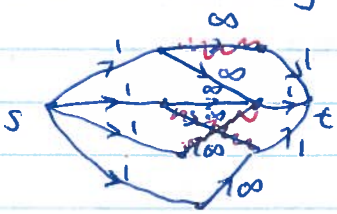
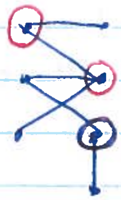
Recall : Matching in Bipartite graphs



FF can be used to find larges matching in a Bipartite graph.

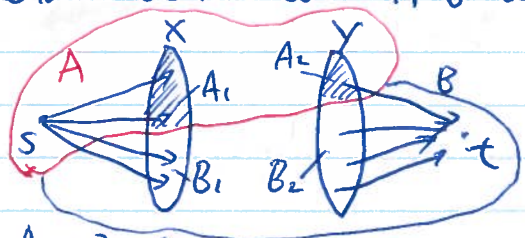


Vertex Cover: the set of vertices s.t deleting them will remove all edges.

thm (Min Vertex Cover equals max matching) (Konig's thm)

For every bipartite graph G. min-VC = max-matching

Pf: Consider a min Cut - A-B in the constructed flow network



min-cut $< \infty$
(for example, $\{s\}, V-\{s\}$ is a cut with cap $< \infty$)

$A = \{s\} \cup A_1 \cup A_2$
$B = \{t\} \cup B_1 \cup B_2$

No edges from A_1 to B_2 as otherwise the cap would be ∞. Thus $B_1 \cup A_2$ is a vertex cover. in the Original graph. Its size is $|B_1| + |A_2|$.

On the other hand, $\mathrm{cap}(A,B) = \sum_{\substack{e=su \\ u \in B_1}} c_e + \sum_{\substack{e=vt \\ v \in A_2}} c_e$

$= |A_2| + |B_1|$

We showed there is a vertex cover $(B_1 \cup A_2)$ whose size is equal to min-cut (A,B).

We need to show next min-cut $\leq$ min-VC

So Let S be the smallest vertex Cover

$S = S_1 \cup S_2$

Let $A = (X \setminus S_1) \cup S_2 \cup \{s\}$ $\quad B = A^c$

$Cap(A,B) = |S_1| + |S_2| = |S|$

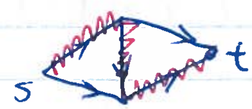
we conclude . max flow = min-cut = min-VC = max-matching

Disjoint Path in directed graph.

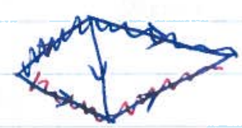
Input: A directed graph and two distinct node are marked as s and t.

Goal: find the maximum number of edge-disjoint paths from s to t.

Ex:



done but opt is

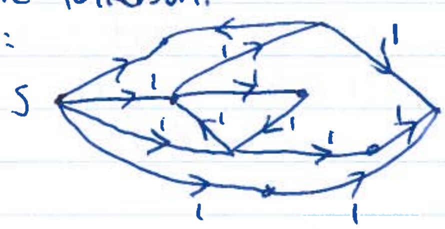


Pick shortest path first may not work as well

ex:

We assign capacity 1 to all edges and run ford-fulkerson.

Ex:



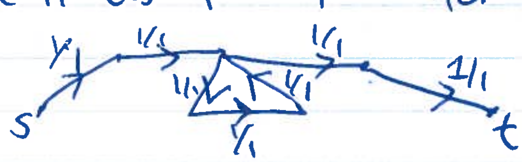
Let k be the max-flow, WTS there are k-edge disjoint path.

Let's start with k=1.

In this case, we have a flow of 1.

We start from s and trace this one unit of flow.

Every time we enter an internal node. we can leave it as $f^{in} = f^{out}$ for such node



We continue in this manner using only new edges.

We remove the loop, we obtain a path from S to t.
What about when $K > 1$?
Apply induction after removing the path.
We will have flow $K-1$ so we have $K-1$ edge disjoint path. Adding the removed path ~~p~~ gives the result.

We proved that max-disjoint $\geq$ max-flow
To prove inequality, note that given ~~a~~ r-edge disjoint path, there is a flow of value at least r
$\Rightarrow$ max-flow $\geq$ max-disjoint path.