Magneto Optics and Optical Activity

Time reciprocity

As discussed in Section 23 one cannot achieve complete isolation using the birefringence. For example, using a λ/4 plate and a linear polarizer one can complete reject the reflected light, as shown in Fig.23.21, as long as the reflection does not affect polarization state. however, depolarized by the reflection light will still pass through. In fact, as long the system is linear, non-magnetic and lossless it is fully reciprocal. This can be understood if one considers the equations governing the propagation, i.e. wave equation

$$\nabla^2\mathbf{E}-\frac{\varepsilon_r}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=0 \tag{25.1}$$

And medium response (Lorentz model)

$$\frac{d^2\mathbf{r}}{dt^2}=-\omega_0^2\mathbf{r}-\frac{e}{m}\mathbf{E} \tag{25.2}$$

Since material polarization is $\mathbf{P}=-Ne\mathbf{r}$ and $\varepsilon_r\mathbf{E}=(1+\chi)\mathbf{E}=\mathbf{E}+\mathbf{P}/\varepsilon_0$ we obtain two coupled second order differential equations

$$\begin{aligned}\nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}&=\frac{1}{\varepsilon_0 c^2}\frac{\partial^2\mathbf{P}}{\partial t^2}\\ \frac{d^2\mathbf{P}}{dt^2}&=-\omega_0^2\mathbf{P}+\frac{Ne^2}{m}\mathbf{E}\end{aligned} \tag{25.3}$$

If we reverse the sign of time both equations will stay unchanged – hence there is full time reciprocity if only electric forces are considered.

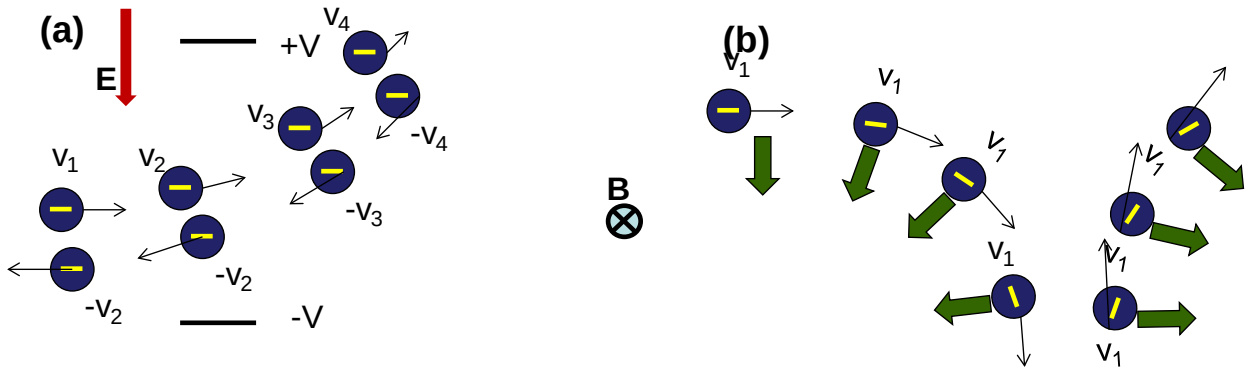


Figure 25.1 Toy model of (a) time reciprocity and in electric field, and (b) nonreciprocity in the presence of magnetic field.

One can also understand full reciprocity using a "toy" model of electron moving in the DC electric field $\mathbf{E}$ as shown Fig.25.1(a) – the electron originally moving horizontally with velocity $\mathbf{v}_1$ will curve upward. When the direction of motion is reversed at the other end the electron will retrace its original trajectory backward and end up exactly where it has started. If, on the other hand the same electron moves in the DC magnetic field $\mathbf{B}$ normal to the direction of motion, as shown in Fig.25.1(b), the electron will turn clockwise and bend downward according to the equation of motion

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{e}{m}\frac{d\mathbf{r}}{dt}\times\mathbf{B} \tag{25.4}$$

When the direction of motion is reversed the electron will still continue to turn in the clockwise direction and thus it will not go back thus breaking time reciprocity. This follows from the presence of the first order time derivative in (25.4) which obviously changes its sign when the time changes its sign.

Motion in the presence of the DC magnetic field

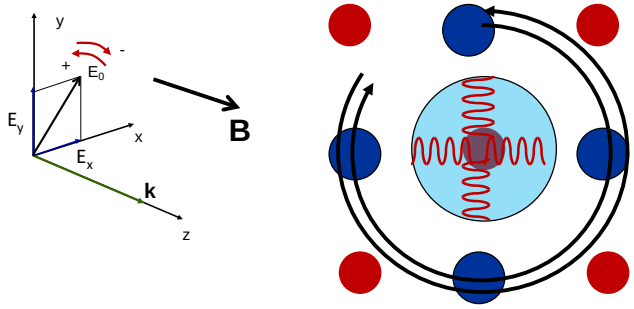


Figure 25.2 Motion of electron in Lorentz model with DC magnetic field and circularly polarized light

For simplicity we start by considering isotropic medium (Lorentz model) placed in the DC magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ and subject to the electromagnetic wave propagating along the direction of $\mathbf{B}$ as shown in Fig. 25.2 . We expect that DC Magnetic field will try to impose counter-clock-wise circular motion onto the electron therefore we consider circularly polarized light.

