

Lecture 24 Electro-optic effect

As we have studied in Lecture 3 the Lorentz model shown in Fig.24.1a used for estimation of the material susceptibility takes into account only the lowest order term, and the equation of motion is

$$m_0 \frac{d^2 x}{dt^2} = -Kx \quad (24.1)$$

where the restoring force coefficient K is a constant. Solution of this equation is obviously a harmonic oscillatory motion

$$x \sim x_0 e^{-j\omega_0 t} + c.c. \quad (24.2)$$

where $\omega_0 = \sqrt{K / m_0}$. Equation of motion (24.1) corresponds to the electron moving in the parabolic potential energy well

$$U(x) = -U_0 + \frac{Kx^2}{2} \quad (24.3)$$

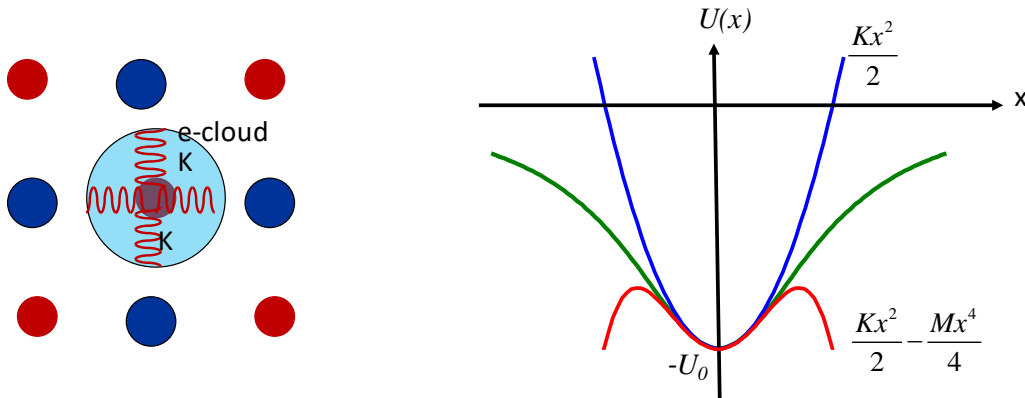


Figure 24.1 (a) Electron in a symmetric environment (b) Electron potential (red line), its parabolic approximation (blue line) and the first two terms in series expansion (black line)

as shown in Fig.24.1 b (blue line). There is a problem with this potential as it assumes infinitely large potential away from the equilibrium position of the electron which is of course unphysical – the real potential shown by a red line smoothly approaches a steady energy (zero level in our example) and if the energy becomes positive electron gets ionized. Obviously, parabolic potential is just an approximation. One can expand the potential into a power series that contains only the even orders of x since potential has inversion symmetry, i.e. $U(-x) = U(x)$,

$$U(x) = -U_0 + \frac{Kx^2}{2} - \frac{Mx^4}{4} + \dots \quad (24.4)$$

The new equation of motion is

$$m_0 \frac{d^2 x}{dt^2} = -Kx + Mx^3 + \dots \quad (24.5)$$

and it contains only the odd order terms. The potential (24.4) is non-parabolic or one can say that is and the electron motion is no longer harmonic and behaves as anharmonic oscillator. One can see that the third order term becomes important when the amplitude of oscillation increases which of course happens when the driving field is strong. Every material starts behaving as an anharmonic oscillator at high optical fields i.e. every material possesses odd order nonlinear terms in its response. We shall look at the lowest of these, the third order response later on, but for now we are concerned with finding the materials that have even order nonlinear terms, starting with the largest at most relative second order response.

Consider material that does have center of inversion symmetry, for instance the one in Fig.24.2(a) – clearly the electron sees different environments on the left and on the right, hence the potential confining this electron is asymmetric as shown in Fig.24.2 (b) and as such it cannot be approximated by an even order curve – its series expansion must contain odd terms,

$$U(x) = -U_0 + \frac{Kx^2}{2} + \frac{Lx^3}{3} + \dots \quad (24.6)$$

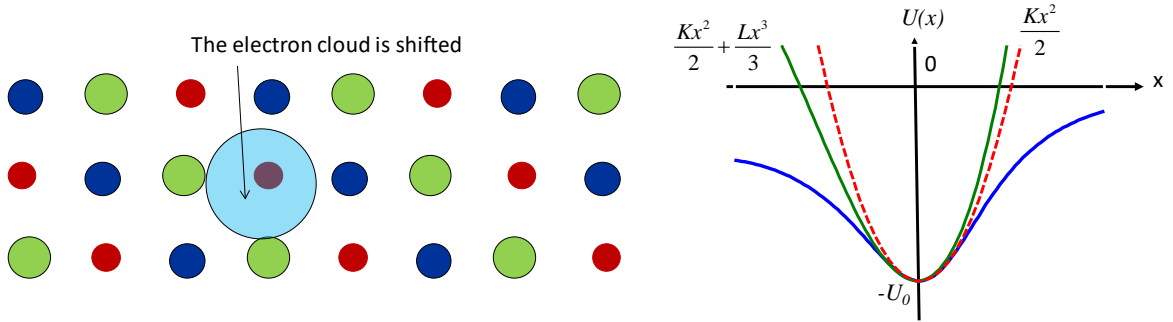


Figure 24.1 (a) Electron in an environment with no inversion symmetry (b) Electron potential (blue line), its parabolic approximation (dashed black line) and the first two terms in series expansion (red line)

The equation of motion (keeping two lowest order terms) is now

$$m_0 \frac{d^2 x}{dt^2} = -Kx - Lx^2 + \dots \quad (24.7)$$

Let us see now what happens when we applied a DC field in the x direction as shown in Fig.24.2 (a). In fact, under DC we actually mean low frequency relative to optical frequency, i.e. up to 1THz. The impact of DC (low frequency) field will depend on its sign (i.e. direction). Let us solve equation of motion (24.7) in the presence of DC and AC fields

$$m_0 \frac{d^2 x}{dt^2} = -Kx - Lx^2 - eE_{DC} - eE(t) \quad (24.8)$$

where $E(t) = E_\omega e^{-j\omega t}$. We shall look for the solution that contains separate optical and DC responses and the mixed response

$$x(t) = aE_\omega e^{-j\omega t} + bE_{DC} + cE_{DC}E_\omega e^{-j\omega t} + \dots \quad (24.9)$$

