Mobile Robot Vehicles (2)

KOM4520 Fundamentals of Robotic Vision

Today's lecture

  - ...
  - Car-Like Mobile Robots, Following a trajectory application
- Differentially-Steered Mobile Robots

Car-Like Mobile Robots – Following a trajectory

- We can let the vehicle follow a trajectory that is a timed sequence of points on the xy-plane.
- A simple and effective algorithm for trajectory following is pure pursuit in which the goal point $(x_T(t),\ y_T(t))$ moves along the trajectory, in its simplest form at constant speed.
- The vehicle always heads toward the goal.

- This problem is very similar to the control problem of moving to a point, except here the point is moving.

- The robot maintains a distance d_T behind the pursuit point and we state the error as

$$e = \sqrt{(x_T - x)^2 + (y_T - y)^2} - d_T$$

Car-Like Mobile Robots – Following a trajectory

We can regulate it to zero by controlling the robot's velocity using a proportional-integral (PI) controller

$$v_T = K_v e + K_i \int e dt$$

The integral term is added here to provide a non zero velocity when the instant error is zero.

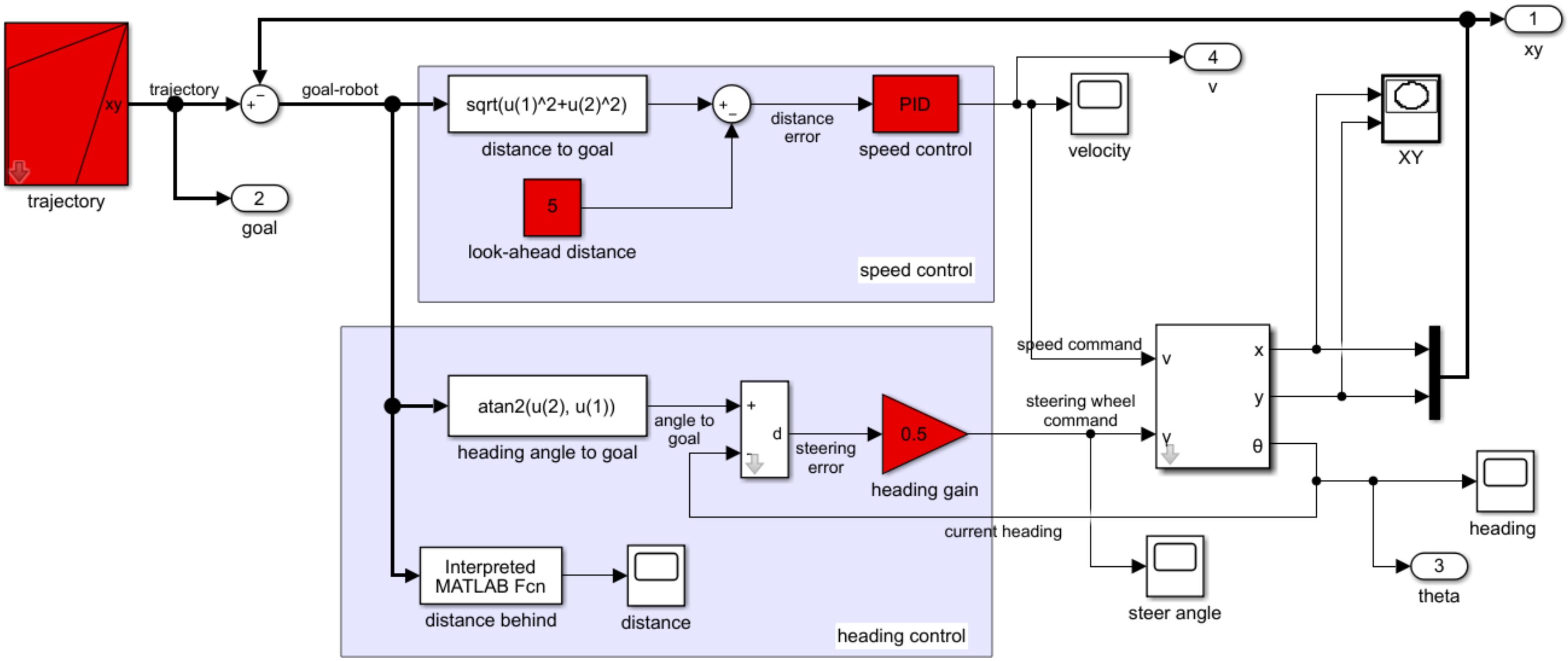
The second controller will be for steering

