CHAPTER 8

Analyzing Sentence Structure

Earlier chapters focused on words: how to identify them, analyze their structure, assign them to lexical categories, and access their meanings. We have also seen how to identify patterns in word sequences or n-grams. However, these methods only scratch the surface of the complex constraints that govern sentences. We need a way to deal with the ambiguity that natural language is famous for. We also need to be able to cope with the fact that there are an unlimited number of possible sentences, and we can only write finite programs to analyze their structures and discover their meanings.

The goal of this chapter is to answer the following questions:

1. How can we use a formal grammar to describe the structure of an unlimited set of sentences?
2. How do we represent the structure of sentences using syntax trees?
3. How do parsers analyze a sentence and automatically build a syntax tree?

Along the way, we will cover the fundamentals of English syntax, and see that there are systematic aspects of meaning that are much easier to capture once we have identified the structure of sentences.

8.1 Some Grammatical Dilemmas

Linguistic Data and Unlimited Possibilities

Previous chapters have shown you how to process and analyze text corpora, and we have stressed the challenges for NLP in dealing with the vast amount of electronic language data that is growing daily. Let's consider this data more closely, and make the thought experiment that we have a gigantic corpus consisting of everything that has been either uttered or written in English over, say, the last 50 years. Would we be justified in calling this corpus "the language of modern English"? There are a number of reasons why we might answer no. Recall that in Chapter 3, we asked you to search the Web for instances of the pattern *the of*. Although it is easy to find examples on the Web containing this word sequence, such as *New man at the of IMG* (see *http://www.telegraph.co.uk/sport/2387900/New-man-at-the-of-IMG.html*), speakers of English will say that most such examples are errors, and therefore not part of English after all.

Accordingly, we can argue that "modern English" is not equivalent to the very big set of word sequences in our imaginary corpus. Speakers of English can make judgments about these sequences, and will reject some of them as being ungrammatical.

Equally, it is easy to compose a new sentence and have speakers agree that it is perfectly good English. For example, sentences have an interesting property that they can be embedded inside larger sentences. Consider the following sentences:

(1) a. Usain Bolt broke the 100m record.
    b. The Jamaica Observer reported that Usain Bolt broke the 100m record.
    c. Andre said The Jamaica Observer reported that Usain Bolt broke the 100m record.
    d. I think Andre said the Jamaica Observer reported that Usain Bolt broke the 100m record.

If we replaced whole sentences with the symbol `S`, we would see patterns like *Andre said* `S` and *I think* `S`. These are templates for taking a sentence and constructing a bigger sentence. There are other templates we can use, such as `S` *but* `S` and `S` *when* `S`. With a bit of ingenuity we can construct some really long sentences using these templates. Here's an impressive example from a Winnie the Pooh story by A.A. Milne, *In Which Piglet Is Entirely Surrounded by Water*:

> [You can imagine Piglet's joy when at last the ship came in sight of him.] In after-years he liked to think that he had been in Very Great Danger during the Terrible Flood, but the only danger he had really been in was the last half-hour of his imprisonment, when Owl, who had just flown up, sat on a branch of his tree to comfort him, and told him a very long story about an aunt who had once laid a seagull's egg by mistake, and the story went on and on, rather like this sentence, until Piglet who was listening out of his window without much hope, went to sleep quietly and naturally, slipping slowly out of the window towards the water until he was only hanging on by his toes, at which moment,

> luckily, a sudden loud squawk from Owl, which was really part of the story, being what his aunt said, woke the Piglet up and just gave him time to jerk himself back into safety and say, "How interesting, and did she?" when—well, you can imagine his joy when at last he saw the good ship, Brain of Pooh (Captain, C. Robin; 1st Mate, P. Bear) coming over the sea to rescue him...

This long sentence actually has a simple structure that begins *S but S when S*. We can see from this example that language provides us with constructions which seem to allow us to extend sentences indefinitely. It is also striking that we can understand sentences of arbitrary length that we've never heard before: it's not hard to concoct an entirely novel sentence, one that has probably never been used before in the history of the language, yet all speakers of the language will understand it.

The purpose of a grammar is to give an explicit description of a language. But the way in which we think of a grammar is closely intertwined with what we consider to be a language. Is it a large but finite set of observed utterances and written texts? Is it something more abstract like the implicit knowledge that competent speakers have about grammatical sentences? Or is it some combination of the two? We won't take a stand on this issue, but instead will introduce the main approaches.

In this chapter, we will adopt the formal framework of "generative grammar," in which a "language" is considered to be nothing more than an enormous collection of all grammatical sentences, and a grammar is a formal notation that can be used for "generating" the members of this set. Grammars use recursive **productions** of the form `S` → `S` *and* `S`, as we will explore in Section 8.3. In Chapter 10 we will extend this, to automatically build up the meaning of a sentence out of the meanings of its parts.

Ubiquitous Ambiguity

A well-known example of ambiguity is shown in (2), from the Groucho Marx movie, *Animal Crackers* (1930):

(2) While hunting in Africa, I shot an elephant in my pajamas. How an elephant got into my pajamas I'll never know.

Let's take a closer look at the ambiguity in the phrase: *I shot an elephant in my pajamas*. First we need to define a simple grammar:

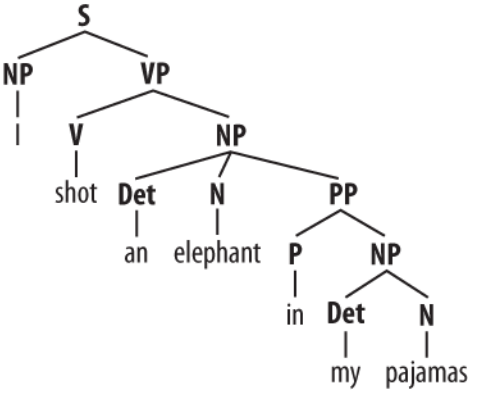
```
>>> groucho_grammar = nltk.parse_cfg("""
... S -> NP VP
... PP -> P NP
... NP -> Det N | Det N PP | 'I'
... VP -> V NP | VP PP
... Det -> 'an' | 'my'
... N -> 'elephant' | 'pajamas'
... V -> 'shot'
... P -> 'in'
... """)
```

This grammar permits the sentence to be analyzed in two ways, depending on whether the prepositional phrase *in my pajamas* describes the elephant or the shooting event.

