Prof. Dr. Daniel Huybrechts
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Winter term 2015/16

Exam, Algebraic Geometry II

Problem A. (3 points)
Describe examples of smooth projective varieties X over a field k of dimension at least two for which i) the canonical bundle $\omega_{X/k}$ is ample, ii) for which $\omega^*_{X/k}$ is ample, and iii) for which neither of the two holds.

Problem B. (6 points)
Let k be an algebraically closed field with $\mathrm{char}(k) \neq 2$. Discuss degree two morphisms $X \to \mathbb{P}^1_k$ with X a smooth projective curve of genus g over k.

Problem C. (3 points)
Let C_1, C_2 be two smooth projective irreducible curves over a field k of genus g_1 and g_2, respectively. Compute all Hodge numbers of $C_1 \times C_2$.

Problem D. (5 points)
Let $\pi\colon X \to \mathbb{P}^1 \times \mathbb{P}^1$ be the blow-up of a closed point $y = ([0 : 1], [s : t]) \in \mathbb{P}^1 \times \mathbb{P}^1$. Consider the exceptional divisor $E := \pi^{-1}(y)$ and the strict transform $\tilde{\Delta} \subset X$ of the diagonal $\Delta \subset \mathbb{P}^1 \times \mathbb{P}^1$ (i.e. the closure of $\pi^{-1}(\Delta \setminus y)$).

Compute $\chi(X_t, \mathcal{O}(\tilde{\Delta}+2E)|_{X_t})$ and $h^0(X_t, \mathcal{O}(\tilde{\Delta}+2E)|_{X_t})$ for the fibres X_t of the composition $f = \mathrm{pr}_1 \circ \pi\colon X \to \mathbb{P}^1$.

Problem E. (6 points)
Compare the notion étale and unramified for a morphism $f\colon X \to Y$.

Problem F. (5 points)
Let $f\colon X \to Y$ be a projective morphism of Noetherian schemes and let $\mathcal{F}$ be a coherent sheaf on X which is flat over Y.

(i) Show that $f_*\mathcal{F}$ is torsion free.

(ii) What can be said about the higher direct images $R^i f_*\mathcal{F}$?

(iii) Find examples that show that the flatness in (i) is essential.

Problem G. (5 points)
Let $\varphi\colon C \to C$ be an endomorphism of a smooth projective connected curve C over a field k. Consider the graph and the diagonal $\Gamma_\varphi, \Delta \subset C \times C$. Decide under which conditions the invertible sheaf $\mathcal{O}(\Delta - \Gamma_\varphi)$ is of the form $\mathrm{pr}_1^*\mathcal{L}$ for some invertible sheaf $\mathcal{L}$ on C.

Klausureinsicht (review of corrected exam): Wednesday 3.08.2016, 15.00, room 3.003
The retry exam will take place: 28.09.2016, 9.00-11.00, GHS