$$\theta_T = \tan^{-1} \frac{y_T - y}{x_T - x}$$

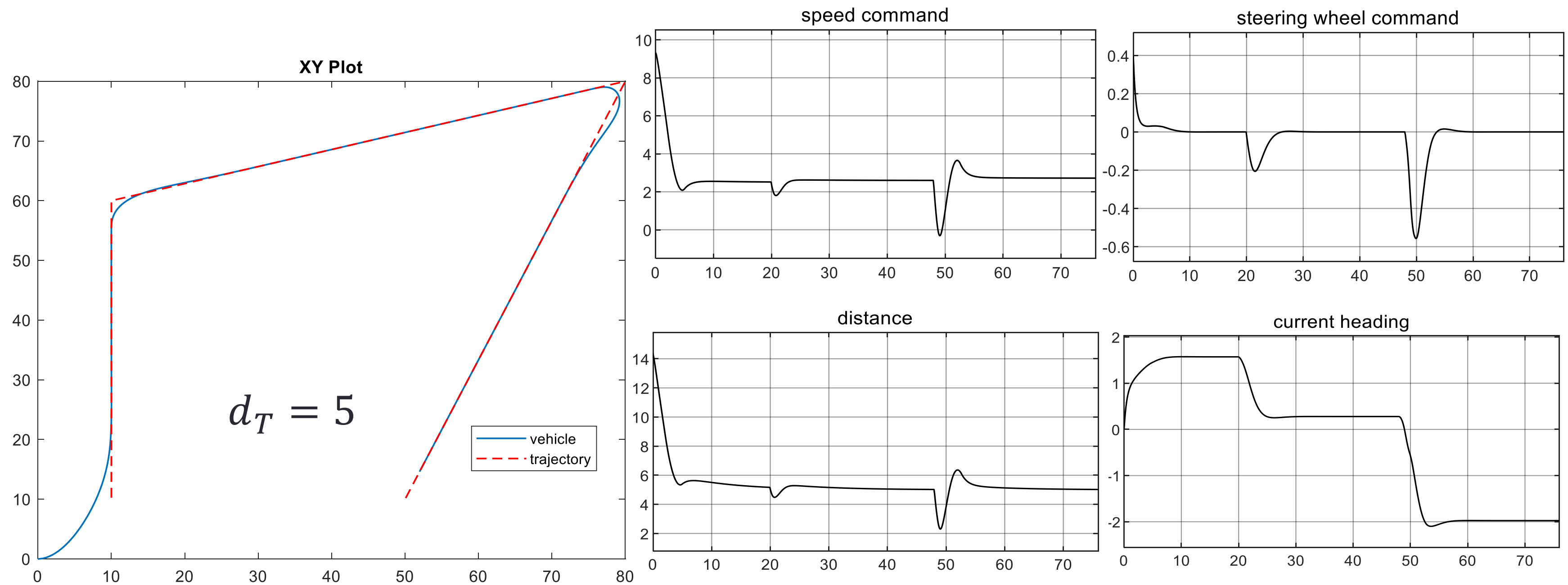
And we can use a simple proportional controller to turn the steering wheel so the vehicle can drive the robot to the target.

