A Macroeconomic Model of the Cross-section of Currencies

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motivation

What drives the cross section of bilateral exchange rates?

- **Finance view:** Verdelhan (2018), Lustig, Roussanov, and Verdelhan (2011, 2014), ...
  - two factors account for most of variation
  - countries heterogeneously exposed to factors
- **This paper:**
  - bottom-up approach: a macro model with fundamental shocks to construct factors
  - fundamental heterogeneity between countries to generate exposure to factors
  - build on recent advances (Itskhoki and Mukhin (2021), Kekre and Lenel (2024))
  - offer a structural interpretation of the factor structure of exchange rates

what we do

Empirical analysis:

- "dollar factor" and "commodity factor" explain more than 50% of variation in ΔXR
- dollar-denominated assets explain country loadings on "dollar factor"
- commodity share of exports explains country loadings on "commodity factor"

Macro model of the cross-section of currencies:

- global economy with heterogeneous countries, imperfect capital mobility
- fundamental shocks to asset intermediation, output $\longrightarrow$ global financial and business cycles
- countries heterogeneous in dollar assets and exposure to commodity prices

Model interpretation of factor structure of exchange rates:

- factors reflect combination of global risk premium and economic activity fluctuations
- country loadings explained by the two sources of heterogeneity

related literature

- factor structure in exchange rates: Verdelhan (2018), Lustig, Roussanov, and Verdelhan (2011), Lustig, Roussanov, and Verdelhan (2014), Lettau, Maggiori, and Weber (2014)
- cross-country heterogeneity: Hassan (2013), Ready, Roussanov, and Ward (2017), Richmond (2019), Lustig and Richmond (2020), Koijen and Yogo (2020)
- macroeconomic models of exchange rates: Gabaix and Maggiori (2015), Kekre and Lenel (2024), Kekre and Lenel (2025), Itskhoki and Mukhin (2021), Itskhoki and Mukhin (2025), Engel and Wu (2024), Chahrour, Cormun, De Leo, Guerrón-Quintana, and Valchev (2024)

model overview

Global economy with heterogeneous countries, global financial intermediaries

Small open economies with traded and non-traded endowments, segmented asset markets

Fluctuations come from

- risk aversion of global intermediaries (global financial cycle)
- tradable or "commodity" endowments, non-tradable endowments (global business cycle)

Countries are permanently heterogeneous in

- exposure to commodity cycle;
- accumulated dollar assets

households: preferences and endowments

Continuum of small open economies indexed by $i \in [0,1]$, household + central bank

$$\max \mathbb{E}_0 \sum_{t=0}^{t=\infty} \beta^t u(\mathcal{C}(C_{it}^N, C_{it}^T))$$

$$P_{it}^N C_{it}^N + C_{it}^T = P_{it}^N Y_{it}^N + Y_{it}^T + Q_{it} B_{it} - R_{it}^{-1} Q_{it} B_{i,t+1} + T_{it} + \Pi_{it}$$

- P_{it}^N price of non-traded good, Q_{it} price of consumption basket $\mathcal{C}(C_{it}^N, C_{it}^T) = (C_{it}^N)^\alpha (C_{it}^T)^{1-\alpha}$
- save in local currency bond $B_{i,t+1}$ at R_{it}, get fiscal rebate T_{it}, profits of intermediraies Π_{it}
- non-traded endowment: $N_{it} = N(1 + x_{it})$, individual shocks x_{it}
- tradable endowment $Y_{it}^T = 1 + e_i z_t$: country-specific exposure e_i to global shock z_t
- microfoundation: raw materials and tradable final goods, high raw endowment $\longrightarrow$ high e_i

central banks

Issue local currency bonds $D_{i,t+1}$, buy foreign reserves $M_{i,t+1}$ denominated in USD

$$T_{it} = R_{it}^{-1} Q_{it} D_{i,t+1} - Q_{it} D_{it} + Q_{ut} M_{it} - R_{ut}^{-1} Q_{ut} M_{i,t+1}$$