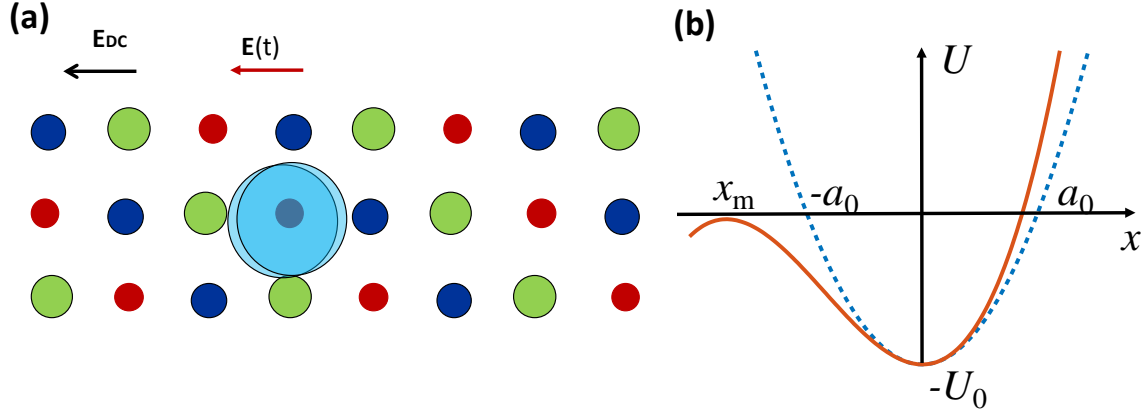


Figure 24.3. (a) Electro optic crystal simultaneously subjected to optical and DC fields (b) explaining the order of magnitude of coefficients K and L in (24.7)

Mixed term is much smaller than the first two. Substituting (24.9) into (24.8) we obtain

$$\begin{aligned} -a\omega^2 E_\omega e^{-j\omega t} - c\omega^2 E_{DC} E_\omega e^{-j\omega t} &= -\frac{K}{m_0} [aE_\omega e^{-j\omega t} + bE_{DC} + cE_{DC}E_\omega e^{-j\omega t}] - \\ &-\frac{L}{m_0} [a^2 E_\omega^2 e^{-2j\omega t} + c^2 E_{DC}^2 e^{-2j\omega t} + 2acE_\omega E_{DC} e^{-2j\omega t} + b^2 E_{DC}^2 + 2abE_{DC}E_\omega e^{-j\omega t} + 2bcE_{DC}^2 E_\omega e^{-j\omega t}] - \\ &-\frac{e}{m_0} E_{DC} - \frac{e}{m_0} E_\omega e^{-j\omega t} \end{aligned} \quad (24.10)$$

We omit all the second harmonic terms containing $\exp(-2j\omega t)$ which are responsible for the second harmonic generation (SHG) because right now are only interested in the electro-optic effect. We shall return to SHG later in the course. We can also neglect the last term in the second square brackets because it is of the third order in the electric fields. That leaves us with

$$\begin{aligned} -a\omega^2 E_\omega e^{-j\omega t} - c\omega^2 E_{DC} E_\omega e^{-j\omega t} &= -\omega_0^2 [aE_\omega e^{-j\omega t} + bE_{DC} + cE_{DC}E_\omega e^{-j\omega t}] - \\ &-\frac{L}{m_0} [b^2 E_{DC}^2 + 2abE_{DC}E_\omega e^{-j\omega t} + \dots] - \frac{e}{m_0} E_{DC} - \frac{e}{m_0} E_\omega e^{-j\omega t} \end{aligned} \quad (24.11)$$

We now equate the terms of the same order and time dependence.

First we have

$$a(\omega_0^2 - \omega^2)E_\omega e^{-j\omega t} = -\frac{e}{m_0}E_\omega e^{-j\omega t} \quad (24.12)$$

This has solution

$$a = -\frac{e}{m_0(\omega_0^2 - \omega^2)} \quad (24.13)$$

And represent previously studied linear motion at optical frequency

Second, we have the linear motion due to DC (low frequency) field

$$\omega_0^2 b E_{DC} = -\frac{e}{m_0}E_{DC} \quad (24.14)$$

with a solution

$$b = -\frac{e}{m_0\omega_0^2} \quad (24.15)$$

And finally we have the remaining terms

$$(\omega_0^2 - \omega^2)cE_{DC}E_\omega e^{-j\omega t} = -\frac{2L}{m_0}abE_{DC}E_\omega e^{-j\omega t} \quad (24.16)$$

which give us the solution for the mixed term

$$c = -\frac{2L}{m_0} \frac{ab}{\omega_0^2 - \omega^2} = -\frac{2L}{m_0^3} \frac{e^2}{(\omega_0^2 - \omega^2)^2 \omega_0^2} \quad (24.17)$$

Total polarization at optical frequency can then be found as

$$P_\omega = -Nex_\omega = -NeaE_\omega - ecE_{DC}E_\omega = \left[\frac{Ne^2}{m_0(\omega_0^2 - \omega^2)} + \frac{2L}{m_0^3} \frac{Ne^3E_{DC}}{(\omega_0^2 - \omega^2)^2 \omega_0^2} \right] E_\omega = \varepsilon_0\chi(\omega)E_\omega + 2\varepsilon_0\chi^{(2)}(\omega)E_{DC}E_\omega \quad (24.18)$$

where the previously studied linear (or first order) susceptibility is

$$\chi(\omega) \equiv \chi^{(1)}(\omega) = \frac{\omega_p^2}{\omega_0^2 - \omega^2} \quad (24.19)$$

$\omega_p = \sqrt{Ne^2 / \varepsilon_0 m_0}$ is the previously introduced in Lecture 3 plasma frequency,

and nonlinear (second order) susceptibility is

$$\chi^{(2)}(\omega) = \frac{Le}{m_0^2} \frac{\omega_p^2}{(\omega_0^2 - \omega^2)^2 \omega_0^2} \quad (24.20)$$

The units of second order susceptibility is m/V, i.e. inverse of the electric field.

