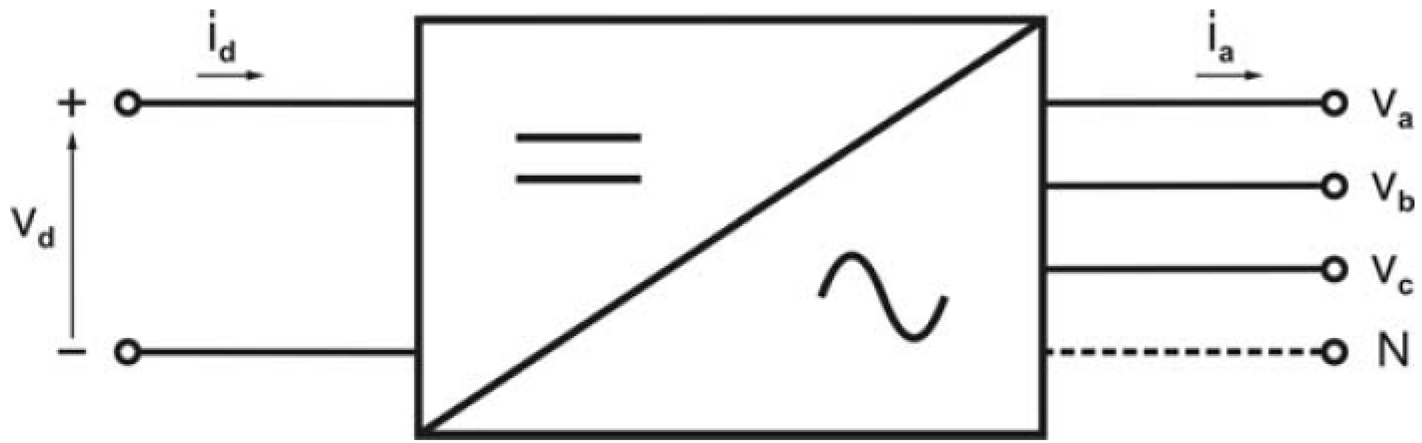
# KOM3560
# INDUSTRIAL ELECTRONICS

**Spring : 2023**

**Lecture 5**

# DC to AC Converters : Inverters

Dc-to-ac converters are known as inverters. The function of an inverter is to change a dc input voltage to a symmetric ac output voltage of desired magnitude and frequency. The output voltage could be fixed or variable at a fixed or variable frequency. A variable output voltage can be obtained by varying the input dc voltage and maintaining the gain of the inverter constant. The inverter gain may be defined as the ratio of the ac output voltage to dc input voltage.

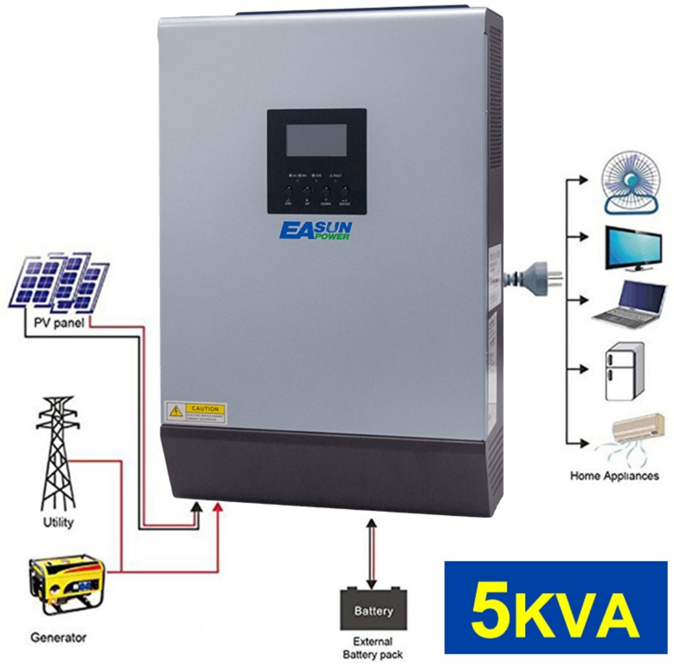



$\mathbf{v_d}$ : DC input voltage
$\mathbf{i_d}$ :DC input current
$\mathbf{i_a, i_b, i_c}$: AC output phase currents
$\mathbf{v_a, v_b, v_c}$ : AC output phase voltages
**N :** Neutral output return line

# DC to AC Converters : Inverters

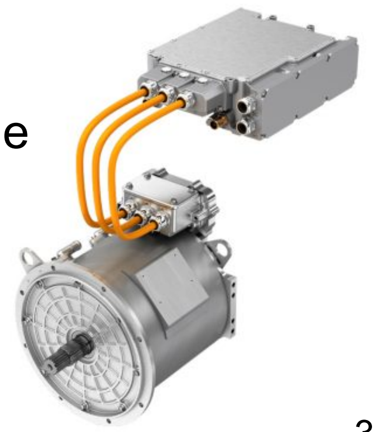
## Main application areas of DC-AC converters

- AC motor control
- SMPS – Switching mode power supply
- UPS – Uniterruptible power supply
- Induction heating and hardening
- Electronic balasts
- AC power regulators and welding devices
- Renewable energy systems
- …




## Main features of DC-AC converters:

→It is forced commutated.
→It is realized with fully controlled switching devices.
→It is controlled by AC - PWM control method.
→A fixed or regulated and non-insulated or isolated AC voltage is generated
→Depending on the control technique, harmonics may occur in the AC output voltage.
→Significant fluctuation may also occur in the DC input source current.



# DC to AC Converters : Inverters

**A)** DC giriş veya besleme kaynağı açısından,

**a)** Gerilim kaynaklı inverterler
**b)** Akım kaynaklı inverterler

**B)** Faz sayısına göre,

**a)** Tek fazlı inverterler
**b)** Üç fazlı inverterler

**C)** Uygulanan kontrol yöntemine göre,

**a)** Kare dalga inverterler
**b)** Kısmi veya boşluklu kare dalga inverterler
**c)** PWM inverterler

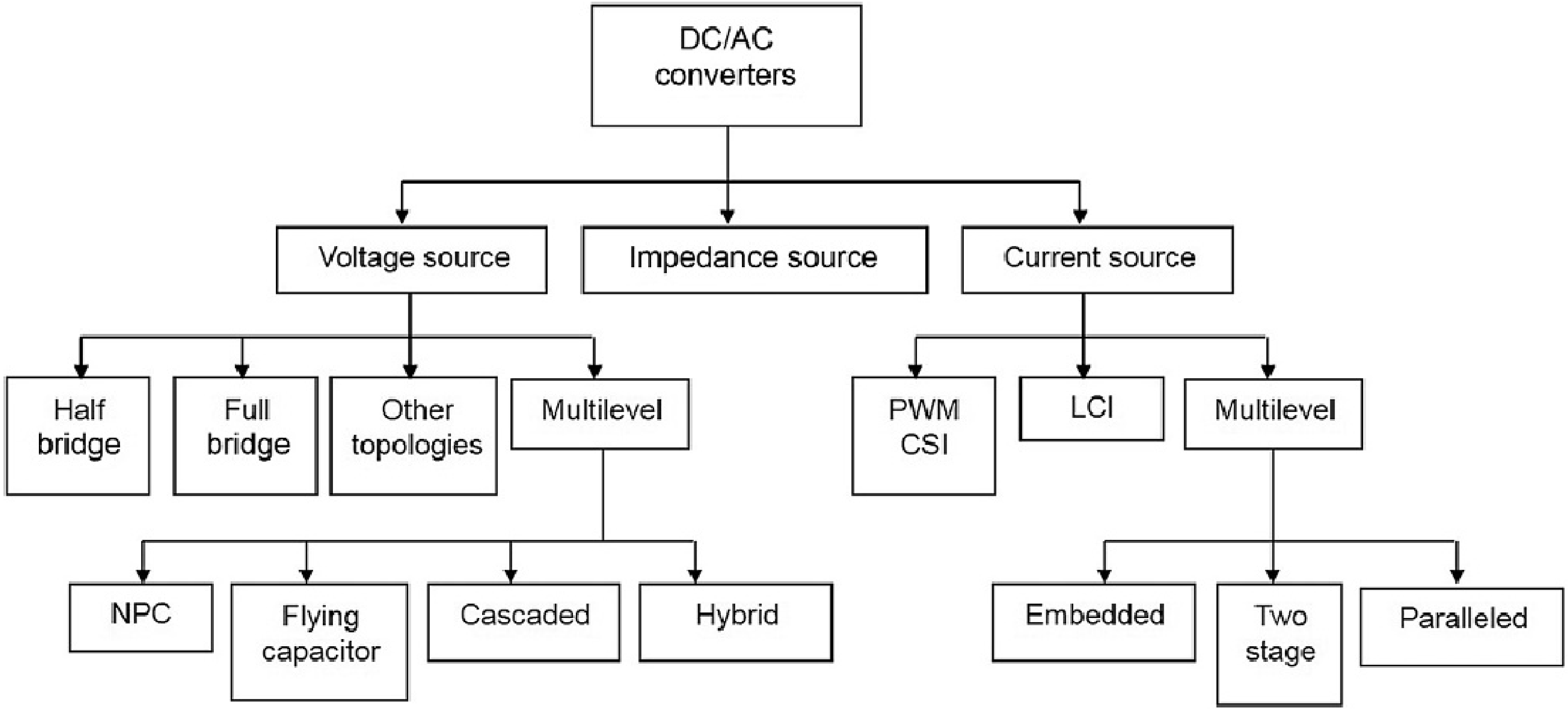
**E)** Tek fazlı inverterler devre yapısına göre,

**a)** Yarım köprü inverterler
**b)** Tam köprü inverterler
**c)** Push-pull inverterler

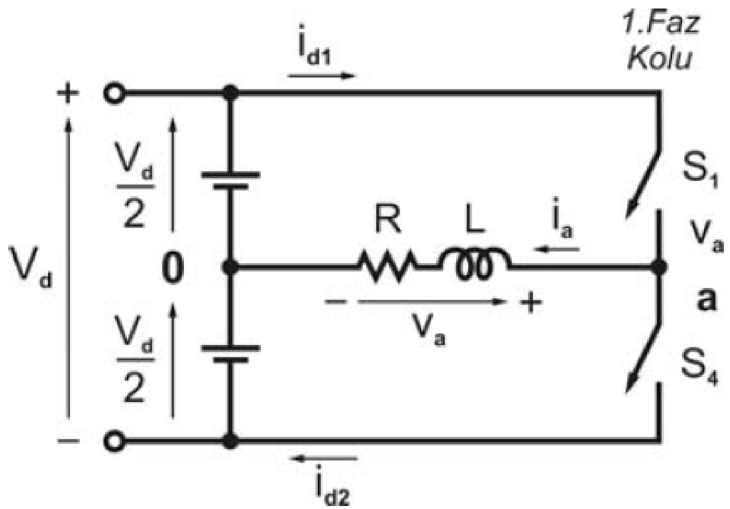
**D)** İzolasyon açısından,

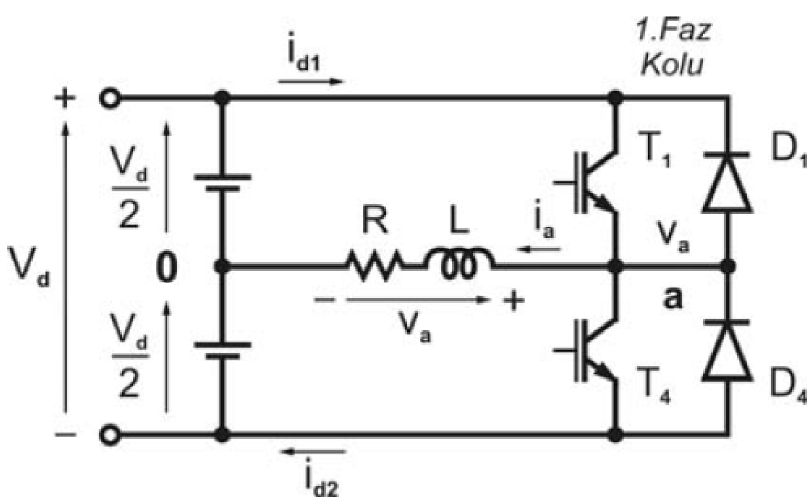
**a)** İzoleli inverterler
**b)** İzolesiz inverterler