Country resource constraint:

$$Q_{it} C_{it} + Q_{it}(D_{it} - B_{it}) = P_{it}^{N} Y_{it}^{N} + Y_{it}^{T} + R_{it}^{-1} Q_{it}(D_{i,t+1} - B_{i,t+1}) + Q_{ut}(M_{it} - R_{ut}^{-1} M_{i,t+1}) + \Pi_{it}$$

Reserve policy (dollar bonds):

$$R_{ut}^{-1} Q_{ut} M_{i,t+1} = \tau \cdot R_{t}^{-1} Q_{t} m_{i} + (1 - \tau) \cdot R_{ut}^{-1} Q_{ut} m_{i}$$

- here $Q_t \equiv \int_0^1 Q_{it}$ and $R_t^{-1} \equiv \int_0^1 R_{it}^{-1} di$, reaction parameter τ
- exogenous heterogeneity in reserve level m_i
- US is the same, except no government

global financial intermediaries

Borrow in dollars from all countries, invest in local currency in all countries + the US

- liabilities L_{t+1}: dollar reserves, taken as given
- USD value of local currency bonds $L_{i,t+1}$
- care about dollar returns $X_{i,t+1} \equiv \frac{R_{it}}{R_{ut}} \frac{Q_{i,t+1}}{Q_{u,t+1}} \frac{Q_{ut}}{Q_{it}} - 1$
- rebate profits $\Pi_{it} = Q_{ut} L_{it} X_{it}$

Choose portfolio of bonds $\{L_{i,t+1}\}$ and $L_{u,t+1}$ in different currencies: maximize

$$\mathbb{E}_t \left[\int L_{i,t+1} X_{i,t+1} di \right] - \frac{\Gamma_t}{2} \mathbb{V}_t \left[\int L_{i,t+1} X_{i,t+1} di \right] \underbrace{- \frac{1}{2\chi} \int \left(L_{i,t+1} - \int L_{j,t+1} dj \right)^2 di - \frac{1}{2\chi} L_{u,t+1}^2}_{\text{portfolio management cost}}$$

$$\text{s.t. } L_{t+1} = \int L_{i,t+1} di + L_{u,t+1}$$

Here $\Gamma_t \equiv \Gamma(1 + \gamma_t)$ is risk aversion

intermediary portfolio choice

Optimal portfolio:

$$L_{i,t+1} = L_{t+1} + \chi\left(\mathbb{E}_t[X_{i,t+1}] - \Gamma_t \mathbb{C}_t\left[X_{i,t+1}, \underbrace{\int X_{j,t+1} L_{j,t+1} dj}_{\text{total profit}}\right]\right)$$

Dollar bonds $L_{u,t+1}$: no excess returns, chosen residually purely for risk-management

$$L_{u,t+1} = \chi\left(\Gamma_t \mathbb{C}_t\left[\underbrace{\int X_{j,t+1} dj}_{\text{global ret.}}, \underbrace{\int X_{j,t+1} L_{j,t+1} dj}_{\text{total profit}}\right] - \mathbb{E}_t\left[\underbrace{\int X_{j,t+1} dj}_{\text{global ret.}}\right]\right)$$

equilibrium

Given $\{D_{it}\}$, equilibrium is a set of processes $\{P_{it}^N, Q_{it}, R_{it}\}$ and $\{C_{it}^N, C_{it}^T, B_{i,t+1}, L_{i,t+1}, M_{i,t+1}\}$ such that the quantities solve the optimization problems and the following markets clear:

$$
\begin{aligned}
C_{it}^N &= Y_{it}^N \\
\int C_{it}^T di &= \int Y_{it}^T di \\
\int L_{i,t+1} + L_{u,t+1} &= \int M_{i,t+1} \\
\frac{R_{ut}^{-1} Q_{ut}}{R_{it}^{-1} Q_{it}} L_{i,t+1} + B_{i,t+1} &= D_{i,t+1}
\end{aligned}
$$

Shocks: random walks $z_{t+1} = z_t + \sigma_z \epsilon_{z,t+1}$ and $x_{i,t+1} = x_{it} + \sigma_x \epsilon_{i,t+1}$ and

$$\gamma_{t+1} = (1 - \rho_\gamma)\gamma_t + \sigma_\gamma \epsilon_{\gamma,t+1}$$

solution

First-order approx. around steady state in the limit of small shocks, large risk aversion

