KOM3712 Control Systems Design

Spring 2020

Design via State Space Methods – 3 of 3: *Integral Control*

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Textbooks followed mostly for the Design in the State Space are,

- **Modern Control Engineering** (5th ed.), Katsuhiko Ogata, Chap. 9
- **Control Systems Engineering** (7th ed.), Norman S. Nise, Chap. 12

State feedback controller design

- Remember the past!
  - We dealt with the controller design problem in state space.
  - An example state feedback control design was for the plant,

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)}$$

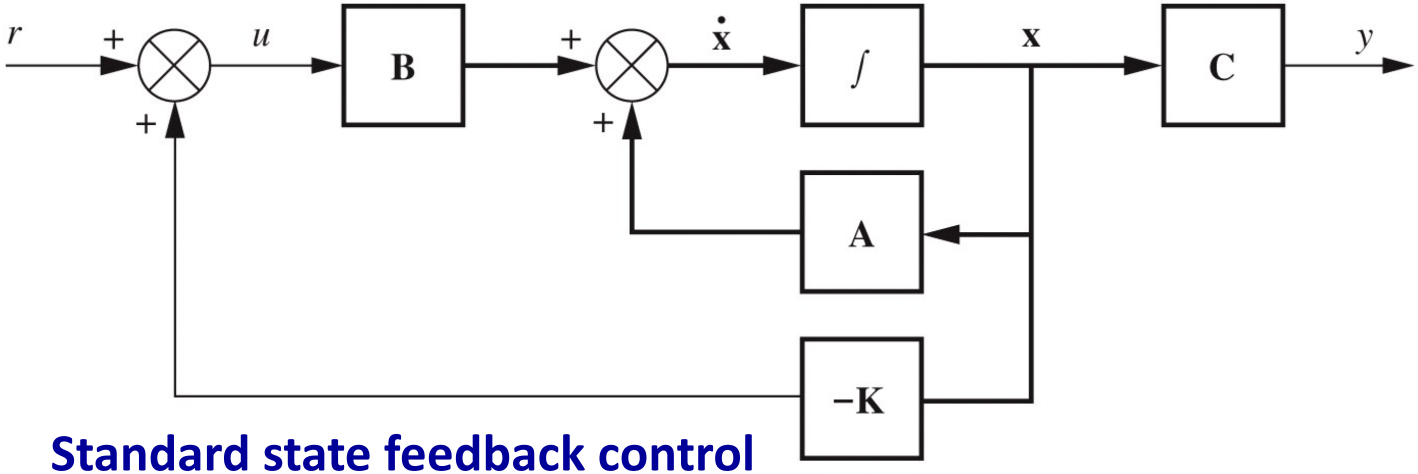
  Design the phase variable feedback gains to yield 9.5% overshoot and a settling time of 0.74 seconds.

- We then compared the compensated/uncompensated systems response through simulation.
- Simulations revealed that the compensated system performed according to design specifications.