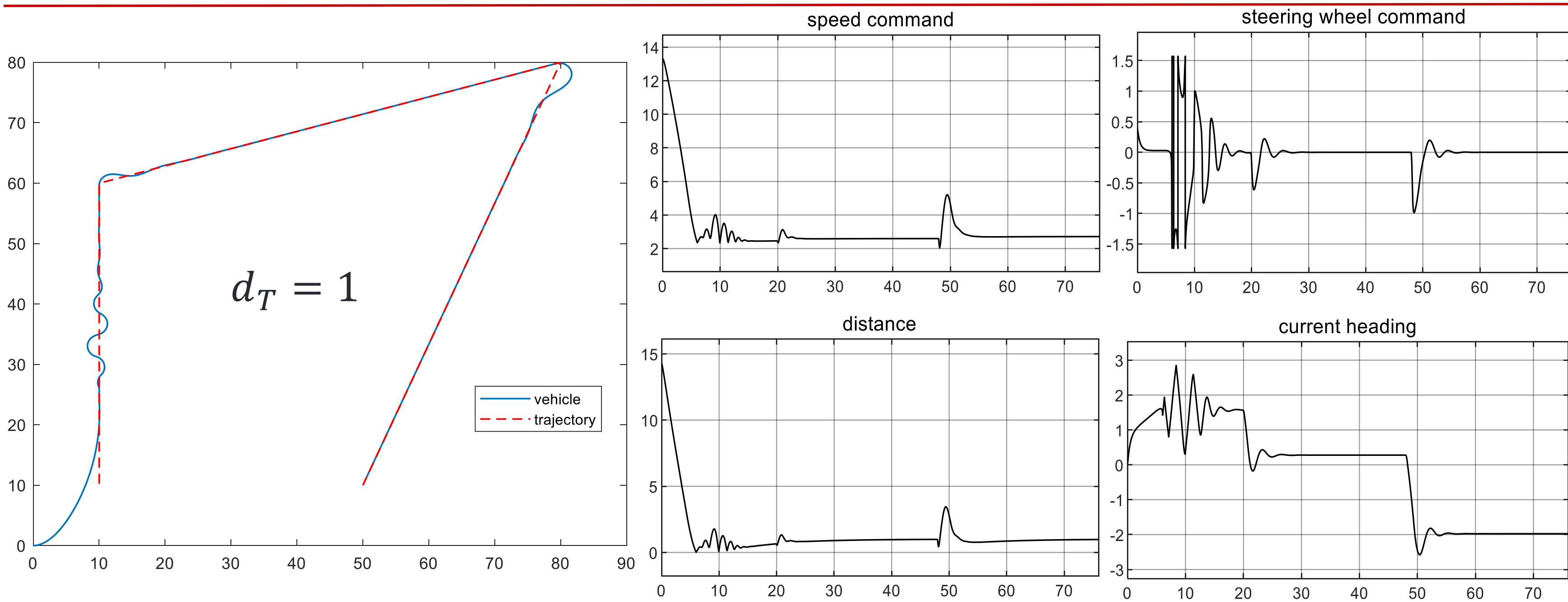
$$\gamma = K_h(\theta_T - \theta) \quad K_h > 0$$

Car-Like Mobile Robots – Following a trajectory



sl_pursuit


XY Plot
$d_T = 5$
vehicle
trajectory
speed command
steering wheel command
distance
current heading


$d_T = 1$
vehicle
trajectory
speed command
steering wheel command
distance
current heading

Differentially-Steered Vehicle

- Having steerable wheels as in a car-like vehicle is mechanically complex. Differential steering does away with this and steers by independently controlling the speed of the wheels on each side of the vehicle – if the speeds are not equal the vehicle will turn.
- Very simple differential steer robots have two driven wheels and a front and back castor to provide stability.
- Larger differential steer vehicles employ a pair of wheels on each side, with each pair sharing a drive motor via some mechanical transmission.

- Very large differential steer vehicles such as bulldozers and tanks sometimes employ caterpillar tracks instead of wheels.
- The vehicle's velocity is by definition v in the vehicle's *x*-direction, and zero in the *y*-direction since the wheels cannot slip sideways. In the vehicle frame $\{B\}$ this is

$$^{B}\boldsymbol{v} = (v, 0)$$

- The pose of the vehicle is represented by its body coordinate frame $\{B\}$ shown in with its x-axis in the vehicle's forward direction and its origin at the center of four wheels. The *configuration* of the vehicle is represented by the generalized coordinates

