Mobile Robot Vehicles

KOM4520 Fundamentals of Robotic Vision

Today's lecture

- Wheeled Mobile Robots
- Kinematic model of Car-Like Mobile Robots (Bicycle Model)
  - Car-Like Mobile Robots, Changing lane application
  - Car-Like Mobile Robots, Moving to a point application
  - Car-Like Mobile Robots, Following a line application
  - Car-Like Mobile Robots, Following a trajectory application
- Differentially-Steered Mobile Robots

Wheeled Mobile Robots

- The effectiveness of car like vehicles and our familiarity with them, makes them the most natural choice for robot platforms that move on the ground.
- In daily life we can notice that vehicles move with a limited capacity.

A few examples:

- It is not possible to drive sideways

  But with some practice we can learn to follow a path that results in the vehicle being to one side of its initial position : parallel parking.
- A car cannot rotate on the spot

  But we can follow a path that results in the vehicle being at the same position but rotated by 180° – a three-point turn.

- The necessity to perform such maneuvers is the hall mark of a system that is nonholonomic.

- Despite these minor limitations the car is the simplest and most effective means of moving in a planar world that we have yet found.

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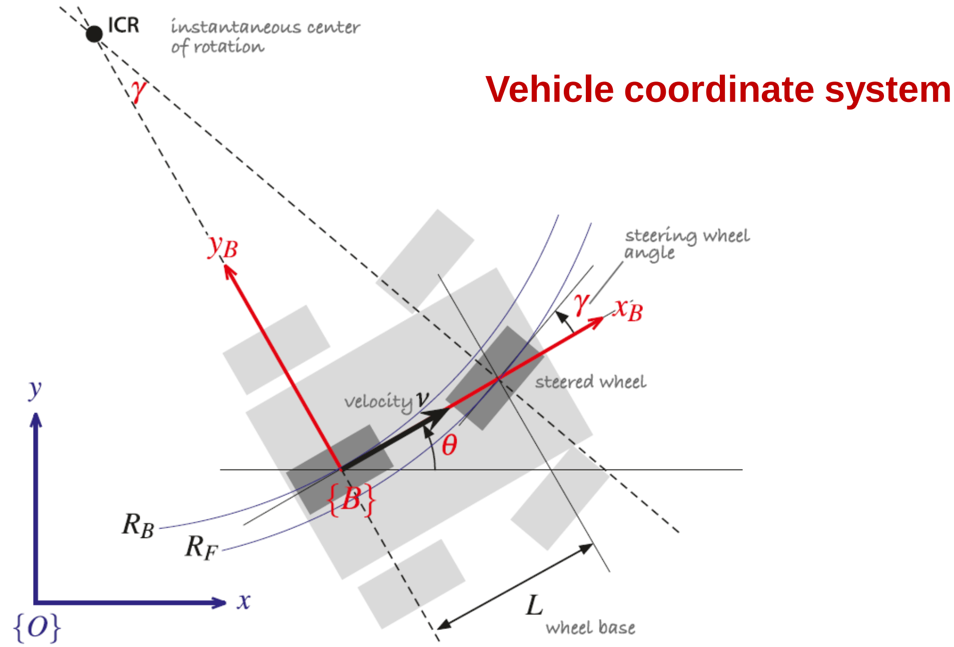
1. The car's motion model and the challenges it raises for control will be discussed.
2. Differentially-steered vehicles will be discussed which are mechanically simpler than cars and do not have steered wheels (A common configuration for small mobile robots

Car-Like Mobile Robots

- Cars with steerable wheels
  1. The model for a car-like vehicle
  2. The controllers that can
     - drive the car to a point,
     - along a line,
     - follow an arbitrary trajectory,
     - drive **to** a specific pose.



Bicycle model of a car.

- The car : *light grey*,
- The bicycle approximation: *dark grey*.
- The vehicle's body frame: *red*
- The world coordinate frame: *blue*.
- The steering wheel angle : γ
- The velocity of the back wheel, in the *x*-direction, is $\boldsymbol{v}$.

- The two wheel axes are extended as dashed lines and intersect at the Instantaneous Center of Rotation (ICR)
- The distance from the ICR to the back and front wheels is $\mathrm{R_B}$ and $\mathrm{R_F}$ respectively.
- L is the wheel base.

Car-Like Mobile Robots

- The coordinate system that we will use, and a common one for vehicles of all sorts is that the *x*-axis is forward (longitudinal motion), the *y*-axis is to the left side (lateral motion) which implies that the *z*-axis is upward.