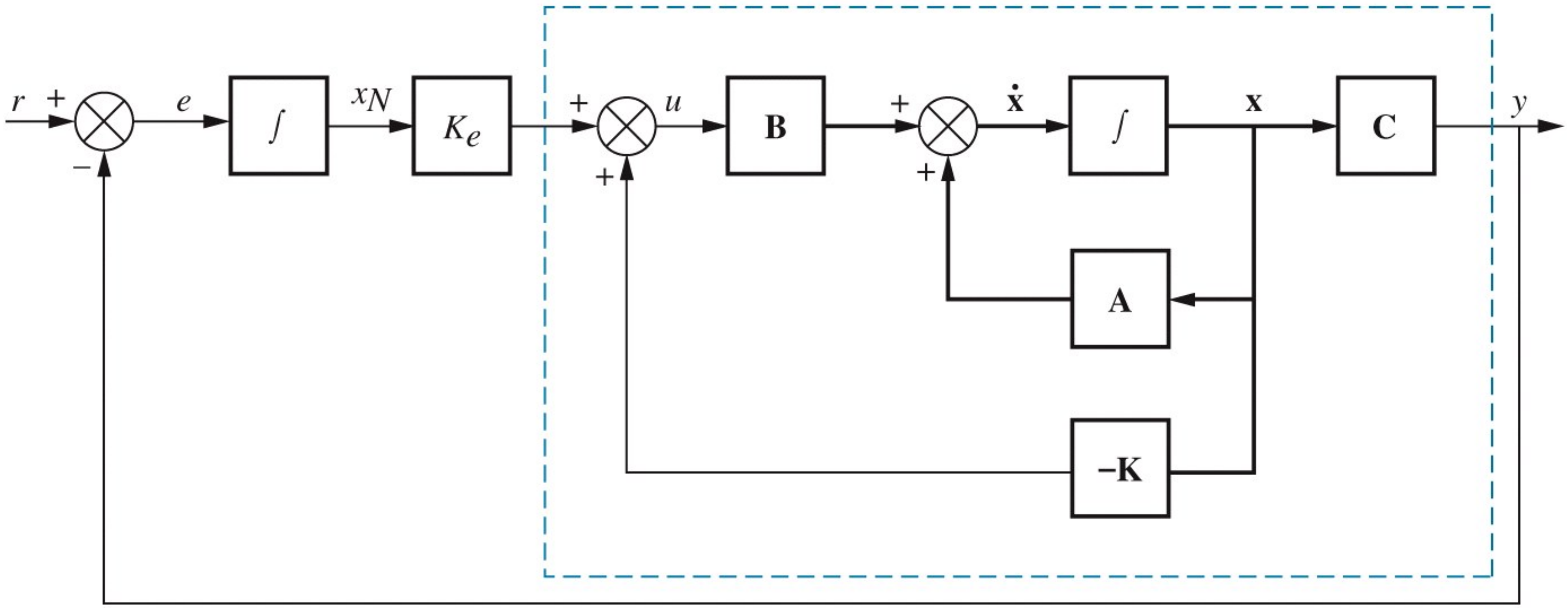
State feedback controller design

- However, the compensated system had a large **steady-state error**.
- Main reason: steady-state accuracy was **not** part of design specifications.
- So far, transient response design specifications are considered for controller design.
- Transient response requirements only allow dominant closed-loop pole placement (steady-state accuracy not addressed).
- Steady-state accuracy **requirement** should explicitly be part of design specifications.