Let us find the order of magnitude of the upper limit of second order nonlinearity. As one can see from Fig. 24.3 b the plot of the potential (24.7) in which just the first two terms are kept, when the derivative of the potential reaches 0, which happens at $x_m = -K/L$ the electron is no longer confined, The electron is confined within characteristic size a_0 from the equilibrium position. If x_m were significantly less than a_0 it would be very easy to “eject” the electron. So $x_m \geq a_0$ and

$$L \leq K / a_0 \quad (24.21)$$

. Also looking at the dashed line for parabolic approximation

$$U(a_0) = -U_0 + Ka_0^2 / 2 \approx 0 \quad (24.22)$$

and $K \sim 2U_0 / a_0^2$. Now, substitute (24.21) and (24.22) into (24.20) and obtain

$$\chi^{(2)}(\omega) = \frac{Le}{m_0^2 \omega_0^4} \frac{\omega_0^2}{\omega_0^2 - \omega^2} \chi^{(1)}(\omega) = \frac{Le}{K^2} \frac{\omega_0^2}{\omega_0^2 - \omega^2} \chi^{(1)}(\omega) < \frac{ea_0^2}{U_0} \frac{1}{a_0} \frac{\omega_0^2}{\omega_0^2 - \omega^2} \chi^{(1)}(\omega) \approx \frac{1}{E_a} \chi^{(1)}(\omega) \quad (24.23)$$

where

$$E_a = \frac{U_0}{ea_0} \quad (24.24)$$

Is the “intrinsic or “atomic” field representing the binding force that holds crystal together. Typically the binding energy U_0 is a few eV and the distance a_0 is a few angstroms so that $E_a \sim 10^{10} \text{ V/m}$. Since $\chi^{(1)}(\omega) = n^2(\omega) \sim 1-10$, maximum conceivable value of $\chi^{(2)}(\omega) < 10^{-9} \text{ m/V}$ in the visible and near IR regions. In reality it is much less since the degree of asymmetry L/K is usually much less than extreme value of $1/a_0$

Impermeability tensor

The prior analysis was all one dimensional. In three dimensions the susceptibility is no longer a scalar, but a third-rank tensor, i.e. 3x3x3 matrix, and polarization is found as

$$\mathbf{P}_\omega = \varepsilon_0 \chi^{(1)}(\omega) \mathbf{E}_\omega + \varepsilon_0 \chi^{(2)}(\omega) \mathbf{E}_{DC} \mathbf{E}_\omega = \varepsilon_0 \left[\chi^{(1)}(\omega) + 2\chi^{(2)}(\omega) \mathbf{E}_{DC} \right] \mathbf{E}_\omega \quad (24.25)$$

Dielectric constant is obviously also a third rank tensor

$$\varepsilon_r(\omega) = [1 + \chi^{(1)}(\omega)] + 2\chi^{(2)}(\omega) \mathbf{E}_{DC} = n^2(\omega) + 2\chi^{(2)}(\omega) \mathbf{E}_{DC} \quad (24.26)$$

In the electro-optics one usually introduces impermeability tensor inverse to the dielectric constant $\epsilon_r^{-1}(\omega)$. This is done by inverting the material relation $\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E}$

$$\epsilon_0 \mathbf{E} = \epsilon_r^{-1} \mathbf{D} \quad (24.27)$$

or explicitly

$$\epsilon_0 \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} (\epsilon_r^{-1})_{11} & (\epsilon_r^{-1})_{12} & (\epsilon_r^{-1})_{13} \\ (\epsilon_r^{-1})_{21} & (\epsilon_r^{-1})_{22} & (\epsilon_r^{-1})_{23} \\ (\epsilon_r^{-1})_{31} & (\epsilon_r^{-1})_{32} & (\epsilon_r^{-1})_{33} \end{pmatrix} \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} \quad (24.28)$$

The tensor is symmetric $(\epsilon_r^{-1})_{ij} = (\epsilon_r^{-1})_{ji}$ so it has only 6 independent components. Write it as

$$\epsilon_r^{-1} = \begin{pmatrix} (\epsilon_r^{-1})_{11} & (\epsilon_r^{-1})_{12} & (\epsilon_r^{-1})_{13} \\ (\epsilon_r^{-1})_{21} & (\epsilon_r^{-1})_{22} & (\epsilon_r^{-1})_{23} \\ (\epsilon_r^{-1})_{31} & (\epsilon_r^{-1})_{32} & (\epsilon_r^{-1})_{33} \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{n^2}\right)_1 & \left(\frac{1}{n^2}\right)_6 & \left(\frac{1}{n^2}\right)_5 \\ \left(\frac{1}{n^2}\right)_6 & \left(\frac{1}{n^2}\right)_2 & \left(\frac{1}{n^2}\right)_4 \\ \left(\frac{1}{n^2}\right)_5 & \left(\frac{1}{n^2}\right)_4 & \left(\frac{1}{n^2}\right)_3 \end{pmatrix} \quad (24.29)$$

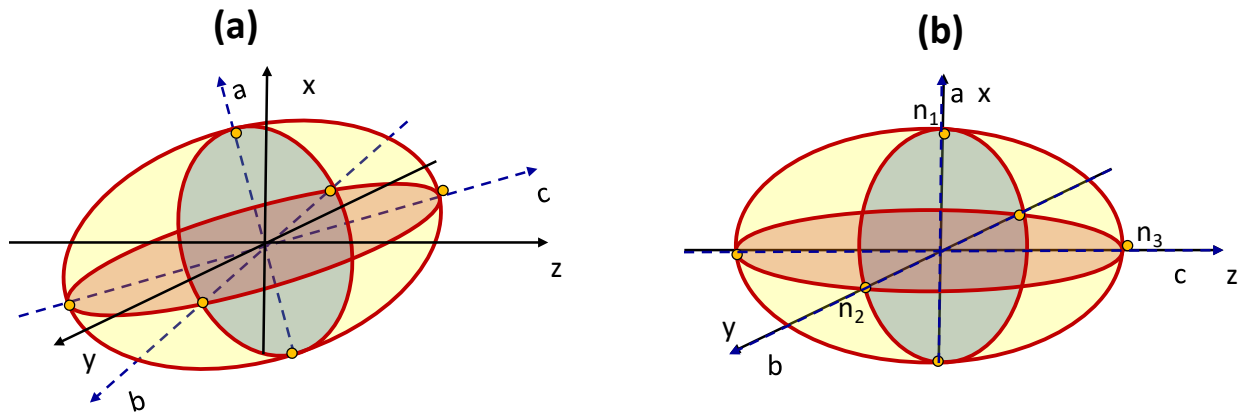


Figure 24.4 Index ellipsoid and its relation to impermeability tensor. (a) arbitrary system of coordinates (b) principal system of coordinates