- We assume that the velocity of each wheel is in the plane of the wheel, and that the wheel rolls without slipping sideways

$$^{B}\boldsymbol{v} = (v, 0)$$

- The pose of the vehicle is represented by its body coordinate frame $\{B\}$ shown in with its x-axis in the vehicle's forward direction and its origin at the center of the rear axle. The *configuration* of the vehicle is represented by the generalized coordinates

$$q = (x, y, \theta) \in \mathcal{C} \; where \; \mathcal{C} \subset R^2 \times S^1.$$

- The dashed lines show the direction along which the wheels cannot move, the lines of no motion, and these intersect at a point known as the Instantaneous Center of Rotation (ICR).

Car-Like Mobile Robots

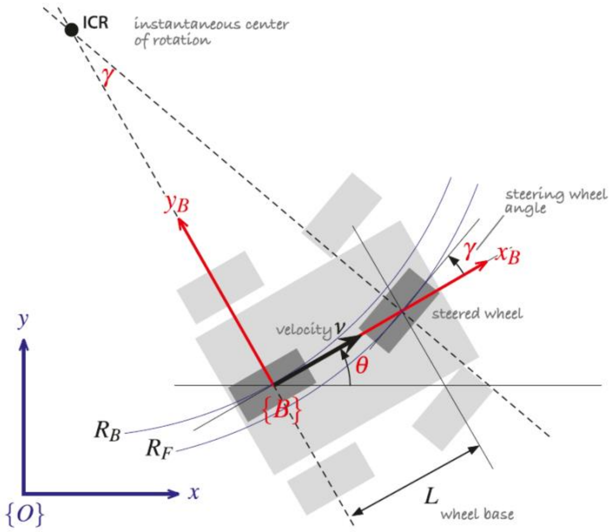
- The dashed lines show the direction along which the wheels cannot move, the lines of no motion, and these intersect at a point known as the Instantaneous Center of Rotation (ICR).
- The reference point of the vehicle thus follows a circular path and its angular velocity is

$$\dot{\theta} = \frac{v}{R_B}$$

and

$$\tan\gamma = L / R_B$$

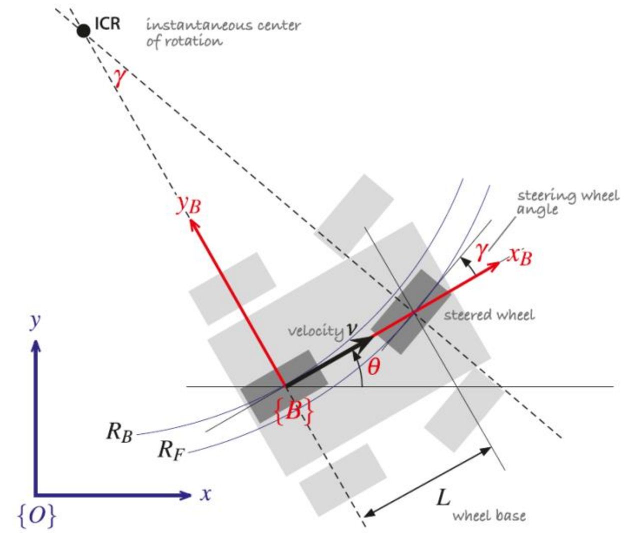
- As we would expect the turning circle increases with vehicle length. The steering angle γ is typically limited mechanically and its maximum value dictates the minimum value of R_B. For a fixed steering wheel angle the car moves along a circular arc.
- Thus, the curves on roads are circular arcs or clothoids which makes life easier for the driver since constant or smoothly varying steering wheel angle allow the car to follow the road.
- $R_F > R_B$ which means the front wheel must follow a longer path and therefore rotate more quickly than the back wheel.



Car-Like Mobile Robots

The velocity in the world coordinates

$$\boxed{\begin{aligned} \dot{x} &= v\, cos\theta \\ \dot{y} &= v\, sin\theta \\ \\ \dot{\theta} &= \frac{v}{L}\tan\gamma \end{aligned}}$$



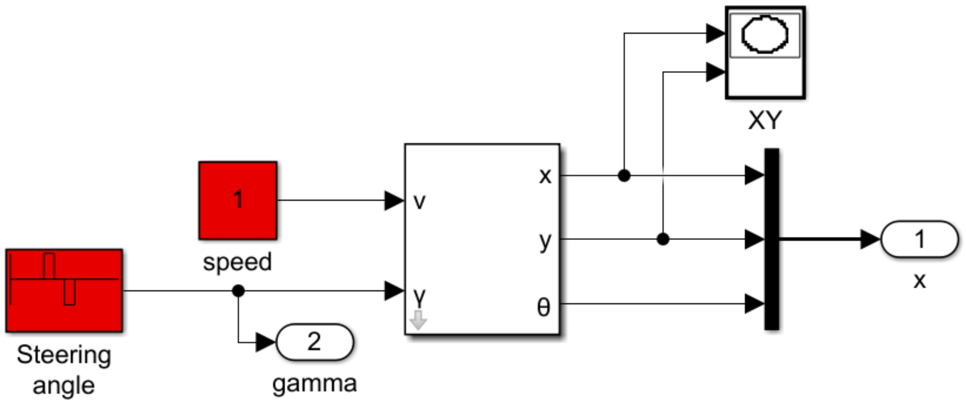
- This is a **kinematic model**; it only describes the velocities of the vehicle but not the forces or torques that cause the velocity