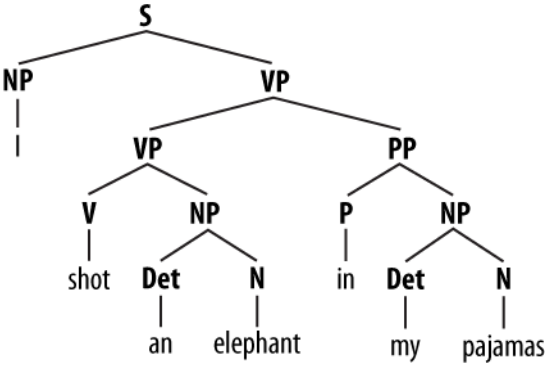
```
>>> sent = ['I', 'shot', 'an', 'elephant', 'in', 'my', 'pajamas']
>>> parser = nltk.ChartParser(groucho_grammar)
>>> trees = parser.nbest_parse(sent)
>>> for tree in trees:
...     print tree
(S
  (NP I)
  (VP
    (V shot)
    (NP (Det an) (N elephant) (PP (P in) (NP (Det my) (N pajamas))))))
(S
  (NP I)
  (VP
    (VP (V shot) (NP (Det an) (N elephant)))
    (PP (P in) (NP (Det my) (N pajamas)))))
```

The program produces two bracketed structures, which we can depict as trees, as shown in (3):

(3) a.



b.



Notice that there's no ambiguity concerning the meaning of any of the words; e.g., the word *shot* doesn't refer to the act of using a gun in the first sentence and using a camera in the second sentence.

Your Turn: Consider the following sentences and see if you can think of two quite different interpretations: *Fighting animals could be dangerous. Visiting relatives can be tiresome.* Is ambiguity of the individual words to blame? If not, what is the cause of the ambiguity?

This chapter presents grammars and parsing, as the formal and computational methods for investigating and modeling the linguistic phenomena we have been discussing. As we shall see, patterns of well-formedness and ill-formedness in a sequence of words can be understood with respect to the phrase structure and dependencies. We can develop formal models of these structures using grammars and parsers. As before, a key motivation is natural language *understanding*. How much more of the meaning of a text can we access when we can reliably recognize the linguistic structures it contains? Having read in a text, can a program "understand" it enough to be able to answer simple questions about "what happened" or "who did what to whom"? Also as before, we will develop simple programs to process annotated corpora and perform useful tasks.

8.2 What's the Use of Syntax?

Beyond n-grams

We gave an example in Chapter 2 of how to use the frequency information in bigrams to generate text that seems perfectly acceptable for small sequences of words but rapidly degenerates into nonsense. Here's another pair of examples that we created by computing the bigrams over the text of a children's story, *The Adventures of Buster Brown* (included in the Project Gutenberg Selection Corpus):

(4) a. He roared with me the pail slip down his back
    b. The worst part and clumsy looking for whoever heard light

You intuitively know that these sequences are "word-salad," but you probably find it hard to pin down what's wrong with them. One benefit of studying grammar is that it provides a conceptual framework and vocabulary for spelling out these intuitions. Let's take a closer look at the sequence *the worst part and clumsy looking*. This looks like a **coordinate structure**, where two phrases are joined by a coordinating conjunction such as *and*, *but*, or *or*. Here's an informal (and simplified) statement of how coordination works syntactically:

Coordinate Structure: if v_1 and v_2 are both phrases of grammatical category X, then v_1 *and* v_2 is also a phrase of category X.

Here are a couple of examples. In the first, two NPs (noun phrases) have been conjoined to make an NP, while in the second, two APs (adjective phrases) have been conjoined to make an AP.

(5) a. The book's ending was (NP *the worst part and the best part*) for me.
b. On land they are (AP *slow and clumsy looking*).

What we *can't* do is conjoin an NP and an AP, which is why *the worst part and clumsy looking* is ungrammatical. Before we can formalize these ideas, we need to understand the concept of **constituent structure**.

Constituent structure is based on the observation that words combine with other words to form units. The evidence that a sequence of words forms such a unit is given by substitutability—that is, a sequence of words in a well-formed sentence can be replaced by a shorter sequence without rendering the sentence ill-formed. To clarify this idea, consider the following sentence:

(6) The little bear saw the fine fat trout in the brook.

The fact that we can substitute *He* for *The little bear* indicates that the latter sequence is a unit. By contrast, we cannot replace *little bear saw* in the same way. (We use an asterisk at the start of a sentence to indicate that it is ungrammatical.)

(7) a. He saw the fine fat trout in the brook.
b. *The he the fine fat trout in the brook.

In Figure 8-1, we systematically substitute longer sequences by shorter ones in a way which preserves grammaticality. Each sequence that forms a unit can in fact be replaced by a single word, and we end up with just two elements.

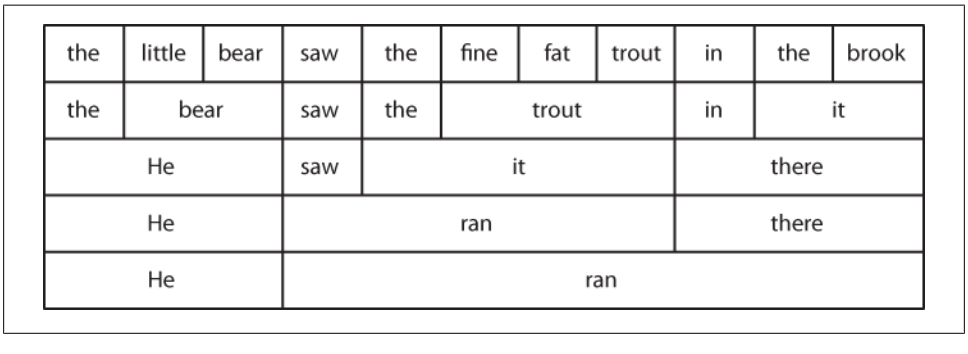


Figure 8-1. Substitution of word sequences: Working from the top row, we can replace particular sequences of words (e.g., the brook*) with individual words (e.g.,* it*); repeating this process, we arrive at a grammatical two-word sentence.*

