Research overview

Andrey Soldatenkov

Main highlights of academic career

- 2014: Obtained a PhD in mathematics from the National Research University HSE, Moscow, under the supervision of Misha Verbitsky. Thesis title: "Geometry of hypercomplex manifolds"
- 2014–2018: Postdoc at the Max Planck institute in Bonn and at the University of Bonn
- 2019–2021: Postdoc at the Humboldt University of Berlin
- 2022: Habilitation from the Humboldt University of Berlin, thesis title: "The Kuga–Satake construction for hyperkähler manifolds and its applications"
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- Differential geometry
- Algebraic geometry
- Hodge theory

My research is mainly focused on the geometric structures related to the algebra of quaternions. These structures give rise to the notions of hypercomplex and hyperkähler manifolds.

The algebra of quaternions:

$$\mathbb{H} = \{a + bi + cj + dk \mid a, b, c, d \in \mathbb{R}\}$$
$$i^2 = j^2 = k^2 = -1, \quad ij = -ji = k$$

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Reminder: Kähler manifolds

Let X be a C^∞-manifold (without boundary)

Definition

A complex structure on X is an endomorphism

$$I\colon TX \to TX,$$

such that $I^2 = -Id$, and I is integrable, meaning that

$$[T^{1,0}X, T^{1,0}X] \subset T^{1,0}X,$$

where $T^{1,0}X \subset TX \otimes \mathbb{C}$ is the eigenbundle of I with eigenvalue $\sqrt{-1}$, so that for $V \in T^{1,0}X$ we have $IV = \sqrt{-1}V$.

Equivalently: a complex structure on X is an atlas with local charts in $\mathbb{C}^n$, such that the transition functions are holomorphic.

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Let (X, I) be a complex manifold.

Definition

A Hermitian metric on (X, I) is a Riemannian metric $g \in S^2T^*X$, such that

$$g(Iu, Iv) = g(u, v)$$

for all $u, v \in TX$.

For a Hermitian metric g define $\omega(u, v) = g(Iu, v)$. Then $\omega \in \Lambda^2 X$.

Definition

The Hermitian metric is Kähler if $d\omega = 0$
A Kähler manifold = a complex manifold with a Kähler metric.

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Hyperkähler manifolds

Definition

A hyperkähler manifold is a C^∞-manifold X with complex structures I, J, K and a Riemannian metric g such that:

- $IJ = -JI = K$;
- g *is Kähler with respect to* I, J *e* K.

We have three Kähler forms: ω_I, ω_J and ω_K.

The two-form $\sigma_I = \omega_J + \sqrt{-1}\omega_K$ is a holomorphic symplectic form on (X, I).

If X is compact, simply connected and the space of holomorphic two-forms on (X, I) is spanned by σ_I, then X is called irreducible holomorphic symplectic (IHS).

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Examples of hyperkähler manifolds

- $\mathbb{H}$ = the quaternion algebra, $\mathbb{Z}^4 \subset \mathbb{H}$ a lattice.
  Then $T = \mathbb{H}/\mathbb{Z}^4$ is a hyperkähler manifold:
  I, J, K are given by quaternionic multiplication and g is the standard flat Hermitian metric.
- Let S be a K3 surface, for example

$$S = \{(x_0 : \ldots : x_3) \mid x_0^4 + \ldots + x_3^4 = 0\} \subset \mathbb{C}P^3.$$

  $S = (X, I)$, where X is a C^∞-manifold underlying S and I is induced by the complex structure on $\mathbb{C}P^3$.
  The Calabi-Yau theorem $\Rightarrow$ there exists a hyperkähler metric on S, meaning that there exist J, K and g as in the definition above.
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Examples of hyperkähler manifolds

- Let S be a K3 surface. The symmetric power $S^{(n)} = S^n/\Sigma_n$ parametrizes n-tuples of points in S. The variety $S^{(n)}$ is singular, but there exists a resolution of singularities
$$r\colon S^{[n]} \to S^{(n)},$$
and $S^{[n]}$ is called the Hilbert scheme of n points on S. The manifold $S^{[n]}$ admits a hyperkähler metric, and $S^{[n]}$ is IHS.
- Let $T = \mathbb{C}^2/\mathbb{Z}^4$ be a complex two-dimensional torus. The Albanese morphism:
$$a\colon T^{[n+1]} \to T, \quad (x_0, \ldots, x_n) \mapsto \sum x_i$$
Then $K^nT = a^{-1}(0)$ is the generalized Kummer variety, it is IHS
- There exist two more types of IHS manifolds of dimensions 6 and 10 constructed by O'Grady
- All other known examples are deformations of the above

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The main results

- We prove structure results for the Kähler cone of the IHS manifolds;
- We study the rigid currents (a notion important for holomorphic denamics) on hyperkähler manifolds. We provide a general construction of rigid currents on IHS manifolds;
- We prove that the cohomology groups of hyperkähler manifolds (with their Hodge structures) admit embeddings into the cohomology of complex tori;
- We prove that André motives of the IHS manifolds of the known deformation types are abelian; We apply this to the study of the Mumford–Tate conjecture and absolute Hodge classes on IHS manifolds;
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- We prove that the monodromy action on all cohomology groups of IHS manifolds is determined by its action on H^2 (up to a finite ambiguity); we determine the limit mixed Hodge structures for the maximal degenerations of the IHS manifolds;
- We generalize the results about André motives to hyperkähler orbifolds;
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The Hodge conjecture and absolute Hodge classes

Let X be a complex projective manifold
that is a manifold admitting an embedding $X \hookrightarrow \mathbb{C}P^N$.

We have:

$$H^k(X,\mathbb{C}) = \oplus_{p+q=k} H^{p,q}(X)$$

Any class $\alpha \in H^{2p}(X,\mathbb{Q}) \cap H^{p,p}(X)$ is called the Hodge class.

The Hodge conjecture:
If α is a Hodge class, then α is algebraic, meaning that there exist subvarieties $V_j \subset X$, $j = 1, \ldots, L$, such that $\alpha = \sum_{j=1}^{L} a_j [V_j]$, where $a_j \in \mathbb{Q}$ and $[V_j]$ are fundamental classes of V_j.

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Since X is projective, there exist homogeneous polynomials

$$P_j(Y_0, \dots, Y_N) = \sum_s P_{j,s} Y_0^{s_0} \dots Y_N^{s_N},$$

where $j = 1, \dots, M$ e $P_{j,s} \in \mathbb{C}$, such that

$$X = \{(x_0 : \dots : x_N) \in \mathbb{C}P^N \mid P_j(x_0, \dots, x_N) = 0, j = 1, \dots, M\}.$$

Let $\tau \in \mathrm{Aut}(\mathbb{C}/\mathbb{Q})$. We can define

$$P_j^\tau(Y_0, \dots, Y_N) = \sum_s \tau(P_{j,s}) Y_0^{s_0} \dots Y_N^{s_N},$$

and the τ-conjugate variety:

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We have a canonical isomorphism

$$\varphi_\tau \colon H^k(X,\mathbb{C}) \xrightarrow{\sim} H^k(X^\tau,\mathbb{C}).$$

Definition

A class $\alpha \in H^{2p}(X,\mathbb{C})$ is absolute Hodge if $\varphi_\tau(\alpha)$ is a Hodge class for all $\tau \in \mathrm{Aut}(\mathbb{C}/\mathbb{Q})$.

The Hodge conjecture consists of two parts:

1. Every Hodge class is absolute;
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Let X be an abelian variety (that is a complex projective torus). Then all Hodge classes on X are absolute.

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Thank you!