# DC to AC Converters : Inverters

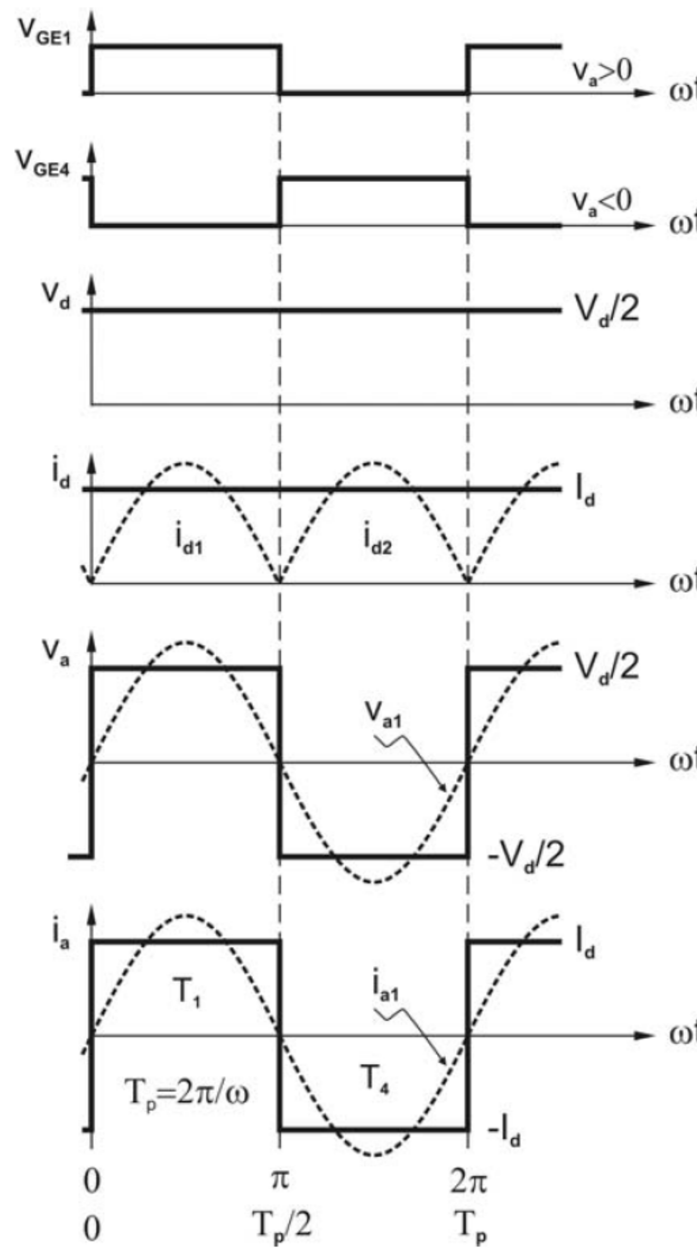


# Single Phase Half-Bridge Square Wave Inverter

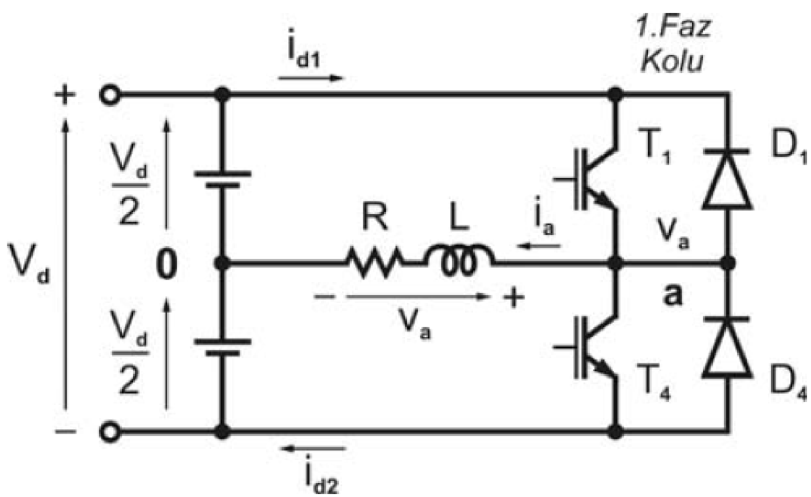



## Pure Ohmics Load



# Single Phase Half-Bridge Square Wave Inverter




$$v_a = \begin{cases} V_d/2 & 0<\omega t<\pi \text{ aralığı, } S_1 \text{ kapalı, } T_1 \text{ veya } D_1 \text{ iletimde} \\ -V_d/2 & \pi<\omega t<2\pi \text{ aralığı, } S_4 \text{ kapalı, } T_4 \text{ veya } D_4 \text{ iletimde} \end{cases}$$

Effective value of output phase voltage

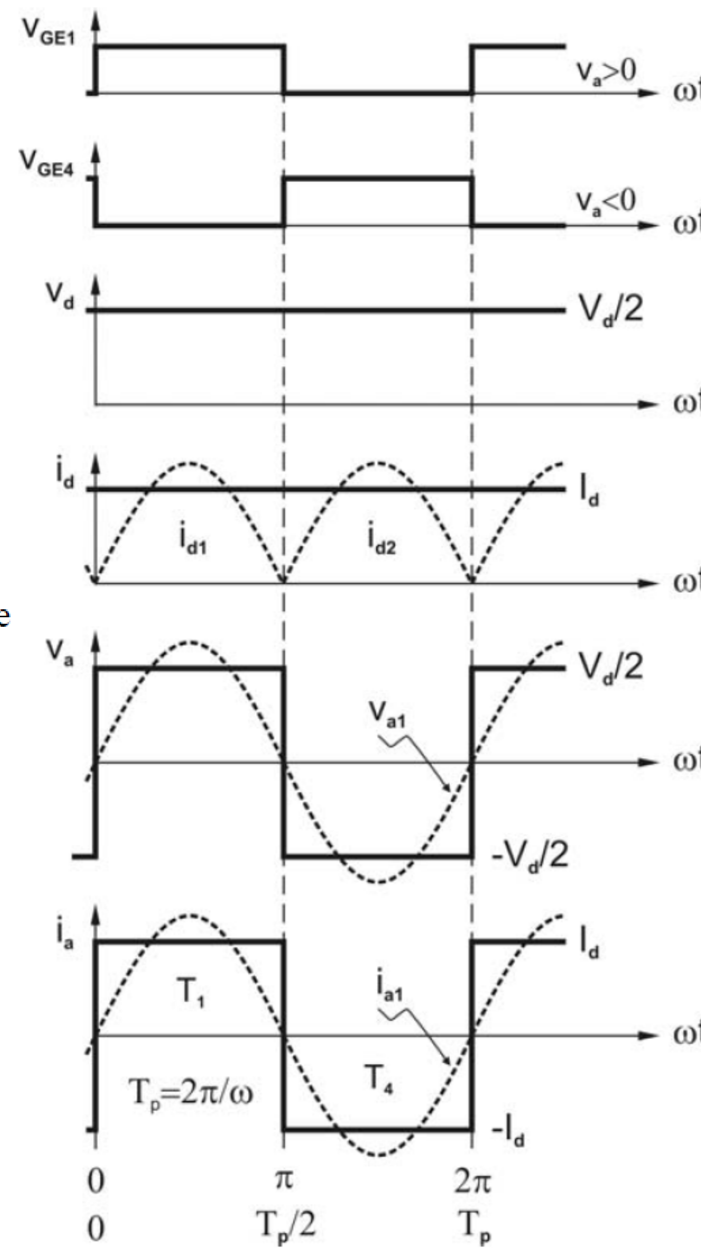
$$V_a = \sqrt{\frac{1}{\pi}\int_0^{\pi}\left(\frac{V_d}{2}\right)^2 d(\omega t)} = \frac{V_d}{2}\sqrt{\frac{1}{\pi}\left|(\omega t)\right|_0^{\pi}}$$

$$V_a = V_f = \frac{V_d}{2}$$

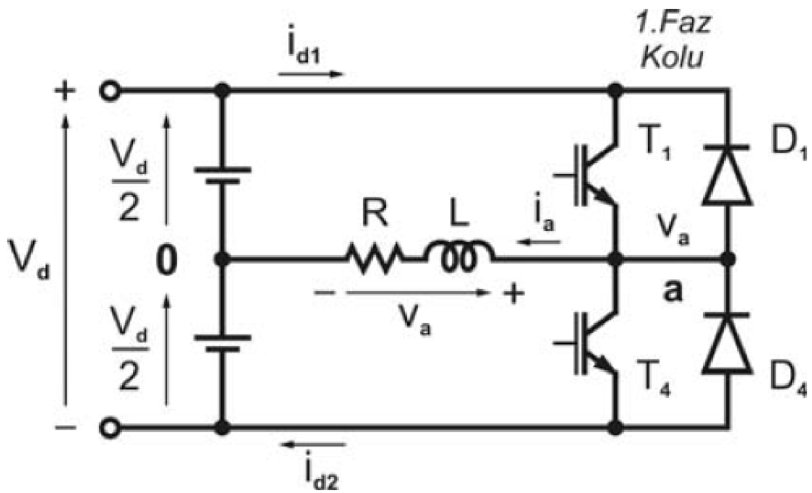
$$V_{a1} = \frac{1}{\sqrt{2}}\frac{2}{\pi}\int_0^{\pi}\frac{V_d}{2}\sin(\omega t)d(\omega t) = \frac{1}{\sqrt{2}}\frac{2}{\pi}\frac{V_d}{2}\left|-\cos(\omega t)\right|_0^{\pi}$$

$$\boxed{V_{a1} = V_{f1} = \frac{2\sqrt{2}}{\pi}\cdot\frac{V_d}{2}}$$

## Pure Ohmics Load



# Single Phase Half-Bridge Square Wave Inverter




$$v_a = \begin{cases} V_d/2 & 0<\omega t<\pi \text{ aralığı, } S_1 \text{ kapalı, } T_1 \text{ veya } D_1 \text{ iletimde} \\ -V_d/2 & \pi<\omega t<2\pi \text{ aralığı, } S_4 \text{ kapalı, } T_4 \text{ veya } D_4 \text{ iletimde} \end{cases}$$

Effective value of output phase voltage

$$V_a = \sqrt{\frac{1}{\pi}\int_0^{\pi}\left(\frac{V_d}{2}\right)^2 d(\omega t)} = \frac{V_d}{2}\sqrt{\frac{1}{\pi}\left|(\omega t)\right|_0^{\pi}}$$

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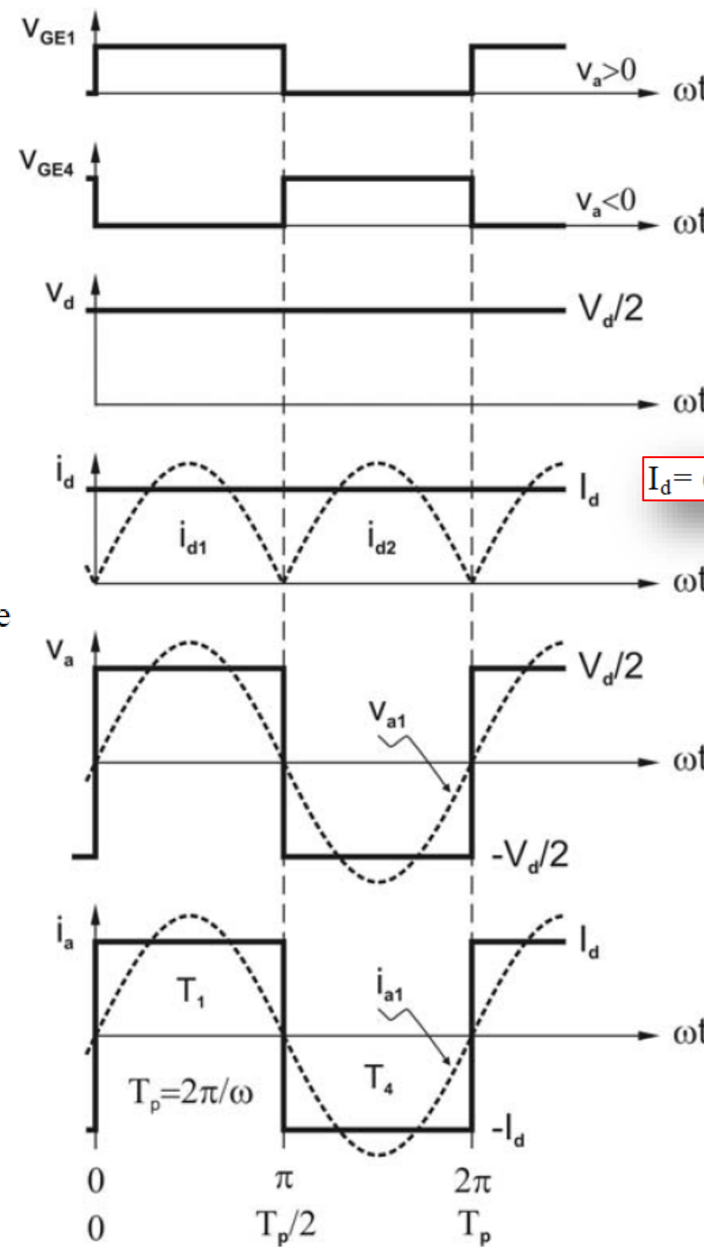
$$\boxed{V_{a1} = V_{f1} = \frac{2\sqrt{2}}{\pi}\cdot\frac{V_d}{2}} \rightarrow \boxed{v_{a1} = \sqrt{2}V_{f1}\sin\omega t}$$