- The rate of change of heading $\dot{\theta}$ is referred to as turn rate, heading rate or yaw rate and can be measured by a gyroscope.
- It can also be deduced from the angular velocity of the non-driven wheels on the left- and right-hand sides of the vehicle which follow arcs of different radius, and therefore rotate at different speeds.

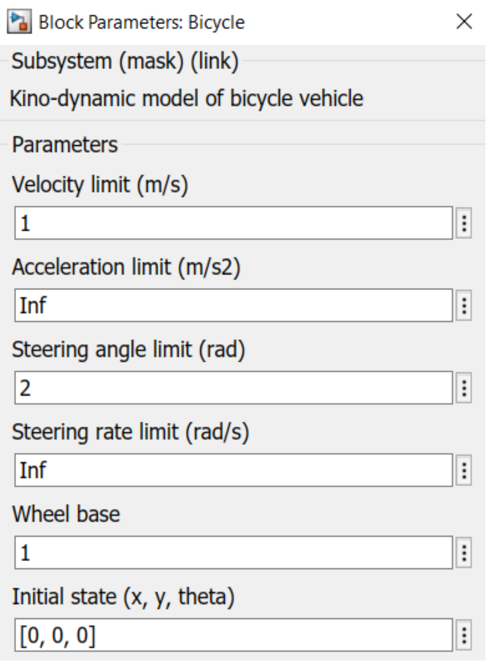
- When v equals to zero : $\dot{\theta}$ is zero.
- In the world coordinate frame we can write an expression for velocity in the vehicle's *y*-direction

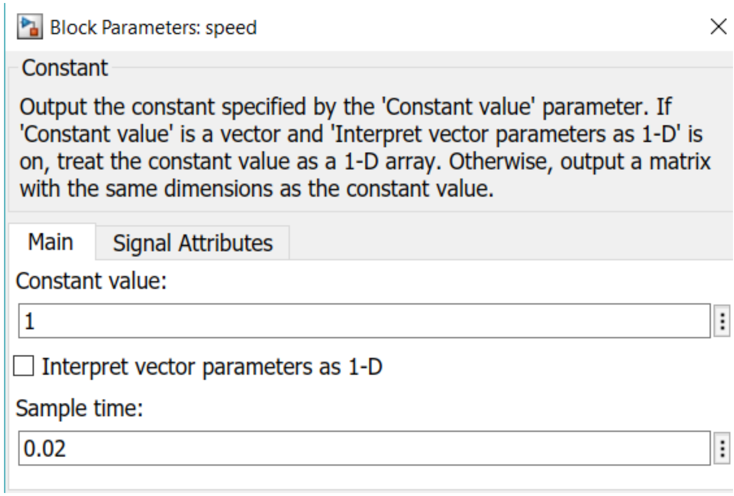
$$\dot{x}\, sin\theta - \dot{y}\, cos\theta = 0$$

