hw3

1. 分别用带有前向检验、MRV和最少约束值启发式的回溯算法手工求解**图6.2**中的密码算数问题

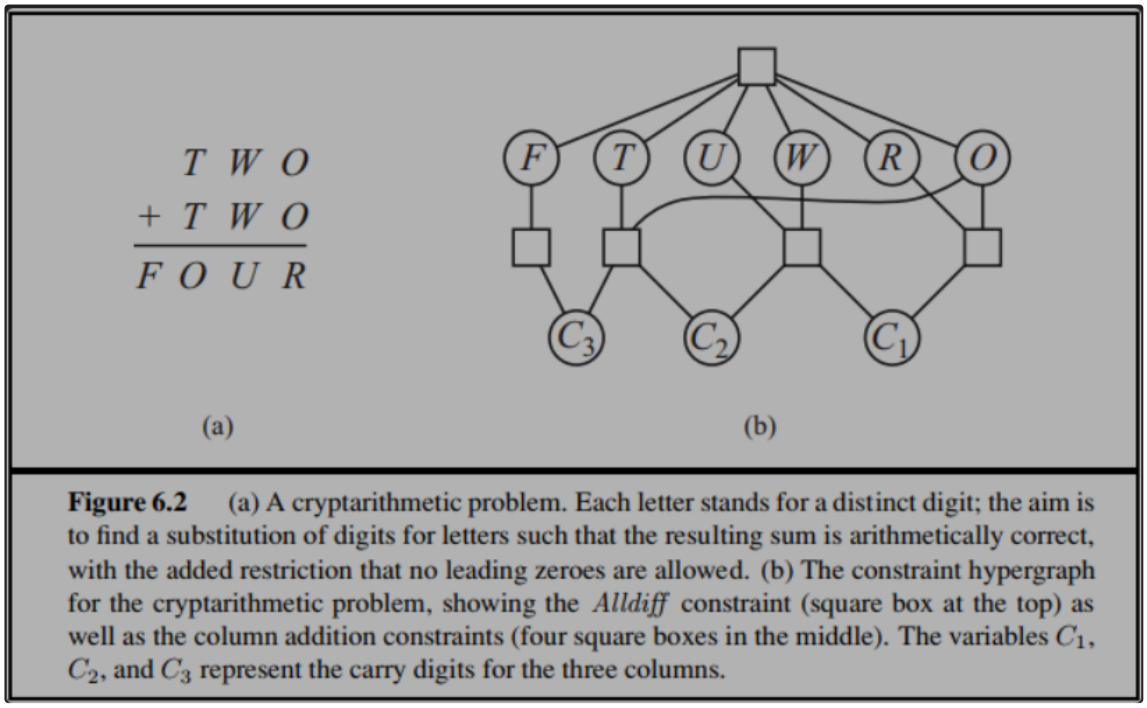

Figure 6.2 (a) A cryptarithmetic problem. Each letter stands for a distinct digit; the aim is to find a substitution of digits for letters such that the resulting sum is arithmetically correct, with the added restriction that no leading zeroes are allowed. (b) The constraint hypergraph for the cryptarithmetic problem, showing the *Alldiff* constraint (square box at the top) as well as the column addition constraints (four square boxes in the middle). The variables C_1, C_2, and C_3 represent the carry digits for the three columns.

形式化定义：

变量：$\{F, T, U, W, R, O, C1, C2, C3\}$

值域：$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

约束：

$$O + O = R + 10 \times C1$$
$$W + W + C1 = U + 10 \times C2$$
$$T + T + C2 = O + 10 \times C3$$
$$F = C3, T \neq 0, F \neq 0$$
$$0 <= C1, C2, C3 < 2$$
$$F \neq O \neq U \neq R \neq T \neq W$$

使用前向检验的回溯法：

先对F赋值

F	T	U	W	R	O	C1	C2	C3
1	(5-9)	(0-9)	(0-9)	(0-9)	(0-9)	(0,1)	(0,1)	1

再对T赋值

F	T	U	W	R	O	C1	C2	C3
1	7	(0-9)	(0-9)	(8,0)	(4,5)	(0,1)	(0,1)	1

对W赋值

F	T	U	W	R	O	C1	C2	C3
1	7	6,7	8	(8,0)	(4,5)	(0,1)	1	1

对O赋值

F	T	U	W	R	O	C1	C2	C3
1	7	U无值可选	8	0	5	1	1	1
1	7	6,7	8	无值可选	4	(0,1)	1	1

此事回溯

对W赋值

F	T	U	W	R	O	C1	C2	C3
1	7	2,3	6	(8,0)	(4,5)	(0,1)	1	1

对O赋值

F	T	U	W	R	O	C1	C2	C3
1	7	3	6	0	5	1	1	1

求得结果

使用带MRV的回溯

考虑到不相等的约束没有值的约束强

先对C3赋值(约束数为4)

F	T	U	W	R	O	C1	C2	C3
1								1

再对C2赋值(约束数为3)

F	T	U	W	R	O	C1	C2	C3
1							1	1

再对C1赋值(约束数为3)

F	T	U	W	R	O	C1	C2	C3
1						1	1	1

再对O赋值(约束数为3)