State feedback controller design



Standard state feedback control

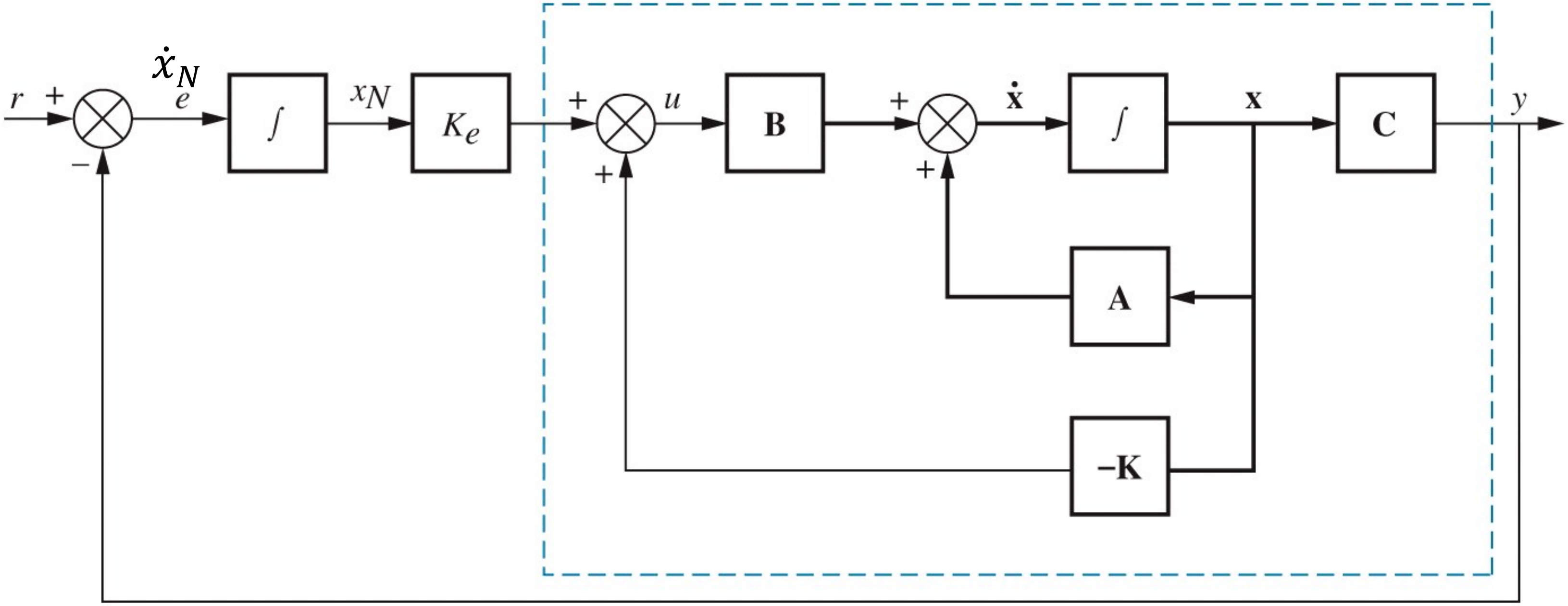


State feedback with *integral control*

State feedback with integral controller design

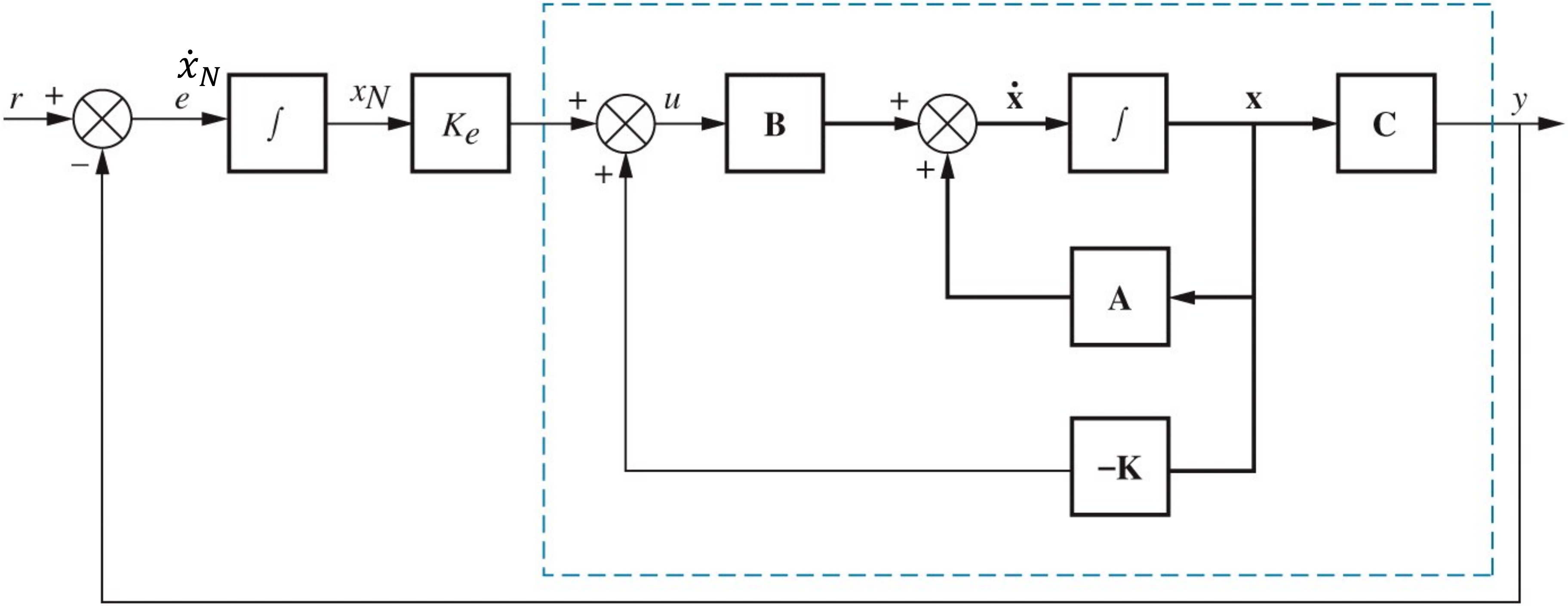
- To design considering both the steady-state accuracy and transient response specifications, state feedback control is augmented with integral control.
- Integral control eliminates the steady-state error, but *also increases the system type.*
- The model of the system with state feedback and integral control is slightly different, so we have to develop it.

State feedback with integral controller design



- A feedback path from the output has been added to form the error, e, which is fed forward to the controlled plant via an integrator.
- The integrator increases the system type and reduces the previous finite error to zero.
- Let's derive the state equations for this new system and then use that form to design a controller.
- Thus, we will be able to design a system for zero steady-state error for a step input as well as design the desired transient response.

State feedback with integral controller design



State feedback with integral control

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u; \; y = \mathbf{C}\mathbf{x}$$

$$\dot{x}_N = -\mathbf{C}\mathbf{x} + r;$$

System model:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 0 \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} r$$

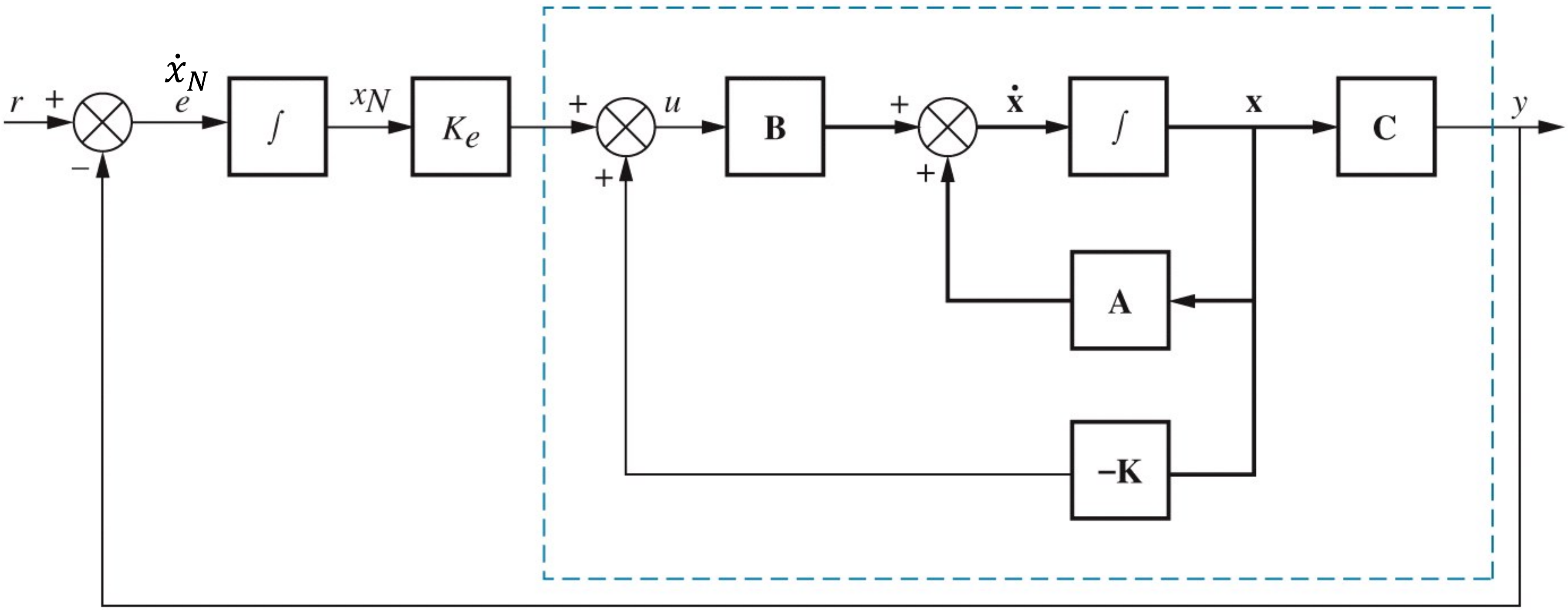
$$y = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix}$$