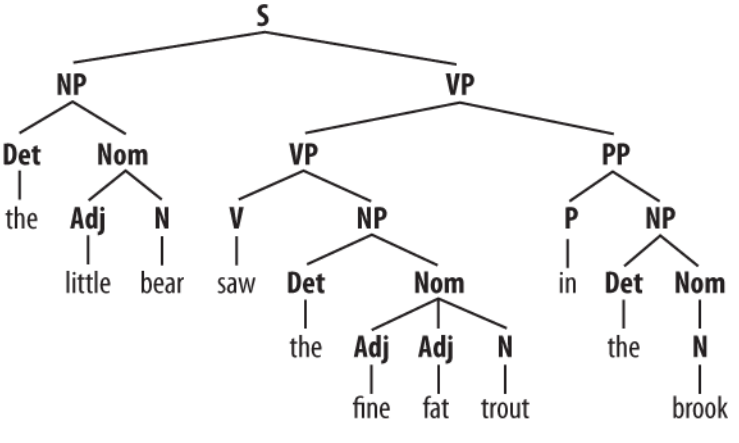
Det the	Adj little	N bear	V saw	Det the	Adj fine	Adj fat	N trout	P in	Det the	N brook
Det the	Nom bear		V saw	Det the	Nom trout			P in	NP it	
NP He			V saw	NP it				PP there		
NP He			VP ran					PP there		
NP He			VP ran							

Figure 8-2. Substitution of word sequences plus grammatical categories: This diagram reproduces Figure 8-1 along with grammatical categories corresponding to noun phrases (NP), verb phrases (VP), prepositional phrases (PP), and nominals (Nom).

In Figure 8-2, we have added grammatical category labels to the words we saw in the earlier figure. The labels NP, VP, and PP stand for **noun phrase**, **verb phrase**, and **prepositional phrase**, respectively.

If we now strip out the words apart from the topmost row, add an S node, and flip the figure over, we end up with a standard phrase structure tree, shown in (8). Each node in this tree (including the words) is called a **constituent**. The **immediate constituents** of S are NP and VP.

(8)



As we saw in Section 8.1, sentences can have arbitrary length. Consequently, phrase structure trees can have arbitrary *depth*. The cascaded chunk parsers we saw in Section 7.4 can only produce structures of bounded depth, so chunking methods aren't applicable here.

As we will see in the next section, a grammar specifies how the sentence can be subdivided into its immediate constituents, and how these can be further subdivided until we reach the level of individual words.

8.3 Context-Free Grammar

A Simple Grammar

Let's start off by looking at a simple **context-free grammar** (CFG). By convention, the lefthand side of the first production is the **start-symbol** of the grammar, typically `S`, and all well-formed trees must have this symbol as their root label. In NLTK, context-free grammars are defined in the `nltk.grammar` module. In Example 8-1 we define a grammar and show how to parse a simple sentence admitted by the grammar.

Example 8-1. A simple context-free grammar.

```
grammar1 = nltk.parse_cfg("""
  S -> NP VP
  VP -> V NP | V NP PP
  PP -> P NP
  V -> "saw" | "ate" | "walked"
  NP -> "John" | "Mary" | "Bob" | Det N | Det N PP
  Det -> "a" | "an" | "the" | "my"
  N -> "man" | "dog" | "cat" | "telescope" | "park"
  P -> "in" | "on" | "by" | "with"
  """)

>>> sent = "Mary saw Bob".split()
>>> rd_parser = nltk.RecursiveDescentParser(grammar1)
>>> for tree in rd_parser.nbest_parse(sent):
...      print tree
(S (NP Mary) (VP (V saw) (NP Bob)))
```

The grammar in Example 8-1 contains productions involving various syntactic categories, as laid out in Table 8-1. The recursive descent parser used here can also be inspected via a graphical interface, as illustrated in Figure 8-3; we discuss this parser in more detail in Section 8.4.

Table 8-1. Syntactic categories

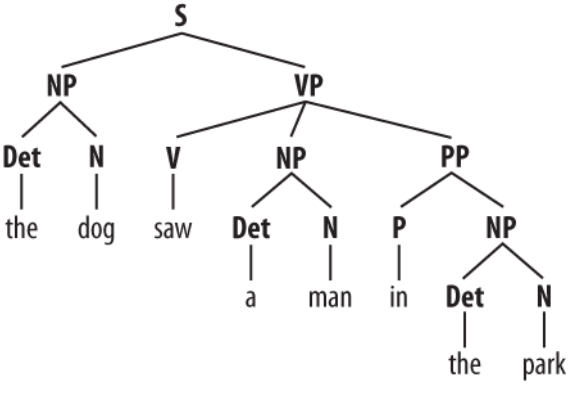
Symbol	Meaning	Example
S	sentence	*the man walked*
NP	noun phrase	*a dog*
VP	verb phrase	*saw a park*
PP	prepositional phrase	*with a telescope*
Det	determiner	*the*
N	noun	*dog*

Symbol	Meaning	Example
V	verb	*walked*
P	preposition	*in*

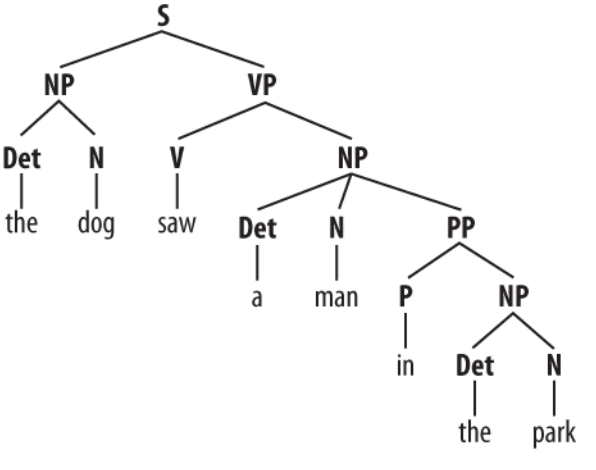
A production like `VP -> V NP | V NP PP` has a disjunction on the righthand side, shown by the `|`, and is an abbreviation for the two productions `VP -> V NP` and `VP -> V NP PP`.

