(\gamma(t), \nu(t), \hat{\nu}(t))$ that rationalize aggregate fluctuations in capital flows and asset prices. I then use these estimates in Section 5 and Section 6, which describe the aggregate and distributional impact of these shocks in detail.

I eliminate heterogeneity in output between countries by setting $\hat{\nu}(t) = \nu(t)$. Section 6 explains the practicality of this assumption, showing that shocks to $\nu(t)$ and $\hat{\nu}(t)$ have very similar

Table 4: Model parameters.

parameter	value	meaning
regular countries		
ρ	0.0793	discount rate
λ	0.0177	emigration rate
ν	0.0600	output rate
σ	0.0647	output volatility
$\overline{\theta}$	0.7059	upper limit on risky asset share
special country		
$\hat{\rho}$	0.0844	discount rate
$\hat{\lambda}$	0.0384	emigration rate
$\hat{\nu}$	0.0600	output rate
$\hat{q}$	0.3096	asset stock
ζ	0.3824	country weight intercept
γ	0.6698	risk-taking capacity

consequences due to financial contagion. I then postulate the following processes for $\gamma(t)$ and $\nu(t)$:

$$d\gamma(t) = (\zeta_\gamma - \gamma(t))\mu_\gamma dt + \sigma_\gamma \cdot dW \tag{46}$$

$$d\nu(t) = (\zeta_\nu - \nu(t))\mu_\nu dt + \sigma_\nu \cdot dW \tag{47}$$

Here ζ_γ and ζ_ν are the long-run values of $\gamma(t)$ and $\nu(t)$ equal to the parameters in my steady-state calibration. Persistence is governed by μ_γ and μ_ν, and vectors σ_γ and σ_ν determine the standard size of the shocks coming from standard Brownian increments $dW = (dW_1, dW_2)$.

To estimate the model, I linearize it around $\sigma_\gamma = \sigma_\nu = (0, 0)$ and work with the first-order approximation. Appendix H describes the linearization procedure. I compute sequence-space Jacobians. They combine several forward-looking and backward-looking components that solve systems of linear partial differential equations, which I tackle by discretizing the grids and using insights from Auclert et al. (2021) and Bhandari et al. (2023). I then construct the likelihood function using these Jacobians and estimate parameters through likelihood maximization.

I use two aggregate series $(X_M(t), X_P(t))$ for estimation. The first one is total outward investment normalized by total external assets:

$$X_M(t) = \frac{\sum_{i \in \mathcal{I}(t)} a_{i,t}}{\sum_{i \in \mathcal{I}(t)} A_{i,t-1}} \tag{48}$$

Here $\mathcal{I}(t)$ is the set of countries i in the sample at time t. The second one, $X_P(t)$, is a detrended version of the risky asset price factor from Habib and Venditti (2019). The model analogs of these series are the normalized first-order deviations of deposits $M(t) = \int l(w,t)dG(w,t)$ and prices $P(t) = \int p(w,t)dG(w,t)$ from the steady state normalized by the steady-state values.

I estimate persistence parameters (μ_γ, μ_ν) and loadings $(\sigma_{\gamma 1}, \sigma_{\gamma 2}, \sigma_{\nu 1}, \sigma_{\nu 2})$. I normalize $\sigma_{\nu 1} = 0$, so that innovations to $\nu(t)$ are proportional to dW_2, and σ_{γ_2} regulates the correlation between $d\gamma(t)$ and $d\nu(t)$. This leaves five parameters $(\mu_\gamma, \mu_\nu, \sigma_{\gamma 1}, \sigma_{\gamma 2}, \sigma_{\nu 2})$, to which I add standard deviations of normal error terms for $X_M(t)$ and $X_P(t)$. Table 12 presents the estimates.

Table 5: Estimation results.

μ_γ	μ_ν	$\sigma_{\gamma 1}$	$\sigma_{\gamma 2}$	$\sigma_{\nu 2}$
1.987	3.148	1.762	-0.805	0.032
(0.404)	(0.311)	(0.306)	(0.172)	(0.002)

The negative sign of $\sigma_{\gamma 2}$ means that innovations to $d\gamma(t)$ and $d\nu(t)$ are negatively correlated. A pulse in dW_1 will have a positive effect on risk-taking capacity and no effect on output. A pulse in dW_2 will have a negative effect on risk-taking capacity and a positive effect on output.

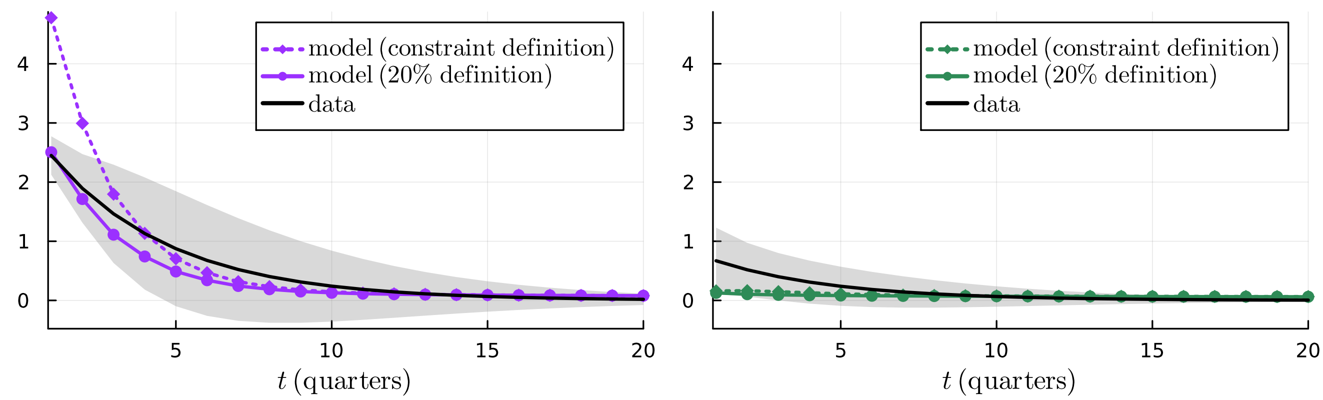


Figure 7: Impulse responses in advanced economies (left) and emerging markets (right). Advanced economies in the model classified in two ways: (i) as those with the constraint $\theta(w) \leq \overline{\theta}$ slack in the steady state and (ii) as the richest 25%. Shaded areas represent 95% confidence intervals.

Untargeted responses. The estimation procedure described above only takes into account aggregate capital flows and asset prices. However, the resulting impulse responses are able to capture the difference between advanced economies and emerging markets in the data. I demonstrate this by estimating the following specification:

$$\overline{a}_t^i = \alpha_i + \beta_i f_t + \epsilon_{it} \tag{49}$$

$$f_t = \alpha + \beta f_{t-1} + \epsilon_t \tag{50}$$

Here $\overline{a}_{it}$ is net asset acquisition in quarter t normalized by external assets at $t-1$ averaged over countries in one of the two groups $i \in \{AE, EM\}$, and f_t is the principal component of net asset acquisition. These variables are defined in Section 2. I estimate β_{AE}, β_{EM}, and β and compute responses of $\overline{a}_s^{AE}$ and $\overline{a}_s^{EM}$ for $s \geq t$ to a standard shock to ϵ_t.

The analog in the model is the expected impulse response of $l(w,t)$ normalized to the steady-state value and averaged over the respective groups, where the expectation is taken over the future path of idiosyncratic shocks. See Appendix G for details of the computation of cross-sectional expectations of future quantities in the presence of idiosyncratic shocks. For this exercise, I drop the first-order approximation and compute full non-linear impulse responses. The shock in the model is a standard pulse to dW_1 that generates an unanticipated exponentially decaying shock to $\gamma(t)$ considered in Section 5.

Figure 7 compares results for the model and the data. I use two different definitions of advanced economies in the model. The first one counts a country as an advanced economy if its portfolio constraint is slack in the steady state. The second one counts the top 25% of the wealth distribution as advanced economies, as opposed to 12% in the benchmark definition. It turns out to be closer to the data, reflecting the fact that advanced economies are overrepresented in my sample.

Figure 7 captures the main qualitative feature of the model: outward flows in advanced economies respond to aggregates more than in emerging markets. A strict definition of advanced economies even makes the model exaggerate the difference somewhat, while a more conservative definition puts the responses roughly in the empirical ballpark. In the next two sections, I zoom in on the shocks to γ, ν, and $\hat{\nu}$ separately.

5 Shock to Risk-taking Capacity

In this section, I consider a one-time unanticipated transitory negative shock to γ. It has the form

$$\gamma(t) = \gamma - \epsilon_\gamma e^{-\mu_\gamma t} \mathbb{1}\{t \geq 0\} \tag{51}$$

I set the persistence parameter $\mu_\gamma = 1.99$ in accordance with my estimation results from Section 4. I choose the size of the shock ϵ_γ at 28.4% of the steady-state value of γ to replicate a standard pulse in dW_1 that translates to $\gamma(0) = \sigma_{\gamma 1}\sqrt{dt}$, where $dt = 0.02$ in my weekly discretization.

The shock to $\gamma(t)$ has an immediate effect on the global factor $\varphi(t) = \gamma(t)\hat{w}(t)$. The intermediary loses its appetite for risky assets and withdraws demand from all markets in regular countries. Expected excess returns rise. Since dividends are constant, these expected returns come from a drift in prices $p(w,t)$. They fall on impact and are expected to recover back to normal gradually.

As investors lose money on risky assets, they feel poorer and want to consume less. But there is no shock to output, so the interest rate $r(t)$ has to fall to convince them to consume the old amount on aggregate. This revalues the safe asset: the price of the special country's tree $\hat{p}(t)$ increases on impact. Other assets are revalued, too, so the fall in $r(t)$ limits the overall fall in prices. The fall

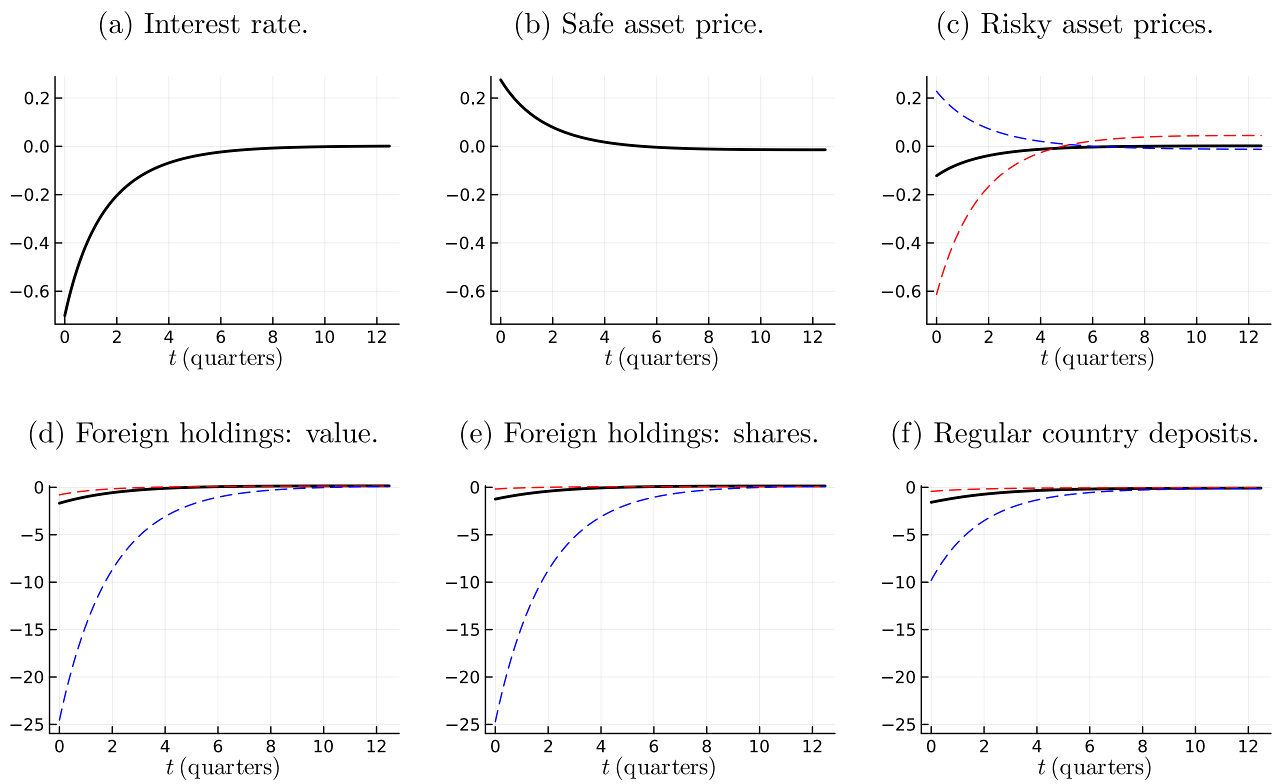


Figure 8: Impulse responses of prices and holdings. Panel (a): global interest rate $r(t)$, percentage points. Panel (b): safe asset price $\hat{p}(t)$, percent of the steady-state value. Panel (c), solid line: average risky asset price $p(w,t)$, percent of the steady-state average. Dashed lines: responses of the prices at the 5-th percentile of the wealth distribution (in red) and at the 95-th percentile (in blue). Panel (d): market value of risky assets in risky countries held by the intermediary $p(w,t)\hat{h}(w,t)$. Panel (e): shares of risky assets in regular countries held by the intermediary $\hat{h}(w,t)$. Panel (f): deposits of regular countries $l(w,t) = (1-\theta(w,t))w$.

in the intermediary's risk-taking capacity thus generates a fall in the global interest rate and a rise in the price of safe assets.

Figure 8 shows the responses of $r(t)$, $\hat{p}(t)$, and the average risky asset price $p(w,t)$. In addition to the average, panel (c) has the responses of prices at the 5-th and 95−th percentiles of the wealth distribution. They are quite different and even have the opposite signs. At the 95-th percentile, the tree gains value on impact, so its response is closer to that of the safe asset. Bottom panels complement this with responses of flows, showing the source of this heterogeneity.

Panel (d) shows the fall in the intermediary's holdings of risky assets in regular countries. There is substantial heterogeneity in the background: the intermediary disproportionately sells trees in rich countries. At the 95-th percentile of the wealth distribution, it decreases its position by more than 20%, and there is a much smaller decrease at the 5-th percentile. Panel (e) shows the same decrease in holdings, this time measured in shares instead of market value. The picture is largely the same, which means that holdings change on impact because of trades and not because of revaluation. Rich countries see much larger outflows of foreign investment than poor ones.

Panel (f) shows the change in deposits, the obverse of the intermediary's risky asset holdings. Deposits fall on impact, reflecting retrenchment by local investors. Seeing wider spreads at home, they sell their foreign assets and buy domestic ones. This is concentrated in rich countries, where local investors have accumulated large savings and readily replace foreign demand. Local demand in these countries is more elastic, keeping spreads low while capital flows are large on impact: both inflows and outflows fall. There is a sudden stop and retrenchment at the same time.

In poor countries, quantities do not change much, and prices have to adjust instead. Domestic savers are up against their portfolio constraint, so they cannot buy trees from the intermediary. To convince the intermediary to keep the old position, expected returns rise, which is achieved through a fall in prices and their future increase back to normal. Capital flows are minimal. They happen to a very limited extent and only because savers take losses on trees, so trees at $t = 0$ occupy less than $\overline{\theta}$ of the total portfolio, and they can buy a bit more.

(a) Change in prices $p(w, t)$ on impact.

(b) Change in holdings $h(w, t)$ on impact.

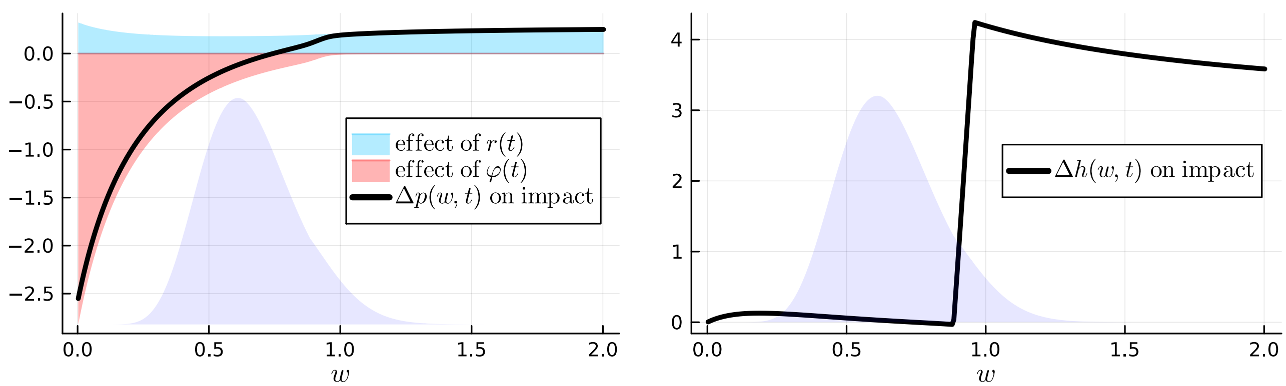


Figure 9: Cross-section of the changes in risky asset prices and domestic asset holdings on impact. Panel (a): percentage change in $p(w, t)$ relative to the steady state decomposed into the effect of interest rate $r(t)$ and foreign demand $\varphi(t)$. Panel (b): changes in tree holdings by domestic investors $h(w, t)$ as a percentage of the total supply. Steady-state distribution in the background.

Figure 9 shows the cross-section of price and flow responses on impact. Panel (a) decomposes the response of prices into parts that come from the falling interest rate $r(t)$ and from the falling foreign demand $\varphi(t)$. The effect of $\varphi(t)$ is only visible in the poorer part of the distribution. This is because, as panel (b) shows, domestic investors in rich countries retrench and compensate for the intermediary leaving. In poor countries, there are no flows, and prices absorb the shock.

Risky assets issued by rich countries thus endogenously behave as a good substitute for safe assets since they react positively to falling risk-taking capacity. Since retrenchment fully neutralizes the fall in foreign demand, these assets only react to $r(t)$. In the limit of $w \to \infty$, excess returns stay at zero, while absolute returns fall, leading to appreciation.

Figure 10: Gains and losses of the intermediary and local savers on impact. Measured in percent of global GDP, weighted by density at $t = +0$

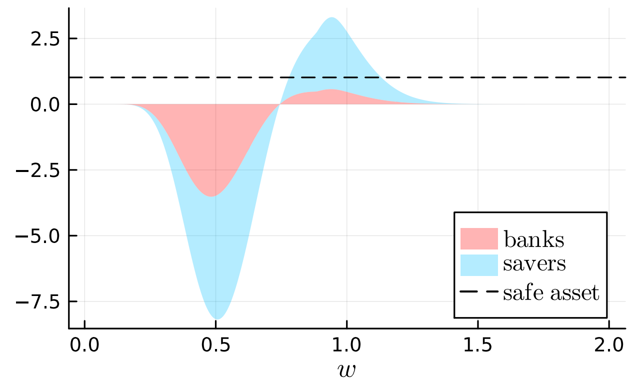


Table 6: Distribution of gains and losses on impact in percent of global GDP. Low w countries are those constrained in steady state, $\theta(w) = \overline{\theta}$

intermediary (safe asset)	0.81%
intermediary (risky assets, low w)	−0.50%
intermediary (risky assets, high w)	0.04%
savers in low w countries	−0.61%
savers in high w countries	0.18%

The safe asset, which is the special country's tree, appreciates as well. In fact, the increase in its price $\hat{p}(t)$ is enough to increase the special country's wealth $\hat{w}(t)$ on impact. This is consistent with the results in Dahlquist et al. (2022) and Jiang et al. (2022). The special country loses money on its foreign holdings but ends up increasing its wealth share.

What would happen without the constraint $\theta(w,t) \leq \overline{\theta}$? More investors would buy domestic assets from the intermediary on impact. Of course, at the lower end of the distribution they cannot buy much, so prices would still have to adjust instead of quantities around $w = 0$. But aggregate effects would be quantitatively different: the fall in the interest rate required to prop up the prices and induce consumption would be less dramatic.

Importantly, without the constraint, some countries would borrow from the intermediary to buy assets, so there would be a rise in lending to poorer countries during global downturns. There is not much empirical support for this. In my calibration, $\overline{\theta} = 0.71$ means that regular countries cannot borrow at all, and there is no spike in lending during busts.

Distributional impact. What do risky assets issued by different countries contribute to gains and losses of the global intermediary's portfolio? And how are gains and losses distributed among investors in regular countries? Figure 10 shows them in cross-section, measured in percent of global GDP and weighted by the steady-state density so that they can be integrated directly to compute the totals. Table 6 aggregates gains and losses into those made by the intermediary on the safe assets and risky assets from countries that are initially constrained and unconstrained. Constrained countries have low wealth, so they map into emerging markets. Table 6 also aggregates the impact of the shock on the wealth of local investors.

Two things stand out. First, total wealth revaluation is close to zero. Because the elasticity of intertemporal substitution is equal to one, it always holds that

$$\int \rho w dG(w,t) + \hat{\rho}\hat{w}(t) = \nu(t) + \hat{q}\hat{\nu}(t) \tag{52}$$

The shock to risk-taking capacity $\gamma(t)$ does not affect output. This means that the interest rate in general equilibrium will adjust to revalue assets enough to keep the weighted sum of global wealth on the left-hand side of equation (52) constant. An implication is that the intermediary's wealth and that of the rest of the world cannot go down at the same time, and there has to be redistribution with an approximately zero sum. If $\rho = \hat{\rho}$, it is exactly zero-sum.

Second, there is substantial heterogeneity. Losses are concentrated in poorer countries. In contrast, risky assets issued in rich countries endogenously behave as almost safe since domestic investors step in and support demand when the intermediary wants to leave. At the same time, the intermediary shares in the losses in poor countries more than in the gains that rich countries make. This is because the intermediary's portfolio is skewed towards high-yielding emerging markets.

Losses on risky assets in regular countries translate into a fall in the special country's net foreign asset position. Thus, this country makes a wealth transfer to the rest of the world by absorbing part of their losses, consistent with observations in Gourinchas and Rey (2022).

On the other hand, the intermediary's exposure to the safe asset and risky assets from rich countries limits its losses, keeps its net worth afloat, and makes its wealth share increase on impact. Rich countries contribute to stabilizing the intermediary's net worth $\hat{w}(t)$ and, through that, help arrest the fall in $\varphi(t) = \gamma(t)\hat{w}(t)$. The upshot is that rich countries partly insure the global intermediary, which in turn partly insures the poor countries. This aspect of the global risk-sharing arrangement is only visible with multiple countries in the model.

Adjustment in the special country. How does the special country adjust to the shock? Its net foreign assets position (NFA) falls on impact because of the losses it makes on its risky portfolio. Panel (a) on Figure 11 shows the path of NFA over the special country's GDP over time.

