Lecture 23 Polarization Optics

Jones vectors

Consider arbitrary polarized light (Fig.23.1 (a))

$$\mathbf{E}(t) = \mathrm{Re}\left[\left(E_{0x}\hat{\mathbf{x}} + E_{0y}\hat{\mathbf{y}}\right)e^{-j\omega t}\right] \tag{23.1}$$

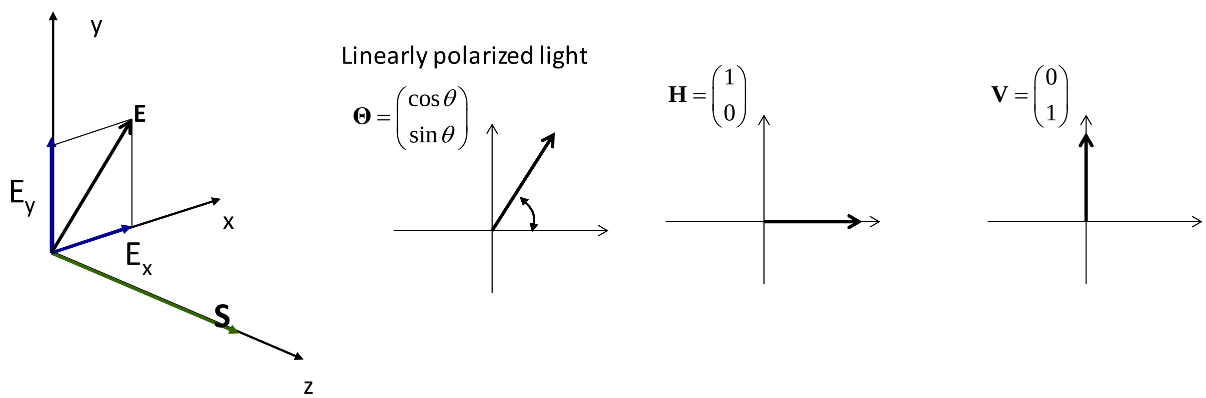


Figure 23.1 (a) Arbitrary polarized light (b),(c),(d) –Jones vectors of linearly polarized light

And represent it as a column vector

$$\mathbf{E}(t) = \mathrm{Re}\left[\left(E_{0x}\hat{\mathbf{x}} + E_{0y}\hat{\mathbf{y}}\right)e^{-j\omega t}\right] = \mathrm{Re}\left[\begin{pmatrix} E_{0x} \\ E_{0y} \end{pmatrix} e^{-j\omega t}\right] = E_0 \,\mathrm{Re}\left[\begin{pmatrix} e_x \\ e_y \end{pmatrix} e^{-j\omega t}\right] \tag{23.2}$$

where $\left|e_x\right|^2 + \left|e_y\right|^2 = 1$. The Jones vector $\mathbf{J} = \begin{pmatrix} e_x \\ e_y \end{pmatrix}$ with generally complex components represents the state of polarization. Linearly polarized light (Fig.23.1.(b)) is thus represented by the Jones vector

$$\mathbf{\Theta} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \tag{23.3}$$

Horizontally-polarized (Fig.23.1(c) light is represented by the Jones vector

$$\mathbf{H} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{23.4}$$

And vertically polarized light (Fig.23.1.(d) by

$$\mathbf{V} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{23.5}$$

Obviously the two linear polarizations are orthogonal

$$\mathbf{H}^* \cdot \mathbf{V} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0 \tag{23.6}$$

Circular polarization

Consider the wave that can be described as

$$\mathbf{E}(t) = E_0 \hat{\mathbf{x}} \cos \omega t \mp E_0 \hat{\mathbf{y}} \sin \omega t \tag{23.7}$$

shown in Fig.23.2. a The sign "-"corresponds to the clockwise rotating wave – called right circularly polarized and sign "+" wave rotates counterclockwise and is called left circularly polarized. Note that direction of rotation is seen looking at the incoming wave. In Jones notation the wave becomes

$$\mathbf{E}(t) = E_0 \hat{\mathbf{x}} \operatorname{Re}\left(e^{-j\omega t}\right) \pm E_0 \hat{\mathbf{y}} \operatorname{Im}\left(e^{-j\omega t}\right) = E_0 \hat{\mathbf{x}} \operatorname{Re}\left(e^{-j\omega t}\right) \mp E_0 \hat{\mathbf{y}} \operatorname{Re}\left(j \cdot e^{-j\omega t}\right) = E_0 \operatorname{Re}\left[\begin{pmatrix} 1 \\ \mp j \end{pmatrix} e^{-j\omega t}\right] \tag{23.8}$$

The right polarized light is therefore described as

$$\mathbf{R} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -j \end{pmatrix} \tag{23.9}$$

and left polarized light as

$$\mathbf{L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ j \end{pmatrix} \tag{23.10}$$

These two vectors are normalized

$$\mathbf{R}^* \cdot \mathbf{R} = \frac{1}{2} \begin{pmatrix} 1 & j \end{pmatrix} \begin{pmatrix} 1 \\ -j \end{pmatrix} = 1 = \mathbf{L}^* \cdot \mathbf{L} \tag{23.11}$$

and orthogonal to each other

$$\mathbf{R}^* \cdot \mathbf{L} = \frac{1}{2} \begin{pmatrix} 1 & j \end{pmatrix} \begin{pmatrix} 1 \\ j \end{pmatrix} = 0 \tag{23.12}$$

One can use either linearly or circularly polarized light as a basis in which one can represent arbitrary polarization. In fact, one can easily make a transition between two. Evaluate

$$\begin{aligned} \mathbf{R} + \mathbf{L} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -j \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ j \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \sqrt{2}\mathbf{H} \\ \mathbf{R} - \mathbf{L} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -j \end{pmatrix} - \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ j \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -2j \end{pmatrix} = -\sqrt{2} j \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -j\sqrt{2}\mathbf{V} \end{aligned} \tag{23.13}$$

Then we have these relations:

$$\mathbf{R} = (\mathbf{H} - j\mathbf{V})/\sqrt{2} \tag{23.14}$$

$$\mathbf{L} = (\mathbf{H} + j\mathbf{V})/\sqrt{2} \tag{23.15}$$

and

$$\begin{aligned} \mathbf{H} &= (\mathbf{R} + \mathbf{L})/\sqrt{2} \\ \mathbf{V} &= j((\mathbf{R} - \mathbf{L}))/\sqrt{2} \end{aligned} \tag{23.16}$$

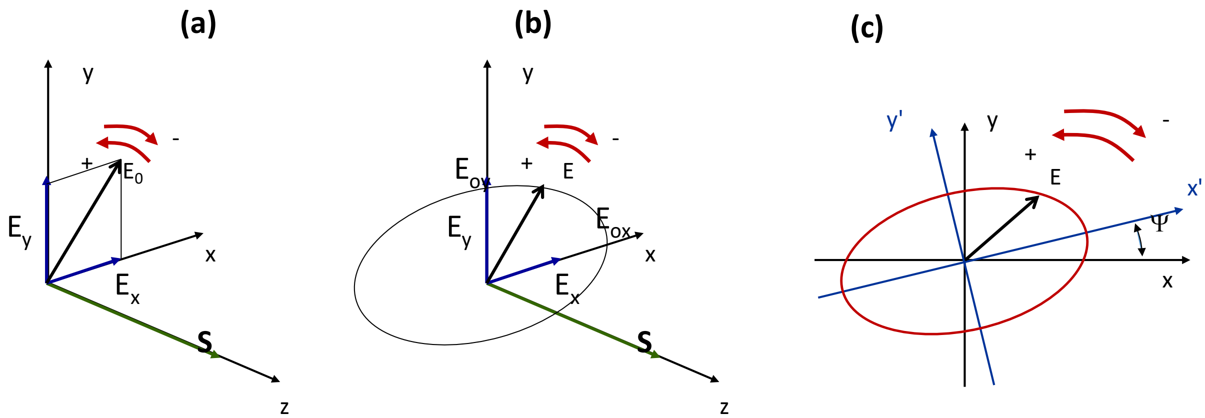


Figure 23.2 (a) Circularly polarized light (b) Elliptically polarized light (c) Elliptically polarized light tilted to the laboratory system of coordinates