Car-Like Mobile Robots – Changing Lane

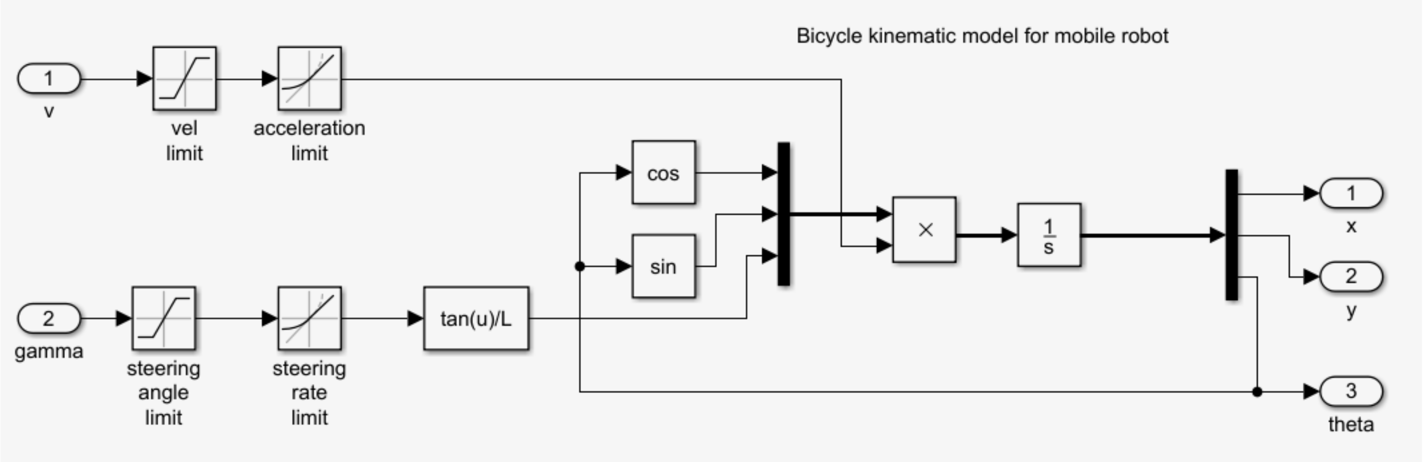


sl_lanechange

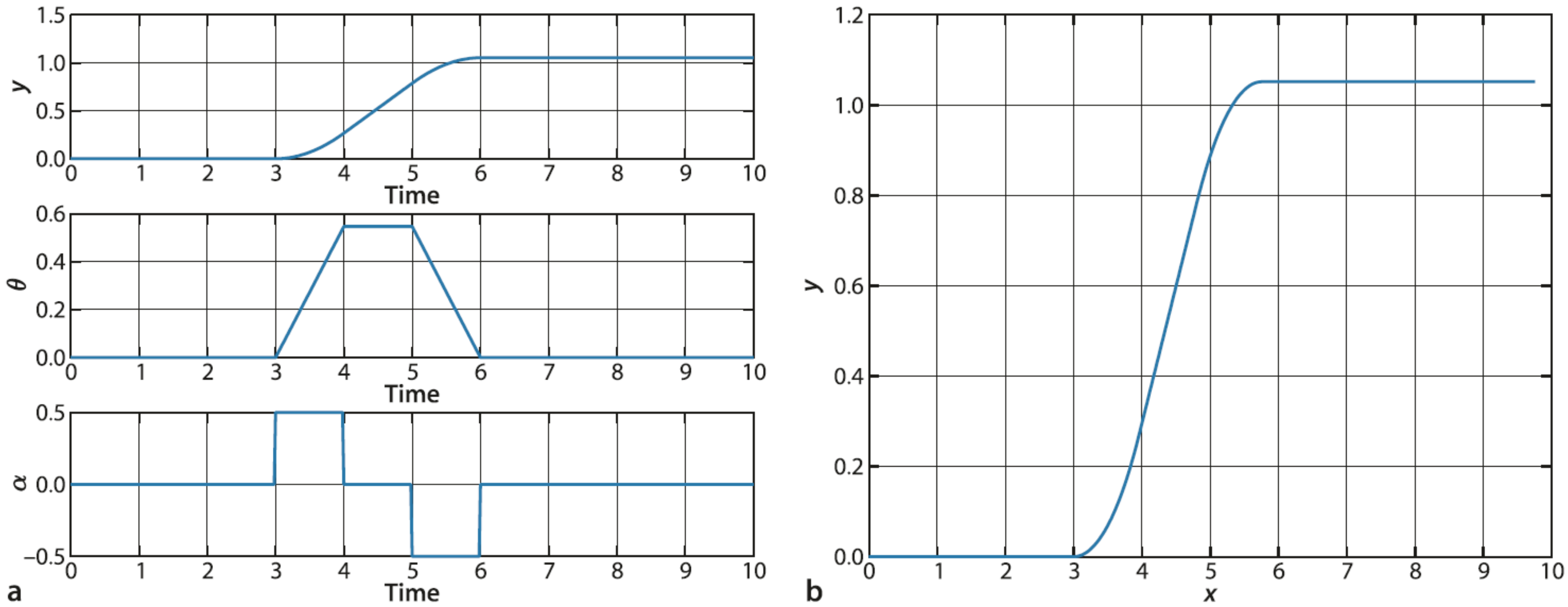




Car-Like Mobile Robots – Changing Lane



Car-Like Mobile Robots – Changing Lane



Car-Like Mobile Robots – Moving to a Point

- Moving to a target point (x_T, y_T) is required.
- We can control the robot's velocity to be proportional to its distance from the target

