

IN.5022 — Concurrent and Distributed Computing

Representing Distributed Algorithms

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Agenda



- How can we represent distributed algorithms?
- Abstract syntax and guarded actions
- Ensuring important properties

The representation and most examples are based on [Ghosh: Distributed Systems...]

The importance of a representation

- An abstract notation helps in understanding the basic problems that are typical in distributed algorithms, in particular the three properties we will tackle today
 - Non-determinism
 - Atomicity 
 - Scheduling and fairness
- Those notions are not built in common programming languages like Java, C++, etc.
 - Languages such as Erlang (or Elixir) are adapted to express distributed algorithms based on these properties

Syntax and semantics

- We will introduce the syntax and semantics for an abstract distributed algorithm representation
 - Many alternative, we will use the one from [Gosh] (simplified)
- Example of headers and simple assignments

```
program name                                % name of program

define var1, var2, var3: integer          % definitions

initially var1 = 0                          % initialisation

var1 := 0                                    % assignments
var2, var3 := 1, 2                        % (compound...
var1, var2 := var2, var1                % ...and swap)
```

Guards

- Sometimes, an action **A** should occur only when some condition **G** holds
 - **G** is called a guard
 - **A** is a guarded action

```
<guard G> → <action A> % guard
```

- This is somewhat equivalent to a conditional statement

```
if G then A % “equivalent” conditional statement
```

Alternative constructs

- Multiple guards can be combined in an alternative construct
 - Action A_i occurs only when guard G_i is true
 - If several guards are true, the choice of action is arbitrary
 - If no guard is true, then nothing is done (*skip*)
- *Alternatives can be used to express **non-determinism** in distributed programs*



```
if                                     % alternative
>> G0 → A0                          % guards...
>> G1 → A1
...
>> Gn → An
fi
```

Repetitive constructs

- Loops can be expressed with a repetitive construct
 - Action A_i occurs only when guard G_i is true
 - If several guards are true, the choice of action is arbitrary
 - The loop is repeated as long as *at least one guard is true*
 - The loop ends *when all guards are false*
- *Repetitions also support **non-determinism***

```
do                                     % repetition
>>  $G_0 \rightarrow A_0$                  % guards...
>>  $G_1 \rightarrow A_1$ 
...
>>  $G_n \rightarrow A_n$ 
od
```



Example: non-determinism

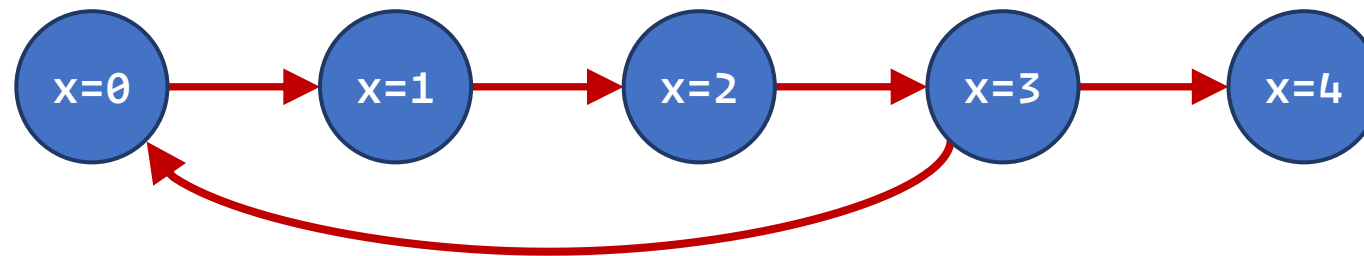
- What happens when this program executes?
 - Will it always terminate?
 - Why?

```
program uncertain  
  
define x: integer  
  
initially x = 0  
  
do  
  >> x < 4 → x := x + 1  
  >> x = 3 → x := 0  
od
```