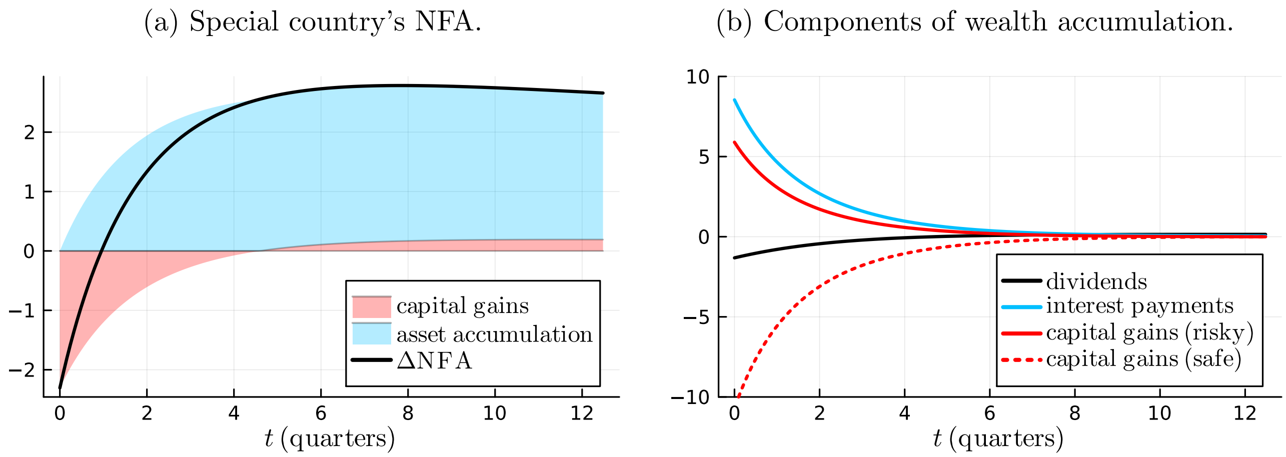


Figure 11: Responses of the special country's NFA and components of net income, percent of GDP. Panel (a): change in NFA decomposed into price changes and net assets accumulation. Panel (b): changes in dividends from regular countries, interest payments to them, and asset prices.

The evolution after $t = 0$ is due to changes in asset prices and accumulation of assets net of incurrence of liabilities. The capital gains component $k(t)$ is given by a $k(0) < 0$ and

$$\dot{k}(t) = \int \mu_p(w,t)\hat{h}(w,t)dG(w,t) - \int \mu_p(w)\hat{h}(w)dG(w) \tag{53}$$

The subtracted term corresponds to the steady state (hence no time argument). This component starts off negative at $t = 0$, reflecting the losses made on impact. NFA then reverses, and valuation changes contribute to the reversal.

Asset accumulation accounts for purchases of shares in risky assets that do not result in additional deposits made by regular countries. Panel (b) sheds light on the sources of these purchases. It plots the changes in four components of the intermediary's wealth accumulation: dividends from assets in regular countries, interest payments (this component contributes negatively to net income), capital gains on risky assets, and capital gains on the safe asset.

Interest payments to regular countries decline by more than dividends flowing out of those countries. The intermediary uses this difference to buy back the shares that local investors in rich countries purchased on impact while retrenching. Changes in wealth, and hence consumption, are much smaller in magnitude, so almost all of this extra income goes to financing asset purchases. Dividends from the safe asset remain constant and are not shown in the picture. Capital gains on risky assets are positive, and those on the safe asset are negative as prices revert back to normal.

(a) Domestic asset holdings. (b) Net income components. (c) Required excess returns.

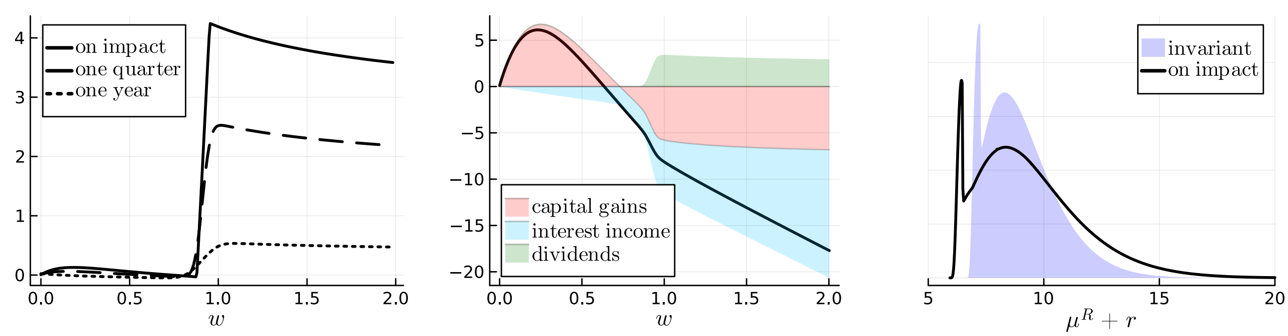


Figure 12: Panel (a): holdings of domestic assets by local investors in percent of total supply (difference relative to the steady state). Solid line for the change on impact, dashed for the expected change one quarter out, dotted for the expected change one year out. Panel (b): changes in components of expected wealth accumulation over the first quarter. Changes in prices in red, changes in interest income in blue, changes in dividends in green. Horizontal axis corresponds to wealth at $t = 0$. Panel (c): distributions of required returns on risky assets (in *pp*).

Adjustment in regular countries. How do regular countries adjust? Panel (a) on Figure 12 shows the expected changes in holdings of domestic assets relative to the steady state. Expectations here are taken with respect to idiosyncratic shocks, and the horizontal axis corresponds to wealth at $t = 0$. See details in Appendix G.

Investors in rich countries retrench on impact and buy about 3% of the total supply of domestic assets. They then gradually sell them back in transition. In expectation, after the first quarter, they liquidate about a third of initial purchases. After one year, they sell almost everything they bought. These transactions, of course, are the other side of the intermediary's asset purchases.

Panel (b) decomposes expected wealth accumulation in regular countries over the first quarter. Changes in asset prices are positive for poor countries and negative for rich ones as the world reverts back to normal. Interest income declines for everyone, but disproportionately so for rich investors. Dividend income increases for rich investors since they retrench and have more risky assets in portfolios right after the shock. Consumption over the first quarter does not change much relative to the steady state (not shown in the picture). To finance it, investors from rich countries have to sell assets since their interest income declines more than their dividend income rises.

The world becomes more unequal in terms of wealth and in terms of spreads after the shock. Realized returns right after the shock are negative in poorer countries and close to zero in richer countries. This is consistent with the results in Chari et al. (2020), who show that the left tail of the return distribution moves more than the right tail during risk-off episodes. The distribution of required returns in the model also becomes more dispersed and skewed: panel (c) on Figure 12 shows that the right tail shifts out, while the low-return bunch, the rich countries, shifts left.

6 Shock to Output and Contagion

Shocks to risk-taking capacity cause an increase in risk premia and a fall in risky asset prices. However, these effects are concentrated in poor countries, while in rich countries, large capital flows stabilize asset prices. Wealth is redistributed rather than destroyed on impact. In particular, the global intermediary's net worth does not take a hit and even increases.

In this section, I describe another type of shock: a persistent decline in output. I hit the economy separately with two unanticipated shocks, one to output in regular countries and one to the special country's output:

$$\nu(t) = \nu - \epsilon_\nu e^{-\mu_\nu t}\mathbb{1}\{t \geq 0\} \tag{54}$$

$$\hat{\nu}(t) = \nu - \hat{\epsilon}_\nu e^{-\mu_\nu t}\mathbb{1}\{t \geq 0\} \tag{55}$$

I set the persistence parameter $\mu_\nu = 3.15$ in accordance with my estimation results and set the size of the shock ϵ_ν and $\hat{\epsilon}_\nu$ to 6.2% of the steady-state value ν, replicating a standard pulse in dW_2.

The aggregate effects of these shocks are qualitatively similar. Output falls on impact, so the interest rate has to rise to make agents consume less. Asset prices fall because of lower future dividends, and the rising interest rate depresses them even further. Magnitudes differ by about three times: the output share of the special country is approximately one-quarter of the total.

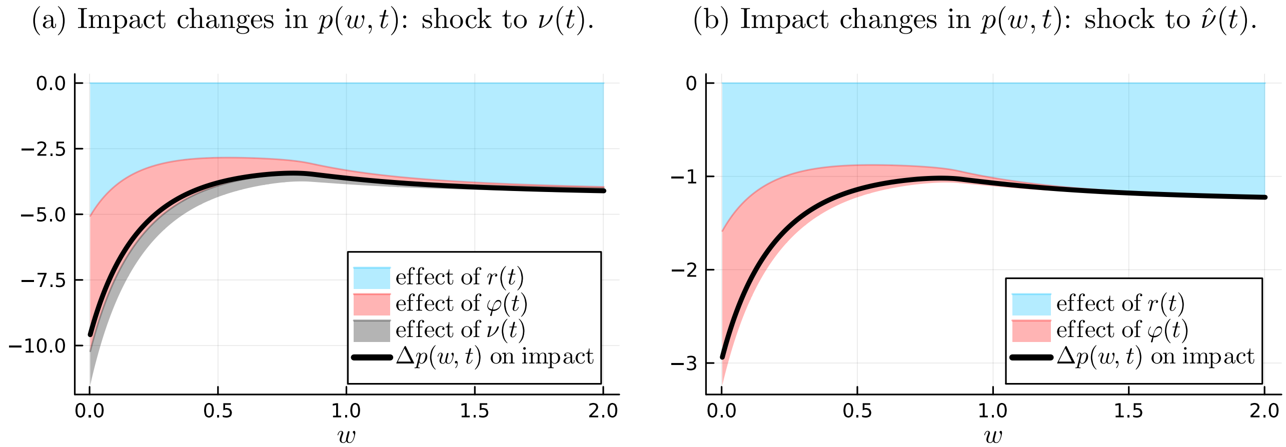


Figure 13: Cross-section of the changes in risky asset prices on impact: percentage change in $p(w,t)$ relative to the steady state decomposed into the effect of interest rate $r(t)$, foreign demand $\varphi(t)$, and dividends $\nu(t)$. Shock to $\nu(t)$ on panel (a) and to $\hat{\nu}(t)$ on panel (b).

The falling asset prices revalue everyone's wealth. In particular, the intermediary's net worth $\hat{w}(t)$ takes a hit, which affects the global factor $\varphi(t) = \gamma(t)\hat{w}(t)$. From the perspective of the equilibrium condition on excess returns in equation (34), it looks like a fall in the intermediary's risk-taking capacity. Importantly, this happens in both cases. It is natural to expect losses to be distributed between both regular country investors and the special country in the case of $\nu(t)$: both local investors and the global intermediary hold risky assets. The shock to $\hat{\nu}(t)$, on the other hand, hits dividends on the safe asset, which is only held by the intermediary. The fact that it spreads to regular countries through $\varphi(t)$ and $r(t)$ reflects strong contagion forces.

Figure 13 shows the cross-section of impact changes in asset prices. Unlike with a shock to risk-taking capacity $\gamma(t)$, prices fall everywhere because the interest rate rises. Panel (a) shows a rough decomposition of the impact effect into parts that come from a jump in $r(t)$, a fall in $\varphi(t)$, and a fall in $\nu(t)$. The interest rates drive asset prices in the right tail of the wealth distribution. In poor countries, prices respond to foreign demand $\varphi(t)$ and discounted future cash flows more. The decomposition is not exactly additive since the price function is highly non-linear.

How are losses distributed between countries? Figure 14 and Figure 15 show density-weighted losses made on assets as a percentage of global GDP. The distributions are remarkably similar. Losses on risky assets are shared between regular countries and the intermediary in roughly the same proportions after both shocks, and the fall in the safe asset price $\hat{p}(t)$ is only marginally larger (relative to other assets) when its dividends $\hat{\nu}(t)$ are hit.

This shows how important the global intermediary is for risk-sharing. Even though it is the only owner of the safe asset, the fall in the safe asset's dividends has largely the same consequences for the global wealth distribution as a shock to $\nu(t)$, adjusting for size. The intermediary's exposure to risky assets generates contagion. This force is the reverse side of insurance that it provides to other countries by absorbing a part of their losses.

Figure 14: Shock to $\nu(t)$: gains and losses on of the intermediary and local savers on impact (percent of global GDP, weighted by density)

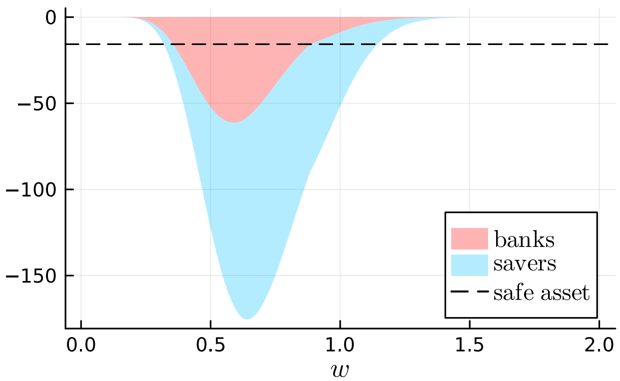


Figure 15: Shock to $\hat{\nu}(t)$: gains and losses on of the intermediary and local savers on impact (percent of global GDP, weighted by density)

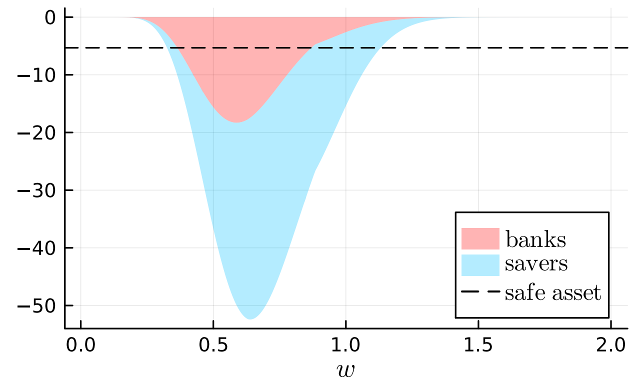


Table 7: Shock to $\nu(t)$: gains on impact (percent of global GDP). Low w countries are those constrained in steady state, $\theta(w) = \overline{\theta}$

intermediary (safe asset)	-15.71%
intermediary (risky assets, low w)	-12.76%
intermediary (risky assets, high w)	-0.90%
savers in low w countries	-23.71%
savers in high w countries	-4.37%

Table 8: Shock to $\hat{\nu}(t)$: gains on impact (percent of global GDP). Low w countries are those constrained in steady state, $\theta(w) = \overline{\theta}$

intermediary (safe asset)	-5.32%
intermediary (risky assets, low w)	-3.82%
intermediary (risky assets, high w)	-0.27%
savers in low w countries	-7.09%
savers in high w countries	-1.30%

Empirically, US shocks are an important driver of global dynamics. Boehm and Kroner (2023) show that news about the US economy has strong effects on risky asset prices globally. Miranda-Agrippino and Rey (2020) provide evidence for the global impact of contractionary monetary policy in the US. My model does not have a nominal dimension and monetary policy, but the real projection of a richer nominal model could generate spillovers from the special country to the rest of the world through the same mechanism.

Another contrast between these shocks against the shock to $\gamma(t)$ is capital flows on impact. Shocks to $\nu(t)$ and $\hat{\nu}(t)$ generate minimal flows since they do not raise disagreement on the risk properties of assets. In other words, wealth-weighted coefficients of risk aversion after the shock do not differ much between the intermediary and local savers. The wealth-weighted risk-aversion of the intermediary is the global factor $\varphi(t) = \gamma(t)\hat{w}(t)$, and that of the local investor is simply w. They fall by similar amounts, since $\gamma(t)$ is constant, and losses are shared roughly equally. With a shock to $\gamma(t)$, this is not the case: the fall in $\gamma(t)$ dominates movements in wealth, so the shock opens a large gap between wealth-weighted risk aversion coefficients, generating trades.

Trades that do happen are only caused by slightly asymmetric wealth revaluation and different effective risk aversion: relative risk aversion of local savers is one, and that of the intermediary is $\gamma(t)$. The fact that there are no capital flows on impact means that the portfolio constraint $\theta(w,t) \leq \overline{\theta}$ does not play a major role in generating heterogeneous responses. Heterogeneity in price changes is mainly driven by subjective discount rates.

Table 9: Summarized qualitative facts about negative shocks to $\gamma(t)$ and $(\nu(t), \hat{\nu}(t))$

	fall in $\gamma(t)$	fall in $\nu(t)$ or $\hat{\nu}(t)$
interest rate	-	+
safe asset	+	-
risky assets, rich countries	+	-
risky assets, poor countries	-	-
retrenchment flows, rich countries	+	0
retrenchment flows, poor countries	0	0

Table 9 collects the differences between the shocks to $\gamma(t)$ and $(\nu(t), \hat{\nu}(t))$. The shock to the global intermediary's risk-taking capacity is essential to generate large capital flows on impact, and these flows only happen in deep markets with rich and unconstrained domestic investors. The shock to output is essential to generate a fall in asset prices that is not confined to poor countries with a shallow investor base. The safe asset appreciates when the intermediary loses appetite for risk and depreciates when the interest rate rises to accommodate a fall in production.

7 Conclusion

I develop a model of the world economy with two tiers of heterogeneity between countries. The special country occupies the place of a global intermediary and issues safe assets. Regular countries issue risky assets and endogenously differ in wealth. The differences in wealth lead to different responses to global shocks that drive risk premia and capital flows.

In particular, when the intermediary's risk-taking capacity decreases and it sells risky assets around the world, investors in rich countries retrench and support the prices of domestic assets. Poor countries do not have enough wealth and hit the constraint that prevents them from issuing safe debt. In their markets, prices adjust instead of quantities and drop sharply. The falling global interest rate revalues assets, appreciating those issued by rich countries, which effectively makes them good substitutes for safe assets. The distributional impact is regressive.

I also show that shocks to the special country's output, to which regular countries are not exposed directly, lead to a global fall in risky asset prices. This is due to contagion through the global intermediary's net worth.

The model is an exchange economy without differentiated goods, nominal rigidities, investment, policymakers, or within-country heterogeneity. Still, it is rich enough to study co-movement and heterogeneity in risk premia and gross flows, while parsimonious enough to summarize foreign demand for assets by a single factor and express excess returns in closed form. It can be a useful first step in understanding international heterogeneity in exposure to the global financial cycle.

Adding differentiated goods and nominal rigidities can help incorporate monetary policy. Bursts of inflation and conventional and unconventional monetary policy in the special country have spillovers to the rest of the world. Advanced economies and emerging markets experience these in different ways, as shown by Kalemli-Özcan (2019). Their own policy responses will be endogenously different too, and this heterogeneity could have aggregate implications. The fact that all countries issue assets denominated in dollars as well as their local currency generates another contagion mechanism, as pointed out by Jiang et al. (2020). Tracking the distribution of countries and gross capital flows is important for studying these questions.

Investment and, more generally, endogenous supply of assets is necessary to model more realistic macro adjustment to short-run shocks and long-run growth. When the supply of assets is not fixed, gross flows matter for borrowing costs and have an important connection to the real economy. Adding these elements to the model can open the door to studying capital controls, industrial policy, and their external spillovers.

Another set of research questions that requires a closer look at heterogeneity concerns trade and global imbalances. Imbalances in trade and related international capital flows are an outcome of a combination of local shocks and policies around the world. Asset bubbles that might form in large countries as a result of accumulated imbalances might have spillovers to other regions. Tracking the distribution of countries and accounting for gross flows can help us make progress in understanding and quantifying these phenomena.

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A Details for empirical facts

In this section, I describe my empirical calculations. I use IMF data on assets and liabilities from the International Investment Position dataset and financial accounts from the Balance of Payments dataset. I combine "portfolio investment" (both debt and equity) with "other investment", which contains bank loans. The GDP data come from the World Bank.

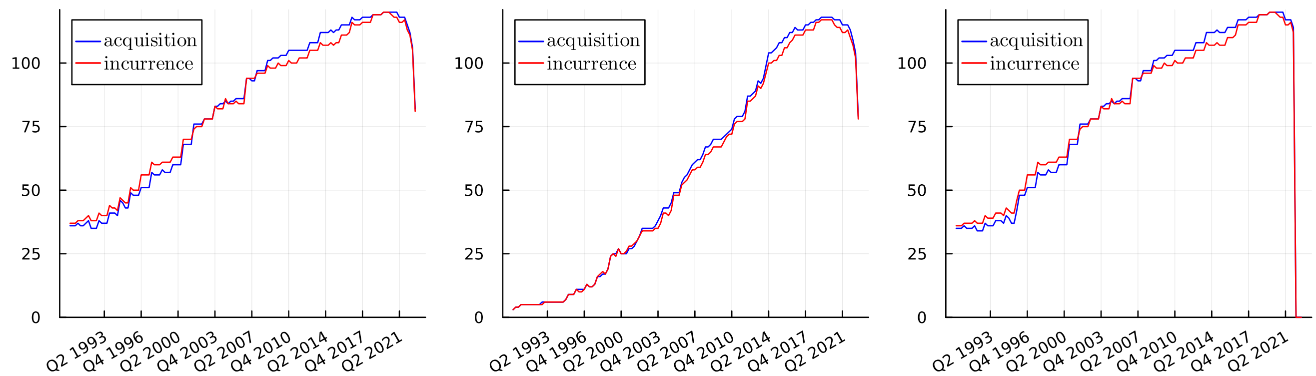

Figure A.1: Number of countries in the sample. Left: acquisition of external assets and incurrence of external liabilities. Center: acquisition of assets and incurrence of liabilities relative to assets and liabilities outstanding. Right: acquisition of assets and incurrence of liabilities to GDP.

The panels are highly unbalanced. Figure A.1 shows the time-varying size of the cross-section. To extract the outflow factor, I choose the time period from 2003 Q1 to 2022 Q3. The country groups for this exercise are:

- Advanced economies: Australia, Canada, Croatia, Czechia, Denmark, Estonia, Finland, France, Germany, Greece, Hong Kong, Iceland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Netherlands, New Zealand, Norway, Portugal, Singapore, Slovak Rep., Slovenia, Spain, Sweden, Switzerland, United Kingdom

- Emerging markets: Argentina, Armenia, Bangladesh, Belarus, Bolivia, Bosnia and Herzegovina, Brazil, Bulgaria, Cambodia, Chile, Colombia, Costa Rica, Ecuador, El Salvador, Fiji, Georgia, Guatemala, Hungary, India, Indonesia, Kazakhstan, Kyrgyz Rep., Malaysia, Mexico, Moldova, Namibia, North Macedonia, Paraguay, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Türkiye, Ukraine, Uruguay

I designate the United States, Cyprus, and Malta, who are also available, as offshores. The balanced panel includes $N = 70$ countries in total and contains $T = 79$ quarters. The factor is

$$\boldsymbol{a} = F\Lambda + \boldsymbol{\varepsilon} \tag{A.1}$$

Here $\boldsymbol{a}$ is a $T \times N$ matrix that collects cross-sections as rows, $\boldsymbol{a}_{ti} = a_{it}$. The matrix F is $T \times r$, where r is the number of factors. The matrix Λ is $r \times N$ and contains factor loadings. Error terms are in the $T \times N$ matrix $\boldsymbol{\epsilon}$.

I extract the principal component following Doz et al. (2012). The estimate S of the variance-covariance matrix is

$$S = \frac{1}{T}\boldsymbol{a}'\boldsymbol{a} \tag{A.2}$$

This matrix is $N \times N$. I denote by $\mathbb{D}$ the diagonal $r \times r$ matrix collecting r of its largest eigenvalues, and by $\mathbb{W}$ the $N \times r$ matrix collecting the corresponding eigenvectors as columns. The first r estimated components comprise the columns of the $T \times r$ matrix $\hat{F}$

$$\hat{F} = \boldsymbol{a}\mathbb{W}\mathbb{D}^{-1/2} \tag{A.3}$$

I denote the first component by f_t.

Table 1 shows correlations between in-group averages $\{a_t^{AE}, a_t^{EM}, \overline{a}_t^{AE}, \overline{a}_t^{EM}, \underline{a}_t^{AE}, \underline{a}_t^{EM}\}$. I download the quarterly VIX from the FRED website. It is available until Q4 2022. The asset price factor from Miranda-Agrippino et al. (2020) is available until Q4 2018, the intermediary factor from He et al. (2017) until Q4 2022, and the treasury basis from Jiang et al. (2021) until Q2 2017. The starting date for all these series, except f_t, is Q1 1990.