Now we can write the equation for the index ellipsoid as

$$\text{Tr}[\epsilon_r^{-1}(\mathbf{r} \otimes \mathbf{r})] = 1 \quad (24.30)$$

or explicitly

$$Tr \left[\begin{pmatrix} \left(\frac{1}{n^2}\right)_1 & \left(\frac{1}{n^2}\right)_6 & \left(\frac{1}{n^2}\right)_5 \\ \left(\frac{1}{n^2}\right)_6 & \left(\frac{1}{n^2}\right)_2 & \left(\frac{1}{n^2}\right)_4 \\ \left(\frac{1}{n^2}\right)_5 & \left(\frac{1}{n^2}\right)_4 & \left(\frac{1}{n^2}\right)_3 \end{pmatrix} \begin{pmatrix} x^2 & xy & xz \\ yx & y^2 & yz \\ zx & zy & z^2 \end{pmatrix} \right] = 1 \quad (24.31)$$

Performing multiplication and finding the sum of diagonal elements we obtain

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 yz + 2\left(\frac{1}{n^2}\right)_5 xz + 2\left(\frac{1}{n^2}\right)_6 xy = 1 \quad (24.32)$$

Which is indeed the equation of ellipsoid whose axes are rotated relative to the system of coordinates as shown in Fig.24.4 a. One can always find the principal system of coordinates in which the tensor becomes diagonal, i.e.

$$\varepsilon_r^{-1} = \begin{pmatrix} 1/n_1^2 & 0 & 0 \\ 0 & 1/n_2^2 & 0 \\ 0 & 0 & 1/n_3^2 \end{pmatrix} \quad (24.33)$$

In these coordinates the ellipsoid becomes the familiar

$$\frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} = 1 \quad (24.34)$$

as shown in Fig24.4b.

Electro-optic (Pockels) tensor

According to (24.26) $\varepsilon_r(\omega) = n^2(\omega) + 2\chi^{(2)}(\omega)\mathbf{E}_{DC}$ and one can see that describe second order susceptibility as DC-field induced change in the dielectric constant

$$2\chi^{(2)}(\omega) = \frac{\partial \varepsilon_r(\omega)}{\partial \mathbf{E}_{DC}} \quad (24.35)$$

Introduce tensor of *electro-optic (Pockels) coefficients* as

$$r(\omega) = \frac{\partial \varepsilon_r^{-1}(\omega)}{\partial \mathbf{E}_{DC}} \approx -\frac{1}{(\bar{\varepsilon})^2} \frac{\partial \varepsilon_r(\omega)}{\partial \mathbf{E}_{DC}} = -\frac{2\chi^{(2)}(\omega)}{\bar{n}^4} \quad (24.36)$$

Where $\bar{n} = \sqrt{\bar{\varepsilon}_r}$ is the average refractive index. Then

$$\varepsilon_r^{-1}(\omega) = \varepsilon_{r,0}^{-1}(\omega) + r(\omega)\mathbf{E}_{DC} \quad (24.37)$$

The problem with $r(\omega)$ is that it is a third rank tensor and as such is difficult to represent on 2D sheet of paper. So we make a transformation of convenience, using the fact that only 6 components of the second rank tensor (3×3 matrix) ε_r^{-1} are independent, write it as a first rank tensor, i.e. 1×6 vector and then write a third rank tensor ($3 \times 3 \times 3$ matrix) of $r(\omega)$ as a second write tensor (3×6) matrix. Then (24.37) assumes this explicit form

$$\begin{pmatrix} (1/n^2)_1 \\ (1/n^2)_2 \\ (1/n^2)_3 \\ (1/n^2)_4 \\ (1/n^2)_5 \\ (1/n^2)_6 \end{pmatrix} = \begin{pmatrix} (1/n^2)_{1,0} \\ (1/n^2)_{2,0} \\ (1/n^2)_{3,0} \\ (1/n^2)_{4,0} \\ (1/n^2)_{5,0} \\ (1/n^2)_{6,0} \end{pmatrix} + \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} \begin{pmatrix} E_{DC,x} \\ E_{DC,y} \\ E_{DC,z} \end{pmatrix} \quad (24.38)$$

While at first glance it looks intimidating, it is not that complicated since depending on the symmetry of a given crystal many of the matrix elements are zero or equal to each other.

The Pockels coefficients of various EO crystals are given in Table 24.1

Characteristics of some electro-optic materials used in Pockels cells				
Material	Linear electro-optic coefficient, r (pm V ⁻¹)	n_o^\dagger	n_e^\dagger	Relative permittivity, ϵ_r
KH ₂ PO ₄ (KDP)	10.6	1.51	1.47	42
KD ₂ PO ₄ (KD*P)	26.4	1.51	1.47	50
AH ₂ PO ₄ (ADP)	8.5	1.52	1.48	12
Cadmium telluride (CdTe)	6.8	2.6		7.3
Lithium tantalate (LiTaO ₃)	30.3	2.175	2.180	43
Lithium niobate (LiNbO ₃)	30.8	2.29	2.20	18
Gallium arsenide (GaAs)	1.6	3.6		11.5
Zinc sulfide (ZnS)	2.1	2.32		16

[†] Values near a wavelength of 550 nm.

[‡] Low frequency values.

Table 24.1 Electro optic materials

Ionic contribution to EO effect

Consider LiNbO₃ with $r_{33} = 30 \text{ pm/V}$ - according to (24.36) $\chi^{(2)}(\omega) = n^4 r_{33} / 2 \approx 350 \text{ pm/V}$ This is way too much considering that only a few electronic bonds are actually non-centro-symmetric. There must be another contribution – ionic. As shown in Fig. 24.4*, when the electric field is applied, the bond length changes.

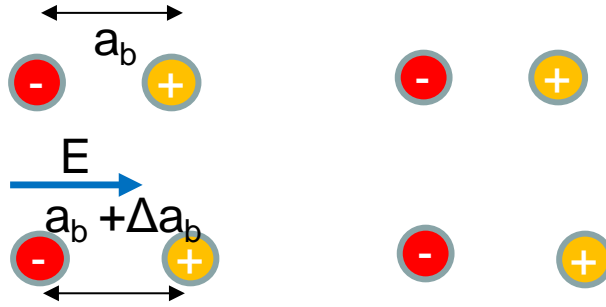


Figure 24.4* Bond length change with applied field

When bond length increases (decreases), the restoring force K in the Lorentz model decreases (increases), hence the resonant frequency $\omega_0^2 = K / m_0$ also decreases (increases) and the optical permittivity and refractive index increase (decrease). The easier it is to move ions, the larger is the change. The measure of “easiness” of ion motion is ionic polarizability, i.e. static dielectric constant ϵ_{DC} . Therefore good electro-optic materials have all large static dielectric constant and are often ferroelectric. In some materials, like GaAs electronic and ionic contributions nearly cancel each other.