Here the input is defined as

$$u = -\mathbf{K}\mathbf{x} + K_e x_N$$

$$= -\begin{bmatrix} \mathbf{K} & -K_e \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix}$$

State feedback with integral controller design



State feedback with integral control

$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 0 \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} r$ and

$u = -[\mathbf{K} \quad -K_e] \begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix}$ gives the system model as,

System model:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K} & \mathbf{B}K_e \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} r$$

$$y = [\mathbf{C} \quad 0] \begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix}$$

$$\boxed{\begin{aligned} \dot{\hat{\mathbf{x}}} &= \widehat{\mathbf{A}}\hat{\mathbf{x}} + \mathbf{B}r \\ y &= \widehat{\mathbf{C}}\hat{\mathbf{x}} \end{aligned}}$$

Closed-loop char. polynomial:

$$\det\left(sI - \widehat{\mathbf{A}}\right) = 0$$

Analysis via Final Value Theorem

A single-input, single-output system represented in state space can be analyzed for steady-state error using the final value theorem and the closed-loop transfer function, Eq. (3.73), derived in terms of the state-space representation. Consider the closed-loop system represented in state space:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}r \tag{7.84a}$$

$$y = \mathbf{C}\mathbf{x} \tag{7.84b}$$

The Laplace transform of the error is

$$E(s) = R(s) - Y(s) \tag{7.85}$$

But

$$Y(s) = R(s)T(s) \tag{7.86}$$

where $T(s)$ is the closed-loop transfer function. Substituting Eq. (7.86) into (7.85), we obtain

$$E(s) = R(s)[1 - T(s)] \tag{7.87}$$

Using Eq. (3.73) for $T(s)$, we find

$$E(s) = R(s)[1 - \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}] \tag{7.88}$$

Applying the final value theorem, we have

$$\boxed{\lim_{s\to 0} sE(s) = \lim_{s\to 0} sR(s)[1 - \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}]} \tag{7.89}$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}r \tag{7.84a}$$

$$y = \mathbf{C}\mathbf{x} \tag{7.84b}$$

Step Inputs. Given the state Eqs. (7.84), if the input is a unit step where $r = 1$, a steady-state solution, $\mathbf{x}_{ss}$, for $\mathbf{x}$, is

$$\mathbf{x}_{ss} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \mathbf{V} \tag{7.92}$$

where V_i is constant. Also,

$$\dot{\mathbf{x}}_{ss} = \mathbf{0} \tag{7.93}$$

Substituting $r = 1$, a unit step, along with Eqs. (7.92) and (7.93), into Eqs. (7.84) yields

$$\mathbf{0} = \mathbf{A}\mathbf{V} + \mathbf{B} \tag{7.94a}$$

$$y_{ss} = \mathbf{C}\mathbf{V} \tag{7.94b}$$

where y_{ss} is the steady-state output. Solving for $\mathbf{V}$ yields

$$\mathbf{V} = -\mathbf{A}^{-1}\mathbf{B} \tag{7.95}$$

But the steady-state error is the difference between the steady-state input and the steady-state output. The final result for the steady-state error for a unit step input into a system represented in state space is

$$\boxed{e(\infty) = 1 - y_{ss} = 1 - \mathbf{C}\mathbf{V} = 1 + \mathbf{C}\mathbf{A}^{-1}\mathbf{B}} \tag{7.96}$$

Ramp Inputs. For unit ramp inputs, $r = t$, a steady-state solution for $\mathbf{x}$ is

$$\mathbf{x}_{ss} = \begin{bmatrix} V_1 t + W_1 \\ V_2 t + W_2 \\ \vdots \\ V_n t + W_n \end{bmatrix} = \mathbf{V}t + \mathbf{W} \tag{7.97}$$

where V_i and W_i are constants. Hence,

$$\dot{\mathbf{x}}_{ss} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \mathbf{V} \tag{7.98}$$

