Homework 2
CSE 351 – Signals and Systems, Spring 2020

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1 Problems

1.1 Consider a system with the following transfer function. (*50pts*)

$H(s) = \frac{2s+3}{s^2+5s+6}$

1.1.1 Determine it's zero-state response if the input $f(t) = e^{-3t}u(t)$.

1.1.2 Write down the differential equation relating output $y(t)$ to the input $f(t)$.

1.1.3 Find the inverse Laplace transform of $\frac{s+2}{s(s+1)^2}$.

Answer:

(i) Part a.

$$f(t) = e^{-3t}u(t)F(s) = \frac{1}{s+3}$$

$$Y(s) = H(s)F(s) = \frac{2s+3}{(s+3)(s^2+5s+6)} = \frac{2s+3}{(s+2)(s+3)^2} = \frac{k}{s+2} + \frac{a_0}{(s+3)^2} + \frac{a_1}{s+3}$$

$$(s=-2) \implies k = \frac{2s+3}{(s+3)^2} = -1 \qquad (s=-3) \implies a_0 = \frac{2s+3}{s+2} = 3$$

$$Y(s) = \frac{2s+3}{(s+2)(s+3)^2} = \frac{-1}{s+2} + \frac{3}{(s+3)^2} + \frac{a_1}{s+3}\bigg|_{s\to\infty} \qquad 0 = -1 + 0 + a_1 \implies a_1 = 1$$

$$Y(s) = \frac{-1}{s+1} + \frac{3}{(s+3)^2} + \frac{1}{s+3} \quad and \quad Y(t) = (-e^{-2t} + (1+3t)3^{-3t})u(t)$$

(ii) Part b.

$$Y(s) = \frac{2s+3}{s^2+5s+6}F(s)$$

$$(D^2+5D+6)Y(D) = (2D+3)F(D) \implies \frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y(t) = 2\frac{df}{dt} + 3f(t)$$

(iii) Part c.
We will use partial fraction expansion method for this question.

$$H(s) = \frac{s+2}{s(s+1)^2} = \frac{k}{s} + \frac{a_0}{(s+1)^2} + \frac{a_1}{s+1} \quad \textbf{using the method} \implies k = 2 \quad a_0 = -1$$

$$H(s) = \frac{s+2}{s(s+1)^2} = \frac{2}{s} - \frac{1}{(s+1)^2} + \frac{a_1}{s+1}\Bigg|_{s\to\infty} \quad 0 = 2 + 0 + a_1 \implies a_1 = -2$$

$$H(s) = \frac{2}{s} - \frac{1}{(s+1)^2} - \frac{2}{s+1} \quad and \quad h(t) = (2 - (2+t)e^{-t})u(t)$$

1.2 For a an LTID system with the following differential equation, *(30pts)*

$2y[k+2] - 3y[k+1] + y[k] = 4f[k+2] - 3f[k-1]$

Find the output y[k] if the input is $f[k] = (4)^{-k}u[k]$ and initial conditions are $y[-1] = 0$ and $y[-2] = 1$.

Answer:
Equation in delay form:

$$2y[k] - 3y[k-1] + y[k-2] = 4f[k] - 3f[k-1] \implies Y[k] \Leftrightarrow Y[z], \quad Y[k-1] \Leftrightarrow \frac{1}{z}Y[z], \quad Y[k-2] \Leftrightarrow \frac{1}{z^2}Y[z] + 1$$

$$2Y[z] - \frac{3}{z}Y[z] + \frac{1}{z^2} + 1 = \frac{4}{z-0.25} - \frac{3}{z-0.25} = (2 - \frac{3}{z} + \frac{1}{z^2})Y[z] = -1 + \frac{4z-3}{z-0.25} = \frac{3z-2.75}{z-0.25}$$

$$\frac{Y[z]}{z} = \frac{z(3z-2.75)}{(2z^2-3z+1)(z-0.25)} = \frac{z(3z-2.75}{z(z-0.5)(z-1)(z-0.25)} = \frac{\frac{5}{2}}{z-\frac{1}{2}} - \frac{\frac{1}{3}}{z-1} - \frac{\frac{4}{3}}{z-0.25}$$

$$Y[k] = (\frac{1}{3} + \frac{5}{2}(2)^{-k} - \frac{1}{3}(4)^{-k})u[k]$$

1.3 Find the inverse z-transform of $\frac{z(-5z+22)}{(z+1)(z-2)^2}$ *(30pts)*

Answer:

$$\frac{H[z]}{z} = \frac{-5z+22}{(z+1)(z-2)^2} = \frac{3}{z+1} + \frac{k}{z-2} + \frac{4}{(z-2)^2}\Bigg|_{s\to\infty} \quad 0 = 3 + k + 0 \implies k = -3$$

$$H[z] = 3\frac{z}{z+1} - 3\frac{z}{z-2} + 4\frac{z}{(z-2)^2} \quad H[z] \Leftrightarrow h[k] \quad hence \quad h[k] = (3(-1)^k - 3(2)^k + 2k(2)^k)u[k]$$
