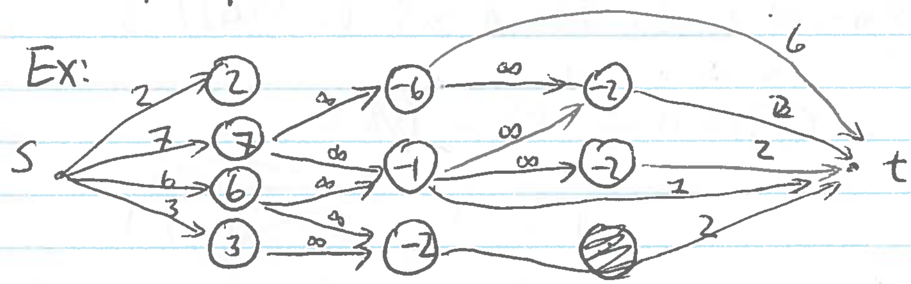
There exist a set of teams T that provide a proof: $M < \frac{\#\text{ edges in } T + \sum_{x \in T} P_x}{|T|}$

Project selection:

- We are given a set of project.
- Each project X has a revenue P_x: If $P_x > 0 \Rightarrow$ it provide a profit
  If $P_x < 0 \Rightarrow$ it provide a loss.
- Some project are prerequisites for other projects. An edge from $x \to y$ means that y is prerequisite for x (If we choose x, we have to choose y).

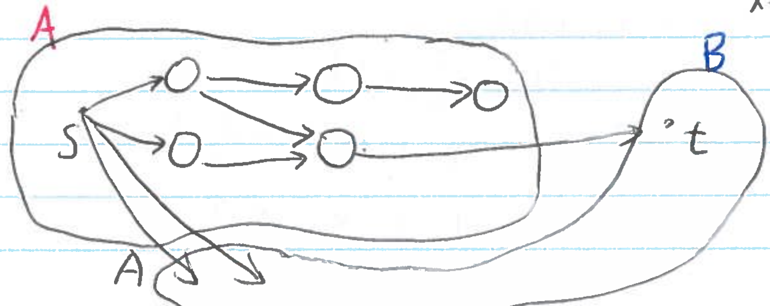
Goal: Select a subset of projects that respects all the prerequisites and maximizes the total revenues.

Ex:

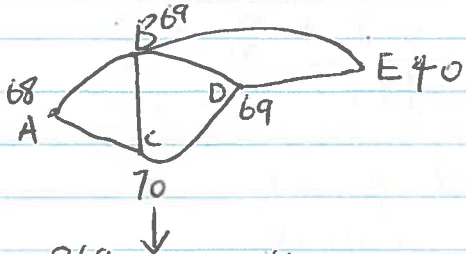


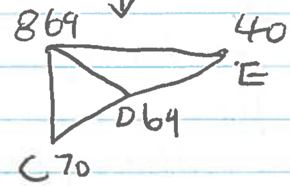
- We assign ∞ capacity to all the edges. This way if a project X is in part A of a min-cut and have the prereq $X \to Y$ then Y also has to be in A.
- We add source S, sink t, edges from S to x for projects with $P_x > 0$ (capacity P_x)
- edges from x with $P_x < 0$ to t with Cap $|P_x|$.

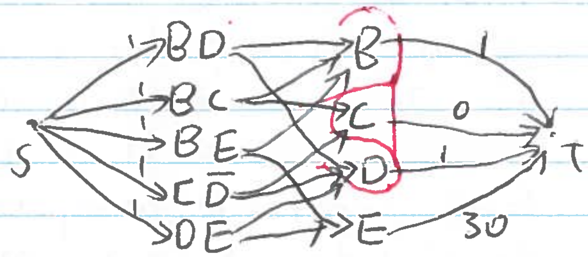
Let (A, B) be a min-cut and let $M = \sum_{x: P_x > 0} P_x$



Baseball Elimination







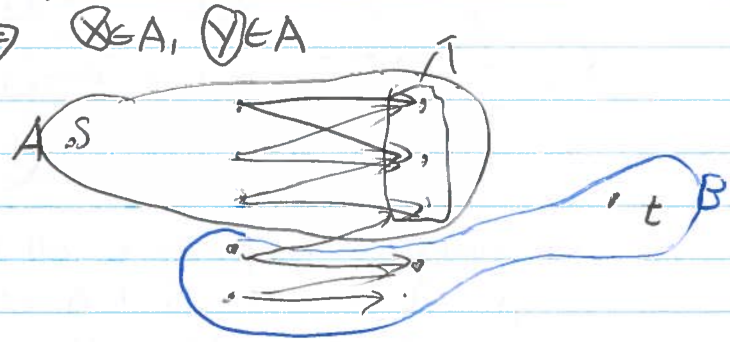
D F-F and if max flow = # remaining edges ⇒ not eliminated.

What can we say about min-cut?

Min-cut $\neq \infty$ (Take $A = \{s\}$ $B = V(G) / \{s\}$)

- Consider min-cut

If $(x, y) \in A \Rightarrow x \in A, y \in A$



$Cap(A, B) = \sum_{x \in T} (M - P_x) +$ number of matches xy with cut lenght one of X or Y not in T.

$= \text{max-flow}.$

If max-flow < # of edges ⇒ our team is eliminated

$\text{maxflow} = cap(A, B)$

$= M \times |T| - \sum_{x \in T} P_x + K$

$\Leftrightarrow M \times |T| - \sum_{x \in T} P_x + K < \#\text{ of edges}$

$\Leftrightarrow M \times |T| < \#\text{ edges in } T + \sum_{x \in T} P_x$

$$\begin{aligned}Cap(A,B) &= \sum_{\substack{x\in B\\ P_x>0}} P_x + \sum_{\substack{x\in A\\ P_x<0}} |P_x| \quad \# \\ &= \sum_{\substack{x\in A\\ P_x<0}} -P_x + \sum_{\substack{x\notin A\\ P_x>0}} P_x \\ &= \sum_{\substack{x\in A\\ P_x<0}} -P_x + \Big(M - \sum_{\substack{x\in A\\ P_x\geq 0}} P_x\Big) \\ &= M - \underline{\sum_{x\in A} P_x} \leftarrow \text{maximized in order to have a min-cut.}\end{aligned}$$

We also know that the projects in A respect the prereq Condition.

The total profit we can make