Elliptical Polarization

Shown in Fig.23.2(b) the elliptically polarized wave has different amplitudes for the vertical and horizontal directions,

$$\mathbf{E}(t) = E_{0x}\hat{\mathbf{x}}\cos\omega t \mp E_{0y}\hat{\mathbf{y}}\sin\omega t \tag{23.17}$$

And is represented by a Jones vector

$$\mathbf{J} = \begin{pmatrix} \cos\theta \\ \mp j\sin\theta \end{pmatrix} \tag{23.18}$$

More generally the axes of the ellipse x', y' can be rotated by angle Ψ relative to the laboratory axes x, y as shown in Fig. 23.2(c). Then in the laboratory coordinates Jones vector becomes $\mathbf{J}_{xy} = R_\Psi \mathbf{J}_{x'y'}$ where the coordinate rotation matrix is

$$R_\Psi = \begin{pmatrix} \cos\Psi & -\sin\Psi \\ \sin\Psi & \cos\Psi \end{pmatrix} \tag{23.19}$$

Therefore for elliptically polarized light with axes rotated by Ψ

$$\mathbf{J}_{\Psi} = \begin{pmatrix} \cos\Psi & -\sin\Psi \\ \sin\Psi & \cos\Psi \end{pmatrix} \begin{pmatrix} \cos\theta \\ \mp j\sin\theta \end{pmatrix} = \begin{pmatrix} \cos\Psi\cos\theta \pm j\sin\Psi\sin\theta \\ \sin\Psi\cos\theta \mp j\cos\Psi\sin\theta \end{pmatrix} = \begin{pmatrix} \cos\gamma \\ \sin\gamma e^{\mp j\delta} \end{pmatrix} \quad (23.20)$$

where

$$\begin{aligned} \cos^2\gamma &= \cos^2\Psi\cos^2\theta + \sin^2\Psi\sin^2\theta \\ \tan\delta &= \frac{\tan\theta}{\tan\Psi}\frac{\tan^2\Psi - 1}{1+\tan^2\theta} \end{aligned} \quad (23.21)$$

Suppose we need to solve the inverse problem: we have detected the two polarizations of light in xy coordinate system as

$$\mathbf{J}_{xy} = \begin{pmatrix} \cos\gamma \\ \sin\gamma e^{-j\delta} \end{pmatrix} = \begin{pmatrix} \cos\gamma \\ \sin\gamma\cos\delta \end{pmatrix} - j\begin{pmatrix} 0 \\ \sin\delta \end{pmatrix} \quad (23.22)$$

and now we want to find what is the system of coordinates $x'y'$ along which the axes on polarization ellipse lay, i.e. we want to represent

$$\mathbf{J}_{xy} = e^{j\beta}\left(\mathbf{A} - j\mathbf{B}\right) = e^{j\beta}\left(a\mathbf{X}' - jb\mathbf{Y}'\right) = e^{j\beta}\left[a\begin{pmatrix} \cos\Psi \\ \sin\Psi \end{pmatrix} - jb\begin{pmatrix} -\sin\Psi \\ \cos\Psi \end{pmatrix}\right] \quad (23.23)$$

where β is some irrelevant common phase and a and b are real numbers and therefore $\mathbf{A}$ and $\mathbf{B}$ are real vectors.

Dot multiply $\mathbf{J}_{xy}$ by itself (not its complex conjugate so we are not getting an absolute value!)

$$\mathbf{J}_{xy} \cdot \mathbf{J}_{xy} = e^{2j\beta}\left(\mathbf{A} - j\mathbf{B}\right)\left(\mathbf{A} - j\mathbf{B}\right) = e^{2j\beta}\left(\mathbf{A}^2 - \mathbf{B}^2\right) \quad (23.24)$$

The expression $\left(\mathbf{A}^2 - \mathbf{B}^2\right)$ is real and positive (we have assumed a is major axis) so

$$2\beta = \arg(\mathbf{J}_{xy} \cdot \mathbf{J}_{xy}) \quad (23.25)$$

Take absolute value of (23.24)

$$\left|\mathbf{J}_{xy} \cdot \mathbf{J}_{xy}\right| = \left(\mathbf{A}^2 - \mathbf{B}^2\right) \quad (23.26)$$

Therefore

$$e^{2j\beta} = \frac{\mathbf{J}_{xy} \cdot \mathbf{J}_{xy}}{\mathbf{A}^2 - \mathbf{B}^2} = \frac{\mathbf{J}_{xy} \cdot \mathbf{J}_{xy}}{\left|\mathbf{J}_{xy} \cdot \mathbf{J}_{xy}\right|} \quad (23.27)$$

and

$$e^{-j\beta} = \sqrt{\frac{\mathbf{J}^*_{xy} \cdot \mathbf{J}^*_{xy}}{\left|\mathbf{J}_{xy} \cdot \mathbf{J}_{xy}\right|}} \tag{23.28}$$

Substitute it into (23.23) to obtain

$$\mathbf{A} - j\mathbf{B} = \mathbf{J}_{xy}\sqrt{\frac{\mathbf{J}^*_{xy} \cdot \mathbf{J}^*_{xy}}{\left|\mathbf{J}_{xy} \cdot \mathbf{J}_{xy}\right|}} \tag{23.29}$$

and $\mathbf{A}$ and $\mathbf{B}$ are found as real and imaginary parts.

Producing linearly polarized light

One can simply relay on Fresnel reflection use multiple Brewster angle $\theta_B = \tan^{-1}(n_2 / n_1)$ reflections as shown in 23.3 (a) in order to reflect s-polarized light. That works well for really high powered lasers.The most often used method is polarizing beam splitter cube shown in Fig.23.3b in which a stack of thin layers have indices n_H an d n_L and the two right angle prisms constituting the cube have index n_C. The Brewster angle for the interfaces n_H / n_L is $\theta_B = \tan^{-1}(n_H / n_L)$ and the incident angle is $\theta_i = \pi / 4$ so that $n_L \sin\theta_B = (1/\sqrt{2})n_C$ from which one can find n_C

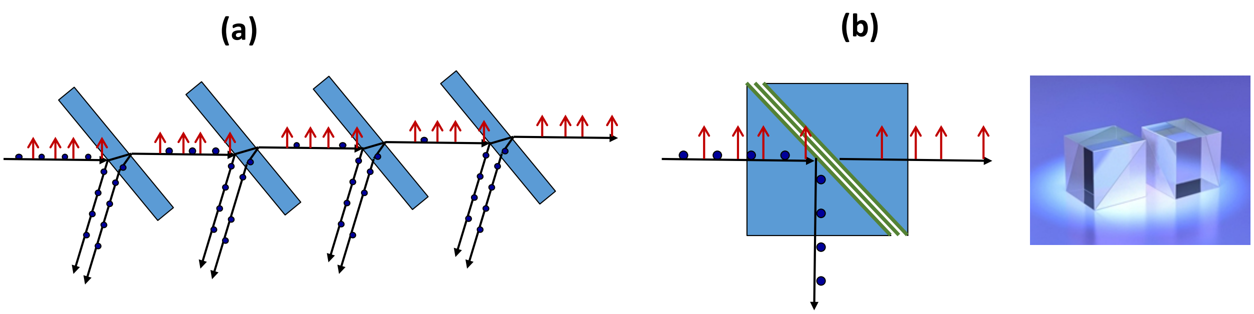


Figure 23.3 Polarizers based on reflection at Brewster angle of incidence (a) multiple plates (b) polarizing beamsplitter cube

Then one can rely on the selective absorption or dichroism he property of some crystals of absorbing one of two plane-polarized components of transmitted light more strongly than the other. Dichroism can be explained using wire grid polarizer quite popular in microwave technology and shown in Fig.23.4 Vertical conductivity is much larger than a horizontal one. In other words, imaginary part of vertical dielectric constant is larger than the horizontal one $\mathrm{Im}(\varepsilon_V) > \mathrm{Im}(\varepsilon_H)$. Vertically polarized component interacts with electrons in wires and gets absorbed, while the horizontal component passes “unmolested”, This polarizer can be characterized by the Jones matrix

$$P_H = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad (23.30)$$

(note that the determinant is not equal to 1 because the polarizer is not a lossless element) . If we apply polarizer to the arbitrary polarized light

$$P_H \boldsymbol{\Theta} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \cos\theta \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \cos\theta \mathbf{H} \qquad (23.31)$$