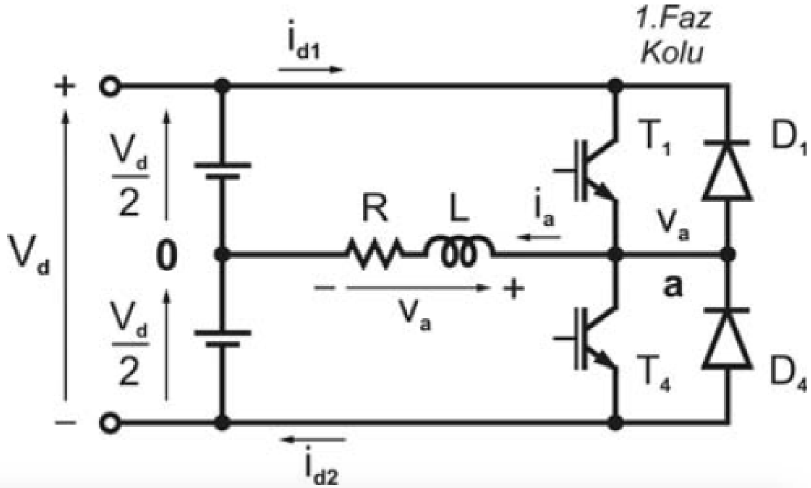
## Pure Ohmics Load




$\boxed{I_d = (V_d/2)/R}$

# Single Phase Half-Bridge Square Wave Inverter

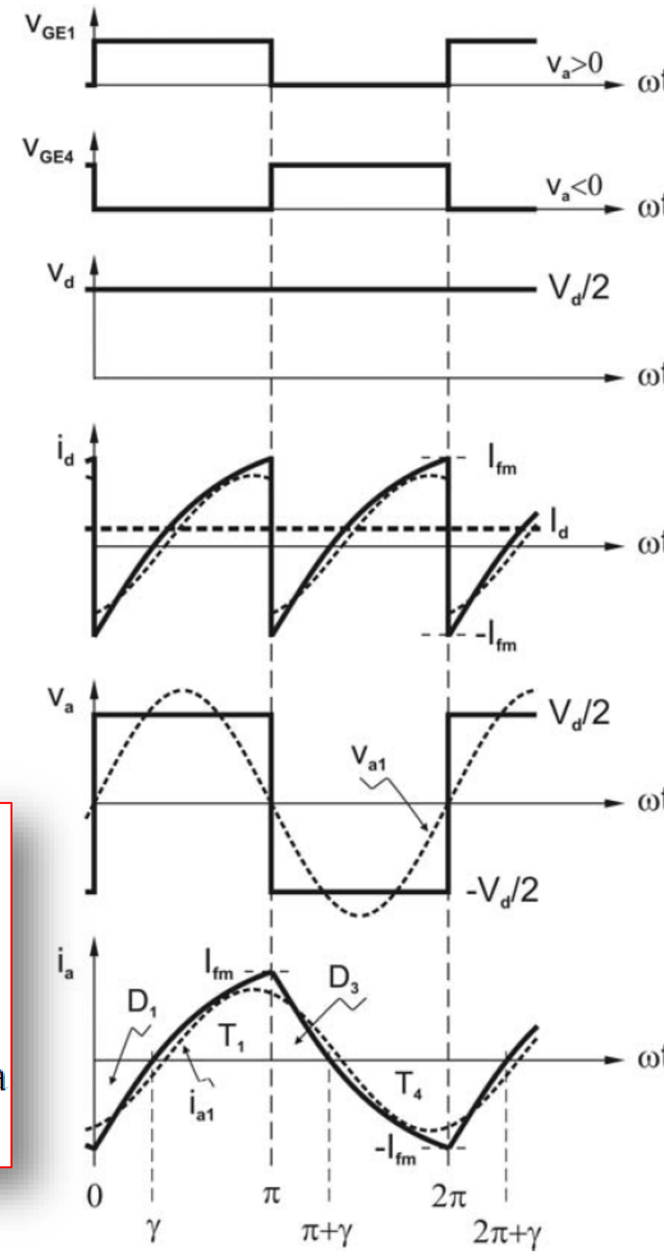
## Ohmic-Inductive Load



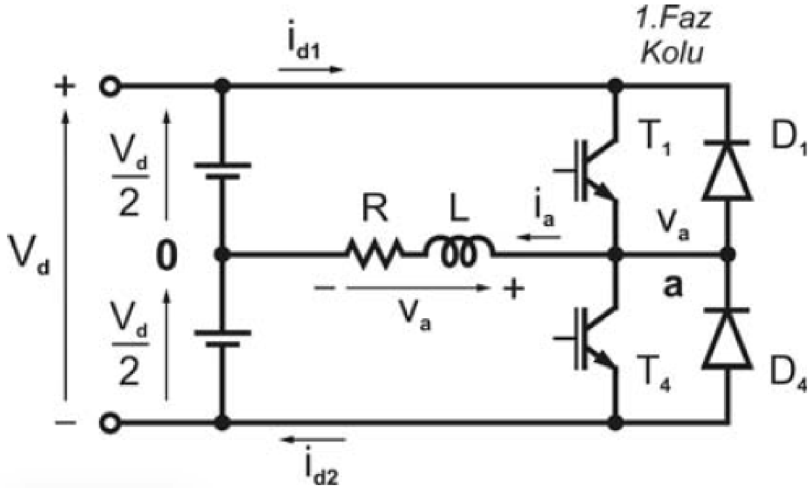

$$\frac{V_d}{2} = L\frac{di_a}{dt} + Ri_a \quad \Big| \quad 0<\omega t<\pi \text{ aralığında}$$

$$-\frac{V_d}{2} = L\frac{di_a}{dt} + Ri_a \quad \Big| \quad \pi<\omega t<2\pi \text{ aralığında}$$

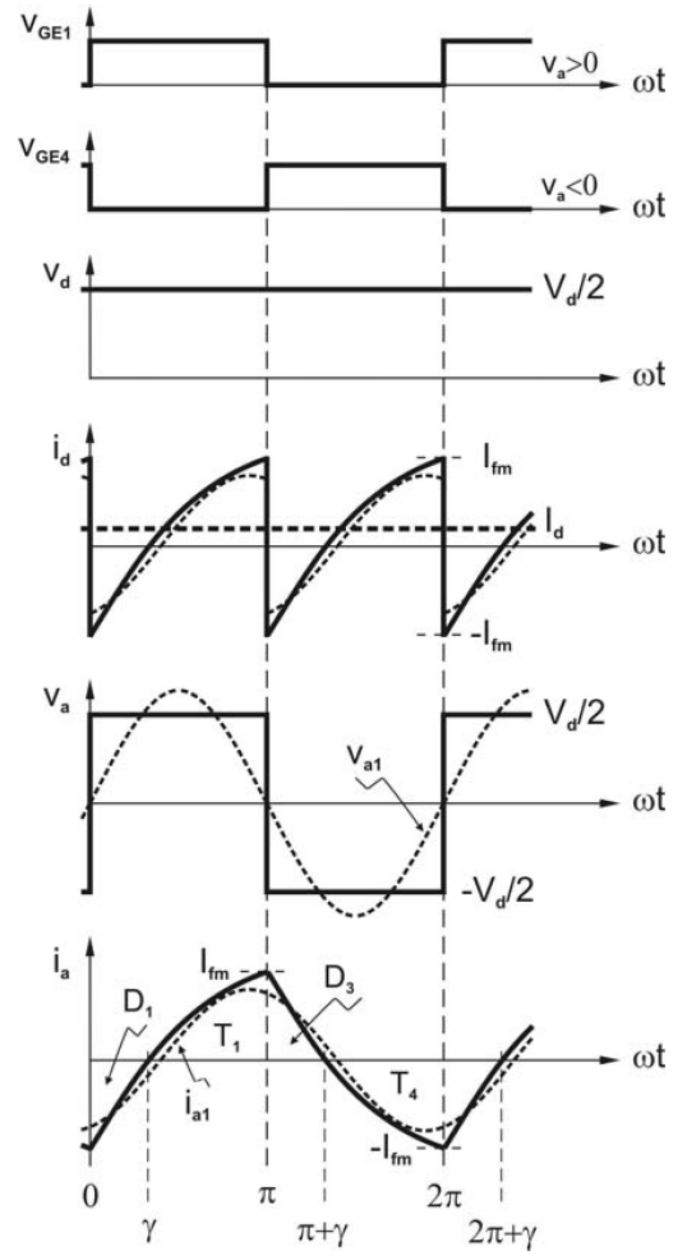
$$i_a = \begin{cases} \dfrac{V_d/2}{R} - \left(\dfrac{V_d/2}{R} + I_{fm}\right)e^{-\frac{1}{\tau}t} & \Big| \quad 0<\omega t<\pi \text{ aralığında} \\ -\dfrac{V_d/2}{R} + \left(\dfrac{V_d/2}{R} + I_{fm}\right)e^{-\frac{1}{\tau}t} & \Big| \quad \pi<\omega t<2\pi \text{ aralığında} \end{cases}$$



# Single Phase Half-Bridge Square Wave Inverter




## Ohmic-Inductive Load




$$\tau = \frac{L}{R}$$

Max value of the phase current

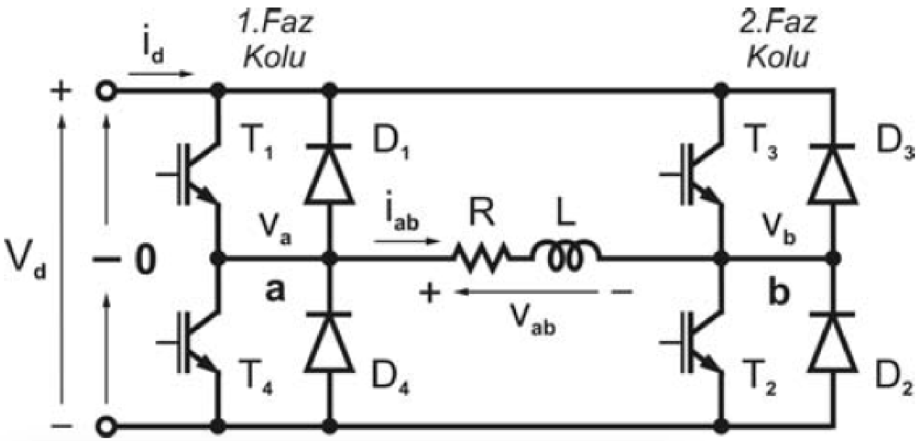
$$I_{fm} = \frac{V_d/2}{R} \cdot \frac{1 - e^{-\frac{1}{\tau}t_\pi}}{1 + e^{-\frac{1}{\tau}t_\pi}}$$

$$t_\gamma = \frac{L}{R} \cdot \ln\left(1 + \frac{RI_{fm}}{V_d/2}\right) = \tau \cdot \ln\left(1 + \frac{RI_{fm}}{V_d/2}\right)$$