Substituting $r = t$ along with Eqs. (7.97) and (7.98) into Eqs. (7.84) yields

$$\mathbf{V} = \mathbf{A}(\mathbf{V}t + \mathbf{W}) + \mathbf{B}t \tag{7.99a}$$

$$y_{ss} = \mathbf{C}(\mathbf{V}t + \mathbf{W}) \tag{7.99b}$$

In order to balance Eq. (7.99a), we equate the matrix coefficients of t, $\mathbf{AV} = -\mathbf{B}$, or

$$\mathbf{V} = -\mathbf{A}^{-1}\mathbf{B} \tag{7.100}$$

Equating constant terms in Eq. (7.99a), we have $\mathbf{AW} = \mathbf{V}$, or

$$\mathbf{W} = \mathbf{A}^{-1}\mathbf{V} \tag{7.101}$$

Substituting Eqs. (7.100) and (7.101) into (7.99b) yields

$$y_{ss} = \mathbf{C}[-\mathbf{A}^{-1}\mathbf{B}t + \mathbf{A}^{-1}(-\mathbf{A}^{-1}\mathbf{B})] = -\mathbf{C}[\mathbf{A}^{-1}\mathbf{B}t + (\mathbf{A}^{-1})^2\mathbf{B}] \tag{7.102}$$

The steady-state error is therefore

$$\boxed{e(\infty) = \lim_{t\to\infty}(t - y_{ss}) = \lim_{t\to\infty}[(1 + \mathbf{CA}^{-1}\mathbf{B})t + \mathbf{C}(\mathbf{A}^{-1})^2\mathbf{B}]} \tag{7.103}$$

Notice that in order to use this method, $\mathbf{A}^{-1}$ must exist. That is, $\det\mathbf{A} \neq 0$.

State feedback with integral controller design: **Example-1**

Consider the plant

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u; \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$

(**a**) Design a state feedback controller <u>without integral control</u> to yield a **10%** *overshoot* and **0.5 sec** *settling time.* Evaluate the steady-state error for a unit step input.

(**b**) Redesign the state feedback controller <u>with integral control</u>; evaluate the steady-state error for a unit step input

<u>Procedure</u>:

(**1**) State space representation

(**2**) $\det\left(s\mathbf{I} - \widehat{\mathbf{A}}\right)$

(**3**) Desired closed-loop characteristic polynomial

(**4**) Compare (**2**) with (**3**)

State feedback with integral controller design: **Example-1**, *cont.'s*

Consider the plant

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u; \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$

(**a**) Design a state feedback controller without integral control to yield a 10% *OS* and $T_s = 0.5$ sec. Find e_{ss} for a unit step input.

Solution (b): (*by following the design steps*)

(**1**) State space representation is given.

(**2**) $\det\left(s\mathbf{I} - \widehat{\mathbf{A}}\right) = \det(s\mathbf{I} - \mathbf{A} + \mathbf{BK}) = \begin{vmatrix} s & -1 \\ 3 + k_1 & s + 5 + k_2 \end{vmatrix} =$

$= s^2 + (5 + k_2)s + 3 + k_1$

(**3**) $10\%\ OS$ ➔ $\zeta = 0.59$, $T_s = 0.5 = \frac{4}{\zeta\omega_n}$ ➔ $\omega_n = 13.53$ rad/s

$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2} = -8 \pm j10.92$ ➔ $D(s) = s^2 + 16s + 183.14$

(**4**) Compare (**2**) with (**3**):

$3 + k_1 = 183.14$ ➔ $k_1 = 180.14$; $5 + k_2 = 16$ ➔ $k_2 = 11$

State feedback with integral controller design: **Example-1**, *cont.'s*

Consider the plant

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u; \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$

(a) Design a state feedback controller without integral control to yield a 10% *OS* and $T_s = 0.5$ sec. Find e_{ss} for a unit step input.

Solution (a): (*by following the design steps*)

$$\mathbf{K} = \begin{bmatrix} 180.14 & 11 \end{bmatrix}$$

Since the control law is defined as $u = -\mathbf{Kx} + r$ for pole placement,

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}u; \quad \mathbf{y} = \mathbf{Cx}$$

$$= \mathbf{Ax} + \mathbf{B}(-\mathbf{Kx} + r) \;\Rightarrow\; \dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BK})\mathbf{x} + \mathbf{B}r$$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -183.14 & -16 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}r; \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$

We can now find the steady-state error for a step input,

$$e_{ss} = 1 + \mathbf{C}(\mathbf{A} - \mathbf{BK})^{-1}\mathbf{B}$$

$$= 1 + \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -183.14 & -16 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boldsymbol{e_{ss} = 0.9945}$$

Example-1, *cont.'s* – MATLAB *Solution*

Solution (a):

