Prof. Dr. Daniel Huybrechts
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Summer term 2016

Retry-Exam, Algebraic Geometry II

Problem A. (3 points)
Let X be the product $\prod C_i$ of smooth projective curves C_i over a field k. Decide under which condition the canonical bundle $\omega_{X/k}$ is trivial or ample. Is in the remaining cases $\omega_{X/k}^*$ automatically ample?

Problem B. (6 points)
Discuss the notion of flatness for morphisms and its properties, give examples. Explain the relation of flatness to the Hilbert polynomial along the fibres of a projective morphism (with the idea of proof).

Problem C. (4 points)
Let $X \subset \mathbb{P}_k^3$ be a smooth hypersurface of degree four. Compute the Hodge numbers $h^{p,q}(X)$.

Problem D. (3 points)
Let $\sigma\colon X \to \mathbb{P}_k^2$ be the blow up in the two points $p_1 := [0:0:1]$ and $p_2 := [0:1:1]$ with the exceptional divisor $E = E_1 \sqcup E_2$. Compute the degree of $\mathcal{O}(E)|_{\widetilde{C}}$ for the strict transform $\widetilde{C}$ (i.e. the closure of $\sigma^{-1}(C \setminus \{p_1, p_2\})$) of the three curves $C = C_1, C_2$, and C_3, where C_1 is the line through p_1, p_2, $C_2 = Z(x_0 - x_1)$, and $C_3 = Z(x_0^2 - x_1^2)$.

Problem E. (3 points)
Discuss the notion of smoothness for morphisms of schemes, give examples of smooth and non-smooth morphisms.

Problem F. (6 points)
Let $f\colon C \to D$ be a morphism between smooth projective curves over a field k and let $\mathcal{L}$ be an invertible sheaf on C.

(i) Show that for f finite the direct image sheaf $f_*\mathcal{L}$ is a locally free sheaf of rank $\deg(f)$.

(ii) What can be said about the higher direct images $R^i f_*\mathcal{L}$?

(iii) Describe an example that shows that the smoothness of D is essential in (i). What about the smoothness of C?

Problem G. (5 points)
Consider the morphism $\varphi\colon \mathbb{P}_k^1 \to \mathbb{P}_k^2$, $[x_0 : x_1] \mapsto [x_0^2 : x_0x_1 : x_1^2]$. Describe the normal bundle of its graph $\Gamma_\varphi \subset \mathbb{P}_k^1 \times \mathbb{P}_k^2$ as a sum of line bundles.

Klausureinsicht (review of corrected exam): Thursday 29.09.2016, 14.00, room 3.003