$$\begin{aligned} E_x(t) &= E_0\cos(kz-\omega t) = \frac{1}{2}E_0 e^{j(kz-\omega t)} + c.c. \\ E_y(t) &= \pm E_0\sin(kz-\omega t) = \mp\frac{1}{2}jE_0 e^{j(kz-\omega t)} + c.c. \end{aligned} \tag{25.5}$$

The equation of motion combines the force of electric field (25.2) and Lorentz force (25.4) as

$$\frac{d^2\mathbf{r}}{dt^2} = -\omega_0^2\mathbf{r} - \frac{e}{m}\left(\mathbf{E}_{\mp}(t) + \frac{d\mathbf{r}}{dt}\times\mathbf{B}\right) \tag{25.6}$$

where ω_0^2 is a scalar since the medium is isotropic. Introduce the Larmore frequency

$$\Omega_L = -\frac{e}{2m}B \tag{25.7}$$

Then

$$\frac{d^2\mathbf{r}}{dt^2}+2\Omega_L\frac{d\mathbf{r}}{dt}\times\hat{\mathbf{z}}+\omega_0^2\mathbf{r}=-\frac{e}{m}E_0\left(\cos(kz-\omega t)\hat{\mathbf{x}}\pm\sin(kz-\omega t)\hat{\mathbf{y}}\right) \tag{25.8}$$

Write now the equations of motion for the two projections of $\mathbf{r}$

$$\begin{aligned}\frac{d^2x}{dt^2}+2\Omega_L\frac{dy}{dt}+\omega_0^2x&=-\frac{e}{m}E_0\cos(kz-\omega t)\\ \frac{d^2y}{dt^2}-2\Omega_L\frac{dx}{dt}+\omega_0^2y&=\mp\frac{e}{m}E_0\sin(kz-\omega t)\end{aligned} \tag{25.9}$$

Now multiply the second equation in (25.9) by j and add two equations to obtain

$$\frac{d^2(x+jy)}{dt^2}+2\Omega_L\frac{d(y-jx)}{dt}+\omega_0^2(x+jy)=-\frac{e}{m}E_0e^{\pm j(kz-\omega t)} \tag{25.10}$$

Introduce a complex variable $\sigma=x+jy$, then $-j\sigma=y-jx$ and (25.10) becomes

$$\frac{d^2\sigma}{dt^2}-2j\Omega_L\frac{d\sigma}{dt}+\omega_0^2\sigma=-\frac{e}{m}E_0e^{\pm j(kz-\omega t)} \tag{25.11}$$

We shall look for a harmonic solution, i.e.

$$\sigma(t)=\sigma_0e^{\pm j(kz-\omega t)} \tag{25.12}$$

It substitution into (25.11) immediately results in

$$-\omega^2\sigma_0\mp2\Omega_L\omega\sigma_0+\omega_0^2\sigma_0=-\frac{e}{m}E_0 \tag{25.13}$$

and yields the solution for the amplitude

$$\sigma_0=\frac{-(e/m)E_0}{\omega_0^2-\omega^2\mp2\Omega_L\omega} \tag{25.14}$$

Therefore,

$$\sigma(t)=\sigma_0e^{\pm j(kz-\omega t)}=\frac{-(e/m)E_0}{\omega_0^2-\omega^2\mp2\Omega_L\omega}e^{\pm j(kz-\omega t)} \tag{25.15}$$

But since $\sigma=x+jy$ we can obtain

$$\begin{aligned}x(t)=\mathrm{Re}[\sigma(t)]&=\frac{-(e/m)E_0}{\omega_0^2-\omega^2\mp2\Omega_L\omega}\cos(kz-\omega t)\\ y(t)=\mathrm{Im}[\sigma(t)]&=\pm\frac{-(e/m)E_0}{\omega_0^2-\omega^2\mp2\Omega_L\omega}\sin(kz-\omega t)\end{aligned} \tag{25.16}$$

Now we can describe the motion of the electron in Lorentz model as circular,

$$\mathbf{r}_{\pm}(t) = x(t)\hat{\mathbf{x}} + y(t)\hat{\mathbf{y}} = \frac{-(e/m)E_0}{\omega_0^2 - \omega^2 \mp 2\Omega_L\omega}\left[\cos(kz-\omega t)\hat{\mathbf{x}} \pm \sin(kz-\omega t)\hat{\mathbf{y}}\right] \tag{25.17}$$

That produces circular polarization of material

$$\mathbf{P}_{\pm} = -Ne\mathbf{r} = \frac{(e^2/m)E_0}{\omega_0^2 - \omega^2 \mp 2\Omega_L\omega}\left[\cos(kz-\omega t)\hat{\mathbf{x}} \pm \sin(kz-\omega t)\hat{\mathbf{y}}\right] = \varepsilon_0\chi_{\pm}\mathbf{E}_{\pm} \tag{25.18}$$

Where we have introduced two different susceptibilities for the different helicitie of circularly polarized light.

$$\chi_{\pm}(\omega, B) = \frac{(e^2/m\varepsilon_0)}{\omega_0^2 - \omega^2 \mp 2\Omega_L\omega} \tag{25.19}$$

Let us see what is happening. The magnetic field tries to force counterclockwise rotation of the electron. The electric field of circularly polarized light also tries to rotate the electron but in two opposite directions depending on whether it is right (-) or left (+) polarized light. Clearly, Lorentz force assists the electric force for the left polarized wave and impedes the rotation when it is a right polarized wave. Naturally, the right polarized wave will result in larger radius of rotation of electron and therefore will have larger susceptibility.