- simultaneously $\beta \longrightarrow 1$, $(\sigma_z, \sigma_x, \sigma_\gamma) \longrightarrow 0$ and $\Gamma \longrightarrow \infty$ with $\Gamma \cdot \overline{\sigma}_z^2 \longrightarrow \Gamma_z$ and $\Gamma \cdot \overline{\sigma}_x^2 \longrightarrow \Gamma_x$
- steady state symmetric in prices, $Q_i = P_i = 1$, and consumption $C_i = 1$
- steady-state financial inflows $\{l_i\}_i \in [0,1]$ and l_u are pinned down and asymmetric

Proposition 1

Steady-state holdings of US bonds are positive: $l_u > 0$. Holdings in other countries are

$$l_i = L - l_u - \psi(e_i - e) + \tau l_u (m_i - m)$$

Here the coefficient ψ is a function of parameters and has <u>the same sign</u> as $e - e_u$.

exchange rates and capital flows

Exchange rate determined by endowments, inflows $\Delta l_{i,t+1}$, and reserve accumulation Δm_{t+1}

$$q_{it} = \alpha e_i z_t + \alpha x_{it} + \underbrace{\alpha l_i \Delta l_{i,t+1}}_{\text{inflows}} - \underbrace{\alpha m_i \Delta m_{i,t+1}}_{\text{reserves}} + \alpha(m_i - l_i) r_{ut}$$

Reserve accumulation $\Delta m_{i,t+1} \equiv \Delta m_{t+1}$:

$$\Delta m_{t+1} = \tau(\underbrace{q_t - q_{ut}}_{\text{USD}\downarrow} + r_{ut} - r_t)$$

Capital inflows:

$$\alpha l_i \Delta l_{i,t+1} = \underbrace{\alpha(l_i - L)\gamma_t}_{\text{risk aversion}} + \underbrace{\alpha L \Delta m_{t+1}}_{\text{liabilities}} + \underbrace{\chi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}]}_{\text{expected excess returns}}$$

Euler equations

solving the model

Look for a linear equilibrium:

$$q_{it} = \xi_i x_{it} + \underbrace{\theta_i z_t + \theta_{iu} x_{ut} + \mu_i \gamma_t}_{\text{global exogenous shocks}} + \underbrace{\omega_i l_{it} + \delta_i l_{ut} + \zeta_i m_t}_{\text{endogenous states}}$$

$$q_{ut} = \theta_u z_t + \theta_{uu} x_{ut} + \mu_u \gamma_t + \omega_u l_{ut} + \zeta_u m_t$$

US non-traded endowment shock:

- does not affect global demand for goods
- affect global finance through US exchange rate

commodity shock z_t

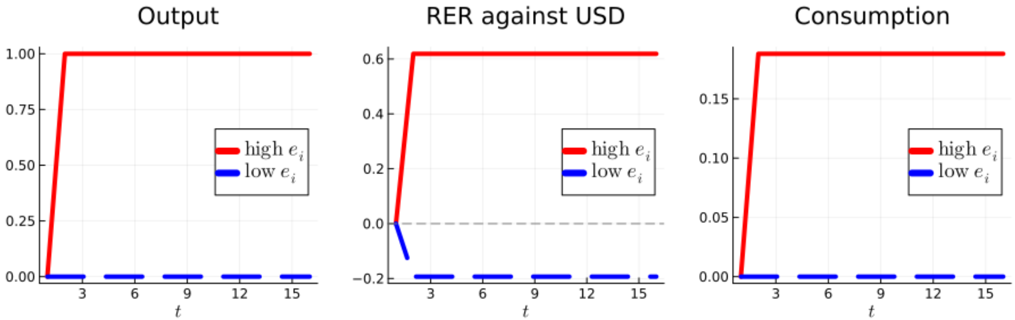


Figure: responses to a permanent innovation in the commodity shock z_t.