If we parse the sentence *The dog saw a man in the park* using the grammar shown in Example 8-1, we end up with two trees, similar to those we saw for (3):

(9) a.



b.



Since our grammar licenses two trees for this sentence, the sentence is said to be **structurally ambiguous**. The ambiguity in question is called a **prepositional phrase attachment ambiguity**, as we saw earlier in this chapter. As you may recall, it is an ambiguity about attachment since the `PP` *in the park* needs to be attached to one of two places in the tree: either as a child of `VP` or else as a child of `NP`. When the `PP` is attached to `VP`, the intended interpretation is that the seeing event happened in the park.

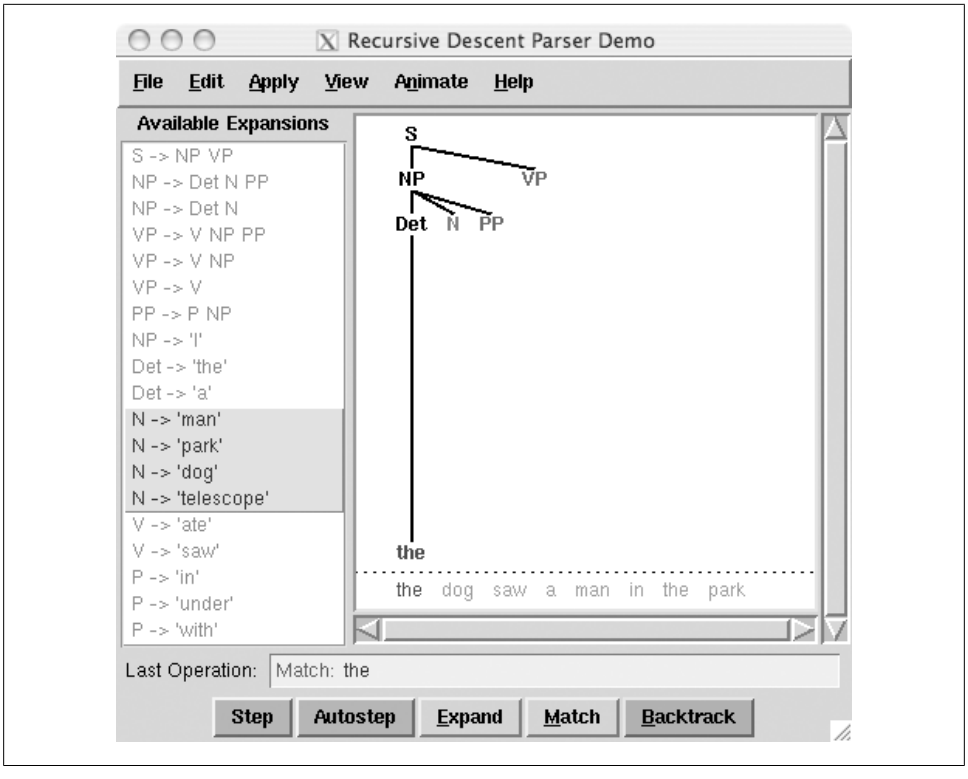


Figure 8-3. Recursive descent parser demo: This tool allows you to watch the operation of a recursive descent parser as it grows the parse tree and matches it against the input words.

However, if the `PP` is attached to `NP`, then it was the man who was in the park, and the agent of the seeing (the dog) might have been sitting on the balcony of an apartment overlooking the park.

Writing Your Own Grammars

If you are interested in experimenting with writing CFGs, you will find it helpful to create and edit your grammar in a text file, say, *mygrammar.cfg*. You can then load it into NLTK and parse with it as follows:

```
>>> grammar1 = nltk.data.load('file:mygrammar.cfg')
>>> sent = "Mary saw Bob".split()
>>> rd_parser = nltk.RecursiveDescentParser(grammar1)
>>> for tree in rd_parser.nbest_parse(sent):
...      print tree
```

Make sure that you put a *.cfg* suffix on the filename, and that there are no spaces in the string `'file:mygrammar.cfg'`. If the command `print tree` produces no output, this is probably because your sentence `sent` is not admitted by your grammar. In this case, call the parser with tracing set to be on: `rd_parser = nltk.RecursiveDescent`

`Parser(grammar1, trace=2)`. You can also check what productions are currently in the grammar with the command `for p in grammar1.productions(): print p`.

When you write CFGs for parsing in NLTK, you cannot combine grammatical categories with lexical items on the righthand side of the same production. Thus, a production such as `PP -> 'of' NP` is disallowed. In addition, you are not permitted to place multiword lexical items on the righthand side of a production. So rather than writing `NP -> 'New York'`, you have to resort to something like `NP -> 'New_York'` instead.

Recursion in Syntactic Structure

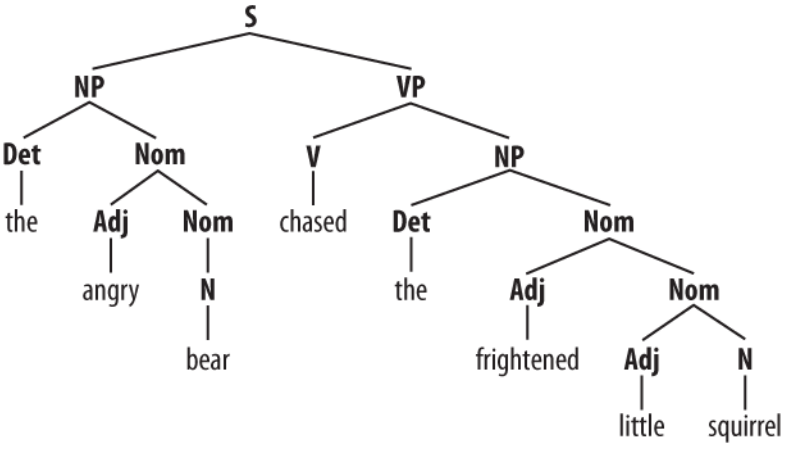
A grammar is said to be **recursive** if a category occurring on the lefthand side of a production also appears on the righthand side of a production, as illustrated in Example 8-2. The production `Nom -> Adj Nom` (where `Nom` is the category of nominals) involves direct recursion on the category `Nom`, whereas indirect recursion on `S` arises from the combination of two productions, namely `S -> NP VP` and `VP -> V S`.