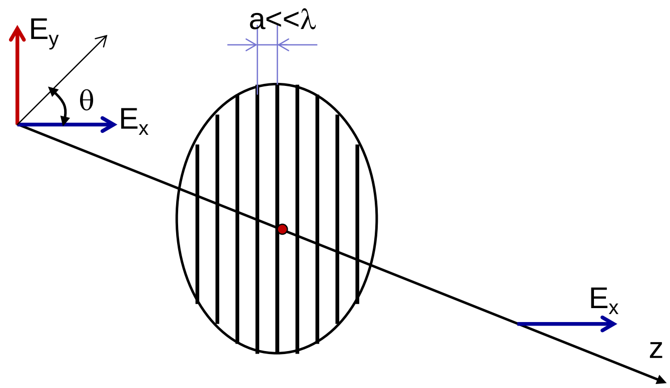


Figure 23.4 Wire grid polarizer

Now, in the optical range the “wires” must be very close to each other but there exist materials with natural fibrous structure such as tourmaline containing Al_2O_3, B_2O_3, and SiO_2 shown in Fig.23.5(a). One can also use polaroid film - long chains of hydrocarbons with iodine supplying electrons that can move along these long chains shown in Fig.23.5. b

(a) **(b)**

Figure 23.5 (a) dichroic material – tourmaline (b) polaroid film

Polarizers based on double refraction and total internal reflection

These polarizers are shown in Fig.23.6. First of them is *Nicol prism* shown in Fig.23.6 a - it consists two prisms made from calcite $CaCO_3$ (n_o=1.658; n_e=1.486) separated by a thin layer of Canada Balsam with refractive index n=1.55 midway between the ordinary and extraordinary indices of calcite. Optical axis is in plane of the figure so the vertical ray is extraordinary and since its index is less than that of Canada balsam, it goes through while the horizontally polarized ordinary ray gets totally internally reflected off Canada balsam and gets absorbed at blackened bottom. The disadvantages of Nicol prism are the narrow field of view and the fact that light is partially elliptically polarized due to inclined face

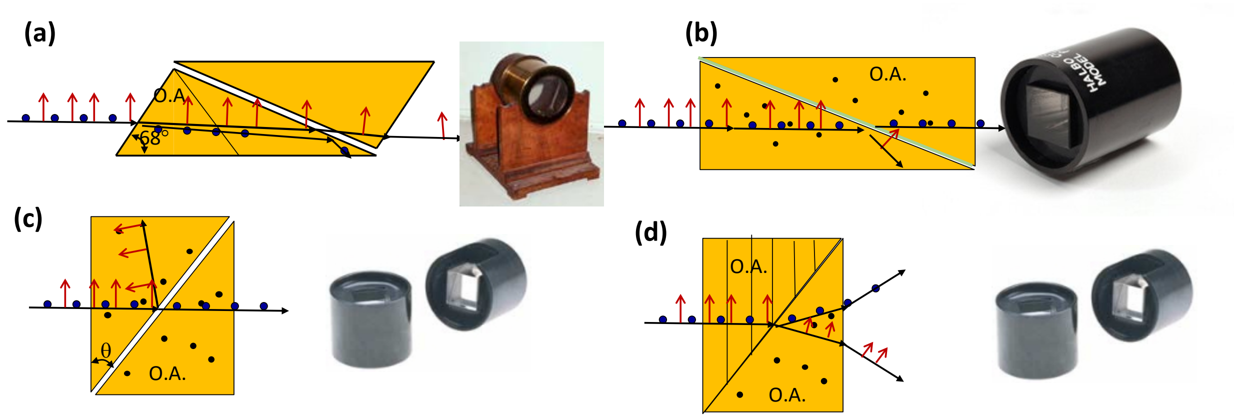


Figure 23.6 Polarizers: (a) Nicol Prism (b) Glan Thompson Prism (c) Glan-Foucault (or Glan Air) Prism (d) Wollaston Prism

In *Glan Thompson* prism shown in Fig. 23.6 b the optcal axis is normal to the plane so now it is the horizontally polarized light that is extraordinary and thus goes through. Field of view is larger than in Nicol prism but the polarizer is quite long. Nowadays synthetic cement is used in place of Canada balsam which comes from fir tree.

In *Glan-Foucault* (or Glan Air) Prism shown in Fig.23.6 c air gap is used in place of balsam. The critical angle for the ordinary ray is $\theta_{c,o} = \sin^{-1} n_o^{-1} = 37.1^\circ$ and critical angle for the extraordinary ray is $\theta_{c,e} = \sin^{-1} n_e^{-1} = 42.3^\circ$. Therefore, the apex angle must fit in between the two $\theta_{c,o} < \theta < \theta_{c,e}$. Typically θ=39°. Advantage of Glan Air prism is its small size and a decent aspect ratio, disadvantage is a very narrow field of view (about 8 degrees)

Finally, a polarizing beamsplitter in which both orthogonally polarized rays emerge at different angles – Wollaston prism is shown in Fig. 23.6 d. There is no air gap here- but two prisms have different orientations of optical axis. The horizontally-polarized ray goes from high ordinary index into low extraordinary index and is bent upward, while the vertically-polarized ray goes from low extraordinary index into high ordinary index and Iis therefore bent downward.

Malus law

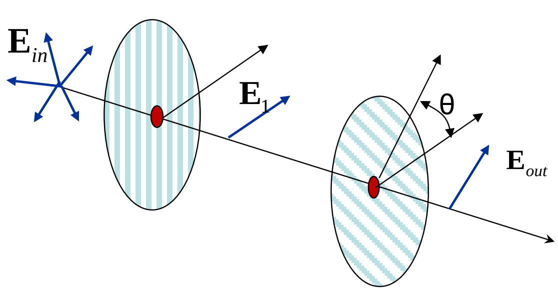


Figure 23.7 Malus Lau and variable optical attenuator based on this law

Consider two polarizers whose directions of polarization are rotated by angle θ (without loss of generality we assume the first polarizer to produce horizontally polarized light as in Fig.23.7. The first polarizer is described by the matrix (23.30). The second polarizer is rotated by an angle θ so its matrix is

$$P_\theta = R_\theta P_H = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{pmatrix} \tag{23.32}$$

The matrix of the entire assembly is

$$R_\theta P_H = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ 0 & 0 \end{pmatrix} \tag{23.33}$$

Assuming that the input light is not polarized

$$I_{out} = \frac{1}{2} I_{in} \cos^2\theta \tag{23.34}$$

One can use Malus law to build a variable optical attenuator consisting of two polarizers one of which can be rotated, as also shown in Fig.23.7

Retardation plates

Retardation plates are phase-shifting devices capable of turning linearly polarized light into circular/elliptical one and vice versa. To take advantage of maximum birefringence the optical axis us parallel to the face. Whether crystal is positive or negative, the larger of two indices is referred to as "slow" index n_s and the smaller one as "fast" index n_f . Accordingly, the direction of polarization along which the refractive index is large is called *slow* axis and the orthogonal direction is called *fast* axis., as shown in Fig. 23.8 a. The phase retardation for the slow component is

$$\Phi_s = \frac{2\pi}{\lambda} n_s d \tag{23.35}$$

And for the fast component

$$\Phi_f = \frac{2\pi}{\lambda} n_f d \tag{23.36}$$

If the incoming light's polarization is described by Jones vector $\begin{pmatrix} e_{s,in} \\ e_{f,in} \end{pmatrix}$, the polarization state of output light is

$$\begin{pmatrix} e_{s,out} \\ e_{f,out} \end{pmatrix} = \begin{pmatrix} e^{j\Phi_s} & 0 \\ 0 & e^{j\Phi_f} \end{pmatrix}\begin{pmatrix} e_{s,in} \\ e_{f,in} \end{pmatrix} = e^{j\bar{\Phi}/2}\begin{pmatrix} e^{j\Delta\Phi/2} & 0 \\ 0 & e^{-j\Delta\Phi/2} \end{pmatrix}\begin{pmatrix} e_{s,in} \\ e_{f,in} \end{pmatrix} \tag{23.37}$$