Conduction time interval of the diode

# Single Phase Full-Bridge Square Wave Inverter

## Ohmic-Inductive Load



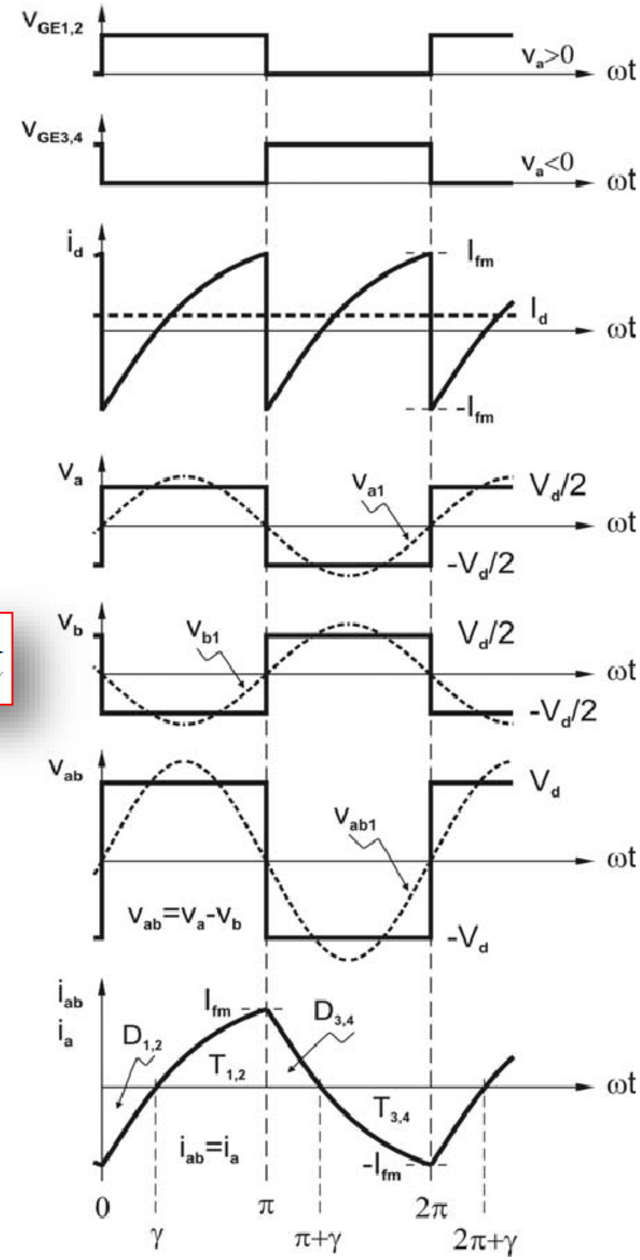

$$V_{ab} = V_h = V_d = 2V_f$$

$$V_{ab1} = V_{h1} = \frac{2\sqrt{2}}{\pi} \cdot V_d = 2V_{f1}$$

$$v_{ab1} = \sqrt{2} V_{h1} \sin \omega t$$

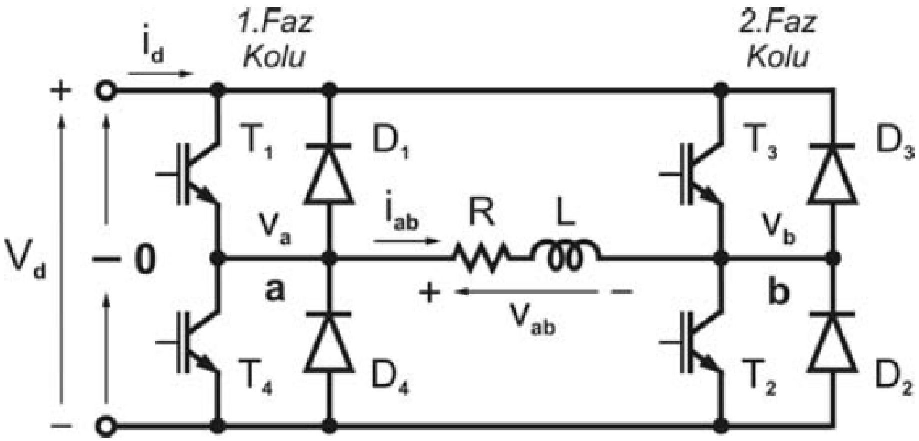
$$V_d = L\frac{di_a}{dt} + Ri_a \quad \Big| \quad 0<\omega t<\pi \text{ aralığında}$$

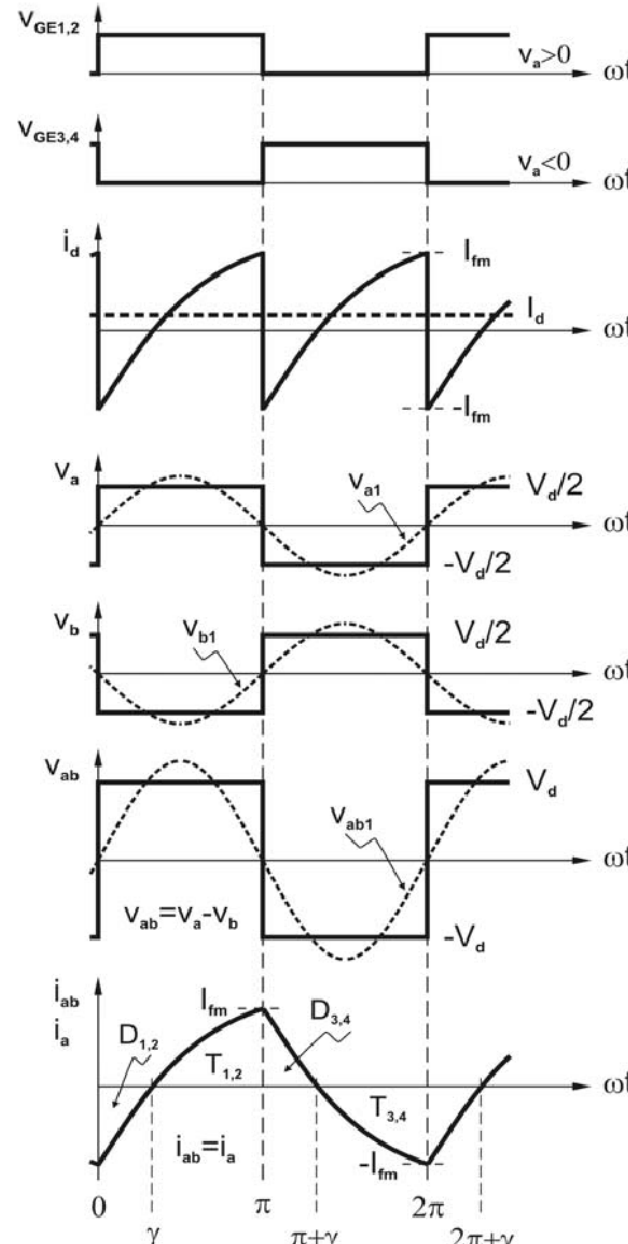
$$-V_d = L\frac{di_a}{dt} + Ri_a \quad \Big| \quad \pi<\omega t<2\pi \text{ aralığında}$$



# Single Phase Full-Bridge Square Wave Inverter

## Ohmic-Inductive Load




$$\tau = \frac{L}{R}$$

Max value of the phase current

$$I_{fm} = \frac{V_d}{R} \cdot \frac{1 - e^{-\frac{1}{\tau}t_\pi}}{1 + e^{-\frac{1}{\tau}t_\pi}}$$

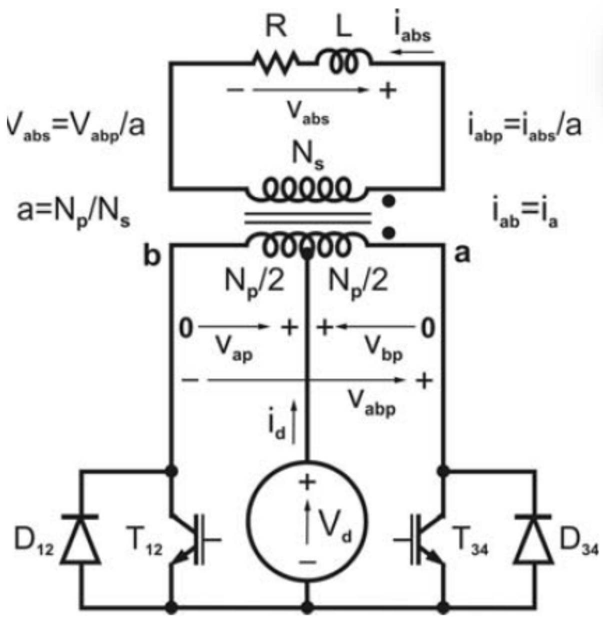
$$t_\gamma = \frac{L}{R} \cdot \ln\left(1 + \frac{RI_{fm}}{V_d}\right) = \tau \cdot \ln\left(1 + \frac{RI_{fm}}{V_d}\right)$$

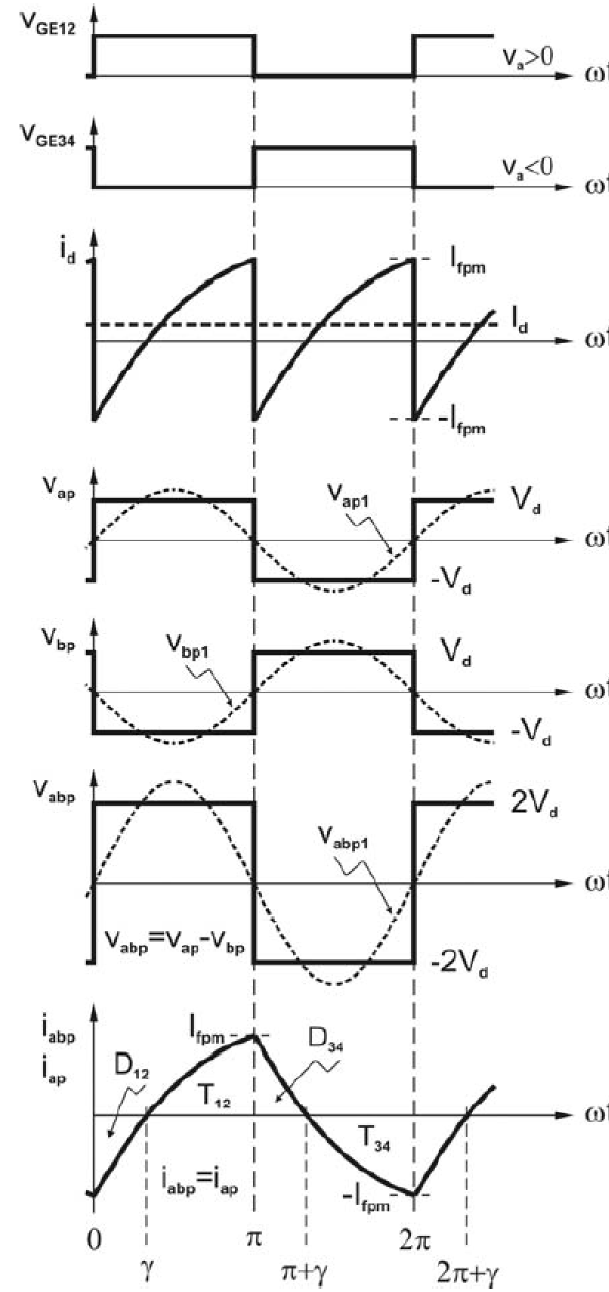
Conduction time interval of the diode

# Single Phase Push-Pull Square Wave Inverter

## Ohmic-Inductive Load

$a = N_p / N_s$




$$V_{abs} = V_{hs} = \frac{2V_d}{a} = 2V_{fs}$$

$$V_{abs1} = V_{hs1} = \frac{2\sqrt{2}}{\pi} \cdot \frac{2V_d}{a} = 2V_{fs1}$$

Analysis methodology is the same with half-bridge inverter except substituting in place of $\frac{V_d}{2}$ to $2V_d$

# Single Phase Push-Pull Square Wave Inverter

## ÖRNEK 6.1

*10 Ω'luk bir direnci 200 V'luk efektif bir gerilimle beslemek üzere, tek fazlı yarım köprü, tam köprü ve a=1 olan push-pull türü inverterlerin kullanılması durumunda, giriş gerilimi, giriş akımı, aktif eleman sayısı, bir elemanın maruz kaldığı ortalama akım ve maksimum gerilim değerlerini bularak bir tabloda özetleyiniz. Devre kayıplarını ihmal ediniz.*

## ÇÖZÜM

**Tablo 6.Ö.1.** *Tek fazlı inverterlerin sayısal bir karşılaştırma örneği*

| | Yarım Köprü | Tam Köprü | Push-Pull |
|---|---|---|---|
| Giriş Gerilimi (V) | 400 | 200 | 100 |
| Giriş Akımı (A) | 10 | 20 | 40 |
| Aktif Eleman Sayısı | 2 | 4 | 2 |
| Ortalama Eleman Akımı (A) | 10 | 10 | 20 |
| Maksimum Eleman Gerilimi (V) | 400 | 200 | 200 |

## ÖRNEK 6.2 Single Phase Push-Pull Square Wave Inverter

*Giriş gerilimi 300 V ve frekansı 50 Hz olan yarım köprü türü bir inverter ile 15 Ω değerinde bir direnç beslenmektedir. Devre kayıplarını ihmal ederek,*

**a)** *Yükün efektif gerilimi ve akımı ile gücünü hesaplayınız.*

**b)** *Giriş akımı ile bir transistörden geçen akımın ortalama ve efektif değerlerini bulunuz.*

## ÇÖZÜM

**a)** *(6.2) bağıntısı kullanılarak, yükün efektif gerilimi ve akımı ile gücü,*

$$V_a = V_f = \frac{V_d}{2} = \frac{300}{2} = 150\ \mathrm{V}\,, \quad I_f = \frac{V_f}{R} = \frac{150}{15} = 10\ \mathrm{A}$$

$$P = \frac{V_f^2}{R} = \frac{150^2}{15} = 1500\ \mathrm{W} = 1{,}5\ \mathrm{kW}\ \textit{olarak bulunur.}$$

