Prof. Dr. Daniel Huybrechts
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Winter term 2015/16

Exam, Algebraic Geometry I

Problem A. (4 points)
Let $\varphi\colon \mathcal{L} \to \mathcal{M}$ be a homomorphism of invertible sheaves on a scheme (or, more generally, a locally ringed space). Show that φ is an isomorphism if φ is surjective and give an example where φ is injective but not an isomorphism.

Problem B. (2 points)
Let X be a projective regular curve over an algebraically closed field k and let $x_1, x_2 \in X$ be two closed points. Show that $\chi(X, \mathcal{O}_X(x_1 - x_2))$ is independent of the chosen points x_1, x_2.

Problem C. (4 points)
Consider the standard open set $D(x) \subset \mathbb{A}^1_k = \mathrm{Spec}(k[x])$ and the coherent sheaf $\mathcal{F} = \tilde{M}$, where M is the $k[x, x^{-1}]$-module $k[x]/(x-1) \oplus k[x, x^{-1}]$. Describe a coherent extension of $\mathcal{F}$ to $\mathbb{A}^1_k$, i.e. a coherent sheaf $\mathcal{G}$ on $\mathbb{A}^1_k$ with $\mathcal{G}|_{D(x)} \cong \mathcal{F}$. Is this extension unique?

Problem D. (6 points)
Let k be an algebraically closed field, $\mathrm{char}(k) \neq 3$. Consider the scheme $X \subset \mathbb{P}^1_k \times \mathbb{P}^2_k$ for which the fibres of the first projection $X \to \mathbb{P}^1_k$ over closed points $[t_0 : t_1]$ are the curves $X_{[t_0:t_1]} \subset \mathbb{P}^2_k$ given by the equation $t_0(x_0^3 + x_1^3 + x_2^3) + t_1 x_0 x_1 x_2 = 0$. Describe X as the zero locus of a section of a line bundle. (The scheme X is the total space of the Hesse pencil.) Find a closed point $[t_0 : t_1]$ for which the fibre is irreducible and a closed point for which the fibre is reducible. Is the generic fibre integral? (You will get extra points if you will consider this last question for the geometric generic fibre.)

Problem E. (4 points)
Let $\varphi\colon \mathbb{P}^n_k \to X$ be a morphism of projective k-schemes. Show that either the image of φ consists of a single point or that φ is quasi-finite.

Problem F. (7 points)
Let $X = \mathbb{P}^1_k$ over a field k. Consider the short exact sequences $0 \to \mathcal{O}_X \to \mathcal{K}_X \to \mathcal{K}_X/\mathcal{O}_X \to 0$ and $0 \to \mathcal{O}_X^* \to \mathcal{K}_X^* \to \mathcal{K}_X^*/\mathcal{O}_X^* \to 0$. Do they define flasque resolutions of $\mathcal{O}_X$ and $\mathcal{O}_X^*$, respectively? What can you conclude for the cohomology H^i, $i > 1$ of $\mathcal{O}_X$ and $\mathcal{O}_X^*$?

Problem G. (3 points)
Let X be a projective scheme over $k = \bar{k}$ and let $\mathcal{L} \in \mathrm{Pic}(X)$. Show that the base locus $\mathrm{Bs}(\mathcal{L})$ of $\mathcal{L}$ contains the base locus $\mathrm{Bs}(\mathcal{L}^n)$ of any power $\mathcal{L}^n$, $n > 0$. Do you think it is true that for $n > m$ one has $\mathrm{Bs}(\mathcal{L}^n) \subset \mathrm{Bs}(\mathcal{L}^m)$?

Results will not be known before Friday February 26.
Klausureinsicht (review of corrected exam): Monday 7.03.2016, 11.15 - 12.15, seminar room 0.011
The retry exam will take place on 31.03.2016, 9.00 - 11.00 in Kleiner Hörsaal, Wegelerstr. 10.