Example: non-determinism

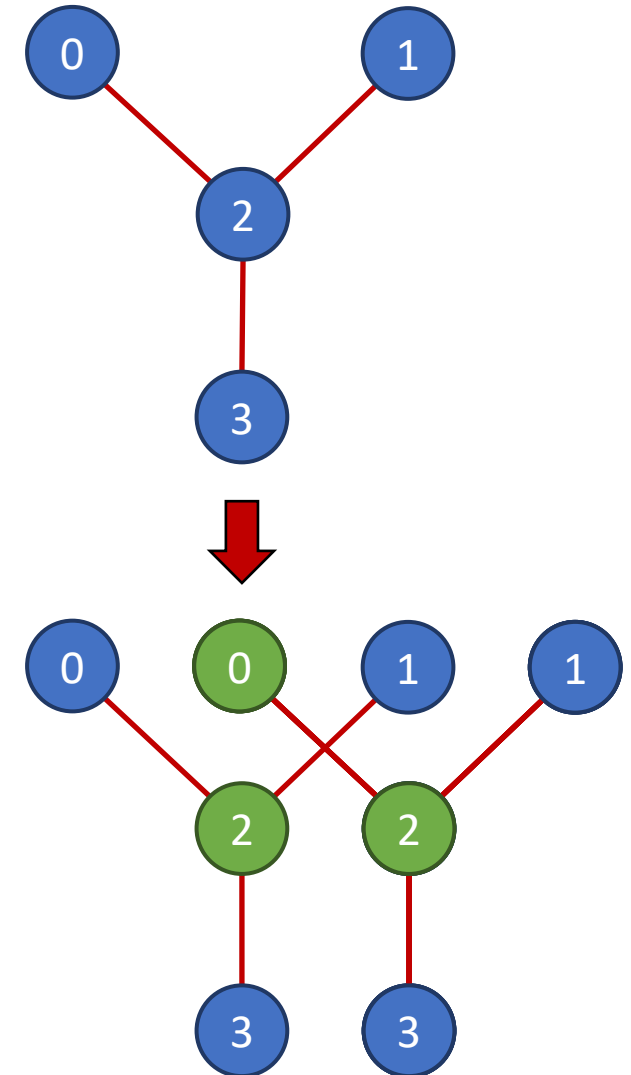
- State-transition diagram is not deterministic
 - Non-deterministic finite automaton



Example: graph colouring

- Four processes and two colours (0, 1)
 - The system must reach a configuration in which no two neighbouring processes have the same colour
 - We assume a central scheduler (one step at a time)
 - Will the program terminate?

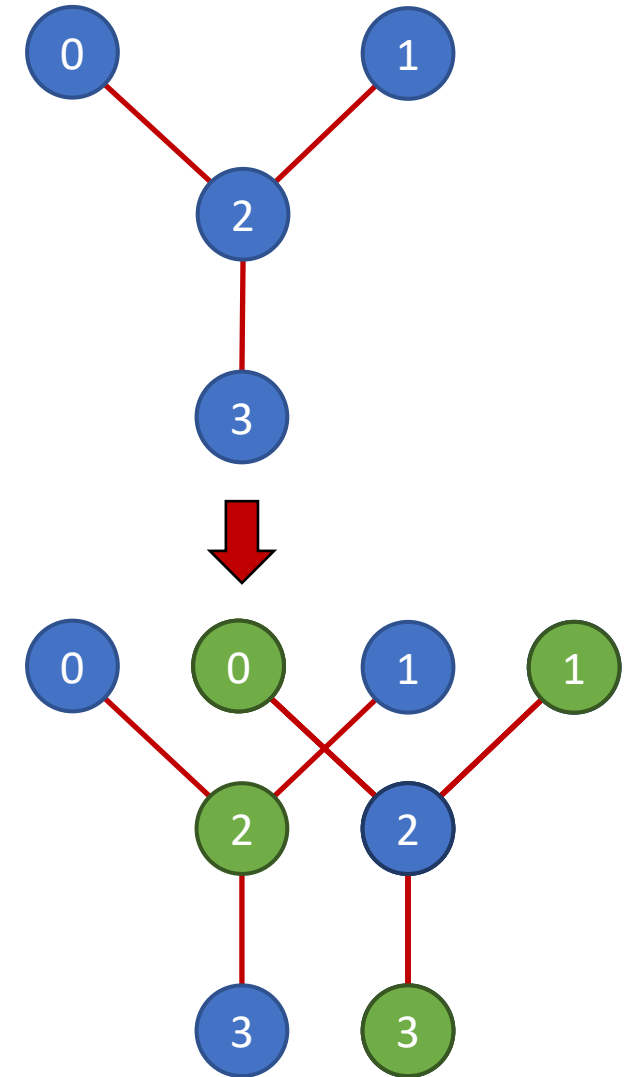
```
program colouring
define c(i): colour
                                % program for process i
do
  >>  $\exists j \in N(i) : c(i) = c(j) \rightarrow c(i) := 1 - c(i)$ 
od
```



Example: graph colouring

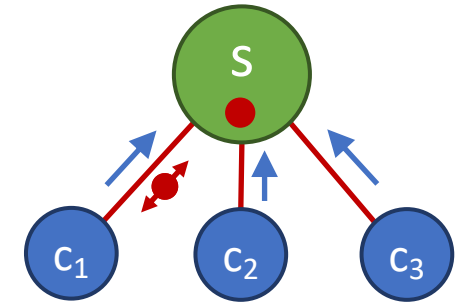
- Let's change the protocol...
 - Node will change its colour only if it is identical to that of *all* its neighbouring nodes
 - We assume a central scheduler (one step at a time)
 - Will the program terminate?

```
program colouring
define c(i): colour
                                % program for process i
do
  >>  $\forall j \in N(i) : c(i) = c(j) \rightarrow c(i) := 1 - c(i)$ 
od
```



Determinism...