where the phase difference $\Delta\Phi = \Phi_s - \Phi_f$ and the mean phase $\bar{\Phi} = \dfrac{\Phi_s + \Phi_f}{2}$ is largely irrelevant.,

hence the retardation plate is described by a Jones matrix

$$W_{\Delta\Phi} = \begin{pmatrix} e^{j\Delta\Phi/2} & 0 \\ 0 & e^{-j\Delta\Phi/2} \end{pmatrix} \tag{23.38}$$

where

$$\Delta\Phi = \frac{2\pi}{\lambda}\left(n_s - n_f\right)d \tag{23.39}$$

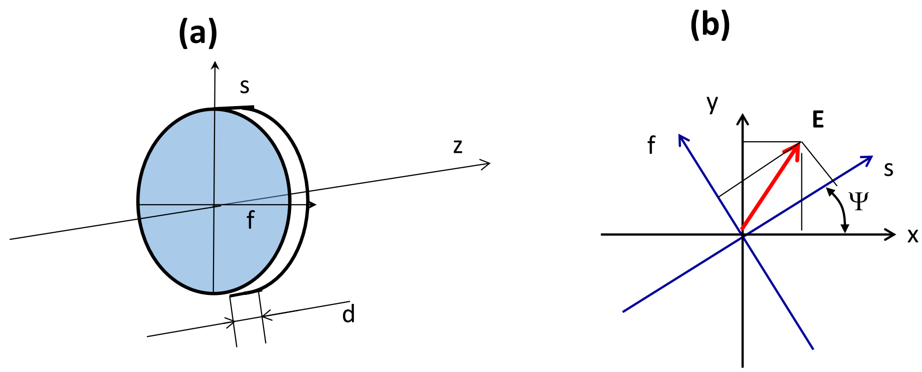


Figure 23.8 (a) Birefringent waveplate (b) Transforming coordinates for the waveplate

As always, there are two important cases of retardation plates.

Half-wave plate (HWP) for which $\Delta\Phi = (2N-1)\pi$ (N is the order of waveplate) . The thickness of HWP is

$$d_{\lambda/2} = \frac{\lambda(2N-1)}{2\left(n_s - n_f\right)} \tag{23.40}$$

and its Jones matrix is

$$W_{\lambda/2} = \begin{pmatrix} e^{j\pi/2} & 0 \\ 0 & e^{-j\pi/2} \end{pmatrix} = \begin{pmatrix} j & 0 \\ 0 & -j \end{pmatrix} = j\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{23.41}$$

(90 deg. phase shift in front can be disregarded). HWP reverses relative phase shift between s and f components

Quarter-wave plate(QWP) for which $\Delta\Phi = (2N-1)\pi/2$ (N is the order of waveplate) . The thickness of HWP is

$$d_{\lambda/4} = \frac{\lambda(2N-1)}{4\left(n_s - n_f\right)} \tag{23.42}$$

and its Jones matrix is

$$W_{\lambda/4} = \begin{pmatrix} e^{j\pi/4} & 0 \\ 0 & e^{-j\pi/4} \end{pmatrix} = e^{j\pi/4}\begin{pmatrix} e^{0} & 0 \\ 0 & e^{-j\pi/2} \end{pmatrix} = e^{j\pi/4}\begin{pmatrix} 1 & 0 \\ 0 & -j \end{pmatrix} \tag{23.43}$$

The common phase of 45 degrees is irrelevant. The QWP introduces 90 degrees shift between s and f components.

Rotated Waveplates

Jones Matrix describes Retardation plate in s-f coordinates but we need to use it in x-y coordinates, rotated by an angle Ψ as in Fig. 23.8 b. To transform Jones vector from x-y system to s-f system use rotation matrix

$$\mathbf{E}_{sf} = \begin{pmatrix} e_s \\ e_f \end{pmatrix} = \begin{pmatrix} \cos\Psi & \sin\Psi \\ -\sin\Psi & \cos\Psi \end{pmatrix}\begin{pmatrix} e_x \\ e_y \end{pmatrix} = R_{\Psi}\begin{pmatrix} e_x \\ e_y \end{pmatrix} = R_{\Psi}\mathbf{E}_{xy} \tag{23.44}$$

And to transfer it back to x y system of coordinates

$$\mathbf{E}_{xy} = \begin{pmatrix} e_x \\ e_y \end{pmatrix} = \begin{pmatrix} \cos\Psi & -\sin\Psi \\ \sin\Psi & \cos\Psi \end{pmatrix}\begin{pmatrix} e_s \\ e_f \end{pmatrix} = R_{-\Psi}\begin{pmatrix} e_s \\ e_f \end{pmatrix} = R_{-\Psi}\mathbf{E}_{sf} \tag{23.45}$$

Therefore, retardation plate in xy coordinates can be found from

$$\mathbf{E}_{xy,out} = R_{-\Psi}\mathbf{E}_{sf,out} = R_{-\Psi}W_{\Delta\Phi,sf}E_{sf,in} = \left(R_{-\Psi}W_{\Delta\Phi,sf}R_{\Psi}\right)\mathbf{E}_{xy,in} = W_{\Delta\Phi,xy}\mathbf{E}_{xy,in} \tag{23.46}$$

i.e.

$$W_{\Delta\Phi,xy} = R_{-\Psi}W_{\Delta\Phi,sf}R_{\Psi} \tag{23.47}$$

The most important is the case hen the rotation angle Ψ is equal to 45 degrees and

$$\begin{aligned} R_{45^\circ} &= \begin{pmatrix} \cos\Psi & \sin\Psi \\ -\sin\Psi & \cos\Psi \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \\ R_{-45^\circ} &= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \end{aligned} \tag{23.48}$$

Therefore, for an arbitrary retardation plate rotted by 45 degrees relative to xy coordinates.

$$W_{\Delta\Phi,xy} = R_{-45^\circ} W_{\Delta\Phi,sf} R_{45^\circ} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} e^{j\Delta\Phi/2} & 0 \\ 0 & e^{-j\Delta\Phi/2} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} =$$
$$= \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} e^{j\Delta\Phi/2} & e^{j\Delta\Phi/2} \\ -e^{-j\Delta\Phi/2} & e^{-j\Delta\Phi/2} \end{pmatrix} = \begin{pmatrix} \cos(\Delta\Phi/2) & j\sin(\Delta\Phi/2) \\ j\sin(\Delta\Phi/2) & \cos(\Delta\Phi/2) \end{pmatrix} \tag{23.49}$$

Considers a HWP rotated by 45 degrees.

$$W_{\lambda/2,xy} = \begin{pmatrix} \cos(\pi/2) & j\sin(\pi/2) \\ j\sin(\pi/2) & \cos(\pi/2) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{23.50}$$

Where we dropped common 90 degrees phase shift. Therefore, consider horizontally polarized input wave as in Fig.23.10 (a). Then the output wave is

$$W_{\lambda/2,xy}\mathbf{H} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \mathbf{V} \tag{23.51}$$

And obviously $W_{\lambda/2,xy}\mathbf{V} = \mathbf{H}$ - hence polarization is rotted by 90 degrees from vertical to horizontal and vice versa.

If we consider a circularly polarized input wave as in Fig.23.10 b, then

$$\begin{aligned} W_{\lambda/2,xy}\mathbf{R} &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -j \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -j \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ j \end{pmatrix} = \mathbf{L} \\ W_{\lambda/2,xy}\mathbf{L} &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ j \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} j \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -j \end{pmatrix} = \mathbf{R} \end{aligned} \tag{23.52}$$

The helicity of light gets reversed.

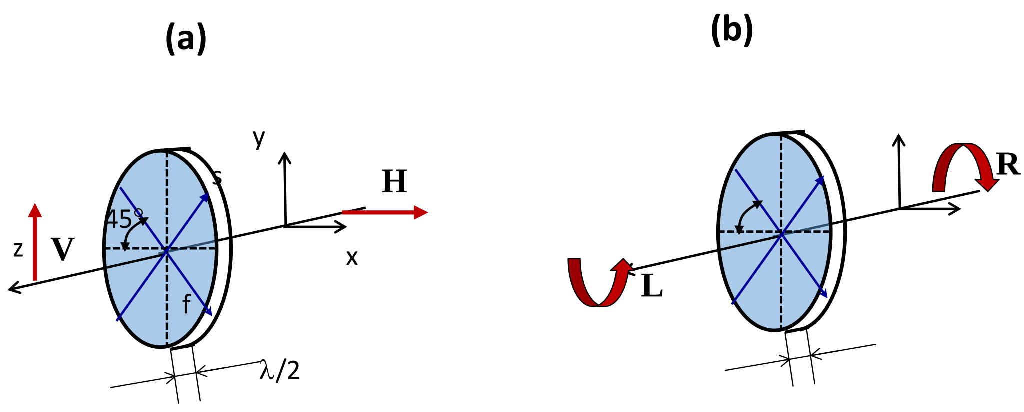


Figure 23.10 HWP rotated by 90 deg. acting on (a) linearly and (b) circularly polarized light.