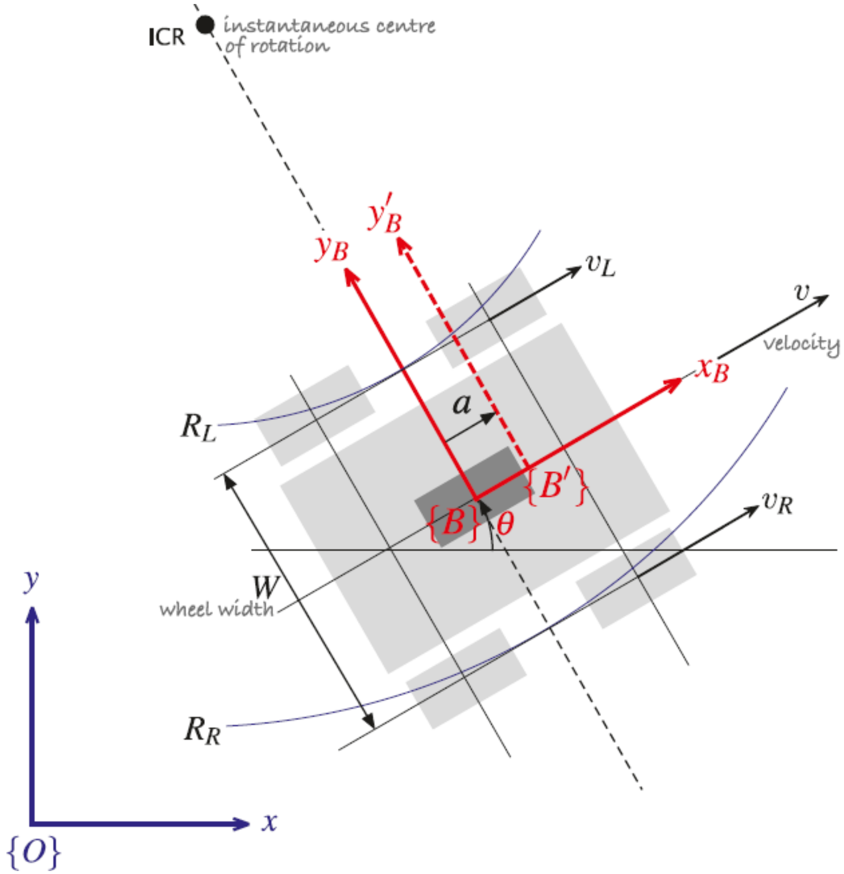
$$q \ = \ (x, y, \theta\ ) \in \ \mathcal{C}\ where\ \mathcal{C} \subset \ R^2 \times \ S^1.$$

- The vehicle follows a curved path centered on the Instantaneous Center of Rotation Instantaneous Center of Rotation (ICR).

Differentially-Steered Vehicle

- Differential drive robot is shown in *light grey*.
- The vehicle's body coordinate frame is shown in *red*
- The world coordinate frame in *blue*.

- The vehicle follows a path around the (ICR) and the distance from the ICR to the left and right wheels is R_L and R_R respectively.
- The left-hand wheels move at a speed of v_L along an arc with a radius of R_L while the right-hand wheels move at a speed of v_R along an arc with a radius of R_R.
- The angular velocity of {*B*} is



Differentially-Steered Vehicle

$$\dot{\theta} = \frac{v_L}{R_L} = \frac{v_R}{R_R}$$

since $R_R = R_L + W$

$$\dot{\theta} = \frac{v_R - v_L}{W}$$

Angular velocity of the vehicle depends on differential velocity and wheel separation

The velocity in the world coordinates

$$\dot{x} = \frac{v_R + v_L}{2} cos\theta$$

$$\dot{y} = \frac{v_R + v_L}{2} sin\theta$$

$$\dot{\theta} = \frac{v_R - v_L}{W}$$

- For a desired speed v and turn rate $\dot{\theta}$ we can solve v_R and v_L
- This model is named **unicycle model**.

Differentially-Steered Vehicle

If we move the vehicle's reference frame to $\{B'\}$ and ignore orientation we can rewrite the in matrix form as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} cos\theta & -asin\theta \\ sin\theta & acos\theta \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

and if $a \neq 0$ this can be inverted as

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -\frac{1}{a}sin\theta & \frac{1}{a}cos\theta \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

to give the required forward speed and turn rate to achieve an arbitrary velocity $(\dot{x}, \dot{y})$ for the origin of frame $\{B'\}$.