Index Ellipsoid in the Electric Field

A very common kind of crystals KDP (KH_2PO_4), KD*P (KD_2PO_4), ADP (NH_4PO_4). They have just three non-zero electro-optic coefficients $r_{41}=r_{52}$ and r_{63} (* stands for deuterium instead of hydrogen)

Crystal	KDP	KD*P	ADP
r_{41} (pm/V)	8	9	23
r_{63} (pm/V)	10	27	8

These crystals have high quality, high damage threshold, and can be grown in large volume. An example of KDP crystal (although grown for a different application is shown in Fig.24.5 a. In the absence of applied field these crystals are uniaxial with impermeability tensor

$$\epsilon_{r,0}^{-1} = \begin{pmatrix} 1/n_o^2 & 0 & 0 \\ 0 & 1/n_o^2 & 0 \\ 0 & 0 & 1/n_e^2 \end{pmatrix} = \begin{pmatrix} 1/n_o^2 \\ 1/n_o^2 \\ 1/n_e^2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (24.39)$$

and the usual index ellipsoid equation

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1 \quad (24.40)$$

Next we apply bias along the optical axis direction as in Fig. 24.5 b and obtain according to (24.38)

$$\varepsilon_r^{-1} = \begin{pmatrix} 1/n_o^2 \\ 1/n_o^2 \\ 1/n_e^2 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ E_{DC} \end{pmatrix} = \begin{pmatrix} 1/n_o^2 \\ 1/n_o^2 \\ 1/n_e^2 \\ 0 \\ 0 \\ r_{63}E_{DC} \end{pmatrix} = \begin{pmatrix} 1/n_o^2 & r_{63}E_{DC} & 0 \\ r_{63}E_{DC} & 1/n_o^2 & 0 \\ 0 & 0 & 1/n_e^2 \end{pmatrix} \quad (24.41)$$

Clearly the crystal becomes biaxial with index ellipsoid

$$\frac{x^2}{n_o^2} + 2r_{63}E_{DC}xy + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1 \quad (24.42)$$

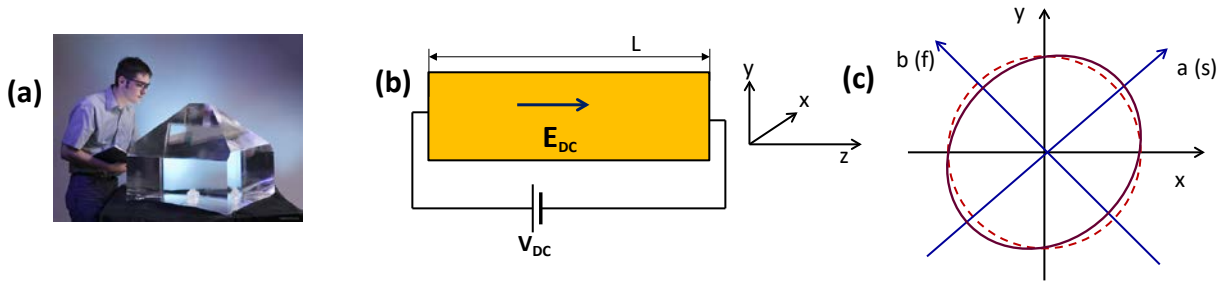


Figure 24.5 (a) KD*P crystal at National Ignition Facility (b) KDP crystal with applied bias (c) change in the index ellipsoid cross-section – induced birefringence

An examination shows that this is an ellipsoid rotated 45 degrees around z-axis as shown in Fig.24.5.c. Indeed, apply the rotate the co-ordinate system from xyz into abz ,

$$\begin{aligned} \varepsilon_{r,ab}^{-1} &= R_{-45^\circ} \varepsilon_{r,xy}^{-1} R_{45^\circ} = \frac{1}{2} \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/n_o^2 & r_{63}E_{DC} & 0 \\ r_{63}E_{DC} & 1/n_o^2 & 0 \\ 0 & 0 & 1/n_e^2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \\ &= \frac{1}{2} \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/n_o^2 - r_{63}E_{DC} & 1/n_o^2 + r_{63}E_{DC} & 0 \\ r_{63}E_{DC} - 1/n_o^2 & 1/n_o^2 + r_{63}E_{DC} & 0 \\ 0 & 0 & 1/n_e^2 \end{pmatrix} = \\ &= \begin{pmatrix} 1/n_o^2 - r_{63}E_{DC} & 0 & 0 \\ 0 & 1/n_o^2 + r_{63}E_{DC} & 0 \\ 0 & 0 & 1/n_e^2 \end{pmatrix} = \begin{pmatrix} 1/n_s^2 & 0 & 0 \\ 0 & 1/n_f^2 & 0 \\ 0 & 0 & 1/n_e^2 \end{pmatrix} \end{aligned} \quad (24.43)$$

Where we have introduced new slow and fast indices

$$1/n_{s,f}^2 = 1/n_o^2 \mp r_{63}E_{DC} \quad (24.44)$$

and therefore

$$n_{s,f} \approx n_0 \pm \frac{1}{2} n_o^3 r_{63} E_{DC} \quad (24.45)$$

Now, Jones Matrix in new sf coordinates is obviously

$$W_{\Delta\Phi, sf} = \begin{pmatrix} e^{j\Delta\Phi/2} & 0 \\ 0 & e^{-j\Delta\Phi/2} \end{pmatrix} \quad (24.46)$$