Example 8-2. A recursive context-free grammar.

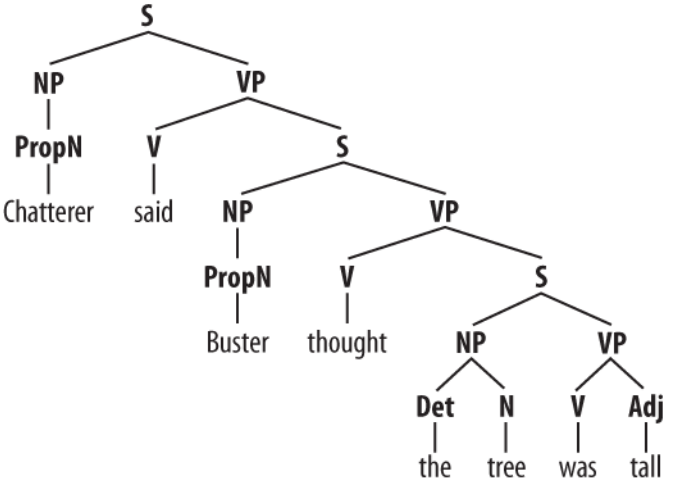
```
grammar2 = nltk.parse_cfg("""
  S  -> NP VP
  NP -> Det Nom | PropN
  Nom -> Adj Nom | N
  VP -> V Adj | V NP | V S | V NP PP
  PP -> P NP
  PropN -> 'Buster' | 'Chatterer' | 'Joe'
  Det -> 'the' | 'a'
  N -> 'bear' | 'squirrel' | 'tree' | 'fish' | 'log'
  Adj  -> 'angry' | 'frightened' |  'little' | 'tall'
  V ->  'chased'  | 'saw' | 'said' | 'thought' | 'was' | 'put'
  P -> 'on'
  """)
```

To see how recursion arises from this grammar, consider the following trees. (10a) involves nested nominal phrases, while (10b) contains nested sentences.

(10) a.



b.



We've only illustrated two levels of recursion here, but there's no upper limit on the depth. You can experiment with parsing sentences that involve more deeply nested structures. Beware that the `RecursiveDescentParser` is unable to handle **left-recursive** productions of the form `X -> X Y`; we will return to this in Section 8.4.

8.4 Parsing with Context-Free Grammar

A **parser** processes input sentences according to the productions of a grammar, and builds one or more constituent structures that conform to the grammar. A grammar is a declarative specification of well-formedness—it is actually just a string, not a program. A parser is a procedural interpretation of the grammar. It searches through the space of trees licensed by a grammar to find one that has the required sentence along its fringe.

A parser permits a grammar to be evaluated against a collection of test sentences, helping linguists to discover mistakes in their grammatical analysis. A parser can serve as a model of psycholinguistic processing, helping to explain the difficulties that humans have with processing certain syntactic constructions. Many natural language applications involve parsing at some point; for example, we would expect the natural language questions submitted to a question-answering system to undergo parsing as an initial step.

In this section, we see two simple parsing algorithms, a top-down method called recursive descent parsing, and a bottom-up method called shift-reduce parsing. We also see some more sophisticated algorithms, a top-down method with bottom-up filtering called left-corner parsing, and a dynamic programming technique called chart parsing.

Recursive Descent Parsing

The simplest kind of parser interprets a grammar as a specification of how to break a high-level goal into several lower-level subgoals. The top-level goal is to find an `S`. The `S → NP VP` production permits the parser to replace this goal with two subgoals: find an `NP`, then find a `VP`. Each of these subgoals can be replaced in turn by sub-subgoals, using productions that have `NP` and `VP` on their lefthand side. Eventually, this expansion process leads to subgoals such as: find the word *telescope*. Such subgoals can be directly compared against the input sequence, and succeed if the next word is matched. If there is no match, the parser must back up and try a different alternative.

The recursive descent parser builds a parse tree during this process. With the initial goal (find an `S`), the `S` root node is created. As the process recursively expands its goals using the productions of the grammar, the parse tree is extended downwards (hence the name *recursive descent*). We can see this in action using the graphical demonstration `nltk.app.rdparser()`. Six stages of the execution of this parser are shown in Figure 8-4.

During this process, the parser is often forced to choose between several possible productions. For example, in going from step 3 to step 4, it tries to find productions with `N` on the lefthand side. The first of these is `N →` *man*. When this does not work it **backtracks**, and tries other `N` productions in order, until it gets to `N →` *dog*, which matches the next word in the input sentence. Much later, as shown in step 5, it finds a complete parse. This is a tree that covers the entire sentence, without any dangling edges. Once a parse has been found, we can get the parser to look for additional parses. Again it will backtrack and explore other choices of production in case any of them result in a parse.

NLTK provides a recursive descent parser:

```
>>> rd_parser = nltk.RecursiveDescentParser(grammar1)
>>> sent = 'Mary saw a dog'.split()
>>> for t in rd_parser.nbest_parse(sent):
...     print t
(S (NP Mary) (VP (V saw) (NP (Det a) (N dog))))
```