Now, Larmore frequency is much smaller than the optical frequency – even for $B = 10T$, $\Omega_L \sim 10^{12}\, s^{-1} << \omega$, and therefore

$$\chi_{\pm}(\omega, B) = \frac{\omega_p^2}{\omega_0^2 - \omega^2 \mp 2\Omega_L\omega - \Omega_L^2 + \Omega_L^2} \approx \frac{\omega_p^2}{\omega_0^2 - \left(\omega \pm \Omega_L\right)^2} = \chi_0(\omega \pm \Omega_L) \tag{25.20}$$

Where we have neglected Ω_L^2 and $\chi_0(\omega)$ is the susceptibility in the absence of magnetic field. Obviously the same can be said about the dielectric constant and refractive index

$$\begin{aligned} \varepsilon_{r,\pm}(\omega, B) &= \varepsilon_{r,0}(\omega \pm \Omega_L) \\ n_{\pm}(\omega, B) &= n_0(\omega \pm \Omega_L) \end{aligned} \tag{25.21}$$

In other words, dispersion curves shift left or right by Larmore frequency as shown in Fig.25.3. This phenomenon, of right and left circularly polarized light having different indices is called *circular birefringence.*

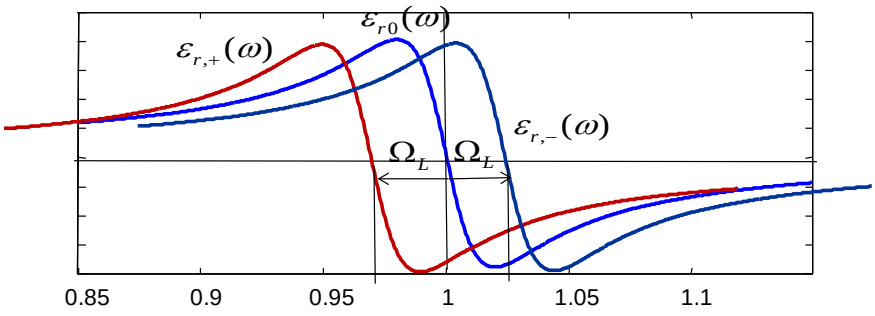


Figure 25.3 Dispersion of dielectric constant for right and left polarized light in the presence of DC magnetic field

Verdet constant

It is easy to see then that change of the refractive index depends on the material dispersion, as we obtain from (25.21)

$$n_{\pm}(\omega, B) = n(\omega \pm \Omega_L) \approx n(\omega) \pm \frac{dn}{d\omega}\Omega_L = n(\omega) \pm \frac{dn}{d\omega}\frac{e}{2m}B \tag{25.22}$$

where

$$\frac{dn}{d\omega} = \frac{dn}{d\lambda}\left[\frac{d\omega}{d\lambda}\right]^{-1} = \frac{dn}{d\lambda}\left[\frac{d\left(2\pi c/\lambda\right)}{d\lambda}\right]^{-1} = -\frac{dn}{d\lambda}\frac{\lambda^2}{2\pi c} \tag{25.23}$$

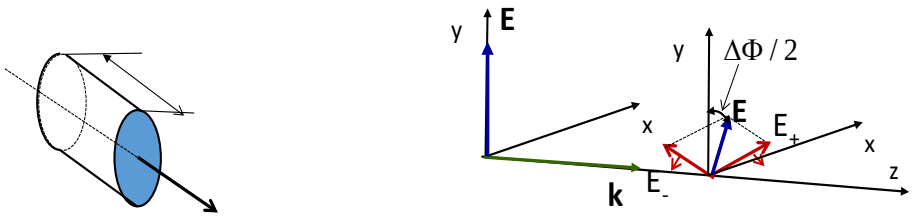


Figure 25.4 (a) Faraday Cell (b) Rotation of linearly polarized light in it

Consider now a Faraday cell in Fig.25.4(a) and calculate the phase shift between the light of two orthogonal circular polarizations

$$\Delta\Phi_{\pm}(\omega, B) = \frac{2\pi}{\lambda}\left[n_{+}(\omega) - n_{-}(\omega)\right]d = -2\frac{2\pi}{\lambda}\frac{dn}{d\lambda}\frac{\lambda^2}{2\pi c}\frac{e}{2m}Bd = -2\lambda\frac{dn}{d\lambda}\frac{e}{2mc}Bd = 2VBd \tag{25.24}$$

where we have introduced *Verdet constant*

$$V = -\lambda\frac{dn}{d\lambda}\frac{e}{2mc} = \frac{2\pi}{\lambda}\frac{dn}{d\omega}\frac{e}{2m} \tag{25.25}$$

As one can see the Verdet constant depends on wavelength but otherwise it does not change all that much. It is highly desirable to have material with large permeability μ_r to increase B. Here are the Verdet constants for different materials, typically diamagnetic materials have positive V's and paramagnetic have negative V's.

