第七章

51. 解：(1) $EX=3-5\theta \Rightarrow \hat{\theta}_M=\frac{3-\bar{X}}{5}=\frac{2}{5}$

$L(\theta)=(\frac{\theta}{2})^{n_0}\theta^{n_1}(\frac{3\theta}{2})^{n_2}(1-3\theta)^{n_3}$

……

$\Rightarrow \hat{\theta}_L=\frac{n-n_3}{3n}=\frac{4}{15}$

(2) $\hat{\theta}_M$ 与 $\hat{\theta}_L$ 皆无偏

(3) $Var(X)=E(X^2)-(EX)^2=10\theta-25\theta^2$

$Var(\hat{\theta}_M)=\frac{1}{25}Var(\bar{X})=\frac{1}{25}\theta-\frac{1}{10}\theta^2$

$n_3\sim B(n,1-3\theta) \Rightarrow Var(n_3)=3n\theta(1-3\theta)$

$Var(\hat{\theta}_L)=\frac{1}{9n^2}Var(n_3)=\frac{1}{30}\theta-\frac{1}{10}\theta^2$

$Var(\hat{\theta}_L)<Var(\hat{\theta}_M)$

$\hat{\theta}_L$ 更有效

54. 证明: $EX_i=\theta$ $Var(X_i)=\frac{1}{3}$

$E(\hat{\theta})=\theta$, $Var(\hat{\theta})=\frac{1}{3n}$

由中心极限定理可推得

$\sqrt{3n}(\hat{\theta}-\theta)\xrightarrow{d}N(0,1)$

56. (1) 证明: $Y_1=2\lambda X_1$

$F(y)=P(2\lambda X_1<y)=P(X_1<\frac{y}{2\lambda})=\int_0^{\frac{y}{2\lambda}}\lambda e^{-\lambda x}dx$

故 $f(y)=\begin{cases}\frac{1}{2}e^{-\frac{y}{2}}, & y>0\\ 0 & y\le 0\end{cases}$

即 $Y_1=2\lambda X_1\sim\chi_2^2$

从而 $\sum_{i=1}^{n}2\lambda X_i\sim\chi_{2n}^2$

(2) 枢轴变量取 $T=2\lambda n\bar{X}$

$P(a\le 2\lambda n\bar{X}\le b)=1-\alpha$

由 $2\lambda n\bar{X}\sim\chi_{2n}^2$ 知

$a=\chi_{2n}^2(1-\frac{\alpha}{2})$ $b=\chi_{2n}^2(\frac{\alpha}{2})$

从而 λ 的 $1-\alpha$ 置信区间为

$$\left[\frac{\chi_{2n}^2(1-\frac{\alpha}{2})}{2n\bar{X}},\frac{\chi_{2n}^2(\frac{\alpha}{2})}{2n\bar{X}}\right]$$

61. 解: $\left[\bar{X}-\frac{\sigma}{\sqrt{n}}u_{\alpha/2},\ \bar{X}+\frac{\sigma}{\sqrt{n}}u_{\alpha/2}\right]$

$=[46.76,\ 49.24]$

64. 解: $\left[\bar{X}-\frac{S}{\sqrt{n}}t_{n-1}(\frac{\alpha}{2}),\ \bar{X}+\frac{S}{\sqrt{n}}t_{n-1}(\frac{\alpha}{2})\right]$

(1) $[119.796,\ 124.538]$

(2) $[118.688,\ 127.534]$

(3) $[118.430,\ 124.014]$

71. (1) $\left[\frac{nS_\mu^2}{\chi_n^2(\frac{\alpha}{2})},\ \frac{nS_\mu^2}{\chi_n^2(1-\frac{\alpha}{2})}\right]$ $S_\mu^2=\frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2$

$=[0.141,\ 0.893]$

(2) $\left[\frac{(n-1)S^2}{\chi_{n-1}^2(\frac{\alpha}{2})},\ \frac{(n-1)S^2}{\chi_{n-1}^2(1-\frac{\alpha}{2})}\right]$