$$v_T = K_v\sqrt{(x_T - x)^2 + (y_T - y)^2}, \qquad K_v > 0$$

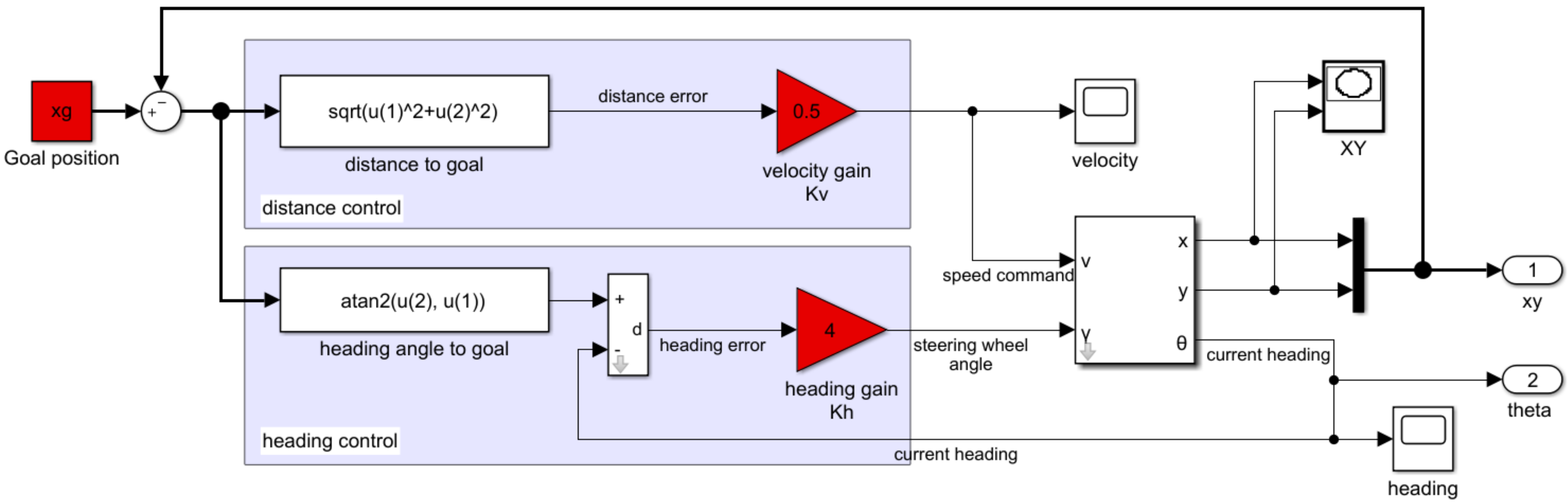
And we can steer toward the goal (vehicle relative angle θ_T) which is going to be

$$\theta_T = \tan^{-1}\frac{y_T - y}{x_T - x}$$

$$\gamma = K_h(\theta_T \ominus \theta), \qquad K_h > 0$$

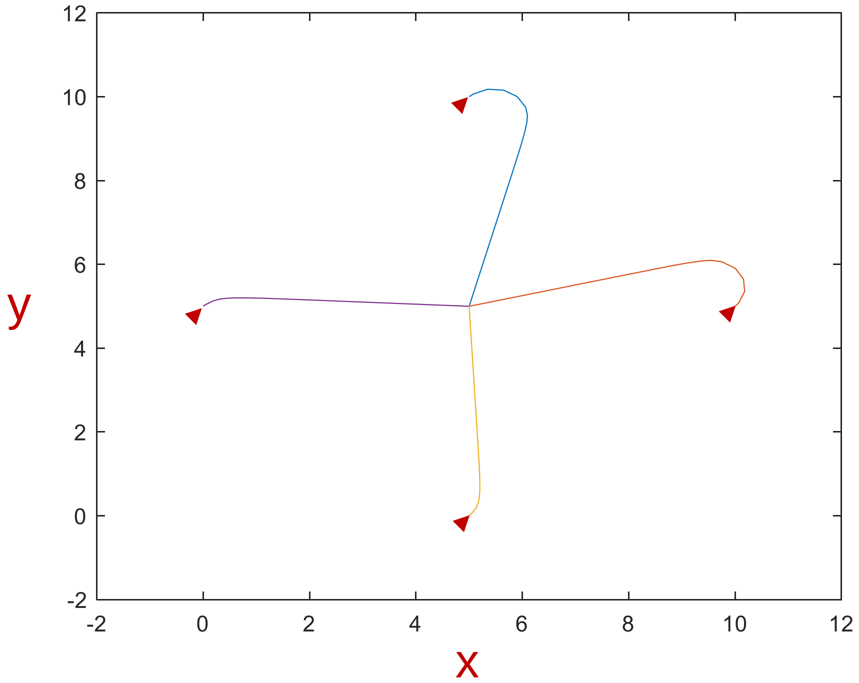
$\ominus$: the result wrapped on the interval [-pi,pi].