**b)** *Giriş akımı ile bir transistörden geçen akımın ortalama ve efektif değerleri,*

$$I_d = \frac{V_d/2}{R} = \frac{300/2}{15} = 10\ \mathrm{A} = I_f\,, \quad I_{TAV} = \frac{1}{2} I_d = \frac{1}{2} \cdot 10 = 5\ \mathrm{A}$$

$$I_{TEF} = \frac{1}{\sqrt{2}} I_d = \frac{1}{\sqrt{2}} \cdot 10 \cong 7{,}071\ \mathrm{A}\ \textit{şeklinde bulunur.}$$

# Single Phase Push-Pull Square Wave Inverter

**ÖRNEK 6.3**

*Giriş gerilimi 200 V ve frekansı 50 Hz olan tek fazlı tam köprü türü bir inverter vasıtasıyla, seri bağlı 10 Ω'luk bir direnç ve 50 mH'lik bir endüktanstan oluşan bir yük beslenmektedir. Devre kayıplarını ihmal ederek,*

**a)** *Yükün efektif gerilimi ve akımı ile gücünü hesaplayınız.*

**b)** *Giriş akımı ile bir transistör ve bir diyottan geçen akımın ortalama ve efektif değerlerini bulunuz.*

**ÇÖZÜM**

**a)** *(6.9) ve (6.6) bağıntılarından, yükün efektif gerilimi ile zaman sabiti,*

$$V_{ab} = V_h = V_d = 200\ \mathrm{V}\,,\quad \tau = \frac{L}{R} = \frac{50\cdot 10^{-3}}{10} = 5\cdot 10^{-3}\ \mathrm{s} \quad \Rightarrow \quad \frac{1}{\tau} = 200$$

*f=50 Hz için, periyot ve yarı periyot,*

$$T_p = t_{2\pi} = 1/50 = 20\cdot 10^{-3}\ \mathrm{s},\quad T_p/2 = t_\pi = 10\cdot 10^{-3}\ \mathrm{s}$$

*yük akımının maksimum değeri, ( , $V_d/2$ yerine $V_d$ alınarak,*

$$I_{hm} = I_{fm} = \frac{V_d}{R}\cdot\frac{1-e^{-\frac{1}{\tau}t_\pi}}{1+e^{-\frac{1}{\tau}t_\pi}} = 20\cdot\frac{1-e^{-200\cdot 10\cdot 10^{-3}}}{1+e^{-200\cdot 10\cdot 10^{-3}}} \cong 15{,}23\ \mathrm{A}$$

*diyodun iletim süresi,(6.8)'de $V_d/2$ yerine $V_d$ alınarak,*

$$t_\gamma = \tau \cdot \ln(1+\frac{RI_{fm}}{V_d}) = 5\cdot 10^{-3}\ln(1+\frac{10\cdot 15{,}23}{200}) \cong 2{,}831\cdot 10^{-3}\ \mathrm{s}$$

*$0<\omega t<\pi$ ve $\pi<\omega t<2\pi$ aralıklarındaki yük akımı değerleri, (6.5) bağıntılarında $V_d/2$ yerine $V_d$ alınarak,*

$$i_h = i_f = \begin{cases} \dfrac{V_d}{R} - (\dfrac{V_d}{R} + I_{fm})e^{-\frac{1}{\tau}t} = 20 - 35{,}23\cdot e^{-200\cdot t}\Big|_{t_0 - t_\pi} \\ -\dfrac{V_d}{R} + (\dfrac{V_d}{R} + I_{fm})e^{-\frac{1}{\tau}t} = -20 + 35{,}23\cdot e^{-200\cdot t}\Big|_{t_\pi - t_{2\pi}} \end{cases}$$

*ve efektif yük akımı ile yükün gücü,*

$$I_h = I_f = \sqrt{\frac{1}{t_\pi}\int_0^{t_\pi}(20 - 35{,}23\cdot e^{-200\cdot t})^2 d(t)} = \cdots\cdots \cong 9{,}767\ \mathrm{A}$$

$$P = RI_f^2 = 10\cdot 9{,}767^2 \cong 953{,}9\ \mathrm{W}$$

*olarak bulunur.*

# Single Phase Push-Pull Square Wave Inverter

**b)** *Giriş akımı ile bir transistör ve bir diyottan geçen akımın ortalama ve efektif değerleri,*

$$I_d = \frac{1}{t_\pi}\int_0^{t_\pi}(20-35{,}23\cdot e^{-200\cdot t})d(t) = \cdots\cdots \cong 4{,}769\ \text{A}$$

$$I_{TAV} = \frac{1}{t_{2\pi}}\int_{t_\gamma}^{t_\pi}(20-35{,}23\cdot e^{-200\cdot t})d(t) = \cdots\cdots \cong 3{,}361\ \text{A}$$

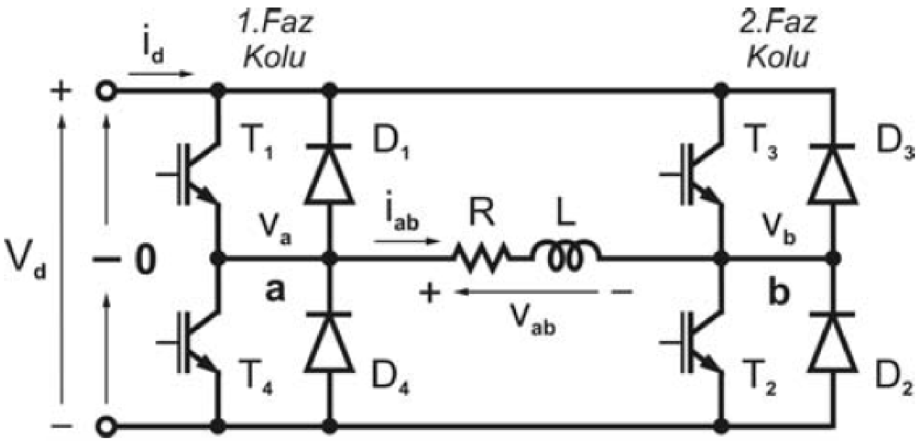
$$I_{TEF} = \sqrt{\frac{1}{t_{2\pi}}\int_{t_\gamma}^{t_\pi}(20-35{,}23\cdot e^{-200\cdot t})^2\, d(t)} = \cdots\cdots \cong 6{,}182\ \text{A}$$

$$I_{DAV} = \frac{1}{t_{2\pi}}\int_0^{t_\gamma}(20-35{,}23\cdot e^{-200\cdot t})d(t) = \cdots\cdots \cong -0{,}9767\ \text{A}$$

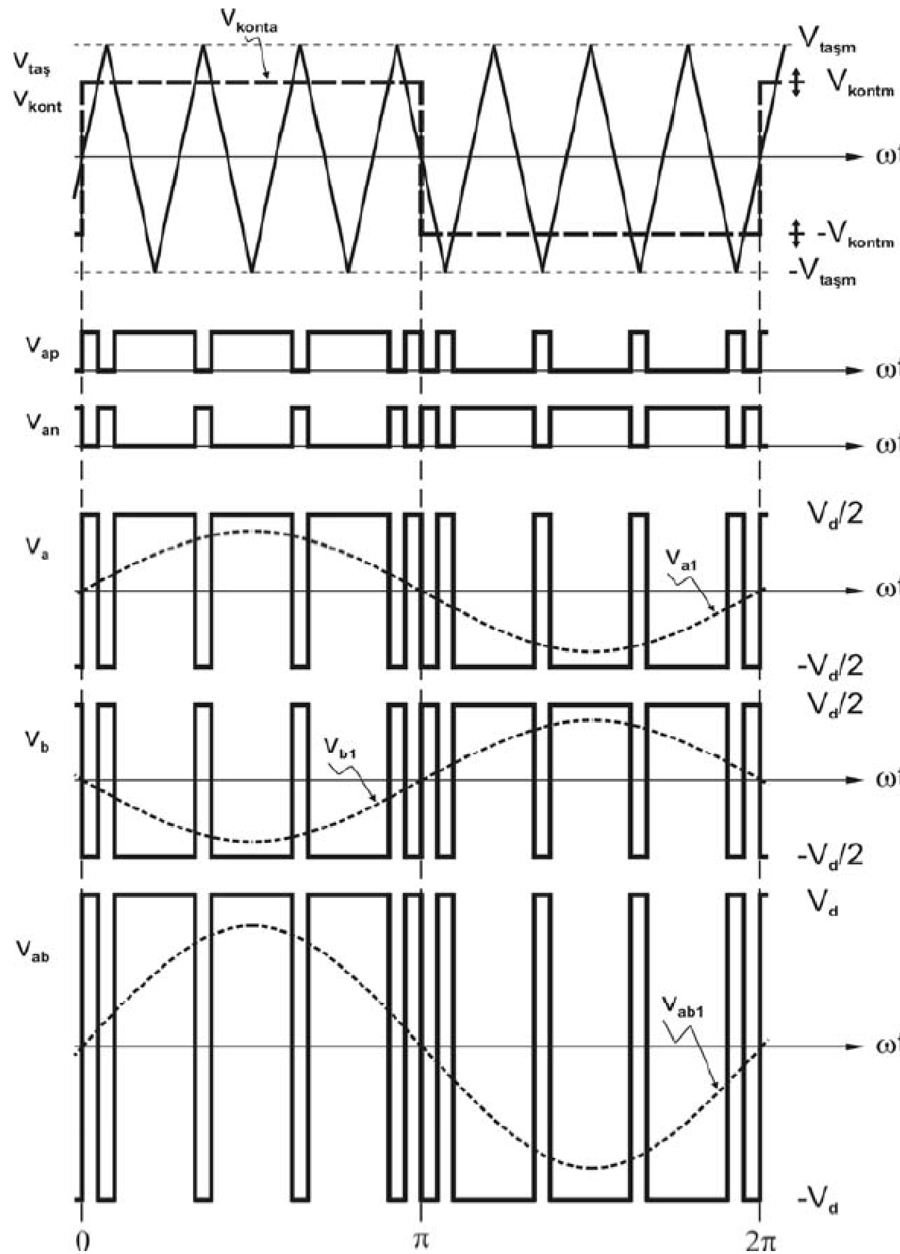
$$I_{DEF} = \frac{1}{t_{2\pi}}\int_0^{t_\gamma}(20-35{,}23\cdot e^{-200\cdot t})^2\, d(t) = \cdots\cdots \cong 3{,}077\ \text{A}$$

*şeklinde bulunur.*

# Single Phase PWM Controlled Inverter




## Bi-directional control

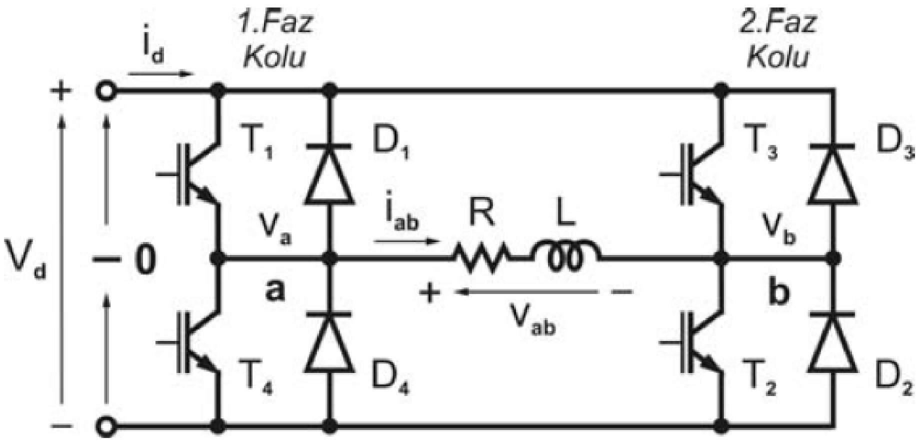



$$
v_{ab} = \begin{cases} V_d & v_{konta} > v_{taş}\ ,\ T_1 \text{ ve } T_2 \text{ iletimde} \\ -V_d & v_{konta} < v_{taş}\ ,\ T_3 \text{ ve } T_4 \text{ iletimde} \end{cases}
$$

$$
V_a = V_b = V_f = V_d / 2 = \text{Sabit}
$$

$$
V_{ab} = V_h = V_d = 2V_f = \text{Sabit}
$$

# Single Phase PWM Controlled Inverter




$$
v_{ab} = \begin{cases} V_d & \text{T}_1 \text{ ve } \text{T}_2 \\ 0 & \text{T}_1 \text{ ve } \text{T}_3 \text{ veya } \text{T}_2 \text{ ve } \text{T}_4 \text{ iletimde veya kesimde} \\ -V_d & \text{T}_3 \text{ ve } \text{T}_4 \end{cases}
$$