Material	λ(nm)	V (rad T^{-1})
Water	589.3	3.81
Diamond	589.3	4.68
Quartz	589.3	4.84
Light flint glass	589.3	9.23
CS_2	589.3	12.3
Pr^{3+}-B glass	670	-70.7
Pr^{3+}-Al- Si glass	700	-57.6
Tb^{3+} - Al- Si glass	700	-62.8
Dy^{3+} - Al-Si glass	700	-79.1
Pr^{3+} - P glass	700	-35.8
Tb^{3+} - P glass	700	-43.6
Ce^{3+} -P glass	500	-94.7
	700	-38.4
Pure silica glass	532	4.77
	632.8	3.13
	785	2.58
$Tb_3Ga_5O_{12}$	500	-218
	632.8	- 134
	800	-65
	1300 -	-20

Polarization rotation

Let us now find the Jones matrix of Faraday's cell. Introduce the vector in which polarization state is a superposition of right and left circular polarizations,

$$\begin{pmatrix} e_r \\ e_l \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -j \\ 1 & j \end{pmatrix} \begin{pmatrix} e_x \\ e_y \end{pmatrix} \tag{25.26}$$

In this basis the Faraday's cell is nothing but a retardation plate,

$$W_{\Delta\Phi} = \begin{pmatrix} e^{j\Delta\Phi/2} & 0 \\ 0 & e^{-j\Delta\Phi/2} \end{pmatrix} \tag{25.27}$$

Therefore in the original xy basis

$$W_{\Delta\Phi} = \frac{1}{2}\begin{pmatrix}1 & 1\\ j & -j\end{pmatrix}\begin{pmatrix}e^{j\Delta\Phi/2} & 0\\ 0 & e^{-j\Delta\Phi/2}\end{pmatrix}\begin{pmatrix}1 & -j\\ 1 & j\end{pmatrix} =$$
$$= \frac{1}{2}\begin{pmatrix}1 & 1\\ j & -j\end{pmatrix}\begin{pmatrix}e^{j\Delta\Phi/2} & -je^{j\Delta\Phi/2}\\ e^{-j\Delta\Phi/2} & je^{-j\Delta\Phi/2}\end{pmatrix} = \begin{pmatrix}\cos(\Delta\Phi/2) & -\sin(\Delta\Phi/2)\\ \sin(\Delta\Phi/2) & \cos(\Delta\Phi/2)\end{pmatrix} = R_{-\Delta\Phi/2} \quad (25.28)$$

which is simply rotation by the angle

$$\theta = \Delta\Phi/2 = VBL \quad (25.29)$$

which explains what Verdet constant means – it is specific rotation per unit length per unit magnetic induction. It is very easy to understand the rotating action from Fig.25.4(b)where the originally vertically polarized light can be represented as sum of two circular polarizations propagating with different phase velocities. As a result, the electric field right polarization rotates more than electric field of left polarization and the total field rotates slowly clockwise.

Quite often one uses permanent magnets made from a ferromagnetic or ferromagnetic material with internal magnetization $\mathbf{M}$. Then polarization rotation is simply

$$\theta_F = \rho_F \frac{M}{M_{sat}} L \quad (25.30)$$

where M_{sat} is saturation magnetization of the material and ρ_F is specific Faraday rotation. The values are are shown below . Since many of these materials absorb strongly, hence one can introduce the figure of merit – rotation per 1 dB loss. The best material is a doped yttrium iron garnet YIG –$Y_3Fe_5O_{12}$

Material	Wavelength λ (nm)	Specific rotation ρ_F (° cm^{-1})	Absorption coefficient α (dB cm^{-1})	Figure of merit ρ_F/α (° dB^{-1})
Fe[a]	546	3.5×10^5	3.3×10^6	0.11
Co	546	3.6×10^5	3.7×10^6	0.10
Ni	400	7.2×10^5	9.1×10^5	0.79
MnBi	632.8	5.3×10^5	3.3×10^6	0.16
YIG[b]	1064	280	65	4.3
	1150	250	54	4.6
	1200	240	50	4.8
	1310	224	35	6.4
	1550	216	23.8	9.1
YbBi : YIG[c]	1310	760	38	20
	1550	404	15.7	25.8
Bi : YIG[d]	1550	−1250	2.7	463
Ce : YIG[e]	1310	−2510	9.8	256
	1550	−1310	2.7	486

Dielectric constant

Let us see what happens with the tensor of dielectric constant in presence of DC magnetic field. The equations for the projections of the electron motion (25.9) in the presence of arbitrary electric field can be written as

$$\frac{d^2x}{dt^2}+2\Omega_L\frac{dy}{dt}+\omega_0^2x=-\frac{e}{m}E_x(t)$$
$$\frac{d^2y}{dt^2}-2\Omega_L\frac{dx}{dt}+\omega_0^2y=\mp\frac{e}{m}E_y(t) \tag{25.31}$$