Next consider a QWP rotated by 45 degrees

$$W_{\lambda/4,xy} = \begin{pmatrix} \cos(\pi/4) & j\sin(\pi/4) \\ j\sin(\pi/4) & \cos(\pi/4) \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & j \\ j & 1 \end{pmatrix} \tag{23.53}$$

Now consider horizontally polarized input wave as in Fig.23.11 (a).

$$W_{\lambda/4,xy}\mathbf{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & j \\ j & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ j \end{pmatrix} = \mathbf{L} \tag{23.54}$$

Linearly polarized light becomes circularly polarized. Of course, if the input light is circularly polarized as in Fig.23.11(b) it becomes linearly polarized

$$W_{\lambda/4,xy}\mathbf{R} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & j \\ j & 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -j \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \mathbf{H} \tag{23.55}$$

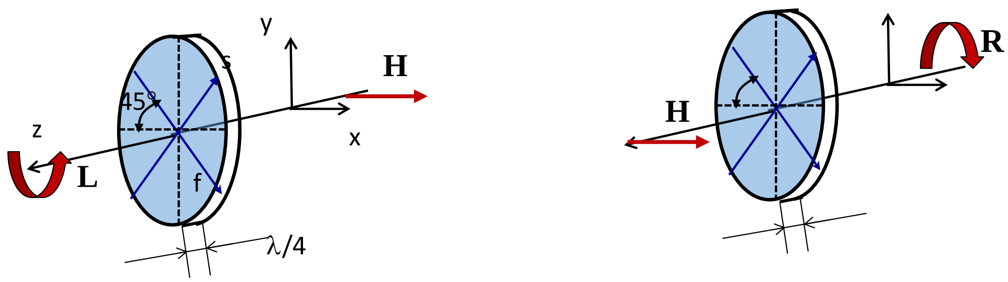


Figure 23.11 QWP rotated by 90 deg. acting on (a) linearly and (b) circularly polarized light.

Variable optical attenuator based on rotating HWP

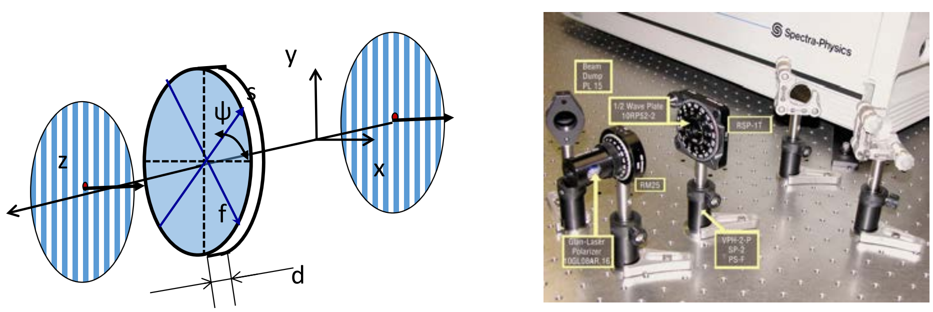


Figure 23.12 Variable attenuator based on rotating HWP

Consider a HWP rotated by an arbitrary angle θ relative to horizontal axis as in Fi.23.12 . The matrix is

$$W_{\lambda/2,\psi} = R_{-\psi} W_{\lambda/2,\psi} R_{\psi} = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} =$$

$$= \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi \\ \sin\psi & -\cos\psi \end{pmatrix} = \begin{pmatrix} \cos^2\psi - \sin^2\psi & 2\sin\psi\cos\psi \\ 2\sin\psi\cos\psi & \sin^2\psi - \cos^2\psi \end{pmatrix} = \begin{pmatrix} \cos 2\psi & \sin 2\psi \\ \sin 2\psi & -\cos 2\psi \end{pmatrix}$$

(23.56)

Place the wave plate between two horizontal polarizers

$$M = HW_{\lambda/2,\psi}H = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos 2\psi & \sin 2\psi \\ \sin 2\psi & -\cos 2\psi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \cos 2\psi & 0 \\ 0 & 0 \end{pmatrix} \tag{23.57}$$

So the transmittance is proportional to $I_{out} \sim I_{in}\cos^2 2\theta$ and one can make a variable attenuator by rotating the plate relative to the polarizers. If the laser light is used it is usually already polarized so one only needs a single polarizer as shown in Fig.. 23.12

Lyot Filter.

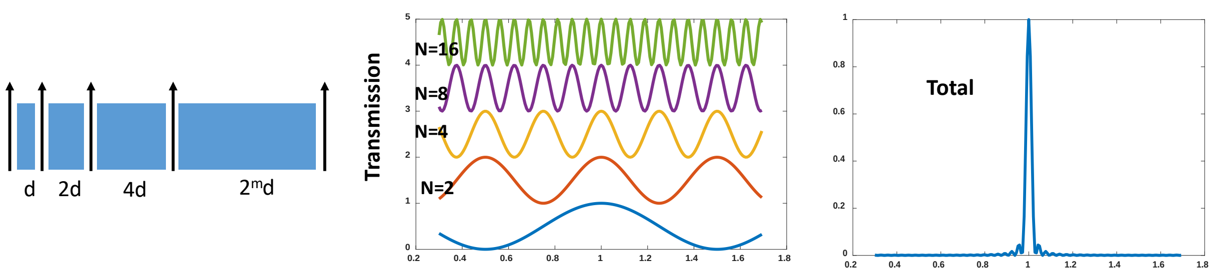


Figure 23.13 (a) Lyot Filter (b) Transmission spectra of stages (c) Transmission spectrum of the entire filter

Consider the property of the full wave plate (FWP) of N-th order whose thickness is

$$d_N = \frac{N\lambda_0}{n_s - n_f} \tag{23.58}$$

where λ_0 is the wavelength for which the FWP is designed. The phase shift between slow and fast wave at arbitrary wavelength λ according to (23.39) is

$$\Delta\Phi_N(\lambda) = \frac{2\pi}{\lambda}\left(n_s - n_f\right)d_N = 2N\pi\frac{\lambda_0}{\lambda} \tag{23.59}$$

If the FWP is rotated by 45 degrees its matrix according to (23.49) is

$$W_N(\lambda) = \begin{pmatrix} \cos(\Delta\Phi_N/2) & j\sin(\Delta\Phi_N/2) \\ j\sin(\Delta\Phi_N/2) & \cos(\Delta\Phi_N/2) \end{pmatrix} \tag{23.60}$$

Obviously at the design wavelength FWP is represented by a unity matrix but it is not true for other wavelength, If we palce the FWPO between two aligned polarizers (does not matter whether vertical or horizontal) , transmission at arbitrary wavelength is

$$T_N(\lambda) = \cos^2 \frac{\Delta\Phi_N(\lambda)}{2} = \cos^2 N\pi \frac{\lambda_0}{\lambda} = \cos^2 N\pi \frac{\nu}{\nu_0} \tag{23.61}$$

Next we create a sequence of FWP separated by aligned poalrizers in such a way that each next one is twice the thickness of the current one, i.e, $N = 1, 2, 4 ... 2^m$. The design, known as Lyot filter is shown in Fig.23.13 a Fig. The transmission spectra for each stage are shown in Fig.23.13.b. The total transmission is a product of the transmissions of individual stages,

$$T(\lambda) = \prod_{m=0}^{M} T_{2^m}(\lambda) \tag{23.62}$$

And is shown in Fig.23.13 c. Note that the filter combines narrow pass band determined by the longest stage with large free spectral range determined by the shortest stage. Lyot filter can be tuned mechanically or electrically using electro-optic effect. This filter is often used inside the lasers to select one wavelength. Originally Bernard Lyot who was an astronomer had invented this filter to observe sun's corona in the absence of full eclipse with most of the sun's emission being blocked by the very by a very narrow filter. Note that each stage of Lyo filter is essentially a Mach Zehnder interferometer where instead of being split into two different optical paths light is split into two orthogonal polarizations which are then forced to interfere after the polarizer.