Where

$$\Delta\Phi = \frac{2\pi}{\lambda} (n_s - n_f) L = \frac{2\pi}{\lambda} n_o^3 r_{63} E_{DC} L = \pi \frac{V_{DC}}{V_\pi} \quad (24.47)$$

where we have introduced the half-wave voltage

$$V_\pi = \frac{\lambda}{2n_o^3 r_{63}} \quad (24.48)$$

i.e. voltage that induces 180 degrees' phase shift, i.e. turns the crystal into a half waveplate. Now, in xy coordinates

$$\begin{aligned} W_{\Delta\Phi, xy} &= R_{-45^\circ} W_{\Delta\Phi, sf} R_{45^\circ} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{j\Delta\Phi/2} & 0 \\ 0 & e^{-j\Delta\Phi/2} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \\ &= \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{j\Delta\Phi/2} & e^{j\Delta\Phi/2} \\ -e^{-j\Delta\Phi/2} & e^{-j\Delta\Phi/2} \end{pmatrix} = \begin{pmatrix} \cos(\Delta\Phi/2) & j\sin(\Delta\Phi/2) \\ j\sin(\Delta\Phi/2) & \cos(\Delta\Phi/2) \end{pmatrix} \end{aligned} \quad (24.49)$$

and when we place the crystal between two crossed polarizers as shown in Fig. 24.6a we end up with a Jones matrix

$$\begin{aligned} M &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos(\Delta\Phi/2) & j\sin(\Delta\Phi/2) \\ j\sin(\Delta\Phi/2) & \cos(\Delta\Phi/2) \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & j\sin(\Delta\Phi/2) \\ 0 & \cos(\Delta\Phi/2) \end{pmatrix} = j \begin{pmatrix} 0 & \sin(\Delta\Phi/2) \\ 0 & 0 \end{pmatrix} \end{aligned} \quad (24.50)$$

Therefore, transmission of the vertically polarized light is

$$E_{out} = E_{in} \sin(\Delta\Phi/2) \quad (24.51)$$

Or for optical power

$$S_{out} = S_{in} \sin^2(\Delta\Phi/2) = S_{in} \sin^2\left(\frac{\pi V_{DC}}{2V_\pi}\right) \quad (24.52)$$

as shown in Fig.24.6(b)

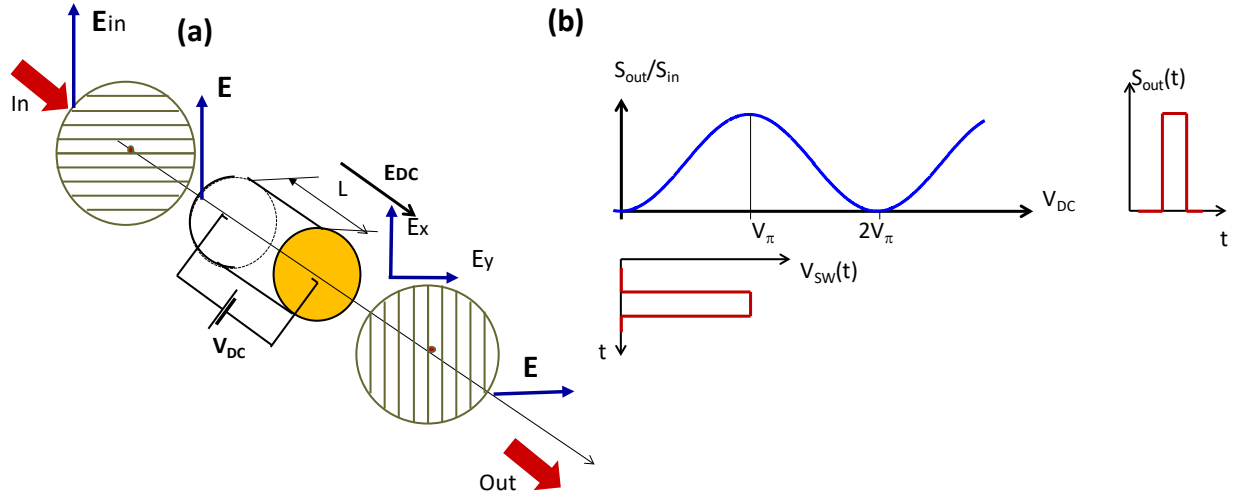


Figure 24.6 (a) Longitudinal electro-optic modulator (b) Input-Output characteristics and use for switching

So, the meaning of V_π is that it is a voltage that fully switches the device on. The problem though is that while the device functions very well as a switch it is not a very good modulator due to quadratic input-output dependence for low power signals. But the modulator can be easily linearized by adding a QWP as shown in Fig.24.7 (a) that shifts the input-output characteristic by 90 degrees which can be seen in Fig.24.7 (b)

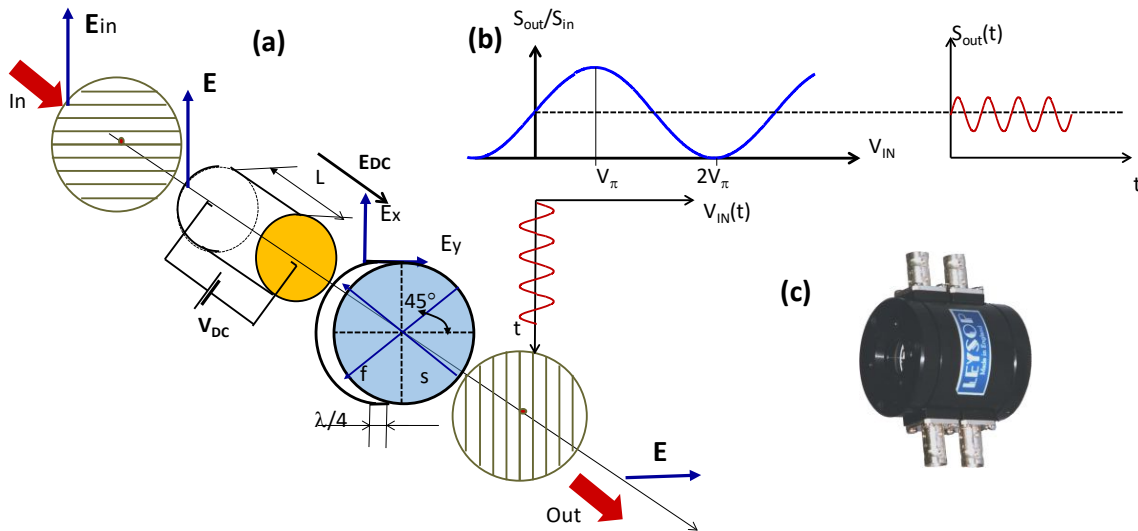


Figure 24.7 (a) Linearized electro-optical modulator (b) Its modulation characteristics (c) EO modulator photo.