And if the field is harmonic with frequency ω we obtain a system of equations

$$-\omega^2x-2j\omega y+\omega_0^2x=-\frac{e}{m}E_x$$
$$-\omega^2y+2j\omega\Omega_Lx+\omega_0^2y=\mp\frac{e}{m}E_y \tag{25.32}$$

or

$$\begin{pmatrix}\omega_0^2-\omega^2 & -2j\omega\Omega_L\\ 2j\omega\Omega_L & \omega_0^2-\omega^2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=-\frac{e}{m}\begin{pmatrix}E_x\\ E_y\end{pmatrix} \tag{25.33}$$

which has solution

$$x=-\frac{e}{m}\frac{\left(\omega_0^2-\omega^2\right)E_x+2j\omega\Omega_LE_y}{\left(\omega_0^2-\omega^2\right)^2-4\omega^2\Omega_L^2}$$
$$y=-\frac{e}{m}\frac{\left(\omega_0^2-\omega^2\right)E_x-2j\omega\Omega_LE_x}{\left(\omega_0^2-\omega^2\right)^2-4\omega^2\Omega_L^2} \tag{25.34}$$

Then one can find two projections of the material polarization

$$P_x=\frac{Ne^2}{m(\omega_0^2-\omega^2)}E_x+\frac{2Ne^2j\omega\Omega_L}{m\left(\omega_0^2-\omega^2\right)^2}E_y$$
$$P_y=\frac{Ne^2}{m(\omega_0^2-\omega^2)}E_y-\frac{2Ne^2j\omega\Omega_L}{m\left(\omega_0^2-\omega^2\right)^2}E_x \tag{25.35}$$

Introduce dielectric constant in te absence of magnetic field

$$\varepsilon_{r0}=1+\frac{Ne^2}{m\varepsilon_0(\omega_0^2-\omega^2)} \tag{25.36}$$

and

$$\delta=\frac{2Ne^2j\omega\Omega_L}{m\varepsilon_0\left(\omega_0^2-\omega^2\right)^2} \tag{25.37}$$

then one can find displacement as $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}(B) = \varepsilon_0 \varepsilon_r(B)\mathbf{E}$ where tensor of magnetic field dependent dielectric constant is

$$\varepsilon_r(B) = \begin{pmatrix} \varepsilon_{r0} & j\delta & 0 \\ -j\delta & \varepsilon_{r0} & 0 \\ 0 & 0 & \varepsilon_{r0} \end{pmatrix} \tag{25.38}$$

Note that the derivative (dispersion) of (25.36) is

$$\frac{d\varepsilon_{r0}}{d\omega} = \frac{2Ne^2\omega}{m\varepsilon_0(\omega_0^2 - \omega^2)^2} \tag{25.39}$$

Therefore,

$$\delta = \frac{d\varepsilon_{r0}}{d\omega}\Omega_L = 2n\frac{dn}{d\omega}\frac{e}{2m}B \tag{25.40}$$

where we used $\varepsilon_r = n^2$. Using definition of Verdet constant (25.25) we obtain

$$\delta = \frac{n\lambda}{\pi}VB = \frac{n\lambda}{\pi}\rho \tag{25.41}$$

where $\rho = VB\,(25.29)$ is specific rotation. Therefore, δ is rotation over the length $L = n\lambda/\pi$. To see what happens when circularly polarized light propagates in the chiral media substitute (25.38) into the wave equation

$$k_\pm^2 \mathbf{E}_\pm = \frac{\omega^2}{c^2}\varepsilon_r(B)\mathbf{E}_\pm \tag{25.42}$$

and obtain

$$k_\pm^2 E_0 \begin{pmatrix} 1 \\ \mp j \\ 0 \end{pmatrix} = \frac{\omega^2}{c^2}\begin{pmatrix} \varepsilon_{r0} & j\delta & 0 \\ -j\delta & \varepsilon_{r0} & 0 \\ 0 & 0 & \varepsilon_{r0} \end{pmatrix} E_0 \begin{pmatrix} 1 \\ \mp j \\ 0 \end{pmatrix} = \frac{\omega^2}{c^2}E_0 \begin{pmatrix} \varepsilon_{r0} \pm \delta \\ \mp j(\varepsilon_{r0} \pm \delta) \\ 0 \end{pmatrix} = \frac{\omega^2}{c^2}(\varepsilon_{r0} \pm \delta)E_0 \begin{pmatrix} 1 \\ \mp j \\ 0 \end{pmatrix} = \frac{\omega^2}{c^2}n_\pm^2(B)E_0 \begin{pmatrix} 1 \\ \mp j \\ 0 \end{pmatrix} \tag{25.43}$$

Thus two orthogonally circularly polarized waves are two eigensolutions of wave equation with refractive indices

$$n_\pm(\omega, B) = \sqrt{n^2 \pm \delta} \approx n(\omega) \pm \frac{\delta}{2n} = n(\omega) \pm \frac{dn}{d\omega}\frac{e}{2m}B \tag{25.44}$$

which is exactly (25.22)

Now, let us perform some sanity check – we have got accustomed to the fact that presence of imaginary part of the dielectric constant indicates loss. But now we do have imaginary of-diagonal terms in (25.38),

even though we have not introduced any loss mechanism. So, let us calculate the work done by the combination of DC magnetic field and optical field. The displacement is

$$\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E} = \varepsilon_0 \begin{pmatrix} \varepsilon_{r0} E_x + j\delta E_y \\ \varepsilon_{r0} E_y - j\delta E_x \end{pmatrix} \tag{25.45}$$