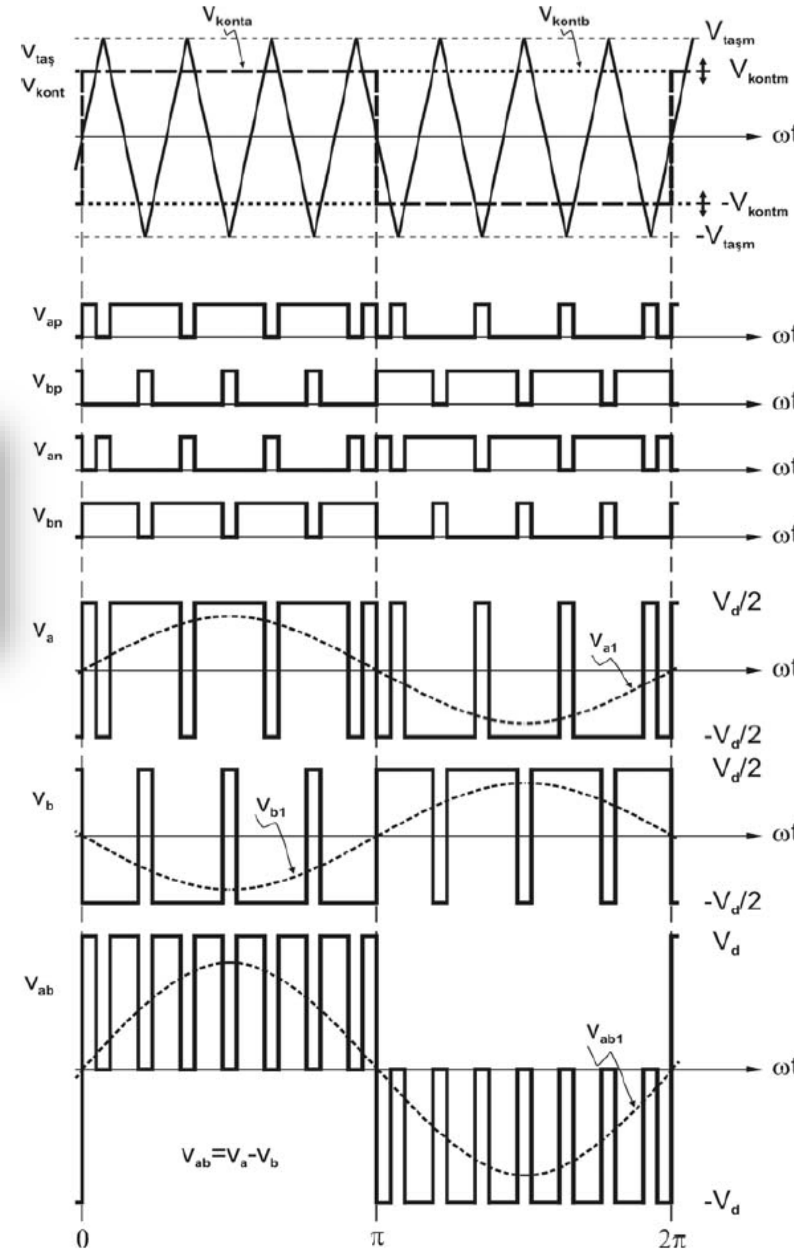
$$V_a = V_b = V_f = V_d / 2 = \text{Sabit}$$

$$V_{ab} = V_h = \sqrt{m_a} \cdot V_d = \sqrt{m_d} \cdot V_d = \sqrt{m_d} \cdot 2V_f$$

$$m_a = \frac{V_{kontm}}{V_{taşm}} = \frac{\beta}{\pi} = m_d$$

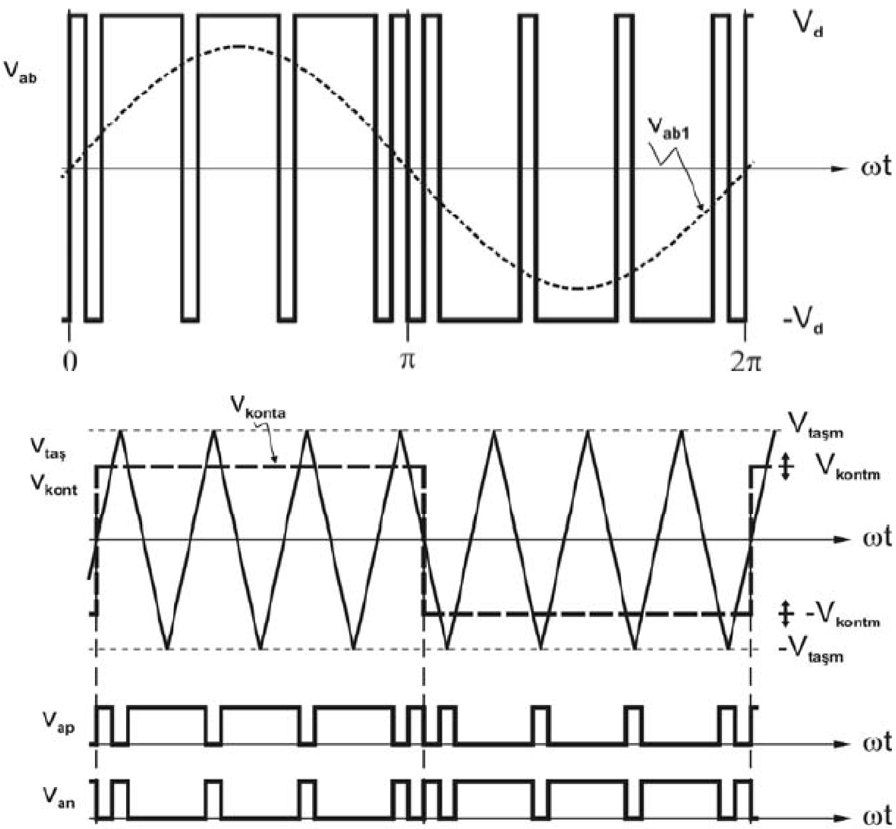
$$m_f = \frac{f_{taş}}{f_{kont}} = \frac{f_p}{f_1} = \frac{\omega_p}{\omega_1} = 2p$$

## Uni-directional control

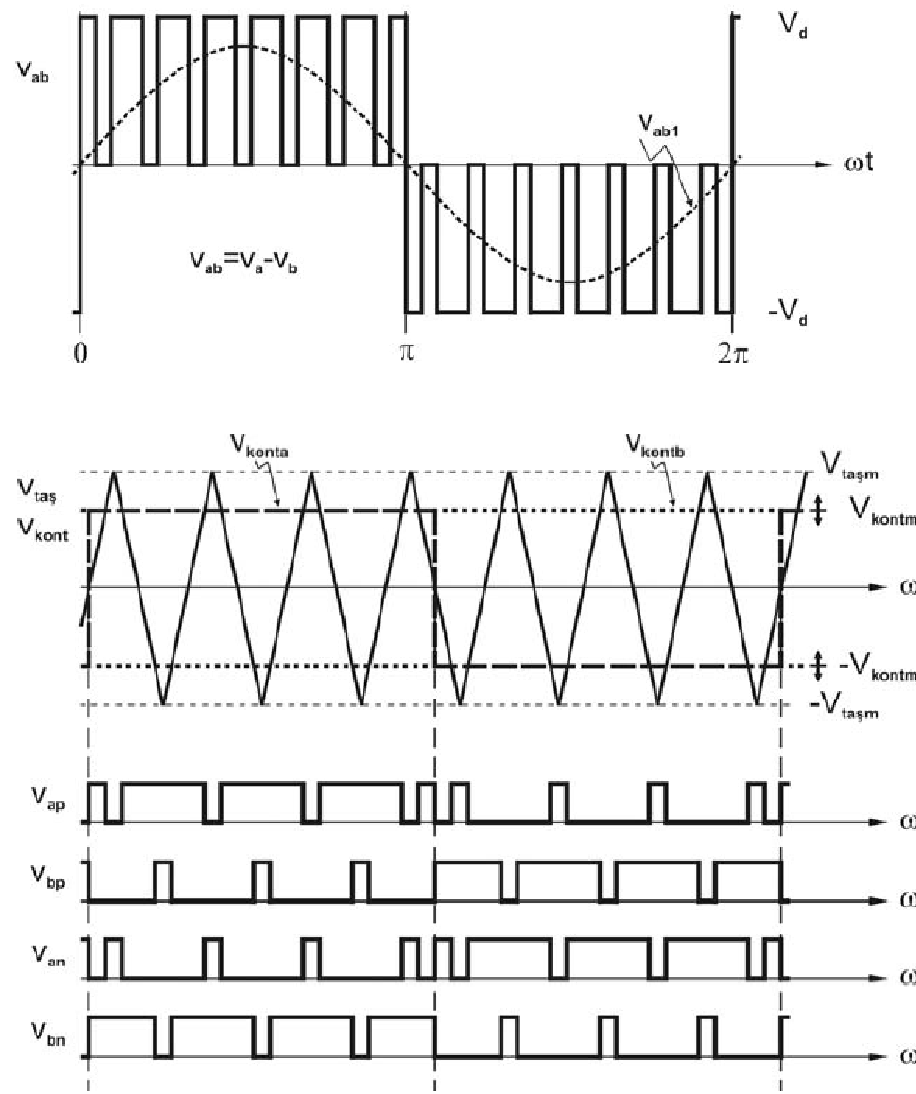


# Single Phase PWM Controlled Inverter

## Bi-directional control




## Uni-directional control




**Harmonic content of the output voltage form of uni-directional control is relatively lower than that of bi-directional one!!!**

# Single Phase PWM Controlled Inverter

**ÖRNEK 6.5**

*DC giriş gerilimi 200 V ve frekansı 50 Hz olan tek fazlı tam köprü türü bir inverterle, $m_f = 33$ olmak üzere iki ve tek yönlü PWM kontrolü uygulanarak, 10 Ω değerinde bir direnç beslenmektedir. Devre kayıplarını ihmal ederek,*

**a)** *Anahtarlama frekansı ile darbe sayısını bulunuz.*

**b)** *Yükün maksimum efektif gerilimi ve gücünü hesaplayınız.*

**c)** *$m_a$=1/4 için, yükün efektif gerilimi ile gücünü bulunuz.*

**ÇÖZÜM**

**a)** *(6.24) bağıntısı kullanılarak, anahtarlama frekansı ile darbe sayısı,*

$$m_f = \frac{f_{taş}}{f_{kont}} \Rightarrow 33 = \frac{f_{taş}}{50} \Rightarrow f_{taş} = 1650 \text{ Hz}$$

$$m_f = 2p \Rightarrow 33 = 2p \Rightarrow p = 16{,}5$$

*olarak bulunur.*

**b)** *İki yönlü kontrolde (6.29)'a göre daima ve tek yönlü kontrolde (6.34)'e göre $m_a$=1 için maksimum olan yükün efektif gerilimi ile gücü,*

$$V_h = V_d = 200 \text{ V}$$

$$P = V_h^2 / R = 200^2 / 10 = 4000 \text{ W} = 4 \text{ kW}$$

*şeklinde bulunur.*

**c)** *İki yönlü kontrolde (6.29)'a ve tek yönlü kontrolde (6.34)'e göre $m_a$=1/4 için yükün efektif gerilimi ile gücü, sırasıyla aşağıdaki gibi bulunur.*