Note that of instead of simple polarizers polarization splitters, such as Wollaston prism, is used in Lyot filter, then one can split the white light into 2^M rays – this is used in hyperspectral imaging.

Šolc (pronounced "Sholtz") Filter

This is another type of the filter that uses interference of polarized light. To start, consider a HWP rotated by an arbitrary angle ψ relative to horizontal axis as shown in Fig. 23.14 a . The matrix $W_{\lambda/2,\psi}$ is described by (23.56) and if the incoming light is polarized at arbitrary angle θ the output light Jones vector is

$$\mathbf{E}_{out} = W_{\lambda/2,\psi}\mathbf{\Theta} = \begin{pmatrix} \cos 2\psi & \sin 2\psi \\ \sin 2\psi & -\cos 2\psi \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} \cos(2\psi - \theta) \\ \sin(2\psi - \theta) \end{pmatrix} \tag{23.63}$$

As one can see the polarization of the incoming light is rotated by an angle

$$\Delta\theta = \theta_1 - \theta = 2(\psi - \theta) \tag{23.64}$$

or as shown in Fig.23.14 b

$$\theta_1 - \psi = \psi - \theta \tag{23.65}$$

meaning that the polarization direction is "reflected" from the fast axis.

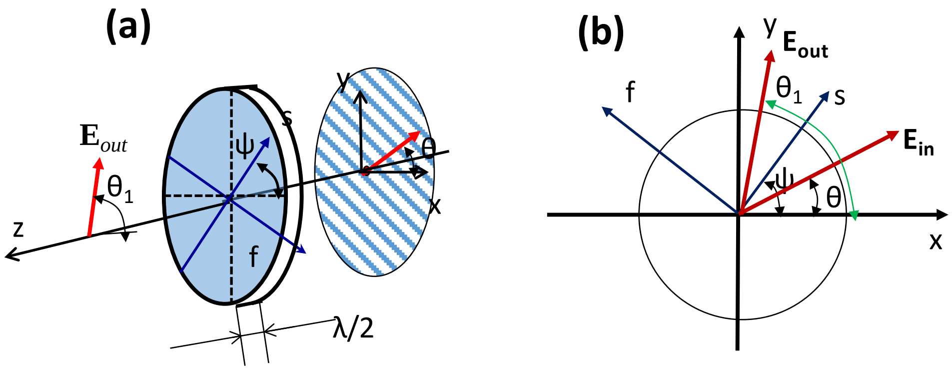


Figure 23.14 (a) light propagation though a rotated HWP (b) polarization rotation.

The Šolc Filter shown in Fig. 23.15 (a) consists of N pairs of HWP's rotated alternatively by +ψ and -ψ relative to the horizontal axis and placed between two polarizers. At the design wavelength λ_0 after each plate light keeps linear polarization and polarization rotates, as shown in Fig. 23.15 (b).

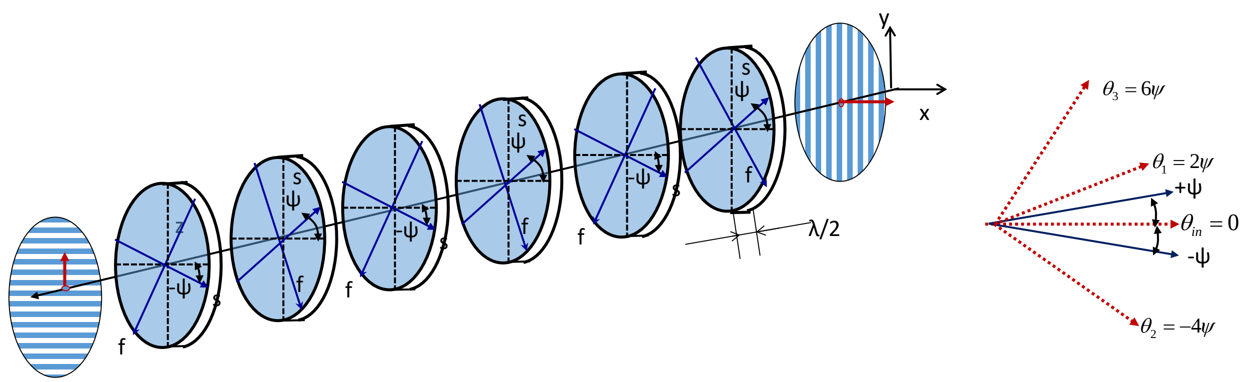


Figure 23.15 (a) Šolc Filter (b) stage by stage rotation of the polarization angle

Starting with horizontal polarization, after the 1st plate θ_1=2ψ, after the 2nd plate θ_2=-4ψ, after the 3nd plate θ_2=6ψ,…and, finally, after the Nth plate

$$\theta_N = (-1)^{N-1} \times 2N\psi \tag{23.66}$$

If we choose

$$\psi = \frac{\pi}{4N} \tag{23.67}$$

$\theta_N = -\pi/2$, i.e. the light is vertically polarized and will through the second polarizer. Obviously this will be true only at the design wavelength. At other wavelengths $\Delta\Phi(\lambda) = \pi\lambda_0/\lambda$ and the transmission quickly falls as shown in Fig.23.16 . The spectrum looks quite similar to that of diffraction grating and gets sharper with the increase of the number of stages N.

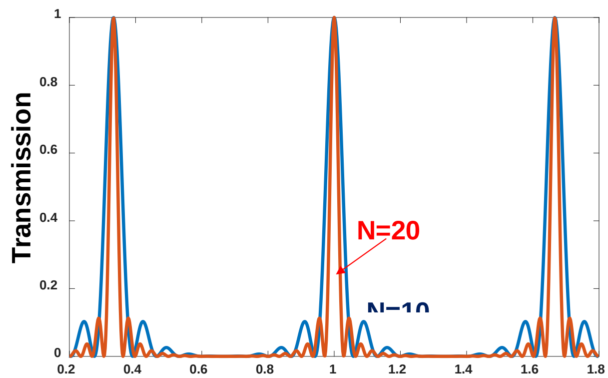


Figure 23.16 Transmission of the Šolc Filter

Polarization controllers

In order to transform one elliptical polarization to another one need to control two independent parameters in Jones vector – relative amplitude and phase. In principle two birefringent elements can do the job, but usually one uses a combination of 3 elements QWP followed by HWP followed by another QWP The plates, and hence directions of s and f axes can rotate as shown in Fig.23.17

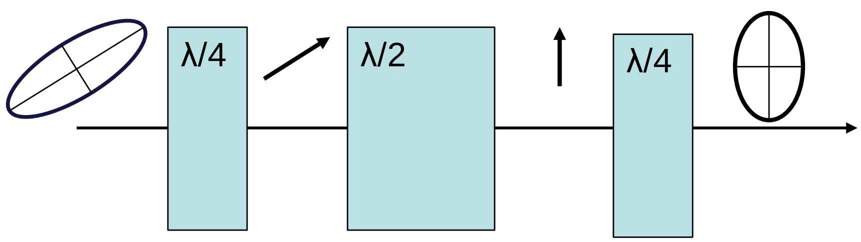


Figure 23.17 Polarization controller

The first QWP transforms elliptical polarization into linear one, the HQP rotates linear polarization, and the second QWP (if necessary) transform linear polarization into another elliptical polarization.

Polarization control ifs of great importance in fiber optical communications where two a orthogonal polarizations carry two different communication channels. Normally optical fiber (Fig. 23.18) possesses circular (cylindrical) symmetry and is therefore isotropic – Fig.13.17 (a). But when optical fiber is bent which happens when it is wound into a coil (Fig.23.18 b) it is strained and no longer symmetric – two orthogonal modes with different effective indices can now propagate in it (Fig.23.18. c) and by selecting a certain length of the fiber in the coil one can crete any phase delay between two polarizations. Thus a ceratin length of coiled fiber can serve as QWP or a HWP. Then one can make a widely popular 3 paddles (or “bat ear”) polarization controller shown in Fig.23. 18 d.