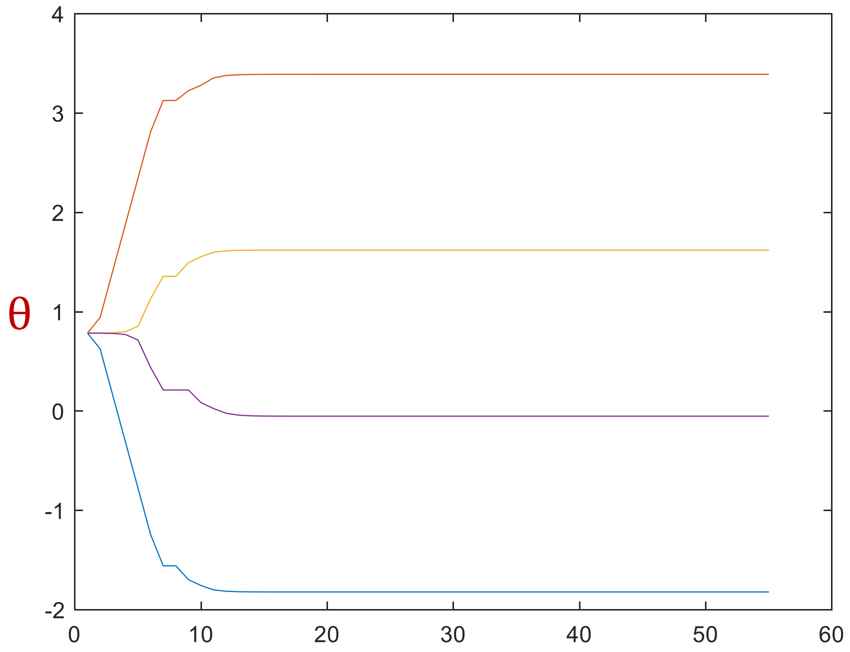
Car-Like Mobile Robots – Moving to a Point