US non-traded output shock x_{ut}

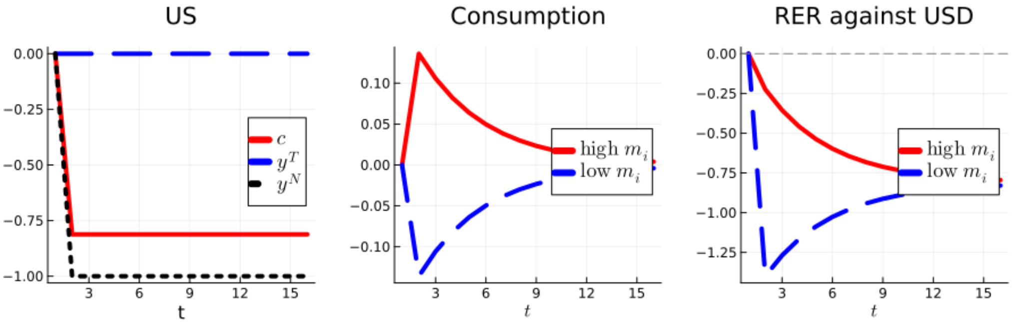


Figure: responses to a permanent innovation in the US non-traded output shock x_{ut}.

risk-aversion shock γ_t

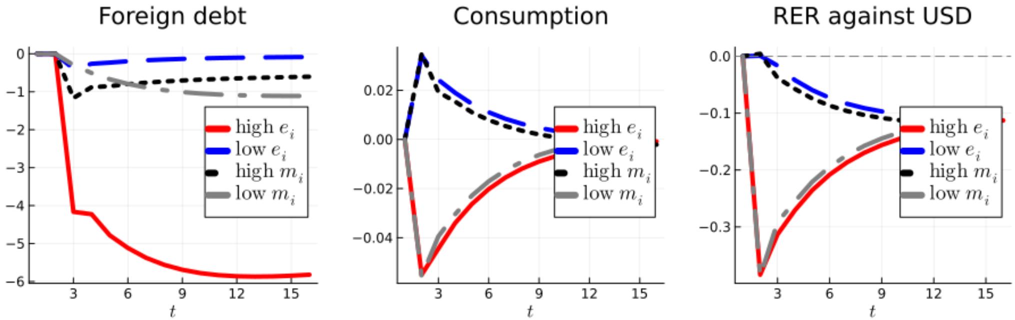


Figure: responses to a transitory risk-aversion shock γ_t.

dollar and commodity factors

Let $E_L = [\underline{e}, \text{med}\{e\}]$ and $E_H = [\text{med}\{e\}, \bar{e}]$

- let e_h and e_l be the averages over E_H and E_L

Define $\Delta s_{i,t+1} \equiv \Delta q_{i,t+1} - \Delta q_{u,t+1}$ and define the dollar and commodity factors as

$$d_{t+1} \equiv \int_{\underline{e}}^{\bar{e}} \Delta s_{i,t+1} di$$
$$f_{t+1} \equiv \int_{E_H} \Delta s_{i,t+1} di - \int_{E_L} \Delta s_{i,t+1} di$$

factor construction

Proposition 2

Take limits $\alpha \longrightarrow 1$, $\chi \longrightarrow 0$, $(\chi\Gamma_z, \chi\Gamma_x) \longrightarrow (\chi_z, \chi_x) > 0$. Define $\mathcal{D}\gamma_{t+1} \equiv \Delta\gamma_{t+1} - \Delta\gamma_t$. The dollar and commodity factors have the following composition (here $\psi \propto e - e_u$):

$$d_{t+1} = (e_u - e) \cdot \Delta z_{t+1} + \Delta z_{u,t+1} + 2l_u \cdot \mathcal{D}\gamma_{t+1}$$

$$f_{t+1} = (e_h - e_l) \cdot \Delta z_{t+1} - \psi(e_h - e_l) \cdot \mathcal{D}\gamma_{t+1}$$