If the applied field is modulated by low RF frequency Ω , the phase difference is

$$\Delta\Phi(t) = \frac{\pi V_\Omega \sin \Omega t}{V_\pi} + \frac{\pi}{2} \quad (24.53)$$

And the output power is

$$\begin{aligned}
S_{out}(t) &= S_{in} \sin^2 \left(\frac{\pi}{4} + \frac{\pi V_{\Omega} \sin \Omega t}{2V_{\pi}} \right) = \frac{S_{in}}{2} \left[1 - \cos \left(\frac{\pi}{2} + \frac{\pi V_{\Omega} \sin \Omega t}{V_{\pi}} \right) \right] = \\
&= \frac{S_{in}}{2} + \frac{S_{in}}{2} \sin \left(\frac{\pi V_{\Omega} \sin \Omega t}{V_{\pi}} \right) \approx \frac{\pi S_{in}}{2} + \frac{S_{in}}{2} \frac{V_{\Omega}}{V_{\pi}} V_{\Omega} \sin \Omega t
\end{aligned} \tag{24.54}$$

i.e. for small enough input signal modulation is linear.

Longitudinal EO modulator has an advantage of large aperture, but it also has large half-wave voltage. Also it requires transparent electrodes on its faces. Consider then transverse EO modulator based on lithium niobate crystal LiNbO_3 and closely related LiNbO_3 and LiTaO_3 all of them arranged in a lattice called perovskite and shown in Fig. 24.8 (a). These crystals are ferroelectric, i.e. they have permanent polarization. EO tensor of perovskites is

$$\begin{pmatrix} 0 & r_{12} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix} \tag{24.55}$$

and the values of the most important coefficients are

EO coefficient (pm/V)	LiNbO_3	LiTaO_3
r_{13}	9.6	8.4
r_{22}	6.8	-0.2
r_{33}	31	30.5
r_{51}	32	15

The transverse EO modulator shown in Fig.24.8(b) includes two crossed polarizers and also a compensator to compensate natural birefringence.

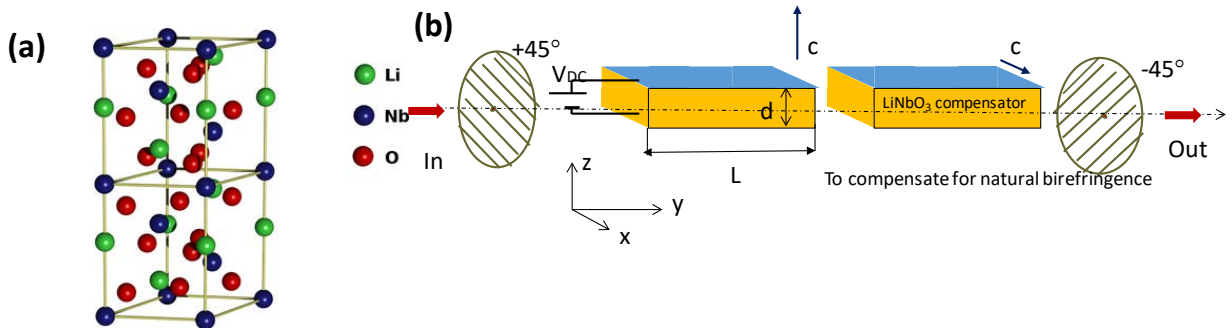


Figure 24.8 (a) Lithium niobate crystal structure (b) Lithium Niobate transverse modulator

Let us apply (24.37) to this modulator and find impermeability tensor

$$\mathcal{E}_r^{-1} = \begin{pmatrix} 1/n_o^2 \\ 1/n_o^2 \\ 1/n_e^2 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & r_{12} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ E_{DC} \end{pmatrix} = \begin{pmatrix} 1/n_o^2 + r_{13}E_{DC} \\ 1/n_o^2 + r_{13}E_{DC} \\ 1/n_e^2 + r_{33}E_{DC} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/n_o^2 + r_{13}E_{DC} & 0 & 0 \\ 0 & 1/n_o^2 + r_{13}E_{DC} & 0 \\ 0 & 0 & 1/n_e^2 + r_{33}E_{DC} \end{pmatrix} \quad (24.56)$$

Let us find indices for x and z polarizations

$$\begin{aligned} 1/n_{x,y}^2 &= 1/n_o^2 + r_{13}E_{DC} \\ n_{x,y} &\approx n_o - \frac{1}{2}n_o^3r_{13}E_{DC} \end{aligned} \quad (24.57)$$

and

$$\begin{aligned} 1/n_z^2 &= 1/n_e^2 + r_{33}E_{DC} \\ n_z &\approx n_e - \frac{1}{2}n_e^3r_{33}E_{DC} \end{aligned} \quad (24.58)$$

The relative phase delay between the vertical and horizontally polarized components is

$$\Delta\Phi = \frac{2\pi}{\lambda}(n_e - n_o)L - \frac{\pi}{\lambda}n_e^3r_{33}E_{DC}L + \frac{\pi}{\lambda}n_o^3r_{13}E_{DC}L + \frac{2\pi}{\lambda}(n_o - n_e)L \quad (24.59)$$

where the last term is the delay in compensator that cancels the first term, therefore

$$\Delta\Phi = \frac{\pi}{\lambda}(n_e^3r_{33} - n_o^3r_{13})\frac{V_{DC}L}{d} = \frac{\pi V_{DC}}{V_\pi} \quad (24.60)$$

where half wave voltage is

$$V_\pi = \frac{\lambda}{n_e^3r_{33} - n_o^3r_{13}} \frac{d}{L} \quad (24.61)$$

And is reduced compared to the longitudinal modulator by the aspect ratio d/L . Consider the example of $LiNbO_3$ modulator with $d = 0.25mm$, $L = 10mm$ operating at $\lambda = 500nm$ - we obtain $V_\pi = 64V$ which is much less than in longitudinal modulator but comes at a price of reduced aperture.