- Token server with 3 clients and 1 token
 - Clients request token, perform some processing and return token to server
 - What's the problem (if any)?

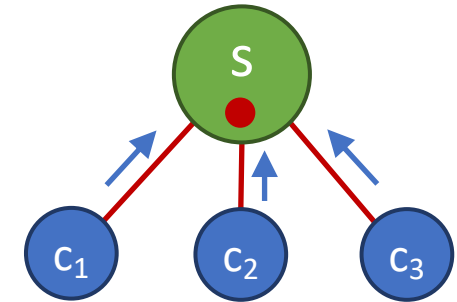


```
program token_server1
do
  if
    >> req1 ∧ token → give token to c1, wait for token back
  else if
    >> req2 ∧ token → give token to c2, wait for token back
  else if
    >> req3 ∧ token → give token to c3, wait for token back
  fi
od
```



...vs. non-determinism


- What if we use non-deterministic choices instead of “if-else”?
 - Deterministic choices are a subset of all possible non-deterministic executions
 - Will this protocol work?



```
program token_server2
do
  >> req1 ∧ token → give token to c1, wait for token back
  >> req2 ∧ token → give token to c2, wait for token back
  >> req3 ∧ token → give token to c3, wait for token back
od
```

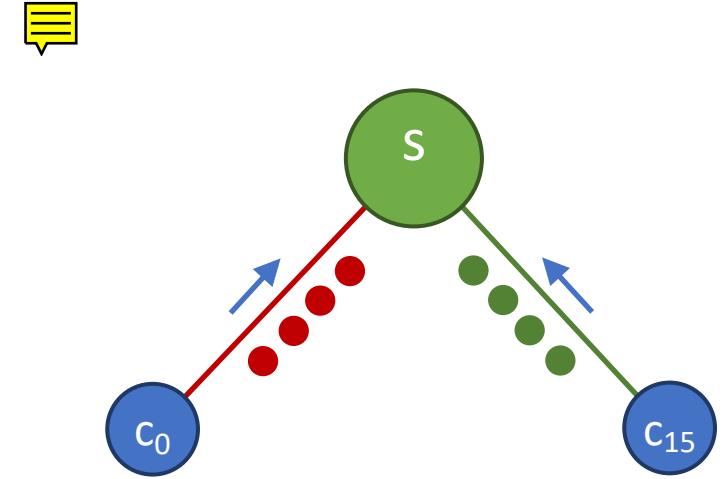


Determinism vs. non-determinism

- Should we use deterministic or non-deterministic choices?
 - Deterministic programs have the same behaviour in every run of the program
 - Non-deterministic programs might exhibit a different behaviour at each run, since the scheduler has a discretionary choice about alternative actions
- Both options are good, depending on the problem to be solved
- A system that is proven correct with non-deterministic choices will also be correct with deterministic choices 

Atomicity

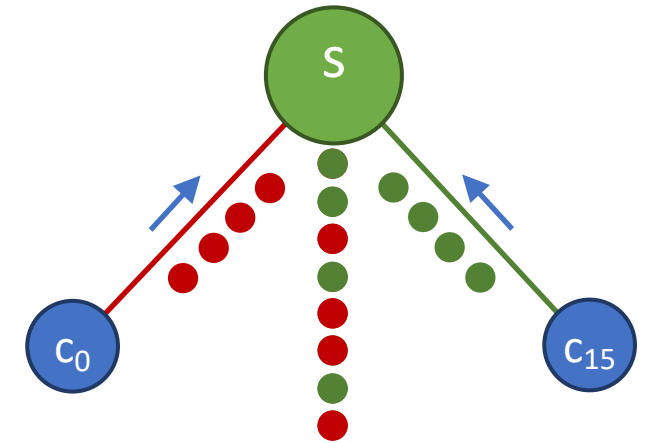
- Incoming stream of messages (each 4 bits) arriving from 2 channels
 - Red bits (0) and green bits (1)
 - Regardless non-determinism, we would expect that the value of x will be an arbitrary sequence of 0's and 15's
 - Is that so?



```
program binary
do
  >> -empty(red) → x := 0           % consume 4 red bits (0000 = 0)
  >> -empty(green) → x := 15       % consume 4 green bits (1111 = 15)
od
```