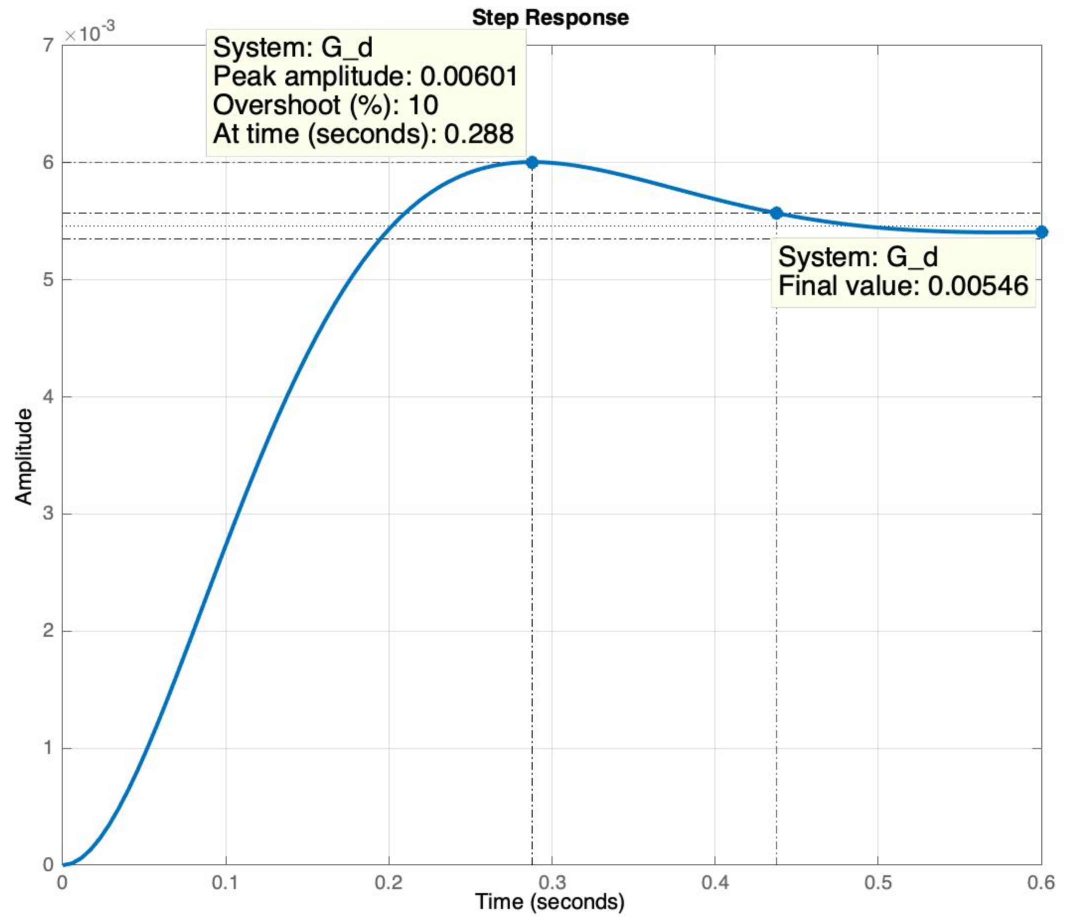
```
>> A=[0 1; -3  -5], B=[0; 1], C=[1 0], D=0
>> pOS=10; >> zeta=-log(pOS/100)/sqrt(pi^2 + (log(pOS/100))^2)
>> Ts=0.5; wn=4/zeta/Ts ➔ zeta = 0.5912;  wn = 13.5328;
>> s1=-zeta*wn-j*wn*sqrt(1-zeta^2); s2=-zeta*wn+j*wn*sqrt(1-zeta^2)
     s1 =  -8.0000 -10.9150i; s2 =  -8.0000 +10.9150i; P=[s1 s2];
>> K=acker(A, B, P)
➔ K =  180.1375   11.0000

>> ess=1+C*(A-B*K)^-1*B
➔ ess =    0.9945

>> G_d=ss(A-B*K,B,C,D);
>> step(G_d)
```

From the figure,
css=0.00546;

```
>> ess=1-css
➔ ess =    0.9945
```



State feedback with integral controller design: **Example-1**, *cont.'s*

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u; \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$

Solution (b): We make use of the following equations

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK} & \mathbf{B}K_e \\ -\mathbf{C} & 0 \end{bmatrix}\begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix}\begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} \left(\begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} k_1 & k_2 \end{bmatrix}\right) & \begin{bmatrix} 0 \\ 1 \end{bmatrix}K_e \\ -\begin{bmatrix} 1 & 0 \end{bmatrix} & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_N \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -(3+k_1) & -(5+k_2) & K_e \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_N \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_N \end{bmatrix}$$

State feedback with integral controller design: **Example-1**, *cont.'s*

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u; \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$