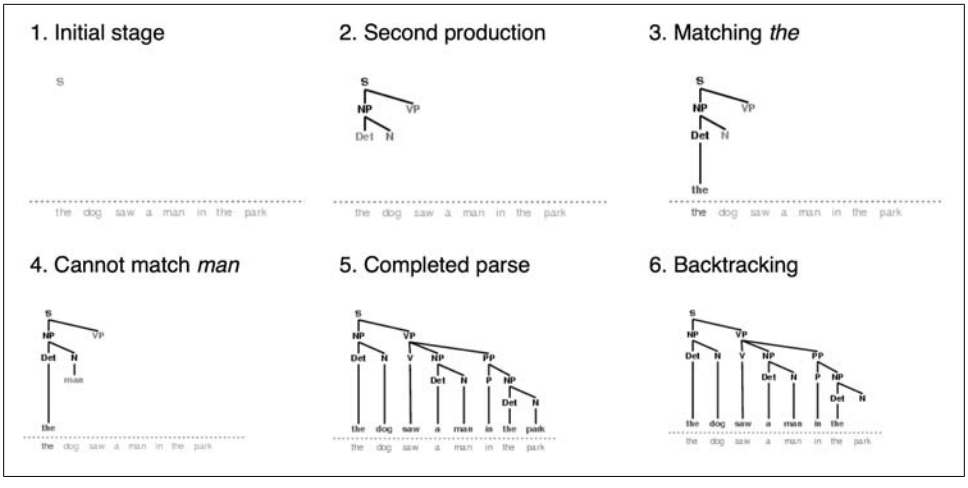


Figure 8-4. Six stages of a recursive descent parser: The parser begins with a tree consisting of the node S; at each stage it consults the grammar to find a production that can be used to enlarge the tree; when a lexical production is encountered, its word is compared against the input; after a complete parse has been found, the parser backtracks to look for more parses.

`RecursiveDescentParser()` takes an optional parameter `trace`. If `trace` is greater than zero, then the parser will report the steps that it takes as it parses a text.

Recursive descent parsing has three key shortcomings. First, left-recursive productions like `NP -> NP PP` send it into an infinite loop. Second, the parser wastes a lot of time considering words and structures that do not correspond to the input sentence. Third, the backtracking process may discard parsed constituents that will need to be rebuilt again later. For example, backtracking over `VP -> V NP` will discard the subtree created for the `NP`. If the parser then proceeds with `VP -> V NP PP`, then the `NP` subtree must be created all over again.

Recursive descent parsing is a kind of **top-down parsing**. Top-down parsers use a grammar to *predict* what the input will be, before inspecting the input! However, since the input is available to the parser all along, it would be more sensible to consider the input sentence from the very beginning. This approach is called **bottom-up parsing**, and we will see an example in the next section.

Shift-Reduce Parsing

A simple kind of bottom-up parser is the **shift-reduce parser**. In common with all bottom-up parsers, a shift-reduce parser tries to find sequences of words and phrases that correspond to the *righthand* side of a grammar production, and replace them with the lefthand side, until the whole sentence is reduced to an `S`.

The shift-reduce parser repeatedly pushes the next input word onto a stack (Section 4.1); this is the **shift** operation. If the top *n* items on the stack match the *n* items on the righthand side of some production, then they are all popped off the stack, and the item on the lefthand side of the production is pushed onto the stack. This replacement of the top *n* items with a single item is the **reduce** operation. The operation may be applied only to the top of the stack; reducing items lower in the stack must be done before later items are pushed onto the stack. The parser finishes when all the input is consumed and there is only one item remaining on the stack, a parse tree with an `S` node as its root. The shift-reduce parser builds a parse tree during the above process. Each time it pops *n* items off the stack, it combines them into a partial parse tree, and pushes this back onto the stack. We can see the shift-reduce parsing algorithm in action using the graphical demonstration `nltk.app.srparser()`. Six stages of the execution of this parser are shown in Figure 8-5.

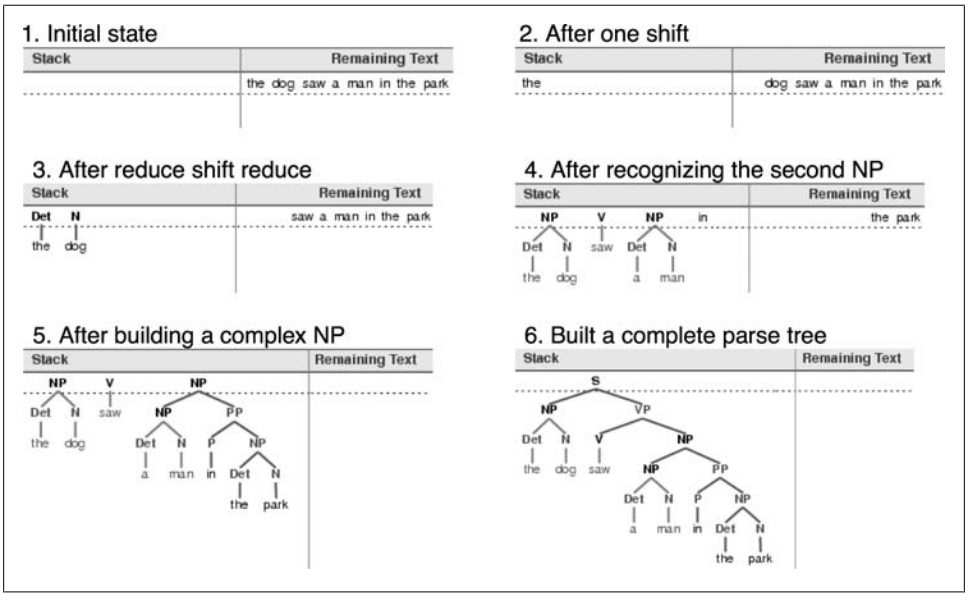


Figure 8-5. Six stages of a shift-reduce parser: The parser begins by shifting the first input word onto its stack; once the top items on the stack match the righthand side of a grammar production, they can be replaced with the lefthand side of that production; the parser succeeds once all input is consumed and one S item remains on the stack.

NLTK provides `ShiftReduceParser()`, a simple implementation of a shift-reduce parser. This parser does not implement any backtracking, so it is not guaranteed to find a parse for a text, even if one exists. Furthermore, it will only find at most one parse, even if more parses exist. We can provide an optional `trace` parameter that controls how verbosely the parser reports the steps that it takes as it parses a text:

```
>>> sr_parse = nltk.ShiftReduceParser(grammar1)
>>> sent = 'Mary saw a dog'.split()
>>> print sr_parse.parse(sent)
  (S (NP Mary) (VP (V saw) (NP (Det a) (N dog))))
```

Your Turn: Run this parser in tracing mode to see the sequence of shift and reduce operations, using `sr_parse = nltk.ShiftReduceParser(grammar1, trace=2)`.