Transit time limitation

Consider the modulator in Fig.24.9(a) driven by an RF voltage $V_{RF} = V_\Omega \sin \Omega t$. Since index of refraction is time dependent the light encounters different indices as it propagates. Output phase for each polarization is

$$\Phi_{out}^{(e,o)} = \Phi_0^{(e,o)} + k_0 \int_0^L n_{e,o}(z) dz \quad (24.62)$$

And the induced retardation (phase difference) after the propagation is

$$\Delta\Phi_{out} = \Delta\Phi_0 + k_0 \int_0^L [n_e(z) - n_o(z)] dz \quad (24.63)$$

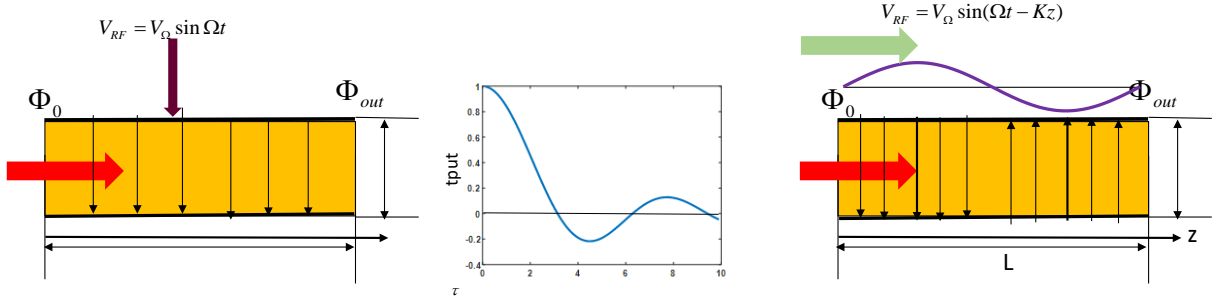


Figure 24.9 (a) Lumped EO modulator driven by AC voltage (b) Frequency response (c) Traveling wave EO modulator.

Now the position of light (photon) is $z = v_g(t - t_{in})$ where t ranges from t_{in} to $t_{out} = t_{in} + \tau_t$ and $\tau_t = L / v_g$ is a transit time. Then one can write (24.63) as

$$\Delta\Phi_{out} = \Delta\Phi_0 + k_0 v_g \int_{t_{in}}^{t_{in} + \tau_t} [n_e(t) - n_o(t)] dt = \Delta\Phi_0' + \frac{\pi v_g}{\lambda d} \int_{t_{in}}^{t_{in} + \tau_t} [n_e^3 r_{33} - n_o^3 r_{13}] V_\Omega \sin \Omega t dt = \Delta\Phi_0' + \pi \frac{V_\Omega}{V_\pi} \tau_t^{-1} \int_{t_{in}}^{t_{in} + \tau_t} \sin \Omega t dt \quad (24.64)$$

where all the terms that do not depend on time have been incorporated into $\Delta\Phi_0'$. Taking the integral

$$\tau_t^{-1} \int_{t_{in}}^{t_{in} + \tau_t} \sin \Omega t dt = \frac{\cos(\Omega t_{in}) - \cos(\Omega t_{in} + \Omega \tau_t)}{\Omega \tau_t} = \frac{\sin(\Omega \tau_t / 2)}{\Omega \tau_t / 2} \sin[\Omega(t_{in} + \tau_t / 2)] \quad (24.65)$$

And if $\Delta\Phi_0' = \pi / 2$ then modulation response (24.54) becomes

$$S_{out}(t) \approx \frac{\pi S_{in}}{2} + \frac{S_{in}}{2} \frac{V_\Omega}{V_\pi} \frac{\sin(\pi f \tau_t)}{\pi f \tau_t} \sin(2\pi f t) \quad (24.66)$$

The amplitude of it is shown in Fig.24.9 (b). As one can see modulation disappears when $\Omega \tau_t = 2\pi$ and 3dB bandwidth can be found as roughly

$$f_{3dB} = 2 / \pi \tau_t \quad (24.67)$$

The bandwidth is thus transit time limited, For instance, if $L=1\text{cm}$ $n_{e,o} \sim 2$ $\tau_t \sim 66\text{ps}$ the 3dB bandwidth is about $f_{3dB} \sim 9.5\text{GHz}$

Traveling wave EO modulator

So, the problem with lumped modulator is that as light propagates it sees the change of sign of the field when $\Omega\tau_t = \pi$ i.e. when $\tau_t = T_{RF}/2$ where $T_{RF} = 1/f$ is a period of RF and the modulation gets cancelled completely when $\tau_t = T_{RF}/2$. Consider then traveling RF wave launched along a stripline transmission line (waveguide) as shown in Fig.24.9(c). The voltage in the traveling wave propagates as a wave

$$V_{RF} = V_{\Omega} \sin(\Omega t - Kz) = V_{\Omega} \sin[\Omega(t - z/v_{RF})] \quad (24.68)$$

with velocity $v_{RF} = \Omega/K$. z is the position of the photon that entered the nonlinear crystal at time t_{in} ,

$$z = v_g(t - t_{in}) \quad (24.69)$$

And substituting it into (24.68) we obtain the voltage seen by the photon

$$V_{RF}(t) = V_{\Omega} \sin\left[\Omega\left(t - (t - t_{in})v_g/v_{RF}\right)\right] = V_{\Omega} \sin\left[\Omega\left(t(1 - v_g/v_{RF}) + t_{in}v_g/v_{RF}\right)\right] \quad (24.70)$$

According to (24.64)

$$\begin{aligned} \Delta\Phi_{out} &= \Delta\Phi_0' + \pi \frac{V_{\Omega}}{V_{\pi}} \tau_t^{-1} \int_{t_{in}}^{t_{in}+\tau_t} \sin\left[\Omega\left(t(1 - v_g/v_{RF}) + t_{in}v_g/v_{RF}\right)\right] dt = \\ &= \Delta\Phi_0' + \pi \frac{V_{\Omega}}{V_{\pi}} \frac{\sin\left(\Omega(1 - v_g/v_{RF})\tau_t/2\right)}{\Omega(1 - v_g/v_{RF})\tau_t/2} \sin\left[\Omega(t_{in} + \tau_t(1 - v_g/v_{RF})/2)\right] \end{aligned} \quad (24.71)$$

As one can see the bandwidth expands as $(1 - v_g/v_{RF})^{-1}$ in comparison to (24.65) – the closer are the two velocities the larger is the bandwidth –when two velocities are equal the propagating light sees the same field through the entire length of the modulator. Note that matching the velocities is not easy because the velocities for optical and RF waves can be quite different. For instance in LiNbO_3 refractive indices are $n_o = 2.28$, $n_e = 2.21$, but microwave refractive indices are $n_{o,RF} = 9.2$, $n_{e,RF} = 5.2$ which means that RF wave propagates much slower than optical wave. To “accelerate” microwaves one has to design the stripline in which field leaks into the air which reduces overlap with optical mode and efficiency