Solution (b): We make use of the following equations

- The transfer function of the plant is $G(s) = \frac{1}{s^2+5s+3}$
- The desired characteristic polynomial for the closed-loop integral-controlled system is $G_d(s) = \frac{1}{s^2+16s+183.14}$
- Since the plant has no zeros, we assume no zeros for the closed-loop system and hence augment $G_d(s)$ with a third pole, $(s+100)$, which has a real part greater than five times that of the desired dominant second-order poles ($-8 \pm j10.92$).
- Now, the desired 3rd-order closed-loop system characteristic poly. Is,

$$(s+100)(s^2+16s+183.14) = s^3+116s^2+1783s+18314$$

- The characteristic polynomial for the integral controlled system from the last equation (from the augmented system matrix),

$$s^3+(5+k_2)s^2+(3+k_1)s+K_e$$

- Matching the coefficients, $k_1 = 1780$; $k_2 = 111$; $K_e = 18314$

State feedback with integral controller design: **Example-1**, *cont.'s*

- Substituting these gain values into the equation below yields the following closed-loop integral-controlled system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -(3+k_1) & -(5+k_2) & K_e \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_N \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r = \begin{bmatrix} 0 & 1 & 0 \\ -1783.1 & -116 & 18{,}310 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_N \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_N \end{bmatrix}$$

$$T(s) = \mathbf{C}(\mathbf{sI} - \mathbf{A})^{-1}\mathbf{B} = \frac{18310}{s^3+116s^2+1783s+18310} \Rightarrow e_{ss} = 1 - 1 = 0$$

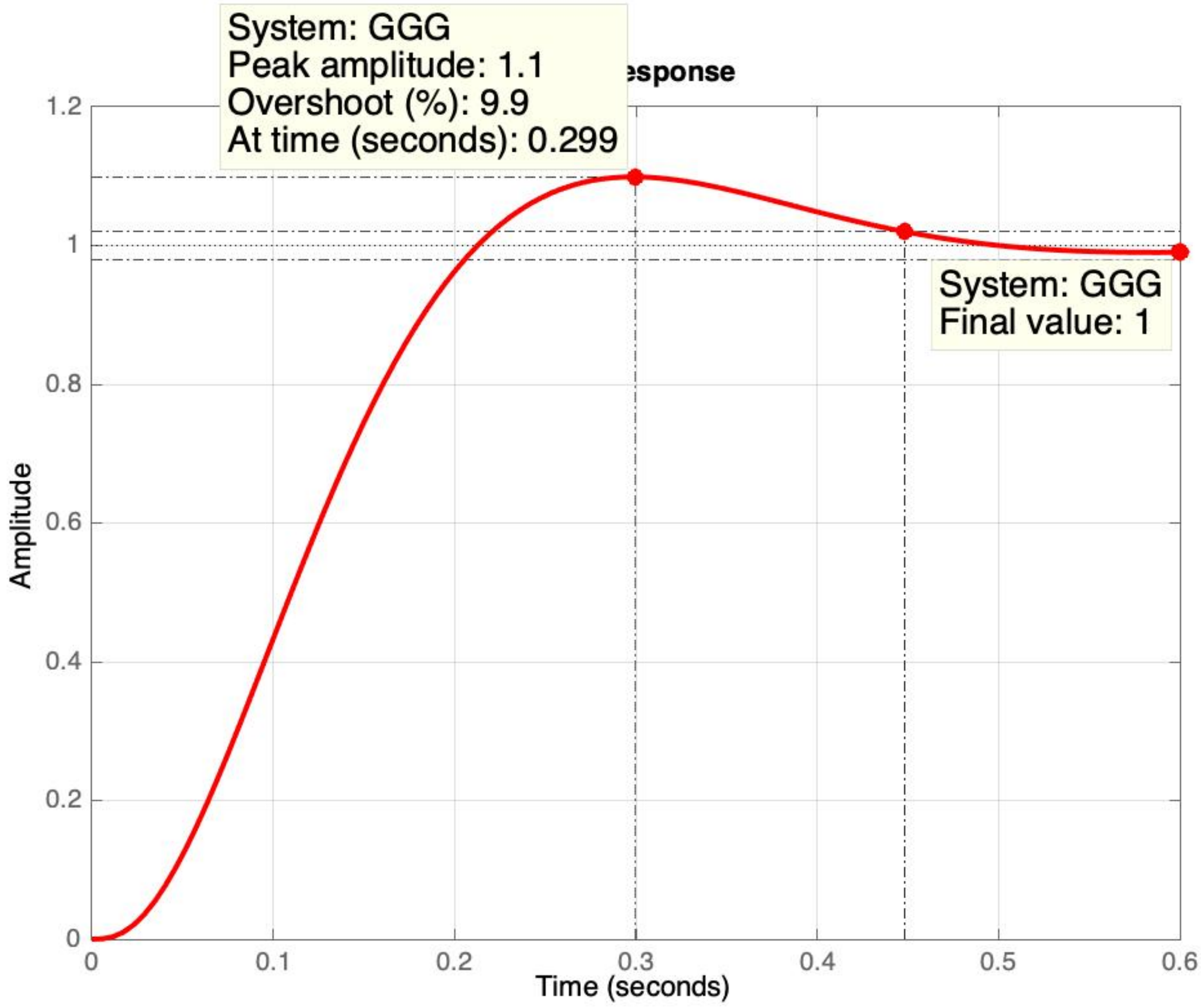
Alternatively, the steady-state error can be found as follows,

$$e_{ss} = e(\infty) = 1 + \mathbf{CA}^{-1}\mathbf{B}$$

$$e(\infty) = 1 + \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1783.1 & -116 & 18{,}310 \\ -1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

Thus, the system behaves like a Type 1 system.