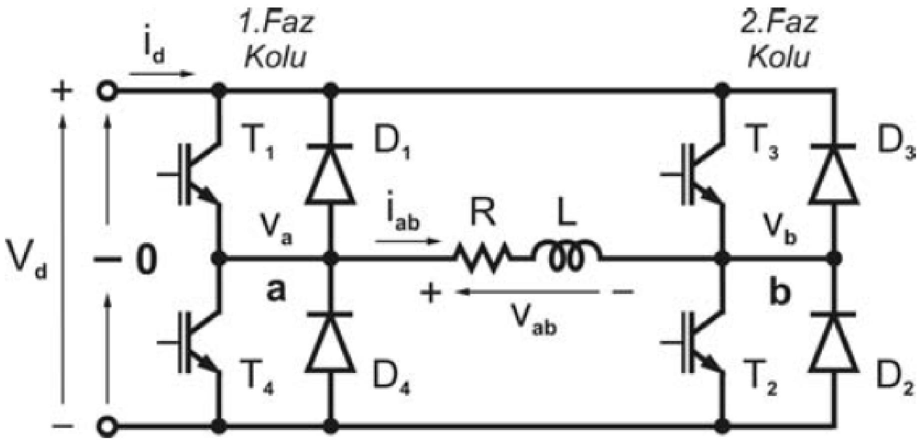
$$V_h = V_d = 200 \text{ V}$$

$$\Rightarrow P = V_h^2 / R = 200^2 / 10 = 4000 \text{ W} = 4 \text{ kW}$$

$$V_h = \sqrt{m_a} \cdot V_d = \sqrt{1/4} \cdot 200 = 100 \text{ V}$$

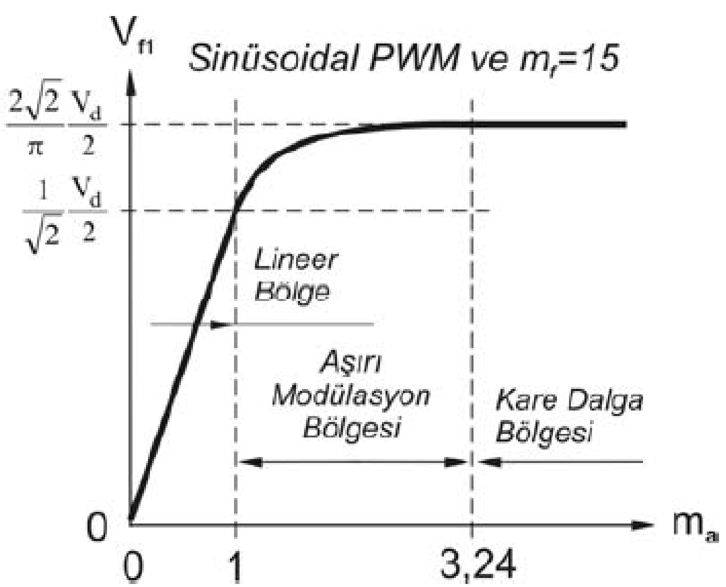
$$\Rightarrow P = 100^2 / 10 = 1000 \text{ W} = 1 \text{ kW}$$

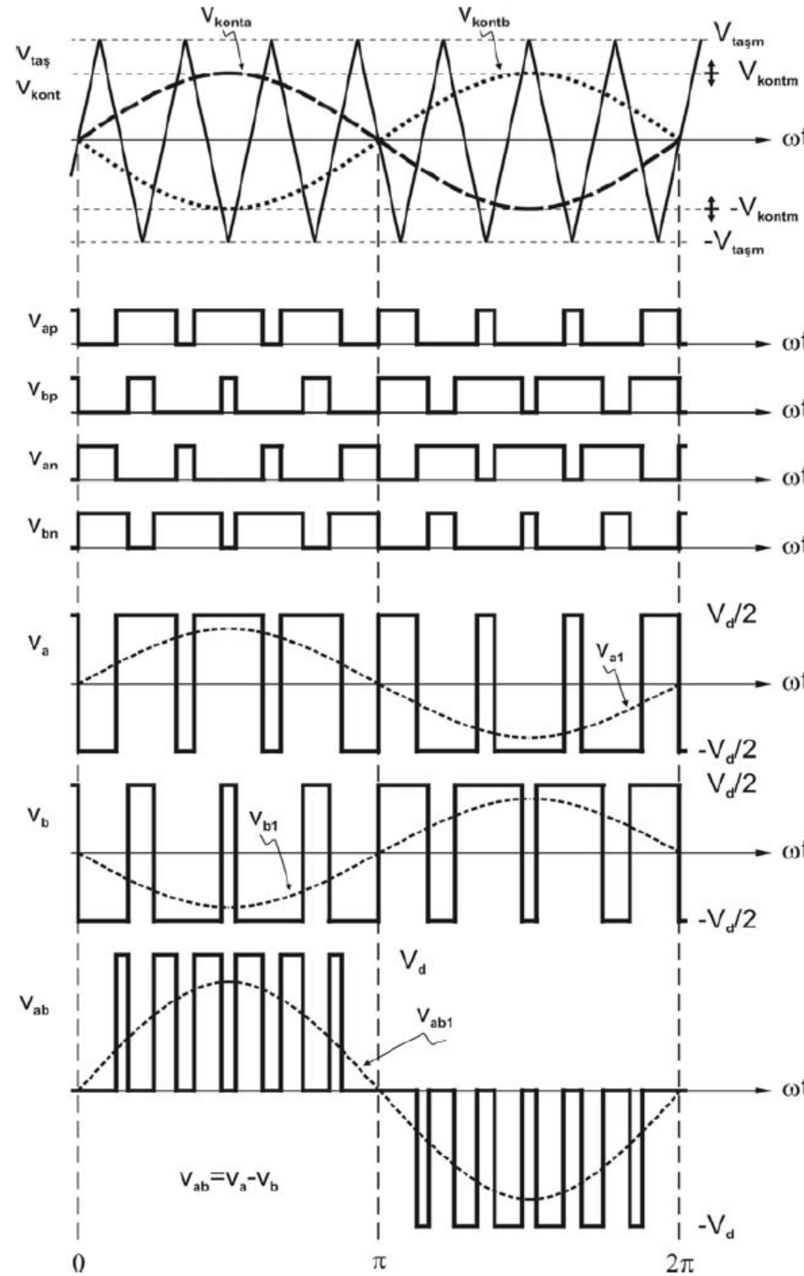
# Single Phase Sinusoidally Controlled Inverter



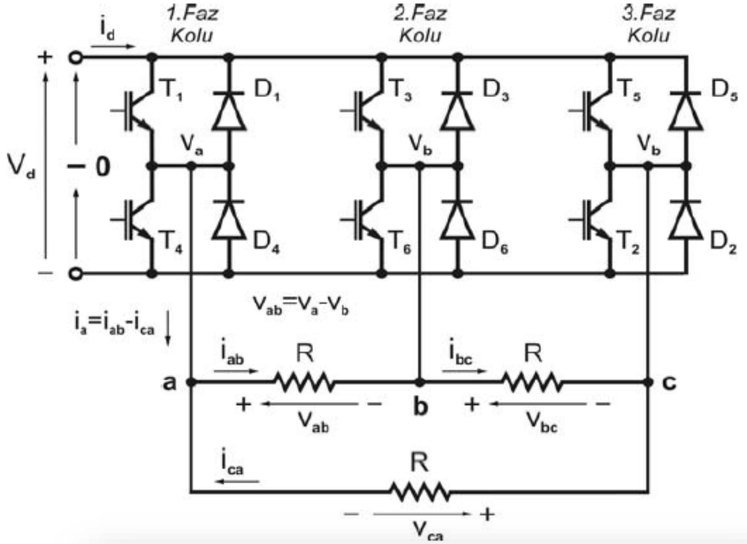

$$V_a = V_b = V_f = V_d / 2 = \text{Sabit}$$

$$V_{ab} = V_h = \sqrt{m_d} \cdot V_d = \sqrt{\frac{2}{\pi} m_a} \cdot V_d = \sqrt{\frac{2}{\pi} m_a} \cdot 2V_f$$



# Three Phase 120$^0$ Controlled Inverter




$$V_a = V_b = V_c = V_f = \sqrt{\frac{2}{3}} \cdot \frac{V_d}{2}$$

$$V_{ab} = V_{bc} = V_{ca} = V_h = \frac{1}{\sqrt{2}} \cdot V_d = \sqrt{3} V_f$$

$$V_{a1} = V_{b1} = V_{c1} = V_{f1} = \frac{\sqrt{6}}{\pi} \cdot \frac{V_d}{2}$$

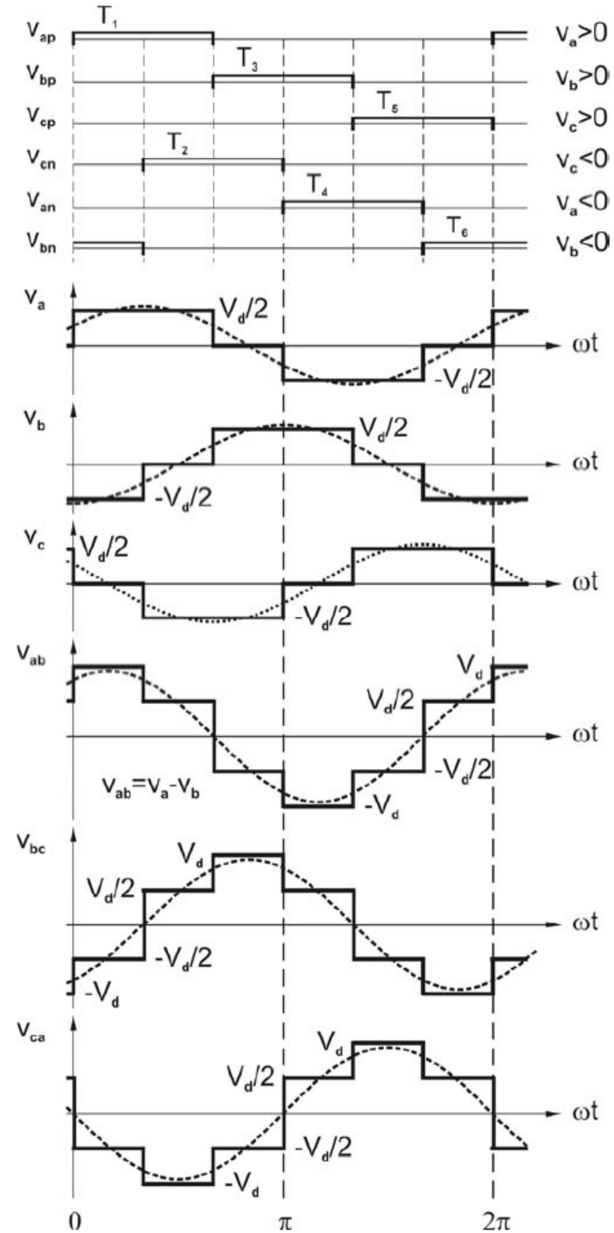
$$V_{ab1} = V_{bc1} = V_{ca1} = V_{h1} = \frac{3}{\sqrt{2}\pi} \cdot V_d = \sqrt{3} V_{f1}$$

$$v_{a1} = \sqrt{2} V_{f1} \sin(\omega t + \pi / 6)$$

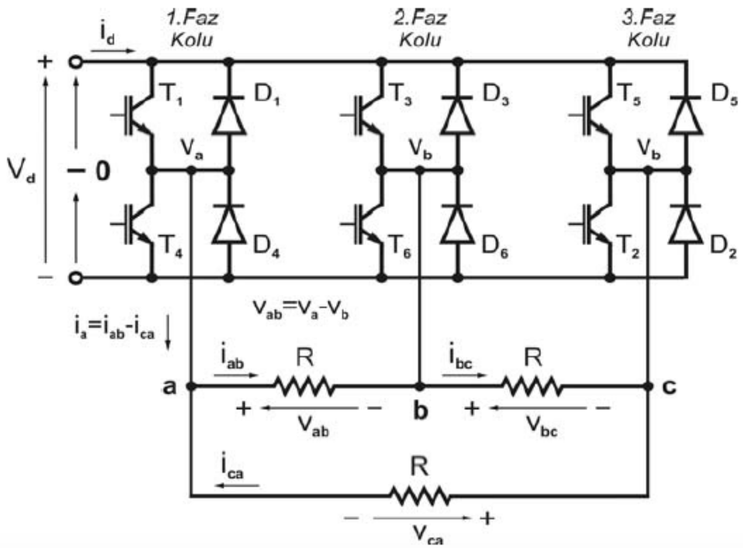
$$v_{b1} = \sqrt{2} V_{f1} \sin(\omega t - 3\pi / 6)$$