A shift-reduce parser can reach a dead end and fail to find any parse, even if the input sentence is well-formed according to the grammar. When this happens, no input remains, and the stack contains items that cannot be reduced to an `S`. The problem arises because there are choices made earlier that cannot be undone by the parser (although users of the graphical demonstration can undo their choices). There are two kinds of choices to be made by the parser: (a) which reduction to do when more than one is possible and (b) whether to shift or reduce when either action is possible.

A shift-reduce parser may be extended to implement policies for resolving such conflicts. For example, it may address shift-reduce conflicts by shifting only when no reductions are possible, and it may address reduce-reduce conflicts by favoring the reduction operation that removes the most items from the stack. (A generalization of the shift-reduce parser, a "lookahead LR parser," is commonly used in programming language compilers.)

The advantages of shift-reduce parsers over recursive descent parsers is that they only build structure that corresponds to the words in the input. Furthermore, they only build each substructure once; e.g., `NP(Det(the), N(man))` is only built and pushed onto the stack a single time, regardless of whether it will later be used by the `VP -> V NP PP` reduction or the `NP -> NP PP` reduction.

The Left-Corner Parser

One of the problems with the recursive descent parser is that it goes into an infinite loop when it encounters a left-recursive production. This is because it applies the grammar productions blindly, without considering the actual input sentence. A left-corner parser is a hybrid between the bottom-up and top-down approaches we have seen.

A **left-corner parser** is a top-down parser with bottom-up filtering. Unlike an ordinary recursive descent parser, it does not get trapped in left-recursive productions. Before starting its work, a left-corner parser preprocesses the context-free grammar to build a table where each row contains two cells, the first holding a non-terminal, and the second holding the collection of possible left corners of that non-terminal. Table 8-2 illustrates this for the grammar from `grammar2`.

Table 8-2. Left corners in grammar2

Category	Left corners (pre-terminals)
S	NP
NP	Det, PropN
VP	V
PP	P

Each time a production is considered by the parser, it checks that the next input word is compatible with at least one of the pre-terminal categories in the left-corner table.

Well-Formed Substring Tables

The simple parsers discussed in the previous sections suffer from limitations in both completeness and efficiency. In order to remedy these, we will apply the algorithm design technique of **dynamic programming** to the parsing problem. As we saw in Section 4.7, dynamic programming stores intermediate results and reuses them when appropriate, achieving significant efficiency gains. This technique can be applied to syntactic parsing, allowing us to store partial solutions to the parsing task and then look them up as necessary in order to efficiently arrive at a complete solution. This approach to parsing is known as **chart parsing**. We introduce the main idea in this section; see the online materials available for this chapter for more implementation details.

Dynamic programming allows us to build the PP *in my pajamas* just once. The first time we build it we save it in a table, then we look it up when we need to use it as a subconstituent of either the object NP or the higher VP. This table is known as a **well-formed substring table**, or WFST for short. (The term "substring" refers to a contiguous sequence of words within a sentence.) We will show how to construct the WFST bottom-up so as to systematically record what syntactic constituents have been found.

Let's set our input to be the sentence in (2). The numerically specified spans of the WFST are reminiscent of Python's slice notation (Section 3.2). Another way to think about the data structure is shown in Figure 8-6, a data structure known as a **chart**.

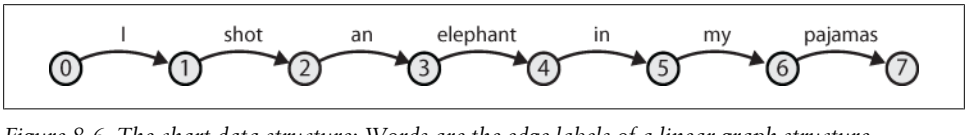


Figure 8-6. The chart data structure: Words are the edge labels of a linear graph structure.

In a WFST, we record the position of the words by filling in cells in a triangular matrix: the vertical axis will denote the start position of a substring, while the horizontal axis will denote the end position (thus *shot* will appear in the cell with coordinates (1, 2)). To simplify this presentation, we will assume each word has a unique lexical category,

and we will store this (not the word) in the matrix. So cell (1, 2) will contain the entry `V`. More generally, if our input string is $a_1 a_2 \ldots a_n$, and our grammar contains a production of the form $A \rightarrow a_i$, then we add A to the cell (i-1, i).

So, for every word in `text`, we can look up in our grammar what category it belongs to.

```
>>> text = ['I', 'shot', 'an', 'elephant', 'in', 'my', 'pajamas']
[V -> 'shot']
```

For our WFST, we create an (n-1) × (n-1) matrix as a list of lists in Python, and initialize it with the lexical categories of each token in the `init_wfst()` function in Example 8-3. We also define a utility function `display()` to pretty-print the WFST for us. As expected, there is a `V` in cell (1, 2).