- dollar factor decreases in Δz_{t+1} as long as $e > e_u$
- dollar factor increases in $\mathcal{D}\gamma_{t+1}$: priced dollar risk + flight to safety
- commodity factor increases in Δz_{t+1} by construction
- commodity factor decreases in $\mathcal{D}\gamma_{t+1}$ due to priced global productivity risk

factor loadings

Proposition 3

Bilateral exchange rate appreciation against the dollar has the form

$$\Delta s_{i,t+1} = \beta_{i,d} \cdot d_{t+1} + \beta_{i,f} \cdot f_{t+1} + \underbrace{\epsilon_{i,t+1} - \tau l_u (m_i - m) \cdot \mathcal{D}\gamma_{t+1}}_{\text{residual}}$$

Here factor loadings are

$$\beta_{i,d} = \tau(m_i - m) - 1$$

$$\beta_{i,f} = \frac{e_i - e}{e_h - e_l}$$

empirical model

Bilateral exchange rate appreciation against the USD $\Delta s_{i,t+1}$ regressed on the factors

$$\Delta s_{i,t+1} = \beta_{i,d} d_{t+1} + \beta_{i,f} f_{t+1} + \epsilon_{i,t+1}$$

Verdelhan (2018) shows that two factors explain more than 50% of cross-sectional variation

- uses dollar factor and carry factor (based on interest differentials)
- we replace carry factor with a commodity factor, same explanatory power
- correlation between carry and commodity factors ~ 0.5

	extended sample	baseline sample	Verdelhan (2018) sample
R^2 : dollar + commodity	0.456	0.507	0.522
R^2 : two PC	0.558	0.600	0.619

commodity factor loadings | dollar factor loadings | explanatory power

cross-sectional determinants

Factor loadings regressed on the cross-sectional measures:

- USD portfolio assets u_i (normalized by total assets)
- share of raw materials in exports f_i

Two specifications:

$$\beta_{i,d} = \theta_{d,u} u_i + \theta_{d,r} f_i + \epsilon_{d,i}$$

$$\beta_{i,f} = \theta_{f,u} u_i + \theta_{f,r} f_i + \epsilon_{f,i}$$

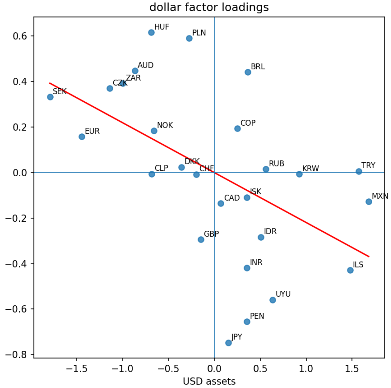
regression results

Table: cross-sectional regressions of $\beta_{i,d}$ and $\beta_{i,f}$ on USD assets and raw materials

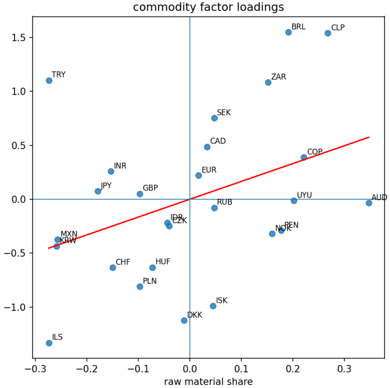
	$\beta_{i,d}$	$\beta_{i,f}$
USD assets	-0.22	0.64
	[-2.95]	[3.90]
Raw material share	0.54	1.65
	[1.46]	[2.03]
Constant	0.85	-0.41
	[7.95]	[-1.76]
N	26	26
R^2	0.27	0.61

why USD enters commodity loadings | targeting slopes in the model

factor loadings in cross-section



(a) regression of $\beta_{i,d}$ on USD assets (controlling for raw material share)



(b) regression of $\beta_{i,f}$ on raw material share (controlling for USD assets)

cross-partials

calibration

	model	data
$\text{std}(\Delta y_{t+1})$	1.96%	1.33%
$\text{std}(\Delta c_{t+1})$	1.28%	1.47%
$\text{std}(\Delta s_{t+1})$	4.03%	4.82%
$\text{corr}(\Delta s_{t+1}, \Delta y_{t+1})$	0.14	0.09
$\text{corr}(\Delta s_{t+1}, \Delta c_{t+1})$	0.03	0.03
$\text{corr}(\Delta s_{t+1}, \Delta c_{t+1} - \Delta c^{\text{US}}_{t+1})$	-0.02	0.07
$\text{corr}(\Delta s_{t+1}, \Delta s_t)$	0.09	0.21
$\text{corr}(\Delta y_{t+1}, \Delta y^{\text{US}}_{t+1})$	0.50	0.71
US NFA/GDP	-1.11	-1.00

macro calibration targets ($s_t \equiv q_t - q_t^{\text{US}}$).