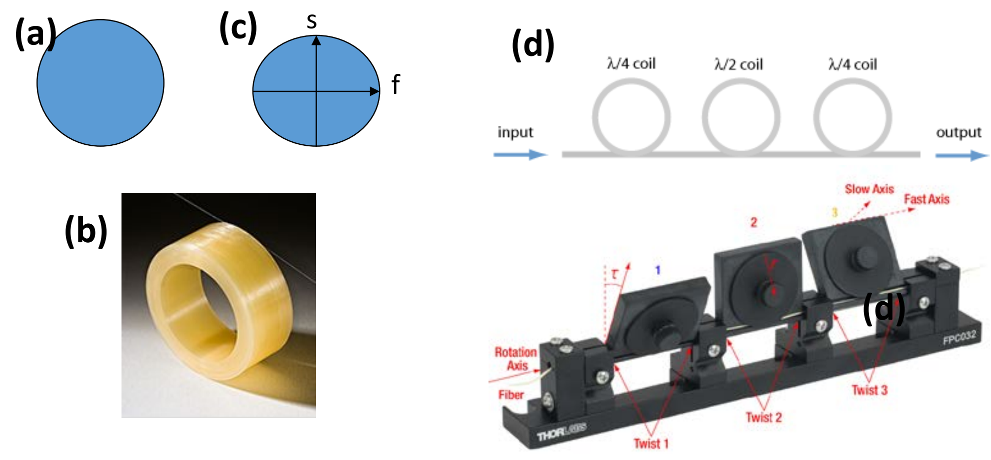


Figure 23.18 (a) unstrained optical fiber cross section (b) fiber coil (c) strained fiber cross-section (d) 3 paddles polarizer

Babinet and Soleil Compensators.

They are used to adjust the phase shift of the waveplate. Babinet compensator shown in Fig.23.19 a consists of two prisms with optcal axes rotated by 90 degrees, so the light that is fast in one prism becomes slow in the other. The relative phase shift is

$$\Delta\Phi = \frac{2\pi}{\lambda}\left(n_s - n_f\right)\left(d_1 - d_2\right) \tag{23.68}$$

and of course it depends on the position of the beam. In Babinet-Soleil compensator the phase shift is the same over the entire aperture. Advantage of using a compensator in place of simple waveplate is that a plate of the lowest (N=1) order can be made that is not exceptionally thin.

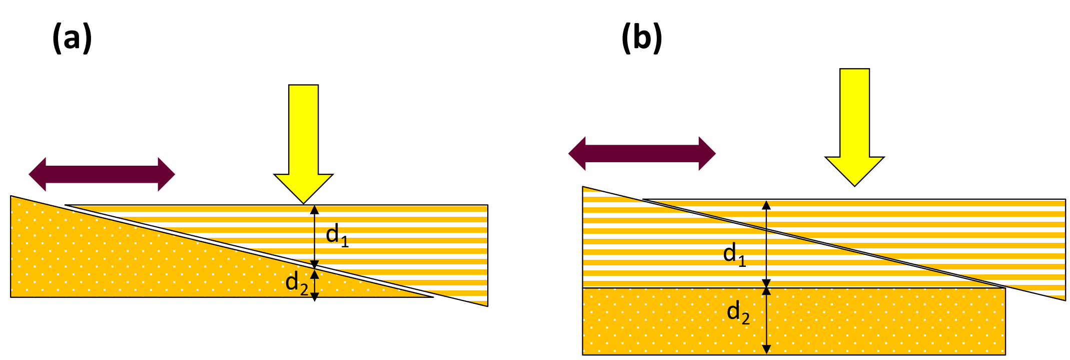


Figure 23.19 (a) Babinet compensator (b) Babinet-Soleil compensator

Optical Isolation

Very often one is faced with a situation when one needs to detect the light reflected straight back at normal incidence and wants to direct all the reflected light to the detector rather than back to the source. If (as often is) the source is the laser getting light reflected back at it not only creates waste but establishes a parasitic feedback mechanism that may cause unstable laser operation. An example can be a readout in the DVD players and other optical storage devices shown in Fig. 23.20 (a) The isolation is accomplished by a combination of a polarizing beamsplitter prism and QWP as show schematically in Fig.23.20 (b). The laser light tis already linearly polarized, so the linear polarizer at the input is not needed. The horizontally polarized laser light is directed towards the reflecting surface of the disc and After one pass through λ/4 plate (rotated by 45 degrees) , according to (23.54)

$$W_{\lambda/4,xy}\mathbf{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & j \\ j & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ j \end{pmatrix} = \mathbf{L} \tag{23.69}$$

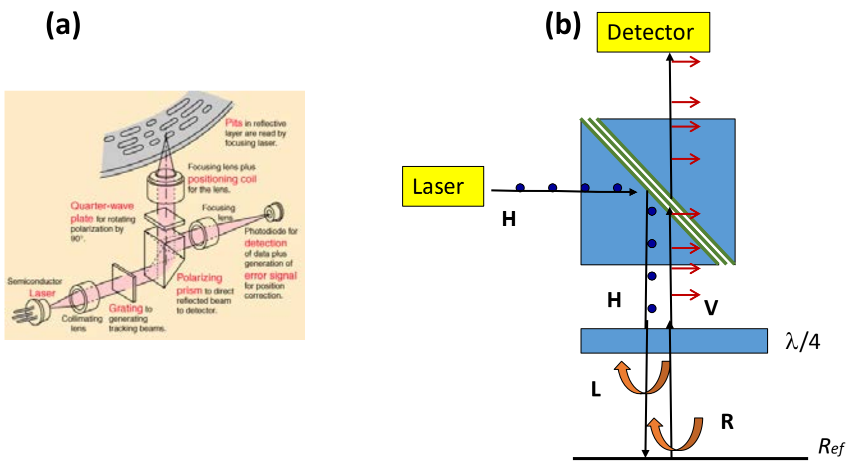


Figure 23.20 (a) optical disc readout (b) optical isolation in it

When light is reflected back, to maintain right hand Cartesian system of coordinates one of the coordinates needs to change sign , $\bar{y} = -y$ - this is represented by a reflection matrix

$$\mathrm{R}_{ef} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{23.70}$$

So that after reflection the light polarization changes helicity

$$\mathrm{R}_{ef}\,\mathbf{L} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ j \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -j \end{pmatrix} = \mathbf{R} \tag{23.71}$$

The QWP for the backward light is still quarter-wave plate

$$W_{\lambda/4,x\bar{y}} = R_{ef}^{-1}W_{\lambda/4,xy}R_{ef} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & j \\ j & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix} \tag{23.72}$$

And after the second pass through QWP

$$W_{\lambda/4,x\bar{y}}\mathbf{R} = \frac{1}{2}\begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -j \end{pmatrix} = \begin{pmatrix} 0 \\ -j \end{pmatrix} = -j\mathbf{V} \tag{23.73}$$

the light becomes vertically polarized and after the beamsplitter goes towards the detector. Of course, one can understand it in a simple way – two passes through QWP amount to one pass through a HWP so a linear polarization is rotated by 90 degrees.

Anti-Glare Screen

One can reduce the amount of reflected from the screen light while still allowing at least half of the light coming from the screen to go through. This had been used a lot on CRT monitors. A soen can see from Fig. 23.21 (a) all the straight light gets absorbed but the emitted unpolarized light is not affected by the QWP and one half of the power comes out. So this is not really an isolator.

