Homework 3
CSE 351 – Signals and Systems, Spring 2020

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1 Problems

1.1 Consider the following periodic signal x(t) shown below. (*50pts*)

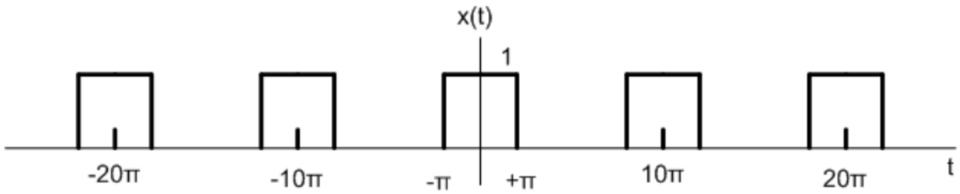

Figure 1

1.1.1 Find the compact trigonometric Fourier series and sketch the amplitude and phase spectra.

1.1.2 Find the exponential Fourier series and sketch the corresponding spectra.

Answer:

(i) Part a.

$$T_0 = 10\pi \qquad \omega_0 = \frac{2\pi}{T_0} = \frac{1}{5}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(\frac{nt}{5}) + b_n \sin(\frac{nt}{5}) \implies \quad a_0 = \frac{1}{5} \quad a_n = \frac{2}{10\pi}\int_{-\pi}^{\pi} \cos(\frac{nt}{5})dt = \frac{1}{5\pi}(\frac{5}{n})\sin\frac{nt}{5}\Big|_{-\pi}^{\pi} = \frac{2}{\pi n}\sin\frac{n\pi}{5}$$

$$\implies \quad b_n = \frac{2}{10\pi}\int_{-\pi}^{\pi}\sin\frac{nt}{5}dt = 0 \qquad c_n = a_n, \quad n = 0, 1, 2, 3, ...$$

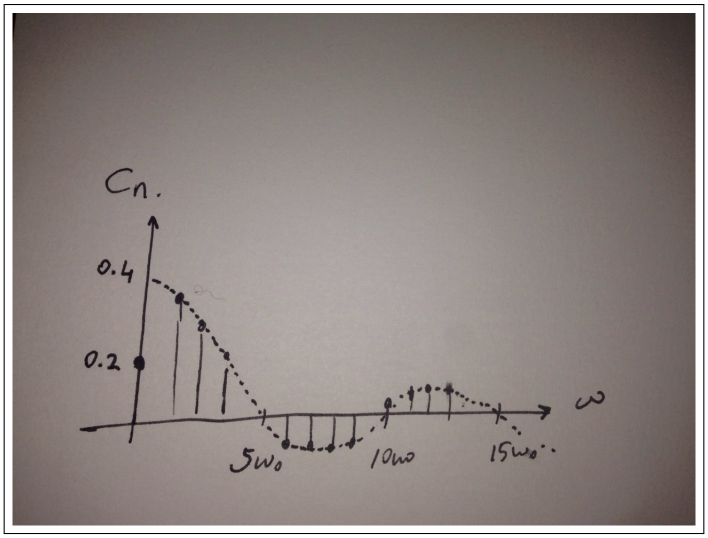

Figure 2

(ii) Part b.

$$T_o = 10\pi \quad \omega_0 = \frac{2\pi}{10\pi} = \frac{1}{5} \qquad f(t) = \sum_{n=-\infty}^{\infty} D_n e^{-j\frac{n}{5}t}$$

$$D_n = \frac{1}{10\pi}\int_{-\pi}^{\pi} e^{-j\frac{nt}{5}}\,dt = \frac{1}{\pi n}\sin\left(\frac{n\pi}{5}\right)$$

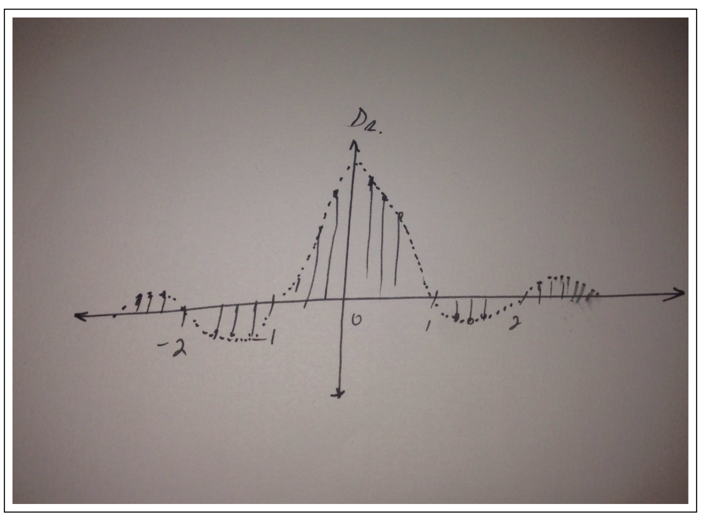

Figure 3

1.2 Consider the following LTIC system: $(D^2 + 3D + 2)y(t) = (D + 3)f(t)$, (*30pts*)

Find the zero-state response if the input f(t) is e$^{-3t}u(t) using Fourier Transform.$

Answer:

$$H(\omega) = \frac{j\omega + 3}{(j\omega)^2 + 3j\omega + 2} = \frac{j\omega + 3}{(j\omega + 1)(j\omega + 2)} \qquad Y(\omega) = F(\omega)H(\omega)$$

$$f(t) = e^{-3t}u(t) \iff F(\omega) = \frac{1}{j\omega + 3}$$

PPE method.

$$\frac{1}{j\omega + 3}[\frac{j\omega + 3}{(j\omega + 1)(j\omega + 2)}] = \frac{1}{(j\omega + 1)(j\omega + 2)} = \frac{1}{j\omega + 1} - \frac{1}{j\omega + 2} \quad Y(t) \iff Y(\omega) \implies Y(t) = (e^{-t} - e{-}2tu(t))$$

1.3 A TV signal (video and audio) has a bandwidth of 4.5 MHz. This signal is sampled, quantized, and binary-coded to obtain a PCM (pulse code modulated) signal. (*30pts*)

1.3.1 Determine the sampling rate if the signal is to be sampled at a rate 20% above the Nyquist rate.

1.3.2 Determine the number of binary pulses required to encode each sample if the quantization level is 1024.

1.3.3 Determine the binary pulse rate (bits/second) of the binary coded signal.

Answer:

B = 4.5MHz

1. Part a.
   $f_s \geq 2B$ Hence,
   Nyquist rate;

$$f_s = 2 \times B = 9MHz$$

   (Actual) Sampling rate;

$$1.2f_s = 1.2 \times 9 = 10.8MHz$$

2. Part b.

$$1024 = 2^N \implies N = 10bits.$$

   10 binary pulses are required to encode each sample.

3. Part c.
   Binary pulse rate;

$$= 10.8MHz \times 10bits = 10.8 \times 10^6(\frac{1}{5}) \times 10bits = 108Mbit/sec$$
