Aslan Oztreves

CS 383

Matthew Burlick

Homework #3

Theory Problems

1. (10pts) Consider the following data:

$$\begin{bmatrix} -2 & 1 \\ -5 & -4 \\ -3 & 1 \\ 0 & 3 \\ -8 & 11 \\ -2 & 5 \\ 1 & 0 \\ 5 & -1 \\ -1 & -3 \\ 6 & 1 \end{bmatrix}$$

(a) Compute the coefficients for the linear regression using least squares estimate (LSE), where the second value (column) is the dependent variable (the value to be predicted) and the first column is the sole feature. Show your work and remember to add a bias feature and to standardize the features. Compute this model using **all** of the data (don't worry about separating into training and testing sets).

1. Standardizing the data
   a. $\mu = \frac{-2-5-3+0-8-2+1+5-1+6}{10} = -0.9$
   b. $\sigma = \frac{(-2+0.9)^2+(-5+0.9)^2+(-3+0.9)^2}{10} = 4.23$
2. Computer the weights by using $\theta = (X^T X)^{-1} X^T Y$

$$X = \begin{bmatrix} 1 & -0.2602 \\ 1 & -0.9697 \\ 1 & -0.4967 \\ 1 & +0.2129 \\ 1 & -1.6792 \\ 1 & -0.2602 \\ 1 & +0.4494 \\ 1 & +1.3954 \\ 1 & -0.0237 \\ 1 & +1.6319 \end{bmatrix}$$

$$(X^T X)^{-1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1111 \end{bmatrix}$$

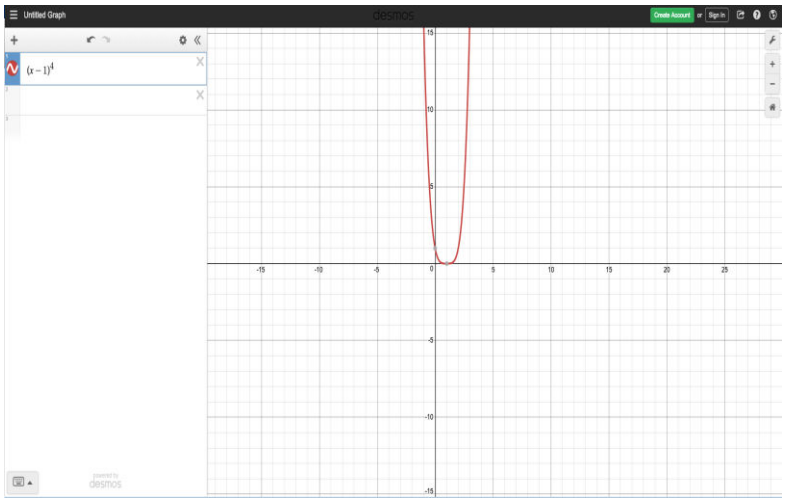
$$\theta = \begin{bmatrix} 1.4 \\ -1.7449 \end{bmatrix}$$

$$y = 1.4 - 1.7449x_1$$

2. For the function $g(x) = (x-1)^4$, where x is a single value (not a vector or matrix):
   (a) What is the gradient with respect to x? Show your work to support your answer.
   (b) What is the global minima for $g(x)$? Show your work to support your answer.
   (c) Plot x vs $g(x)$ use a software package of your choosing.

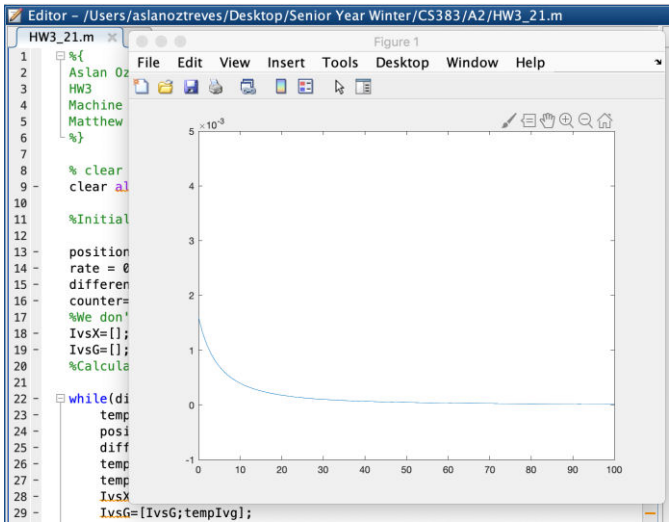
3. We have to take the derivate of the function g(x).