sl_drivepoint

Car-Like Mobile Robots – Moving to a Point



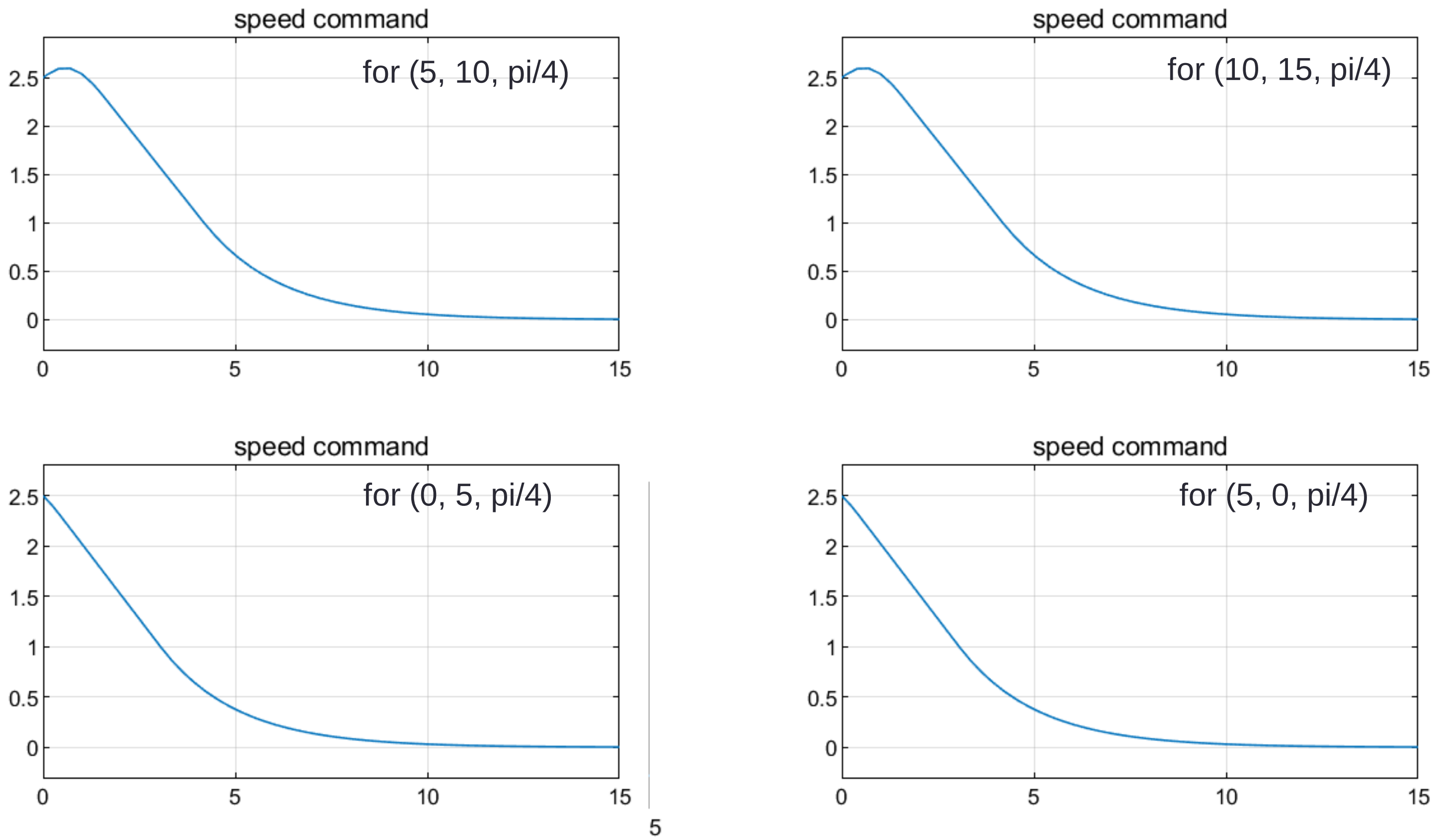


- Simulation results for sl_drivepoint for different initial poses:

(5, 10, pi/4) (10, 5, pi/4) (5, 0, pi/4) (0, 5, pi/4) The goal is to reach (5, 5)

- In each case the vehicle has moved forward and turned onto a path toward the goal point. The fi nal part of each path is a straight line and the fi nal orientation therefore depends on the starting point.

Car-Like Mobile Robots – Moving to a Point



Car-Like Mobile Robots – Following a line

- Following a line with equation $ax + by + c = 0$.
- This requires two controllers to adjust steering.

$$\alpha_d = -K_d d, \qquad K_d > 0$$

Turns the robot toward the line to minimize the robot's normal distance from the line

$$d = \frac{(a, b, c) \cdot (x, y, 1)}{\sqrt{a^2 + b^2}}$$

The second controller adjusts the heading angle or orientation of the vehicle to be parallel to the line

$$\theta_T = \tan^{-1} \frac{-a}{b}$$

Using the proportional controller

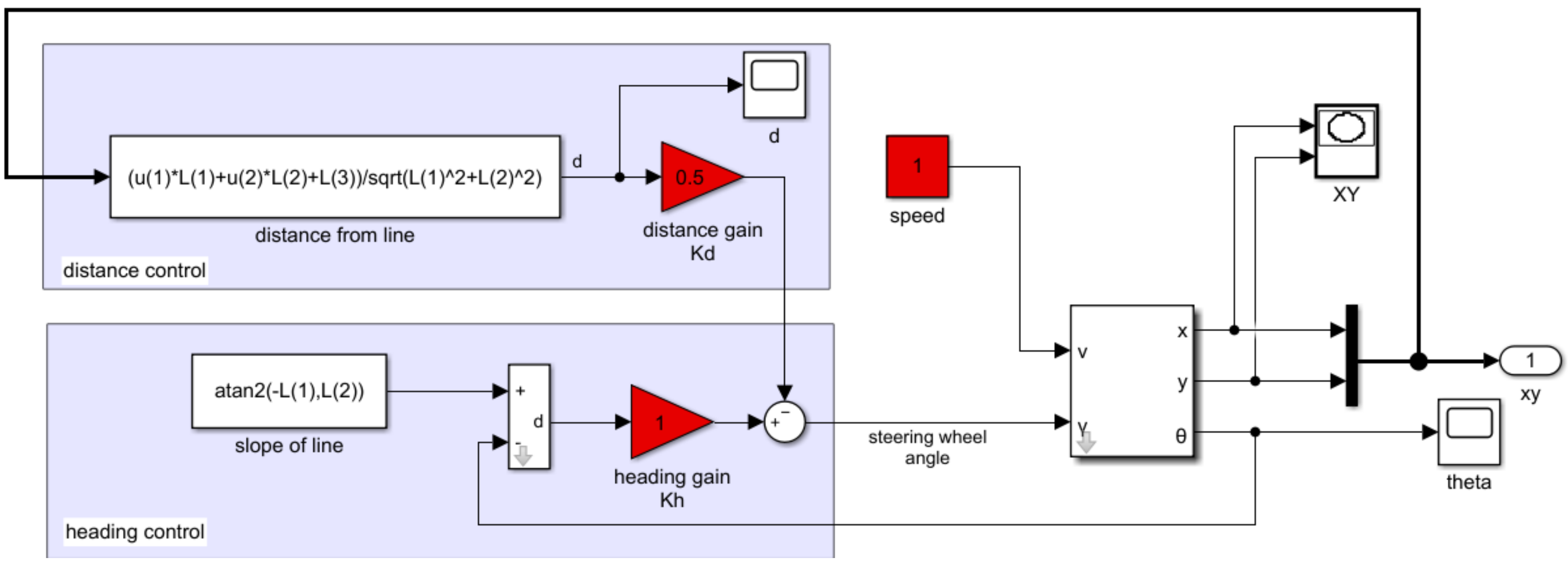
$$\alpha_h = K_h(\theta_T \ominus \theta), \qquad K_h > 0$$