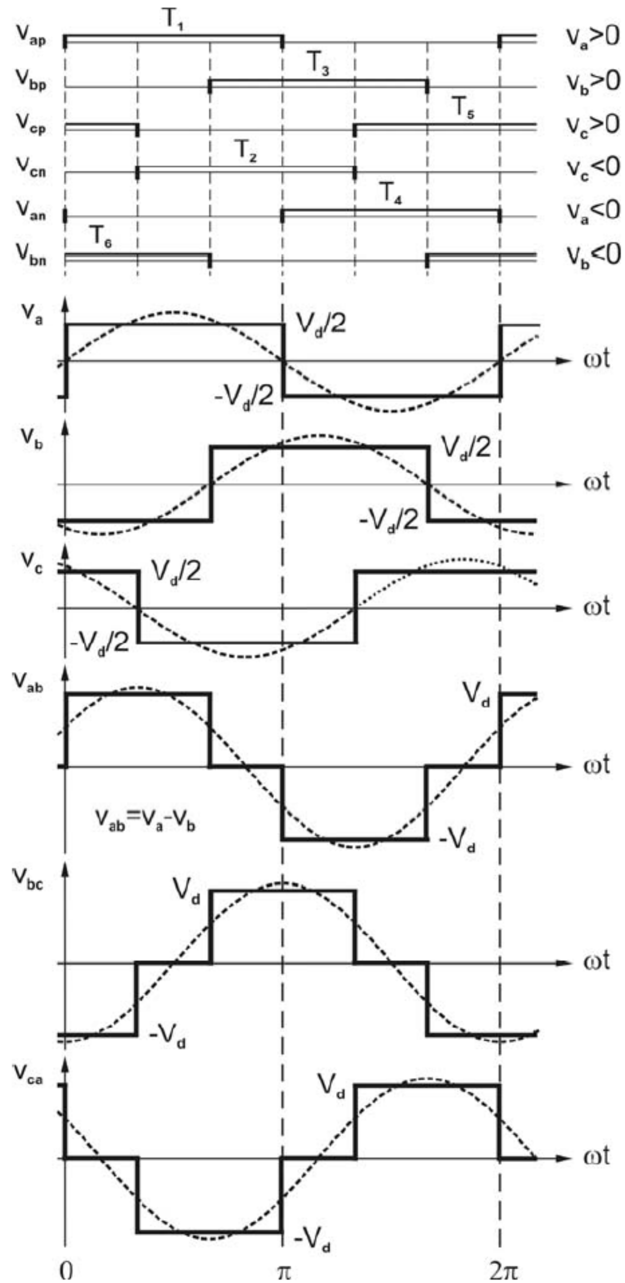
$$v_{c1} = \sqrt{2} V_{f1} \sin(\omega t - 7\pi / 6)$$

$$v_{ab1} = \sqrt{2} V_{h1} \sin(\omega t + \pi / 3)$$



# Three Phase 180⁰ Controlled Inverter



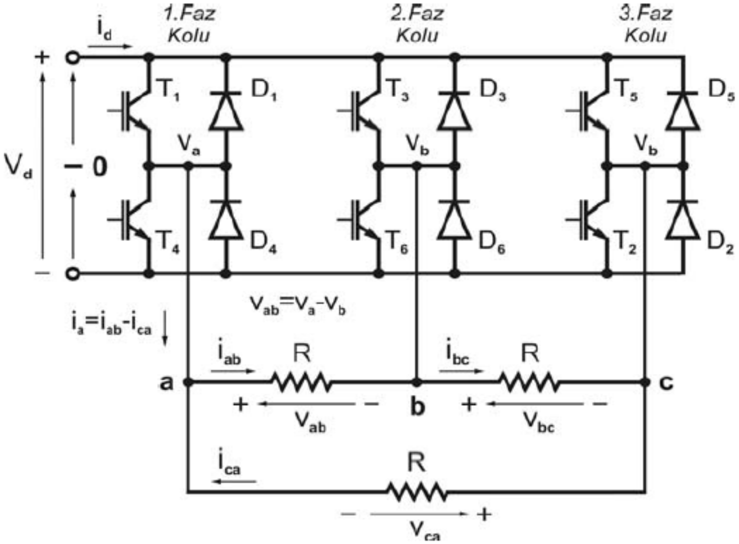

$$V_a = V_b = V_c = V_f = \frac{V_d}{2}$$

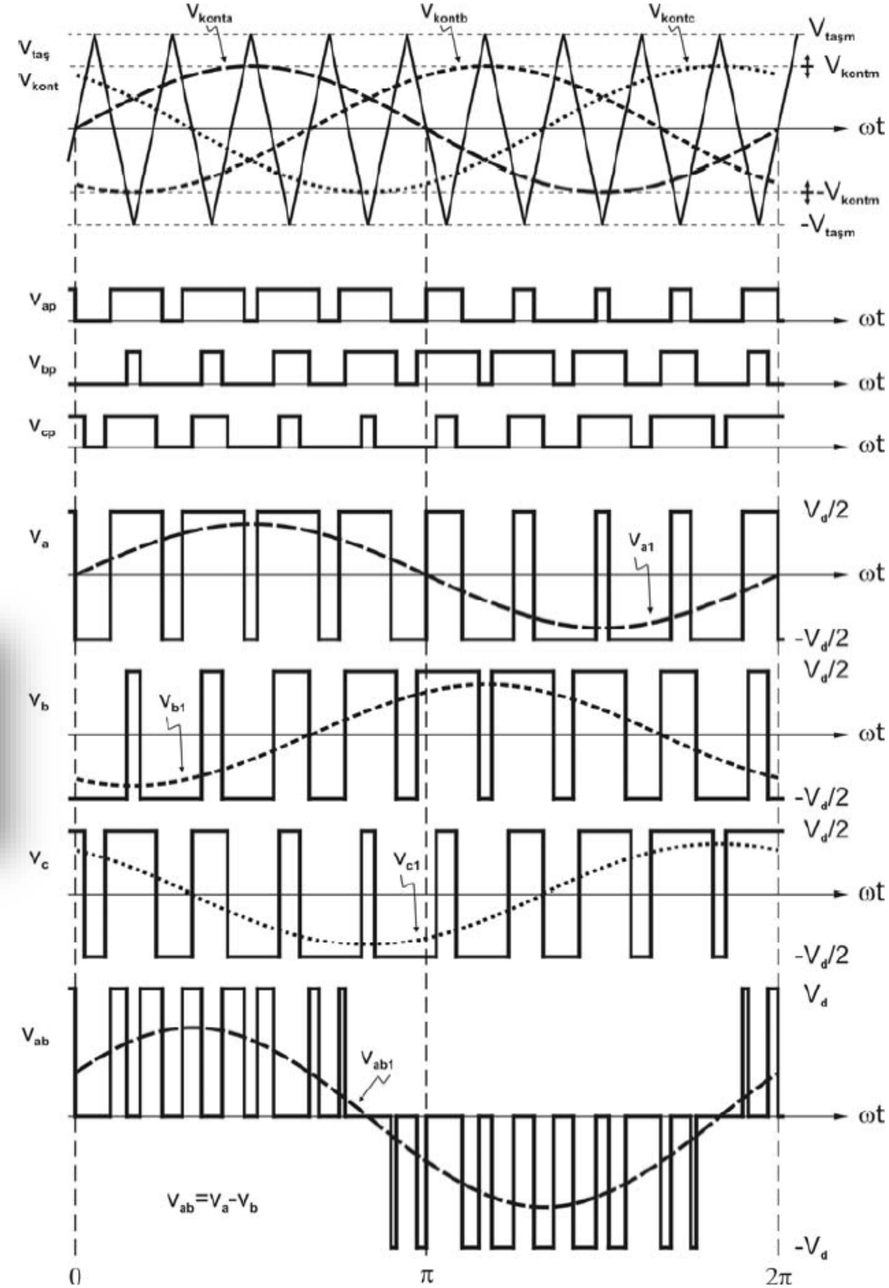
$$V_{ab} = V_{bc} = V_{ca} = V_h = \sqrt{\frac{2}{3}} \cdot V_d \neq \sqrt{3} V_f$$

$$V_{a1} = V_{b1} = V_{c1} = V_{f1} = \frac{2\sqrt{2}}{\pi} \cdot \frac{V_d}{2}$$

$$V_{ab1} = V_{bc1} = V_{ca1} = V_{h1} = \frac{\sqrt{6}}{\pi} \cdot V_d = \sqrt{3} V_{f1}$$

# Three Phase Sinusoidally Controlled Inverter

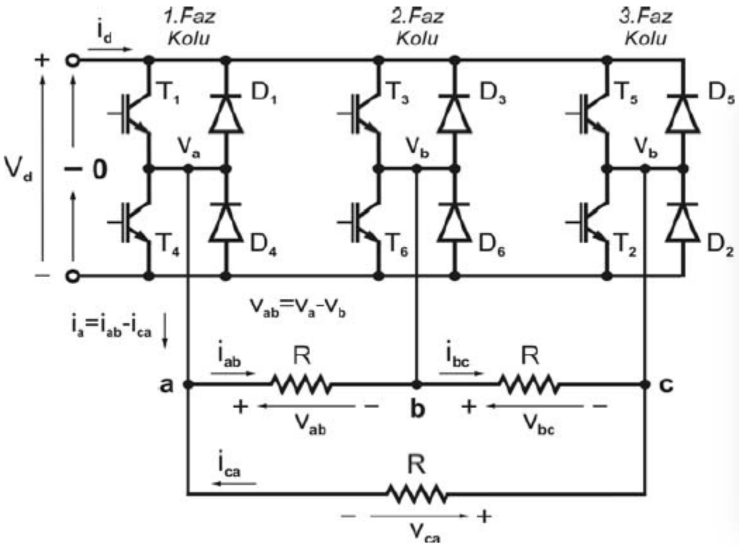


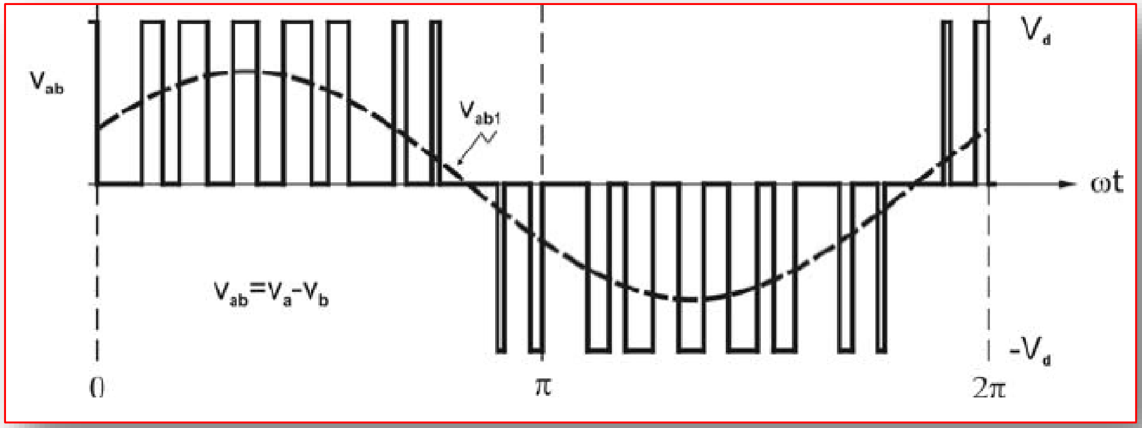

$$V_a = V_b = V_c = V_f = V_d / 2 = \text{Sabit}$$

$$V_{ab} = V_{bc} = V_{ca} = V_h = \sqrt{m_d} \cdot V_d = \sqrt{\frac{2}{\pi} m_a} \cdot \sqrt{3} V_f$$

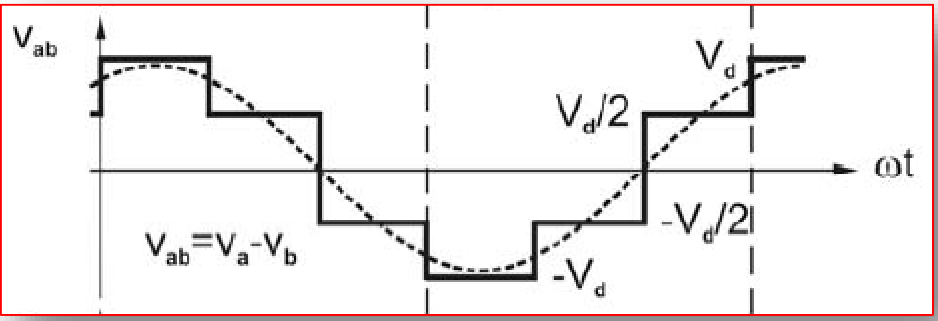
# Three Phase $180^0$ Controlled Inverter



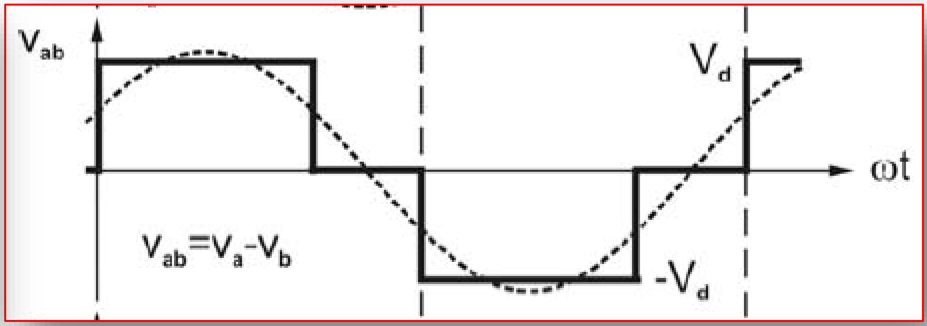

Sinusoidal




$120^0$




$180^0$



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