And the displacement current can be written as a sum of two components

$$\mathbf{J}_D = -j\omega \mathbf{D} = -j\omega\varepsilon_0 \begin{pmatrix} \varepsilon_{r0} E_x \\ \varepsilon_{r0} E_y \end{pmatrix} + \omega\varepsilon_0 \begin{pmatrix} \delta E_y \\ -\delta E_x \end{pmatrix} = \mathbf{J}_D^{'} + \mathbf{J}_D^{''} \tag{25.46}$$

The imaginary part of the displacement current $\mathbf{J}_D^{'}$ is 90 degrees out of phase with the electric field and thus does not perform any work. The real part $\mathbf{J}_D^{''}$ is in phase with the electric field and thus may perform work and dissipate power, but as can be seen here

$$\left\langle P_J \right\rangle_t = \left\langle \mathbf{J}_D^{''}(t) \cdot \mathbf{E}(t) \right\rangle_t = \frac{1}{2} \mathbf{J}_D^{''} \cdot \mathbf{E} = \varepsilon_0 \omega \begin{pmatrix} \delta E_y & -\delta E_x \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = 0 \tag{25.47}$$

no dissipation takes place. In fact, the off-diagonal elements of permittivity tensors do not have to be real as long as $\varepsilon_{r,ij} = \varepsilon_{r,ji}^*$, i.e. the tensor is Hermitian. Also we can see that the energy density is

$$U = \frac{1}{2} \mathbf{D}^* \cdot \mathbf{E} = \frac{1}{2} \varepsilon_0 \varepsilon_{r0} \left| E \right|^2 \tag{25.48}$$

The energy does not change as it should be – DC magnetic field does not change energy.

Optical isolator

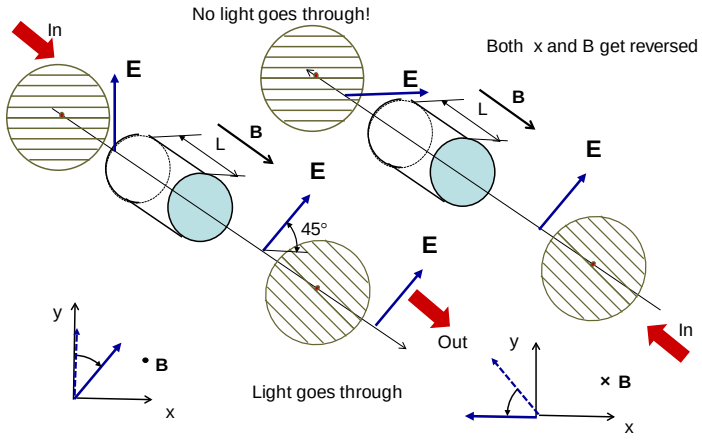


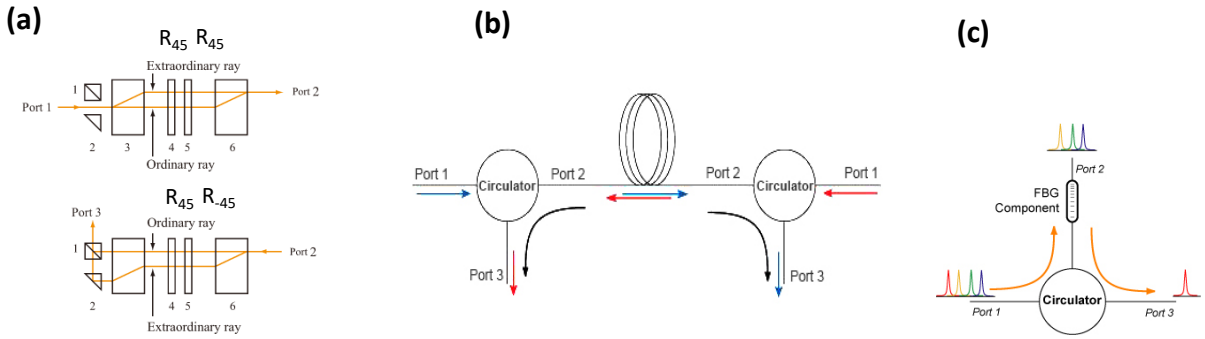
Figure 25.5 Optical isolator

Shown in Fig.25.5 optical isolator consists of a Faraday cell and two polarizers rotated by 45 degrees. When forward propagating signal arrives it first gets polarized horizontally and then its polarization

direction is rotated by 45 degrees clockwise so it goes through the second polarizer. But when the backward propagating light arrives and gets linearly polarized at 45 digress, Faraday cell further rotates its polarization direction by 45 degrees in the same clockwise (looking from the same position) direction. So now the light arrives at the front polarizer at 90 degrees and gets absorbed (or reflected sideways if it's a polarizing beamsplitter). There are two ways to look at it. The rotation depends only on the direction of magnetic field (or magnetization if it is a permanent magnet) and not on the direction of propagation. The other way to look at it – if you look at the incoming light, then after reflection both direction of one axis (x) and direction of B change so the sense of rotation remains the same. This is non-reciprocity at its best.