	model	data
$\text{std}(d_{t+1})$	3.50	2.20
$\text{std}(f_{t+1})$	0.60	2.37
$\partial\beta_{i,f}/\partial r_i$	2.82	1.65
$\partial\beta_{i,d}/\partial u_i$	0.19	0.22

factor slope targets.

variance decomposition of factors and exchange rates

Table: variance decomposition (shares of R^2)

	z_t	x_{ut}	γ_t	x_{it}
commodity factor f_{t+1}	0.09	0.00	<u>0.91</u>	
dollar factor d_{t+1}	0.01	0.06	<u>0.93</u>	
high e_i country	0.04	0.03	<u>0.89</u>	0.03
low e_i country	0.06	<u>0.47</u>	0.00	<u>0.46</u>
high m_i country	0.00	0.00	<u>0.60</u>	<u>0.40</u>
low m_i country	0.02	0.08	<u>0.88</u>	0.02

conclusion

Macroeconomic model:

- output + financial shocks $\mapsto$ dollar and commodity factors
- heterogeneity in exposure to commodities and dollar assets $\mapsto$ loadings on factors

Replicate factors and loadings in the model

Decompose variation in exchange rates into fundamental sources

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Euler equations and Backus-Smith

Consumption and exchange rate are related:

$$c_{it} = \frac{1-\alpha}{\alpha} q_{it} + x_{it}$$

Backus-Smith covariance:

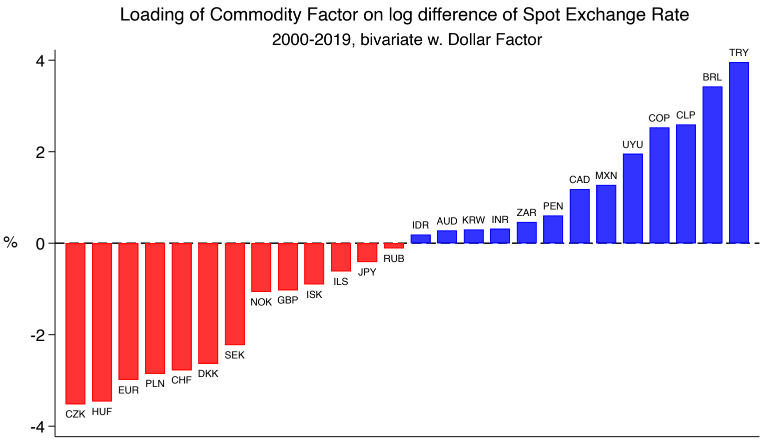
$$\mathbb{C}[c_{it} - c_{ut}, q_{it} - q_{ut}] = \frac{1-\alpha}{\alpha} \mathbb{V}[q_{it} - q_{ut}] + \mathbb{C}[q_{it} - q_{ut}, x_{it} - x_{ut}]$$

Euler equation leads to

$$r_{it} = \frac{1-\alpha}{\alpha} \mathbb{E}_t[\Delta q_{i,t+1}] + \mathbb{E}_t[\Delta x_{i,t+1}]$$

back

commodity factor loadings



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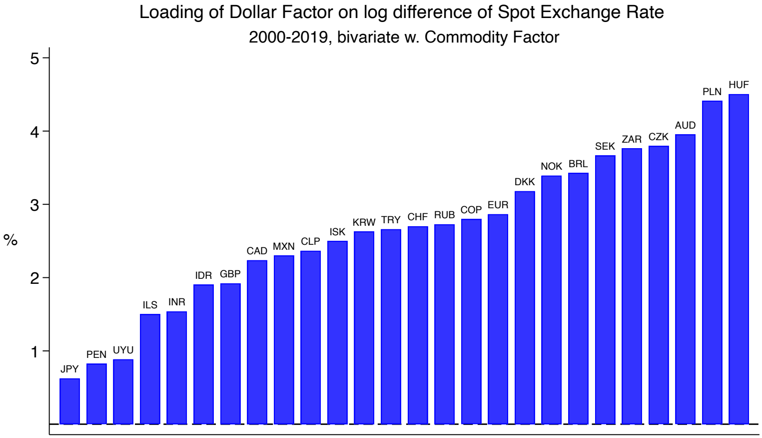
dollar factor loadings