$=[0.149,\ 1.050]$

73. $\sigma^2: \left[\frac{(n-1)s^2}{\chi^2_{n-1}(\frac{\alpha}{2})}, \frac{(n-1)s^2}{\chi^2_{n-1}(1-\frac{\alpha}{2})}\right]$

$= [78.579, 344.930]$

$\sigma: \left[\left(\frac{(n-1)s^2}{\chi^2_{n-1}(\frac{\alpha}{2})}\right)^{\frac{1}{2}}, \left(\frac{(n-1)s^2}{\chi^2_{n-1}(1-\frac{\alpha}{2})}\right)^{\frac{1}{2}}\right]$

$= [8.864, 18.572]$

76. $\left[\bar{X} - u_{\alpha/2}\sqrt{\frac{\bar{X}(1-\bar{X})}{n}}, \bar{X} + u_{\alpha/2}\sqrt{\frac{\bar{X}(1-\bar{X})}{n}}\right]$

$= [0.015, 0.045]$

78.(1) $\left[\frac{t_0}{S}-u_{\alpha/2}\sqrt{\frac{\frac{t_0}{S}(1-\frac{t_0}{S})}{S}}\ ,\ \frac{t_0}{S}+u_{\alpha/2}\sqrt{\frac{\frac{t_0}{S}(1-\frac{t_0}{S})}{S}}\right]$

(2) $\left[\frac{r}{\frac{t_0}{S}+u_{\alpha/2}\sqrt{\frac{\frac{t_0}{S}(1-\frac{t_0}{S})}{S}}}\ ,\ \frac{r}{\frac{t_0}{S}-u_{\alpha/2}\sqrt{\frac{\frac{t_0}{S}(1-\frac{t_0}{S})}{S}}}\right]$

80. 解: $\hat{\theta}_{MLE}=X_{(1)}$，令 $Y=\frac{X_{(1)}}{\theta}$

$f_{X_{(1)}}(x)=n(1-F(x))^{n-1}f(x)$

$=-n\frac{x^{n-1}}{\theta^n}I(\theta<x<0)$

$f_Y(y)=\cdots$

算得 $f(y)=ny^{n-1}$，$0<y<1$ 与参数θ无关

取 $Y=\frac{X_{(1)}}{\theta}$ 为枢轴变量

$F(y)=y^n$，$0<y<1$

$P(\frac{X_{(1)}}{\theta}\le a)=1-\alpha$ $\quad P(\frac{X_{(1)}}{\theta}\ge b)=1-\alpha$ (*)

$\Rightarrow a=(1-\alpha)^{\frac{1}{n}}$ $\quad b=\alpha^{\frac{1}{n}}$

由(*)式 进而得到参数θ的 $1-\alpha$ 置信下限和上限 $(\theta<0)$

即由 $P(\theta\ge\frac{X_{(1)}}{\alpha^{\frac{1}{n}}})=1-\alpha$ $\quad P(\theta\le\frac{X_{(1)}}{(1-\alpha)^{\frac{1}{n}}})=1-\alpha$

$\Rightarrow \hat{\theta}_L=\frac{X_{(1)}}{\alpha^{\frac{1}{n}}}$ $\quad \hat{\theta}_U=\frac{X_{(1)}}{(1-\alpha)^{\frac{1}{n}}}$

83. 求63题中···

(1) $\frac{nS_\mu^2}{\chi_n^2(1-\alpha)}$ ， $S_\mu^2=\frac{1}{n}\sum(X_i-\mu)^2$

$=0.0368$

(2) $\frac{(n-1)S^2}{\chi_{n-1}^2(1-\alpha)}=0.0340$

85. $\bar{X}-\frac{S}{\sqrt{n}}t_{n-1}(\alpha)=41147.53$