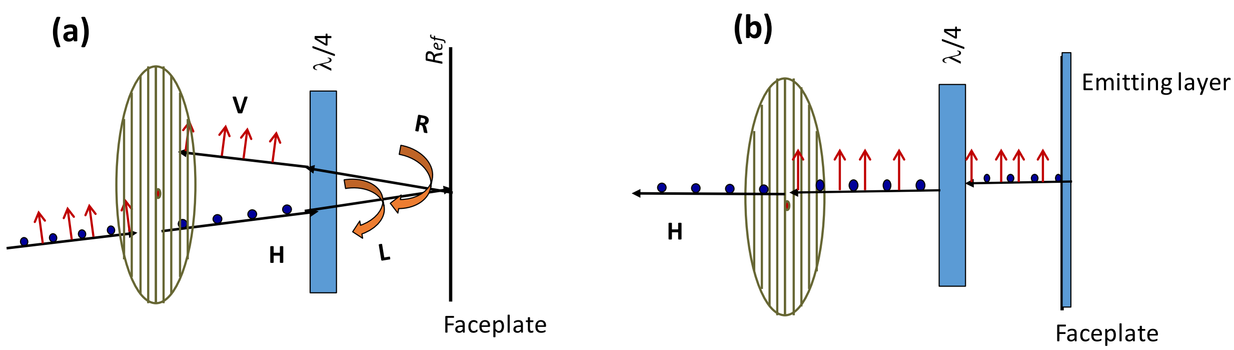


Figure 23.21 Antiglare screen for SRT. (a) reflected light is blocked (b) emitted light goes through

3D displays

One of the ways to create separate images for right and left eyes was using linearly polarized light. However, this method suffers from a nearly fatal flaw – if one tilts the head obviously the isolation is no longer perfect. That is why modern 3D displays use circularly polarized light, no matter how one rotates the coordinates the helicity of the light remains the same.

Consider combination of P_H=polarizer and λ/4 plate with axes rotated 45 degrees relative to the horizontal axis as in Fig. 23.22

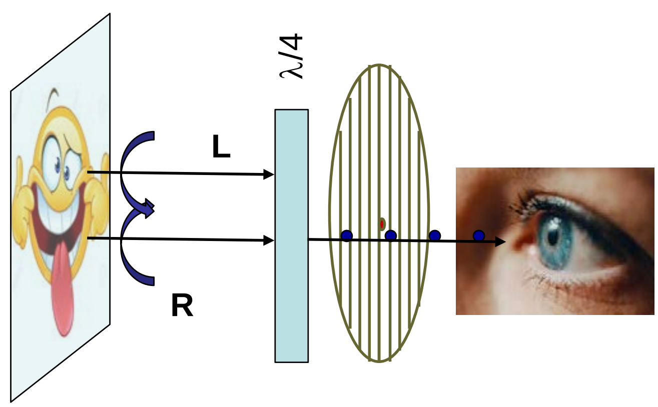


Figure 23.22 polarizers used to see 3D movie

The Jones matrix is

$$M = P_H W_{\lambda/4,45^\circ} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & j \\ j & 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & j \\ 0 & 0 \end{pmatrix} \tag{23.74}$$

The right circularly polarized light becomes horizontally polarized and passes through

$$M\mathbf{R} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & j \\ 0 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -j \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \mathbf{H} \tag{23.75}$$

while left polarized light becomes vertically polarized and gets absorbed

$$M\mathbf{L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & j \\ 0 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ j \end{pmatrix} = 0 \tag{23.76}$$

Now, let us see what happens if the head is tilted and the entire assembly rotates

$$MR_\Psi = \begin{pmatrix} 1 & j \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\Psi & -\sin\Psi \\ \sin\Psi & \cos\Psi \end{pmatrix} = \begin{pmatrix} \cos\Psi + j\sin\Psi & j\cos\Psi - \sin\Psi \\ 0 & 0 \end{pmatrix} = e^{j\Psi} M \tag{23.77}$$

Same matrix M – right polarized light still goes through while left polarized light gets blocked.

Polarizing microscope

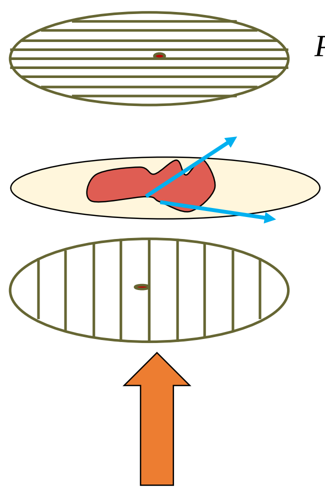
Often the object is perfectly transparent but has index variations and birefringent. Then one can "see" the changes in phase using crossed polarizers as shown in Fig.23.23 (a) and (b). The object is birefringent with and is placed on rotating stage. If the slow and fast axes of object are aligned with the polarizer, i.e. XY then

$$P_X O_\Phi P_Y = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} e^{j\Delta\Phi/2} & 0 \\ 0 & e^{-j\Delta\Phi/2} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & e^{-j\Delta\Phi/2} \end{pmatrix} = 0 \tag{23.78}$$

and the object is invisible. But if the axes are rotated by 45 degrees then

$$\begin{aligned} P_X O_\Phi P_Y &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos(\Delta\Phi/2) & j\sin(\Delta\Phi/2) \\ j\sin(\Delta\Phi/2) & \cos(\Delta\Phi/2) \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & j\sin(\Delta\Phi/2) \\ 0 & \cos(\Delta\Phi/2) \end{pmatrix} = \begin{pmatrix} 0 & j\sin(\Delta\Phi/2) \\ 0 & 0 \end{pmatrix} \end{aligned} \tag{23.79}$$

Object is visible when illuminated with unpolarized light – the light intensity is proportional to $\sin^2(\Delta\Phi/2)$. Quite often anisotropy is caused by strain – this way one can "see" the strain. We can sense the phase (index) changes by probing phase in two orthogonal polarizations and then interfering two rays



$$P_X = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_Y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

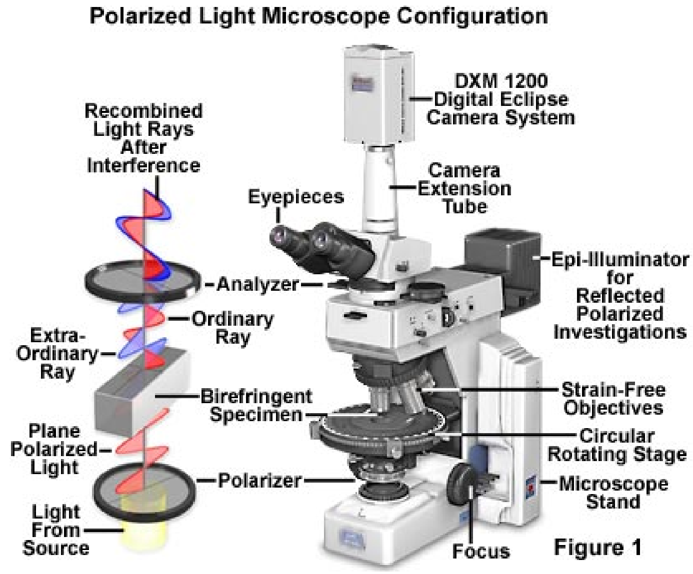
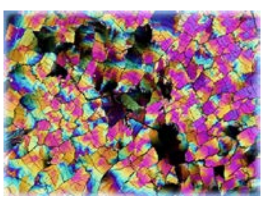
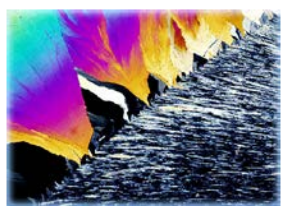


Figure 23.23 (a) the idea behind polarizing microscope. (b) its schematic

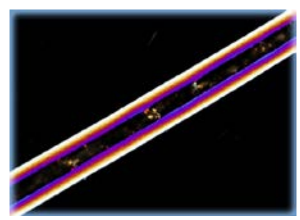
One can see some images obtained with a polarizing microscope in Fig.23.24



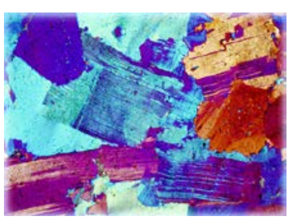
Cholesterol



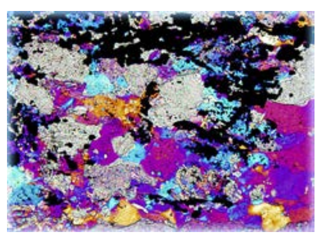
Vitamin E



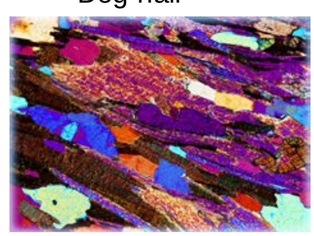
Dog hair



Granite



Marble



Mica

Figure 23.24 Images obtained with polarizing microscope