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Required Background:

- Tree
- graph. DFS, BFS
- Greedy, Dynamic, divide Conquere, recursion
- Running Time analysis.
- Big O

Some problems:

1) S is a set of positive integer. Define:

$$A = \sum_{x\in S} x^2$$

$$B = \sum_{\substack{\cancel{x\in S}\\ x^2\in S}} x$$

If $S=\{1,2,4,5\}$ then $A= 1+4+16+25 = 46$
$B = 1+2 = 3$

2) M is an $n\times n$ matrix,
M_{ij} denote the (i,j) entry of M.
Suppose the total sum of the entries of M is 100

$$\sum_{i=1}^{n}\sum_{j\in\{1,\dots,n\}\setminus\{i\}} \sum_{r=1}^{n} M_{ir} = (n-1)\,100,$$

Here, the sum is $(n-1)100$ b/c each M_{ir} is sumed $(n-1)$ time as j goes from 1 to n skipping i.

3) Binary expansion/ representation
How many digits are in the binary expansion of n.
→ It is $\lceil \log_2 n\rceil$
let $n = a_m 2^m + a_{m-1}2^{m-1} + \cdots + a_1 2 + a_0$

$$\sum_{n=0}^{k} 2^n = 2^{k+1} - 1$$

4) Let $S = (a_1, a_2, \ldots, a_n)$ a sequence of integers. E is the set of even numbers from 1 to n.

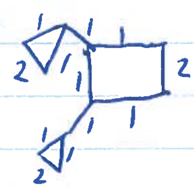
$A = \sum_{i \in E} a_i$.

- If $S = (1, 3, 2, 5, 4)$, then $A = 8$

5) Let $G = (V, E)$ be an undirected graph. Suppose to every edge uv, a number C_{uv} is assigned.

What does the following statement mean?

"$\exists c \ \forall u \in V \ \sum_{uv \in E} C_{uv} = c$"

Ex: 

The statement translates to, in english, "$\exists$ a number c s.t for any vertex in G, the sum of C_{uv} for all uv, edges adjacent to u, is c.

6) Let $G = (V, E)$ undirected graph, degree of every vertex is 10.

Suppose to every vertex $v \in V$ a positive integer a_v is assigned.

If $\sum_{v \in V} a_v = 5$ then what is

$\sum_{u \in V} \sum_{\substack{w \in V \\ uw \in E}} a_w = ? = \sum_{u \in V} 10 \cdot a_w = 10 \cdot \sum_{v \in V} a_w = 10 \cdot 5 = 50$.

Topic Covered

- Network Flow
- Linear programming
- Midterm
- Linear programming

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- NP Completeteness.
- Approximation algorithms
- Randomized algorithms.

Network Flows

Max Flow problem:

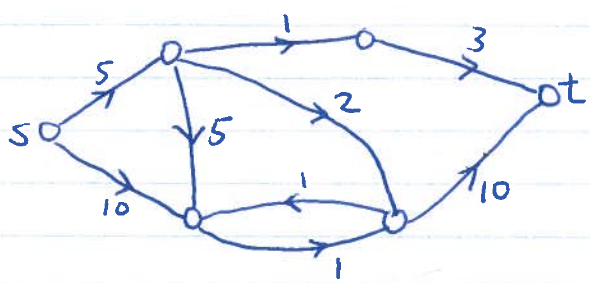
Def: A Flow network is a directed graph $G=(V,E)$ s.t

(1) Every edge e has a capacity $C_e \geq 0$

(2) There is a source $s \in V$

(3) There is a sink $t \in V$, $t \neq s$

Ex:



Remark: For the sake of convenience, we make the following assumption:

① No edge enters S

② No edge leaves t

③ All capacities are integer (This is for convenience.

④ The is at ~~most~~ least one edge incident to every vertex.

Def: A flow is a function $f: E \to \mathbb{R}^+$ such that

1) $f(v) \leq C_v$, $f(v) \geq 0$ $\forall v \in E(G)$

2) Conservation:

$\forall v \in V(G)$ $v \neq s, v \neq t,$

$$\sum_{uv \in E} f(uv) = \sum_{vu \in E} f(vu)$$

For Convenience, we write $f^{in}(u) = \sum_{vu \in E} f(vu)$

$f^{out}(u) = \sum_{uv \in E} f(uv)$

Def: $\mathrm{val}(f) = \sum_{su \in E} f(su) = f^{out}(s)$

Max Flow problem: Given a flow network, find a flow with largest possible value.