This sample uses more countries. The groups are

- Advanced economies: Australia, Austria, Belgium, Canada, Croatia, Czechia, Denmark, Estonia, Finland, France, Germany, Greece, Hong Kong, Iceland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Netherlands, New Zealand, Norway, Portugal, Singapore, Slovak Rep., Slovenia, Spain, Sweden, Switzerland, United Kingdom

- Emerging markets: Afghanistan, Albania, Angola, Argentina, Armenia, Aruba, Azerbaijan, Bahamas, Bangladesh, Belarus, Belize, Bhutan, Bolivia, Bosnia and Herzegovina, Brazil, Brunei, Bulgaria, Cabo Verde, Cambodia, Cameroon, Chile, China, Colombia, Congo, Costa Rica, Curaçao and Sint Maarten, Djibouti, Dominican Rep., Ecuador, Egypt, El Salvador, Ethiopia, Fiji, Georgia, Guatemala, Guinea, Guyana, Haiti, Honduras, Hungary, India, Indonesia, Iraq, Jamaica, Jordan, Kazakhstan, Kiribati, Kosovo, Kuwait, Kyrgyz Rep., Laos, Lebanon, Lesotho, Madagascar, Malaysia, Mauritania, Mauritius, Mexico, Moldova, Mongolia, Montenegro, Morocco, Mozambique, Myanmar, Namibia, Nepal, Nicaragua, Nigeria, North Macedonia, Pakistan, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Qatar, Romania, Russia, Rwanda, Samoa, Saudi Arabia, Serbia, Sint Maarten, Solomon Islands, South Africa, Sri Lanka, Sudan, Suriname, São Tomé and Príncipe, Tajikistan, Tanzania, Thailand, Tonga, Trinidad and Tobago, Türkiye, Uganda, Ukraine, Uruguay, Uzbekistan, Vanuatu, Venezuela, Vietnam, West Bank and Gaza, Yemen, Zambia, Zimbabwe

- Offshores: Bahrain, Bermuda, Cyprus, Ireland, Luxembourg, Malta, Panama, Seychelles, Timor-Leste, United States

Table 10: Decomposition of variance in $\underline{a}_{it}$ between advanced economies and emerging markets.

	standard deviation of $\overline{a}_{it}$	total variance	aggregate	idiosyncratic
advanced economies	0.17	0.0297	**0.0060**	0.0237
emerging markets	0.05	0.0026	**0.0001**	0.0025

Table 10 is the analog of Table 2 for flows adjusted by GDP. The differences between advanced economies and emerging markets are so much more pronounced because advanced economies have more external assets relative to GDP. Figure A.2 shows the ratio between average assets over GDP in advanced economies and emerging markets, $\underline{a}_t^{AE}/\underline{a}_t^{EM}$. It also shows the same ratio for liabilities over GDP. Advanced economies accumulate more assets relative to the size of their economies, so it is not particularly surprising that purchases and sales of foreign assets adjusted for output are larger in magnitude in these countries.

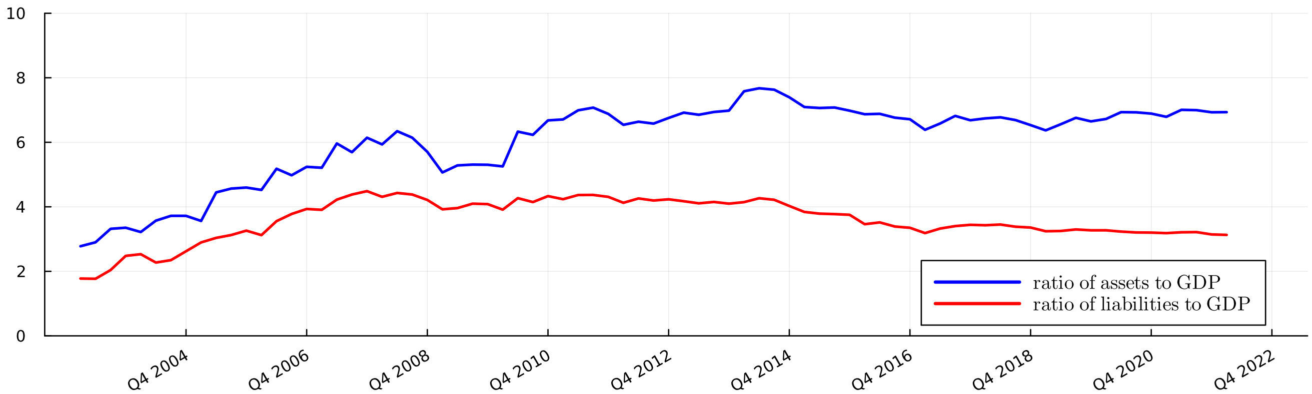

Figure A.2: Ratio of average assets-to-GDP and liabilities-to-GDP in advanced economies and emerging markets.

The synchronization of GDP-adjusted measures of flow with the global flows is also more pronounced in advanced economies. Figure A.3 shows the analog of Figure 3 for $\underline{a}_t^{AE}$ and $\underline{a}_t^{EM}$. Emerging markets look even less synchronized with the global capital flows than on Figure 3 since f_t on these pictures is normalized to have a standard deviation that is the average of that of $\underline{a}_t^{AE}$ and $\underline{a}_t^{EM}$, and these two are substantially different. Using position-adjusted flows, as I do in Section 2, is more informative because they are commensurate in size across advanced and emerging economies and because they capture the intensity of trading relative to wealth, which has a very different relationship to output in different countries.

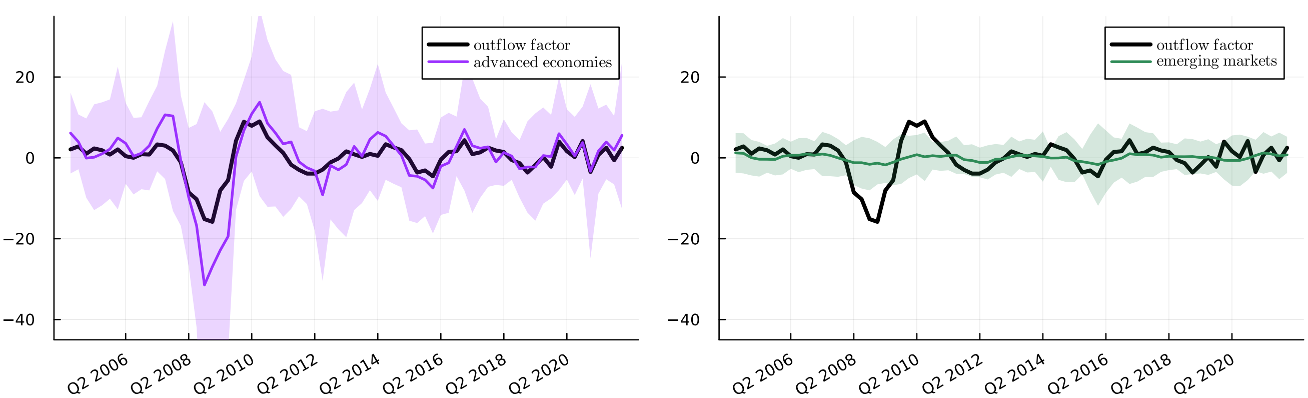

Figure A.3: Aggregate outflows from advanced economies and emerging markets as a percentage of GDP. Black line: outflow factor f_t normalized to have the average standard deviation between $\underline{a}_{it}^{AE}$ and $\underline{a}_{it}^{EM}$. Error bands show cross-sectional standard deviations of $\overline{a}_{it}$ in the two groups for each quarter t.

Figure A.4 presents the time series of cross-sectional size for advanced economies and emerging markets. Advanced economies dominate the sample of position-adjusted outflows before 2008, but the groups are of comparable size. After 2008, emerging markets overtake advanced economies, outnumbering them by slightly more than two times at the peak. The idiosyncratic component of the total variance in the emerging market panel is five times larger. In the sample of outflows normalized by GDP, they always outnumber advanced economies by 2-2.5 times, and the idiosyncratic component of the total variance is 9 times smaller.

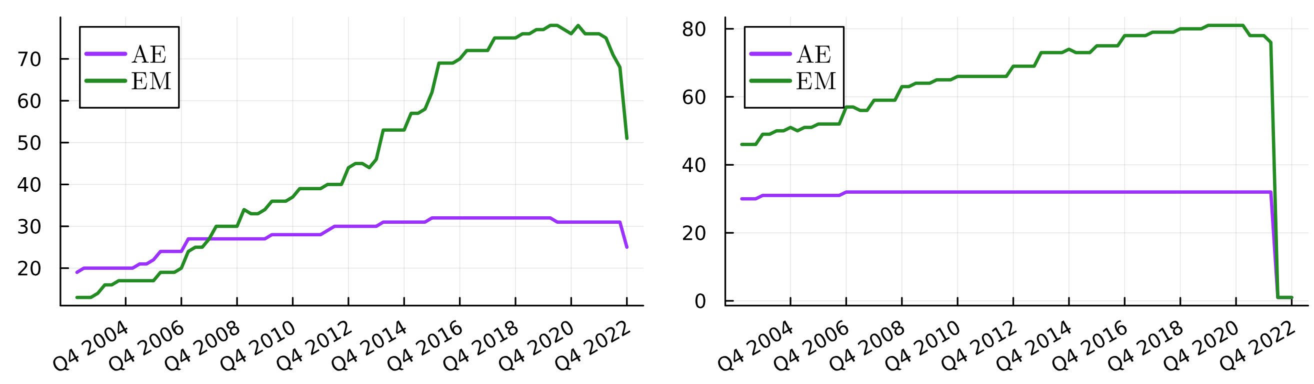

Figure A.4: Number of countries in the sample for the two groups: advanced economies and emerging markets. Left: the panel of outflows normalized by positions. Right: the panel of outflows normalized by GDP.

B Details of portfolio choice

In this section, I derive the portfolio choice of the agents and characterize their value functions.
Regular country savers. I start with the savers from regular countries. The proposition below characterizes the solution to their problem in equation (9).

Proposition 2. Given the time paths of the global interest rate r_t, the special country's wealth $\hat{w}_t$, and the drift and volatility of the excess return process $(\mu_{it}^R, \sigma_{it}^R)$,

$$V_{it} = \log(\rho \underline{w}_{it}) + \kappa(w_{it}, t) \tag{A.4}$$

where $\kappa(w_{it}, t)$ satisfies a partial differential equation. Consumption and portfolio choice are

$$c_{it} = \rho \underline{w}_{it} \tag{A.5}$$

$$\theta_{it} = \min\left\{\overline{\theta}, \frac{\mu_{Rit}}{(\sigma_{Rit})^2}\right\} \tag{A.6}$$

Proof of Proposition 2. Since there is no aggregate uncertainty, state variables for a saver in country i are her own wealth $\underline{w}_{it}$, aggregate wealth of her country w_{it}, and time t. Dropping the subscript i, define the drift and volatility of w_{it} and $\underline{w}_{it}$:

$$dw = \mu_w(w,t)dt + \sigma_w(w,t)dZ \tag{A.7}$$

$$d\underline{w} = \mu_{\underline{w}}(\underline{w}, w, t; c, \theta)dt + \sigma_{\underline{w}}(\underline{w}, w, t; \theta)dZ \tag{A.8}$$

The saver correctly assesses the functions $\mu_w(w,t)$ and $\sigma_w(w,t)$ but does not internalize the effect of her choices on w. The drift and volatility of individual wealth depend on consumption and portfolio choice (c, θ):

$$\mu_{\underline{w}}(\underline{w}, w, t; c, \theta) = r(t)\underline{w} - c + \theta\mu_R(w,t)\underline{w} + \underline{w}\left(\hat{\lambda}\frac{\hat{w}(t)}{w} - \lambda\right) \tag{A.9}$$

$$\sigma_{\underline{w}}(\underline{w}, w, t; \theta) = \theta\sigma_R(w,t)\underline{w} \tag{A.10}$$

Here the dependence on time comes from the drift and volatility of returns $\mu_R(w,t)$ and $\sigma_R(w,t)$ as well as the global interest rate $r(t)$ and the net worth of the special country $\hat{w}(t)$. The HJB equation for the saver's value $V(\underline{w}, w, t)$ is, suppressing the arguments,

$$\begin{aligned}\rho V - \partial_t V = \max_{c, \theta \leq \overline{\theta}}\ & \rho \log(c) + \mu_{\underline{w}}(\underline{w}, w, t; c, \theta)\partial_{\underline{w}} V + \frac{\sigma_{\underline{w}}(\underline{w}, w, t; \theta)^2}{2}\partial_{\underline{w}\underline{w}} V \\ & + \mu_w(w,t)\partial_w V + \frac{\sigma_w(w,t)^2}{2}\partial_{ww} V + \sigma_w(w,t)\sigma_{\underline{w}}(\underline{w}, w, t; \theta)\partial_{\underline{w}w} V\end{aligned} \tag{A.11}$$

Now guess that the value function $V(\underline{w}, w, t)$ has the following form:

$$V(\underline{w}, w, t) = \log(\rho\underline{w}) + \kappa(w, t) \tag{A.12}$$

Plugging this into equation (A.11),

$$\rho\log(\rho\underline{w}) + \rho\kappa(w,t) - \partial_t\kappa(w,t) = \max_{c,\theta\leq\overline{\theta}}\ \rho\log(c) + \frac{\mu_{\underline{w}}(\underline{w},w,t;c,\theta)}{\underline{w}} - \frac{\sigma_{\underline{w}}(\underline{w},w,t;\theta)^2}{2\underline{w}^2} + \mu_w(w,t)\partial_w\kappa(w,t) + \frac{\sigma_w(w,t)^2}{2}\partial_{ww}\kappa(w,t) \tag{A.13}$$

Notice that the cross-derivative term drops out. Now using the functional forms for $\mu_{\underline{w}}(\underline{w},w,t;c,\theta)$ and $\sigma_{\underline{w}}(\underline{w},w,t;\theta)$ from equation (A.9) and equation (A.10), the optimal choices are

$$c^* = \rho\underline{w} \tag{A.14}$$

$$\theta^* = \min\left\{\frac{\mu_R(w,t)}{\sigma_R(w,t)^2}, \overline{\theta}\right\} \tag{A.15}$$

This shows that savers consume a constant fraction of their wealth and choose a mean-variance portfolio whenever they can.

To get the partial differential equation that describes $\kappa(w,t)$, use the consistency requirement $\underline{w} = w$, which also implies $\mu_w(w,t) = \mu_{\underline{w}}(w,w,t;c^*,\theta^*)$ and $\sigma_w(w,t) = \sigma_{\underline{w}}(w,w,t;\theta^*)$. Plugging this and equation (A.14) into equation (A.13),

$$\rho\kappa(w,t) - \partial_t\kappa(w,t) = \frac{\mu_w(w,t)}{w} - \frac{\sigma_w(w,t)^2}{2w^2} + \mu_w(w,t)\partial_w\kappa(w,t) + \frac{\sigma_w(w,t)^2}{2}\partial_{ww}\kappa(w,t) \tag{A.16}$$

Boundary conditions for this equation in general depend on the properties of loadings $\mu_R(w,t)$ and $\sigma_R(w,t)$. Plugging the optimal choice of controls in equation (A.14) and equation (A.15),

$$\mu_w(w,t) = (r(t)-\rho-\lambda)w + \hat{\lambda}\hat{w}(t) + \min\left\{\frac{\mu_R(w,t)}{\sigma_R(w,t)^2}, \overline{\theta}\right\}\mu_R(w,t)w \tag{A.17}$$

$$\sigma_w(w,t) = \min\left\{\frac{\mu_R(w,t)}{\sigma_R(w,t)^2}, \overline{\theta}\right\}\sigma_R(w,t)w \tag{A.18}$$

At $w = 0$, the drift of wealth is not equal to zero. This property helps avoid $w = 0$ being an absorbing state. However, $\kappa(w,t)$ might diverge around small w. Assuming that $\mu_R(w,t)$ is bounded, the limiting behavior of $\kappa(w,t)$ around $w = 0$ is

$$\lim_{x\to 0}\frac{\kappa(x,t)}{\log(x)} = -1 \tag{A.19}$$

Assuming that $\mu_R(w,t)$ and $\sigma_R(w,t)$ approach zero as $w\to\infty$,

$$\lim_{x\to\infty}\rho\kappa(x,t) - \partial_t\kappa(x,t) = r(t) - \rho - \lambda \tag{A.20}$$

The last remaining piece is a suitable initial or terminal condition. In practice, I will use the steady-state value of $\kappa(w,t)$ as the limiting terminal condition at infinity. Assuming that $\mu_R(w,t)$ is bounded and $(\mu_R(w,t),\sigma_R(w,t)) \to (0,0)$ as $w\to\infty$, this completes the characterization of $\kappa(w,t)$ given the general equilibrium objects $r(t)$, $\hat{w}(t)$, $\mu_R(w,t)$, and $\sigma_R(w,t)$. $\square$

Problem of the global bank. The next proposition deals with the problem of the global bank, which is also the special country's saver, given by equation (20).

PROPOSITION 3. Fix the number of regular countries at n. Given the path of the global interest rate r_t and the vector x_t of aggregate wealth in every country including $\hat{w}_t$, the value function of an individual special country saver is

$$\hat{V}_t^{(n)} = \log(\hat{\rho}\underline{\hat{w}}_t) + \hat{\kappa}^{(n)}(x_t, t) \tag{A.21}$$

The function $\hat{\kappa}_t$ solves a first-order ordinary differential equation. The choice of portfolio weights and drift correction for each country i is

$$f_{it}^{(n)} = \frac{\hat{\gamma}_t \eta(w_{it})}{1 + \hat{\gamma}_t \eta(w_{it})} \cdot \frac{\mu_{Rit}}{(\sigma_{Rit})^2} \tag{A.22}$$

$$\xi_{it}^{(n)} = \frac{1}{\hat{\gamma}_t \eta(w_{it})} \cdot f_{it}^{(n)} \sigma_{Rit} = \frac{1}{1 + \hat{\gamma}_t \eta(w_{it})} \cdot \frac{\mu_{it}^R}{\sigma_{Rit}} \tag{A.23}$$

Proof of Proposition 3. Fix the number of regular countries n. The state variables of the global bank are its wealth $\underline{\hat{w}}$, a vector x that combines aggregate special country wealth $\hat{w}$ and all other (w_i), and time t that summarizes all other variables. The evolution of individual wealth $\underline{\hat{w}}$ is

$$d\underline{\hat{w}} = (r_t \underline{\hat{w}}_t - \hat{c}_t)dt + \sum_i f_{it}\underline{\hat{w}}_t((\mu_{it}^R - \sigma_{it}^R \xi_{it})dt + \sigma_{it}^R d\widetilde{Z}_{it}) + \hat{f}_t \underline{\hat{w}}_t d\hat{R}_t + \frac{\underline{\hat{w}}_t}{\hat{w}_t}(\lambda w_t dt - \hat{\lambda}\hat{w}_t)dt \tag{A.24}$$

Combine all $(d\tilde{Z}_{it})$ into a vector $d\tilde{Z}_t$. Aggregate vector x evolves as

$$dx = \mu_x(x,t)dt + \sigma_x(x,t)d\tilde{Z}_t \tag{A.25}$$

The HJB equation for $\hat{V}(\underline{\hat{w}}, x, t)$ is, suppressing arguments,

$$\begin{aligned}\hat{\rho}\hat{V} - \partial_t \hat{V} = \max_{\hat{c}, \hat{f}, (f_i)} \min_{(\xi_i)} \ \hat{\rho}\log(\hat{c}) + \frac{\hat{\gamma}(t)}{2}\sum_i \eta(w_{it})\xi_i^2 + \mu_{\underline{\hat{w}}}(\underline{\hat{w}}, x, t; \hat{c}, \hat{f}, (f_i, \xi_i))\partial_{\underline{\hat{w}}}\hat{V} + \mu_x(x,t)'\partial_x \hat{V} \\ + \frac{\sigma_{\underline{\hat{w}}}(\underline{\hat{w}}, x, t; (f_i))^2}{2}\partial_{\underline{\hat{w}}\underline{\hat{w}}}V + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'}V\sigma_x(x,t)) + \sigma_{\underline{\hat{w}}x}(\underline{\hat{w}}, x, t; (f_i))'\partial_{\underline{\hat{w}}x}V\end{aligned} \tag{A.26}$$

Here the drift and variance of $\underline{\hat{w}}$ conditional on controls c, $\hat{f}$, (f_i), and (ξ_i) are

$$\mu_{\underline{\hat{w}}}(\underline{\hat{w}}, x, t; \hat{c}, \hat{f}, (f_i, \xi_i)) = (r(t) - \hat{\lambda} + \hat{f}\mathbb{E}_t[d\hat{R}_t])\underline{\hat{w}} - c + \sum_i \underline{\hat{w}} f_i(\mu_{it}^R - \xi_i \sigma_{it}^R) + \frac{w(t)}{\hat{w}(t)}\lambda \underline{\hat{w}} \tag{A.27}$$

$$\sigma_{\underline{\hat{w}}}(\underline{\hat{w}}, x, t; (f_i))^2 = \underline{\hat{w}}^2 \sum_i f_i^2 (\sigma_{it}^R)^2 \tag{A.28}$$

$$\sigma_{\underline{\hat{w}}x}(\underline{\hat{w}}, x, t; (f_i)) = \underline{\hat{w}}\sigma_x(x,t)v(x,t,(f_i)) \tag{A.29}$$

The vector $v(x, t, (f_i))$ in equation (A.29) collects the products $(f_i \sigma_{it}^R)$.

The solution for the weight on the special country's tree $\hat{f}$ will only be finite if $\mathbb{E}_t[d\hat{R}_t] = 0$. I assume that this is the case. The optimal weight $\hat{f}^*$ is not determined and does not affect the value of the objective, so I will omit it from the notation for $\mu_{\underline{\hat{w}}}(\underline{\hat{w}}, x, t; \hat{c}, \hat{f}^*, (f_i, \xi_i^*))$ below.

The expressions for variance of $d\hat{\underline{w}}$ and co-variance of $d\hat{\underline{w}}$ and $d\hat{w}$ per unit of time in equation (A.28) and equation (A.29) use independence between $d\tilde{Z}_{it}$.

