习题集第六章

16. 解: $Y_1 \sim N(\mu, \frac{1}{6}\sigma^2)$ $\quad$ $Y_2 \sim N(\mu, \frac{1}{6}\frac{1}{3}\sigma^2)$

$$\sqrt{2}(Y_1 - Y_2) \sim N(0, \sigma^2)$$

$$\frac{2S^2}{\sigma^2} \sim \chi_2^2$$

$$Z = \frac{\sqrt{2}(Y_1 - Y_2)}{S} = \frac{\sqrt{2}(Y_1 - Y_2)}{\sqrt{\frac{2S^2}{2\sigma^2}}} \sim t_2$$

18. 解: (1) $\bar{X} \sim N(a, \frac{\sigma^2}{n})$

(2) $n\bar{X} = \Sigma X_i \sim P(n\lambda)$

$$P(n\bar{X} = k) = \frac{(n\lambda)^k}{k!} e^{-n\lambda}$$

$$\Rightarrow P(\bar{X} = \frac{k}{n}) = \frac{(n\lambda)^k}{k!} e^{-n\lambda}, \quad k = 0, 1, \cdots$$

$$P(\bar{X} = t) = \frac{(n\lambda)^{nt}}{(nt)!} e^{-n\lambda}, \quad t = 0, \frac{1}{n}, \frac{2}{n}, \cdots$$ 规定一下取值

(3) $X_i \sim Exp(\lambda) = \Gamma(1, \lambda)$ $\Rightarrow$ $n\bar{X} = \Sigma X_i \sim \Gamma(n, \lambda)$

$$\bar{X} \sim \Gamma(n, n\lambda)$$

即

$$f_{\bar{X}}(x) = \frac{(n\lambda)^n}{\Gamma(n)} x^{n-1} e^{-n\lambda x}, \quad x \geq 0$$

20. 解: $X_{n+1}-\bar{X} \sim N(0, \frac{n+1}{n}\sigma^2)$

$$\frac{X_{n+1}-\bar{X}}{\sigma}\sqrt{\frac{n}{n+1}} \sim N(0,1)$$

$$\frac{(n-1)S_n^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$\Rightarrow \frac{\frac{X_{n+1}-\bar{X}}{\sigma}\sqrt{\frac{n}{n+1}}}{\frac{S_n}{\sigma}} = \frac{X_{n+1}-\bar{X}}{S_n}\sqrt{\frac{n}{n+1}} \sim t_{n-1}$$

习题集第七章

1. 解: $E(X) = 3-4\theta$

$$\theta = \frac{3-EX}{4}$$

故 θ 矩估计为 $\hat{\theta} = \frac{3-\bar{x}}{4} = 0.38$

3. 解: (1) $EX = \frac{\theta-1}{2} \Rightarrow \theta = 2EX+1$

$$\hat{\theta} = 2\bar{x}+1$$

(2) $EX = m\theta$

$Var(X) = m\theta(1-\theta)$

$$\Rightarrow \theta = 1 - \frac{Var(X)}{E(X)}$$

用 $\bar{x}$ 和 s^2 代入上式 $E(x)$ 与 $Var(x)$

$$\hat{\theta} = 1 - \frac{s^2}{\bar{x}}$$

(3) $E(X)=\sum_{x=2}^{\infty} x(x-1)\theta^2(1-\theta)^{x-2}$

$=\theta^2 \sum_{x=2}^{\infty} x(x-1)(1-\theta)^{x-2}$

$=\theta^2 \cdot \frac{2}{\theta^3}$

$=\frac{2}{\theta}$

$\Rightarrow \hat{\theta}=\frac{2}{\bar{x}}$

(4) $E(X)=\sum_{x=1}^{\infty} -\frac{1}{\ln(1-\theta)}\theta^x=-\frac{\theta}{(1-\theta)\ln(1-\theta)}$

$E(X^2)=\sum_{x=1}^{\infty} -\frac{1}{\ln(1-\theta)}x\theta^x=-\frac{\theta}{(1-\theta)^2\ln(1-\theta)}$

故 $1-\theta=\frac{E(X)}{E(X^2)} \Rightarrow \hat{\theta}=1-\frac{\overline{X}}{\overline{X^2}}=1-\frac{\sum x_i}{\sum x_i^2}$

(5) $E(X)=\theta \Rightarrow \hat{\theta}=\bar{x}$

5.解：(1) $EX=\theta\int_0^{\infty}\frac{4x^3}{\theta^3\sqrt{\pi}}e^{-\frac{x^2}{\theta^2}}d\frac{x}{\theta}$

$$=\frac{\theta}{\sqrt{\pi}}\int_0^{\infty}4x^3e^{-x^2}dx$$

$$=\frac{\theta}{\sqrt{\pi}}\int_0^{\infty}2xe^{-x}dx$$

$$=\frac{2\theta}{\sqrt{\pi}}$$

$$\Rightarrow\hat{\theta}=\frac{\sqrt{\pi}\,\overline{X}}{2}$$

(2) $Var(\hat{\theta})=\frac{\pi}{4}\cdot\frac{1}{n}Var(X_i)$

$$E(X_i^2)=\int_0^{\infty}\frac{4x^4}{\theta^3\sqrt{\pi}}e^{-\frac{x^2}{\theta^2}}dx$$

$$=\frac{4\theta^2}{\sqrt{\pi}}\int_0^{\infty}t^4e^{-t^2}dt$$

$$=\frac{4\theta^2}{\sqrt{\pi}}\cdot\frac{3}{4}\int_0^{\infty}e^{-t^2}dt$$

$$=\frac{3\theta^2}{\sqrt{\pi}}\cdot\frac{1}{2}\int_{-\infty}^{\infty}e^{-t^2}dt$$

$$=\frac{3\theta^2}{2}$$

$$Var(X_i)=E(X_i^2)-(EX_i)^2=\left(\frac{3}{2}-\frac{4}{\pi}\right)\theta^2$$

$$\Rightarrow Var(\hat{\theta})=\left(\frac{3\pi}{8n}-\frac{1}{n}\right)\theta^2$$