Unit step response of State feedback with integral controller



State feedback with integral controller design: **Example-2**

Problem: Given the plant,

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)} \quad \rightarrow \quad G(s) = \frac{20s+100}{s^3+5s^2+4s}$$

(**a**) Design a state feedback controller without integral control to yield **9.5%** *overshoot* and **0.74 sec** *settling time*

(**b**) Redesign the state feedback controller with integral control; evaluate the steady-state error for a unit step input

Procedure:

(**1**) State space representation

(**2**) $\det\left(s\mathbf{I} - \widehat{\mathbf{A}}\right)$

(**3**) Desired closed-loop characteristic polynomial

(**4**) Compare (**2**) with (**3**)

State feedback with integral controller design: example-2

(1) **State space representation of the plant,** $G(s) = \frac{20(s+5)}{s(s+1)(s+4)} = \frac{20s+100}{s^3+5s^2+4s}$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \mathbf{C} = [100 \quad 20 \quad 0], D = 0$$

The closed-loop poles can be found for transient response requirements as,

$$9.5\% \rightarrow \zeta = 0.6;\ T_s = 0.74 = \frac{4}{\zeta\omega_n} \rightarrow \omega_n = 9 \text{ rad/s} \Rightarrow$$

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -5.4 \pm j7.2 \rightarrow$$

$$\Delta(s) = s^2 + 10.81s + 81.26$$

However, the system is of 3rd order. Therefore,

- We need to add a 3rd pole, e.g. to the near location of zero, $s = -5.1$;
- and another pole to $s = -27$ for integral control

(2) **Desired closed-loop characteristic polynomial**:

$$\Delta_{CL-des} = (s+5.1)(s+27)(s^2+10.81s+81.26)$$
$$= s^4 + 42.8s^3 + 561.6s^2 + 4050s + 10935 \quad (2)$$

(3) $\det\left(s\mathbf{I} - \hat{\mathbf{A}}\right) = 0$

$$\Delta_{CL} = s^4 + (5+k_3)s^3 + (4+k_2)s^2 + (20k_e + k_1)s + 100k_e = 0 \quad (3)$$

State feedback with integral controller design: example-2

(4) Compare (2) with (3):

$$\Delta_{CL-des} = s^4 + 42.8s^3 + 561.6s^2 + 4050s + 10935 \ldots (2)$$

$$\Delta_{CL} = s^4 + (5 + k_3)s^3 + (4 + k_2)s^2 + (20k_e + k_1)s + 100k_e = 0 \ldots (3)$$

State feedback with integral control:

$$\mathbf{K} = [k_1 \quad k_2 \quad k_3] = [1863 \quad 557.6 \quad 37.8]$$
$$k_e = 109.35$$

State feedback without integral control:

$$\mathbf{K}_1 = [k_1 \quad k_2 \quad k_3] = [414.48 \quad 132.45 \quad 10.92]$$

In Matlab – No Integral Control:

```
>> A=[0  1  0;  0  0  1; 0  -4  -5]; B=[0; 0; 1]; C=[100 20 0]; D=0;
>> pOS = 9.5000;
>> zeta=-log(pOS/100)/sqrt(pi^2 + (log(pOS/100))^2) ➔ zeta= 0.5996
>> Ts=0.74; wn=4/zeta/Ts ➔ wn = 9.0147
>> s1=-zeta*wn-j*wn*sqrt(1-zeta^2) ➔ s1,2=-5.4054 -/+ 7.2143i; s3=-5.1
>> P=[s1 s2 -5.1];  K=acker(A, B, P)
                    K =  414.4490  132.3996   10.9108
```

In Matlab – With Integral Control:

```
>> A=[0  1  0;  0  0  1; 0  -4  -5]; B=[0; 0; 1]; C=[100 20 0]; D=0;
>> syms s1 k1 k2 k3 ke
>> A_int=[s1 -1 0 0; 0 s1 -1 0; k1 4+k2 s1+5+k3 -ke; 100 20 0 s1]
>> det(A_int)=
100*ke + k1*s1 + 20*ke*s1 + k2*s1^2 + k3*s1^3 + 4*s1^2 +
5*s1^3 + s1^4
```

➔ $\Delta_{CL} = s^4 + (5+k_3)s^3 + (4+k_2)s^2 + (20k_e + k_1)s + 100k_e$

Compare this equation with gains with the desired equation:

```
Delta_CL=conv(conv([1 5.1],[1 27]), [1 10.81 81.26])
Delta_CL =  1.0e+04 *    0.0001    0.0043    0.0566    0.4097    1.1190
```

➔ $\Delta_{CL-des} = (s+5.1)(s+27)(s^2+10.81s+81.26)$

$= s^4 + 43s^3 + 566s^2 + 4097s + 11{,}190$

$$\mathbf{K} = [k_1 \quad k_2 \quad k_3] = [1859 \quad 562 \quad 38]; \;\; k_e = 111.9$$

Step response plots of full-state feedback controlled system and its integrator augmented version are left for students!