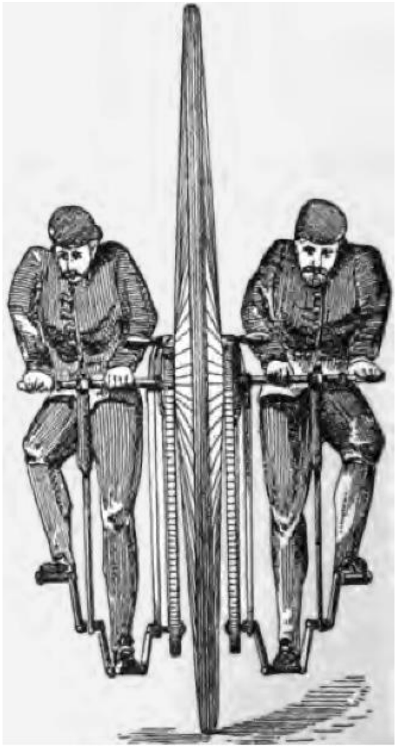
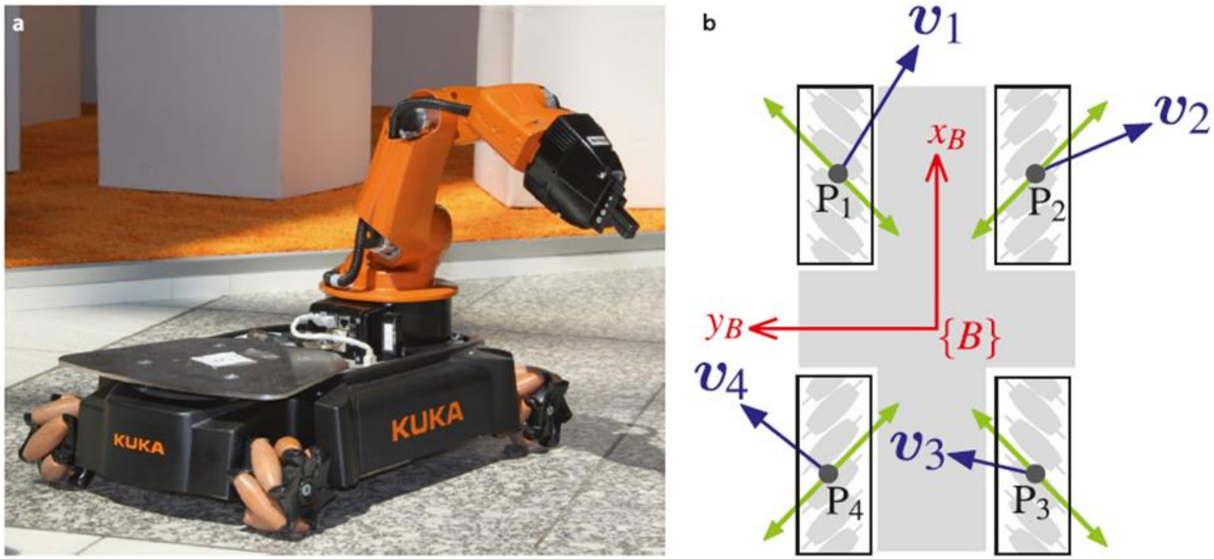
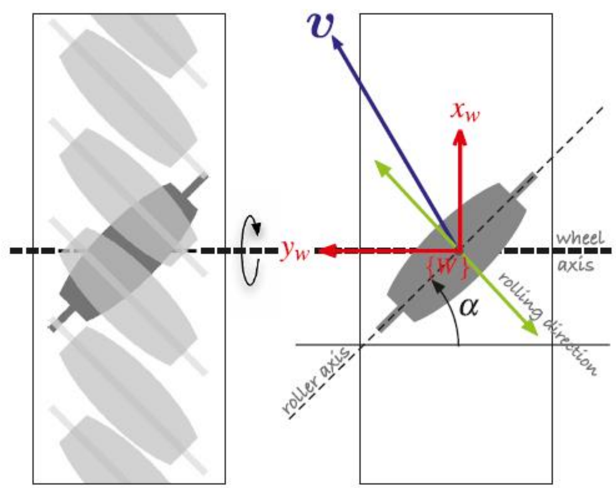


FIG 171.

Omni Directional Vehicle



Kuka youBot, which has has four mecanum wheels (image courtesy youBot Store); **b** schematic of a vehicle with four mecanum wheels in the youBot configuration



Schematic of a mecanum wheel in plan view. The *light rollers* are on top of the wheel, the *dark roller* is in contact with the ground. The *green arrow* indicates the rolling direction