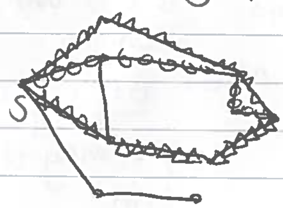
CENTRE
DE RECHERCHES
MATHÉMATIQUES

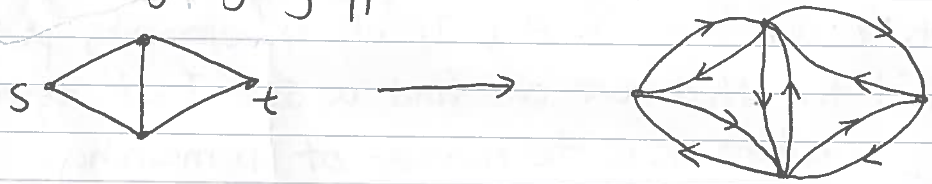
Edge-disjoint path in directed graph

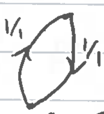
max flow $\geq$ # paths (easy) use the path to direct flow
max flow $\leq$ # paths : we start from the flow and trace one path and remove all edges of this path

Can we solve the same problem on undirected graph?
Goal: Find the maximum number of edge-disjoint s-t path.



We can replace every edge with two directed edges going opposite direction



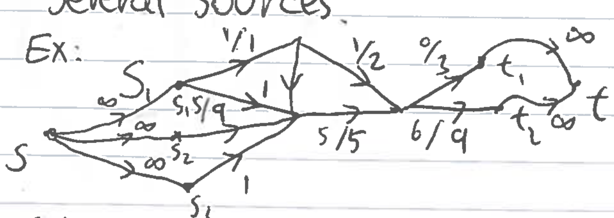
Now we can try to find max number of edge disjoint path in this directed graph using f-f.
- After running f-f if we have  , we remove them.

This does not change the value of flow, and so we will still have max-flow. Using this flow will avoid using shared edges in the undirected graph.

Multi-Source . Multi-sink flow :

Similar to the original max-flow problem, except we now have several sources.

Ex:



We want to generate maximum # of unit flow at the sources.

Solution: add one source S and one sink t. Connect S to S_i and t_i to t. and connect the original sinks to t with ∞-cap edges.

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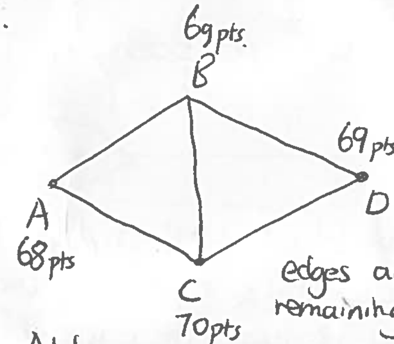
Baseball Elimination Problem

- We have a tournament
- Currently. we are in the middle and each team has some points and some remaining matches.
- We are interested in specific team, A, and we want to know does A have a chance of getting first place.

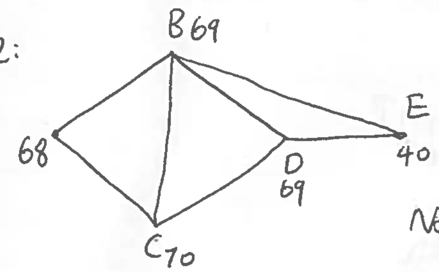
Ex1: The final point between B, C, D is at least $69+69+70+3$. So one of them will have at least 70 pts.



edges are remaining matches.

At least one of B, C, D will get 71 pts.

Ex2:



Again one of B, C, D will get more than 70.

Note: $\frac{B+D+E+C+5}{4} < 70$ and you may think A might win.

Let M be total points A will have if A wins all its remaining point. If A is eliminated, is it true that we can find a set T of teams s.t

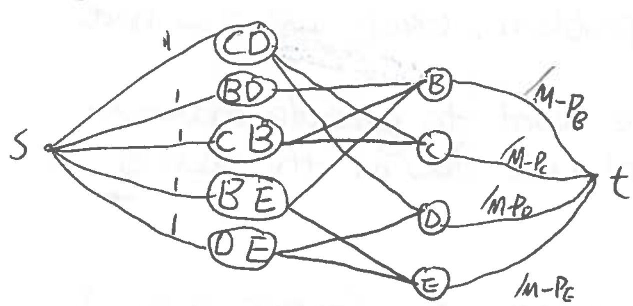
$$\frac{\left(\sum_{k \in T} P_x + k\right)}{|T|} > M?$$

where k is the number of remaining matches between teams in T and P_x is the points of x.

How can we decide weather A is eliminated?

Step1: Let $M := P_A +$ degree of A, remove A.

Step2: Construct the following flow-network: For every edge uv. put a vertex V_{uv} in the network. Also for every team except A add a node.



Step3: Add a source. Connect it to all uv with cap 1 edges. add edges $uv-u$ and $uv,-v$ with ∞ cap. add edges $u-t$ with cap $M-P_u$.

Step 4: Solve max-flow, if its value equals to the outgoing capacity then A is not eliminated. Otherwise it is.