Combined control law will be

$$\gamma = -K_d d + K_h(\theta_T \ominus \theta),$$

and this turns the steering wheel so as to drive the robot toward the line and move along it/

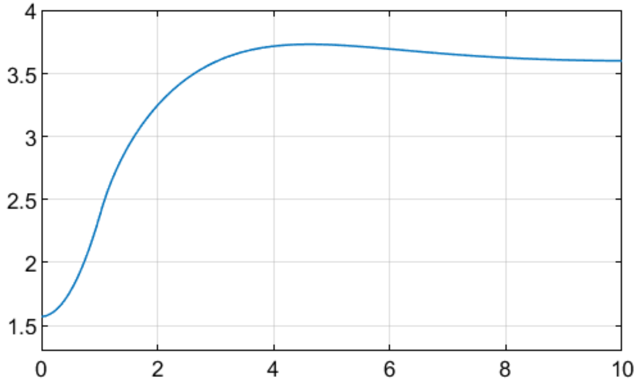
Car-Like Mobile Robots – Following a line



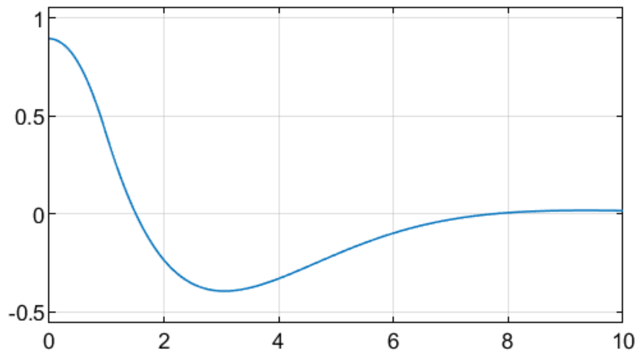
sl_driveline

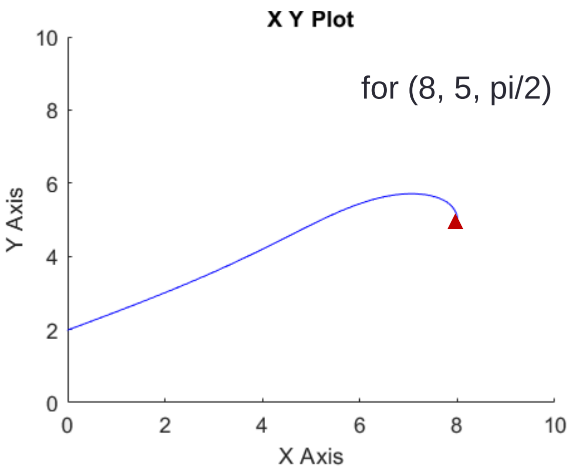
Car-Like Mobile Robots – Following a line

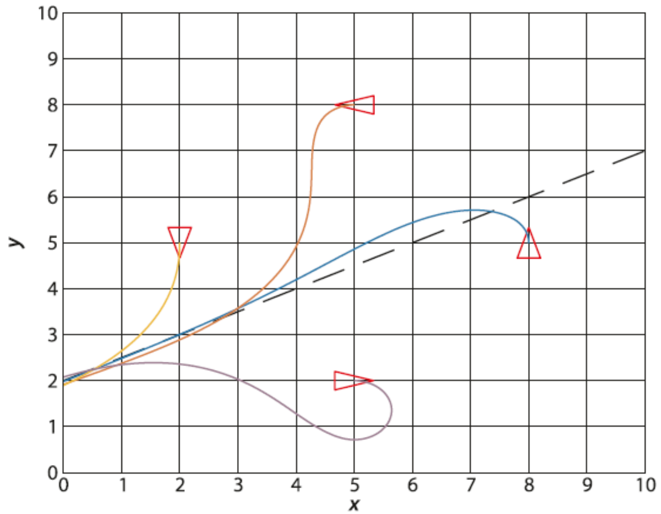
θ



d







Simulation results from different initial poses for the line (1,−2, 4)