F	T	U	W	R	O	C1	C2	C3
1	8			4	7	1	1	1

再对U赋值(约束数为2)

F	T	U	W	R	O	C1	C2	C3
1	8	3	6	4	7	1	1	1

此时有解FOUR = 1734 TWO=867

使用最少约束值启发式，启发式函数可以设为对某次赋值后剩下的未赋值变量之和

先对C3赋值可以减少两个变量的赋值$h(C3) = 7$

F	T	U	W	R	O	C1	C2	C3
1								1

再对C2赋值,$h(C2) = 6$

F	T	U	W	R	O	C1	C2	C3
1							1	1

再对T赋值，$h(T) = 2$

F	T	U	W	R	O	C1	C2	C3
1	7			0	5	1	1	1

最后对U赋值，$h(U) = 0$

F	T	U	W	R	O	C1	C2	C3
1	7	3	6	0	5	1	1	1

即可求解

2. 用**AC-3**算法说明弧相容对**图6.1**中问题能够检测出部分赋值，WA=RED，V=BLUE的不相容

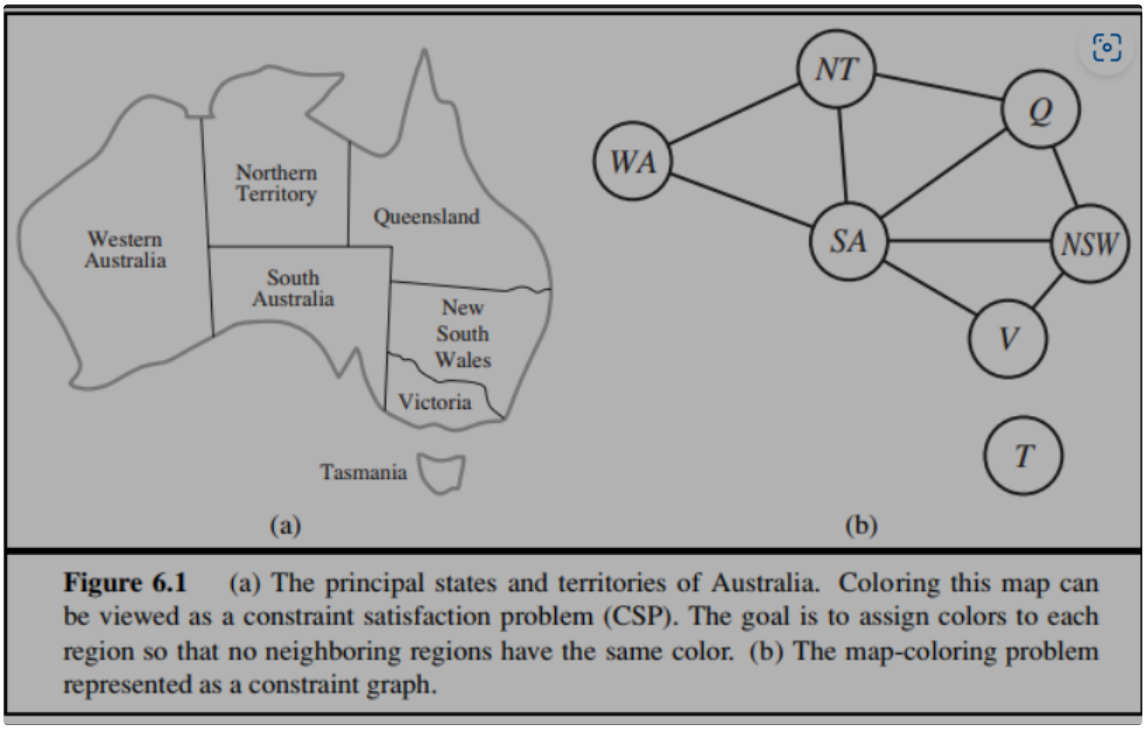

Figure 6.1 (a) The principal states and territories of Australia. Coloring this map can be viewed as a constraint satisfaction problem (CSP). The goal is to assign colors to each region so that no neighboring regions have the same color. (b) The map-coloring problem represented as a constraint graph.

WA	NT	Q	NSW	SA	V
RED					BLUE

弧队列为：WA-SA，SA-V，WA-NT，NT-SA，NT-Q，SA-Q，Q-NSW，SA-NSW，NSW-V

1. pop(WA-NT)，此时从SA值域删掉red
2. pop(SA-V)，此时从SA删掉blue，因此SA只能是GREEN
3. pop(WA-NT)，此时从NT删掉red
4. pop(NT-SA)，此时从NT删掉GREEN，因此NT必为BLUE
5. pop(NT-Q)，此时从Q删掉BLUE
6. pop(SA-Q)，此时从Q删掉GREEN，Q必为RED
7. pop(Q-NSW)，此时从NSW删掉RED
8. pop(SA-NSW)，此时从NSW删掉GREEN
9. pop(NSW-V)，此时从NSW删掉BLUE，NSW无值可选，因此赋值不满足约束

3. 用**AC-3**算法求解树结构**CSP**在最坏情况下的复杂度是多少?

因为树状结构没有环，所以一条边只会被考虑一次，且最多只会移除d个值(d是值域最大的可取值数)

因此最坏情况下，每条边都会被考虑一次。为$O(Ed)$，E是边数