Solving the minimization problem over (ξ_i),

$$\xi_i^* = \frac{1}{\hat{\gamma}(t)\eta(w_{it})} \cdot f_i \sigma_{it}^R \cdot \hat{\underline{w}} \partial_{\hat{\underline{w}}} \hat{V} \tag{A.30}$$

Plugging this into equation (A.46),

$$\mu_{\hat{\underline{w}}}(\hat{\underline{w}}, x, t; \hat{c}, (f_i, \xi_i^*)) = (r(t) - \hat{\lambda})\hat{\underline{w}} - \hat{c} + \sum_i \left(\hat{\underline{w}} f_i \mu_{it}^R - \partial_{\hat{\underline{w}}} \hat{V} \frac{(\hat{\underline{w}} \sigma_{it}^R)^2}{\hat{\gamma}(t)\eta(w_{it})} f_i^2 \right) + \frac{w(t)}{\hat{w}(t)} \lambda \hat{\underline{w}} \tag{A.31}$$

The problem in equation (A.26) is now

$$\begin{aligned} \hat{\rho}\hat{V} - \partial_t \hat{V} = \max_{\hat{c},(f_i)} \ & \hat{\rho}\log(\hat{c}) + \frac{(\partial_{\hat{\underline{w}}}\hat{V})^2}{2\hat{\gamma}(t)} \sum_i \frac{(\hat{\underline{w}}\sigma_{it}^R)^2}{\eta(w_{it})} f_i^2 + \mu_{\hat{\underline{w}}}(\hat{\underline{w}}, x, t; \hat{c}, (f_i, \xi_i^*))\partial_{\hat{\underline{w}}}\hat{V} + \mu_x(x,t)'\partial_x \hat{V} \\ & + \frac{\sigma_{\hat{\underline{w}}}(\hat{\underline{w}}, x, t; (f_i))^2}{2} \partial_{\hat{\underline{w}}\hat{\underline{w}}} \hat{V} + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'} V \sigma_x(x,t)) + \sigma_{\hat{\underline{w}}x}(\hat{\underline{w}}, x, t; (f_i))'\partial_{\hat{\underline{w}}x}\hat{V} \end{aligned} \tag{A.32}$$

Plugging equation (A.31) and equation (A.28) into this,

$$\begin{aligned} \hat{\rho}\hat{V} - \partial_t \hat{V} = \max_{\hat{c},(f_i)} \ & \hat{\rho}\log(\hat{c}) - \hat{c}\partial_{\hat{\underline{w}}}\hat{V} + \partial_{\hat{\underline{w}}}\hat{V} \sum_i \hat{\underline{w}} \mu_{it}^R f_i - \frac{(\partial_{\hat{\underline{w}}}\hat{V})^2}{2\hat{\gamma}(t)} \sum_i \frac{(\hat{\underline{w}}\sigma_{it}^R)^2}{\eta(w_{it})} f_i^2 \\ & + \left(r(t) - \hat{\lambda} + \lambda \frac{w(t)}{\hat{w}(t)} \right) \hat{\underline{w}} \partial_{\hat{\underline{w}}} \hat{V} + \hat{\underline{w}}^2 \sum_i \frac{f_i^2 (\sigma_{it}^R)^2}{2} \partial_{\hat{\underline{w}}\hat{\underline{w}}} V \\ & + \mu_x(x,t)'\partial_x \hat{V} + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'} V \sigma_x(x,t)) + \sigma_{\hat{\underline{w}}x}(\hat{\underline{w}}, x, t; (f_i))'\partial_{\hat{\underline{w}}x} V \end{aligned} \tag{A.33}$$

Guess that the value function $\hat{V}(\hat{\underline{w}}, t)$ has the following form

$$\hat{V}(\hat{\underline{w}}, x, t) = \log(\hat{\rho}\hat{\underline{w}}) + \hat{\kappa}(x,t) \tag{A.34}$$

This immediately leads to the optimal choice of consumption:

$$\hat{c}^* = \hat{\rho}\hat{\underline{w}} \tag{A.35}$$

Replacing this in equation (A.50),

$$\begin{aligned} \hat{\rho}\hat{\kappa}(x,t) - \partial_t \hat{\kappa}(x,t) = \max_{(f_i)} \ & \sum_i \left(\mu_{it}^R f_i - \frac{(\sigma_{it}^R)^2}{2\hat{\gamma}(t)\eta(w_{it})} f_i^2 - \frac{(\sigma_{it}^R)^2}{2} f_i^2 \right) + r(t) - \hat{\rho} - \hat{\lambda} + \frac{\lambda w(t)}{\hat{w}(t)} \\ & + \mu_x(x,t)'\partial_x \hat{\kappa}(x,t) + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'}\hat{\kappa}(x,t)\sigma_x(x,t)) \end{aligned} \tag{A.36}$$

The optimal choice of portfolio weights (f_i) is

$$f_i^* = \frac{\hat{\gamma}(t)\eta(w_{it})}{1 + \hat{\gamma}(t)\eta(w_{it})} \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.37}$$

This leads to the following expression for the optimal drift correction:

$$\xi_i^* = \frac{1}{1 + \hat{\gamma}(t)\eta(w_{it})} \cdot \frac{\mu_{it}^R}{\sigma_{it}^R} \tag{A.38}$$

The differential equation for $\hat{\kappa}(t)$ becomes

$$\begin{aligned}\hat{\rho}\hat{\kappa}(x,t) - \partial_t \hat{\kappa}(\hat{w},t) = \frac{1}{2} \sum_i \frac{\hat{\gamma}(t)\eta(w_{it})}{1 + \hat{\gamma}(t)\eta(w_{it})} \left(\frac{\mu_{it}^R}{\sigma_{it}^R} \right)^2 + r(t) - \hat{\rho} - \hat{\lambda} + \frac{\lambda w(t)}{\hat{w}(t)} \\ + \mu_x(x,t)' \partial_x \hat{\kappa}(x,t) + \frac{1}{2} \mathrm{tr}(\sigma_x(x,t)' \partial_{xx'} \hat{\kappa}(x,t) \sigma_x(x,t))\end{aligned} \tag{A.39}$$

The last remaining piece is a suitable initial or terminal condition. In practice, I will use the steady-state value of $\hat{\kappa}$ as the terminal limit at $t \to \infty$. $\square$

C Alternative interpretations for the global banks

This section provides details of the bank's problem in the continuous limit. I also show alternative ways to model the banks: a VAR-type constraint, robustness concerns with common regional factors, and segmented country-specific departments.

C.1 Continuous limit

The problem of the bank in the continuous world is

$$\hat{V}_t = \max_{(\hat{c}_s, \hat{f}_s, (f_{is}))_{s\geq t}} \inf_{((\xi_{it}))_{s\geq t}} \mathbb{E}_t^{Q(\xi)} \left[\int_t^{\infty} e^{\hat{\rho}(s-t)} \left(\hat{\rho} \log(\hat{c}_s) + \frac{\gamma_s}{2} \int_0^1 \eta(w_{is}) \xi_{is}^2 di \right) ds \right] \tag{A.40}$$

subject to the budget constraint

$$\begin{aligned} d\underline{\hat{w}} = (r_t \underline{\hat{w}}_t - \hat{c}_t)dt + \int_0^1 f_{it} \underline{\hat{w}}_t ((\mu_{it}^R - \sigma_{it}^R \xi_{it})dt + \sigma_{it}^R d\widetilde{Z}_{it})di \\ + \hat{f}_t \underline{\hat{w}}_t d\hat{R}_t + \frac{\underline{\hat{w}}_t}{\hat{w}_t}(\lambda w_t dt - \hat{\lambda}\hat{w}_t)dt \end{aligned} \tag{A.41}$$

The bank has access to a continuum of uncorrelated risky assets, so its payoffs are deterministic. However, the bank has freedom to choose a model for uncertainty over each risky asset in its portfolio. Marginal benefits of considering scenarios with more substantial losses (that is, marginal benefits of increasing ξ_{it}) grow with portfolio weights f_{it}. But as the bank entertains more pessimistic models, marginal benefits of raising f_{it} decline. This brings decreasing returns into the choice of portfolio weights, even though uncertainty can effectively be disregarded.

The following proposition characterizes the solution.

Proposition 4. Given the paths of the global interest rate r_t, the special country's aggregate wealth $\hat{w}_t$, the drift and volatility of the excess return process $(\mu_{it}^R, \sigma_{it}^R)$ for all i, the value function of an individual special country saver is

$$\hat{V}_t = \log(\hat{\rho}\underline{\hat{w}}_t) + \hat{\kappa}_t \tag{A.42}$$

The function $\hat{\kappa}_t$ solves a first-order ordinary differential equation. The choice of drift correction and portfolio weights for each country i is

$$f_{it} = \gamma_t \eta(w_{it}) \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.43}$$

$$\xi_{it} = \frac{1}{\gamma_t \eta(w_{it})} \cdot f_{it} \sigma_{it}^R = \frac{\mu_{it}^R}{\sigma_{it}^R} \tag{A.44}$$

Portfolio choice of the bank given by equation (A.60) is very similar to that of the local savers in regular countries, being proportional to mean over variance of returns. A common factor $\gamma_t \hat{w}_t$ applies to all regular countries. Movements in γ_t map into shifts in foreign demand for risky assets in all countries at once. It can be interpreted as a risk-tolerance parameter. Infinite γ_t would make the bank essentially neutral to risk, willing to take an arbitrarily large position in any asset that offers excess returns.

Interpreted literally as a cost parameter in equation (18), high γ_t means that the penalty for considering alternative models with large losses is prohibitively high. Equation (A.61) shows that, for a fixed f_{it}, high γ_t makes the bank stick closer to the baseline measure. With a smaller correction, marginal costs of increasing f_{it} are lower, and the bank demands more risky assets for a given mean-variance ratio.

Why do correction terms ξ_{it} pick up the volatility of idiosyncratic returns if idiosyncratic shocks wash out in aggregate? This is because alternative models chosen by the bank apply to the fundamental dividend shocks dZ_{it}. A mistake in evaluating the expectation of the dividend shock dZ_{it} translates to a mistake in evaluating the expectation of excess returns $dR_{it} = \mu_{it}^R dt + \sigma_{it}^R dZ_{it}$, which is what really matters for portfolio choice, and this latter mistake scales by σ_{it}^R.

Proof of Proposition 4. Since there is no aggregate uncertainty, the evolution of the aggregate wealth in the special country $\hat{w}_t$ and the distribution of (w_{it}) can be summarized by time. The HJB equation for $\hat{V}(\underline{\hat{w}}, t)$ is, suppressing arguments,

$$\hat{\rho}\hat{V} - \partial_t \hat{V} = \max_{\hat{c}, \hat{f}, (f_i)} \min_{(\xi_i)} \ \hat{\rho}\log(\hat{c}) + \frac{\gamma(t)}{2}\int_0^1 \eta(w_{it})\xi_i^2 di + \mu^{\underline{\hat{w}}}(\underline{\hat{w}}, t; \hat{c}, \hat{f}, (f_i, \xi_i))\partial_{\underline{\hat{w}}}\hat{V} \tag{A.45}$$

Here the drift of $\underline{\hat{w}}$ conditional on controls c, $\hat{f}$, (f_i), and (ξ_i) is

$$\mu^{\underline{\hat{w}}}(\underline{\hat{w}}, t; \hat{c}, \hat{f}, (f_i, \xi_i)) = (r(t) - \hat{\lambda} + \hat{f}\mathbb{E}_t[d\hat{R}_t])\underline{\hat{w}} - c + \int_0^1 \underline{\hat{w}} f_i(\mu_{it}^R - \xi_i \sigma_{it}^R)di + \frac{w(t)}{\hat{w}(t)}\lambda \underline{\hat{w}} \tag{A.46}$$

The solution for the weight on the special country's tree $\hat{f}$ will only be finite if $\mathbb{E}_t[d\hat{R}_t] = 0$. I will assume that this is the case henceforth. The optimal weight $\hat{f}^*$ is not determined and does not affect the value of the objective, so I will omit it from the notation for $\mu^{\underline{\hat{w}}}(\underline{\hat{w}}, t; \hat{c}, \hat{f}^*, (f_i, \xi_i^*))$ below.

Solving the minimization problem over (ξ_i),

$$\xi_i^* = \frac{1}{\gamma(t)\eta(w_{it})} \cdot f_i \sigma_{it}^R \cdot \underline{\hat{w}}\partial_{\underline{\hat{w}}}\hat{V} \tag{A.47}$$

Plugging this into equation (A.46),

$$\mu^{\underline{\hat{w}}}(\underline{\hat{w}}, t; \hat{c}, (f_i, \xi_i^*)) = (r(t) - \hat{\lambda})\underline{\hat{w}} - \hat{c} + \int_0^1 \left(\underline{\hat{w}} f_i \mu_{it}^R - \partial_{\underline{\hat{w}}}\hat{V}\frac{(\underline{\hat{w}}\sigma_{it}^R)^2}{\gamma(t)\eta(w_{it})} f_i^2 \right) di + \frac{w(t)}{\hat{w}(t)}\lambda\underline{\hat{w}} \tag{A.48}$$

The problem in equation (A.45) is now

$$\hat{\rho}\hat{V} - \partial_t \hat{V} = \max_{\hat{c}, (f_i)} \ \hat{\rho}\log(\hat{c}) + \frac{(\partial_{\underline{\hat{w}}}\hat{V})^2}{2\gamma(t)}\int_0^1 \frac{(\underline{\hat{w}}\sigma_{it}^R)^2}{\eta(w_{it})} f_i^2 di + \mu^{\underline{\hat{w}}}(\underline{\hat{w}}, t; \hat{c}, (f_i, \xi_i^*))\partial_{\underline{\hat{w}}}\hat{V} \tag{A.49}$$

Plugging equation (A.48) into this,

$$\hat{\rho}\hat{V} - \partial_t \hat{V} = \max_{\hat{c},(f_i)} \hat{\rho}\log(\hat{c}) - \hat{c}\partial_{\underline{\hat{w}}}\hat{V} + \partial_{\underline{\hat{w}}}\hat{V}\int_0^1 \underline{\hat{w}}\mu_{it}^R f_i di - \frac{(\partial_{\underline{\hat{w}}}\hat{V})^2}{2\gamma(t)}\int_0^1 \frac{(\underline{\hat{w}}\sigma_{it}^R)^2}{\eta(w_{it})} f_i^2 di$$
$$+ \left(r(t) - \hat{\lambda} + \lambda\frac{w(t)}{\hat{w}(t)}\right)\underline{\hat{w}}\partial_{\underline{\hat{w}}}\hat{V} \tag{A.50}$$

Guess that the value function $\hat{V}(\underline{\hat{w}}, t)$ has the following form

$$\hat{V}(\underline{\hat{w}}, t) = \log(\hat{\rho}\underline{\hat{w}}) + \hat{\kappa}(t) \tag{A.51}$$

This immediately leads to the optimal choice of consumption:

$$\hat{c}^* = \hat{\rho}\underline{\hat{w}} \tag{A.52}$$

Replacing this in equation (A.50),

$$\hat{\rho}\hat{\kappa}(t) - \hat{\kappa}'(t) = \max_{(f_i)} \int_0^1 \left(\mu_{it}^R f_i - \frac{(\sigma_{it}^R)^2}{2\gamma(t)\eta(w_{it})} f_i^2\right) di + r(t) - \hat{\rho} - \hat{\lambda} + \frac{\lambda w(t)}{\hat{w}(t)} \tag{A.53}$$

The optimal choice of portfolio weights (f_i) is

$$f_i^* = \gamma(t)\eta(w_{it}) \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.54}$$

The differential equation for $\hat{\kappa}(t)$ becomes

$$\hat{\rho}\hat{\kappa}(t) - \hat{\kappa}'(t) = \frac{\gamma(t)}{2}\int_0^1 \eta(w_{it})\left(\frac{\mu_{it}^R}{\sigma_{it}^R}\right)^2 di + r(t) - \hat{\rho} - \hat{\lambda} + \frac{\lambda w(t)}{\hat{w}(t)} \tag{A.55}$$

The last remaining piece is a suitable initial or terminal condition. In practice, I will use the steady-state value of $\hat{\kappa}$ as the terminal limit at $t \to \infty$. □

C.2 VAR-type constraint

The bank with a VAR-type constraint solves

$$\max_{(\hat{c}_s, \hat{f}_s, (f_{is}))_{s\geq t}} \mathbb{E}_t\left[\int_t^\infty e^{\hat{\rho}(s-t)}\log(\hat{c}_s)ds\right] \tag{A.56}$$

subject to a budget constraint

$$d\underline{\hat{w}}_t = (r_t\underline{\hat{w}}_t - \hat{c}_t)dt + \int_0^1 f_{it}\underline{\hat{w}}_t dR_{it}di + \hat{f}_t\underline{\hat{w}}_t d\hat{R}_t + \left(\lambda\frac{w_t}{\hat{w}_t} - \hat{\lambda}\right)\underline{\hat{w}}_t dt \tag{A.57}$$

and the following institutional or regulatory constraint:

$$\int_0^1 \mathbb{V}_t[f_{it}dR_{it}]\eta(w_{it})^{-1}di \leq \gamma_t \int_0^1 \mathbb{E}_t[f_{it}dR_{it}]di \tag{A.58}$$

The next proposition characterizes the solution.

Proposition 5. Given the path of the global interest rate r_t, the special country's aggregate wealth $\hat{w}_t$, the drift and volatility of the excess return processes $(\mu_{it}^R, \sigma_{it}^R)$ for all i, the value function of an individual special country saver with a VAR-type constraint is

$$\hat{V}_t = \log(\hat{\rho}\underline{\hat{w}}_t) + \hat{\kappa}_t \tag{A.59}$$

The function $\hat{\kappa}_t$ solves a first-order ordinary differential equation. The choice of drift correction and portfolio weights for each country i is

$$f_{it} = \gamma_t \eta(w_{it}) \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.60}$$

$$\xi_{it} = \frac{1}{\gamma_t \eta(w_{it})} \cdot f_{it}\sigma_{it}^R = \frac{\mu_{it}^R}{\sigma_{it}^R} \tag{A.61}$$

Proof of Proposition 5. Without aggregate uncertainty, the evolution of the aggregate special country wealth $\hat{w}_t$ and the cross-section of (w_{it}) can be summarized by time. The HJB equation for $\hat{V}(\underline{\hat{w}}_t, t)$ is, suppressing arguments,

$$\hat{\rho}\hat{V} - \partial_t \hat{V} = \max_{\hat{c}, \hat{f}, (\hat{f}_i)} \hat{\rho}\log(\hat{c}) + \mu^{\underline{\hat{w}}}(\underline{\hat{w}}, t; \hat{c}, \hat{f}, (f_i, \xi_i))\partial_{\underline{\hat{w}}}\hat{V} \tag{A.62}$$

$$\text{s.t.} \quad \int_0^1 f_i^2(\sigma_{it}^R)^2\eta(w_{it})^{-1}di \leq \gamma_t \int_0^1 f_i\mu_{it}^R di \tag{A.63}$$

Here the drift of $\underline{\hat{w}}$ conditional on controls c, $\hat{f}$, (f_i), and (ξ_i) is

$$\mu^{\underline{\hat{w}}}(\underline{\hat{w}}, t; \hat{c}, \hat{f}, (f_i, \xi_i)) = (r(t) - \hat{\lambda} + \hat{f}\mathbb{E}_t[d\hat{R}_t])\underline{\hat{w}} - c + \int_0^1 \underline{\hat{w}} f_i \mu_{it}^R di + \frac{w(t)}{\hat{w}(t)}\lambda\underline{\hat{w}} \tag{A.64}$$

The solution for the weight on the special country's tree $\hat{f}$ will only be finite if $\mathbb{E}_t[d\hat{R}_t] = 0$. I will assume that this is the case henceforth. The optimal weight $\hat{f}^*$ is not determined and does not affect the value of the objective, so I will omit it from the notation for $\mu^{\underline{\hat{w}}}(\underline{\hat{w}}, t; \hat{c}, \hat{f}^*, (f_i, \xi_i^*))$ below.

Guess that the value function $\hat{V}(\underline{\hat{w}}, t)$ has the following form

$$\hat{V}(\underline{\hat{w}}, t) = \log(\hat{\rho}\underline{\hat{w}}) + \hat{\kappa}(t) \tag{A.65}$$

This immediately leads to the optimal choice of consumption:

$$\hat{c}^* = \hat{\rho}\underline{\hat{w}} \tag{A.66}$$

Plugging this into equation (A.62),

$$\hat{\rho}\hat{\kappa}(t) - \hat{\kappa}'(t) = \max_{(\hat{f}_i)} \int_0^1 f_i \mu_{it}^R di + r(t) - \hat{\lambda} - \hat{\rho} + \lambda \frac{w(t)}{\hat{w}(t)} \tag{A.67}$$

$$\text{s.t.} \quad \int_0^1 f_i^2 (\sigma_{it}^R)^2 \eta(w_{it})^{-1} di \leq \gamma_t \int_0^1 f_i \mu_{it}^R di \tag{A.68}$$

Let the multiplier on the constraint be ξ_t. The first-order condition for f_i is

$$f_i = \eta(w_{it}) \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \cdot \frac{1 + \xi_t \gamma_t}{2\xi_t} \tag{A.69}$$

Plugging this into the constraint,

$$\frac{1 - \gamma_t^2 \xi_t^2}{4\xi_t^2} \int_0^1 \eta(w_{it}) \left(\frac{\mu_{it}^R}{\sigma_{it}^R} \right)^2 di \leq 0 \tag{A.70}$$

This holds with equality if $\xi_t > 0$, so $\xi_t = 1/\gamma_t$ and

$$f_i^* = \gamma_t \eta(w_{it}) \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.71}$$

Plugging this back into equation (A.67),

$$\hat{\rho}\hat{\kappa}(t) - \hat{\kappa}'(t) = \gamma_t \int_0^1 \eta_{it} \left(\frac{\mu_{it}^R}{\sigma_{it}^R} \right)^2 di + r(t) - \hat{\lambda} - \hat{\rho} + \lambda \frac{w(t)}{\hat{w}(t)} \tag{A.72}$$

The only remaining bit is the suitable initial or terminal condition for $\hat{\kappa}(\cdot)$. In practice, I will use the steady state as a terminal condition as a limit at $t \to \infty$. $\square$

D Details of equilibrium

In this section, I provide details for the equilibrium section. First, I discuss the steady state and a useful benchmark of unlimited risk-taking capacity in which there is complete risk-sharing and no non-degenerate wealth distribution. This benchmark also illustrates the role of migration between countries. Then, I provide justification for the equilibrium condition in equation (34) and equation (44) and proof for Proposition 1.

Steady state. Define aggregate profit rates from risky assets π and $\hat{\pi}$:

$$\pi \int w dG(w) = \int w\theta(w)\mu_R(w)dG(w) \tag{A.73}$$

$$\hat{\pi}\hat{w} = \int \hat{w} f(w)\mu_R(w)dG(w) \tag{A.74}$$

Here π is the average expected excess return that regular countries receive on their trees, and $\hat{\pi}$ is the total expected excess return that the global bank receives on them. These profit rates allow me to express the steady-state interest rate and aggregate wealth of regular and special countries.

Proposition 6. In the steady state, the interest rate is

$$r = \frac{\rho + \lambda + \hat{\rho} + \hat{\lambda} - \pi - \hat{\pi} - \sqrt{(\rho + \lambda - \hat{\rho} - \hat{\lambda} - \pi + \hat{\pi})^2 + 4\lambda\hat{\lambda}}}{2} \tag{A.75}$$

It decreases in both π and $\hat{\pi}$ and is bounded between $\min\{\rho - \pi, \hat{\rho} - \hat{\pi}\}$ and $\max\{\rho - \pi, \hat{\rho} - \hat{\pi}\}$. The aggregate wealth of regular countries and that of the special country are given by

$$\int w dG(w) = \frac{\hat{\lambda}(\nu + \hat{q}\hat{\nu})}{\rho\hat{\lambda} + \hat{\rho}\lambda + \rho\hat{\rho} - \hat{\rho}(r + \pi)} \tag{A.76}$$

$$\hat{w} = \frac{\lambda(\nu + \hat{q}\hat{\nu})}{\rho\hat{\lambda} + \hat{\rho}\lambda + \rho\hat{\rho} - \rho(r + \hat{\pi})} \tag{A.77}$$

This proposition shows how the interest rate and aggregate wealth of regular and special countries depend on profit rates. This is useful for thinking about the benchmark limit $\gamma \to \infty$. In this limit, any positive expected excess returns lead the bank to assume an infinite position in risky assets. Expected excess returns $\mu_R(w)$ then have to be zero in equilibrium, and $\pi = \hat{\pi} = 0$. Since r decreases in both π and $\hat{\pi}$, limited risk-taking capacity depresses the global interest rate.