   a. $\frac{dg(x)}{dx} = 4(x-1)^3$
   b. $4(x-1)^3 = 0$, minima is x=1, checking the slope for both ends of 1, and find that it changes sign thus it's going to come from negative to positive on the minima.
   c. Plotting x vs g(x)

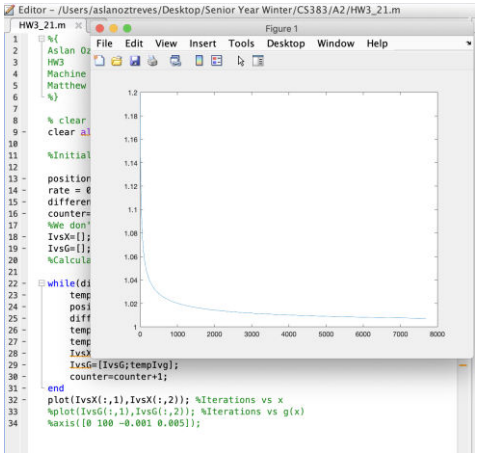


Gradient Descent

2.1.1) Plot Iteration vs g(x)

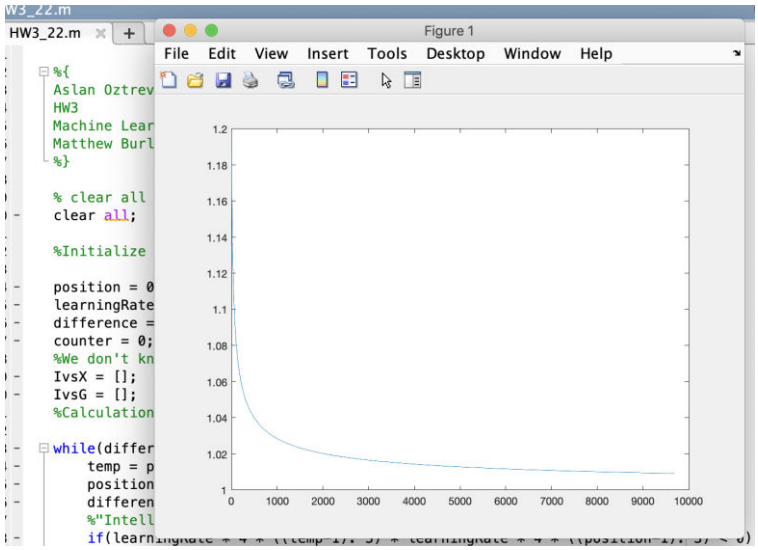


2.1.2) Plot Iterations vs x

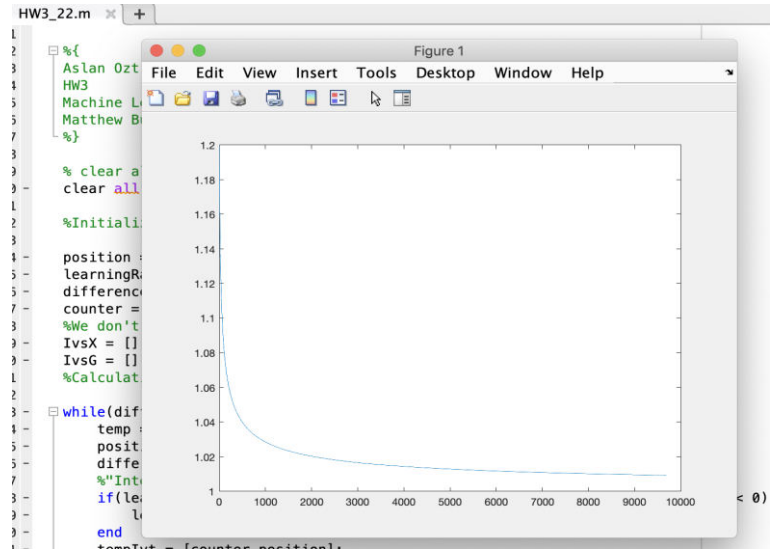


2.1.3) Chosen value of n is 0.3.

2.2.1) Iteration vs g(x).



2.2.2) Iteration vs x



Closed Form Linear Regression

4. Formula

$$y = 3425.66 + 846.94x_1 + (-369.22)x_2$$

5. Values
   a. MSE = 7.2826e+05
   b. RMSE = 853.3806