Example 8-3. Acceptor using well-formed substring table.

```
def init_wfst(tokens, grammar):
    numtokens = len(tokens)
    wfst = [[None for i in range(numtokens+1)] for j in range(numtokens+1)]
    for i in range(numtokens):
        productions = grammar.productions(rhs=tokens[i])
        wfst[i][i+1] = productions[0].lhs()
    return wfst

def complete_wfst(wfst, tokens, grammar, trace=False):
    index = dict((p.rhs(), p.lhs()) for p in grammar.productions())
    numtokens = len(tokens)
    for span in range(2, numtokens+1):
        for start in range(numtokens+1-span):
            end = start + span
            for mid in range(start+1, end):
                nt1, nt2 = wfst[start][mid], wfst[mid][end]
                if nt1 and nt2 and (nt1,nt2) in index:
                    wfst[start][end] = index[(nt1,nt2)]
                    if trace:
                        print "[%s] %3s [%s] %3s [%s] ==> [%s] %3s [%s]" % \
                        (start, nt1, mid, nt2, end, start, index[(nt1,nt2)], end)
    return wfst

def display(wfst, tokens):
    print '\nWFST ' + ' '.join([("%-4d" % i) for i in range(1, len(wfst))])
    for i in range(len(wfst)-1):
        print "%d   " % i,
        for j in range(1, len(wfst)):
            print "%-4s" % (wfst[i][j] or '.'),
        print
```

```
>>> tokens = "I shot an elephant in my pajamas".split()
>>> wfst0 = init_wfst(tokens, groucho_grammar)
>>> display(wfst0, tokens)
WFST 1    2    3    4    5    6    7
0    NP   .    .    .    .    .    .
1    .    V    .    .    .    .    .
2    .    .    Det  .    .    .    .
3    .    .    .    N    .    .    .
```

```
4    .    .    .    .    P    .    .
5    .    .    .    .    .    Det  .
6    .    .    .    .    .    .    N
>>> wfst1 = complete_wfst(wfst0, tokens, groucho_grammar)
>>> display(wfst1, tokens)
WFST 1    2    3    4    5    6    7
0    NP   .    .    S    .    .    S
1    .    V    .    VP   .    .    VP
2    .    .    Det  NP   .    .    .
3    .    .    .    N    .    .    .
4    .    .    .    .    P    .    PP
5    .    .    .    .    .    Det  NP
6    .    .    .    .    .    .    N
```

Returning to our tabular representation, given that we have `Det` in cell (2, 3) for the word *an*, and `N` in cell (3, 4) for the word *elephant*, what should we put into cell (2, 4) for *an elephant*? We need to find a production of the form $A \rightarrow$ `Det N`. Consulting the grammar, we know that we can enter `NP` in cell (0, 2).

More generally, we can enter A in (i, j) if there is a production $A \rightarrow B\ C$, and we find non-terminal B in (i, k) and C in (k, j). The program in Example 8-3 uses this rule to complete the WFST. By setting `trace` to `True` when calling the function `complete_wfst()`, we see tracing output that shows the WFST being constructed:

```
>>> wfst1 = complete_wfst(wfst0, tokens, groucho_grammar, trace=True)
[2] Det [3]   N [4] ==> [2]  NP [4]
[5] Det [6]   N [7] ==> [5]  NP [7]
[1]   V [2]  NP [4] ==> [1]  VP [4]
[4]   P [5]  NP [7] ==> [4]  PP [7]
[0]  NP [1]  VP [4] ==> [0]   S [4]
[1]  VP [4]  PP [7] ==> [1]  VP [7]
[0]  NP [1]  VP [7] ==> [0]   S [7]
```

For example, this says that since we found `Det` at `wfst[0][1]` and `N` at `wfst[1][2]`, we can add `NP` to `wfst[0][2]`.

To help us easily retrieve productions by their righthand sides, we create an index for the grammar. This is an example of a space-time trade-off: we do a reverse lookup on the grammar, instead of having to check through entire list of productions each time we want to look up via the righthand side.

We conclude that there is a parse for the whole input string once we have constructed an `S` node in cell (0, 7), showing that we have found a sentence that covers the whole input. The final state of the WFST is depicted in Figure 8-7.

Notice that we have not used any built-in parsing functions here. We've implemented a complete primitive chart parser from the ground up!

WFSTs have several shortcomings. First, as you can see, the WFST is not itself a parse tree, so the technique is strictly speaking **recognizing** that a sentence is admitted by a

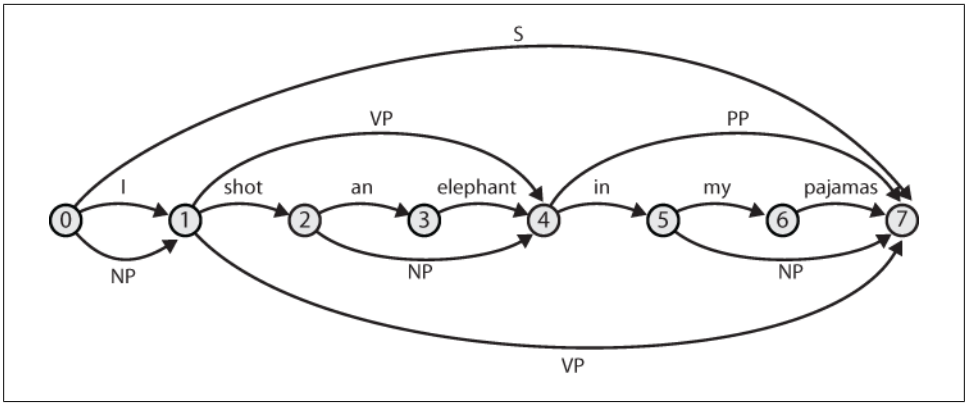


Figure 8-7. The chart data structure: Non-terminals are represented as extra edges in the chart.

grammar, rather than parsing it. Second, it requires every non-lexical grammar production to be *binary*. Although it is possible to convert an arbitrary CFG into this form, we would prefer to use an approach without such a requirement. Third, as a bottom-up approach it is potentially wasteful, being able to propose constituents in locations that would not be licensed by the grammar.

Finally, the WFST did not represent the structural ambiguity in the sentence (i.e., the two verb phrase readings). The `VP` in cell (2,8) was actually entered twice, once for a `V NP` reading, and once for a `VP PP` reading. These are different hypotheses, and the second overwrote the first (as it happens, this didn't matter since the lefthand side was the same). Chart parsers use a slightly richer data structure and some interesting algorithms to solve these problems (see Section 8.8).

Your Turn: Try out the interactive chart parser application `nltk.app.chartparser()`.

8.5 Dependencies and Dependency Grammar

Phrase structure grammar is concerned with how words and sequences of words *combine* to form constituents. A distinct and complementary approach, **dependency grammar**, focuses instead on how words *relate* to other words. Dependency is a binary asymmetric relation that holds between a **head** and its **dependents**. The head of a sentence is usually taken to be the tensed verb, and every other word is either dependent on the sentence head or connects to it through a path of dependencies.

A dependency representation is a labeled directed graph, where the nodes are the lexical items and the labeled arcs represent dependency relations from heads to dependents. Figure 8-8 illustrates a dependency graph, where arrows point from heads to their dependents.