Optical circulator

Optical isolators are used for such tasks as protecting lasers from the back reflections which cause instabilities and also to isolate optical amplifiers in the fiber optics communications – if the light can propagate only in one direction then no parasitic feedback can cause amplifier to start generating noise. But isolators only work for one polarization of light, i..e up to 50% of light gets lost. A more sophisticated and capable device is *optical circulator* shown in Fig. 25.6 (a)



1: Polarization beam splitter (PBS), 2: reflection prism, 3 and 6: Birefringence crystal, 4: Faraday rotator, 5: half-wave plate

Figure 25.6 (a) Optical circulator (b) Circulator-enabled two-way transmission over a single fiber (c) Filtering using fiber Bragg grating (FBG)

The principle is rather simple. The light from Port 1 is split into two orthogonally polarized rays shifted relative to each other using a birefringent crystal with optical axis at some angle to the surface. Then both spatially separated rays pass through a Faraday cell that rotates polarization by 45 degrees and the through a HWP with optical axes rotated by 22.5 degrees which rotates polarization by another 45 degrees. Thus ordinary ray becomes extraordinary and vice versa - hence the second birefringent crystal combines rays into a single ray exiting from Port 2.

But when the light enters through the Port 2 and gets split into two rays, the reciprocal HWP rotates its polarization by -45 degrees and nonreciprocal Faraday cell by +45 degrees. So the polarization of both rays is preserved and extraordinary beam is further shifted. Now the two rays can be combined again using a reflector and polarizing beamsplitter. So the light exists from Port 3.

In Fig.25.6(b) one can see how a couple of circulators can be used to enable two-way communication over a single fiber with sources connected to Ports 1 and detectors to Ports 3. Another application shown in

Fig.25.6(c) is in filtering (channel selection) in wavelength division multiplexing (WDM) optical networks. Fiber Bragg Grating (FBG) provides excellent selectivity by reflecting over a very narrow bandwidth, say 50GHz (0.4nm) corresponding to a single WDM channel. But it only works in reflection so a circulator is needed to send the selected signal to the detector or perhaps launch it into another fiber.

Optical activity

Optical activity is phenomenon of circular birefringence that occurs in certain materials, called *chiral* that occurs without any external field. In chemistry, a molecule or ion is called chiral if it cannot be superposed on its mirror image by any combination of rotations and translations. Chiral molecules (and also some crystals) occur as two distinct enantiomers, dextral (right handed) and sinistral (left handed) that are mirror images of each other as shown in Fig.25.7(a) . Some of the chiral molecules are shown in Fig. 25.7 (b) and the rotation of polarization in optically active medium is shown Fig. 25.7 (c)

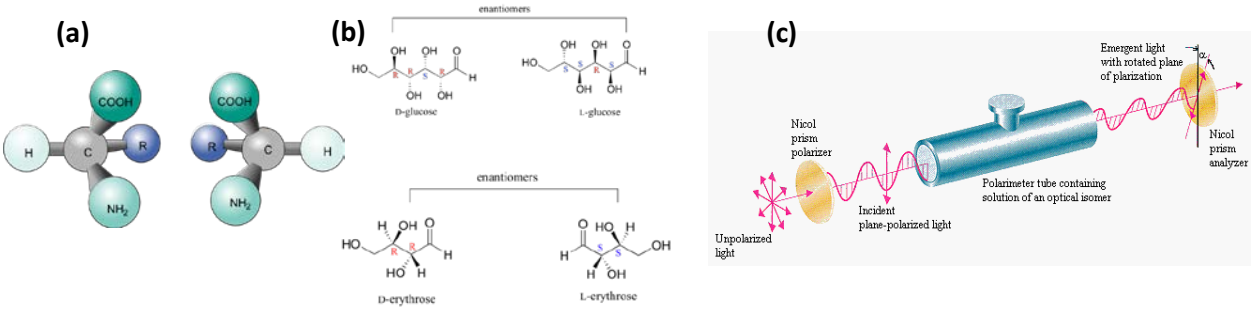


Figure 25.7 (a) Two enantiomers of a tetrahedral complex. (b) Chiral Molecules (c) Polarization rotation in optically active medium

Many different molecules occur only in nature mostly in one of enantiomers. For example, there are two enantiomers of glucose, called D-glucose and L-glucose. The D-enantiomer is the common sugar that our bodies use for energy. We cannot metabolize the left handed sugars. Therefore, by checking polarization rotation one can identify metabolism.

Origin of optical activity

Truth be said, it is not easy to explain optical activity on a classical level, while quantum description is quite easy. But we shall try. First consider a normal, non-chiral molecule in Fig.25.8(a).Electric field of the plane wave induces usual dipole moment along x direction $p_x = \alpha E_x$. There is also a circular current induced by the AC magnetic field $J \sim -dB/dt$ according to Lenz's rule and the magnetic moment, but no additional dipole moment. The situation is different in the chiral molecule shown schematically as a "spring" in Fig.25.8(b) where the current is helical and charges move along y as well resulting with an additional dipole moment $p_y = -\beta \frac{dB_y}{dt}$. Thus response of a chiral entity can be written as

$$\mathbf{p} = \alpha \mathbf{E} - \beta \frac{\partial \mathbf{B}}{\partial t} \tag{25.49}$$

where coefficient β has units of $F \cdot m^3$.

