Semi-logistic model

In[1]:=
```
(* Setup the model *)
$Assumptions = c ≠ 0 && p > 0 && Element[c, Reals];
F[x_] := {a x[[1]] (1 - b x[[1]]) + c x[[2]], d x[[1]] (1 - e x[[1]]) + f x[[2]]}
G[x_] := {{p, 0}, {r, s}}
```

In[4]:=
```
(* Moment equations *)
expr =
    D[x[t]^i y[t]^j, {{x[t], y[t]}}].F[{x[t], y[t]}] +
    1/2 Tr[G[{x[t], y[t]}]^T.D[x[t]^i y[t]^j, {{x[t], y[t]}, 2}].G[{x[t], y[t]}]];
mex[i_, j_] := Evaluate[FullSimplify[Expand[expr] /.
     Flatten[Table[x[t]^(i+p) y[t]^(j+q) → m_(i+p,j+q)[t], {p, -2, 4}, {q, -2, 4}]]]]
meq[i_, j_] := m_(i,j)'[t] == mex[i, j]
msol[i_, j_] := Solve[meq[i + 1, j - 1], m_(i,j)[t]][[1]]
```

In[8]:=
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(* Moment equation - in text *)
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In[9]:=
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mex[i, j]
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Out[9]=
$$\frac{1}{2}(-1+i)\,i\,p^2\,m_{-2+i,j}[t] + i\,j\,p\,r\,m_{-1+i,-1+j}[t] + c\,i\,m_{-1+i,1+j}[t] + \frac{1}{2}(-1+j)\,j\left(r^2+s^2\right)m_{i,-2+j}[t] + (a\,i+f\,j)\,m_{i,j}[t] - a\,b\,i\,m_{1+i,j}[t] + d\,j\left(m_{1+i,-1+j}[t] - e\,m_{2+i,-1+j}[t]\right)$$

First order necessarily satisfied equation

In[10]:=
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(* First equation *)
eq1 = FullSimplify[MultiplySides[meq[0, 1], -c] /. msol[0, 1] /. D[msol[0, 1], t]]
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Out[10]=
$$(a+f)\,m_{1,0}'[t] == (-c\,d+a\,f)\,m_{1,0}[t] + (c\,d\,e-a\,b\,f)\,m_{2,0}[t] + a\,b\,m_{2,0}'[t] + m_{1,0}''[t]$$

Second order necessarily satisfied equation

In[11]:=
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(* Second equation *)
subs1 = FullSimplify[msol[0, 2] /. msol[1, 1] /. msol[2, 1]];
eq2 =
 FullSimplify[meq[0, 2] /. subs1 /. D[subs1, t] /. msol[1, 1] /. D[msol[1, 1], t] /.
      D[msol[1, 1], {t, 2}] /. msol[2, 1] /.
    m0,0[t] → 1 /. Table[D[m0,0[t], {t, k}] → 0, {k, 1, 5}]]
```

Out[12]=

$$12\,(c\,d\,e-a\,b\,f)\,p^2\,m_{1,0}[t]+6\left(f\,(a+f)\,p^2+c^2\left(r^2+s^2\right)+2\,(a+f)\,(-c\,d+a\,f)\,m_{2,0}[t]+2\,c\,d\,(a\,(b+e)+e\,f)\,m_{3,0}[t]+a\,b\left(2\,a\,b\,f\,m_{4,0}[t]+p^2\,m_{1,0}'[t]\right)+2\,c\,d\,m_{2,0}'[t]\right)+2\,a\,b\,(6\,a+11\,f)\,m_{3,0}'[t]+9\,(a+f)\,m_{2,0}''[t]==6\,c\,p\,(d\,p+2\,f\,r)+12\,a\,b\,f\,(2\,a+f)\,m_{3,0}[t]+12\,a\,b\,c\,d\,e\,m_{4,0}[t]+6\left(a^2+4\,a\,f+f^2\right)m_{2,0}'[t]+10\,c\,d\,e\,m_{3,0}'[t]+6\,a^2\,b^2\,m_{4,0}'[t]+8\,a\,b\,m_{3,0}''[t]+3\,m_{2,0}^{(3)}[t]$$