Holdings are well defined in this limit. Local savers are not willing to hold trees since there is fundamental risk in the dividends but expected excess returns are zero. The global bank holds all the risky assets and fully insures the regular countries. There is no wealth distribution among the regular countries because they are not exposed to idiosyncratic shocks. Equation (A.76) in this case shows wealth accumulated by each of them. Simplifying more, $\rho = \hat{\rho}$ leads to $r = \rho = \hat{\rho}$ and

$$\int w dG(w) = \frac{\hat{\lambda}}{\hat{\lambda} + \lambda} \frac{\nu + \hat{q}\hat{\nu}}{\rho} \tag{A.78}$$

$$\hat{w} = \frac{\lambda}{\hat{\lambda} + \lambda} \frac{\nu + \hat{q}\hat{\nu}}{\rho} \tag{A.79}$$

Risk is perfectly diversified and there is no difference in time preferences, so wealth is simply the present value of output split between the intermediary and the savers by migration. This is the only way in which migration affects the aggregates. The technical reason to include it is that, without it, $w = 0$ would be an absorbing state for regular countries, and the special country's income would be linear in its wealth $\hat{w}$, leaving no possibility for a well-defined invariant distribution. In my calibration, I set net migration in the steady state to zero but allow for some gross flows.

Proofs. I now formulate equation (34) as equation (44) propositions and prove them. I also prove Proposition 1 and Proposition 6.

PROPOSITION 7. *Equation (33) implies equation (34). That is,*

$$p(w,t) = w \min\left\{\overline{\theta}, \frac{\mu^R(w,t)}{\sigma^R(w,t)^2}\right\} + \varphi(t)\eta(w)\frac{\mu^R(w,t)}{\sigma^R(w,t)^2} \Longrightarrow \tag{A.80}$$

$$\frac{\mu^R(w,t)}{\sigma^R(w,t)^2} = \max\left\{\frac{p(w,t)}{\varphi(t)\eta(w)+w}, \frac{p(w,t)-\overline{\theta}w}{\varphi(t)\eta(w)}\right\} \tag{A.81}$$

Proof of Proposition 7. Suppose that equation (33) holds. Use the equivalence

$$\frac{p(w,t)}{\varphi(t)\eta(w)+w} \geq \frac{p(w,t)-\overline{\theta}w}{\varphi(t)\eta(w)} \Leftrightarrow \frac{p(w,t)}{\varphi(t)\eta(w)+w} \leq \overline{\theta} \tag{A.82}$$

Suppose $\mu^R(w,t)/\sigma^R(w,t)^2 \leq \overline{\theta}$. Then, from equation (33) it follows that

$$\frac{\mu^R(w,t)}{\sigma^R(w,t)^2} = \frac{p(w,t)}{\varphi(t)\eta(w)+w} \tag{A.83}$$

This means that $p(w,t)/(\varphi(t)\eta(w)+w) \leq \overline{\theta}$, so, from equation (A.82),

$$\max\left\{\frac{p(w,t)}{\varphi(t)\eta(w)+w}, \frac{p(w,t)-\overline{\theta}w}{\varphi(t)\eta(w)}\right\} = \frac{p(w,t)}{\varphi(t)\eta(w)+w} = \frac{\mu^R(w,t)}{\sigma^R(w,t)^2} \tag{A.84}$$

Now suppose that $\mu^R(w,t)/\sigma^R(w,t)^2 > \overline{\theta}$. From equation (33) it follows that

$$\frac{\mu^R(w,t)}{\sigma^R(w,t)^2} = \frac{p(w,t)-\overline{\theta}w}{\varphi(t)\eta(w)} \tag{A.85}$$

This means that $p(w,t) - \overline{\theta}w > \overline{\theta}\varphi(t)\eta(w)$, so $p(w,t)/(w+\varphi(t)\eta(w)) > \overline{\theta}$. Then, from equation (A.82) it follows that

$$\max\left\{\frac{p(w,t)}{\varphi(t)\eta(w)+w}, \frac{p(w,t)-\overline{\theta}w}{\varphi(t)\eta(w)}\right\} = \frac{p(w,t)-\overline{\theta}w}{\varphi(t)\eta(w)} = \frac{\mu^R(w,t)}{\sigma^R(w,t)^2} \tag{A.86}$$

Thus, in any case,

$$\max\left\{\frac{p(w,t)}{\varphi(t)\eta(w)+w}, \frac{p(w,t)-\overline{\theta}w}{\varphi(t)\eta(w)}\right\} = \frac{\mu^R(w,t)}{\sigma^R(w,t)^2} \tag{A.87}$$

This completes the proof. □

Proof of Proposition 1. Start with plugging the expressions for $\mu^R(w,t)$ and $\sigma^R(w,t)$ into equation (34). Rewriting it yields a formula for the risk premium in terms of price dynamics:

$$\frac{\mu^p(w,t)+\nu(t)}{p(w,t)} - r(t) = \frac{(\sigma^p(w,t)+\sigma)^2}{p(w,t)^2} \cdot \max\left\{\frac{p(w,t)}{w+\varphi(t)\eta(w)}, \frac{p(w,t)-\bar{\theta}w}{\varphi(t)\eta(w)}\right\} \tag{A.88}$$

Using Itô's lemma,

$$\mu^p(w,t) = \mu^w(w,t)\partial_w p(w,t) + \frac{\sigma^w(w,t)^2}{2}\partial_{ww}p(w,t) + \partial_t p(w,t) \tag{A.89}$$
$$\sigma^p(w,t) = \sigma^w(w,t)\partial_w p(w,t) \tag{A.90}$$

Multiplying both sides of equation (A.88) by $p(w,t)$,

$$\begin{aligned}\mu^p(w,t)+\nu(t)-p(w,t)r(t)+\partial_t p(w,t) &= (\sigma^p(w,t)+\sigma)^2 \\ &\cdot \max\left\{\frac{1}{w+\varphi(t)\eta(w)}, \frac{1}{\varphi(t)\eta(w)}\left(1-\frac{\bar{\theta}w}{p(w,t)}\right)\right\}\end{aligned} \tag{A.91}$$

Plugging the drift and volatility of prices,

$$\begin{aligned}&\mu^w(w,t)\partial_w p(w,t) + \frac{\sigma^w(w,t)^2}{2}\partial_{ww}p(w,t) + \nu(t) - p(w,t)r(t) + \partial_t p(w,t) \\ &\quad= (\sigma^w(w,t)\partial_w p(w,t)+\sigma)^2 \cdot \max\left\{\frac{1}{w+\varphi(t)\eta(w)}, \frac{1}{\varphi(t)\eta(w)}\left(1-\frac{\bar{\theta}w}{p(w,t)}\right)\right\}\end{aligned} \tag{A.92}$$

Now rewrite the process for a regular country's wealth in equation (10):

$$\begin{aligned}dw &= (r(t)-\rho)wdt + \theta(w,t)wdR(w,t) + (\hat{\lambda}\hat{w}(t)-\lambda w)dt \\ &= (r(t)-\rho-\lambda)wdt + \hat{\lambda}\hat{w}(t)dt + w\theta(w,t)\mu^R(w,t)dt + w\theta(w,t)\sigma^R(w,t)dZ \\ &= \hat{\lambda}\hat{w}(t)dt + \left(r(t)(1-\theta(w,t)) - \rho - \lambda + \theta(w,t)\frac{\mu^p(w,t)+\nu(t)}{p(w,t)}\right)wdt \\ &+ w\theta(w,t)\frac{\sigma^p(w,t)+\sigma}{p(w,t)}dZ\end{aligned} \tag{A.93}$$

From this, it follows that

$$\sigma^w(w,t) = w\theta(w,t)\frac{\sigma^p(w,t)+\sigma}{p(w,t)} \tag{A.94}$$

Plugging equation (A.90),

$$\begin{aligned}\sigma^w(w,t) = w\theta(w,t)\frac{\sigma^w(w,t)\partial_w p(w,t)+\sigma}{p(w,t)} &= \frac{\theta(w,t)w\sigma}{p(w,t)-w\theta(w,t)\partial_w p(w,t)} \\ &= \frac{\theta(w,t)w\sigma}{p(w,t)(1-\theta(w,t)\epsilon(w,t))}\end{aligned} \tag{A.95}$$

Here $\epsilon(w,t) = w/p(w,t) \cdot \partial_w p(w,t)$ is the wealth elasticity of the price. This implies

$$(\sigma^w(w,t)\partial_w p(w,t) + \sigma)^2 = \left(\frac{\sigma}{1-\theta(w,t)\epsilon(w,t)}\right)^2 \tag{A.96}$$

Plugging this into equation (A.92),

$$\begin{aligned}\mu^w(w,t)\partial_w p(w,t) + \frac{\sigma^w(w,t)^2}{2}\partial_{ww}p(w,t) + \nu(t) - p(w,t)r(t) + \partial_t p(w,t)\\ = \left(\frac{\sigma}{1-\theta(w,t)\epsilon(w,t)}\right)^2 \cdot \max\left\{\frac{1}{w+\varphi(t)\eta(w)}, \frac{1}{\varphi(t)\eta(w)}\left(1-\frac{\overline{\theta}w}{p(w,t)}\right)\right\}\end{aligned} \tag{A.97}$$

Define the risk-adjusted payoff $y(w,t)$ as

$$y(w,t) = \nu(t) - \left(\frac{\sigma}{1-\theta(w,t)\epsilon(w,t)}\right)^2 \max\left\{\frac{1}{w+\varphi(t)\eta(w)}, \frac{1}{\varphi(t)\eta(w)}\left(1-\frac{\overline{\theta}w}{p(w,t)}\right)\right\} \tag{A.98}$$

Plugging leads to

$$r(t)p(w,t) - \partial_w p(w,t) = y(w,t) + \mu^w(w,t)\partial_w p(w,t) + \frac{\sigma^w(w,t)^2}{2}\partial_{ww}p(w,t) \tag{A.99}$$

This is the Kolmogorov backward equation (41) for prices. The Kolmogorov forward equation (42) for wealth follows from the fact that the wealth process is a diffusion. □

Proof of Proposition 6. Take the evolution of the special country's wealth and integrate the evolution of the regular countries' wealth to get aggregate dynamics:

$$\begin{aligned} d\hat{w}(t) &= (r(t)-\hat{\rho})\hat{w}(t)dt + \int_0^1 \hat{w}(t)f(w,t)\mu^R(w,t)dG(w,t)dt \\ &\quad + \left(\lambda\int wdG(w,t) - \hat{\lambda}\hat{w}(t)\right)dt \end{aligned} \tag{A.100}$$

$$\begin{aligned} \int dwdG(w,t) &= (r(t)-\rho)\int wdG(w,t)dt + \int w\theta(w,t)\mu^R(w,t)dG(w,t)dt \\ &\quad + \left(\hat{\lambda}\hat{w}(t) - \lambda\int wdG(w,t)\right)dt \end{aligned} \tag{A.101}$$

In the steady state, the left-hand side is zero in both of these equations:

$$\begin{aligned} 0 &= (r-\hat{\rho})\hat{w} + \int_0^1 \hat{w}f(w)\mu^R(w)dG(w) + \left(\lambda\int wdG(w) - \hat{\lambda}\hat{w}\right) \\ &= (r-\hat{\rho})\hat{w} + \hat{\pi}\hat{w} + \lambda\int wdG(w) - \hat{\lambda}\hat{w} \end{aligned} \tag{A.102}$$

$$\begin{aligned} 0 &= (r-\rho)\int wdG(w) + \int w\theta(w)\mu^R(w)dG(w) + \left(\hat{\lambda}\hat{w} - \lambda\int wdG(w)\right) \\ &= (r-\rho)\int wdG(w) + \pi\int wdG(w) + \hat{\lambda}\hat{w} - \lambda\int wdG(w) \end{aligned} \tag{A.103}$$

This leads to

$$\hat{w} = \int wdG(w) \frac{\lambda}{\hat{\lambda} + \hat{\rho} - r - \hat{\pi}} = \hat{w} \frac{\hat{\lambda}}{\lambda + \rho - r - \pi} \frac{\lambda}{\hat{\lambda} + \hat{\rho} - r - \hat{\pi}} \tag{A.104}$$

Reorganize this as a quadratic equation

$$(r + \pi - \rho - \lambda)(r + \hat{\pi} - \hat{\rho} - \hat{\lambda}) = \lambda\hat{\lambda} \tag{A.105}$$

The solution is

$$r = \frac{\alpha + \hat{\alpha} \pm \sqrt{(\alpha - \hat{\alpha})^2 + 4\lambda\hat{\lambda}}}{2} \tag{A.106}$$

Here $\alpha = \rho + \lambda - \pi$ and $\hat{\alpha} = \hat{\rho} + \hat{\lambda} - \hat{\pi}$. Take the root with a plus and consider $r - \alpha$ and $r - \hat{\alpha}$:

$$r - \alpha = \frac{(\hat{\alpha} - \alpha) + \sqrt{(\hat{\alpha} - \alpha)^2 + 4\lambda\hat{\lambda}}}{2} > \hat{\alpha} - \alpha \tag{A.107}$$

$$r - \hat{\alpha} = \frac{(\alpha - \hat{\alpha}) + \sqrt{(\alpha - \hat{\alpha})^2 + 4\lambda\hat{\lambda}}}{2} > \alpha - \hat{\alpha} \tag{A.108}$$

These imply that $r > \alpha$ and $r > \hat{\alpha}$, which is not possible since $\hat{w}$ and $\int wdG(w)$ would be negative in this case. The right root is then that with a minus. Plugging this expression for the interest rate into

$$\hat{w} = \frac{\lambda}{\hat{\lambda} + \hat{\rho} - r - \hat{\pi}} \int wdG(w) \tag{A.109}$$

$$\int wdG = \frac{\hat{\lambda}}{\lambda + \rho - r - \pi} \hat{w} \tag{A.110}$$

completes the proof. □

Proposition 8. *In the steady state,*

$$\hat{c} = \hat{\nu} + \nu \cdot \int \hat{h}(w)dG(w) - r \cdot \int (1 - \theta(w))wdG(w) + \int \mu^p(w)\hat{h}(w)dG(w) \tag{A.111}$$

Proof of Proposition 8 The wealth accumulation of the global bank in the steady state can be rewritten as

$$\begin{aligned} d\hat{w} &= (r\hat{w} - \hat{c})dt + \int f(w)\hat{w}dR(w)dG(w) + (\lambda w - \hat{\lambda}\hat{w})dt \\ &= (r\hat{w} - \hat{c})dt - r\hat{w}dt \cdot \int f(w)dG(w) + \nu\hat{w}dt \cdot \int \hat{h}(w)dG(w) \\ &+ dt \cdot \int \mu^p(w)\hat{h}(w)dG(w) + (\lambda w - \hat{\lambda}\hat{w})dt \end{aligned} \tag{A.112}$$

This uses the fact that $d\hat{R} = 0$ in equilibrium, so the special asset does not contribute to wealth accumulation. The second line simply applies the definition of $dR(w) = (\mu^p(w) + \nu)/p(w) - r$ and the definition of $h(w)$: $\hat{w}f(w) = p(w)h(w)$. The balance sheet of the global bank is

$$\hat{w} = \int p(w)\hat{h}(w)dG(w) + \hat{p}\hat{q} - \int (1 - \theta(w))wdG(w) \tag{A.113}$$

The bank's wealth is its consolidated position in risky assets and its position in the safe asset less deposits outstanding. Plugging this and using the steady-state relations $\hat{p} = \hat{\nu}/r$ (no risk premium on the safe asset), $d\hat{w} = 0$ (no fluctuations in the intermediary's wealth), and $\hat{\lambda}\hat{w} = \lambda w$ (no net migration),

$$\hat{c} = \hat{\nu} + \nu \cdot \int \hat{h}(w)dG(w) - r \cdot \int (1 - \theta(w))wdG(w) + \int \mu^p(w)\hat{h}(w)dG(w) \tag{A.114}$$

This completes the proof. $\square$

E Details for calibration and estimation

In this section, I explain the algorithm for calibration and estimation. I first calibrate the steady-state version of the model using four aggregate moments and a panel of external assets and liabilities from IFS data provided by the IMF. Then, I use two aggregate series and sequence-space Jacobians to estimate the parameters of aggregate shock processes by likelihood maximization.

Calibration. I construct the ratio of assets to liabilities:

$$R_{it} = \frac{A_{it}}{L_{it}} \tag{A.115}$$

I measure the moments of its distribution in the data using the following procedure:

- First, I take unbalanced panels for A_{it} and L_{it} starting in 1990.
- I then smooth out assets and liabilities by replacing the value in each quarter with the mean value over the last four quarters.
- For every country that eventually appears in the sample, I create a weight that is inversely proportional to the duration of its presence in the sample. This allows me to correct for the over-representation of advanced economies with relatively large assets and liabilities

Figure A.5 shows the model fit for the distribution.

Figure A.5: Model-generated distributions (red) and data for the ratio of external assets to external liabilities.

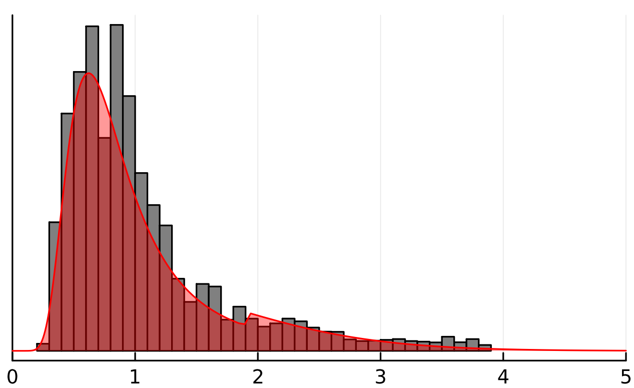

Table 11: Average excess returns in emerging markets and advanced economies. Emerging markets are those constrained: $\theta(w) = \overline{\theta}$

advanced economies	$0.676pp$
emerging markets	$2.893pp$
difference	$2.218pp$

I set the net migration flows to zero in the steady state:

$$\lambda \int w dG(w) = \hat{\lambda}\hat{w} \tag{A.116}$$

Given all other parameters, this pins down $\hat{\lambda}$. I also set $\nu = \hat{\nu}$. The level of dividends can be normalized using the following symmetry property. Consider one model parameterized by eleven parameters $(\rho, \hat{\rho}, \lambda, \hat{\lambda}, \nu, \hat{\nu}, \overline{\theta}, \sigma, \gamma, \hat{q}, \zeta)$ and another one parameterized by $(\rho, \hat{\rho}, \lambda, \hat{\lambda}, \nu^*, \hat{\nu}^*, \overline{\theta}, \sigma^*, \gamma^*, \hat{q}, \zeta^*)$

such that

$$\nu^* = \alpha\nu \tag{A.117}$$

$$\hat{\nu}^* = \alpha\hat{\nu} \tag{A.118}$$

$$\sigma^* = \alpha\sigma \tag{A.119}$$

$$\zeta^* = \frac{\alpha\zeta}{1+(\alpha-1)\zeta} \tag{A.120}$$

$$\gamma^* = \gamma\frac{1+(\alpha-1)\zeta}{\alpha} \tag{A.121}$$

The prices and quantities corresponding to the original and the starred model satisfy the following:

$$\hat{w}^*(t) = \alpha\hat{w}(t) \tag{A.122}$$

$$p^*(\alpha w, t) = \alpha p(w, t) \tag{A.123}$$

$$f^*(\alpha w, t) = f(w, t) \tag{A.124}$$

Multiplying expected dividends and their volatility by the same number simply rescales wealth and asset prices if γ and ζ are suitably transformed. The same transformation as in equation (A.123) applies to the loadings of prices and wealth processes $(\mu_p(w,t), \mu_w(w,t), \sigma_p(w,t), \sigma_w(w,t))$, while instantaneous returns $(\mu_R(w,t), \sigma_R(w,t))$ follow the pattern in equation (A.124). Interest rates remain the same. This can be verified by noting that

$$\varphi(t)\eta(\alpha w) = \hat{w}(t)\gamma(t)\eta + \hat{w}(t)\gamma(t)(1-\eta)\cdot\alpha w \tag{A.125}$$

Applying the transformations above to equation (34),

$$\hat{w}^*(t)\gamma^*(t)\zeta^* = \alpha\hat{w}(t)\gamma(t)\zeta \tag{A.126}$$

$$\hat{w}^*(t)\gamma^*(t)(1-\zeta^*)\cdot\alpha w = \hat{w}(t)\gamma(t)(1-\zeta)\cdot\alpha w \tag{A.127}$$

This means

$$\frac{\mu_R^*(\alpha w,t)}{\sigma_R^*(\alpha w,t)} = \max\left\{\frac{p^*(\alpha w,t)}{\varphi^*(t)\eta^*(\alpha w)+\alpha w}, \frac{p^*(\alpha w,t)-\overline{\theta}\alpha w}{\varphi^*(t)\eta^*(\alpha w)}\right\} \tag{A.128}$$

$$= \max\left\{\frac{\alpha p(w,t)}{\alpha\varphi(t)\eta(w)+\alpha w}, \frac{\alpha p(w,t)-\overline{\theta}\alpha w}{\alpha\varphi(t)\eta(w)}\right\} = \frac{\mu_R(w,t)}{\sigma_R(w,t)} \tag{A.129}$$

Portfolio choice for αw is hence the same under the starred parameterization as that for w under the original one. The upshot is that equation (A.122) and equation (A.123) imply that $(\mu_p(w,t), \mu_w(w,t), \sigma_p(w,t), \sigma_w(w,t))$ transform in the same way as in equation (A.123), which verifies equation (A.124) since consumption follows the same proportional rule under both parametrizations and migration scales with $\hat{w}(t)$. The evolution of $\hat{w}(t)$ then shows that equation (A.122) is satisfied, and equation (41) shows that equation (A.123) holds.

This symmetry means that choosing a number $\nu = \hat{\nu}$ is simply a normalization. The remaining parameters are an outcome of numerical optimization, where I look for a configuration that minimizes the quadratic distance between the targets and their model analogs in Table 3. All moments are assigned the same weight.

Estimation. This subsection explains the use of the linearized version of the model to estimate parameters of shocks to $\gamma(t)$ and $(\nu(t), \hat{\nu}(t))$ that rationalize the data on aggregate outflows and risky asset prices. Since Section 6 shows very similar effects of shocks to $\nu(t)$ and $\hat{\nu}(t)$, I suppose that $\nu(t) = \hat{\nu}(t)$ for simplicity.

The unanticipated exponentially decaying shock to $\gamma(t)$ and $\nu(t)$ from Section 5 and Section 6 can be interpreted as a result of one pulse to dW in

$$d\gamma(t) = (\zeta^\gamma - \gamma(t))\mu_\gamma dt + \sigma_\gamma \cdot dW \tag{A.130}$$

$$d\nu(t) = (\zeta^\nu - \nu(t))\mu_\nu dt + \sigma_\nu \cdot dW \tag{A.131}$$

Here $(\zeta^\gamma, \zeta^\nu)$ are the long-run values of $\gamma(t)$ and $\nu(t)$ equal to the parameters in my steady-state calibration. Persistence is governed by (μ_γ, μ_ν), and vectors σ_γ and σ_ν determine the standard size of the shocks coming from standard Brownian increments $dW = (dW_1, dW_2)$.

The linearization of the model is around $\sigma_\gamma = (0, 0)$ and $\sigma_\nu = (0, 0)$. This means perfect foresight with respect to paths of $\gamma(t)$ and $\nu(t)$. Denote the perfectly foreseen deviations of these exogenous series from the long-run values by $\tilde{\gamma}(t) = \gamma(t) - \zeta^\gamma$ and $\tilde{\nu}(t) = \nu(t) - \zeta^\nu$. For any function $z(w, t)$ and a fixed path of $(\tilde{\gamma}(t), \tilde{\nu}(t))_{t\geq 0}$, one can write the first-order perturbation as

$$\tilde{z}(w, t) = \int_0^\infty J_{z\gamma}(w, t, s)\tilde{\gamma}(s)ds + \int_0^\infty J_{z\nu}(w, t, s)\tilde{\nu}(s)ds \tag{A.132}$$

Here $J_{z\gamma}$ and $J_{z\nu}$ are general equilibrium sequence-space Jacobians. They consist of several components. Some of these components are forward-looking and solve linear Kolmogorov backward equations. Some are backward-looking and solve Kolmogorov forward equations. To compute them, I discretize the grids and use symmetries described in Auclert et al. (2021) and Bhandari et al. (2023). Appendix H describes the equations in detail.