Atomicity

- Not necessarily so...
 - Depending on interleaving of red and green bits (and how assignment is done), value could be anything between 0 and 15
 - $0\mathbf{1}0\mathbf{1}$ (5) then $0\mathbf{1}1\mathbf{0}$ (6)
 - $1\mathbf{1}0\mathbf{1}$ (13) then $0\mathbf{0}1\mathbf{0}$ (2)...
 - It depends on *atomicity* of assignment
 - Atomicity often require dedicated HW or SW support (RMW operations, transactions, critical sections)
 - We henceforth assume that $\mathbf{G} \rightarrow \mathbf{A}$ is atomic



Atomicity

- Consider the following program
 - Will it terminate?
 - And what about if we “split” the first guard?


```
program switch
define a, flag: boolean
initially a = true, flag = false

do
  >> a → flag := true,
    flag := false
  >> flag AND a → a := false
od
```

```
program switch
define a, flag: boolean
initially a = true, flag = false

do
  >> a → flag := true
  >> flag → flag := false
  >> flag AND a → a := false
od
```

Fairness

- In non-deterministic programs, when multiple guards are true, there are more than one action to choose from
 - The choice of which one to take is determined by the notion of fairness 
- Fairness is a property of the scheduler and can affect the behaviour of programs
 - The scheduler is in charge of ordering the activities in a process
 - It makes the (“arbitrary”) choices in scheduling activities

Fairness as an adversarial game

- A distributed computation can be viewed as a game between the system and an adversary
 - The adversary may come up with feasible schedules to challenge the system (and cause “bad things”)
 - A correct algorithm must be able to prevent those bad things from happening
- Fairness defines the **restrictions on the scheduling of actions**
 - No restriction implies an unfair scheduler
- Fair schedulers can provide different levels of fairness
 - Unconditional fairness
 - Weak fairness
 - Strong fairness

Fairness levels



- Consider the following program
 - An **unfair** scheduler may never schedule the 2nd and 3rd actions and **x** may remain zero
 - An **unconditionally** fair scheduler eventually schedule each statement, regardless of the value of its guard
 - A **weakly fair** scheduler eventually schedules every guarded action whose guard becomes true and remains true thereafter (incl. 2nd action but excl. 3rd action)
 - A **strongly fair** scheduler eventually schedules every guarded action whose guard is true infinitely often (incl. 3rd action)


```
program scheduler
define x: integer
    % initial value unknown
do
  >> true → x := 0
  >> x = 0 → x := 1
  >> x = 1 → x := 2
od
```

Summary

- Non-determinism, atomicity and fairness are important aspects of concurrent and distributed programming
 - The semantics of the computation depend on specific assumptions about atomicity, non-determinism and scheduling policies
- Implementing distributed programs in Java, C++, etc. requires to implement not only the guards, but also the intended grain of atomicity, non-determinism and appropriate fairness of scheduler (not trivial!)
 - Languages like Erlang provide built-in support for those

Erlang syntax

- Erlang maps (quite) closely to abstract syntax
 - Example: two processes exchange a message **M** times between each other (“ping” and “pong” messages)
 - Graceful termination (via “finished” message)

```
program pong   
do  
  >> ping → reply pong  
  >> finished → break  
od
```

```
-module(pingpong).  
-export([start/0, ping/2, pong/0]).  
ping(0, Pong_PID) ->  
    Pong_PID ! finished;  
ping(N, Pong_PID) ->  
    Pong_PID ! {ping, self()},  
    receive  
        pong -> io:format("Pong!~n", [])  
    end,  
    ping(N - 1, Pong_PID).  
pong() ->                                     % function  
    receive                                     % receive message  
        {ping, Ping_PID} ->                   % guard  
            io:format("Ping!~n", []),  
            Ping_PID ! pong,                   % send message  
            pong()  
        finished -> true;  
    end.  
start() ->                                     % spawn both processes  
    Pong_PID = spawn(?MODULE, pong, []),  
    spawn(?MODULE, ping, [3, Pong_PID]).
```

Elixir syntax

- *Elixir* maps (quite) closely to abstract syntax
 - Example: two processes exchange a message **M** times between each other (“ping” and “pong” messages)
 - Graceful termination (via “finished” message)

```
program pong
do
  >> ping → reply pong
  >> finished → break
od
```

```
defmodule PingPong do
  def ping(0, pong_pid) do
    send(pong_pid, :finished)
  end
  def ping(n, pong_pid) do
    send(pong_pid, {:ping, self()})
    receive do
      :pong -> IO.puts("Pong!")
    end
    ping(n - 1, pong_pid)
  end
  def pong do
    receive do
      {:ping, ping_pid} ->
        IO.puts("Ping!")
        send(ping_pid, :pong)
        pong()
      :finished -> true
    end
  end
  def start do
    pong_pid = spawn(fn -> pong() end)
    spawn(fn -> ping(3, pong_pid) end)
  end
end
```