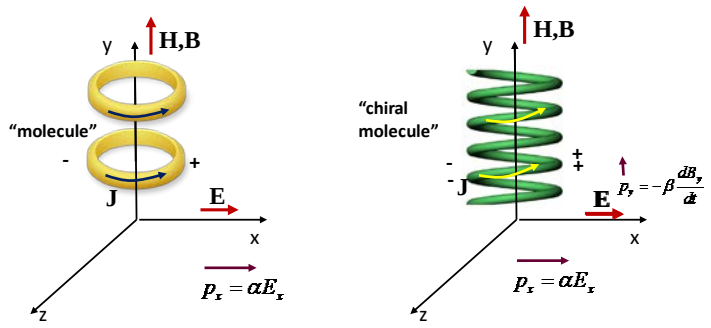


Figure 25.8 Additional dipole moment induced in chiral molecules

We can then find the displacement as

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \mathbf{E} + N\mathbf{p} = \varepsilon_0 \varepsilon_r \mathbf{E} - \beta N \frac{\partial \mathbf{B}}{\partial t} \tag{25.50}$$

where N is the density of the molecules. Substitute $j\mathbf{k} \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ into (25.50) and obtain

$$\mathbf{D} = \varepsilon \varepsilon_r \mathbf{E} + j\beta N \mathbf{k} \times \mathbf{E} = \varepsilon_0 \left[\varepsilon_r \mathbf{E} + j\varepsilon_0^{-1} \beta N \mathbf{k} \times \mathbf{E} \right] = \varepsilon_0 \left[\varepsilon_r \mathbf{E} + j\mathbf{G} \times \mathbf{E} \right] \tag{25.51}$$

Where we have introduced dimensionless optical gyration vector

$$\mathbf{G} = N \varepsilon_0^{-1} \beta \mathbf{k} \tag{25.52}$$

Note that for plane wave propagating in z direction

$$\mathbf{k} \times \mathbf{E} = \pm k \begin{pmatrix} -E_y \\ E_x \end{pmatrix} = \pm k \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \pm k R_{\pi/2} \mathbf{E} \tag{25.53}$$

where $R_{\pi/2}$ is 90 degree polarization rotation around z. Introduce absolute value of the gyration vector

$$\delta = |\boldsymbol{G}| = N \varepsilon_0^{-1} \beta k \tag{25.54}$$

and then gyration center

$$\left[G_{\pm z} \right] = \begin{pmatrix} 0 & \pm j\delta & 0 \\ \mp j\delta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{25.55}$$

Then one can write (25.51) as

$$\mathbf{D} = \varepsilon_0 \left(\varepsilon_r \pm j \left[G_{\pm z} \right] \right) \mathbf{E} = \varepsilon_0 \varepsilon_{r,a} \mathbf{E} \tag{25.56}$$

Where the permittivity tensor of chiral material is

$$\varepsilon_{r,a} = \varepsilon_r \pm j\left[G_{\pm z}\right] = \begin{pmatrix} \varepsilon_{r,11} & \pm j\delta & 0 \\ \mp j\delta & \varepsilon_{r,22} & 0 \\ 0 & 0 & \varepsilon_{r,33} \end{pmatrix} \tag{25.57}$$

Note that according to (25.52) the sign changes with the direction of propagation (wavevector) which means that the chiral material is fully reciprocal and cannot be used for optical isolation. This is dramatically different from magneto optic effect (25.38) where the sign of δ changes with the direction of magnetic field and does not depend on the propagation direction. That makes magneto optic effect non reciprocal.

Just as in Faraday effect (25.44) the indices of refraction for left and right circularly polarized light are different

$$\begin{aligned} n_+(\omega, B) &= \sqrt{n^2 \pm \delta} \approx n(\omega) \pm \frac{\delta}{2n} \\ n_-(\omega, B) &= \sqrt{n^2 \pm \delta} \approx n(\omega) \mp \frac{\delta}{2n} \end{aligned} \tag{25.58}$$

But note that the sign of circular birefringence changes for forward and backward direction. Phase shift between two circular polarizations (25.24) is

$$\Delta\Phi = \frac{2\pi}{\lambda}\left[n_+(\omega) - n_-(\omega)\right]d = \pm\frac{2\pi d}{\lambda}\frac{\delta}{n} \tag{25.59}$$

and the polarization rotation angle is

$$\Delta\theta = \frac{\Delta\Phi}{2} = \pm\frac{\pi d}{\lambda}\frac{\delta}{n} \tag{25.60}$$

As mentioned above, rotation is reversed for backward propagation –hence no isolation! Using (25.54) we can evaluate specific rotation, also called optical rotary power

$$\rho = \frac{d\theta}{dz} = \frac{\pi}{\lambda}\frac{\delta}{n} = 2\frac{N\pi^2}{\varepsilon_0 n\lambda^2}\beta \tag{25.61}$$

Note that rotary power has very strong wavelength dependence and this fact can be used for filtering the light. Crystal materials also can be chiral and their rotary powers are shown below.

Optical Rotatory Powers

	λ (Å)	ρ (degree/mm)
Quartz	4000	49
	4500	37
	5000	31
	5500	26
	6000	22
	6500	17
$AgGaS_2$	4850	950
	4900	700
	4950	600
	5000	500
	5050	430
Se	7500	180
	10000	30
Te	(6 μm)	40
	(10 μm)	15
TeO_2	3698	587
	4382	271
	5300	143
	6328	87
	10000	30