Estimation can be done with aggregate analogs equation (A.132). For any aggregate variable $X(t)$ and a fixed path of $(\tilde{\gamma}(t), \tilde{\nu}(t))_{t\geq 0}$, one can write the first-order perturbation $\tilde{X}(t)$:

$$\tilde{X}(t) = \int_0^\infty J_{x\gamma}(t, s)\tilde{\gamma}(s)ds + \int_0^\infty J_{x\nu}(t, s)\tilde{\nu}(s)ds \tag{A.133}$$

Importantly, the sequences $(\tilde{\gamma}(s), \tilde{\nu}(s))_{s\geq t}$ here extend into the future, so agents have to foresee their paths or form expectations. I add more details below.

Likelihood function. Under the model in equation (A.130) and equation (A.131), sequences of deviations $(\tilde{\gamma}(t), \tilde{\nu}(t))_{t\geq 0}$ can be traced to the underlying shocks. Fix a particular realized sequence $(dW(t))_{t\geq 0}$. The paths of $\tilde{\gamma}(t)$ and $\tilde{\nu}(t)$ with $(\tilde{\gamma}(0), \tilde{\nu}(0)) = (0, 0)$ can be written as

$$\tilde{\gamma}(t) = \int_0^t e^{\mu_\gamma(s-t)}\sigma_\gamma \cdot dW(s) \tag{A.134}$$

$$\tilde{\nu}(t) = \int_0^t e^{\mu_\nu(s-t)}\sigma_\nu \cdot dW(s) \tag{A.135}$$

Discretizing the time axis and imposing a truncation horizon T, one can rewrite these as

$$\boldsymbol{\gamma} = \sigma_{\gamma 1}\mathbb{J}_{\gamma w}(\mu_\gamma)\,\boldsymbol{W}_1 + \sigma_{\gamma 2}\mathbb{J}_{\gamma w}(\mu_\gamma)\,\boldsymbol{W}_2 \tag{A.136}$$

$$\boldsymbol{\nu} = \sigma_{\nu 1}\mathbb{J}_{\nu w}(\mu_\nu)\,\boldsymbol{W}_1 + \sigma_{\nu 2}\mathbb{J}_{\nu w}(\mu_\nu)\,\boldsymbol{W}_2 \tag{A.137}$$

Here the $T \times 1$ vectors $\boldsymbol{\gamma}$ and $\boldsymbol{\nu}$ collect the realized deviations $\tilde{\gamma}(t)$ and $\tilde{\nu}(t)$, and $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ collect the realized shocks $dW_1(t)$ and $dW_2(t)$. The $T \times T$ matrices $\mathbb{J}_{\gamma w}(\mu_\gamma)$ and $\mathbb{J}_{\nu w}(\mu_\nu)$ approximate integration with the discounting factors $e^{\mu_\gamma(s-t)}$ and $e^{\mu_\nu(s-t)}$.

How do agents form their expectations about future parts of $\boldsymbol{\gamma}$ and $\boldsymbol{\nu}$? I assume that they do not anticipate any new shocks dW_1 and dW_2, since linearization is done around $\sigma_\gamma = \sigma_\nu = (0, 0)$. This means that, conditional on seeing $\tilde{\gamma}(t)$ and $\tilde{\nu}(t)$ at t, agents project their evolution as follows:

$$\tilde{\gamma}(s) = e^{\mu_\gamma(t-s)}\tilde{\gamma}(t) \tag{A.138}$$

$$\tilde{\nu}(s) = e^{\mu_\nu(t-s)}\tilde{\nu}(t) \tag{A.139}$$

I further assume that past realizations of $\tilde{\gamma}(t)$ and $\tilde{\nu}(t)$ are forgotten at t. This transforms equation (A.133) into

$$\begin{aligned}
\tilde{X}(t) &= \int_t^\infty J_{x\gamma}(t,s)\tilde{\gamma}(s)ds + \int_t^\infty J_{x\nu}(t,s)\tilde{\nu}(s)ds \\
&= \tilde{\gamma}(t)\int_t^\infty J_{x\gamma}(t,s)e^{\mu_\gamma(t-s)}ds + \tilde{\nu}(t)\int_t^\infty J_{x\nu}(t,s)e^{\mu_\nu(t-s)}ds \\
&= \tilde{\gamma}(t)\hat{J}_{x\gamma}(\mu_\gamma) + \tilde{\nu}(t)\hat{J}_{x\nu}(\mu_\nu)
\end{aligned} \tag{A.140}$$

The integrals $\hat{J}_{x\gamma}(\mu_\gamma)$ and $\hat{J}_{x\nu}(\mu_\nu)$ defined in equation (A.140) depend on steady-state objects through Jacobians $J_{x\gamma}(\cdot)$ and $J_{x\nu}(\cdot)$ and on the persistence parameters μ_γ and μ_ν. The fact that they can be reduced to numbers is due to the linearity of the linearized model and the recursivity of the Ornstein-Uhlenbeck process: the expected evolution of a random variable following this process is always an exponentially decaying sequence. Innovations simply rescale the sequence going forward. The integrals $\hat{J}_{x\gamma}(\mu_\gamma)$ and $\hat{J}_{x\nu}(\mu_\nu)$ encode the first-order response of $\tilde{X}(t)$ to exponentially decaying sequences parametrized by μ_γ and μ_ν and the initial level pinned down at 1. Multiplication by $\tilde{\gamma}(t)$ and $\tilde{\nu}(t)$ does the necessary rescaling.

I assume that $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ are independent and $\sigma_{\nu 1} = 0$. The discretized equation for the sequence of perturbed aggregates $\tilde{X}(t)$ is

$$\begin{aligned}
\boldsymbol{X} &= \mathbb{J}_{x\gamma}(\mu_\gamma)\,\boldsymbol{\gamma} + \mathbb{J}_{x\nu}(\mu_\nu)\,\boldsymbol{\nu} \\
&= \sigma_{\gamma 1}\mathbb{J}_{x\gamma}(\mu_\gamma)\mathbb{J}_{\gamma w}(\mu_\gamma)\,\boldsymbol{W}_1 + (\sigma_{\gamma 2}\mathbb{J}_{x\gamma}(\mu_\gamma)\mathbb{J}_{\gamma w}(\mu_\gamma) + \sigma_{\nu 2}\mathbb{J}_{x\nu}(\mu_\nu)\mathbb{J}_{\nu w}(\mu_\nu))\,\boldsymbol{W}_2
\end{aligned} \tag{A.141}$$

Here $\boldsymbol{X}$ collects the deviations $\tilde{X}(t)$. The matrices $\mathbb{J}_{x\gamma}$ and $\mathbb{J}_{x\nu}$ are diagonal matrices collecting the integrals $\hat{J}_{x\gamma}(\mu_\gamma)$ and $\hat{J}_{x\nu}(\mu_\nu)$. Stacking the shocks into a vector $\mathbf{w} = (\boldsymbol{W}_1', \boldsymbol{W}_2')'$ yields $\boldsymbol{X} = \mathbb{J}_{xw}(\mu_\gamma, \mu_\nu, \sigma_{\gamma 1}, \sigma_{\gamma 2}, \sigma_{\nu 2})\,\mathbf{w}$. Finally, the data are a time-aggregated, quarterly, version of $\boldsymbol{X}$. The observed vector $\mathbf{x}$ is

$$\mathbf{x} = \mathbb{Q}\,\boldsymbol{X} = \mathbb{Q}\mathbb{J}_{xw}(\mu_\gamma, \mu_\nu, \sigma_{\gamma 1}, \sigma_{\gamma 2}, \sigma_{\nu 2})\,\mathbf{w} \tag{A.142}$$

Here $\mathbb{Q}$ is the matrix that aggregates observations into quarters.

The data series in place of $\mathbf{x}$ are the first-order perturbations of the average of risky asset prices $P(t) = \int p(w,t)dG(w,t)$ and external assets of regular countries $M(t) = \int l(w,t)dG(w,t)$, where deposits $l(w,t)$ are given by $l(w,t) = w(1 - \theta(w,t))$. I normalize the first-order perturbations by the steady-state values and compute the corresponding Jacobians $(\mathbb{J}_{p\gamma}, \mathbb{J}_{p\nu}, \mathbb{J}_{m\gamma}, \mathbb{J}_{m\nu})$ that form the Jacobians $(\mathbb{J}_{pw}, \mathbb{J}_{mw})$ from equation (A.142).

I use the principal component of asset returns across countries and aggregate capital outflows normalized by aggregate external assets as the empirical analogs for the deviations of $P(t)$ and $M(t)$. Details are provided below. The model is

$$\begin{pmatrix} \mathbf{x}_p \\ \mathbf{x}_m \end{pmatrix} = \begin{pmatrix} \mathbb{Q}\mathbb{J}_{pw}(\mu_\gamma, \mu_\nu, \sigma_{\gamma 1}, \sigma_{\gamma 2}, \sigma_{\nu 2}) \\ \mathbb{Q}\mathbb{J}_{mw}(\mu_\gamma, \mu_\nu, \sigma_{\gamma 1}, \sigma_{\gamma 2}, \sigma_{\nu 2}) \end{pmatrix} \mathbf{w} + \begin{pmatrix} \sigma_p \, \boldsymbol{\epsilon}_p \\ \sigma_m \, \boldsymbol{\epsilon}_m \end{pmatrix} \tag{A.143}$$

Here $\mathbf{x}_p$ is the asset price factor and $\mathbf{x}_m$ is the outflow serie. The vectors $\boldsymbol{\epsilon}_p$ and $\boldsymbol{\epsilon}_m$ include errors that are independent across dates and series and are standard normal. The scale (σ_p, σ_m) requires estimation.

Stacking observations in a vector $\mathbf{x}$, errors in a vector $\boldsymbol{\epsilon}$, and using a diagonal matrix $\mathbb{J}_\epsilon$ for (σ_p, σ_m) turn equation (A.143) into $\mathbf{x} = \mathbb{J}_w \, \mathbf{w} + \mathbb{J}_\epsilon \, \boldsymbol{\epsilon}$. Assuming $\mathbf{w}$ and $\boldsymbol{\epsilon}$ are jointly normal with independent components, the log-likelihood of $\mathbf{x}$ is proportional to

$$\mathcal{L} = -\log(\det(\mathbb{J}_w \mathbb{J}_w' + \mathbb{J}_\epsilon \mathbb{J}_\epsilon')) - \mathbf{x}'(\mathbb{J}_w \mathbb{J}_w' + \mathbb{J}_\epsilon \mathbb{J}_\epsilon')^{-1} \, \mathbf{x} \tag{A.144}$$

Data. I use the following aggregate series as empirical analogs of $P(t)$ and $M(t)$. First, $X_P(t)$ is the principal component of asset prices, which is a detrended and differenced version of a factor generously provided by Habib and Venditti (2019). I take differences to approximate deviations from a steady state. Second, outflows normalized by the aggregate external assets $X_M(t)$ are

$$X_M(t) = \frac{\sum_{i \in \mathcal{I}(t)} a_{i,t}}{\sum_{i \in \mathcal{I}(t)} A_{i,t-1}} \tag{A.145}$$

Here a_{it} is the detrended net acquisition of assets, a measure of outward flows defined in Section 2. The denominator aggregates external assets one quarter prior. This measure is also detrended as in Forbes and Warnock (2012) and Forbes and Warnock (2021). The sets $\mathcal{I}(t)$ collect countries for which these measures are available at t. The resulting dates in the sample are 73 quarters spanning Q3 2004 through Q3 2022.

Results. I maximize this log-likelihood over $(\mu_\gamma, \mu_\nu, \sigma_{\gamma 1}, \sigma_{\gamma 2}, \sigma_{\nu 2}, \sigma_p, \sigma_m)$. Table 12 presents point estimates. Note that error terms are realized at a quarterly frequency, so their standard deviations (σ_p, σ_m) are not directly comparable to the volatility terms $(\sigma_{\gamma 1}, \sigma_{\gamma 2}, \sigma_{\nu 2})$.

Table 12: Estimation results.

μ_γ	μ_ν	$\sigma_{\gamma 1}$	$\sigma_{\gamma 2}$	$\sigma_{\nu 2}$	σ_p	σ_m
1.987	3.148	1.762	-0.805	0.032	0.117	0.084
(0.404)	(0.311)	(0.306)	(0.172)	(0.002)	(0.007)	(0.010)

These estimates go into the shocks in Section 5 and Section 6. The standard deviation of an increment to $d\gamma(t)$ coming from dW_1 is $\sigma_{\gamma 1}\sqrt{dt}$, which in a weakly discretization $(dt = 0.02)$ corresponds to about 24.9% of the steady-state value of γ. The corresponding size of the shock to $d\nu(t)$ and $d\hat{\nu}(t)$ is about 6.2% of the steady-state value of ν. These shocks are relatively large, so the exact solutions for non-linear dynamics in Section 5 and Section 6 are quite different from the linear responses computed with Jacobians.

Figure A.6 shows some columns of the Jacobian matrix $\mathbb{J}_w$ with the estimated parameters. Each line can be interpreted as follows: suppose there is a pulse of size 1 to $dW_1(s)$ at time s in the future (say, 1 year ahead). The corresponding line on panel (a) shows the time path of $M(t)$ expressed as a percentage deviation from its steady-state value. The corresponding line on panel (c) shows the same for $P(t)$. Since there is perfect foresight in the linearized model, these aggregates might react already at $t = 0$ and will then evolve in accordance with the model. Similarly, the lines on panel (b) and (c) show how $M(t)$ and $P(t)$ react to a pulse to $dW_2(s)$ at some time s.

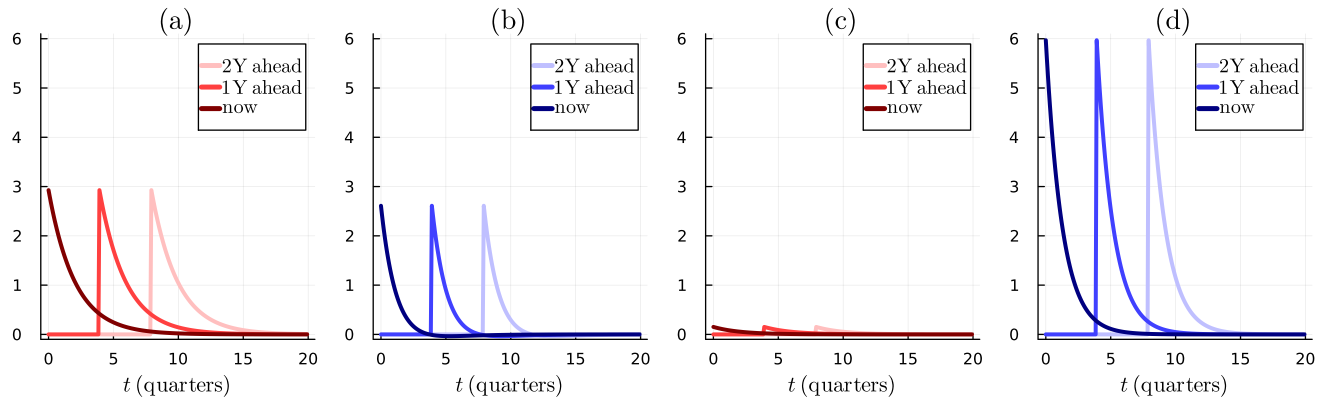

Figure A.6: Columns of Jacobians of outflow sequence $(M(t))_{t\geq 0}$ with respect to $(dW_1(s))_{s\geq 0}$ on panel (a) and $(dW_2(s))_{s\geq 0}$ on panel (b). Same price sequence $(P(t))_{t\geq 0}$ with respect to $(dW_1(s))_{s\geq 0}$ on panel (c) and $(dW_2(s))_{s\geq 0}$ on panel (d). Percentages of steady-state values.

On panel (a), aggregate outflows react positively to a pulse in dW_1, since $\gamma(t)$ loads positively on it. An increase in $\gamma(t)$ makes risky assets more attractive to the global bank. It buys them from local investors, who then increase their foreign holdings. Panel (b) shows that the reaction of outflows to dW_2 is mixed. A pulse to dW_2 increases output since $d\nu(t)$ has a positive loading on it. This increases outflows as prices of risky assets rise and expected returns fall, making investors more willing to hold deposits. They then run them down to buy assets back.

Panel (c) shows a muted but positive reaction of $P(t)$ to a pulse to dW_1. An increase in $\gamma(t)$ raises prices of risky assets issued by poorer countries, but it also raises the interest rate as the global bank scrambles for funding to increase its investments. This adversely affects the prices of assets issued by rich countries. The reason is that these assets are traded in deep markets and react to interest rates much more than to swings in foreign demand. Finally, panel (d) shows that risky asset prices positively react to a pulse in dW_2, since $d\nu(t)$ has a positive loading on it.

Quarterly magnitudes. To get a sense of magnitude for shocks to $\gamma(t)$ and $(\nu(t), \hat{\nu}(t))$ over discrete time periods, one can solve the stochastic differential equation (A.130) and equation (A.131):

$$\tilde{\gamma}(\tau) = e^{-\mu_\gamma \tau}\tilde{\gamma}(0) + \int_0^\tau e^{-\mu_\gamma s}\sigma_{\gamma 1} dW_1(s) + \int_0^\tau e^{-\mu_\gamma s}\sigma_{\gamma 2} dW_2(s)$$
$$\tilde{\nu}(\tau) = e^{-\mu_\nu \tau}\tilde{\nu}(0) + \int_0^\tau e^{-\mu_\nu s}\sigma_{\gamma 2} dW_2(s) \tag{A.146}$$

Setting $\tau = 0.25$ yields a quarterly model and $\tau = 1$ makes it annual. Substituting the stochastic integrals random variables and defining parameters,

$$\tilde{\gamma}_{t+1} = \rho_\gamma \tilde{\gamma}_t + \varsigma_{\gamma 1}\varepsilon_{1,t+1} + \varsigma_{\gamma 2}\varepsilon_{2,t+1} \tag{A.147}$$
$$\tilde{\nu}_{t+1} = \rho_\nu \tilde{\nu}_t + \varsigma_{\nu 2}\varepsilon_{2,t+1} \tag{A.148}$$

Here the persistence parameters are $\rho_\gamma = e^{-\mu_\gamma \tau}$, $\rho_\nu = e^{-\mu_\nu \tau}$, random variables $(\varepsilon_{1,t+1}, \varepsilon_{2,t+1})$ are independent standard normals, and the volatilities of innovations are

$$\varsigma_{\gamma 1} = \sigma_{\gamma 1} \sqrt{\mathbb{V}\left[\int_0^\tau e^{-\mu_\gamma s} dW_1(s)\right]} = \sigma_{\gamma 1} \sqrt{\mathbb{E}\left[\int_0^\tau e^{-2\mu_\gamma s} ds\right]} = \sigma_{\gamma 1} \sqrt{\frac{1 - e^{-2\mu_\gamma \tau}}{2\mu_\gamma}} \tag{A.149}$$

Similarly,

$$\varsigma_{\gamma 2} = \sigma_{\gamma 2} \sqrt{\frac{1 - e^{-2\mu_\gamma \tau}}{2\mu_\gamma}} \tag{A.150}$$

$$\varsigma_{\nu 2} = \sigma_{\nu 2} \sqrt{\frac{1 - e^{-2\mu_\nu \tau}}{2\mu_\nu}} \tag{A.151}$$

It is more natural to report these parameters as a share of the steady-state values γ and ν. Table 13 reports them for the quarterly and annual frequencies.

Table 13: Finite time parameters.

	ρ_γ	ρ_ν	$\varsigma_{\gamma 1}$	$\varsigma_{\gamma 2}$	$\varsigma_{\nu 2}$
quarterly	0.608	0.455	$0.934 \cdot \gamma$	$-0.427 \cdot \gamma$	$0.129 \cdot \nu$
annual	0.137	0.043	$1.387 \cdot \gamma$	$-0.634 \cdot \gamma$	$0.171 \cdot \nu$

The correlation between innovations to $\tilde{\gamma}_{t+1}$ and $\tilde{\nu}_{t+1}$ is -0.42 at all horizons. The negative sign comes from the negative loading of $\gamma(t)$ on dW_2.

F Numerical solution algorithm

Here I briefly describe the algorithm I use to solve the system of partial differential equations in Proposition 1. The system is for prices $p(w,t)$ and density $g(w,t)$:

$$r(t)p(w,t) - \partial_t p(w,t) = y(w,t) + \mu_w(w,t)\partial_w p(w,t) + \frac{1}{2}\sigma_w(w,t)^2\partial_{ww}p(w,t) \tag{A.152}$$

$$\partial_t g(w,t) = -\partial_w[\mu_w(w,t)g(w,t)] + \frac{1}{2}\partial_{ww}[\sigma_w(w,t)^2 p(w,t)] \tag{A.153}$$

Here the function $y(w,t)$ is the risk-adjusted payoff:

$$y(w,t) = \nu(t) - \left(\frac{\sigma}{1-\epsilon(w,t)\theta(w,t)}\right)^2 \max\left\{\frac{1}{w+\varphi(t)\eta(w)}, \frac{1}{\varphi(t)\eta(w)}\left(1-\frac{\bar{\theta}w}{p(w,t)}\right)\right\} \tag{A.154}$$

with $\epsilon(w,t) = w/p(w,t)\cdot\partial_w p(w,t)$ being the wealth elasticity of price.

The partial differential equation (A.152) is non-linear. The price $p(w,t)$ is explicitly included in $y(w,t)$. Moreover, the drift and volatility of wealth $\mu_w(w,t)$ and $\sigma_w(w,t)$ depend on it. Plugging the optimal policy of investors,

$$\mu_w(w,t) = (r(t)-\rho-\lambda)w + \hat{\lambda}\hat{w}(t) + \min\left\{\frac{\mu_R(w,t)}{\sigma_R(w,t)^2}, \bar{\theta}\right\}\mu_R(w,t)w \tag{A.155}$$

$$\sigma_w(w,t) = \min\left\{\frac{\mu_R(w,t)}{\sigma_R(w,t)^2}, \bar{\theta}\right\}\sigma_R(w,t)w \tag{A.156}$$

Here the mean and volatility of returns are

$$\mu_R(w,t) = \frac{\mu_p(w,t)+\nu(t)}{p(w,t)} - r(t) \tag{A.157}$$

$$\sigma_R(w,t) = \frac{\sigma_p(w,t)+\sigma}{p(w,t)} \tag{A.158}$$

The drift and volatility of prices, $\mu_p(w,t)$ and $\sigma_p(w,t)$, in turn can be expressed in terms of $\mu_w(w,t)$ and $\sigma_w(w,t)$ using Itô's lemma:

$$\mu_p(w,t) = \partial_t p(w,t) + \mu_w(w,t)\partial_w p(w,t) + \frac{\sigma_w(w,t)^2}{2}\partial_{ww}p(w,t) \tag{A.159}$$

$$\sigma_p(w,t) = \sigma_w(w,t)\partial_w p(w,t) \tag{A.160}$$

I next describe the iterative algorithm that I use to solve for $p(w,t)$ as a fixed point of equation (A.152). I first describe the steady state, where all functions just have w as their argument, and both equation (A.152) and equation (A.153) are ODE, not PDE. After that, I describe the transition dynamics.

My numerical solution requires a discretization of time and wealth spaces. In addition, I use a monotone transformation of the wealth scale from $[0,\infty)$ onto $[0,1]$ by mapping w to another function $x(w) = 1-e^{-w}$. This allows for better approximations at large levels of wealth, which is useful given that one of the boundary conditions for $p(w,t)$ is at infinity. All equations have to be corrected for this transformation using the chain rule and Itô's lemma.