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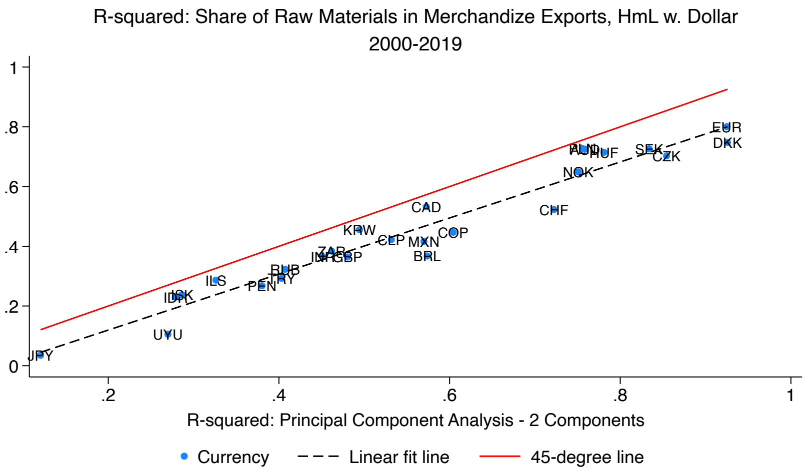
explanatory power



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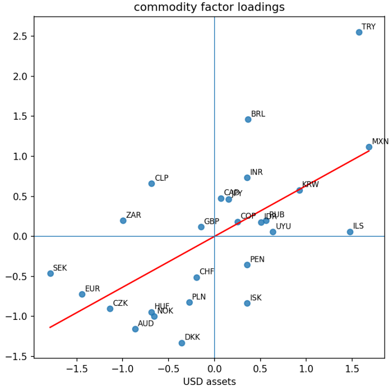
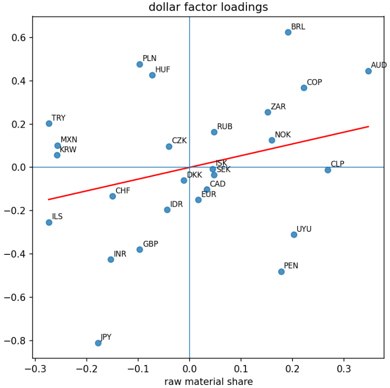
factor loadings in cross-section (cross-partials)



(a) regression of $\beta_{i,f}$ on USD assets (controlling for raw material share)



(b) regression of $\beta_{i,d}$ on raw material share (controlling for USD assets)

back

estimating a two-factor model

Imagine a misspecified model

back

$$\Delta s_{i,t+1} = \tilde{\beta}_{i,d} \cdot d_{t+1} + \tilde{\beta}_{f,i} \cdot f_{t+1} + \tilde{\epsilon}_{i,t+1}$$

Misspecification is in assuming $\mathbb{E}[\tilde{\epsilon}_{i,t+1} d_{t+1}] = \mathbb{E}[\tilde{\epsilon}_{i,t+1} f_{t+1}] = 0$.

Proposition 4

Take the population coefficients $(\tilde{\beta}_{i,f}, \tilde{\beta}_{i,d})$ under the misspecified model. With $\mathbb{C}[e_i, m_i] = 0$,

- the population coefficient $\tilde{\beta}_{i,f}$ is increasing in $e_i - e$ and decreasing in $m_i - m$;
- the population coefficient $\tilde{\beta}_{i,d}$ increases in $m_i - m$;
- the slope $\partial \tilde{\beta}_{i,d} / \partial (m_i - m)$ is biased downwards compared to that of the true coefficient $\beta_{i,d}$.