Steady state. To compute the price functions in the steady state, I use a two-tier loop. In the outer loop, I solve for r, $\hat{w}$, and $G(\cdot)$, the distribution of w that produce all other quantities that clear markets. In the inner loop, I fix r, $\hat{w}$, and $\varphi = \gamma\hat{w}$ and solve for prices. Given these numbers, I iterate on the price functions in the following way:

- guess $p^{(n)}(w)$, $\mu_w^{(n)}(w)$, and $\sigma_w^{(n)}(w)$
- compute $y(w)$ and solve the time-invariant version of equation (A.152), which is on ODE instead of a PDE in the steady state, to get the new guess $p^{(n+1)}(w)$
- use the new guess $p^{(n+1)}(w)$ and old guesses $\mu_w^{(n)}(w)$ and $\sigma_w^{(n)}(w)$ in equation (A.159) and equation (A.160) get $\mu_p(w)$ and $\sigma_p(w)$
- use the new guess $p^{(n+1)}(w)$ and the newly computed $\mu_p(w)$ and $\sigma_p(w)$ to compute $\mu_R(w)$ and $\sigma_R(w)$, the mean and standard deviation of excess returns in equation (A.157) and equation (A.158)
- use the newly computed $\mu_R(w)$ and $\sigma_R(w)$ to compute the new guesses $\mu_w^{(n+1)}(w)$ and $\sigma_w^{(n+1)}(w)$ of the drift and volatility of wealth in equation (A.155) and equation (A.156)
- stop if old and new guesses from $p(w)$, $\mu_w(w)$, and $\sigma_w(w)$ are sufficiently close

In the outer loop, I use the last guesses for $\mu_w(w)$ and $\sigma_w(w)$ to solve equation (A.153). This allows me to compute the steady-state value of the average regular country wealth $\int w dG(w)$ and the total profits of the intermediary, which also takes in the last guess of $\mu_R(w)$. I then compute the steady-state value of $\hat{w}$ using the fact that wealth accumulation in the special country is zero: consumption $\hat{\rho}\hat{w}$ offsets profits, and net migration is zero. Given the new guess for $\hat{w}$, I compute the new guess for r using the intermediary's balance sheet:

$$\hat{w} = \frac{\hat{\nu}}{r}\hat{q} + \int p(w)h(w)dG(w) - \int l(w)dG(w) \tag{A.161}$$

Here the ratio $\hat{\nu}/r$ is the steady-state price of the safe asset.

Transition dynamics. I discretize the time and solve for sequences of $r(t)$ and $\hat{w}(t)$. Given guesses for these sequences, I also have a guess for the sequence $\varphi(t)$.

There is an inner loop at all nodes t of the time grid where I solve for the current price vector $p(w, t)$ and the vectors of wealth drift and volatilities $\mu_w(w, t)$ and $\sigma_w(w, t)$. This inner loop is exactly the same as in solving for the steady state price.

In the outer loop, I compute the flow profits of all investors, the evolution of wealth in all countries, and migration flows using $(\mu_p(w, t), \sigma_p(w, t), \mu_R(w, t), \sigma_R(w, t), \mu_w(w, t), \sigma_w(w, t))_{t\geq 0}$. This calculation leads to the new guess of the path of the special country's wealth $(\hat{w}(t))_{t\geq 0}$ and the global factor $(\varphi(t))_{t\geq 0}$. The new guess of the interest rate sequence $(r(t))_{t\geq 0}$ comes from differentiating the consumption goods market clearing condition with respect to time:

$$\rho \int \mu_w(w, t)dG(w, t) + \hat{\rho}\hat{w}'(t) = \nu'(t) + \hat{q}\hat{\nu}'(t) \tag{A.162}$$

The interest rate can be extracted from this equation given profit flows coming from expected excess returns $\mu_R(w, t)$. I then use the sequence space Jacobians to update the guesses.

G Details for impulse responses

The decomposition of the impact response of asset prices relies on the following fact. It is enough to know the future path of dividends $(\nu(t))_{t\geq 0}$, the interest rate $(r(t))_{t\geq 0}$, the global factor $(\varphi(t))_{t\geq 0}$, and the intermediary's wealth $(\hat{w}(t))_{t\geq 0}$ to calculate the whole path of $(p(w,t))_{t\geq 0}$. The intermediary's wealth only matters for migration flows, given the path of $(\varphi(t))_{t\geq 0}$.

Taking advantage of this, I produce decompositions on Figure 9 and Figure 13. First, for the shock to risk-taking capacity, I compute two counterfactual price sequences: one with $r(t)$ set at the steady-state r (this isolates the effect of $\varphi(t)$), and the other with $\varphi(t)$ set at the steady-state φ, isolates the effect of $r(t)$. In both cases, all other sequences are taken from the baseline general equilibrium transition dynamics.

For the output shock, I compute three counterfactual price sequences. One is with $r(t)$ and $\varphi(t)$ both held at the steady-state levels. This isolates the effect of $\nu(t)$ that directly enters the Kolmogorov backward equation for prices. Another holds $r(t)$ and $\nu(t)$ at the steady-state level, isolating the effect of $\varphi(t)$, and the last one isolates the effect of $r(t)$ by taking in constant (ν, φ).

These decompositions are not additive, since I consider relatively large shocks, and prices are highly non-linear in $r(t)$, $\varphi(t)$, and $\nu(t)$.

Expectations in cross-section. Panel (a) on Figure 12 provides cross-sections of expected holdings at three different points in time, $t = 0$, $t = 0.25$, and $t = 1$, conditional on w_0. The value of holdings at $t = 0$ conditional on w_0 is known. Holdings as a function of (w,t) are known as well. But wealth itself changes between $t = 0$ and $t = 0.25$, both due to aggregate drift and idiosyncratic shocks.

Consider any function $z(w,t)$. The time-s expectation at time t, denoted by $Z(w,t,s)$, is

$$Z(w,t,s) = \mathbb{E}\left[z(w_s,s)\middle|w_t = w\right] \tag{A.163}$$

This object satisfies the following HJB equation:

$$0 = \partial_t Z(w,t,s) + \mu_w(w,t)\partial_w Z(w,t,s) + \frac{\sigma_w(w,t)^2}{2}\partial_{ww} Z(w,t,s) \tag{A.164}$$

The terminal condition is $Z(w,s,s) = z(w,s)$. I compute holdings of domestic assets τ ahead expected at $t = 0$ by solving this partial differential equation numerically and evaluating $Z(w,0,\tau)$.

Another type of cross-section of expectations is an expected average over time. Panel (b) on Figure 12 shows the cross-section of expected wealth accumulation over the first quarter, decomposing it into parts. These expectations can be computed as follows. Take any time-varying function $\tilde{z}(w,t)$. The expected time average between t and s conditional on w_t, denoted by $\tilde{Z}(w,t,s)$, is

$$\tilde{Z}(w,t,s) = \mathbb{E}\left[\int_t^s \tilde{z}(w,\tau)d\tau\middle|w_t = w\right] \tag{A.165}$$

This object satisfies the following HJB equation:

$$0 = \tilde{z}(w,t) - \partial_t \tilde{Z}(w,t,s) + \mu_w(w,t)\partial_w \tilde{Z}(w,t,s) + \frac{\sigma_w(w,t)^2}{2}\partial_{ww}\tilde{Z}(w,t,s) \tag{A.166}$$

The terminal condition is $\tilde{Z}(w,s,s) = 0$. I evaluate $\tilde{Z}(w,0,\tau)$ for panel (b) on Figure 12.

Note that the expectation functions $Z(w,t,s)$ is essentially nested in $\tilde{Z}(w,t,s)$ if one is willing to consider the function $z(w,t)$ that incorporates Dirac's delta function:

$$\tilde{z}(w,t) = \delta(t-s)z(w,t) \tag{A.167}$$

Importantly, when plotting panel (a) and panel (b) on Figure 12, I account for initial wealth revaluation. Specifically, instead of $Z(w,0,\tau)$ and $\tilde{Z}(w,0,\tau)$, I plot $Z(W(w),0,\tau)$ and $\tilde{Z}(W(w),0,\tau)$, where the function $W(\cdot)$ maps wealth just before the shock hits into the level after revaluation.

Revaluation. Wealth revaluation in regular countries happens because asset prices jump on impact. The function $W(\cdot)$ solves the following functional equation:

$$W(w) = l(w) + p(W(w),0)h(w) \tag{A.168}$$

After revaluation, wealth consists of the old, steady-state level of deposits $l(w)$ and old holdings of risky assets $h(w)$ evaluated at the new price $p(W(w),0)$. The holdings have to be taken from just before the shock since revaluation happens before portfolios can be rebalanced. The absence of t as an argument in $h(w)$ and $l(w)$ means that these are steady-state functions. The price is evaluated at $W(w)$ since the country instantly becomes one with wealth $W(w)$ instead of w.

Wealth revaluation thus comes from two sources. First, price as a function of wealth changes relative to the steady state: $p(w,0) \neq p(w)$. Second, wealth itself changes because changing prices revalue it: $W(w) \neq w$. In my numerical procedure, I solve for $W(w)$ given $p(\cdot)$ as a function of (w,t) by iterating on guesses $W^n(\cdot)$:

$$W^{n+1}(w) = l(w) + p(W^n(w),0)h(w) \tag{A.169}$$

I evaluate the new guess $W^{n+1}(\cdot)$ on the grid by interpolation and do it until convergence.

H Details for linearization

In this section, I explain the linearization procedure. The notation convention is that a function $z(w)$ with only one argument corresponds to the steady state. Functions with tildes, like $\tilde{z}(w,t)$, correspond to the first-order deviations.

Step 1: constraint is slack. Take values of w for which the constraint is slack in the steady state. Start with the drift and volatility of the wealth:

$$\mu_w(w,t) = (r(t) - \rho - \lambda)w + \hat{\lambda}n(t) + \mu_R(w,t)\theta(w,t)w \tag{A.170}$$

$$\sigma_w(w,t) = \sqrt{\sigma_R(w,t)}\theta(w,t)w \tag{A.171}$$

Using the optimal choice, $\theta(w,t) = \mu_R(w,t)/\sigma_R(w,t)$, replace the returns in the drift:

$$\mu_w(w,t) = (r(t) - \rho - \lambda)w + \hat{\lambda}n(t) + \frac{\sigma_w(w,t)^2}{w} \tag{A.172}$$

This leads to

$$\tilde{\mu}_w(w,t) = w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) + \frac{2\sigma_w(w)}{w}\tilde{\sigma}_w(w,t) \tag{A.173}$$

Now using the definition $\sqrt{\sigma_R(w,t)} = (\sigma_p(w,t) + \sigma_y)/p(w,t)$ and $\sigma_p(w,t) = \partial_w p(w,t)\sigma_w(w,t)$,

$$\sigma_w(w,t) = \frac{\sigma_y \theta(w,t)w}{p(w,t) - \theta(w,t)\partial_w p(w,t)w} \tag{A.174}$$

Using market clearing $\mu_R(w,t)/\sigma_R(w,t) = p(w,t)/(\varphi(t)\eta(w) + w)$ and $\theta(w,t) = \mu_R(w,t)/\sigma_R(w,t)$,

$$\sigma_w(w,t) = \frac{\sigma_y w}{\varphi(t)\eta(w) + w - \partial_w p(w,t)w} \tag{A.175}$$

Expanding,

$$\tilde{\sigma}_w = \frac{\sigma_w(w)^2}{\sigma_y w}(w\partial_w \tilde{p} - \eta(w)\tilde{\varphi}(t)) \tag{A.176}$$

The market-clearing condition is

$$\mu_p(w,t) + \nu - r(t)p(w,t) = (\sigma_p(w,t) + \sigma_y)^2 \frac{1}{\varphi(t)\eta(w) + w} \tag{A.177}$$

Expanding,

$$\tilde{\mu}_p - \tilde{r}(t)p(w) - r\tilde{p} = \tilde{\sigma}_p \frac{2(\sigma_p(w) + \sigma_y)}{\varphi(t)\eta(w) + w} - \tilde{\varphi}(t)\eta(w)\frac{(\sigma_p(w) + \sigma_y)^2}{(\varphi(t)\eta(w) + w)^2} \tag{A.178}$$

Now using $\sigma_w(w) = w\theta(w)(\sigma_p(w) + \sigma_y)/p(w) = w(\sigma_p(w) + \sigma_y)/(\varphi\eta(w) + w)$,

$$\tilde{\mu}_p - \tilde{\sigma}_p \frac{2\sigma_w(w)}{w} - r\tilde{p} = \tilde{r}(t)p(w) - \tilde{\varphi}(t)\eta(w)\frac{\sigma_w(w)^2}{w^2} \tag{A.179}$$

The expressions for $\tilde{\mu}_p$ and $\tilde{\sigma}_p$ are

$$\begin{aligned}\tilde{\mu}_p &= \partial_t\tilde{p} + \mu_w(w)\partial_w\tilde{p} + p'(w)\tilde{\mu}_w + \tilde{\sigma}_w\sigma_w(w)p''(w) + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} \qquad (A.180)\\ &= \partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \mu_w\partial_w\tilde{p} + p'(w)\left(w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) + \frac{2\sigma_w(w)}{w}\tilde{\sigma}_w\right) + \tilde{\sigma}_w\sigma_w(w)p''(w)\\ \tilde{\sigma}_p &= \partial_w\tilde{p}\sigma_w(w) + p'(w)\tilde{\sigma}_w \qquad (A.181)\end{aligned}$$

Combining,

$$\begin{aligned}\tilde{\mu}_p - \tilde{\sigma}_p\frac{2\sigma_w(w)}{w} &= p'(w)(w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t)) + \partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w(w) - \frac{2\sigma_w(w)^2}{w}\right)\partial_w\tilde{p}\\ &\quad + \tilde{\sigma}_w\sigma_w(w)p''(w)\\ &= p'(w)(w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t)) - \tilde{\varphi}(t)p''(w)\frac{\sigma_w(w)^3\eta(w)}{\sigma_y w}\\ &\quad + \partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w(w) - \frac{2\sigma_w(w)^2}{w} + p''(w)\frac{\sigma_w(w)^3}{\sigma_y}\right)\partial_w\tilde{p} \qquad (A.182)\end{aligned}$$

Plugging this into equation (A.179),

$$\begin{aligned}\frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w(w) - \frac{2\sigma_w(w)^2}{w} + p''(w)\frac{\sigma_w(w)^3}{\sigma_y}\right)\partial_w\tilde{p} - r\tilde{p} &= \tilde{r}(t)(p(w) - p'(w)w) - p'(w)\hat{\lambda}\tilde{n}(t)\\ + \left(p''(w)\frac{\sigma_w(w)^3\eta(w)}{\sigma_y w} - \eta(w)\frac{\sigma_w(w)^2}{w^2}\right)\tilde{\varphi}(t) - \partial_t\tilde{p} \qquad (A.183)\end{aligned}$$

Step 2: the constraint binds. Take values of w for which the constraint binds in the steady state. The drift and volatility of the wealth are

$$\mu_w(w,t) = (r(t) - \rho - \lambda)w + \hat{\lambda}n(t) + \mu_R(w,t)\overline{\theta}w \qquad (A.184)$$
$$\sigma_w(w,t) = \overline{\theta}\sqrt{\sigma_R(w,t)}w \qquad (A.185)$$

Using market clearing $\mu_R(w,t)/\sigma_R(w,t) = (p(w,t) - \overline{\theta}w)/(\varphi(t)\eta(w))$ and the definition of $\sigma_R(w,t)$,

$$\mu_w(w,t) = (r(t) - \rho - \lambda)w + \hat{\lambda}n(t) + \frac{\sigma_w(w,t)^2}{\varphi(t)\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right) \qquad (A.186)$$
$$\sigma_w(w,t) = \frac{\overline{\theta}w\sigma_y}{p(w,t) - \overline{\theta}w\partial_w p(w,t)} \qquad (A.187)$$

Expanding,

$$\tilde{\mu}_w = \tilde{r}(t)w + \hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)^2}{\varphi^2\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right)\tilde{\varphi}(t) + \frac{2\sigma_w(w)}{\varphi\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right)\tilde{\sigma}_w + \frac{\sigma_w(w)^2}{\varphi\overline{\theta}\eta(w)w}\tilde{p} \qquad (A.188)$$
$$\tilde{\sigma}_w = \frac{\sigma_w(w)^2}{\sigma_y}\partial_w\tilde{p} - \frac{\sigma_w(w)^2}{\overline{\theta}w\sigma_y}\tilde{p} \qquad (A.189)$$

The expressions for expanded drift and volatility of the price process are

$$
\begin{aligned}
\tilde{\mu}_p &= \partial_t \tilde{p} + \mu_w(w)\partial_w \tilde{p} + p'(w)\tilde{\mu}_w + \tilde{\sigma}_w \sigma_w(w) p''(w) + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} \\
&= \partial_t \tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \mu_w \partial_w \tilde{p} + p'(w)\left(w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)^2}{\varphi^2 \eta(w)}\left(\frac{p(w,t)}{\overline{\theta} w} - 1\right)\tilde{\varphi}(t)\right) \\
&\quad + p'(w)\left(\frac{2\sigma_w(w)}{\varphi\eta(w)}\left(\frac{p(w,t)}{\overline{\theta} w} - 1\right)\tilde{\sigma}_w + \frac{\sigma_w(w)^2}{\varphi\overline{\theta}\eta(w)w}\tilde{p}\right) + p''(w)\sigma_w(w)\tilde{\sigma}_w && \text{(A.190)} \\
\tilde{\sigma}_p &= \sigma_w(w)\partial_w \tilde{p} + p'(w)\tilde{\sigma}_w && \text{(A.191)}
\end{aligned}
$$

The market clearing condition is

$$
\mu_p(w,t) + y - r(t)p(w,t) = (\sigma_p(w,t) + \sigma_y)^2 \frac{1}{\varphi(t)\eta(w)}\left(1 - \frac{\overline{\theta} w}{p(w,t)}\right) \tag{A.192}
$$

Expanding,

$$
\begin{aligned}
\tilde{\mu}_p - p(w)\tilde{r}(t) - r\tilde{p} &= 2\tilde{\sigma}_p \frac{\sigma_p(w) + \sigma_y}{\varphi\eta(w)}\left(1 - \frac{\overline{\theta} w}{p(w)}\right) - \frac{(\sigma_p(w) + \sigma_y)^2}{\varphi^2\eta(w)}\left(1 - \frac{\overline{\theta} w}{p(w)}\right)\tilde{\varphi}(t) \\
&\quad + \frac{(\sigma_p(w) + \sigma_y)^2}{\varphi\eta(w)}\frac{\overline{\theta} w}{p(w)^2}\tilde{p} && \text{(A.193)}
\end{aligned}
$$

Now using $\sigma_p(w) + \sigma_y = \sigma_w(w)p(w)/(\overline{\theta} w)$,

$$
\begin{aligned}
\tilde{\mu}_p - p(w)\tilde{r}(t) - r\tilde{p} &= \tilde{\sigma}_p \frac{2\sigma_w(w)}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta} w} - 1\right) - \frac{\sigma_w(w)^2 p(w)}{\varphi^2\overline{\theta} w(w) w}\left(\frac{p(w)}{\overline{\theta} w} - 1\right)\tilde{\varphi}(t) \\
&\quad + \frac{\sigma_w(w)^2}{\varphi\overline{\theta}\eta(w)w}\tilde{p} && \text{(A.194)}
\end{aligned}
$$

Now compute

$$
\begin{aligned}
\tilde{\mu}_p - \tilde{\sigma}_p \frac{2\sigma_w(w)}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta} w} - 1\right) &= \partial_t \tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \mu_w \partial_w \tilde{p} \\
&\quad + p'(w)\left(w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)^2}{\varphi^2\eta(w)}\left(\frac{p(w,t)}{\overline{\theta} w} - 1\right)\tilde{\varphi}(t)\right) \\
&\quad - \frac{2\sigma_w(w)^2}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta} w} - 1\right)\partial_w \tilde{p} + p'(w)\frac{\sigma_w(w)^2}{\varphi\overline{\theta}\eta(w)w}\tilde{p} + p''(w)\sigma_w(w)\tilde{\sigma}_w \\
&= \partial_t \tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w - \frac{2\sigma_w(w)^2}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta} w} - 1\right) + p''(w)\frac{\sigma_w(w)^3}{\sigma_y}\right)\partial_w \tilde{p} \\
&\quad + p'(w)\left(w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)^2}{\varphi^2\eta(w)}\left(\frac{p(w,t)}{\overline{\theta} w} - 1\right)\tilde{\varphi}(t)\right) \\
&\quad + \left(p'(w)\frac{\sigma_w(w)^2}{\varphi\overline{\theta}\eta(w)w} - p''(w)\frac{\sigma_w(w)^3}{\overline{\theta} w\sigma_y}\right)\tilde{p} && \text{(A.195)}
\end{aligned}
$$

Plugging,

$$\begin{aligned}\partial_t\tilde{p} + \partial_{ww}\tilde{p}\frac{\sigma_w(w)^2}{2} + \partial_w\tilde{p}\left(\mu_w(w) + \frac{\sigma_w(w)^3}{\sigma_y}p''(w) - \frac{2\sigma_w(w)^2}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\right)& \\ + \tilde{p}\left(\frac{\sigma_w(w)^2(p'(w)-1)}{\varphi\overline{\theta}\eta(w)w} - \frac{\sigma_w(w)^3p''(w)}{\overline{\theta}w\sigma_y} - r\right)& \\ = (p(w) - p'(w)w)\tilde{r}(t) - p'(w)\hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)^2}{\varphi^2\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\left(\frac{p(w)}{\overline{\theta}w} - p'(w)\right)\tilde{\varphi}(t)&\end{aligned} \tag{A.196}$$

Finally, acknowledging that $p(w)/(\overline{\theta}w) - p'(w) = \sigma_y/\sigma_w(w)$,

$$\begin{aligned}\partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w(w) + p''(w)\frac{\sigma_w(w)^3}{\sigma_y} - \frac{2\sigma_w(w)^2}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\right)\partial_w\tilde{p}& \\ + \tilde{p}\left(\frac{\sigma_w(w)^2(p'(w)-1)}{\varphi\overline{\theta}\eta(w)w} - \frac{\sigma_w(w)^3p''(w)}{\overline{\theta}w\sigma_y} - r\right)& \\ = \tilde{r}(t)(p(w) - p'(w)w) - p'(w)\hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)\sigma_y}{\varphi^2\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\tilde{\varphi}(t)&\end{aligned} \tag{A.197}$$

Step 3: putting it together. Now rewrite the equations together. When the constraint is slack,

$$\begin{aligned}\partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w(w) + p''(w)\frac{\sigma_w(w)^3}{\sigma_y} - \frac{2\sigma_w(w)^2}{w}\right)\partial_w\tilde{p} - r\tilde{p}& \\ = \tilde{r}(t)(p(w) - p'(w)w) - p'(w)\hat{\lambda}\tilde{n}(t) + \left(p''(w)\frac{\sigma_w(w)^3\eta(w)}{\sigma_y w} - \frac{\sigma_w(w)^2\eta(w)}{w^2}\right)\tilde{\varphi}(t)&\end{aligned} \tag{A.198}$$

When it binds:

$$\begin{aligned}\partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w(w) + p''(w)\frac{\sigma_w(w)^3}{\sigma_y} - \frac{2\sigma_w(w)^2}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\right)\partial_w\tilde{p}& \\ + \tilde{p}\left(\frac{\sigma_w(w)^2(p'(w)-1)}{\varphi\overline{\theta}\eta(w)w} - \frac{\sigma_w(w)^3p''(w)}{\overline{\theta}w\sigma_y} - r\right)& \\ = \tilde{r}(t)(p(w) - p'(w)w) - p'(w)\hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)\sigma_y}{\varphi^2\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\tilde{\varphi}(t)&\end{aligned} \tag{A.199}$$

At the boundary $\overline{w}$,

$$\lim_{w\to\overline{w}+0}\frac{1}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta}} - w\right) = 1 \tag{A.200}$$

The coefficient on $\partial_w\tilde{p}$ is continuous. Subtracting the $\overline{w}+$ and $\overline{w}-$ limits of A.198 and A.199,

$$\begin{aligned}\frac{\sigma_w(\overline{w})^2}{2}\left(\partial_{ww}\tilde{p}(\overline{w}+) - \partial_{ww}\tilde{p}(\overline{w}-)\right) = \left[p''(\overline{w})\frac{\sigma_w(\overline{w})^3\eta(\overline{w})}{\sigma_y\overline{w}} - \frac{\sigma_w(\overline{w})^2\eta(\overline{w})}{\overline{w}^2} + \frac{\sigma_w(\overline{w})\sigma_y}{\varphi^2\eta(\overline{w})}\left(\frac{p(\overline{w})}{\overline{\theta}\overline{w}} - 1\right)\right]\tilde{\varphi}(t)& \\ + \left[\frac{\sigma_w(\overline{w})^2(p'(\overline{w})-1)}{\varphi\overline{\theta}\eta(\overline{w})\overline{w}} - \frac{\sigma_w(\overline{w})^3p''(\overline{w})}{\overline{\theta}\overline{w}\sigma_y}\right]\tilde{p}&\end{aligned} \tag{A.201}$$

Now using the fact that $p'(\overline{w}) = p(\overline{w})/(\overline{\theta}\overline{w}) - \sigma_y/\sigma_w(\overline{w})$, rewrite the coefficient on $\tilde{p}$:

$$\begin{aligned}\frac{\sigma_w(\overline{w})^2(p'(\overline{w})-1)}{\varphi\overline{\theta}\eta(\overline{w})\overline{w}} - \frac{\sigma_w(\overline{w})^3 p''(\overline{w})}{\overline{\theta}w\sigma_y} &= \frac{\sigma_w(\overline{w})^2}{\overline{\theta}\overline{w}\varphi\eta(\overline{w})}\left(\frac{p(\overline{w})}{\overline{\theta}\overline{w}}-1\right) - \frac{\sigma_w(\overline{w})\sigma_y}{\overline{\theta}\overline{w}\varphi\eta(\overline{w})} - \frac{\sigma_w(\overline{w})^3 p''(\overline{w})}{\overline{\theta}\overline{w}\sigma_y} \\ &= \frac{\sigma_w(\overline{w})^2}{\overline{\theta}\overline{w}} - \frac{\sigma_w(\overline{w})\sigma_y}{\overline{\theta}\overline{w}\varphi\eta(\overline{w})} - \frac{\sigma_w(\overline{w})^3 p''(\overline{w})}{\overline{\theta}\overline{w}\sigma_y} \equiv -\frac{1}{\theta}C \qquad \text{(A.202)}\end{aligned}$$

Rewriting the coefficient on $\tilde{\varphi}(t)$,

$$\begin{aligned}\eta(\overline{w})\left[\frac{p''(\overline{w})\sigma_w(\overline{w})^3}{\sigma_y\overline{w}} - \frac{\sigma_w(\overline{w})^2}{\overline{w}^2} + \frac{\sigma_w(\overline{w})\sigma_y}{\varphi^2\eta(\overline{w})^2}\left(\frac{p(\overline{w})}{\overline{\theta}\overline{w}}-1\right)\right] &= \eta(\overline{w})\left[\frac{p''(\overline{w})\sigma_w(\overline{w})^3}{\sigma_y\overline{w}} - \frac{\sigma_w(\overline{w})^2}{\overline{w}^2} + \frac{\sigma_w(\overline{w})\sigma_y}{\varphi\eta(\overline{w})}\right] \\ &= \eta(\overline{w})C \qquad \text{(A.203)}\end{aligned}$$

Plugging this into the boundary condition equation (A.201),

$$\frac{\sigma_w(\overline{w})^2}{2}\left(\partial_{ww}\tilde{p}(\overline{w}+) - \partial_{ww}\tilde{p}(\overline{w}-)\right) = \left(\eta(\overline{w})\tilde{\varphi}(t) - \frac{\tilde{p}(\overline{w})}{\overline{\theta}}\right)\left[\frac{p''(\overline{w})\sigma_w(\overline{w})^3}{\sigma_y\overline{w}} - \frac{\sigma_w(\overline{w})^2}{\overline{w}^2} + \frac{\sigma_w(\overline{w})\sigma_y}{\varphi\eta(\overline{w})}\right] \qquad \text{(A.204)}$$

Step 4: distribution. Consider the KFE for $g(w,t)$:

$$\partial_t g(w,t) = -\partial_w(\mu_w(w,t)g(w,t)) + \frac{1}{2}\partial_{ww}(\sigma_w(w,t)^2 g(w,t)) \qquad \text{(A.205)}$$

Expanding,

$$\partial_t\tilde{g} = -\partial_w(\mu_w(w)\tilde{g}) + \frac{1}{2}\partial_{ww}(\sigma_w(w)^2\tilde{g}) - \partial_w(\tilde{\mu}_w(w)g(w)) + \partial_{ww}(\tilde{\sigma}_w\sigma_w(w)g(w)) \qquad \text{(A.206)}$$

Having computed $\tilde{p}$, one can plug $\tilde{\mu}_w$ and $\tilde{\sigma}_w$. When the constraint is slack,

$$\tilde{\sigma}_w = \frac{\sigma_w(w)^2}{\sigma_y}\partial_w\tilde{p} - \frac{\sigma_w(w)^2\eta(w)}{\sigma_y w}\tilde{\varphi}(t) \qquad \text{(A.207)}$$

$$\tilde{\mu}_w = \frac{2\sigma_w(w)^3}{\sigma_y w}\partial_w\tilde{p} - \frac{2\sigma_w(w)^3\eta(w)}{\sigma_y w^2}\tilde{\varphi}(t) + w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) \qquad \text{(A.208)}$$

When it binds,

$$\tilde{\sigma}_w = \frac{\sigma_w(w)^2}{\sigma_y}\partial_w\tilde{p} - \frac{\sigma_w(w)^2}{\overline{\theta}w\sigma_y}\tilde{p} \qquad \text{(A.209)}$$

$$\begin{aligned}\tilde{\mu}_w = &\frac{2\sigma_w(w)^3}{\sigma_y\varphi\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w}-1\right)\tilde{\partial}_w\tilde{p} - \frac{\sigma_w(w)^2}{\varphi^2\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w}-1\right)\tilde{\varphi}(t) + w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) \\ &+ \frac{\sigma_w(w)^2}{\varphi\overline{\theta}\eta(w)w}\left[1 - \frac{2\sigma_w(w)}{\sigma_y}\left(\frac{p(w,t)}{\overline{\theta}w}-1\right)\right]\tilde{p} \qquad \text{(A.210)}\end{aligned}$$

Aggregate regular country and intermediary wealth deviations are

$$\tilde{w}(t) = \int \tilde{g} w dw \tag{A.211}$$

$$\tilde{n}(t) = -\frac{\rho}{\hat{\rho}} \tilde{w}(t) \tag{A.212}$$

The US tree price satisfies

$$\hat{p}(t)q = n(t) + w(t) - \int p(w,t)g(w,t)dt \tag{A.213}$$

Expanding,

$$\tilde{\hat{p}} q = \tilde{n}(t)\left(1 - \frac{\rho}{\hat{\rho}}\right) - \int \tilde{p} g(w) dw - \int \tilde{g} p(w) dw \tag{A.214}$$

The deviations in the interest rate and the global factor satisfy

$$\tilde{r}(t) = \frac{\partial_t \tilde{\hat{p}}}{\hat{p}} - \frac{r\tilde{\hat{p}}}{\hat{p}} \tag{A.215}$$

$$\tilde{\varphi}(t) = \gamma \tilde{n}(t) + n\tilde{\gamma}(t) \tag{A.216}$$

These close the linearized model.
Step 5: numerical procedure. The discrete approximation of the PDE for $\tilde{p}$ can be written as

$$\left(A_p - \frac{1}{dt}\right)\mathbf{p}(t) = J_{rhs,r}\,\mathbf{r}(t) + J_{rhs,n}\,\mathbf{n}(t) + J_{rhs,\varphi}\,\boldsymbol{\varphi}(t) - \frac{1}{dt}\,\mathbf{p}(t+1) \tag{A.217}$$

Denoting $M_1 = (A_p - 1/dt)^{-1}$, $M_2 = 1/dt$, and $M_3 = -M_1 M_2$,

$$\begin{aligned}
\mathbf{p}(t) &= M_1 J_{rhs,r}\,\mathbf{r}(t) + M_1 J_{rhs,n}\,\mathbf{n}(t) + M_1 J_{rhs,\varphi}\,\boldsymbol{\varphi}(t) + M_3\,\mathbf{p}(t+1) \\
&= \sum_{s=0}^{T-t} (M_3)^s M_1 J_{rhs,r}\,\mathbf{r}(t+s) + \sum_{s=0}^{T-t} (M_3)^s M_1 J_{rhs,n}\,\mathbf{n}(t+s) + \sum_{s=0}^{T-t} (M_3)^s M_1 J_{rhs,\varphi}\,\boldsymbol{\varphi}(t+s) \\
&= \sum_{s=0}^{T-t} J_{p,r}(s)\,\mathbf{r}(t+s) + \sum_{s=0}^{T-t} J_{p,n}(s)\,\mathbf{n}(t+s) + \sum_{s=0}^{T-t} J_{p,\varphi}(s)\,\boldsymbol{\varphi}(t+s)
\end{aligned} \tag{A.218}$$

Here M_1, M_2, and M_3 have to be corrected to incorporate the boundary condition at $\overline{w}$.

The discrete approximation of the KFE for $\tilde{g}$ is

$$\left(A_g + \frac{1}{dt}\right)\mathbf{g}(t+1) = -A_1\,\boldsymbol{\mu}_w(t) + A_2\,\boldsymbol{\sigma}_w(t) + \frac{1}{dt}\,\mathbf{g}(t) \tag{A.219}$$

Here the matrix A_1 discretizes the operator $\partial_w \cdot g(w)\cdot$, and A_2 discretizes $\partial_{ww} \cdot \sigma_w(w) \cdot g(w)\cdot$.

Denoting $M_4 = (A_g + 1/dt)^{-1}$, $M_5 = -M_4 A_1$, $M_6 = M_4 A_2$, and $M_g = M_4 M_2$,

$$\begin{aligned}
\mathbf{g}(t+1) &= M_5\, \boldsymbol{\mu}_w(t) + M_6\, \boldsymbol{\sigma}_w(t) + M_g\, \mathbf{g}(t) \\
&= \sum_{s=0}^{t-1} (M_g)^s M_5\, \boldsymbol{\mu}_w(t-s) + \sum_{s=0}^{t-1} (M_g)^s M_6\, \boldsymbol{\sigma}_w(t-s) + (M_g)^t\, \mathbf{g}(0) \\
&= \sum_{s=0}^{t-1} J_{g,\mu}(s)\, \boldsymbol{\mu}_w(t-s) + \sum_{s=0}^{t-1} J_{g,\sigma}(s)\, \boldsymbol{\sigma}_w(t-s) + (M_g)^{t-1}\, \mathbf{g}(1)
\end{aligned} \tag{A.220}$$

The drift deviations can be written as

$$\begin{aligned}
\boldsymbol{\mu}_w(t) &= J_{\mu,p}\, \mathbf{p}(t) + J_{\mu,r}^{direct}\, \mathbf{r}(t) + J_{\mu,n}^{direct}\, \mathbf{n}(t) + J_{\mu,\varphi}^{direct}\, \boldsymbol{\varphi}(t) = \left(\sum_{s=0}^{T-t} J_{p,r}(s) + J_{\mu,r}^{direct} \delta_{s,0} \right) \mathbf{r}(t+s) \\
&+ \left(\sum_{s=0}^{T-t} J_{p,n}(s) + J_{\mu,n}^{direct} \delta_{s,0} \right) \mathbf{n}(t+s) + \left(\sum_{s=0}^{T-t} J_{p,\varphi}(s) + J_{\mu,\varphi}^{direct} \delta_{s,0} \right) \boldsymbol{\varphi}(t+s) \\
&= \sum_{s=0}^{T-t} J_{\mu,r}(s)\, \mathbf{r}(t+s) + \sum_{s=0}^{T-t} J_{\mu,n}(s)\, \mathbf{n}(t+s) + \sum_{s=0}^{T-t} J_{\mu,\varphi}(s)\, \boldsymbol{\varphi}(t+s)
\end{aligned} \tag{A.221}$$

The volatility deviations are

$$\begin{aligned}
\boldsymbol{\sigma}_w(t) &= J_{\sigma,p}\, \mathbf{p}(t) + J_{\sigma,r}^{direct}\, \mathbf{r}(t) + J_{\sigma,n}^{direct}\, \mathbf{n}(t) + J_{\sigma,\varphi}^{direct}\, \boldsymbol{\varphi}(t) = \left(\sum_{s=0}^{T-t} J_{p,r}(s) + J_{\sigma,r}^{direct} \delta_{s,0} \right) \mathbf{r}(t+s) \\
&+ \left(\sum_{s=0}^{T-t} J_{p,n}(s) + J_{\sigma,n}^{direct} \delta_{s,0} \right) \mathbf{n}(t+s) + \left(\sum_{s=0}^{T-t} J_{p,\varphi}(s) + J_{\sigma,\varphi}^{direct} \delta_{s,0} \right) \boldsymbol{\varphi}(t+s) \\
&= \sum_{s=0}^{T-t} J_{\sigma,r}(s)\, \mathbf{r}(t+s) + \sum_{s=0}^{T-t} J_{\sigma,n}(s)\, \mathbf{n}(t+s) + \sum_{s=0}^{T-t} J_{\sigma,\varphi}(s)\, \boldsymbol{\varphi}(t+s)
\end{aligned} \tag{A.222}$$

Plugging,

$$\begin{aligned}
\mathbf{g}(t+1) &= \sum_{s=0}^{t-1} J_{g,\mu}(s) \sum_{u=0}^{T-t+s} J_{\mu,r}(u)\, \mathbf{r}(t-s+u) + \sum_{s=0}^{t-1} J_{g,\mu}(s) \sum_{u=0}^{T-t+s} J_{\mu,n}(u)\, \mathbf{n}(t-s+u) \qquad \text{(A.223)} \\
&+ \sum_{s=0}^{t-1} J_{g,\mu}(s) \sum_{u=0}^{T-t+s} J_{\mu,\varphi}(u)\, \boldsymbol{\varphi}(t-s+u) + \sum_{s=0}^{t-1} J_{g,\sigma}(s) \sum_{u=0}^{T-t+s} J_{\sigma,r}(u)\, \mathbf{r}(t-s+u) \\
&+ \sum_{s=0}^{t-1} J_{g,\sigma}(s) \sum_{u=0}^{T-t+s} J_{\sigma,n}(u)\, \mathbf{n}(t-s+u) + \sum_{s=0}^{t-1} J_{g,\sigma}(s) \sum_{u=0}^{T-t+s} J_{\sigma,\varphi}(u)\, \boldsymbol{\varphi}(t-s+u) \\
&+ (M_g)^{t-1}\, \mathbf{g}(0)
\end{aligned}$$

Changing the order of summation,

$$\begin{aligned}\mathbf{g}(t+1) &= \sum_{u=0}^{t-1} J_{g,\mu}(u) J_{\mu,r}(u+s-t)\,\mathbf{r}(s) + \sum_{u=0}^{t-1} J_{g,\mu}(u) J_{\mu,n}(u+s-t)\,\mathbf{n}(s) \\ &+ \sum_{u=0}^{t-1} J_{g,\mu}(u) J_{\mu,\varphi}(u+s-t)\,\boldsymbol{\varphi}(s) + \sum_{u=0}^{t-1} J_{g,\sigma}(u) J_{\sigma,r}(u+s-t)\,\mathbf{r}(s) \\ &+ \sum_{u=0}^{t-1} J_{g,\sigma}(u) J_{\sigma,n}(u+s-t)\,\mathbf{n}(s) + \sum_{u=0}^{t-1} J_{g,\sigma}(u) J_{\sigma,\varphi}(u+s-t)\,\boldsymbol{\varphi}(s) + (M_g)^{t-1}\,\mathbf{g}(0)\end{aligned} \tag{A.224}$$

Initial revaluation $\mathbf{g}(0)$ comes from the jump in prices: $\mathbf{g}(0) = A^{reval}\,\mathbf{p}(0)$, so $\mathbf{g}(t+1)$ is

$$\begin{aligned}\mathbf{g}(t+1) &= \sum_{s=1}^{T} J^{accum}_{g,\mu,r}(t,s)\,\mathbf{r}(s) + \sum_{s=1}^{T} J^{accum}_{g,\mu,n}(t,s)\,\mathbf{n}(s) + \sum_{s=1}^{T} J^{accum}_{g,\mu,\varphi}(t,s)\,\boldsymbol{\varphi}(s) \\ &+ \sum_{s=1}^{T} J^{accum}_{g,\sigma,r}(t,s)\,\mathbf{r}(s) + \sum_{s=1}^{T} J^{accum}_{g,\sigma,n}(t,s)\,\mathbf{n}(s) + \sum_{s=1}^{T} J^{accum}_{g,\sigma,\varphi}(t,s)\,\boldsymbol{\varphi}(s) \\ &+ \sum_{s=1}^{T} J^{reval}_{g,r}(t,s)\,\mathbf{r}(s) + \sum_{s=1}^{T} J^{reval}_{g,n}(t,s)\,\mathbf{n}(s) + \sum_{s=1}^{T} J^{reval}_{g,\varphi}(t,s)\,\boldsymbol{\varphi}(s)\end{aligned} \tag{A.225}$$

Here the accumulation Jacobians are

$$J^{accum}_{g,\mu,r}(t,s) = \sum_{u=0}^{t-1} J_{g,\mu}(u) J_{\mu,r}(u+s-t) = J^{accum}_{g,\mu,r}(t+1,s+1) - J_{g,\mu}(t) J_{\mu,r}(s) \tag{A.226}$$

$$J^{accum}_{g,\mu,n}(t,s) = \sum_{u=0}^{t-1} J_{g,\mu}(u) J_{\mu,n}(u+s-t) = J^{accum}_{g,\mu,n}(t+1,s+1) - J_{g,\mu}(t) J_{\mu,n}(s) \tag{A.227}$$

$$J^{accum}_{g,\mu,\varphi}(t,s) = \sum_{u=0}^{t-1} J_{g,\mu}(u) J_{\mu,\varphi}(u+s-t) = J^{accum}_{g,\mu,\varphi}(t+1,s+1) - J_{g,\mu}(t) J_{\mu,\varphi}(s) \tag{A.228}$$

$$J^{accum}_{g,\sigma,r}(t,s) = \sum_{u=0}^{t-1} J_{g,\sigma}(u) J_{\sigma,r}(u+s-t) = J^{accum}_{g,\sigma,r}(t+1,s+1) - J_{g,\sigma}(t) J_{\sigma,r}(s) \tag{A.229}$$

$$J^{accum}_{g,\sigma,n}(t,s) = \sum_{u=0}^{t-1} J_{g,\sigma}(u) J_{\sigma,n}(u+s-t) = J^{accum}_{g,\sigma,n}(t+1,s+1) - J_{g,\sigma}(t) J_{\sigma,n}(s) \tag{A.230}$$

$$J^{accum}_{g,\sigma,\varphi}(t,s) = \sum_{u=0}^{t-1} J_{g,\sigma}(u) J_{\sigma,\varphi}(u+s-t) = J^{accum}_{g,\sigma,\varphi}(t+1,s+1) - J_{g,\sigma}(t) J_{\sigma,\varphi}(s) \tag{A.231}$$

The revaluation Jacobians are

$$J^{reval}_{g,r}(t,s) = (M_g)^{t-1} A^{reval} J_{p,r}(s) \tag{A.232}$$

$$J^{reval}_{g,n}(t,s) = (M_g)^{t-1} A^{reval} J_{p,n}(s) \tag{A.233}$$

$$J^{reval}_{g,\varphi}(t,s) = (M_g)^{t-1} A^{reval} J_{p,\varphi}(s) \tag{A.234}$$

The accumulation Jacobians can be computed recursively. This recursive procedure is the analog of the fake-news algorithm. The initial conditions are $J^{accum}_{g,\mu,r}(t,1) = J_{g,\mu}(t-1)J_{\mu,r}(0)$ and $J^{accum}_{g,\mu,r}(1,s) = J_{g,\mu}(0)J_{\mu,r}(s-1)$ for $\mathbf{r}$ and analogously for $\mathbf{n}$ and $\boldsymbol{\varphi}$. The initial conditions for $\boldsymbol{\sigma}$ parts of the Jacobian are identical.

With these Jacobians at hand, it is possible to compute the Jacobian of any moment of the distribution and any function of w. Specifically, aggregate wealth of the regular countries is

$$\mathbf{w}(t) = \sum_{s=1}^{T} W' J^{total}_{g,r}(s,t)\,\mathbf{r}(s) + \sum_{s=1}^{T} W' J^{total}_{g,n}(s,t)\,\mathbf{n}(s) + \sum_{s=1}^{T} W' J^{total}_{g,\varphi}(s,t)\,\boldsymbol{\varphi}(s) \tag{A.235}$$

The deviation of the US tree price is

$$\begin{aligned}
\hat{\mathbf{p}}(t) &= \mathbf{w}(t)\frac{1}{q}\left(1-\frac{\hat{\rho}}{\rho}\right) - \frac{1}{q}P'\,\mathbf{g}(t) - \frac{1}{q}G'\,\mathbf{p}(t) \\
&= \frac{1}{q}\sum_{s=1}^{T}\left[\left(\left(1-\frac{\hat{\rho}}{\rho}\right)W' - P'\right)J^{total}_{g,r}(s,t) - G'J_{p,r}(s-t)\right]\mathbf{r}(s) \\
&+ \frac{1}{q}\sum_{s=1}^{T}\left[\left(\left(1-\frac{\hat{\rho}}{\rho}\right)W' - P'\right)J^{total}_{g,n}(s,t) - G'J_{p,n}(s-t)\right]\mathbf{n}(s) \\
&+ \frac{1}{q}\sum_{s=1}^{T}\left[\left(\left(1-\frac{\hat{\rho}}{\rho}\right)W' - P'\right)J^{total}_{g,\varphi}(s,t) - G'J_{p,\varphi}(s-t)\right]\boldsymbol{\varphi}(s)
\end{aligned} \tag{A.236}$$

Here G is the vectorized steady-state density, P is the vectorized steady-state price, and W is the wealth grid. The convention is that $J_{p,r}(t-s) = 0$ whenever $s > t$, and the same for n and φ.

Shocks to output. All equations are very similar when there are shocks to output $\tilde{\nu}(t)$ and $\tilde{\tilde{\nu}}(t)$ instead of risk-taking capacity $\tilde{\gamma}(t)$. Indeed, $\tilde{\gamma}(t)$ only appears explicitly at the very end in $\tilde{\varphi}(t) = \gamma\tilde{n}(t) + n\tilde{\gamma}(t)$. In case of output shocks, deviations $\tilde{\varphi}(t)$ and $\tilde{n}(t)$ still enter all equations separately but the relationship is simpler, $\tilde{\varphi}(t) = \gamma\tilde{n}(t)$.

One substantial difference is that $\tilde{\nu}(t)$ directly enters the Kolmogorov backward equation for risky asset prices, while $\tilde{\tilde{\nu}}(t)$ enters the pricing equation for the safe asset. It is straightforward to add these deviations to all equations.